Cholesky Decomposition of 2D Kernel

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1

In the 1D case, the cholesky decomposition algorithm for a covariance kernel represented by a semiseparable matrix with the form

$$K = A + \mathrm{tril}(UV^T) + \mathrm{triu}(VU^T) \tag{1}$$

where U and V are both N x J matrices and A is an N x N diagonal matrix is given in in section 5.1 of Foreman-Mackey et al, 2017 (FM17). In the 2D case, the covariance matrix is given by

$$K = A' + \mathrm{tril}(U'V'^T) + \mathrm{triu}(V'U'^T) \tag{2}$$

where $U' = U \otimes Q$, $V' = V \otimes I_M$, and $A' = (A - \sigma_n) \otimes Q + \sigma_{nM+i}$ where $Q_{i,j}$ specifies the covariance between coordinates x_i and x_j in the second dimension, and I_M is the identity matrix with M corresponding to the size of Q. Because U' and V' for U and V does not alter the semiseperable property of the matrices, the decomposition can proceed as in the 1D case with the following alterations to equations 43-45 of FM17:

$$\tilde{U}_{(n-1)M+p,(2j-1)M+q} = a_j \cos(d_j t_n) + b_j \sin(d_j t_n) Q_{p,q} \tag{3}$$

$$\tilde{U}_{(n-1)M+p,(2j)M+q} = a_j \cos(d_j t_n) + b_j \sin(d_j t_n) Q_{p,q} \tag{4}$$

$$\tilde{V}_{(n-1)M+p,(2j-1)M+q} = a_j \sin(d_j t_n) - b_j \cos(d_j t_n) \delta_{p,q} \tag{5}$$

$$\tilde{V}_{(n-1)M+p,(2j)M+q} = a_j \sin(d_j t_n) - b_j \cos(d_j t_n) \delta_{p,q} \tag{6}$$

$$A_{(n-1)M+p,(n-1)M+p} = \sigma_{(n-1)M+p} + Q_{p,p} \sum_{j=1}^{J} a_j \tag{7}$$

where $p, q = 1, 2...M$.

$$\phi_{(n-1)M+p,(2j-1)M+q} = \phi_{(n-1)M+p,(2j)M+q} = \begin{cases} e^{-c_j(t_n - t_{n-1})} & p = 1 \\ 1 & p > 1 \end{cases} \tag{8}$$

Still with the constraint that

$$\phi_{1,(2j-1)M+q} = \phi_{1,(2j)M+q} = 0 \tag{9}$$

The algorithm for computing the cholesky decomposition is unaltered from equation 46 of FM17.