Transformation –
Changing position, shape, size or orientation of an object on display is known as transformation.

Basic geometric transformations are:

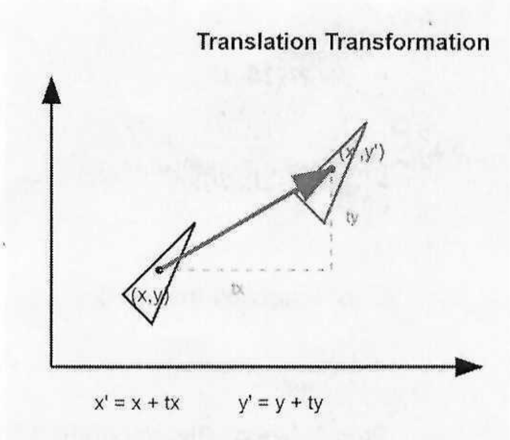
- Translation
- Rotation
- Scaling

Other transformations are –

- Reflection
- Shear

(1) Translation –
It is a transformation that is used to reposition the object along the straight line path from one coordinate location to another.

- We need to translate the whole object.



We translate a 2D point by adding translation distance txt_xtx and tyt_yty to the original coordinate position (x,y) to move it at a new position (x',y') as:

$x' = x + t_x$
$y' = y + t_y$
Translation distance pair (t_x, t_y) is called a **translation vector / shift vector**.

Translation can also be specified by the following transformation matrix –

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Then we can rewrite the formula as:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate the triangle A(10,10), B(15,15), C(20,20) by 2 units in the x-direction and 1 unit in the y-direction.

Step 1: Translation Formula

For a point (x, y), translation is defined as:

$$x' = x + t_x, \quad y' = y + t_y$$

Here,

- $t_x = 2$
- $t_y = 1$

Step 2: Translate Each Vertex

- For $A(10, 10)$:

$$A'(x', y') = (10 + 2, 10 + 1) = (12, 11)$$

- For $B(15, 15)$:

$$B' = (15 + 2, 15 + 1) = (17, 16)$$

- For $C(20, 20)$:

$$C' = (20 + 2, 20 + 1) = (22, 21)$$

So the translated triangle is:

$$A'(12, 11), \ B'(17, 16), \ C'(22, 21)$$

Step 3: Matrix Representation (Homogeneous Coordinates)

Translation matrix:

$$T = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Coordinates of the triangle in matrix form:

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 15 & 20 \\ 10 & 15 & 20 \\ 1 & 1 & 1 \end{bmatrix}$$

Now multiply:

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 & 15 & 20 \\ 10 & 15 & 20 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 17 & 22 \\ 11 & 16 & 21 \\ 1 & 1 & 1 \end{bmatrix}$$

So the new coordinates are exactly the same as calculated directly:

$$A'(12, 11), \ B'(17, 16), \ C'(22, 21)$$

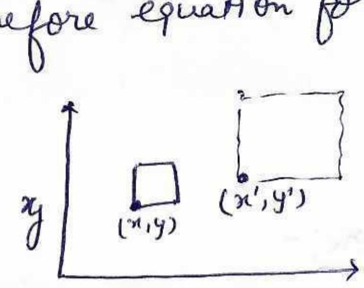
✅ This example shows why matrix representation is powerful:

- Instead of applying formulas individually, we can transform multiple points at once using matrix multiplication.
- It is efficient for computer graphics, where thousands of points (pixels, vertices) are transformed.

Scaling:-

- It is a transformation that is used to change the size of an object.
- This operation is carried out by multiplying coordinate value (x, y) with scaling factor (S_x, S_y) respectively. Therefore equation for scaling is given by

$$x' = x \cdot S_x$$
$$y' = y \cdot S_y$$



- If values of S_x & S_y are less than 1, reduces the size, while values greater than 1 enlarges the size of object.
- Object remains unchanged when $S_x = 1$, $S_y = 1$.
- Same values of S_x & S_y ($S_x = S_y$) will produce uniform scaling. Different values of $S_x \neq S_y$ will produce differential scaling.
- Scaling can also be specified by the following transformation matrix.

$$\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

we can re-write as

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scaling is performed about the origin:-

→ If scale > 1 enlarge the object & move it away from origin

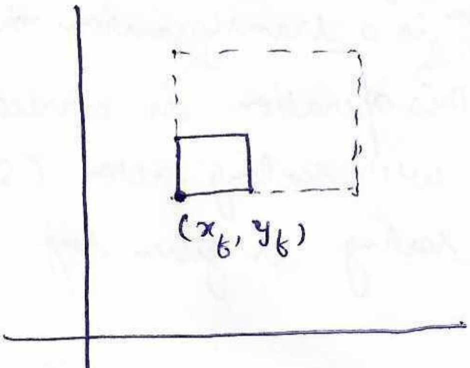
→ If scale = 1 leave the object same.

→ If scale < 1 shrink / reduce the object & move it towards the origin.

Q. Consider square with left-bottom corner at (2,2) & right top corner at (6,6), apply the transformation which makes the size half.

Scaling perform about the fixed point.

$$x' = x_f + (x - x_f)\, S_x$$
$$y' = y_f + (y - y_f)\, S_y$$



3) Rotation:-

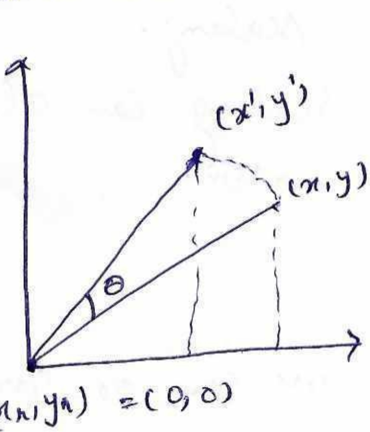
→ It is a transformation that used to reposition the object along the circular path in XY plane.

→ To generate a rotation we specify a rotation angle θ and the position of the rotation point (pivot point) (x_r, y_r) about which the object is to be rotated.

- positive value of the rotation angle define counter clockwise rotation (anticlockwise)

- Negative value of the rotation angle define clockwise rotation.

- Rotated point (x', y') after rotation by angle θ is



$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$

This rotation can also be specified by the following tranformation matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Triangle A(5,4)
B(8,3),
C(8,8)
θ = 90 clockwise

we can re-written the above as

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Transformation about the pivot point (x_r, y_r) is

$$x' = x_r + (x - x_r)\cos\theta - (y - y_r)\sin\theta$$
$$y' = y_r + (x - x_r)\sin\theta + (y - y_r)\cos\theta$$

4) **Reflection:-** It is a transformation that produces a mirror image of an object.

The mirror image for a 2-D reflection is generated relative to an axis of reflection by rotating the object 180° about the reflection axis.

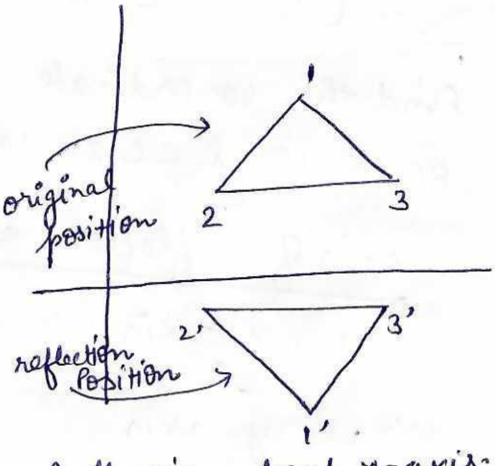
Reflection gives image based on positioning of axis of reflection.

i) <u>Reflection about x-axis:-</u>

$$x' = x$$
$$y' = -y$$

<u>Equation:-</u>

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



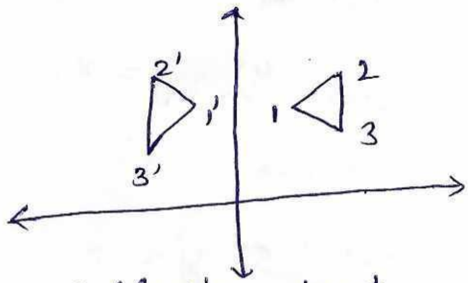
Reflection about x-axis.

ii) <u>Reflection about y-axis:-</u>

$$x' = -x$$
$$y' = y$$

<u>Equation:</u>

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



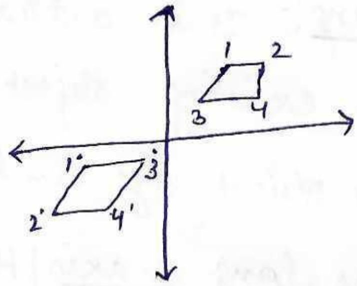
Reflection about y-axis.

iii) <u>Reflection about the origin:-</u>

$$x' = -x$$
$$y' = -y$$

<u>Equation:-</u>

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

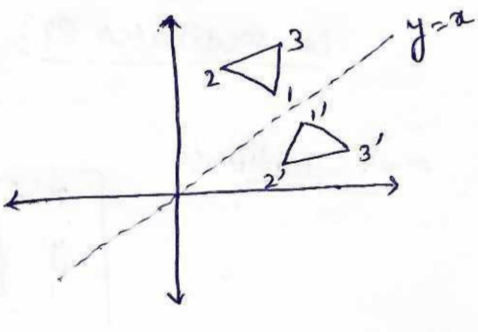


iv) <u>Reflection about the diagnol line $y = x$:-</u>

$$x' = y$$
$$y' = x$$

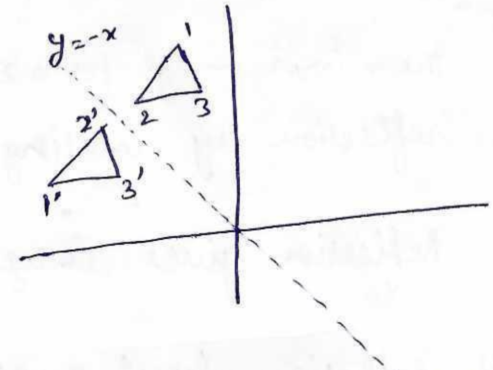
<u>Equation</u>

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$$



v) Reflection about the diagonal line $y = -x$

$$x' = -y$$
$$y' = \cancel{y} - x$$



Equation:-

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$$

Q. Find the co-ordinate after reflection of the triangle A(10,10) B(15,15) & C(20,10) about x-axis.

Axis of reflection about	co-ordinate after Reflection	Matrix Eqn
1) x-axis	$x' = x$, $y' = -y$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
2) y-axis	$x' = -x$, $y' = y$	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
3) origin	$x' = -x$, $y' = -y$	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
4) $y = x$	$x' = \cancel{y}x$, $y' = \cancel{y}x$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
5) $y = -x$	$x' = -y$, $y' = -\cancel{y}x$	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

⑤ Shearing:- It is a transformation that is used to change the shape of an existing object in a 2D plane.

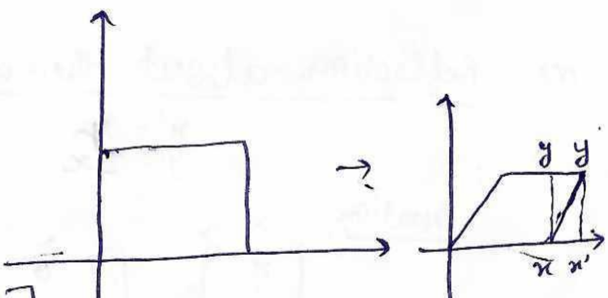
- The object size can be changed along x-axis as well as y-axis.

i) Shearing ~~along~~ x-axis | Horizontal shearing:-

Shearing parameter = Sh_x

Co-ordinates Eqn:- $x' = x + y \cdot Sh_x$

$$y' = y$$



Matn. Equation:-

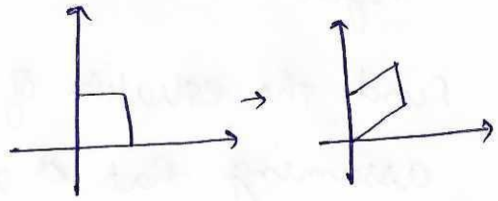
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

ii) Shearing along y-axis ; Vertical shearing :-

shearing parameter = sh_y

Co-ordinates eqⁿ :- $x' = x$

$y' = y + sh_y \cdot x$

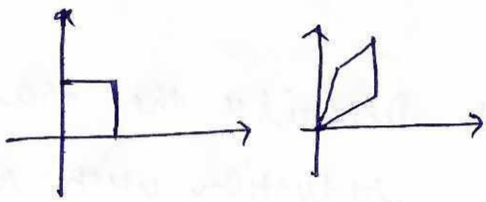


Matⁿ eqⁿ :-

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

iii) Shearing along x-axis & y-axis :-

Shearing parameters := sh_x, sh_y



Co-ordinate eqⁿ :- $x' = x + sh_x \cdot y$

$y' = y + sh_y \cdot x$

Mat. eqⁿ :-

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Properties of All transformations :-

Transformations will be compared ont the bases of

i) shape
ii) size
iii) orientation
iv) Parallelism
v) Distortion

Transformation	Shape	Size	Orientation	Parallelism	Distortion
Translation	Preserved	Preserved	Preserved	Preserved	None
Scaling	Preserved (if uniform)	Changed	Preserved	Preserved	Possible
Rotation	Preserved	Preserved	Changed	Preserved	None
Reflection	Preserved	Preserved	Reversed	Preserved	None
Shearing	Changed	Sometimes	May change	Preserved	Yes

Q. Perform a 60° rotation A (0,0), B (1,1), C (5,2) about origin.

Q. Find the equation of circle $x^2 + y^2 = 25$ in terms of x', y' co-ordinate assuming that x, y co-ordinate system results from a scaling of 'a' units in x-direction and 'b' units in the y-direction.

Q. Perform a magnification of a square with half-size having co-ordinates A (0,0), B(1,0), C (1,1) & D (0,1).

Q. Describe the transformation used in magnification and reduction with respect to origin. Find the new co-ordinates of the triangle A(0,0), B(1,1) & C (5,2) after it has been

a) magnified to twice its size.

b) reduced to half the size.

Q. Find the co-ordinates of a rectangle A(2,1), B(6,1), C(6,4), D(2,4) reflecting about

a) line $y = x$

b) line $y = -x$

c) origin

Q. What are the translated co-ordinates of rectangle A (1,2) B (4,2), C (4,5), D (1,5) with translation vector $I = \langle 3, 2 \rangle$.