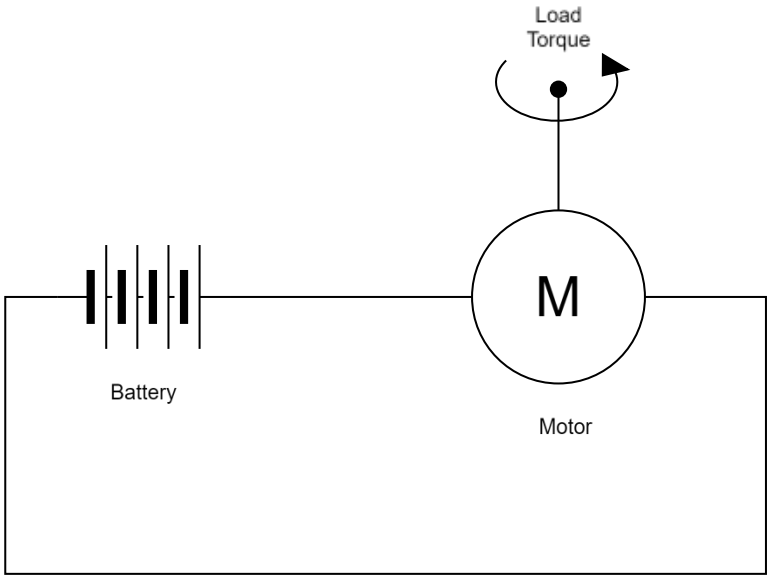
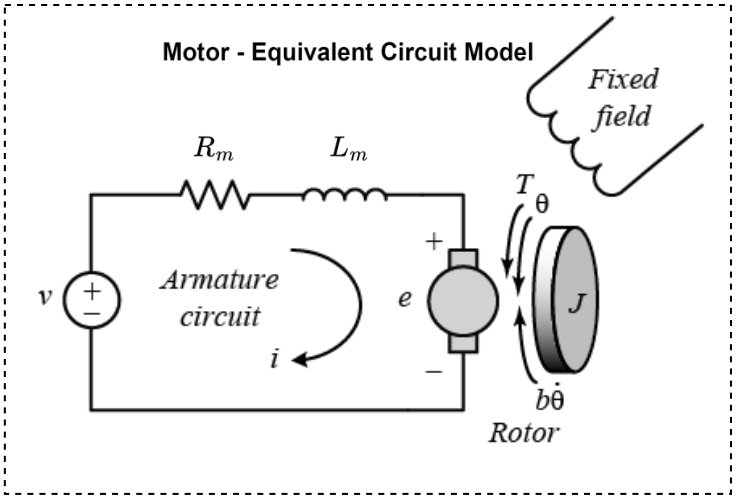
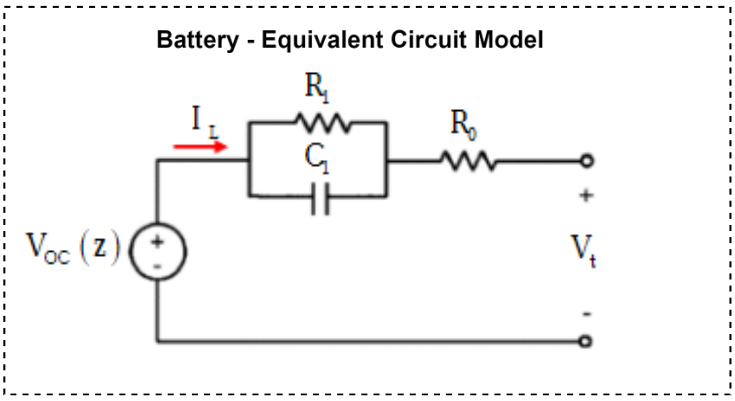
Consider the following case where a battery is connected to a DC motor and an external load torque is applied opposing the relative rotation of the motor armature:







The equivalent circuit models for the motor and the battery are taken from these sources respectively (you can also use these as a guide and for help 😉):

1. Motor Model: https://ctms.engin.umich.edu/CTMS/index.php?example=MotorSpeed&section=SystemModeling

2. Battery Model (you only need information from page (3) – equations 1,2 and 3): https://www.researchgate.net/publication/335570431_A_Nonlinear-Model-Based_Observer_for_a_State-of-Charge_Estimation_of_a_Lithium-Ion_Battery_in_Electric_Vehicles

The corresponding parameters of the motor and battery models are given below:

1. Motor Model:
    a. Rotational moment of inertia (J) = 0.1 kg/m^2
    b. Motor viscous friction constant (b)= 0.05 Nms
    c. Electromotive force constant (Ke) = 0.01 V/rad/sec
    d. Motor toque constant (Kt) = 0.01 Nm/A
    e. Motor electric resistance (Rm) = 0.01 Ohm
    f. Motor electric inductance (Lm) = 0.05 H

2. Battery Model:
    a. Open Circuit Voltage (Voc) = 3.7 V (assume it is constant and does not change with SOC)
    b. Equivalent internal resistance (R0) = 0.005 Ohm
    c. Equivalent resistance of RC pair (R1) = 0.005 Ohm
    d. Equivalent capacitance of RC pair (C1) = 100 F
    e. Nominal capacity of the battery (Cn) = 1 Ah

Using these parameters and the guide (links) given above, answer the following questions after simulating the system for 1 minute assuming the system starts from rest (all initial conditions are 0 unless explicitly stated). Use the simulation step size / sampling time of 0.05 seconds. The SOC (State-of-Charge) of the battery starts at 1.

Note: You are free to use whatever software you like (Python, MATLAB, Simulink, OpenModelica, etc). You can also solve this problem using any method - Transfer Functions, Numerical Integration, State Space Systems, Casual / Acausal Modelling etc.

1. For the first task assume the external load torque is zero.
    a. Plot the angular velocity ($\dot{\theta}$) with respect to time.
    b. Plot the SOC of the battery with respect to time.

2. Now consider the external toque (in Nm) (opposite to the direction of the rotating armature) applied follows the function, **0.05*(1+sin(*time*))**, where *time* continuously changes from 0 to 1 minute as your simulation progresses.
    a. Plot the angular velocity ($\dot{\theta}$) with respect to time.
    b. Plot the SOC of the battery with respect to time.

3. Bonus Question (Optional): Write down the State Space Forms of the individual systems (Motor & Battery) and the combined system. Also explain the states you chose for the system and why?