In the name of God

Sharif University of Technology

School of Electrical Engineering

Convex Optimization

Course Project

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The `.m` file is available in the archive that is uploaded onto the CW. It also available in the appendix of the report.

1 Step 1: Choosing a Portfolio

It can be seen in the figures below that as the *risk* increases, the *total return* also increases which seems logical; If you are willing to risk more, you expect a larger return. But still, I cannot understand the complete monotonic behaviour of the function.

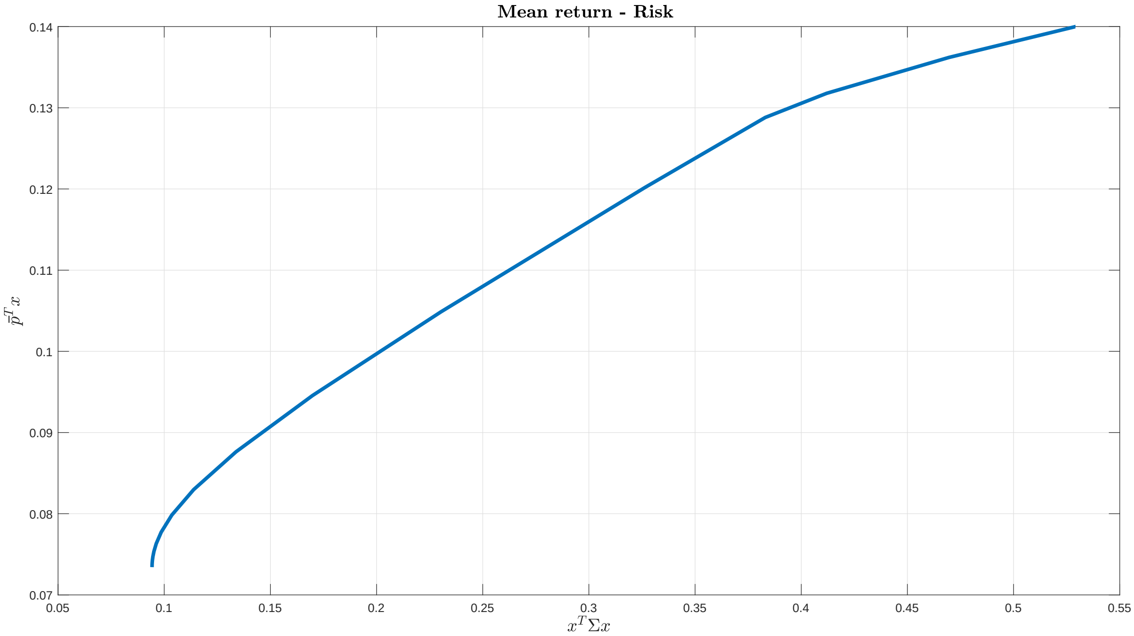

Figure 1: Total return versus the standard deviation for different values of η

From the second figure we can see that as the risk increases i.e. as η increases and the effect of the negative term $x^T \Sigma x$ increases, we do not tend toward a single stock and we can also see that for large values of η, it is best to have a certain amount from each stock so that we could minimize the loss that would occur if we had made an unfortunate decision at buying the stock.

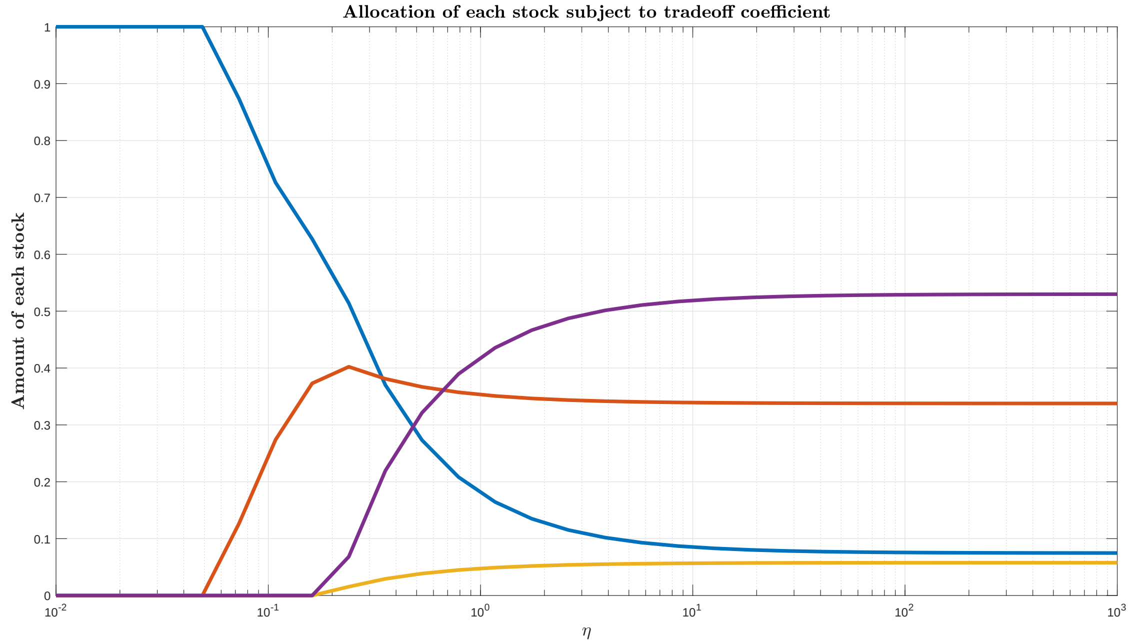

Figure 2: Percentage allocated to each stock with respect to η, logarithmic x-axis

2 Short Positions

In the figure below we can see that for the same risk, we have a greater return with short selling compared to without short selling.

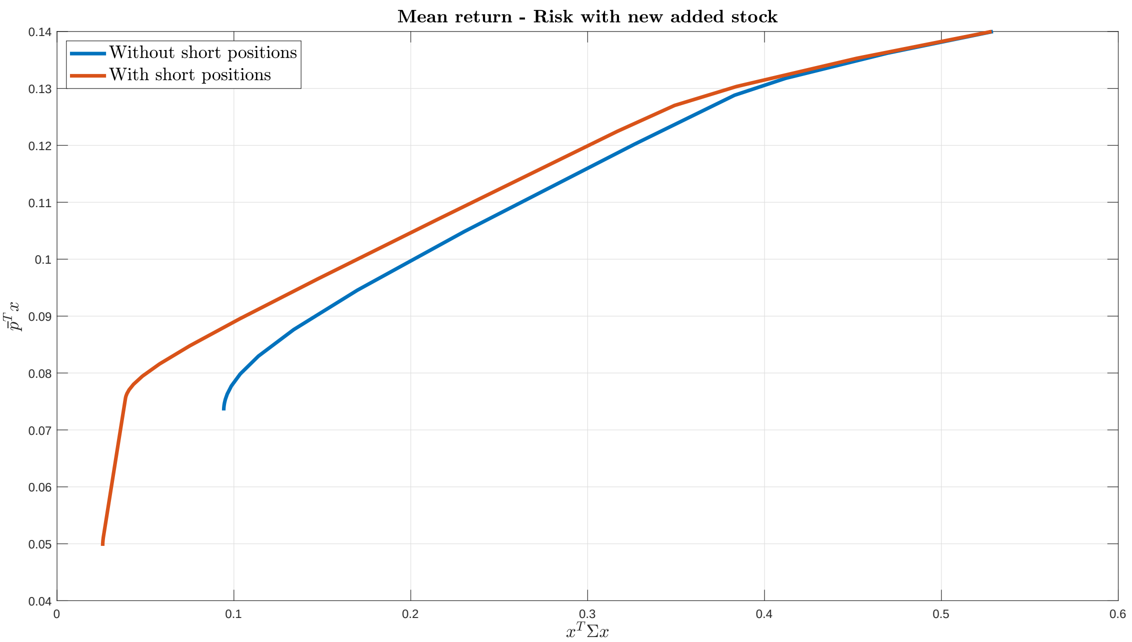

Figure 3: Total return versus the standard deviation for different values of η

3 Portfolio Optimization with Real Data

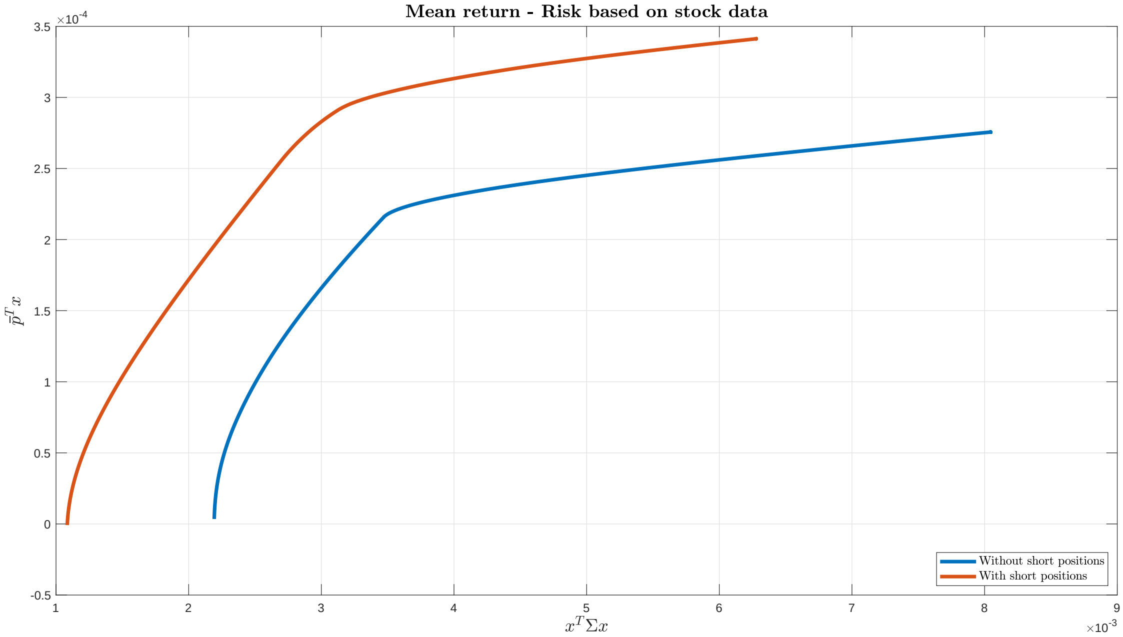

Figure 4: Total return versus the standard deviation for different values of η, real stock data

Since it is a bit hard to choose a point on the plot, the optimal η for each plot is discussed here and the plots will follow afterwards. Below is the optimum point for each portfolio in order of their appearance in the dataset:

1. In this portfolio, the best strategy would be *not to short sell* and the optimum point is the maximum possible *eta* i.e. 1000.

2. In this portfolio, the *short selling* strategy is only good for large values of *eta* e.g. for $\eta > 90$.

3. In this portfolio, the *short selling* strategy is always optimum and the best η is around 8.5.

4. In this portfolio, the *short selling* strategy is only good for large values of *eta* e.g. for $\eta > 60$.

5. In this portfolio, the best strategy would be *not to short sell* and the optimum η is any η that is not in the range 30 to 100.

6. In this portfolio, the *short selling* strategy is always optimum and the best η is any η that is less than 2.

7. In this portfolio, the *short selling* strategy optimum for η smaller than 70 and the optimum η is around 28.

8. In this portfolio, the best strategy would be *not to short sell* and the optimum η is around 13.
9. In this portfolio, the *short selling* strategy is always better and the optimum range for η is any η smaller than 10.
10. In this portfolio, the *short selling* strategy is always better and the optimum value is around 20.
11. In this portfolio, the best strategy would be *not to short sell* for η smaller than 100 but *short selling* would be wise for η larger than the said value.
12. In this portfolio, the *short selling* strategy is always better and the optimum value is around 10.

Below are the returns of each portfolio with respect to η:

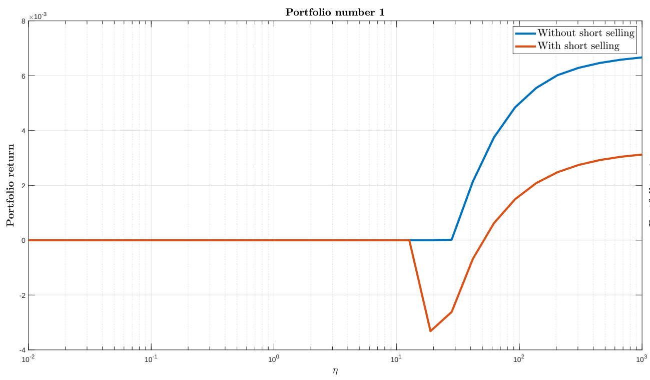

Figure 5: Portfolio 1 cumulative return

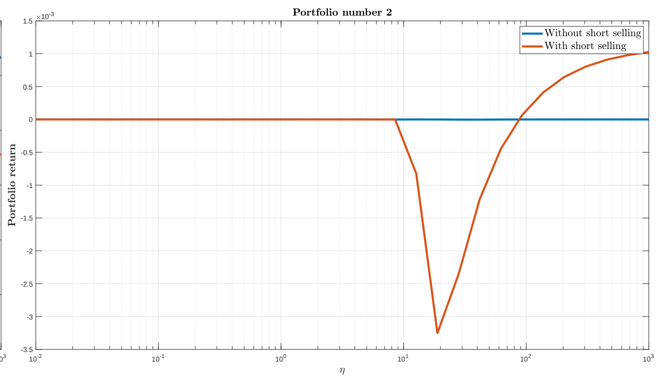

Figure 6: Portfolio 2 cumulative return

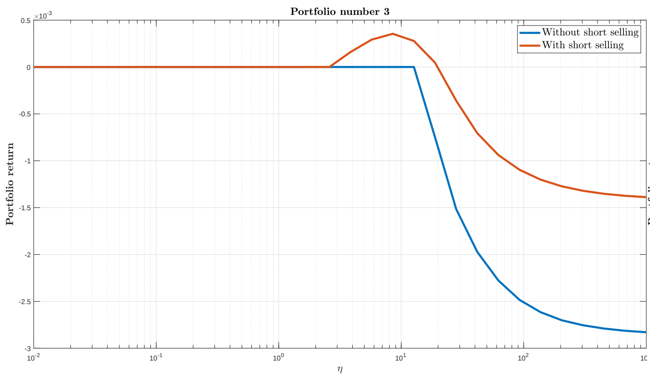

Figure 7: Portfolio 3 cumulative return

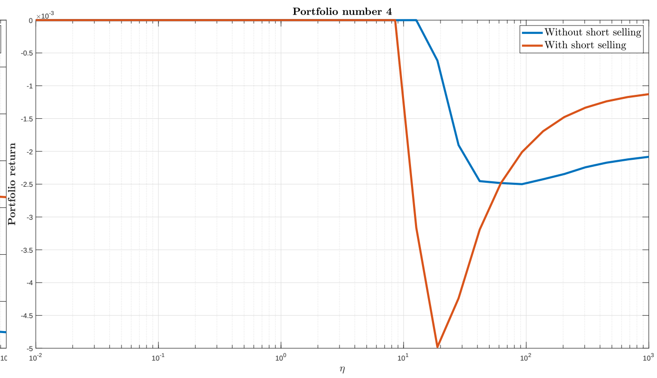

Figure 8: Portfolio 4 cumulative return

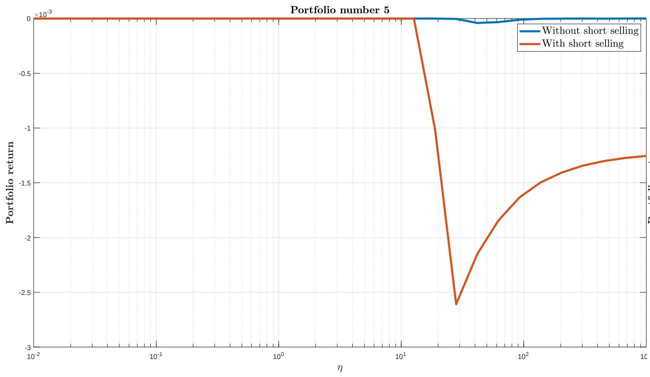

Figure 9: Portfolio 5 cumulative return

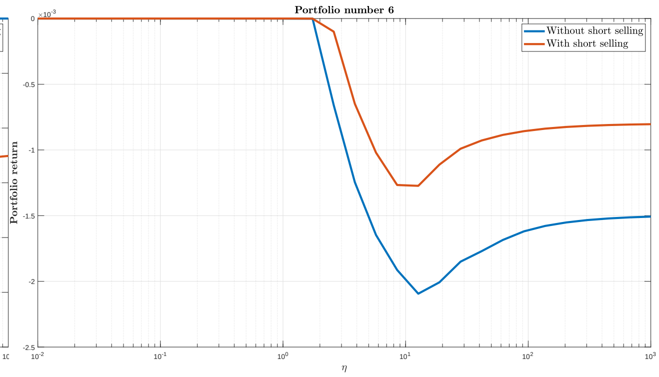

Figure 10: Portfolio 6 cumulative return

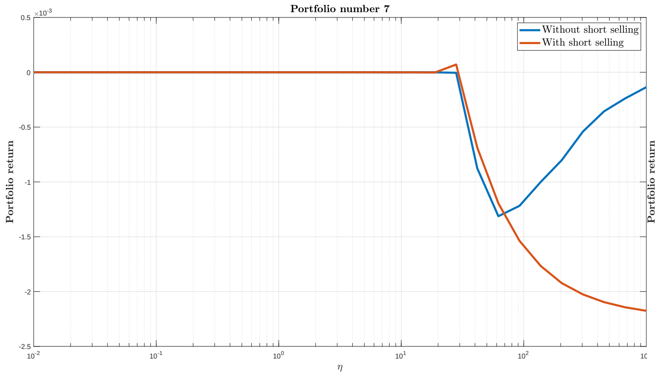

Figure 11: Portfolio 7 cumulative return

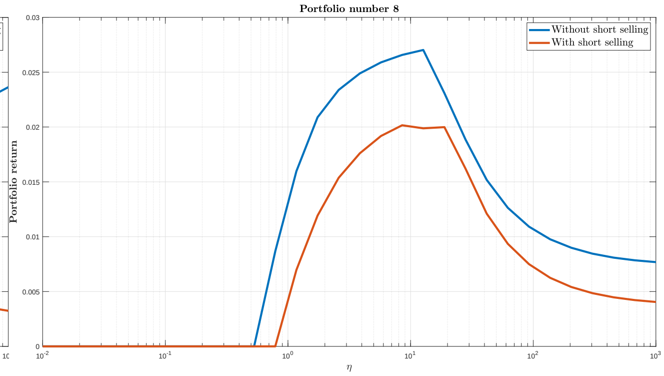

Figure 12: Portfolio 8 cumulative return

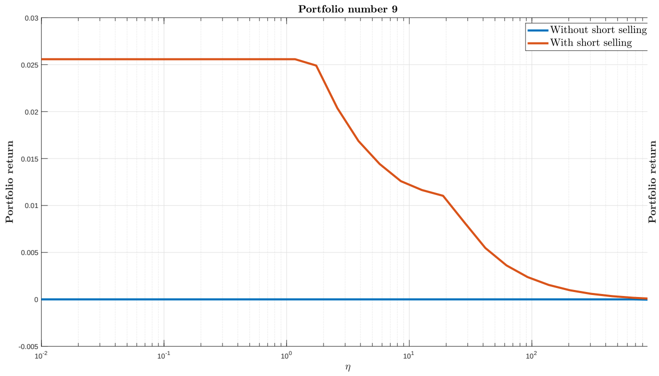

Figure 13: Portfolio 9 cumulative return

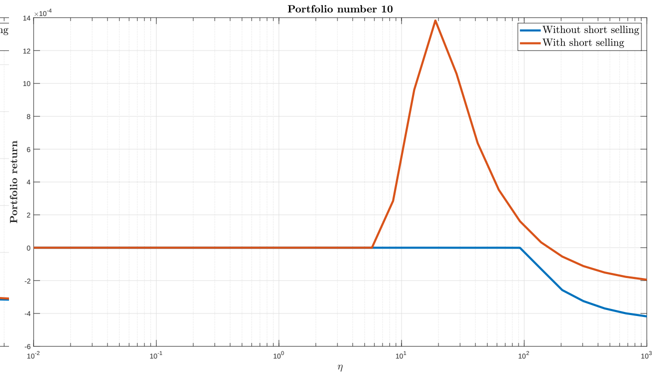

Figure 14: Portfolio 10 cumulative return

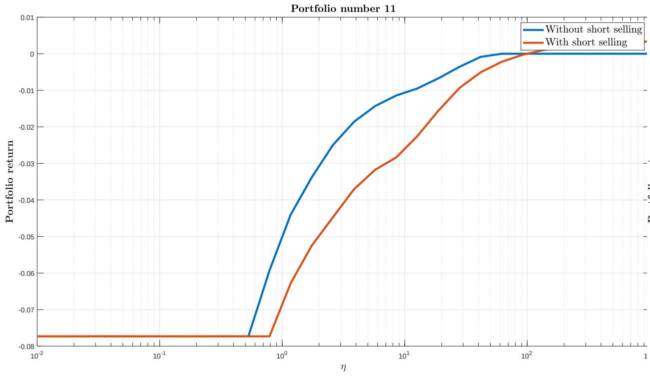

Figure 15: Portfolio 11 cumulative return

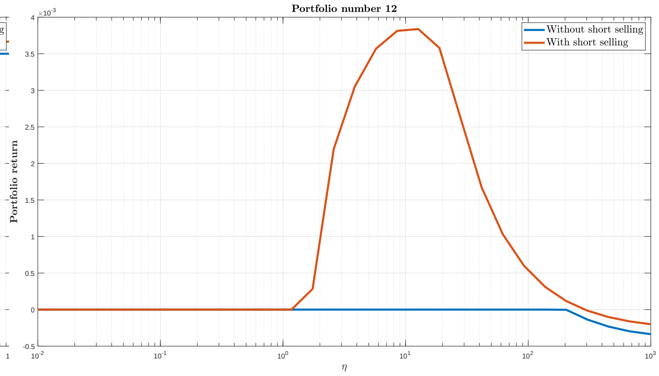

Figure 16: Portfolio 12 cumulative return

The figure below depicts the behaviour of the total return of all portfolios with respect to η. As it can be seen, the strategy of short selling is not optimum for most η and is only desirable for small η e.g. η smaller than 0.8. Above this limit, the optimum strategy would be not to short sell. The optimum η for both the cases of short selling and not short selling is around 12. For all η smaller than 4, we have no gain and the investment results in a loss.

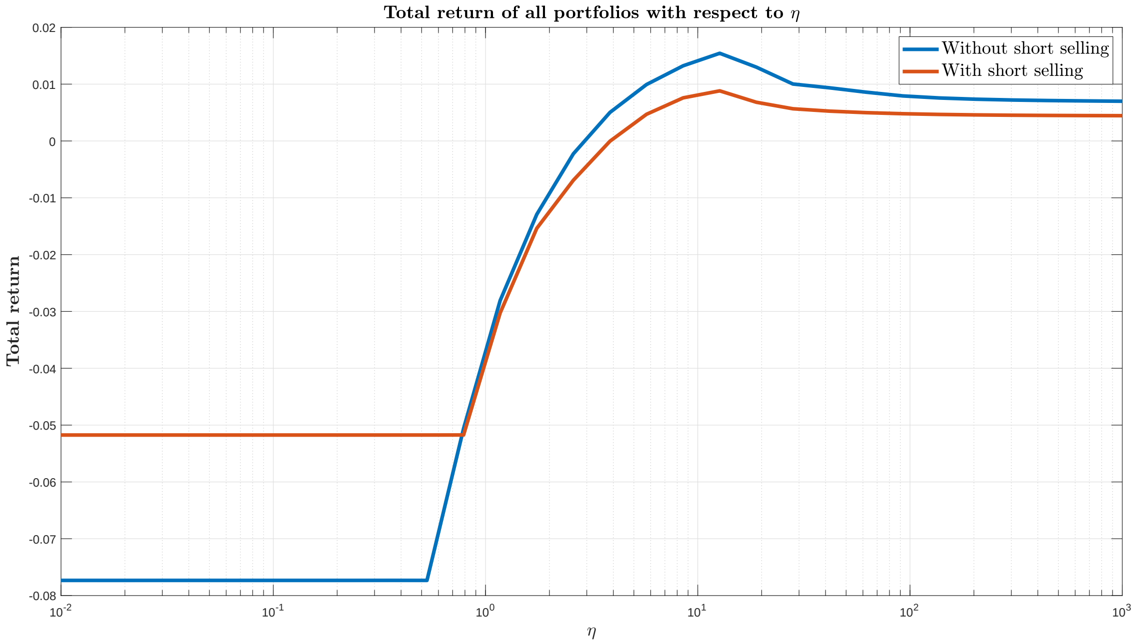

Figure 17: Total return versus η, real stock data

4 Step 4: Improve Your Estimate

The results of this section are only stated for the case without short selling. A general look to the figures below follows up with the deduction that the new estimate predicts worse outcomes i.e. lower return of almost all of the portfolios but if we also heed attention to figure 30,showing the relation between the *total return* and η for both estimates of Σ, we can see that the new estimate predicts that the returns will be less for almost all η meaning that after making our estimate more exact, the data suggests that the expected return will be less.

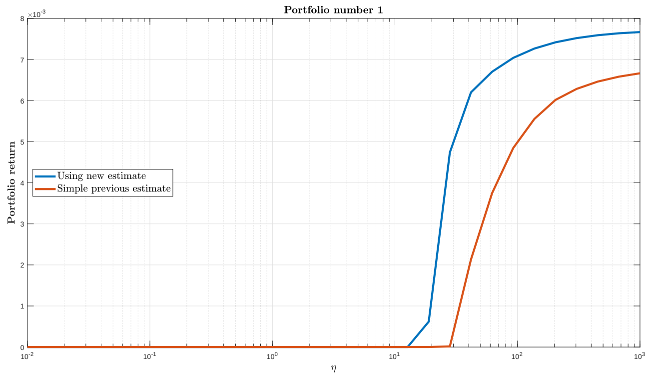

Figure 18: Portfolio 1 cumulative return

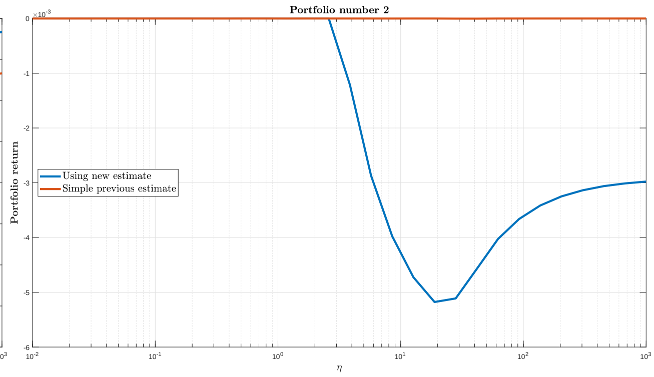

Figure 19: Portfolio 2 cumulative return

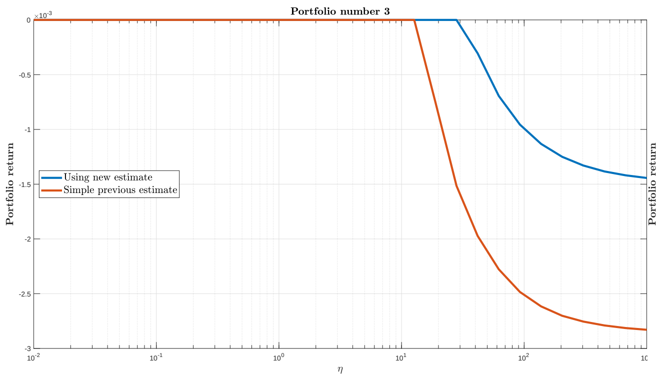

Figure 20: Portfolio 3 cumulative return

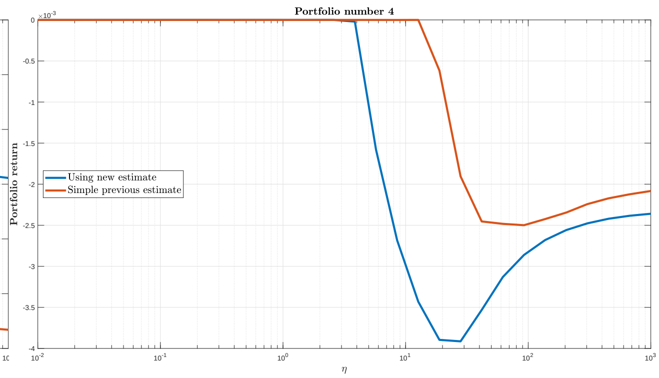

Figure 21: Portfolio 4 cumulative return

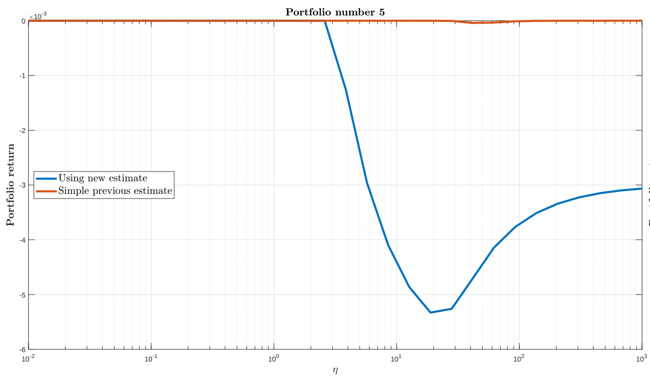

Figure 22: Portfolio 5 cumulative return

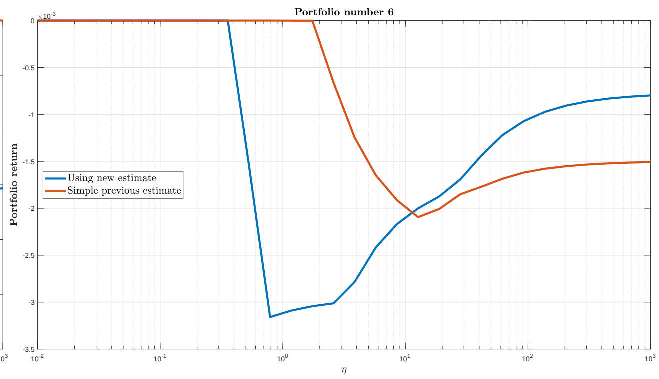

Figure 23: Portfolio 6 cumulative return

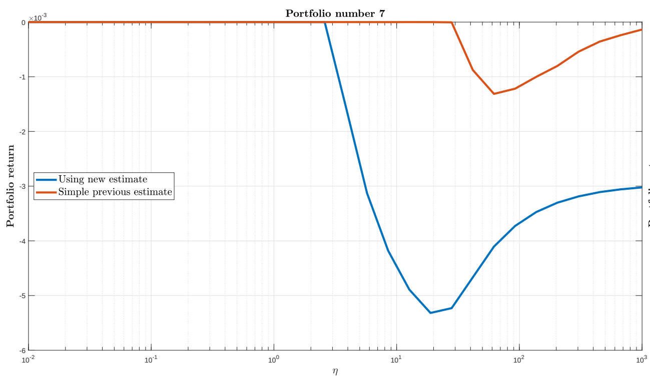

Figure 24: Portfolio 7 cumulative return

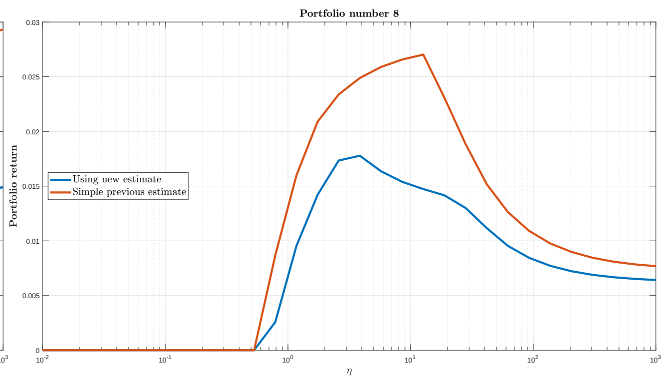

Figure 25: Portfolio 8 cumulative return

The maximization problem that is used to acquire a new estimate of the covariance matrix gives a maximum likelihood estimate of the covariance based

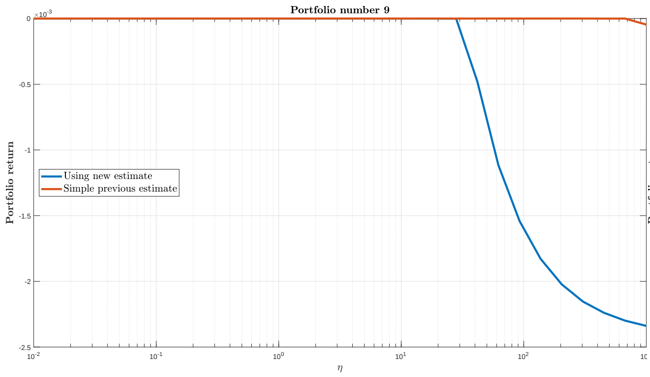

Figure 26: Portfolio 9 cumulative return

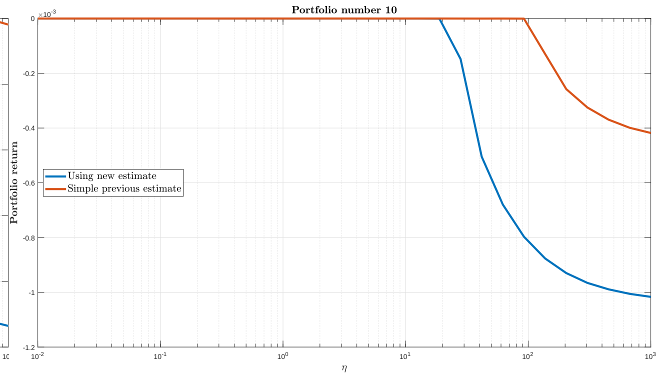

Figure 27: Portfolio 10 cumulative return

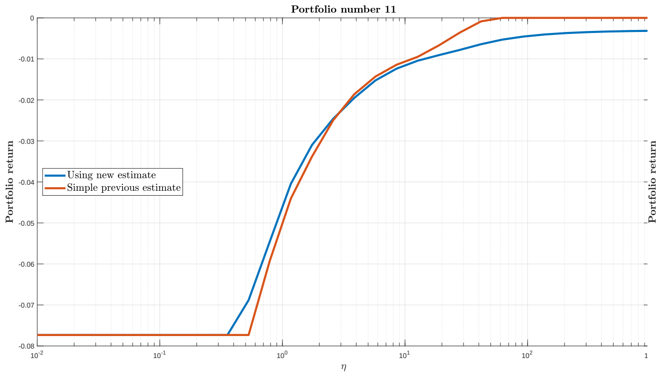

Figure 28: Portfolio 11 cumulative return

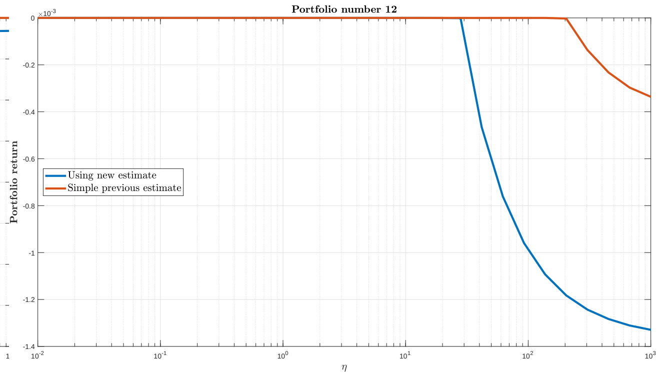

Figure 29: Portfolio 12 cumulative return

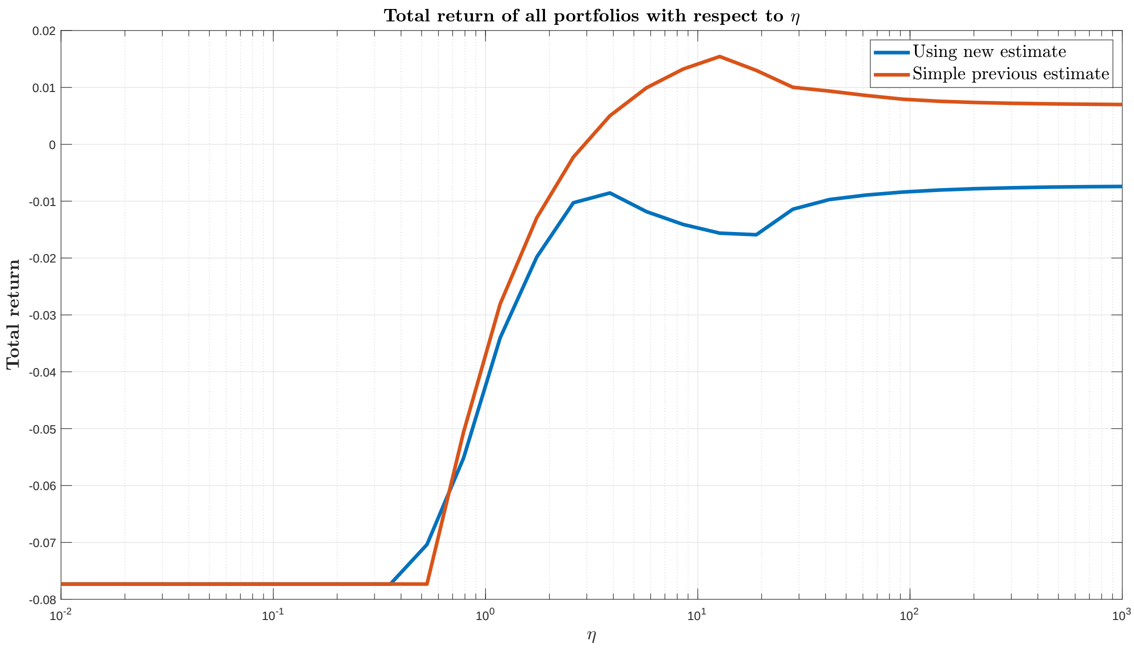

Figure 30: Total return versus η, using the new Σ estimate obtained through optimization

on the current available data, which is the sample covariance.

5 Step 5: Portfolio Optimization with Loss Risk Constraints

As explained in the textbook, we can follow the procedure below to convert the constraint $\mathbf{prob}(r \le 0) \le 0$ to a SOCP constraint:
Since we assume that the $\bar{p}$ is supposed to be a Gaussian random vector, we can write the constraint as:

$$\mathbf{prob}(\frac{r-\bar{r}}{\sigma} \le \frac{-\bar{r}}{\sigma}) <= \beta \tag{1}$$

Since $\dfrac{r-\bar{r}}{\sigma}$ is a zero mean unit variance Gaussian variable, the above probability is simply $\Phi(\dfrac{-\bar{r}}{\sigma})$ where :

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2/2} dt \tag{2}$$

is the CDF of a zero mean unit variance Gaussian variable. Taking the inverse Φ of the given inequality we arrive at:

$$\frac{-\overline{u}}{\sigma} \le \Phi^{-1}(\beta) \tag{3}$$

thus finally arriving at the below inequality

$$\bar{p}^T x + \Phi^{-1}(\beta) \left\| \Sigma^{1/2} x \right\|_2 \ge 0 \tag{4}$$

So the optimization problem can now be stated as:

$$\begin{aligned} \text{maximize} \quad & \bar{p}^T x - \eta x^T \Sigma x \\ \text{subject to} \quad & \sum_{i=1}^{n} x_i = 1, \quad x \ge 0 \\ & \bar{p}^T x + \Phi^{-1}(\beta) \left\| \Sigma^{1/2} x \right\|_2 \ge 0 \end{aligned} \tag{5}$$

To evaluate $\Phi^{-1}(\beta)$ in MATLAB, the function `norminv` is used.
Even though the above optimization problem is convex and is a SOCP, CVX could not solve it. After much testing and changing of parameters I found that if instead of $\mathbf{prob}(r \le 0) \le \beta$, we were to use a slightly different bound on r, CVX could solve the problem. The following results are for the case in which the constraint has been adjusted to $\mathbf{prob}(r \le -0.05) \le \beta$.

From figure 31 we can see that for these four values of η, we will eventually have no gain and will only have losses. A search on the values of the total return for different values of η and β yielded that in the best case, our total return would be 0 meaning that we neither gain anything nor do we lose any

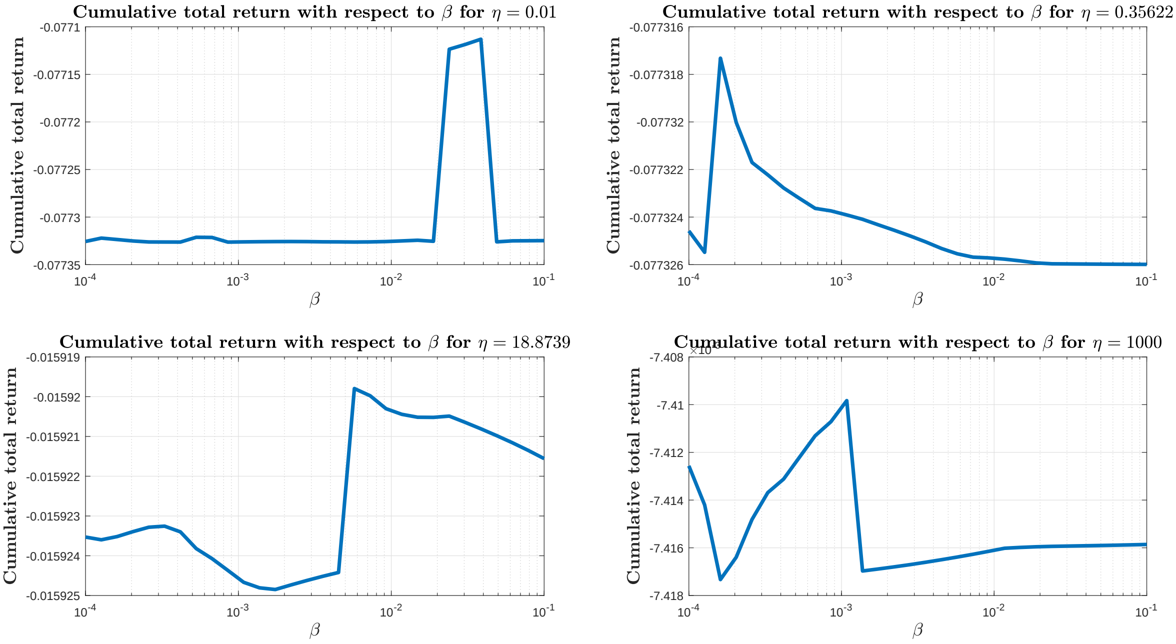

Figure 31: Total return versus β with new constraint $mathbfr <= -0.05 <= \beta$

money. A example of this occurs for $\eta = 1.1721$ and $\beta = 0.0001$. The following section is done with the portfolio for these parameters, which is:

$$
\begin{aligned}
x = [&5.227e-06 \\
&2.829e-05 \\
&3.226e-06 \\
&2.265e-05 \\
&2.836e-05 \\
&0.220 \\
&3.009e-05 \\
&0.256 \\
&3.219e-06 \\
&3.941e-06 \\
&0.522 \\
&3.538e-06]
\end{aligned}
\tag{6}
$$

In the above vector we can see that only 3 stocks have considerable amounts and the rest of the stocks hold tiny infinitesimal amounts. I tried to reason this with $\bar{p}$ but none of their values were significant. I also tried to find a relationship in the covariance matrix Σ but no simple relation was visible.

6 Step 6: Maximum Risk for the Chosen Portfolio

The proposed method can be viewed as an optimization problem of maximizing $x^T \Sigma x$.

$$\begin{aligned} \text{maximize} x^T S x \\ \text{subject to } & S_{ii} = \Sigma_{ii} \\ & S \succeq 0 \\ & sign(S_{ij}) = sign(\Sigma_{ij}) \end{aligned} \tag{7}$$

The answer of this problem is calculated analytically by CVX and we have $\sigma_{wc} = +2.09203e - 05$.
I don't understand what the question is asking is in part b. If the question is to find the minimum risk for all the portfolios calculated in step 5, then by running the code for all portfolios that had the maximum return in step 5 we see that the minimum achievable is $2.5203e - 06$.

7 Appendix: MATLAB Code

```
%% Step 1: Choosing a Portfolio
clc
clear
pbar=load("C:\Users\marson\Dropbox\Convex Optimization\Project\data\pbar.txt");
sigma=load("C:\Users\marson\Dropbox\Convex Optimization\Project\data\sigma.txt")
    ;
m=30;
eta=logspace(-2,3,m);
n=length(pbar);
% Solving for each eta
for i=1:m
    cvx_begin
    variable x(n)
    maximize(pbar'*x-eta(i)*x'*sigma*x)
    subject to
    sum(x)==1;
    x>=0;
    cvx_end
    optimumValue(i)=cvx_optval;
    X(i).x=x;
    standardDeviation(i)=sqrt(x'*sigma*x);
    meanReturn(i)=pbar'*x;
end
% Plotting the figure('units','normalized','outerposition',[0 0 1 1])s
% meanReturn Vs SD
figure('units','normalized','outerposition',[0 0 1 1])
plot(standardDeviation,meanReturn,'LineWidth',3);
title('\textbf{Mean return - Risk }','interpreter','latex','FontSize',15);
xlabel('\textbf{$ x^T\Sigma x$}','interpreter','latex','FontSize',15);
ylabel('\textbf{$ \bar{p}^Tx$}','interpreter','latex','FontSize',15);
grid on
% meanReturn Vs eta times SD on semilogx
figure('units','normalized','outerposition',[0 0 1 1])
semilogx(eta.*standardDeviation,meanReturn,'LineWidth',3);
title('\textbf{Mean return - $\eta$ * Risk}','interpreter','latex','FontSize'
    ,15);
xlabel('\textbf{$ \eta x^T\Sigma x$}','interpreter','latex','FontSize',15);
ylabel('\textbf{$ \bar{p}^Tx$}','interpreter','latex','FontSize',15);
grid on
% Semilogx of portfolio allocation vs eta
figure('units','normalized','outerposition',[0 0 1 1])
c=[X(:).x];
for i=1:n
    semilogx(eta,c(i,:),'LineWidth',3);
    hold on
end
grid on
title('\textbf{Allocation of each stock subject to tradeoff coefficient}','
    interpreter','latex','FontSize',15);
xlabel('\textbf{$\mathbf{\eta}$}','interpreter','latex','FontSize',15);
ylabel('\textbf{Amount of each stock}','interpreter','latex','FontSize',15);
save('step1Data.mat','c','eta','standardDeviation','meanReturn');


%% Step 2: Short Positions
clear
clc
close all
```

```
pbarShort=load("C:\Users\marson\Dropbox\Convex Optimization\Project\data\
    pbar_short.txt");
sigmaShort=load("C:\Users\marson\Dropbox\Convex Optimization\Project\data\
    sigma_short.txt");
m=30;
eta=logspace(-2,3,m);
n=length(pbarShort);
gamma=.5;
% Optimization with short selling
for i=1:m
    cvx_begin
    variables xs(n) xl(n)
    maximize(pbarShort'*(xl-xs)-eta(i)*(xl-xs)'*sigmaShort*(xl-xs))
    subject to
    sum(xl)==1;
    xl>=0;
    xs>=0;
    sum(xs)<=gamma*sum(xl);
    cvx_end
    optimumValue(i)=cvx_optval;
    XShort(i).x=(xl-xs);
    standardDeviationShort(i)=sqrt((xl-xs)'*sigmaShort*(xl-xs));
    meanReturnShort(i)=pbarShort'*(xl-xs);
end
% Optimization problem without short selling
for i=1:m
    cvx_begin
    variable x(n)
    maximize(pbarShort'*x-eta(i)*x'*sigmaShort*x)
    subject to
    sum(x)==1;
    x>=0;
    cvx_end
    optimumValue(i)=cvx_optval;
    X(i).x=x;
    standardDeviation(i)=sqrt(x'*sigmaShort*x);
    meanReturn(i)=pbarShort'*x;
end
% Plotting the figures
figure('units','normalized','outerposition',[0 0 1 1])
plot(standardDeviation,meanReturn,'LineWidth',3);
hold on
plot(standardDeviationShort,meanReturnShort,'LineWidth',3);
title('\textbf{Mean return - Risk with new added stock}','interpreter','latex','
    FontSize',15);
xlabel('\textbf{$ x^T\Sigma x$}','interpreter','latex','FontSize',15);
ylabel('\textbf{$ \bar{p}^Tx$}','interpreter','latex','FontSize',15);
grid on
legend('Without short positions','With short positions','interpreter','latex','
    FontSize',15,'Location','NorthWest')

figure('units','normalized','outerposition',[0 0 1 1])
semilogx(standardDeviation.*eta,meanReturn,'LineWidth',3);
hold on
semilogx(eta.*standardDeviationShort,meanReturnShort,'LineWidth',3);
title('\textbf{Mean return - Risk with new added stock}','interpreter','latex','
    FontSize',15);
xlabel('\textbf{$\eta x^T\Sigma x$}','interpreter','latex','FontSize',15);
ylabel('\textbf{$ \bar{p}^Tx$}','interpreter','latex','FontSize',15);
grid on
legend('Without short positions','With short positions','interpreter','latex','
```

```
    FontSize',15,'Location','NorthWest')


%% Step 3: Portfolio Optimization with Real Data
clear
clc
close all
load("C:\Users\marson\Dropbox\Convex Optimization\Project\data\stock_dataset.mat
    ");
pbarestimate=mean(past_returns,1)';
sigmaestimate=cov(past_returns);
m=30;
eta=logspace(-2,3,m);
n=length(pbarestimate);
gamma=.5;
% Optimization with short selling
for i=1:m
    cvx_begin
    variables xs(n) xl(n)
    maximize(pbarestimate'*(xl-xs)-eta(i)*(xl-xs)'*sigmaestimate*(xl-xs))
    subject to
    sum(xl)==1;
    xl>=0;
    xs>=0;
    sum(xs)<=gamma*sum(xl);
    cvx_end
    optimumValue(i)=cvx_optval;
    XShort(i).x=(xl-xs);
    standardDeviationShort(i)=sqrt((xl-xs)'*sigmaestimate*(xl-xs));
    meanReturnShort(i)=pbarestimate'*(xl-xs);
end
% Optimization problem without short selling
for i=1:m
    cvx_begin
    variable x(n)
    maximize(pbarestimate'*x-eta(i)*x'*sigmaestimate*x)
    subject to
    sum(x)==1;
    x>=0;
    cvx_end
    optimumValue(i)=cvx_optval;
    X(i).x=x;
    standardDeviation(i)=sqrt(x'*sigmaestimate*x);
    meanReturn(i)=pbarestimate'*x;
end
figure('units','normalized','outerposition',[0 0 1 1])
plot(standardDeviation,meanReturn,'LineWidth',3);
hold on
plot(standardDeviationShort,meanReturnShort,'LineWidth',3);
title('\textbf{Mean return - Risk based on stock data}','interpreter','latex','
    FontSize',15);
xlabel('\textbf{$ x^T\Sigma x$}','interpreter','latex','FontSize',15);
ylabel('\textbf{$ \bar{p}^Tx$}','interpreter','latex','FontSize',15);
grid on
legend('Without short positions','With short positions','interpreter','latex','
    FontSize',10,'Location','SouthEast');
% Part d: returns
c=[X(:).x];
cShort=[XShort(:).x];
for i=1:m
    noShortReturns(:,:,i)=future_returns.*c(:,i)';
```

```
    withShortReturns(:,:,i)=future_returns.*cShort(:,i)';
end
cumNoShort=cumsum(noShortReturns,1);
cumWithShort=cumsum(withShortReturns,1);
for i=1:n
    figure('units','normalized','outerposition',[0 0 1 1])
    semilogx(eta,squeeze(cumNoShort(end,i,:)),'LineWidth',3);
    hold on
    semilogx(eta,squeeze(cumWithShort(end,i,:)),'LineWidth',3);
    legend('Without short selling','With short selling','interpreter','latex','FontSize',15,'Location','NorthEast');
    grid on
    txt='Portfolio number %d';
    txt2='portfolio%d';
    txt=sprintf(txt,i);
    txt2=sprintf(txt2,i);
    txt=['\textbf{', txt, '}'];
    title(txt,'interpreter','latex','FontSize',15);
    xlabel('\textbf{$\eta$}','interpreter','latex','FontSize',15);
    ylabel('\textbf{Portfolio return}','interpreter','latex','FontSize',15);
    saveas(gca,txt2,'epsc');
end
% Total return plot
figure('units','normalized','outerposition',[0 0 1 1])
totalReturn=cumsum(future_returns*c,1);
totalReturnShort=cumsum(future_returns*cShort,1);
semilogx(eta,totalReturn(end,:),'LineWidth',3)
hold on
semilogx(eta,totalReturnShort(end,:),'LineWidth',3)
grid on
legend('Without short selling','With short selling','interpreter','latex','FontSize',15);
title('\textbf{Total return of all portfolios with respect to $ \eta$}','interpreter','latex','FontSize',15);
xlabel('\textbf{$\eta$}','interpreter','latex','FontSize',15);
ylabel('\textbf{Total return}','interpreter','latex','FontSize',15);


%% Step 4:  Improve your estimate
clear
clc
close all
load("C:\Users\marson\Dropbox\Convex Optimization\Project\data\stock_dataset.mat");
pbarestimate=mean(past_returns,1)';
S=cov(past_returns);
n=12;
e=ones(12,1);
lambda=10^(-5);
cvx_begin
    variable theta(n,n)
    obj=0;
    for i=1:n
        for j=1:n
            if j==i
                continue
            end
            obj=obj+abs(theta(i,j));
        end
    end
    %maximize(log_det(theta)-trace(S*theta)-lambda*(e'*theta*e-trace(theta)))
```

```
    maximize(log_det(theta)-trace(S*theta)-lambda*obj)
    subject to
    theta>=0;
cvx_end
sigmaestimate=inv(theta);
%
m=30;
eta=logspace(-2,3,m);
n=length(pbarestimate);
gamma=.5;
for i=1:m
    cvx_begin
    variable x(n)
    maximize(pbarestimate'*x-eta(i)*x'*sigmaestimate*x)
    subject to
    sum(x)==1;
    x>=0;
    cvx_end
    optimumValue(i)=cvx_optval;
    X(i).x=x;
    standardDeviation(i)=sqrt(x'*sigmaestimate*x);
    meanReturn(i)=pbarestimate'*x;
end
for i=1:m
    cvx_begin
    variable x(n)
    maximize(pbarestimate'*x-eta(i)*x'*S*x)
    subject to
    sum(x)==1;
    x>=0;
    cvx_end
    optimumValue(i)=cvx_optval;
    XSimple(i).x=x;
    standardDeviationSimple(i)=sqrt(x'*S*x);
    meanReturnSimple(i)=pbarestimate'*x;
end
c=[X(:).x];
cSimple=[XSimple(:).x];
for i=1:m
    noShortReturns(:,:,i)=future_returns.*c(:,i)';
    noShortReturnsSimple(:,:,i)=future_returns.*cSimple(:,i)';
end
cumNoShort=cumsum(noShortReturns,1);
cumNoShortSimple=cumsum(noShortReturnsSimple,1);
for i=1:n
    figure('units','normalized','outerposition',[0 0 1 1])
    semilogx(eta,squeeze(cumNoShort(end,i,:)),'LineWidth',3);
    hold on
    semilogx(eta,squeeze(cumNoShortSimple(end,i,:)),'LineWidth',3);
    legend('Using new estimate','Simple previous estimate','interpreter','latex'
        ,'FontSize',15,'Location','West');
    grid on
    txt='Portfolio number %d';
    txt=sprintf(txt,i);
    txt2='step4_portfolio%d';
    txt2=sprintf(txt2,i);
    txt=['\textbf{', txt, '}'];
    title(txt,'interpreter','latex','FontSize',15);
    xlabel('\textbf{$\eta$}','interpreter','latex','FontSize',15);
    ylabel('\textbf{Portfolio return}','interpreter','latex','FontSize',15);
    saveas(gca,txt2,'epsc');
```

```
end
% Total return plot
figure('units','normalized','outerposition',[0 0 1 1])
totalReturn=cumsum(future_returns*c,1);
totalReturnSimple=cumsum(future_returns*cSimple,1);
semilogx(eta,totalReturn(end,:),'LineWidth',3)
hold on
semilogx(eta,totalReturnSimple(end,:),'LineWidth',3)
grid on
legend('Using new estimate','Simple previous estimate','interpreter','latex','
    FontSize',15);
title('\textbf{Total return of all portfolios with respect to $ \eta$}','
    interpreter','latex','FontSize',15);
xlabel('\textbf{$\eta$}','interpreter','latex','FontSize',15);
ylabel('\textbf{Total return}','interpreter','latex','FontSize',15);


%% Step 5: Portfolio optimization with loss risk constraints

clear
clc
close all
load("C:\Users\marson\Dropbox\Convex Optimization\Project\data\stock_dataset.mat
    ");
pbarestimate=mean(past_returns,1)';
S=cov(past_returns);
n=12;
e=ones(12,1);
lambda=10^(-5);
cvx_begin
    variable theta(n,n)
    obj=0;
    for i=1:n
        for j=1:n
            if j==i
                continue
            end
            obj=obj+abs(theta(i,j));
        end
    end
    %maximize(log_det(theta)-trace(S*theta)-lambda*(e'*theta*e-trace(theta)))
    maximize(log_det(theta)-trace(S*theta)-lambda*obj)
    subject to
    theta>=0;
cvx_end
sigmaestimate=inv(theta);
m=30;
eta=logspace(-2,3,m);
n=length(pbarestimate);
gamma=.5;
q=20;
beta=logspace(-4,-1,q);
squareRootSigma=sqrtm(sigmaestimate);
%
for j=1:q
    for i=1:m
        cvx_begin
            variable x(n)
            maximize(pbarestimate'*x-eta(i)*x'*sigmaestimate*x)
            subject to
            sum(x)==1;
```

```
                x>=0;
                pbarestimate'*x+norminv(beta(j))*norm(squareRootSigma*x)>-.5;
            cvx_end
            optimumValue(i)=cvx_optval;
            X(i,j).x=x;
            standardDeviation(i,j)=sqrt(x'*sigmaestimate*x);
            meanReturn(i,j)=pbarestimate'*x;
        end
end
%
n1=1;
n2=10;
n3=20;
n4=30;
c1=[X(n1,:).x];
c2=[X(n2,:).x];
c3=[X(n3,:).x];
c4=[X(n4,:).x];
for i=1:n
    txt2='Step5_portfolio%d';
    txt2=sprintf(txt2,i);
    figure('units','normalized','outerposition',[0 0 1 1])
    subplot(2,2,1)
    plot(beta,c1(i,:),'LineWidth',3);
    txt=['\textbf{Allocation of portfolio ', num2str(i), ' for $ \eta = $ ',
        num2str(eta(n1)),' }'];
    title(txt,'interpreter','latex','FontSize',15);
    xlabel('\textbf{$\beta$}','interpreter','latex','FontSize',15);
    ylabel('\textbf{Stock amount}','interpreter','latex','FontSize',15);
    grid on
    subplot(2,2,2)
    plot(beta,c2(i,:),'LineWidth',3);
    txt=['\textbf{Allocation of portfolio ', num2str(i), ' for $ \eta = $ ',
        num2str(eta(n2)),' }'];
    title(txt,'interpreter','latex','FontSize',15);
    xlabel('\textbf{$\beta$}','interpreter','latex','FontSize',15);
    ylabel('\textbf{Stock amount}','interpreter','latex','FontSize',15);
    grid on
    subplot(2,2,3)
    plot(beta,c3(i,:),'LineWidth',3);
    txt=['\textbf{Allocation of portfolio ', num2str(i), ' for $ \eta = $ ',
        num2str(eta(n3)),' }'];
    title(txt,'interpreter','latex','FontSize',15);
    xlabel('\textbf{$\beta$}','interpreter','latex','FontSize',15);
    ylabel('\textbf{Stock amount}','interpreter','latex','FontSize',15);
    grid on
    subplot(2,2,4)
    plot(beta,c4(i,:),'LineWidth',3);
    txt=['\textbf{Allocation of portfolio ', num2str(i), ' for $ \eta = $ ',
        num2str(eta(n4)),' }'];
    title(txt,'interpreter','latex','FontSize',15);
    xlabel('\textbf{$\beta$}','interpreter','latex','FontSize',15);
    ylabel('\textbf{Stock amount}','interpreter','latex','FontSize',15);
    grid on
    saveas(gca,txt2,'epsc');
end
%%
return1=cumsum(future_returns*c1,1);
return2=cumsum(future_returns*c2,1);
return3=cumsum(future_returns*c3,1);
return4=cumsum(future_returns*c4,1);
```

```
figure('units','normalized','outerposition',[0 0 1 1])
subplot(2,2,1)
semilogx(beta,return1(end,:),'LineWidth',3)
title(['\textbf{Cumulative total return with respect to $ \beta$ for $ \eta = '
    ,...
    num2str(eta(n1)),' $}'],'interpreter','latex','FontSize',15);
xlabel('\textbf{$\beta$}','interpreter','latex','FontSize',15);
ylabel('\textbf{Cumulative total return}','interpreter','latex','FontSize',15);
grid on
subplot(2,2,2)
semilogx(beta,return2(end,:),'LineWidth',3)
title(['\textbf{Cumulative total return with respect to $ \beta$ for $ \eta = '
    ,...
    num2str(eta(n2)),' $}'],'interpreter','latex','FontSize',15);
xlabel('\textbf{$\beta$}','interpreter','latex','FontSize',15);
ylabel('\textbf{Cumulative total return}','interpreter','latex','FontSize',15);
grid on
subplot(2,2,3)
semilogx(beta,return3(end,:),'LineWidth',3)
title(['\textbf{Cumulative total return with respect to $ \beta$ for $ \eta = '
    ,...
    num2str(eta(n3)),' $}'],'interpreter','latex','FontSize',15);
xlabel('\textbf{$\beta$}','interpreter','latex','FontSize',15);
ylabel('\textbf{Cumulative total return}','interpreter','latex','FontSize',15);
grid on
subplot(2,2,4)
semilogx(beta,return4(end,:),'LineWidth',3)
grid on
title(['\textbf{Cumulative total return with respect to $ \beta$ for $ \eta = '
    ,...
    num2str(eta(n4)),' $}'],'interpreter','latex','FontSize',15);
xlabel('\textbf{$\beta$}','interpreter','latex','FontSize',15);
ylabel('\textbf{Cumulative total return}','interpreter','latex','FontSize',15);
grid on
%%
fr=cumsum(future_returns,1);
fr=fr(end,:);
c=zeros(n,m,q);
for i=1:n
    for j=1:n
        for k=1:q
            c(i,j,k)=X(j,k).x(i);
        end
    end
end
for j=1:m
    for k=1:q
        Returns(j,k)=fr*squeeze(c(:,j,k));
    end
end
figure('units','normalized','outerposition',[0 0 1 1])
[a,b]=meshgrid(beta,eta);
surf(a,b,Returns);
[etamax, betamax]=find(Returns==max(max(Returns)),1);
x=X(etamax,betamax).x;
etamax=eta(etamax);
betamax=beta(betamax);
save('portfolio.mat','x','etamax','betamax','sigmaestimate','pbarestimate');


%% Step 6: maximum risk for the chosen portfolio
```

```
%clear
close all
clc
%load portfolio.mat
n=12;

cvx_begin
    variable S(n,n)
    maximize(x'*S*x)
    subject to
    S>=0;
    diag(S)==diag(sigmaestimate);
    for i=1:12
        for j=1:12
            if i==j
                continue
            elseif sigmaestimate(i,j)>0
                S(i,j)>=0;
            else
                S(i,j)<=0;
            end
        end
    end

cvx_end
```