MATLAB assignment

a) Minimize $\frac{1}{2}x^T Q x + c^T x$

Subject to $Ax = b$

$x \geq 0$

$$\xrightarrow{KKT} \begin{cases} \cdot\ Ax = b \\ \cdot\ x \geq 0 \\ \cdot\ s_i \geq 0 \\ \cdot\ s_i x_i = 0 \\ \cdot\ Qx + c - A^T y - s = 0 \Rightarrow A^T y + s - Qx = c \end{cases} \quad \text{I}$$

b) Minimize $F = \frac{1}{2}x^T Q x + c^T x + \mu \sum -\log(-x_i)$

Subject to $Ax = b$

$$\nabla F = Qx + c - \mu \sum \frac{e_i}{x_i} = Qx + c - \mu X^{-1} e$$

$$\mathcal{L} = F + y^T(b - Ax)$$

$$\nabla \mathcal{L} = \nabla F + A^T y = \underbrace{Qx}_{QXe} + c - \mu X^{-1} e - A^T y = 0$$

$$\text{KKT}: \begin{cases} Ax = b \\ (QX + \mu X^{-1})e + c + A^T y^* = 0 \end{cases} \quad \text{II}$$

by comparing I & II we can see that by choosing $y_{\text{I}} = y^*_{\text{II}}$ & $s = \mu X^{-1} e$

$$\Rightarrow Se = \mu X^{-1} e \Rightarrow XSe = \mu e$$

Using both of the KKT conditions we arrive at the set of equations:

$Ax = b$

$A^T y + s - Qx = c$

$XSe = \mu e$

$X \geq 0,\ S \geq 0$

c) since by choosing $y_{I} = y^*_{II}$ & $s = \mu X^{-1}e$, the ∇L will be zero for the primal problem. thus by using these values & setting $x_{I} = x_{II}(\mu)$ we get: $g_{I}(s, y) = \mathcal{L}(x(\mu), \mu X^{-1}e, y^*)$

Since weak duality always holds we have:

$$p^* \geq \mathcal{L}(x(\mu), \mu X^{-1}e, y^*) = f_0(x^*) - \underbrace{s^T x}_{\mu e^T X^{-1} X e = n} + y^* \underbrace{(Ax^* - b)}_{0}$$

$$= f_0(x^*) - n\mu \Rightarrow \boxed{f_0(x^*) - p^* \leq n\mu}$$