

In the name of God



Sharif University of Technology

School of Electrical Engineering

Convex Optimization

Course Project

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The .m file is available in the archive that is uploaded onto the CW. It also available in the appendix of the report.

1 Step 1: Choosing a Portfolio

It can be seen in the figures below that as the *risk* increases, the *total return* also increases which seems logical; If you are willing to risk more, you expect a larger return. But still, I cannot understand the complete monotonic behaviour of the function.

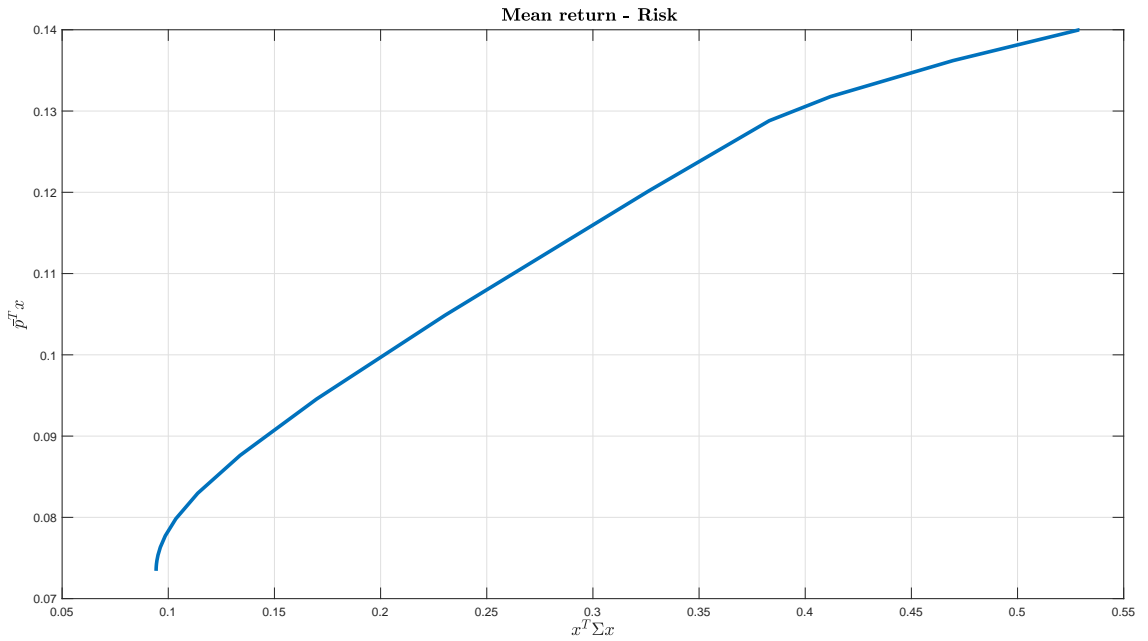


Figure 1: Total return versus the standard deviation for different values of η

From the second figure we can see that as the risk increases i.e. as η increases and the effect of the negative term $x^T \Sigma x$ increases, we do not tend toward a single stock and we can also see that for large values of η , it is best to have a certain amount from each stock so that we could minimize the loss that would occur if we had made an unfortunate decision at buying the stock.

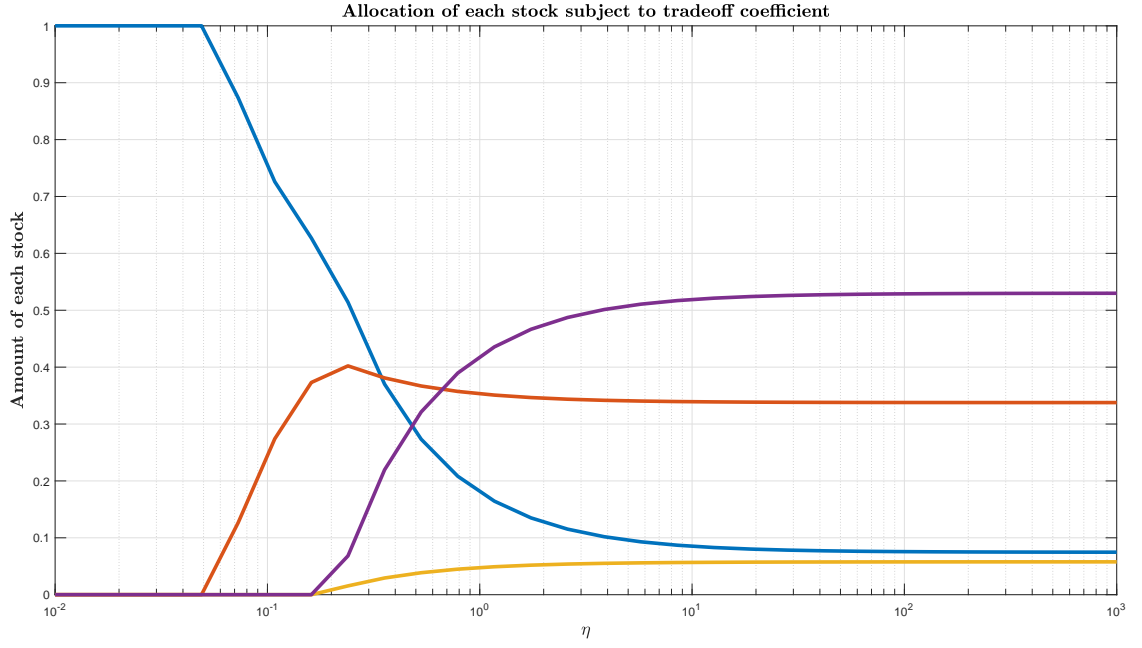


Figure 2: Percentage allocated to each stock with respect to η , logarithmic x-axis

2 Short Positions

In the figure below we can see that for the same risk, we have a greater return with short selling compared to without short selling.

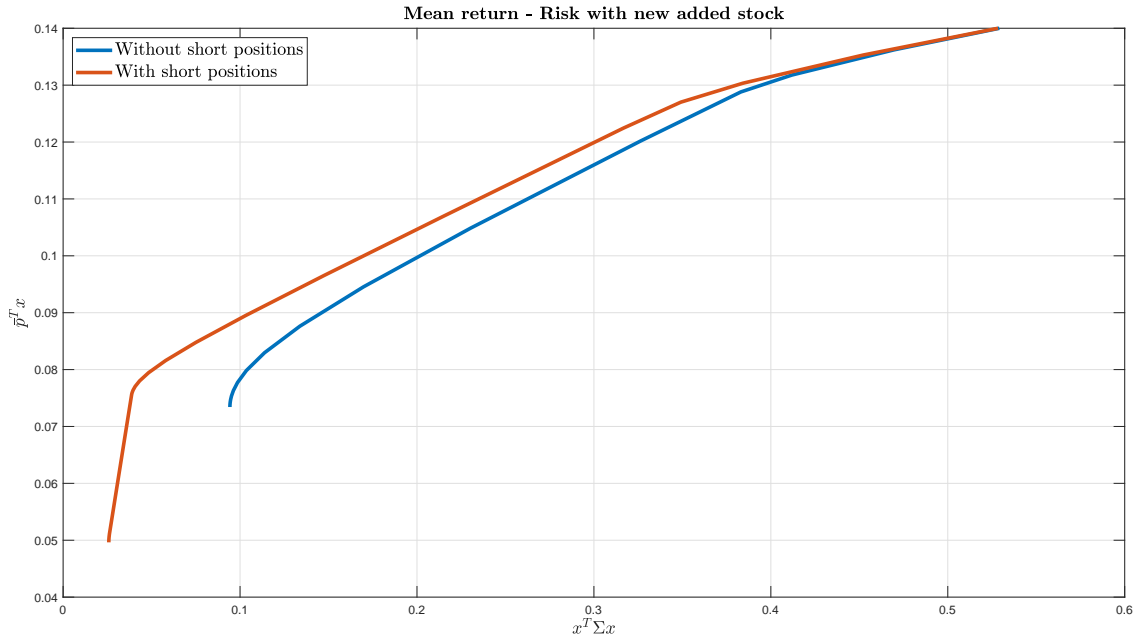


Figure 3: Total return versus the standard deviation for different values of η

3 Portfolio Optimization with Real Data

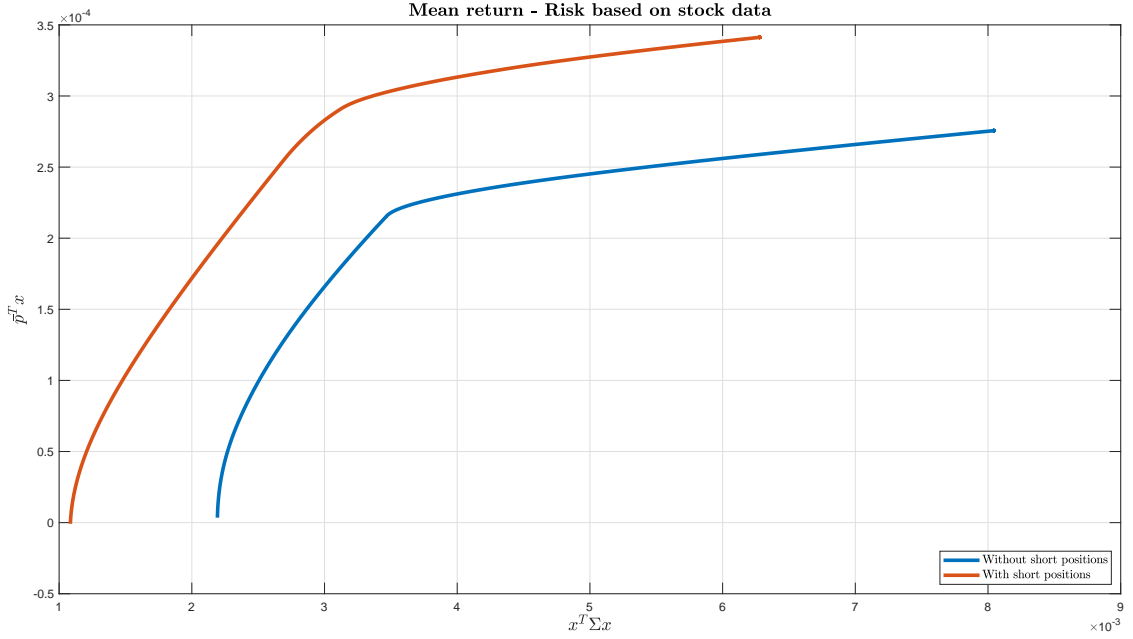


Figure 4: Total return versus the standard deviation for different values of η , real stock data

Since it is a bit hard to choose a point on the plot, the optimal η for each plot is discussed here and the plots will follow afterwards. Below is the optimum point for each portfolio in order of their appearance in the dataset:

1. In this portfolio, the best strategy would be *not to short sell* and the optimum point is the maximum possible *eta* i.e. 1000.
2. In this portfolio, the *short selling* strategy is only good for large values of *eta* e.g. for $\eta > 90$.
3. In this portfolio, the *short selling* strategy is always optimum and the best η is around 8.5.
4. In this portfolio, the *short selling* strategy is only good for large values of *eta* e.g. for $\eta > 60$.
5. In this portfolio, the best strategy would be *not to short sell* and the optimum η is any η that is not in the range 30 to 100.
6. In this portfolio, the *short selling* strategy is always optimum and the best η is any η that is less than 2.
7. In this portfolio, the *short selling* strategy optimum for η smaller than 70 and the optimum η is around 28.

8. In this portfolio, the best strategy would be *not to short sell* and the optimum η is around 13.
9. In this portfolio, the *short selling* strategy is always better and the optimum range for η is any η smaller than 10.
10. In this portfolio, the *short selling* strategy is always better and the optimum value is around 20.
11. In this portfolio, the best strategy would be *not to short sell* for η smaller than 100 but *short selling* would be wise for η larger than the said value.
12. In this portfolio, the *short selling* strategy is always better and the optimum value is around 10.

Below are the returns of each portfolio with respect to η :

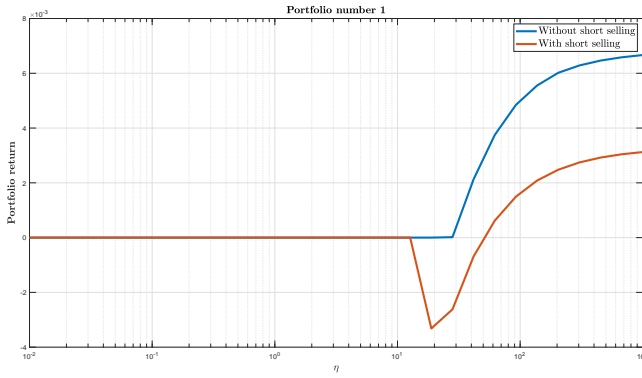


Figure 5: Portfolio 1 cumulative return

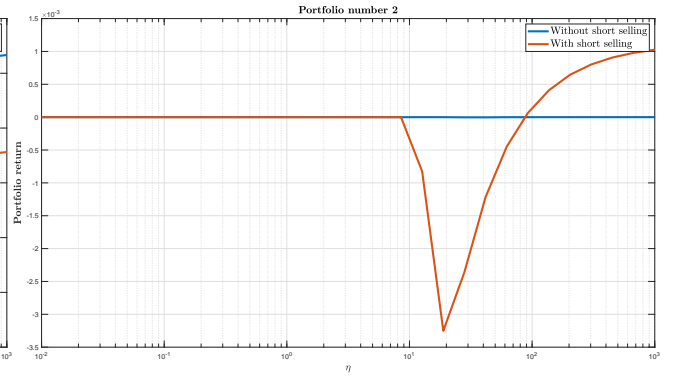


Figure 6: Portfolio 2 cumulative return

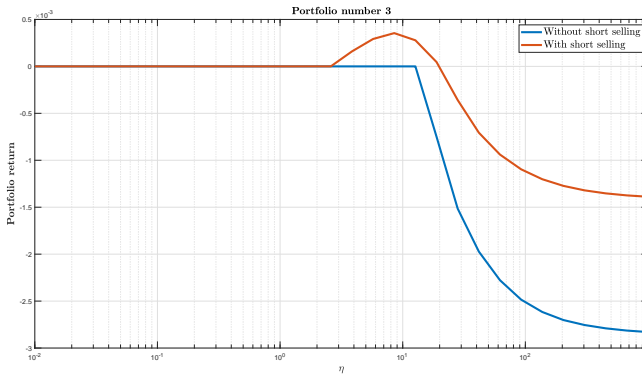


Figure 7: Portfolio 3 cumulative return

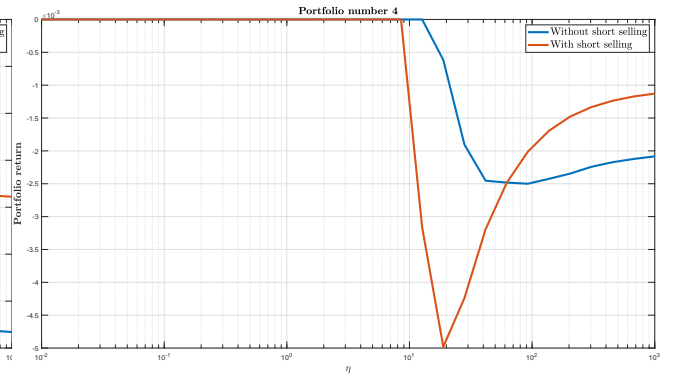


Figure 8: Portfolio 4 cumulative return

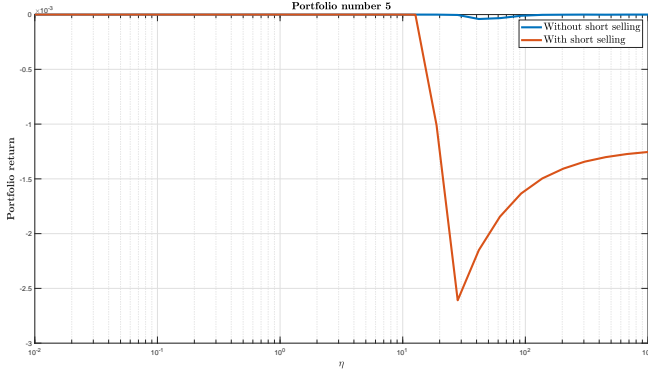


Figure 9: Portfolio 5 cumulative return

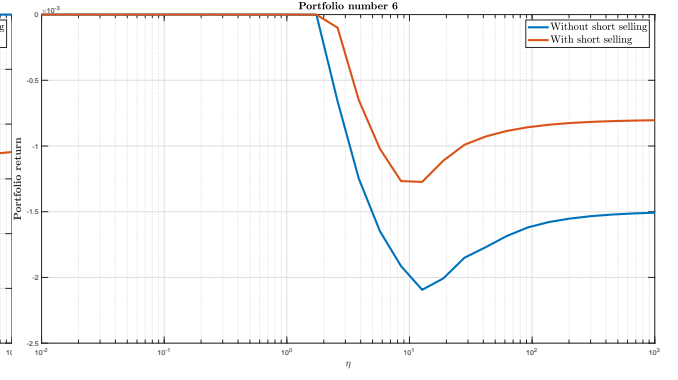


Figure 10: Portfolio 6 cumulative return

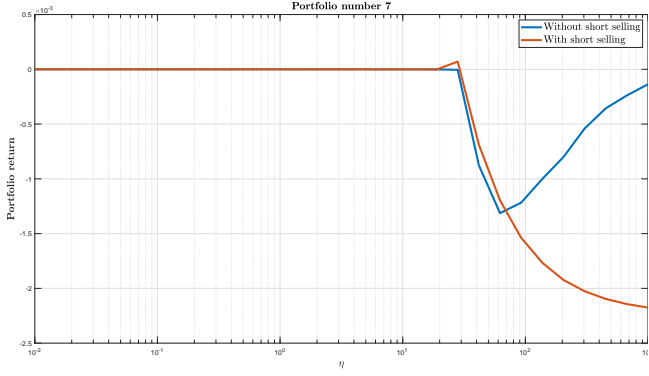


Figure 11: Portfolio 7 cumulative return

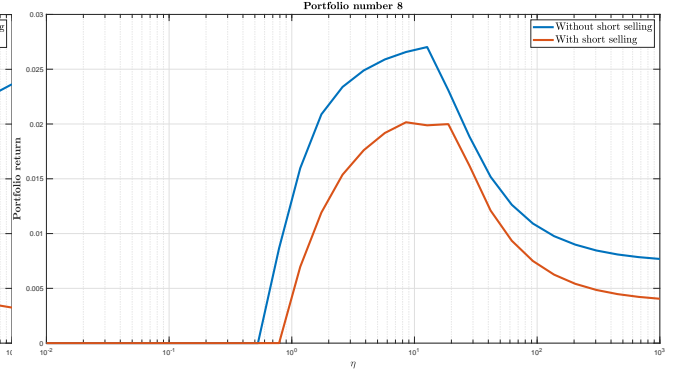


Figure 12: Portfolio 8 cumulative return

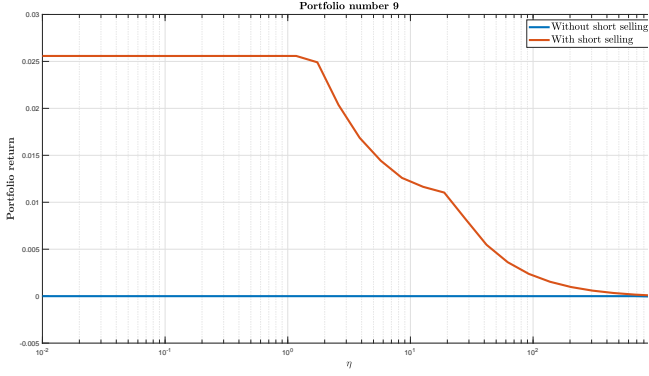


Figure 13: Portfolio 9 cumulative return

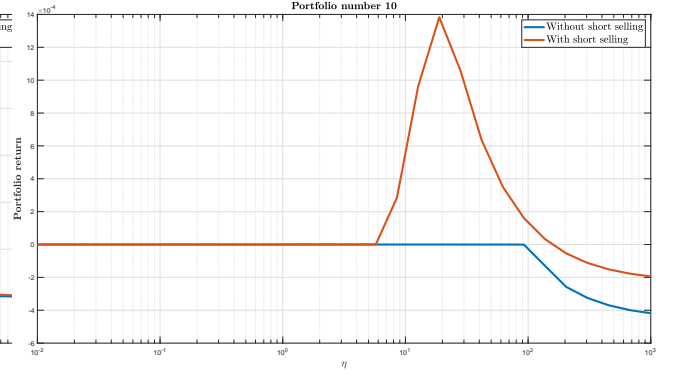


Figure 14: Portfolio 10 cumulative return

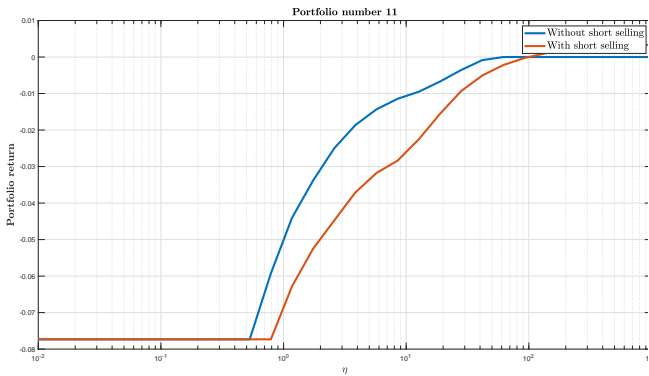


Figure 15: Portfolio 11 cumulative return

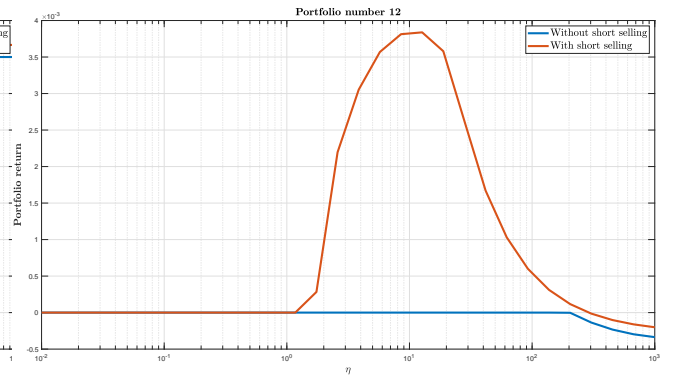


Figure 16: Portfolio 12 cumulative return

The figure below depicts the behaviour of the total return of all portfolios with respect to η . As it can be seen, the strategy of short selling is not optimum

for most η and is only desirable for small η e.g. η smaller than 0.8. Above this limit, the optimum strategy would be not to short sell. The optimum η for both the cases of short selling and not short selling is around 12. For all η smaller than 4, we have no gain and the investment results in a loss.

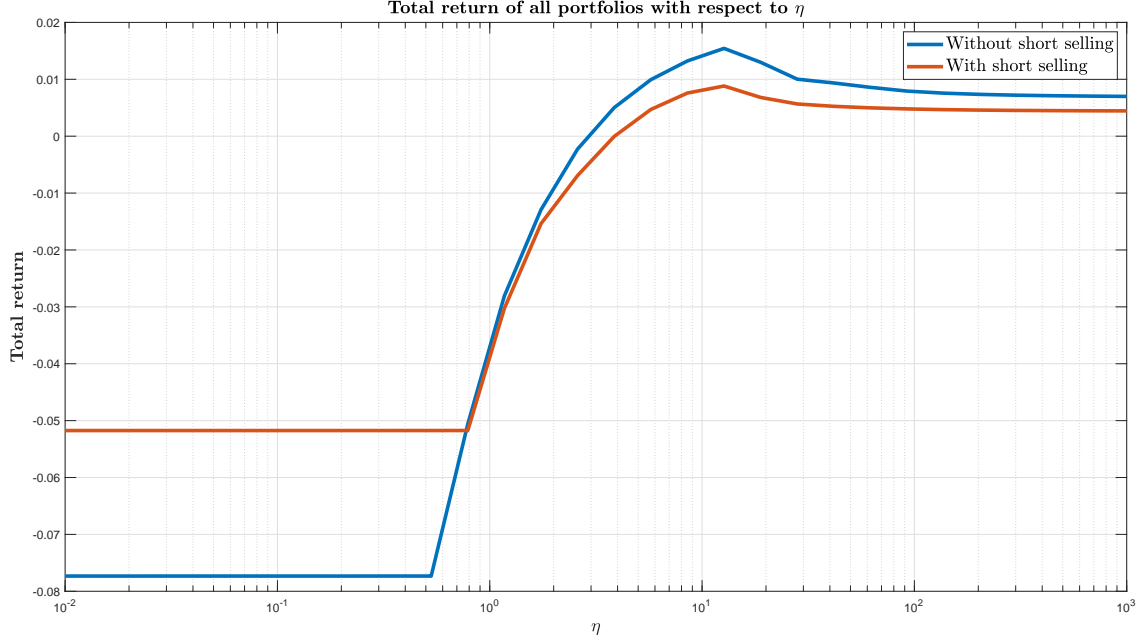


Figure 17: Total return versus η , real stock data

4 Step 4: Improve Your Estimate

The results of this section are only stated for the case without short selling. A general look to the figures below follows up with the deduction that the new estimate predicts worse outcomes i.e. lower return of almost all of the portfolios but if we also heed attention to figure 30, showing the relation between the *total return* and η for both estimates of Σ , we can see that the new estimate predicts that the returns will be less for almost all η meaning that after making our estimate more exact, the data suggests that the expected return will be less.

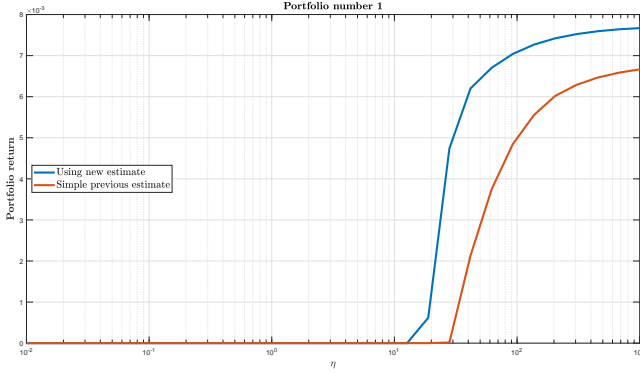


Figure 18: Portfolio 1 cumulative return

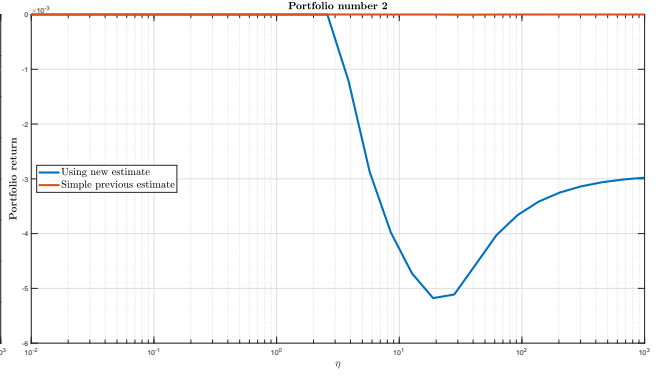


Figure 19: Portfolio 2 cumulative return

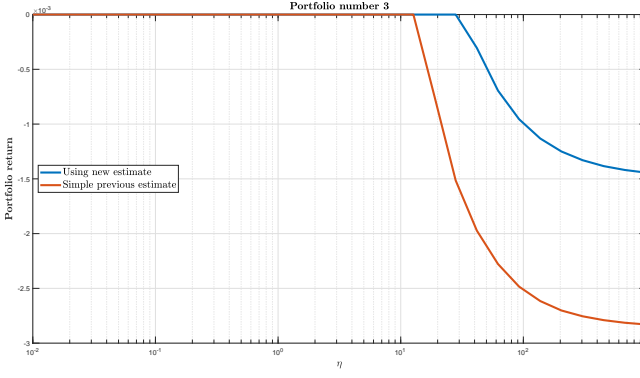


Figure 20: Portfolio 3 cumulative return

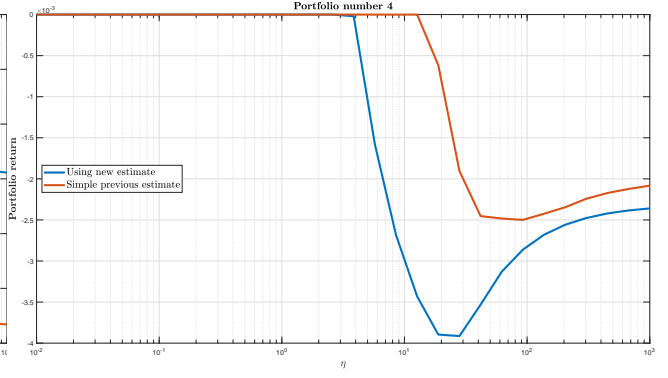


Figure 21: Portfolio 4 cumulative return

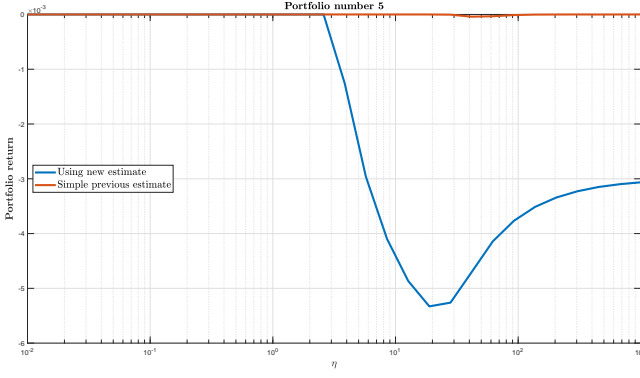


Figure 22: Portfolio 5 cumulative return

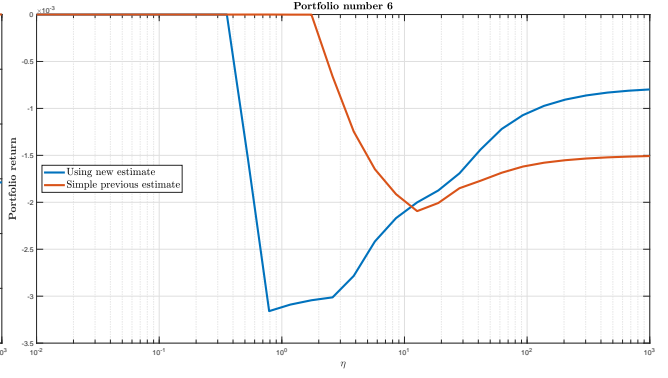


Figure 23: Portfolio 6 cumulative return

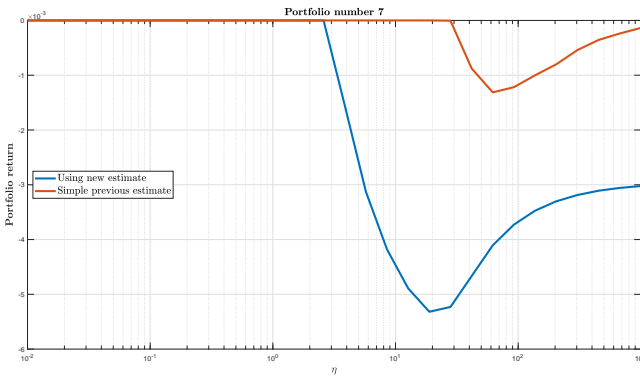


Figure 24: Portfolio 7 cumulative return

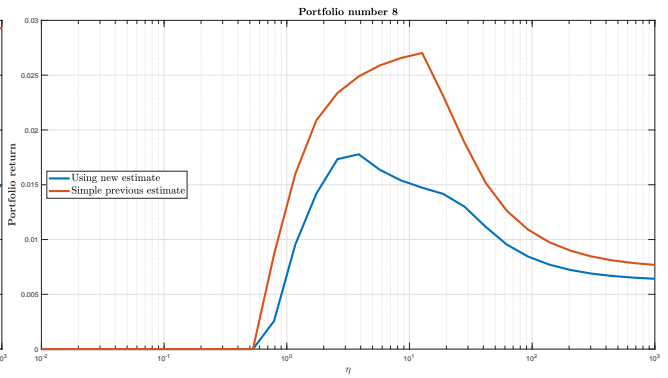


Figure 25: Portfolio 8 cumulative return

The maximization problem that is used to acquire a new estimate of the covariance matrix gives a maximum likelihood estimate of the covariance based

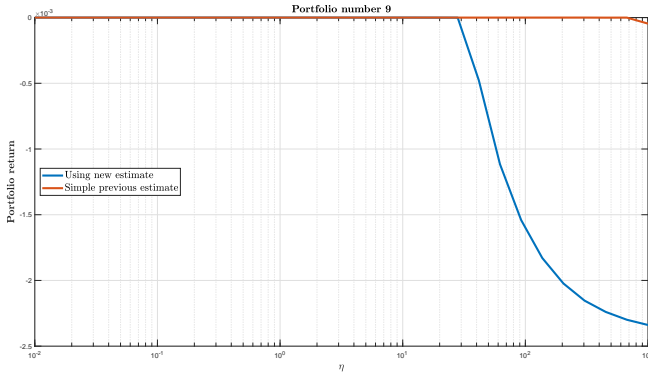


Figure 26: Portfolio 9 cumulative return

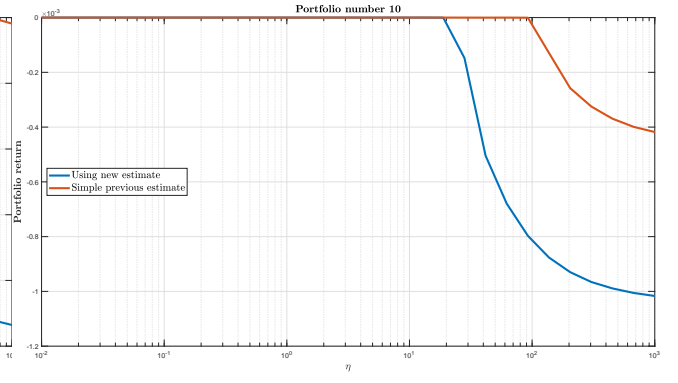


Figure 27: Portfolio 10 cumulative return

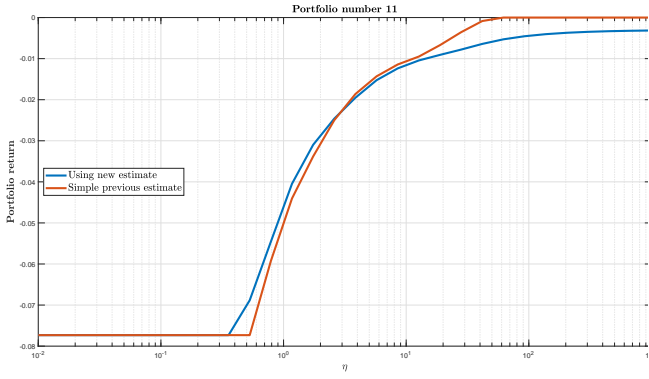


Figure 28: Portfolio 11 cumulative return

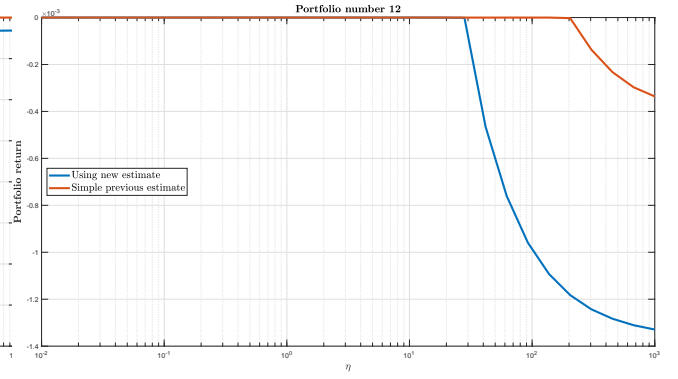
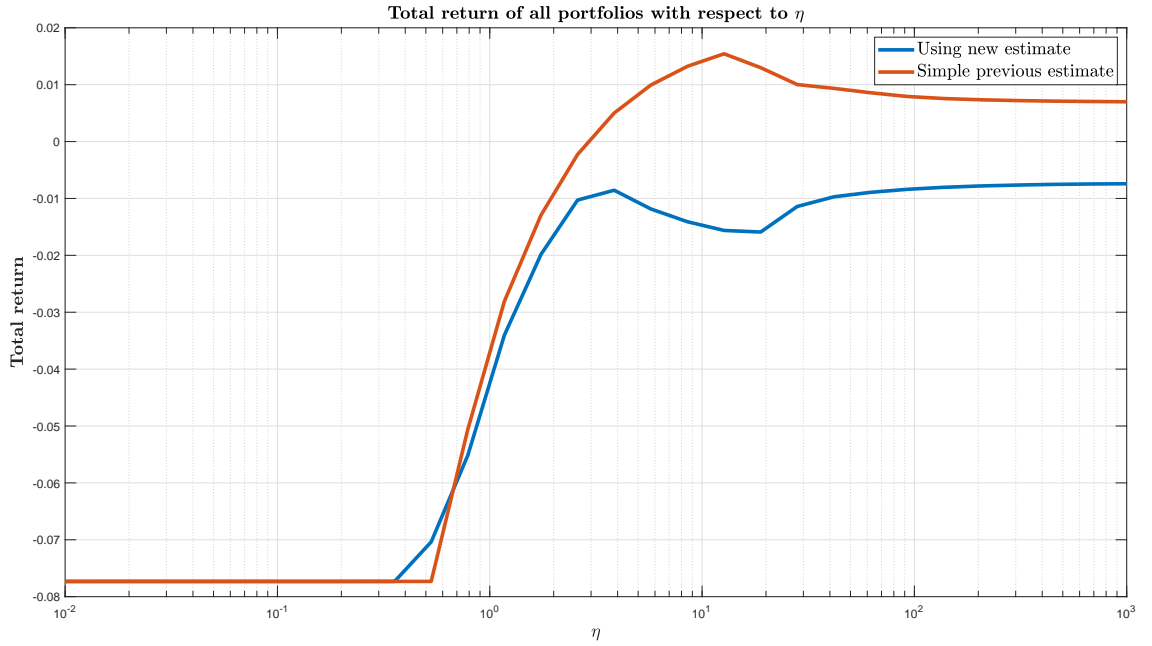


Figure 29: Portfolio 12 cumulative return

Figure 30: Total return versus η , using the new Σ estimate obtained through optimization

on the current available data, which is the sample covariance.

5 Step 5: Portfolio Optimization with Loss Risk Constraints

As explained in the textbook, we can follow the procedure below to convert the constraint $\mathbf{prob}(r \leq 0) \leq \beta$ to a SOCP constraint:

Since we assume that the \bar{p} is supposed to be a Gaussian random vector, we can write the constraint as:

$$\mathbf{prob}\left(\frac{r - \bar{r}}{\sigma} \leq \frac{-\bar{r}}{\sigma}\right) \leq \beta \quad (1)$$

Since $\frac{r - \bar{r}}{\sigma}$ is a zero mean unit variance Gaussian variable, the above probability is simply $\Phi\left(\frac{-\bar{r}}{\sigma}\right)$ where :

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-t^2/2} dt \quad (2)$$

is the CDF of a zero mean unit variance Gaussian variable. Taking the inverse Φ of the given inequality we arrive at:

$$\frac{-\bar{r}}{\sigma} \leq \Phi^{-1}(\beta) \quad (3)$$

thus finally arriving at the below inequality

$$\bar{p}^T x + \Phi^{-1}(\beta) \left\| \Sigma^{1/2} x \right\|_2 \geq 0 \quad (4)$$

So the optimization problem can now be stated as:

$$\begin{aligned} & \text{maximize} \quad \bar{p}^T x - \eta x^T \Sigma x \\ & \text{subject to} \quad \sum_{i=1}^n x_i = 1, \quad x \geq 0 \\ & \quad \bar{p}^T x + \Phi^{-1}(\beta) \left\| \Sigma^{1/2} x \right\|_2 \geq 0 \end{aligned} \quad (5)$$

To evaluate $\Phi^{-1}(\beta)$ in MATLAB, the function `norminv` is used.

Even though the above optimization problem is convex and is a SOCP, CVX could not solve it. After much testing and changing of parameters I found that if instead of $\mathbf{prob}(r \leq 0) \leq \beta$, we were to use a slightly different bound on r , CVX could solve the problem. The following results are for the case in which the constraint has been adjusted to $\mathbf{prob}(r \leq -0.05) \leq \beta$.

From figure 31 we can see that for these four values of η , we will eventually have no gain and will only have losses. A search on the values of the total return for different values of η and β yielded that in the best case, our total return would be 0 meaning that we neither gain anything nor do we lose any

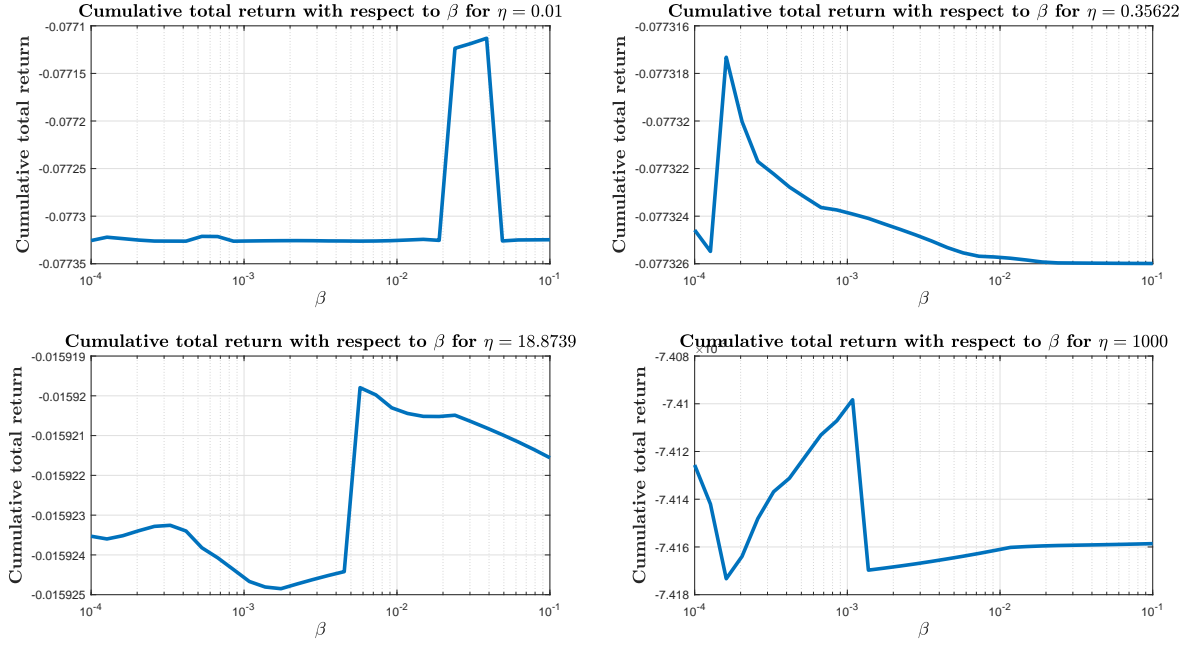


Figure 31: Total return versus β with new constraint $\text{mathbf{r}} \leq -0.05 \leq \beta$

money. A example of this occurs for $\eta = 1.1721$ and $\beta = 0.0001$. The following section is done with the portfolio for these parameters, which is:

$$\begin{aligned}
 x = & \begin{bmatrix} 5.227e - 06 \\ 2.829e - 05 \\ 3.226e - 06 \\ 2.265e - 05 \\ 2.836e - 05 \\ 0.220 \\ 3.009e - 05 \\ 0.256 \\ 3.219e - 06 \\ 3.941e - 06 \\ 0.522 \\ 3.538e - 06 \end{bmatrix}
 \end{aligned} \tag{6}$$

In the above vector we can see that only 3 stocks have considerable amounts and the rest of the stocks hold tiny infinitesimal amounts. I tried to reason this with \bar{p} but none of their values were significant. I also tried to find a relationship in the covariance matrix Σ but no simple relation was visible.

6 Step 6: Maximum Risk for the Chosen Portfolio

The proposed method can be viewed as an optimization problem of maximizing $x^T \Sigma x$.

$$\begin{aligned} & \text{maximize } x^T S x \\ & \text{subject to } S_{ii} = \Sigma_{ii} \\ & \quad S \succeq 0 \\ & \quad \text{sign}(S_{ij}) = \text{sign}(\Sigma_{ij}) \end{aligned} \tag{7}$$

The answer of this problem is calculated analytically by CVX and we have $\sigma_{wc} = +2.09203e - 05$.

I don't understand what the question is asking is in part b. If the question is to find the minimum risk for all the portfolios calculated in step 5, then by running the code for all portfolios that had the maximum return in step 5 we see that the minimum achievable is $2.5203e - 06$.

7 Appendix: MATLAB Code

```

1 %% Step 1: Choosing a Portfolio
2 clc
3 clear
4 pbar=load('C:\Users\marson\Dropbox\Convex Optimization\Project\data\pbar.txt');
5 sigma=load('C:\Users\marson\Dropbox\Convex Optimization\Project\data\sigma.txt')
6 ;
7 m=30;
8 eta=logspace(-2,3,m);
9 n=length(pbar);
10 % Solving for each eta
11 for i=1:m
12     cvx_begin
13         variable x(n)
14         maximize(pbar'*x-eta(i)*x'*sigma*x)
15         subject to
16         sum(x)==1;
17         x>=0;
18     cvx_end
19     optimumValue(i)=cvx_optval;
20     X(i).x=x;
21     standardDeviation(i)=sqrt(x'*sigma*x);
22     meanReturn(i)=pbar'*x;
23 end
24 % Plotting the figure('units','normalized','outerposition',[0 0 1 1])s
25 % meanReturn Vs SD
26 figure('units','normalized','outerposition',[0 0 1 1])
27 plot(standardDeviation,meanReturn,'LineWidth',3);
28 title('\textbf{Mean return - Risk}','interpreter','latex','FontSize',15);
29 xlabel('\textbf{$x^T\Sigma x$}','interpreter','latex','FontSize',15);
30 ylabel('\textbf{$\bar{p}^T x$}','interpreter','latex','FontSize',15);
31 grid on
32 % meanReturn Vs eta times SD on semilogx
33 figure('units','normalized','outerposition',[0 0 1 1])
34 semilogx(eta.*standardDeviation,meanReturn,'LineWidth',3);
35 title('\textbf{Mean return - $\eta$ * Risk}','interpreter','latex','FontSize',15);
36 xlabel('\textbf{$\eta x^T\Sigma x$}','interpreter','latex','FontSize',15);
37 ylabel('\textbf{$\bar{p}^T x$}','interpreter','latex','FontSize',15);
38 grid on
39 % Semilogx of portfolio allocation vs eta
40 figure('units','normalized','outerposition',[0 0 1 1])
41 c=[X(:).x];
42 for i=1:n
43     semilogx(eta,c(i,:), 'LineWidth',3);
44     hold on
45 end
46 grid on
47 title('\textbf{Allocation of each stock subject to tradeoff coefficient}','interpreter','latex','FontSize',15);
48 xlabel('\textbf{$\mathbf{\eta}$}','interpreter','latex','FontSize',15);
49 ylabel('\textbf{Amount of each stock}','interpreter','latex','FontSize',15);
50 save('step1Data.mat','c','eta','standardDeviation','meanReturn');
51
52 %% Step 2: Short Positions
53 clear
54 clc
55 close all

```

```

56 pbarShort=load("C:\Users\marson\Dropbox\Convex Optimization\Project\data\
    pbar_short.txt");
57 sigmaShort=load("C:\Users\marson\Dropbox\Convex Optimization\Project\data\
    sigma_short.txt");
58 m=30;
59 eta=logspace(-2,3,m);
60 n=length(pbarShort);
61 gamma=.5;
62 % Optimization with short selling
63 for i=1:m
64     cvx_begin
65         variables xl(n)
66         maximize(pbarShort'*(xl-xs)-eta(i)*(xl-xs)'*sigmaShort*(xl-xs))
67         subject to
68         sum(xl)==1;
69         xl>=0;
70         xs>=0;
71         sum(xs)<=gamma*sum(xl);
72     cvx_end
73     optimumValue(i)=cvx_optval;
74     XShort(i).x=(xl-xs);
75     standardDeviationShort(i)=sqrt((xl-xs)'*sigmaShort*(xl-xs));
76     meanReturnShort(i)=pbarShort'*(xl-xs);
77 end
78 % Optimization problem without short selling
79 for i=1:m
80     cvx_begin
81         variable x(n)
82         maximize(pbarShort'*x-eta(i)*x'*sigmaShort*x)
83         subject to
84         sum(x)==1;
85         x>=0;
86     cvx_end
87     optimumValue(i)=cvx_optval;
88     X(i).x=x;
89     standardDeviation(i)=sqrt(x'*sigmaShort*x);
90     meanReturn(i)=pbarShort'*x;
91 end
92 % Plotting the figures
93 figure('units','normalized','outerposition',[0 0 1 1])
94 plot(standardDeviation,meanReturn,'LineWidth',3);
95 hold on
96 plot(standardDeviationShort,meanReturnShort,'LineWidth',3);
97 title('\textbf{Mean return - Risk with new added stock}','interpreter','latex','
    FontSize',15);
98 xlabel('\textbf{$x^T\Sigma x$}','interpreter','latex','FontSize',15);
99 ylabel('\textbf{$\bar{p}^T x$}','interpreter','latex','FontSize',15);
100 grid on
101 legend('Without short positions','With short positions','interpreter','latex','
    FontSize',15,'Location','NorthWest')
102
103 figure('units','normalized','outerposition',[0 0 1 1])
104 semilogx(standardDeviation.*eta,meanReturn,'LineWidth',3);
105 hold on
106 semilogx(eta.*standardDeviationShort,meanReturnShort,'LineWidth',3);
107 title('\textbf{Mean return - Risk with new added stock}','interpreter','latex','
    FontSize',15);
108 xlabel('\textbf{$\eta x^T\Sigma x$}','interpreter','latex','FontSize',15);
109 ylabel('\textbf{$\bar{p}^T x$}','interpreter','latex','FontSize',15);
110 grid on
111 legend('Without short positions','With short positions','interpreter','latex','

```

```

    'FontSize',15,'Location','NorthWest')
112
113
114 %% Step 3: Portfolio Optimization with Real Data
115 clear
116 clc
117 close all
118 load('C:\Users\marson\Dropbox\Convex Optimization\Project\data\stock_dataset.mat
    ');
119 pbarestimate=mean(past_returns,1)';
120 sigmaestimate=cov(past_returns);
121 m=30;
122 eta=logspace(-2,3,m);
123 n=length(pbarestimate);
124 gamma=.5;
125 % Optimization with short selling
126 for i=1:m
127     cvx_begin
128         variables xl(n)
129         maximize(pbarestimate'*(xl-xs)-eta(i)*(xl-xs)'*sigmaestimate*(xl-xs))
130         subject to
131         sum(xl)==1;
132         xl>=0;
133         xs>=0;
134         sum(xs)<=gamma*sum(xl);
135     cvx_end
136     optimumValue(i)=cvx_optval;
137     XShort(i).x=(xl-xs);
138     standardDeviationShort(i)=sqrt((xl-xs)'*sigmaestimate*(xl-xs));
139     meanReturnShort(i)=pbarestimate'*(xl-xs);
140 end
141 % Optimization problem without short selling
142 for i=1:m
143     cvx_begin
144         variable x(n)
145         maximize(pbarestimate'*x-eta(i)*x'*sigmaestimate*x)
146         subject to
147         sum(x)==1;
148         x>=0;
149     cvx_end
150     optimumValue(i)=cvx_optval;
151     X(i).x=x;
152     standardDeviation(i)=sqrt(x'*sigmaestimate*x);
153     meanReturn(i)=pbarestimate'*x;
154 end
155 figure('units','normalized','outerposition',[0 0 1 1])
156 plot(standardDeviation,meanReturn,'LineWidth',3);
157 hold on
158 plot(standardDeviationShort,meanReturnShort,'LineWidth',3);
159 title('\textbf{Mean return - Risk based on stock data}','interpreter','latex','
    'FontSize',15);
160 xlabel('\textbf{$x^T\Sigma x$}','interpreter','latex','FontSize',15);
161 ylabel('\textbf{$\bar{p}^T x$}','interpreter','latex','FontSize',15);
162 grid on
163 legend('Without short positions','With short positions','interpreter','latex','
    'FontSize',10,'Location','SouthEast');
164 % Part d: returns
165 c=[X(:).x];
166 cShort=[XShort(:).x];
167 for i=1:m
168     noShortReturns(:, :, i)=future_returns.*c(:, i)';

```

```

169     withShortReturns(:, :, i) = future_returns .* cShort(:, i)';
170 end
171 cumNoShort = cumsum(noShortReturns, 1);
172 cumWithShort = cumsum(withShortReturns, 1);
173 for i = 1:n
174     figure('units', 'normalized', 'outerposition', [0 0 1 1])
175     semilogx(eta, squeeze(cumNoShort(end, i, :)), 'LineWidth', 3);
176     hold on
177     semilogx(eta, squeeze(cumWithShort(end, i, :)), 'LineWidth', 3);
178     legend('Without short selling', 'With short selling', 'interpreter', 'latex', '
        FontSize', 15, 'Location', 'NorthEast');
179     grid on
180     txt = 'Portfolio number %d';
181     txt2 = 'portfolio%d';
182     txt = sprintf(txt, i);
183     txt2 = sprintf(txt2, i);
184     txt = ['\textbf{', txt, '}'];
185     title(txt, 'interpreter', 'latex', 'FontSize', 15);
186     xlabel('\textbf{\$ \eta \$}', 'interpreter', 'latex', 'FontSize', 15);
187     ylabel('\textbf{Portfolio return}', 'interpreter', 'latex', 'FontSize', 15);
188     saveas(gca, txt2, 'eps');
189 end
190 % Total return plot
191 figure('units', 'normalized', 'outerposition', [0 0 1 1])
192 totalReturn = cumsum(future_returns * c, 1);
193 totalReturnShort = cumsum(future_returns * cShort, 1);
194 semilogx(eta, totalReturn(end, :), 'LineWidth', 3)
195 hold on
196 semilogx(eta, totalReturnShort(end, :), 'LineWidth', 3)
197 grid on
198 legend('Without short selling', 'With short selling', 'interpreter', 'latex', '
        FontSize', 15);
199 title('\textbf{Total return of all portfolios with respect to \$ \eta \$}', '
        interpreter', 'latex', 'FontSize', 15);
200 xlabel('\textbf{\$ \eta \$}', 'interpreter', 'latex', 'FontSize', 15);
201 ylabel('\textbf{Total return}', 'interpreter', 'latex', 'FontSize', 15);
202
203
204 %% Step 4: Improve your estimate
205 clear
206 clc
207 close all
208 load('C:\Users\marson\Dropbox\Convex Optimization\Project\data\stock_dataset.mat
        ');
209 pbarestimate = mean(past_returns, 1)';
210 S = cov(past_returns);
211 n = 12;
212 e = ones(12, 1);
213 lambda = 10^(-5);
214 cvx_begin
215     variable theta(n, n)
216     obj = 0;
217     for i = 1:n
218         for j = 1:n
219             if j == i
220                 continue
221             end
222             obj = obj + abs(theta(i, j));
223         end
224     end
225     %maximize(log_det(theta) - trace(S*theta) - lambda*(e'*theta*e - trace(theta)))

```

```

226     maximize(log_det(theta)-trace(S*theta)-lambda*obj)
227     subject to
228     theta >= 0;
229 cvx_end
230 sigmaestimate=inv(theta);
231 %
232 m=30;
233 eta=logspace(-2,3,m);
234 n=length(pbarestimate);
235 gamma=.5;
236 for i=1:m
237     cvx_begin
238     variable x(n)
239     maximize(pbarestimate'*x-eta(i)*x'*sigmaestimate*x)
240     subject to
241     sum(x)==1;
242     x>=0;
243     cvx_end
244     optimumValue(i)=cvx_optval;
245     X(i).x=x;
246     standardDeviation(i)=sqrt(x'*sigmaestimate*x);
247     meanReturn(i)=pbarestimate'*x;
248 end
249 for i=1:m
250     cvx_begin
251     variable x(n)
252     maximize(pbarestimate'*x-eta(i)*x'*S*x)
253     subject to
254     sum(x)==1;
255     x>=0;
256     cvx_end
257     optimumValue(i)=cvx_optval;
258     XSimple(i).x=x;
259     standardDeviationSimple(i)=sqrt(x'*S*x);
260     meanReturnSimple(i)=pbarestimate'*x;
261 end
262 c=[X(:).x];
263 cSimple=[XSimple(:).x];
264 for i=1:m
265     noShortReturns(:,:,i)=future_returns.*c(:,i)';
266     noShortReturnsSimple(:,:,i)=future_returns.*cSimple(:,i)';
267 end
268 cumNoShort=cumsum(noShortReturns,1);
269 cumNoShortSimple=cumsum(noShortReturnsSimple,1);
270 for i=1:n
271     figure('units','normalized','outerposition',[0 0 1 1])
272     semilogx(eta,squeeze(cumNoShort(end,i,:)),'LineWidth',3);
273     hold on
274     semilogx(eta,squeeze(cumNoShortSimple(end,i,:)),'LineWidth',3);
275     legend('Using new estimate','Simple previous estimate','interpreter','latex',
276           'FontSize',15,'Location','West');
276     grid on
277     txt='Portfolio number %d';
278     txt=sprintf(txt,i);
279     txt2='step4_portfolio%d';
280     txt2=sprintf(txt2,i);
281     txt=['\textbf{',txt,'}'];
282     title(txt,'interpreter','latex','FontSize',15);
283     xlabel('\textbf{\eta}','interpreter','latex','FontSize',15);
284     ylabel('\textbf{Portfolio return}','interpreter','latex','FontSize',15);
285     saveas(gca,txt2,'eps');

```

```

286 end
287 % Total return plot
288 figure('units','normalized','outerposition',[0 0 1 1])
289 totalReturn=cumsum(future_returns*c,1);
290 totalReturnSimple=cumsum(future_returns*cSimple,1);
291 semilogx(eta,totalReturn(end,:), 'LineWidth',3)
292 hold on
293 semilogx(eta,totalReturnSimple(end,:), 'LineWidth',3)
294 grid on
295 legend('Using new estimate','Simple previous estimate','interpreter','latex','
    FontSize',15);
296 title('\textbf{Total return of all portfolios with respect to $ \eta$', '
    interpreter','latex','FontSize',15);
297 xlabel('\textbf{$\eta$', 'interpreter','latex','FontSize',15);
298 ylabel('\textbf{Total return}', 'interpreter','latex','FontSize',15);
299
300
301 %% Step 5: Portfolio optimization with loss risk constraints
302
303 clear
304 clc
305 close all
306 load('C:\Users\marson\Dropbox\Convex Optimization\Project\data\stock_dataset.mat
    ');
307 pbarestimate=mean(past_returns,1)';
308 S=cov(past_returns);
309 n=12;
310 e=ones(12,1);
311 lambda=10^(-5);
312 cvx_begin
313     variable theta(n,n)
314     obj=0;
315     for i=1:n
316         for j=1:n
317             if j==i
318                 continue
319             end
320             obj=obj+abs(theta(i,j));
321         end
322     end
323     %maximize(log_det(theta)-trace(S*theta)-lambda*(e'*theta*e-trace(theta)))
324     maximize(log_det(theta)-trace(S*theta)-lambda*obj)
325     subject to
326         theta >= 0;
327 cvx_end
328 sigmaestimate=inv(theta);
329 m=30;
330 eta=logspace(-2,3,m);
331 n=length(pbarestimate);
332 gamma=.5;
333 q=20;
334 beta=logspace(-4,-1,q);
335 squareRootSigma=sqrtm(sigmaestimate);
336 %
337 for j=1:q
338     for i=1:m
339         cvx_begin
340             variable x(n)
341             maximize(pbarestimate'*x-eta(i)*x'*sigmaestimate*x)
342             subject to
343                 sum(x)==1;

```

```

344         x>=0;
345         pbarestimate'*x+norminv(beta(j))*norm(squareRootSigma*x)>-.5;
346     cvx_end
347     optimumValue(i)=cvx_optval;
348     X(i,j).x=x;
349     standardDeviation(i,j)=sqrt(x'*sigmaestimate*x);
350     meanReturn(i,j)=pbarestimate'*x;
351 end
352 end
353 %%
354 n1=1;
355 n2=10;
356 n3=20;
357 n4=30;
358 c1=[X(n1,:)'.x];
359 c2=[X(n2,:)'.x];
360 c3=[X(n3,:)'.x];
361 c4=[X(n4,:)'.x];
362 for i=1:n
363     txt2='Step5_portfolio%d';
364     txt2=sprintf(txt2,i);
365     figure('units','normalized','outerposition',[0 0 1 1])
366     subplot(2,2,1)
367     plot(beta,c1(i,:), 'LineWidth',3);
368     txt=['\textbf{Allocation of portfolio ', num2str(i), ' for $ \eta = $ ',
369         num2str(eta(n1)), ' }'];
369     title(txt, 'interpreter','latex','FontSize',15);
370     xlabel('\textbf{$\beta$}', 'interpreter','latex','FontSize',15);
371     ylabel('\textbf{Stock amount}', 'interpreter','latex','FontSize',15);
372     grid on
373     subplot(2,2,2)
374     plot(beta,c2(i,:), 'LineWidth',3);
375     txt=['\textbf{Allocation of portfolio ', num2str(i), ' for $ \eta = $ ',
376         num2str(eta(n2)), ' }'];
376     title(txt, 'interpreter','latex','FontSize',15);
377     xlabel('\textbf{$\beta$}', 'interpreter','latex','FontSize',15);
378     ylabel('\textbf{Stock amount}', 'interpreter','latex','FontSize',15);
379     grid on
380     subplot(2,2,3)
381     plot(beta,c3(i,:), 'LineWidth',3);
382     txt=['\textbf{Allocation of portfolio ', num2str(i), ' for $ \eta = $ ',
383         num2str(eta(n3)), ' }'];
383     title(txt, 'interpreter','latex','FontSize',15);
384     xlabel('\textbf{$\beta$}', 'interpreter','latex','FontSize',15);
385     ylabel('\textbf{Stock amount}', 'interpreter','latex','FontSize',15);
386     grid on
387     subplot(2,2,4)
388     plot(beta,c4(i,:), 'LineWidth',3);
389     txt=['\textbf{Allocation of portfolio ', num2str(i), ' for $ \eta = $ ',
390         num2str(eta(n4)), ' }'];
390     title(txt, 'interpreter','latex','FontSize',15);
391     xlabel('\textbf{$\beta$}', 'interpreter','latex','FontSize',15);
392     ylabel('\textbf{Stock amount}', 'interpreter','latex','FontSize',15);
393     grid on
394     saveas(gca,txt2,'epsc');
395 end
396 %%
397 return1=cumsum(future_returns*c1,1);
398 return2=cumsum(future_returns*c2,1);
399 return3=cumsum(future_returns*c3,1);
400 return4=cumsum(future_returns*c4,1);

```

```

401 figure('units','normalized','outerposition',[0 0 1 1])
402 subplot(2,2,1)
403 semilogx(beta,return1(end,:), 'LineWidth',3)
404 title(['\textbf{Cumulative total return with respect to $ \beta$ for $ \eta = ',
    ...,
405     num2str(eta(n1)), ' $}'], 'interpreter','latex','FontSize',15);
406 xlabel('\textbf{$\beta$}', 'interpreter','latex','FontSize',15);
407 ylabel('\textbf{Cumulative total return}', 'interpreter','latex','FontSize',15);
408 grid on
409 subplot(2,2,2)
410 semilogx(beta,return2(end,:), 'LineWidth',3)
411 title(['\textbf{Cumulative total return with respect to $ \beta$ for $ \eta = ',
    ...,
412     num2str(eta(n2)), ' $}'], 'interpreter','latex','FontSize',15);
413 xlabel('\textbf{$\beta$}', 'interpreter','latex','FontSize',15);
414 ylabel('\textbf{Cumulative total return}', 'interpreter','latex','FontSize',15);
415 grid on
416 subplot(2,2,3)
417 semilogx(beta,return3(end,:), 'LineWidth',3)
418 title(['\textbf{Cumulative total return with respect to $ \beta$ for $ \eta = ',
    ...,
419     num2str(eta(n3)), ' $}'], 'interpreter','latex','FontSize',15);
420 xlabel('\textbf{$\beta$}', 'interpreter','latex','FontSize',15);
421 ylabel('\textbf{Cumulative total return}', 'interpreter','latex','FontSize',15);
422 grid on
423 subplot(2,2,4)
424 semilogx(beta,return4(end,:), 'LineWidth',3)
425 grid on
426 title(['\textbf{Cumulative total return with respect to $ \beta$ for $ \eta = ',
    ...,
427     num2str(eta(n4)), ' $}'], 'interpreter','latex','FontSize',15);
428 xlabel('\textbf{$\beta$}', 'interpreter','latex','FontSize',15);
429 ylabel('\textbf{Cumulative total return}', 'interpreter','latex','FontSize',15);
430 grid on
431 %%
432 fr=cumsum(future_returns,1);
433 fr=fr(end,:);
434 c=zeros(n,m,q);
435 for i=1:n
436     for j=1:n
437         for k=1:q
438             c(i,j,k)=X(j,k).x(i);
439         end
440     end
441 end
442 for j=1:m
443     for k=1:q
444         Returns(j,k)=fr*squeeze(c(:,j,k));
445     end
446 end
447 figure('units','normalized','outerposition',[0 0 1 1])
448 [a,b]=meshgrid(beta,eta);
449 surf(a,b>Returns);
450 [etamax, betamax]=find(Returns==max(max>Returns),1);
451 x=X(etamax,betamax).x;
452 etamax=eta(etamax);
453 betamax=beta(betamax);
454 save('portfolio.mat','x','etamax','betamax','sigmaestimate','pbareestimate');
455
456
457 %% Step 6: maximum risk for the chosen portfolio

```

```
458 %clear
459 close all
460 clc
461 %load portfolio.mat
462 n=12;
463
464 cvx_begin
465     variable S(n,n)
466     maximize(x'*S*x)
467     subject to
468     S>=0;
469     diag(S)==diag(sigmaestimate);
470     for i=1:12
471         for j=1:12
472             if i==j
473                 continue
474             elseif sigmaestimate(i,j)>0
475                 S(i,j)>=0;
476             else
477                 S(i,j)<=0;
478             end
479         end
480     end
481
482 cvx_end
```