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IFAC PapersOnLine 52-5 (2019) 405–410

Sensitivity Analysis on the Sizing Parameters of a Series-Parallel HEV

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Abstract: As an alternative to power-split hybrid architectures, a simple series-parallel architecture named SPHEV 2 can be realized. In this paper, the sizing process of this architecture is briefly presented and a deeper analysis is made. Mathematical sensitivity analysis studies are conducted on the sizing parameters of the architecture in order to bring more understanding to the optimization results. Local and global sensitivities are performed to understand the influence of the sizing variables on the fuel consumption. An analysis of the sensitivities is also made.

Keywords: Sensitivity Analysis; Hybrid Vehicles; Modelling; Dynamic Programming; Genetic Algorithms.

1. INTRODUCTION

The automotive sector is undergoing significant changes into new forms of mobility. For instance, car manufacturers are proposing more and more electric and hybrid electric models on the market (Groupe PSA, 2016) (Renault-Nissan-Mitsubishi, 2017). These changes aim to meet the stringent regulations, the global fleet CO_2 targets and the clients' new needs and expectations.

Regarding Hybrid Electric Vehicle (HEV), different powertrain architectures exist, different components technologies can be selected, and different sizing can be made. The powertrain operation and fuel consumption on a selected driving cycle will depend on the architecture chosen, the components and their sizing, and lastly on the energy management during the vehicle operation.

Series, parallel and series-parallel (power-split and non-power-split) are the main categories of existing hybrid architectures. Series-parallel combine the advantages of series and parallel, but they can have a relatively costly design and complicated control (Chan, 2007). The simplest series-parallel architecture that can be realised is a configuration where the engine and the electric machines are mounted on a same shaft with clutches in between (Trigui, Vinot and Jeanneret, 2012). This powertrain was proven to be less efficient than the power-split ones (Vinot, 2016). Yet, with the addition of gears or gearbox and with an adequate sizing and control, this architecture can be improved.

In this framework of improving the efficiency of simple series-parallel hybrid powertrains, an architecture named SPHEV 2 was proposed and studied in (B. Kabalan, 2017). This architecture is presented below in *Fig.1*:

- ICE: Internal Combustion Engine
- EM1, EM2: Electric Machines
- C1, C2: Clutches
- G1, G2, G3: Reduction Gears
- FG: Final Gear

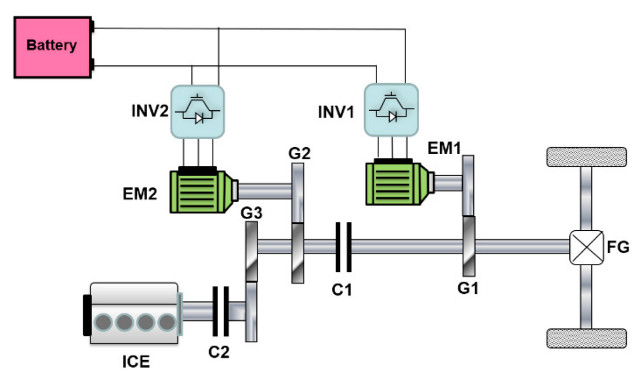

Fig.1. SPHEV 2 architecture

Based on the clutches' states, this architecture can operate in pure electric, series hybrid or parallel hybrid mode. In contrast, powersplit architecures that include a planetary gear (PG) operate always in a power split between both the series path and the parallel path. This continuous split of power allows a speed degree of freedom for the engine for more operation optimization, but involves a continuous use of the series path that is less efficient than the parallel one. This power split is not present in SPHEV 2.

An energetic model was developped using VEHLIB (Vinot *et al.*, 2008) to calculate the performance and fuel comsumption of SPHEV 2. This model has 8 sizing variables:

- Number of Battery Modules	- ICE gear (G3)
- Power of ICE	- EM1 gear (G1)
- Power of EM1	- EM2 gear (G2)
- Power of EM2	- Final Drive Ratio (FDR)

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2019.09.065

In previous work, these variables are optimized and SPHEV2 is compared to other architectures, including a reference power-split (Kabalan *et al.*, 2017). Many works have been conducted on sensitivity analysis using different approaches based on a quantitative method (Yanping, et al., 2011) or a qualitative method (Stroe, et al., 2017). In this paper, the authors make deeper analysis of this architecture. Mathematical sensitivity analysis studies are conducted to bring more understanding to the optimization results. Local and global sensitivities are performed to understand the influence of the sizing variables on the fuel consumption results. This can lead to dismiss some variables or to put more attention on others in future optimization work.

In the following section, the methods that can be generally used to perform local and global sensitivities on a system or a model are screened. In section 3, the model is presented with the methodology that was used for its optimization. In section 4, the sensitivity methods that are going to be applied on our model are selected and explained. Finally, the results are shown in section 5 with an analysis of the design parameters influence.

2. SENSITIVITY CALCULATION

This section presents the methods that are generally used to calculate the local and global sensitivities.

2.1 Local Sensitivity

The local sensitivity of n input variables $(x_1, \dots, x_n)$ on an output function f is the sensitivity of the input variables on the function at a point of study of these input variables. This point is noted $(\widetilde{x_1},\dots,\widetilde{x_n})$. For more understanding, one can refer to (Saltelli *et al.*, 2004).
The analytical method used to calculate this local sensitivity is presented hereafter (Yanping, et al., 2011).

- Derivative Formulas

They are commonly used in physics for sensitivity calculation.
The first formula is given with direct differential method and is expressed as follows:

$$S_{x_i} = f'(x_i)\,,\ i = 1..n \tag{1}$$

This needs the expression of the function or its approximation using the Response Surface. If S_{x_i} is high it means that the slope of the tangent is high and hence a small variation in the parameter x_i leads to a high variation in the output function.
The second formula uses an approximation of the derivative formula and is expressed as follows:

$$S_{x_i} = \frac{f(x_i+\delta x_i)-f(x_i)}{\delta x_i}\,, i = 1..n \tag{2}$$

However, variables' sensitivity should be normalized before comparison in order to have the same range of variation. Thus, a fair comparison needs a third formula of sensitivity that is expressed as follows:

$$S_{x_i} = \frac{[f(x_i+\delta x_i)-f(x_i)].x_i}{\delta x_i.f(x_i)}\ i = 1..n \tag{3}$$

The function f corresponds to the vehicle's fuel consumption of the driving cycle. Thus, there is no case of division by zero because the function is always strictly positive. Both formulas (2) and (3) provide the same ranks of variables' sensitivity at a point $(\widetilde{x_1},\dots,\widetilde{x_n})$ because the value of f at this point is the same when calculating sensitivity of variables.
Instance analysis shows that the sensitivity S_{x_i} calculated using this third formula could lead to good engineering understanding. In addition, in the case of this work, the fuel consumption is not expressed as analytical formula in terms of the input variables. The formula (3) is therefore the most relevant.

2.2 Global Sensitivity

It is the study of how the uncertainty in the model output (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input. The mainly used analytical methods are presented below. More information can be found in (Saltelli *et al.*, 2008).

- Linear Regression

In order to calculate global sensitivity, we use Taylor-Mac Laurin series. Three models are presented: Degree 2 (4), Degree 1 with interactions (5), and Degree 1 (6). The coefficients $(\beta_1, \dots, \beta_n)$ represent the sensitivity of the normalized variables $(x_1, \dots, x_n)$.

$$y = \beta_0 + \beta_1.x_1 + \beta_2.x_2 + \dots + \beta_n.x_n + \sum\beta_i.\beta_j.x_i.x_j + \sum\beta_i.{x_i}^2 \tag{4}$$

$$y = \beta_0 + \beta_1.x_1 + \beta_2.x_2 + \dots + \beta_n.x_n + \sum\beta_i.\beta_j.x_i.x_j \tag{5}$$

$$y = \beta_0 + \beta_1.x_1 + \beta_2.x_2 + \dots + \beta_n.x_n \tag{6}$$

The calculation of the coefficients $(\beta_1, \dots, \beta_n)$ is done by Least Squares Method (Saltelli *et al.*, 2008) with either analytical or with numerical method. In this work, we used Degree 1 model (6). The computation time of Degree 1 model is the least among the three models though its accuracy is the worst. The choice of combinations (points) is done with Monte Carlo Method. This method consists of generating all possible combinations of the n variables' value by mixing different parameter's value range.

Another method of choosing combinations (Goupy and Creighton, 2006) consists on using combinations with extreme values of each variable (after normalization). For n variables, there will be 2^n combinations. These combinations are used to determine the coefficients of the formulas above that correspond to variables' sensitivity.

3. MODELS AND OPTIMIZATION

When designing a hybrid vehicle powertrain, three interdependent dimensions should be taken into account. In fact, the powertrain architecture, the components choice and sizing, and the related energy management, strongly interfere in the design process (Silvas *et al.*, 2017). To address this problem, a bi-level framework (*Fig. 2*) was developed in

(Vinot, Reinbold and Trigui, 2015). Once the choice of the architecture is done, a genetic algorithm (NSGA-2, Deb et al. 2002) will act on different parameters of the vehicle. An explanation of the entire bi-level optimization can be found in (Kabalan *et al.*, 2017).

One of the optimization objectives of this algorithm is the fuel consumption. It is assessed on a known-in-advance driving cycle using optimal energy management (dynamic programming method in this paper) to guarantee the minimum fuel consumption while respecting a constraint on the state of charge variation. An energetic systemic model is used to assess, on a backward manner, the fuel consumption going upstream from wheel to engine and battery. The gears model consists of a constant ratio and efficiency, the engine modeled by a Brake Specific Fuel Consumption map (BSFC) (*Fig. 3*), the electrical machines by efficiency maps (*Fig. 3*) and the battery by an equivalent electrical circuit. This circuit consists of an open circuit voltage, an internal resistance, and a faradic efficiency. These three parameters depend on the battery state of charge, on the current, and on the temperature.

The model or system in this study is the entire vehicle model which has eight optimization variables: the Number of Battery Modules, the powers of engine and electrical machines, and the gear ratios (final drive FG, engine gear G3, EM1 gear G1, and EM2 gear G2). The sizes of the ICE and EMs change according to scaling factors applied on the torque. For EMs, the maximum torque, the losses, and the weight are multiplied by a scaling factor k_{EM}. The inertia is multiplied by $(k_{EM})^{5/3}$. For the ICE, the maximum torque, the fuel consumption, and the weight are multiplied by k_{ICE}. The inertia is multiplied by $(k_{ICE})^{5/3}$. The shape of the BSFC maps does not change; it is only the torque scale that is changed. Such scaling technique is a classical method used for the components sizing in the context of powertrain optimization (Vinot, 2016). For more accurate sizing, large experimental data of component maps is needed. Another alternative is to use precise models which takes into account the size of the component (Reinbold *et al.*, 2016; Zhao, 2017). A compromise is to be done between the accuracy of the models and the calculation time.

4. APPLICATION TO OUR MODEL

In order to explain the optimization results obtained, sensitivity analysis is conducted. The input variables are the eight sizing variables of SPHEV2. The output function is the fuel consumption of SPHEV 2, calculated by simulating the vehicle model in mixed driving conditions. Three representative cycles for different driving conditions are used: urban, rural, and highway. Weighting factors multiplies the fuel consumption on each driving condition:

$$FC = \alpha.FC_{urban} + \beta.FC_{road} + \gamma.FC_{highway}$$

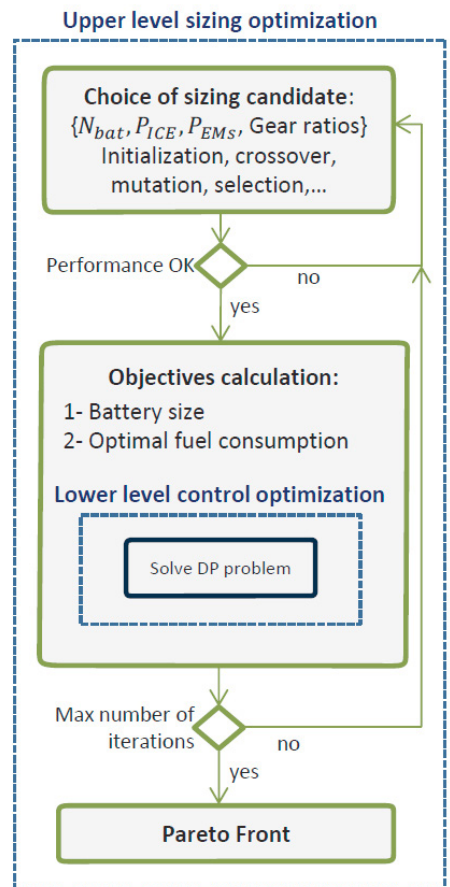

Fig. 2. Bi-level optimization framework

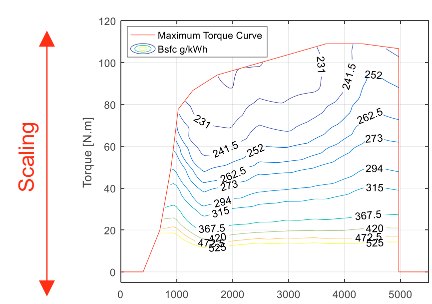

Fig. 3. ICE reference map before scaling

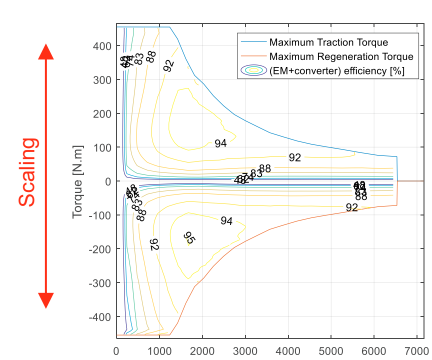

Fig. 4. EM Reference map before scaling

Where the α, β, and γ are coefficients calculated from the mean traveled distance in urban, rural road, and highway conditions by the French population. These values are respectively 0.4, 0.3, and 0.3 (Eurostat, no date).

For the calculation of the local sensitivity, the derivative formula requires the evaluation of 20 combinations for each variable. In fact, to calculate local sensitivity of each variable x_i, the step δx_i is taken equal to a variation from -10% to 10% of $\tilde{x_i}$ (with a step of 1%), $\tilde{x_i}$ being the local value of x_i. Local sensitivity is calculated as the mean (in absolute values) of the sensitivities obtained for each value of δx_i.

For the calculation of the global sensitivity, a choice is to be made for the sizing combinations that need to be evaluated. This can be usually done using Design of Experiments (DOE). In the case of this application, the design space is constrained by the performance requirements. For instance, if the combinations of extreme sizing values are considered ($2^8 = 256$ combinations), only 9 combinations among the 256 are able to pass the performance requirement constraints (maximum speed, acceleration time from 0 to 100km/h and overtaking capability at 80km/h) and to be evaluated.

This is why the choice of combinations is done using Monte-Carlo method instead of DOE. First, the variables are normalized, then the $[0,1]$ interval of each variable is divided in three intervals $[0,\frac{1}{3}]$, $[\frac{1}{3},\frac{2}{3}]$, and $[\frac{2}{3},1]$. A uniform probability is used to choose a value in each of the three intervals. Then, all possible combinations that mix different parameter's value range (low, medium, high) are generated. Each of the 8 variables can take three possible values; hence, $3^8 = 6561$ combinations can be generated. Not knowing a priori the design space, the performance constraints are checked for all those combinations. The fuel consumption is calculated only for the feasible combinations (in this case 842 out of 6561). Linear Regression's coefficients are then calculated either numerically or analytically.

5. RESULTS AND ANALYSIS

5.1 Sensitivity results

The local sensitivity is calculated on highway cycle at two points of sizing that correspond to the minimum and maximum of fuel consumption in the Pareto front (*Fig. 5*) of SPHEV 2 (Kabalan *et al.*, 2017). This Pareto front corresponds to an optimization on mixed driving condition.

Fig. 6 and Fig. 7 show the results of sensitivity for the 8 variables at the 2 points shown in red arrow in *Fig. 5*.

The global sensitivity of the variables is calculated on highway cycle using the linear regression method. *Fig. 8* shows the influence and the sign of the influence of the 8 variables. A negative sensitivity means that the consumption and the variable have an opposite sign of variation. The Final Drive Ratio and the gear for EM1 appear to be the 2 most significant variables.

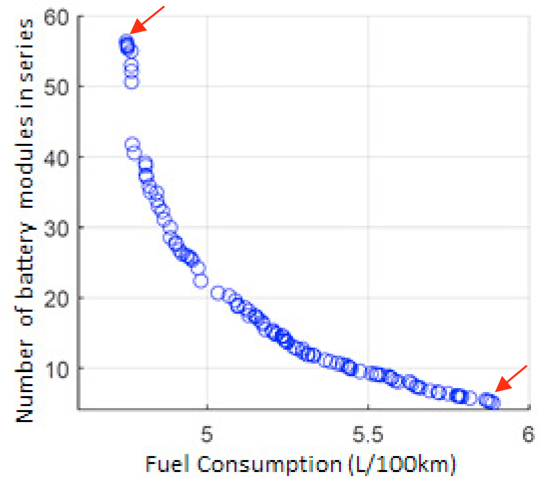

Fig. 5. Pareto front of SPHEV2

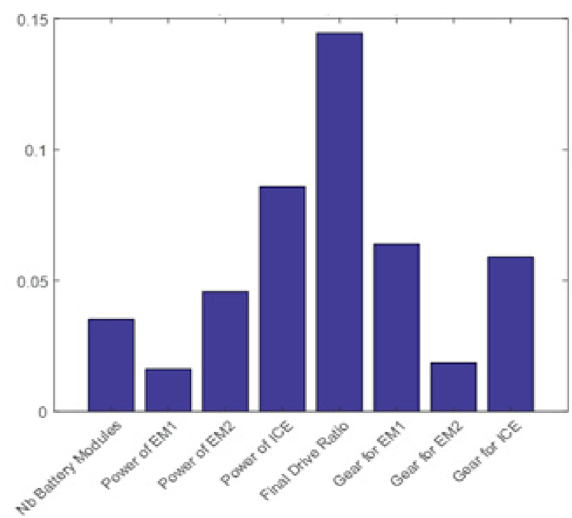

Fig. 6. Local sensitivity of consumption at the point of minimum consumption

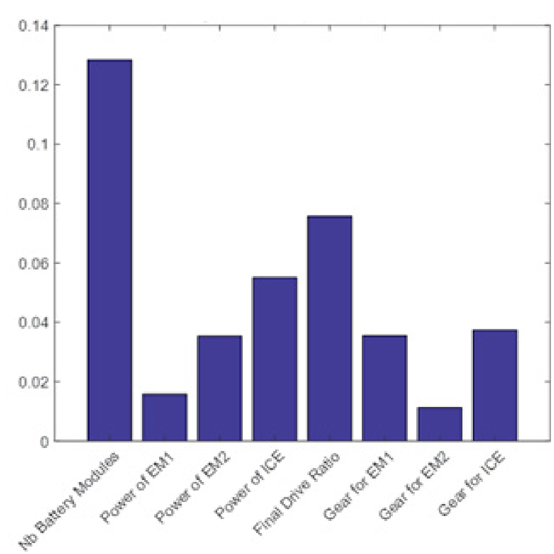

Fig. 7. Local sensitivity of consumption at the point of maximum consumption

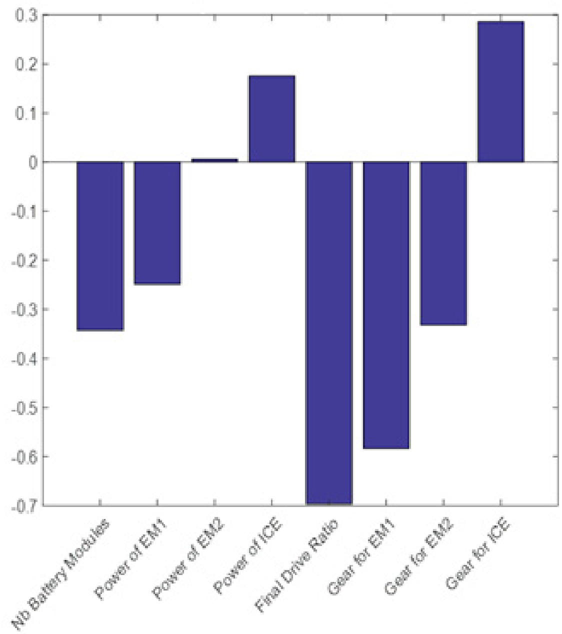

Fig. 8. Global sensitivity of consumption

5.2 *Discussion for each variable*

- **Number of Battery Modules**

This variable appears to be very influential at the point of maximum of consumption (minimum Number of Battery Modules) and less influential at the point of minimum of consumption (maximum Number of Battery Modules) Globally, a middle impact is depicted.
For a small number of battery modules, the size of the battery does not allow to recover all the braking energy and constraint the operation in electric mode. Therefore, increasing the Number of Battery Modules here will have an important influence on the fuel consumption.
On the other hand, for high Number of Battery Modules, the electric operations of the powertrain are not affected by the size of the battery pack.
Since the size of the battery greatly influences the price of the powertrain, a trade off needs to be find between cost and fuel consumption improvement.

- **The final drive ratio**

It is the most sensitive variable locally and globally. This is due to the fact that it is linked to all other components thus a variation in final drive ratio leads to speed variation and will affect all other components' operating points. Globally, an increase in final drive ratio leads to a decrease in fuel consumption.

- **The power of ICE**

It is very sensitive locally and globally. In fact, a variation in the power of ICE leads to a change of the BSFC areas in which the engine operating points are located. Similarly, a variation in the gear of ICE leads to a change in the ICE operating points, what explains the sensitivity of ICE gear to fuel consumption locally and globally.

- **The powers and gears of the electrical machines**

Their impacts on fuel consumption is different from local to global study. As for the ICE, those variables have an impact on the location of the operating points inside the efficiency maps. EM1 power and gear are more sensitive than those of EM2, because EM1 is more involved in the electric mode than EM2.

5.3 *Analysis on powertrain operation*

For a better understanding of the influence of these variables on the fuel consumption, a closer look should be made on the system operating modes and the global efficiencies of the components. Three modes are present:

- Pure Electric: The components involved are EM1 only, EM2 only or EM1 and EM2.
- Series: ICE and EM2 are used as electric generator, only EM1 provides mechanical power to the wheels.
- Parallel: ICE is mechanically connected to the wheels. EM1 and/or EM2 are also connected mechanically to provide or absorb power.

Table 1 presents the split between these modes and the global efficiencies of the EMs when the variables are changed. A reference sizing is shown in the first column ('Sizing of Minimum Fuel Consumption'). It corresponds to the minimum fuel consumption point (Maximum Number of Battery Modules). The other columns correspond to the component sizing where only one variable at a time is changed compared to reference one. For example, the column 'FDR' is the sizing similar to the reference, with only variation on the FDR.

Table 1. EMs losses and time/energy sharing between modes for different sizings

		Sizing of Minimum of Fuel Consumption	FDR	Power of EM1	Power of EM2	Power of ICE
%Time %Energy	Electric	29.09% 3.13%	27.78% 2.56%	29.37% 3.29%	29% 3.07%	26.38% 2%
	Series	1.31% 1.34%	1.68% 1.72%	1.12% 1.18%	1.22% 1.18%	1.96% 1.8%
	Parallel	69.6% 95.53%	70.53% 95.73%	69.5% 95.53%	69.78% 95.75%	71.66% 96.18%
EM1 Losses (Wh/km) Average Efficiency		11.74 78.64%	11.58 79.64%	10.9 79.95%	11.7 78.67%	11.78% 79.21%
EM2 Losses (Wh/km) Average Efficiency		11.72 84.35%	1.8 83.85%	1.77 82.9%	1.89 84.55%	1.89 85.38%

		Gear of EM1	Gear of EM2	Gear of ICE	Battery
%Time %Energy	Electric	29.09% 3.13%	29% 3.1%	27.78% 2.55%	29.65% 3.41%
	Series	1.4% 1.48%	1.12% 1.18%	1.68% 1.72%	1.5% 1.69%
	Parallel	69.5% 95.39%	69.88% 95.72%	70.53% 95.73%	68.85% 94.9%
EM1 Losses (Wh/km) Average Efficiency		11.92 78.33%	11.63 78.67%	11.8 79.27%	11.72% 78.46%
EM2 Losses (Wh/km) Average Efficiency		1.76 84.81%	1.91 83.61%	1.797 84.97%	1.75 84.7%

In the rows, the percentage of time and consumed energy in each of the three modes (Electric, Series, and Parallel) are shown, in addition to the electrical losses and average efficiency of both electrical machines over the entire driving cycle. For example, the column 'FDR' shows that 1.68% of the time is spent in the 'Series' mode. This corresponds to 1.72% of the global traction energy transferred to the wheels during the driving cycle. The total losses of EM1 on all the driving cycle are 11.58 Wh/km and its average efficiency is 79.64%.

The red cells correspond to the noticeable variation from the reference. It shows that the final drive ratio and the power of ICE are the variables that have important impacts on electrical losses (11.58 vs 11.74 and 1.89 vs 1.72) and on time and energy sharing between different operating modes (power of ICE increases the series mode from 1.31% to 1.96% of the time). This can be explained by the fact that the change in the power of ICE results in more efficient series operation (improvement of mean BSFC in series).

6. CONCLUSIONS

Through a sensitivity analysis conducted on the sizing variables of the SPHEV 2 architecture, this work allows more understanding of the optimization results previously done on this architecture. Moreover, this paper allows the identification of the most influential variables on the fuel consumption of SPHEV 2, like the final drive ratio for instance. Such information can be used later on to exclude some variables from the optimization for a gain in optimization time, or to put more efforts on other variables because they are more influential. This work can be extended to any other architecture. Future work will also include conducting such sensitivity analysis on the energy management variables that are decided by Dynamic Programming in the case of this work. This can give more understanding to the decisions made by Dynamic Programming and can also lead to a detection of the most influential variables that should be given more attention in any online simplified energy management technique.

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