EEC 332 - Analog Integrated Circuits

Lecture 5
MOSFET Models, Basic Configurations and Current Mirrors

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1 MOSFET small signal model

ignoring channel length modulation ($\lambda = 0$), therefore :

$$i_D = \frac{1}{2} k'_n \left(\frac{W}{L}\right) (v_{GS} - V_{tn})^2 \qquad (1)$$

the following figure presents a graphical interpretation of the small-signal operation of the MOSFET amplifier.

Note that g_m is equal to the slope of the $i_D - v_{GS}$ characteristic at the bias point

$$g_m \equiv \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{v_{GS}=v_{GS}} \qquad (2)$$

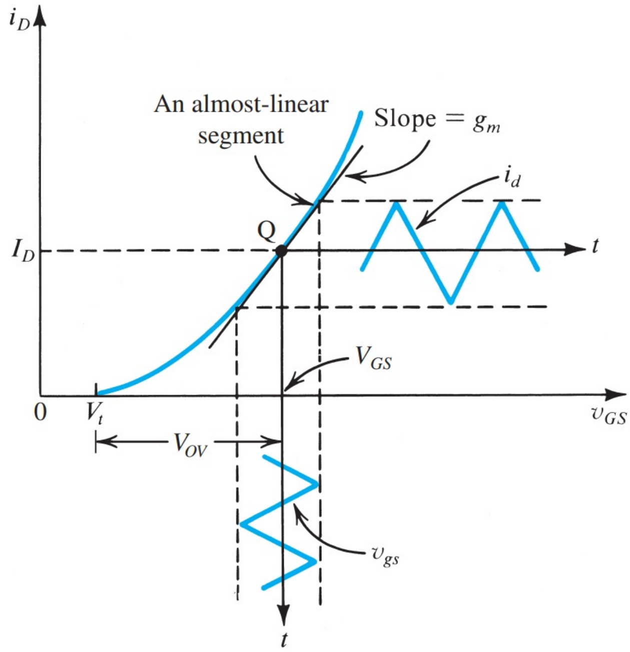


Figure 1: Small-signal operation of the MOSFET amplifier.

2 Small-Signal Equivalent-Circuit Models

From a signal point of view, the MOSFET behaves as a voltage-controlled current source. It accepts a signal v_{gs} between its gate and source and provides a current $g_m v_{gs}$ at the drain terminal.

2.1 The π Equivalent-Circuit Model

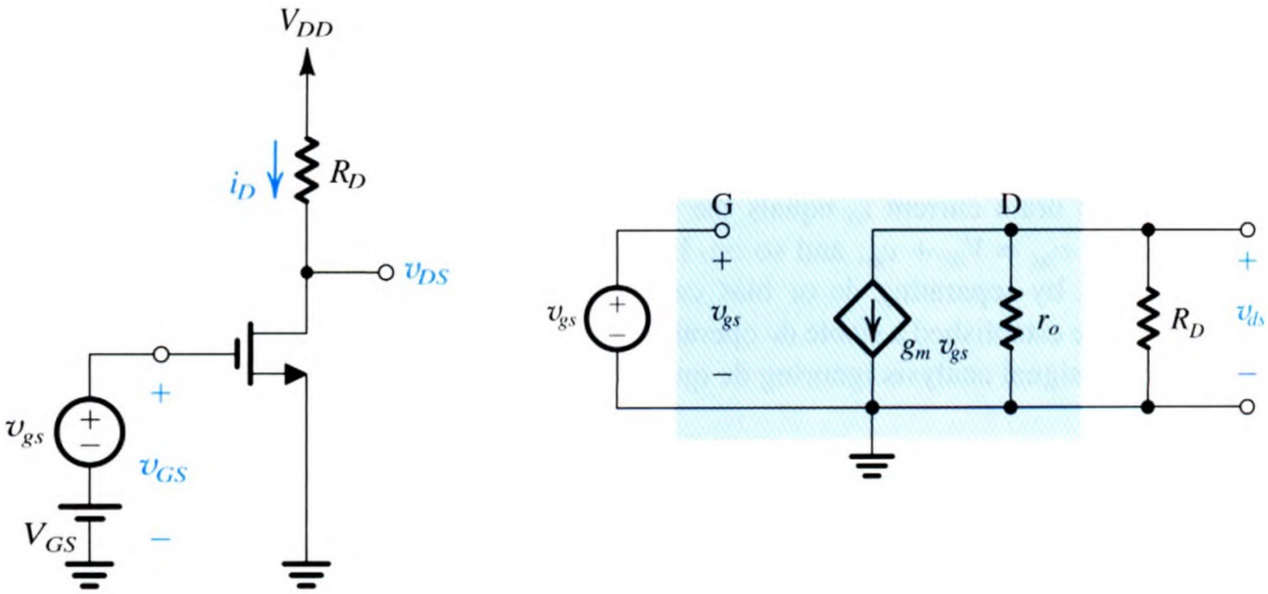


Figure 2: Small-signal equivalent $\boldsymbol{\pi}$ model .

3 The T Equivalent-Circuit Model

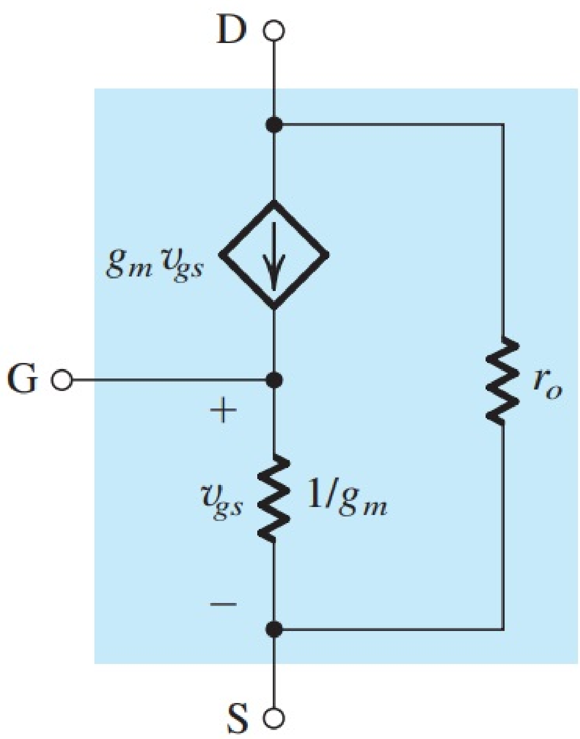


Figure 3: Small-signal equivalent $\boldsymbol{T}$ model .

4 The Three Basic Configurations of MOSFET Circuits

There are three basic configurations for connecting a MOSFET as an amplifier. Each of these configurations is obtained by connecting one of the device terminals to ground, thus creating a two-port network with the grounded terminal being common to the input and output ports.

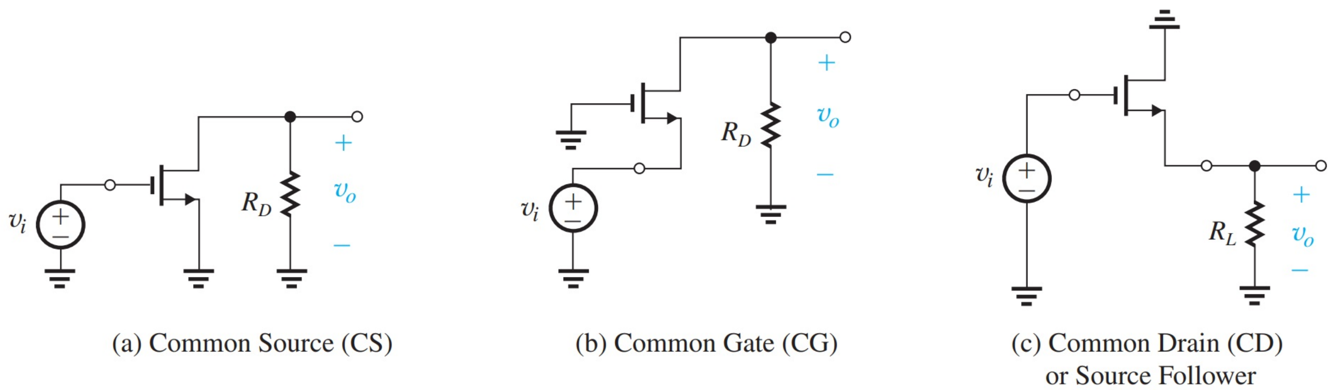
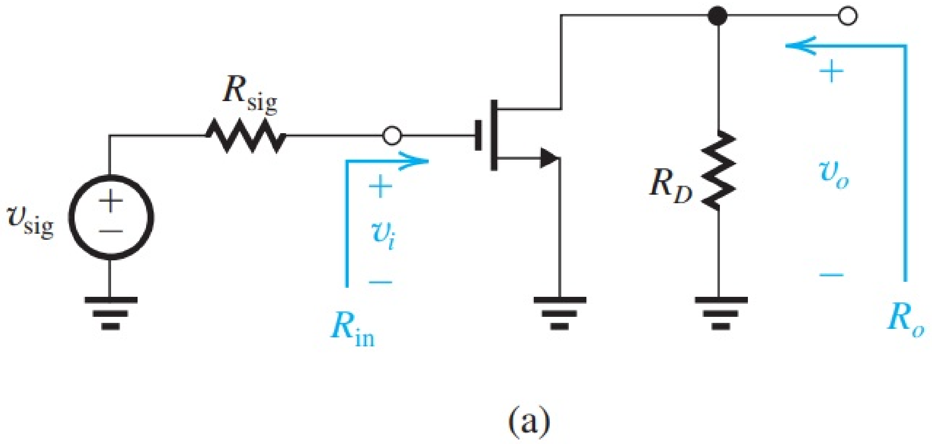


Figure 4: The basic configurations of transistor amplifiers

4.1 The Common-Source (CS) Amplifiers



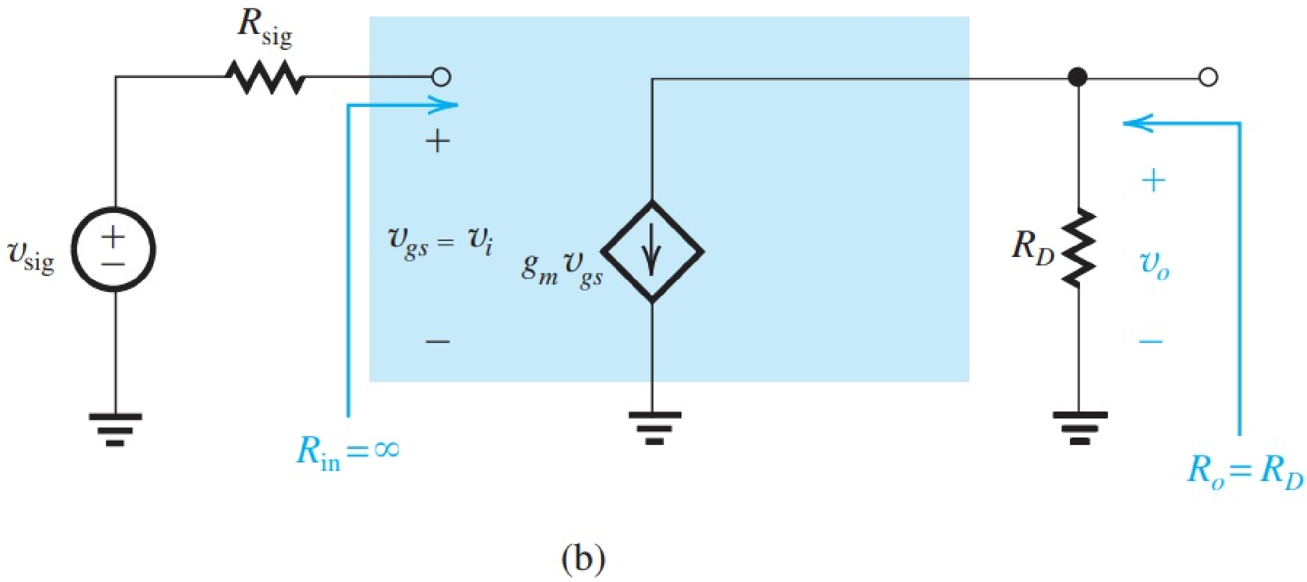


Figure 5: (a) Common-source amplifier fed with a signal $\boldsymbol{v_{sig}}$ from a generator with a resistance $\boldsymbol{R_{sig}}$ (b) The common-source amplifier with the MOSFET replaced with its $\boldsymbol{\pi}$ model.

from the circuit model:

$$R_{\text{in}} = \infty \tag{3}$$

$$v_{\text{sig}} = v_i = v_{gs} \tag{4}$$

$$v_o = -g_m v_{gs} R_D \tag{5}$$

$$A_{vo} \equiv \frac{v_o}{v_i} = \frac{-g_m v_{gs} R_D}{v_{gs}} = -g_m R_D \tag{6}$$

$$R_o = R_D \tag{7}$$

If a load resistance R_L is connected across R_D, the voltage gain A_v can be obtained from

$$A_v = A_{vo}\frac{R_L}{R_L + R_o} = -g_m R_D \frac{R_L}{R_L + R_o} = -g_m\left(R_D \| R_L\right) \tag{8}$$

since $R_{in} = \infty$ and thus $v_i = v_{sig}$, the overall voltage gain G_v is equal to A_v.

$$G_v \equiv \frac{v_o}{v_{\text{sig}}} = -g_m\left(R_D \| R_L\right) \tag{9}$$

4.2 The Common-Gate (CG) Amplifiers

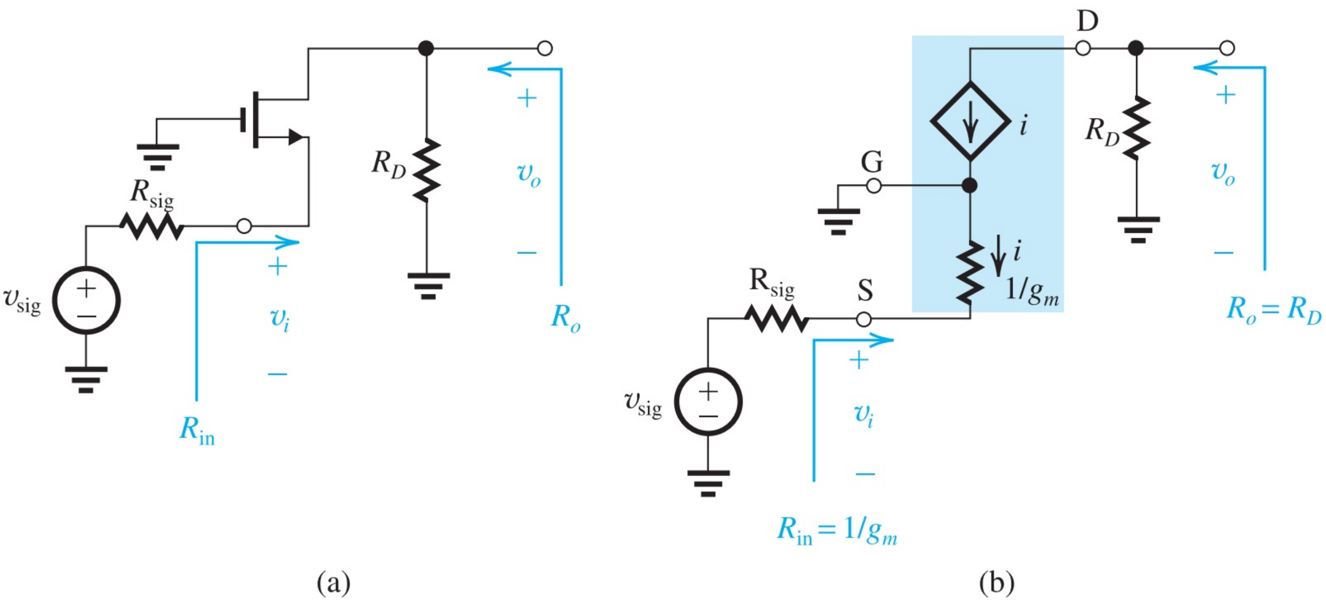


Figure 6: (a) Common-gate amplifier (b) Equivalent circuit of the common-gate replaced with its $\boldsymbol{T}$ model.

$$R_{\text{in}} = \frac{1}{g_m} \tag{10}$$

$$v_o = -iR_D \tag{11}$$

$$i = -\frac{v_i}{1/g_m} \tag{12}$$

$$v_i = -i \times 1/g_m \tag{13}$$

$$A_{vo} \equiv \frac{v_o}{v_i} = \frac{-iR_D}{-i \times 1/g_m} = g_m R_D \tag{14}$$

$$R_o = R_D \tag{15}$$

$$\frac{v_i}{v_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} = \frac{1/g_m}{1/g_m + R_{\text{sig}}} \tag{16}$$

$$G_v = \frac{v_o}{v_{\text{sig}}} = \frac{v_i}{v_{\text{sig}}} \times \frac{v_o}{v_i} \tag{17}$$

In case of resistance R_L connected at the output

$$\frac{v_o}{v_i} = g_m \left(R_D \| R_L\right) \tag{18}$$

$$G_v = \frac{1/g_m}{R_{\text{sig}} + 1/g_m} \left[g_m \left(R_D \| R_L\right)\right] \tag{19}$$

$$\therefore G_v = \frac{R_D \| R_L}{R_{\text{sig}} + 1/g_m} \tag{20}$$

$$\tag{21}$$

The Need for Voltage Buffers

An amplifier can be used to connect a signal source to a load resistance, preventing severe signal attenuation. A unity-gain buffer amplifier with high input resistance and low output resistance can be used for this purpose, as it allows almost all of the signal to appear at the input and a significant portion of the signal to appear across the load. The source follower can be used to implement such an amplifier.

in the following figure , notice how without a voltage buffer $v_o \approx 1$ mV, after using the voltage buffer, $v_o \approx 0.9$ V

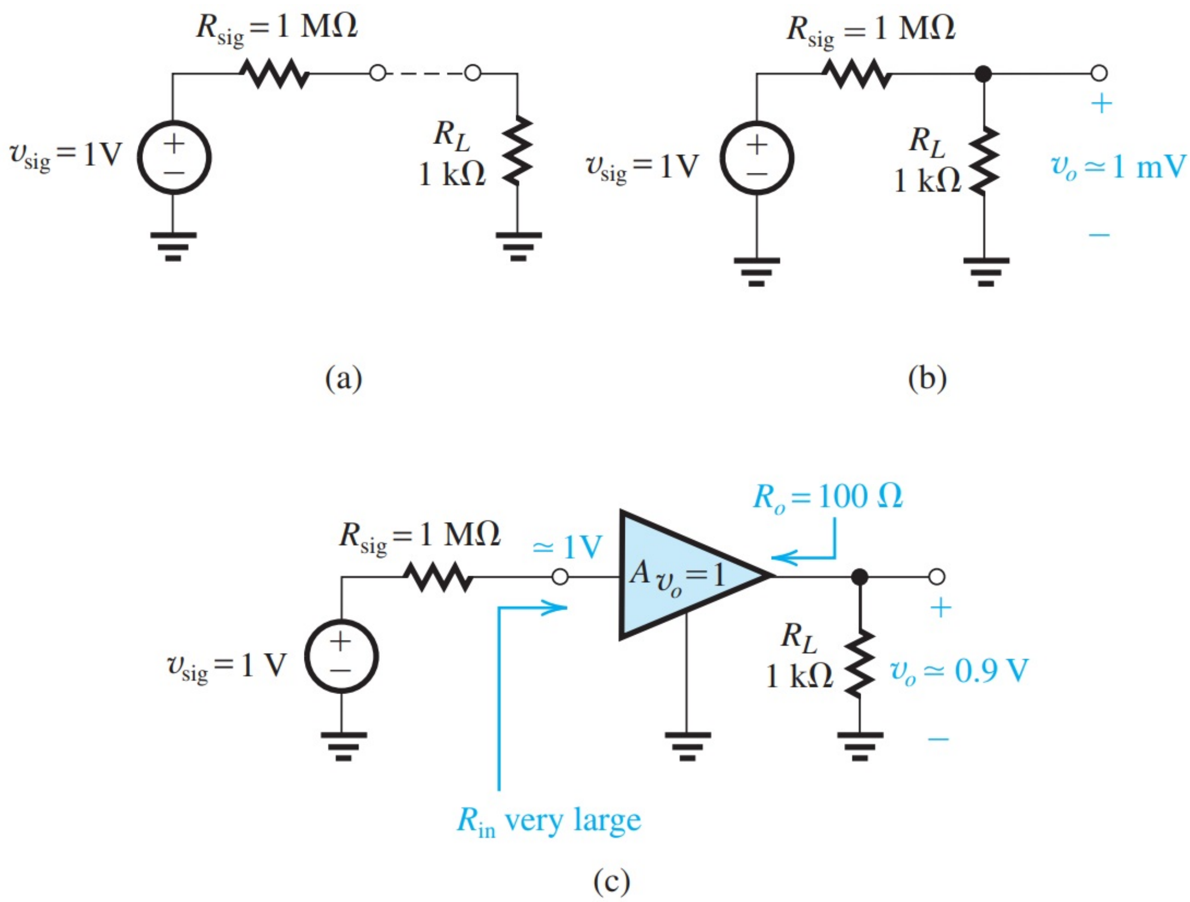
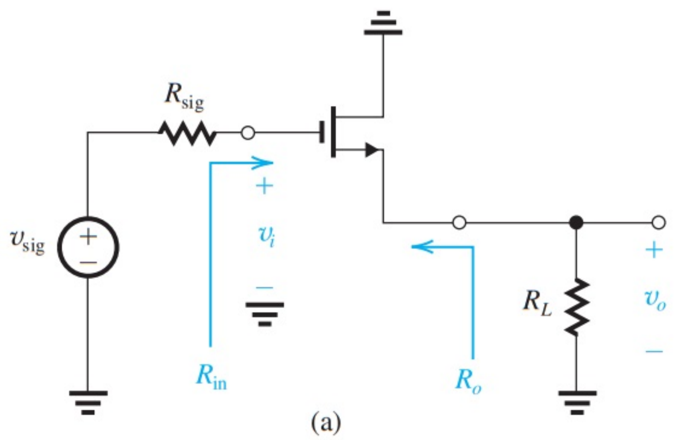


Figure 7: Illustrating the need for a unity-gain voltage buffer amplifier.

4.3 The Common-Drain (CD) Amplifiers (Source Follower)



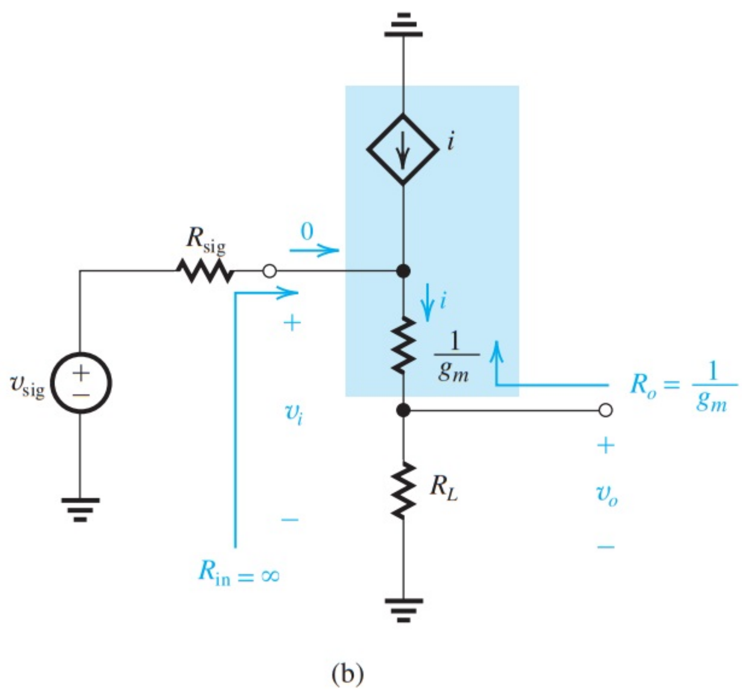


Figure 8: (a) Common-drain amplifier or source follower (b) Equivalent circuit of the source follower obtained by replacing the MOSFET with its $\boldsymbol{T}$ model.

$$\boldsymbol{R_{\text{in}} = \infty \quad (\text{input current} = 0)} \tag{22}$$

$$\boldsymbol{A_v \equiv \frac{v_o}{v_i} = \frac{R_L}{R_L + 1/g_m}} \tag{23}$$

Setting $R_L = \infty$, we obtain :

$$\boldsymbol{A_{vo} = 1} \tag{24}$$

$$\boldsymbol{R_o = 1/g_m} \tag{25}$$

since $\boldsymbol{R_{in} = \infty}$ and thus $\boldsymbol{v_i = v_{sig}}$, the overall voltage gain $\boldsymbol{G_v}$ is equal to $\boldsymbol{A_v}$.

$$\boldsymbol{G_v = A_v = \frac{R_L}{R_L + 1/g_m}} \tag{26}$$

5 The Basic MOSFET Current Source

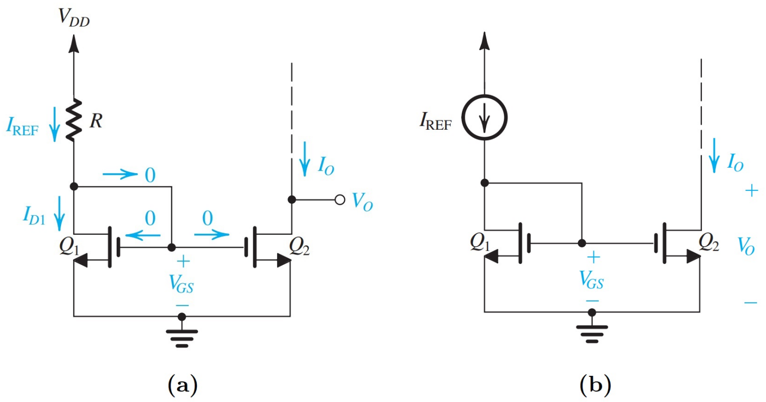


Figure 9: (a) Circuit for a basic MOSFET constant current source ; (b) Basic MOSFET current mirror.

the previous figure shows the circuit of a simple MOS constant-current source. The heart of the circuit is transistor Q_1 , the drain of which is shorted to its gate[1], thereby forcing it to operate in the saturation mode with

$$I_{D1} = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_1 (V_{GS} - V_{tn})^2 \tag{27}$$

where we have neglected channel-length modulation. The drain current of is supplied by V_{DD} through resistor R, since the gate currents are zero,

$$I_{D1} = I_{\text{REF}} = \frac{V_{DD} - V_{GS}}{R} \tag{28}$$

Similarly

$$I_O = I_{D2} = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_2 (V_{GS} - V_{tn})^2 \tag{29}$$

Equations (28) and (29) enable us to relate the output current I_O to the reference current I_{REF} as follows:

$$\frac{I_O}{I_{\text{REF}}} = \frac{(W/L)_2}{(W/L)_1} \tag{30}$$

[1]Such a transistor is said to be *diode connected.*

6 The MOS Current Mirror

To ensure that Q_2 is saturated, the circuit to which the drain of Q_2 is to be connected must establish a drain voltage V_o that satisfies the relationship

$$V_O \geq V_{GS} - V_{tn} \tag{31}$$

or, equivalently, in terms of the overdrive voltage V_{OV} of Q_1 and Q_2,

$$V_O \geq V_{OV} \tag{32}$$

Although thus far neglected, channel-length modulation can have a significant effect on the operation of the current mirror. For the mirror circuit, we see that the output current will be at its nominal value of

$$I_O|_{\text{nominal}} = \frac{(W/L)_2}{(W/L)_1} I_{\text{REF}} \tag{33}$$

at the value of V_O that makes Q_1 and Q_2 have the same V_{DS}; that is, $V_O = V_{GS}$. As V_O deviates from this value, I_O will deviate from the nominal value by ΔI_O,

$$\Delta I_o = \frac{\Delta V_O}{r_{o2}} = \frac{V_o - V_{GS}}{r_{o2}} \tag{34}$$

where r_{o2} is the output resistance of Q_2,

$$r_{o2} = \frac{V_{A2}}{I_O|_{\text{nominal}}} \tag{35}$$

and where V_{A2} is the Early voltage of Q_2. Equations (33), (34), and (35) can be used to find I_O at an arbitrary V_O (that is greater than V_{OV}) as

$$I_O = \frac{(W/L)_2}{(W/L)_1} I_{\text{REF}} \left[1 + \frac{V_O - V_{GS}}{V_{A2}}\right]$$

This analysis is illustrated graphically in the following figure , which shows I_O versus V_O. Note that this is the $i_D - v_{DS}$ characteristic curve of Q_2 that corresponds to $v_{GS2} = V_{GS}$. Finally, observe from Eq. 35 that the output resistance of the mirror, r_{O_2}, is proportional to the Early voltage of Q_2, V_{A2}. Since for a given process technology, V_{A2} is proportional to the channel length of Q_2, to obtain a high output resistance, we normally give the mirror transistor Q_2 a relatively long channel.

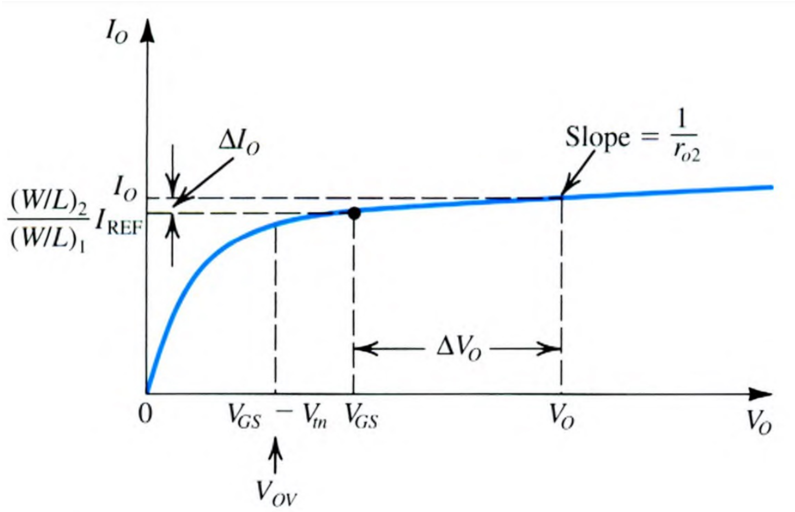


Figure 10: Output characteristic of the current mirror

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1. Q **Given $V_{DD} = 3$ V and using $I_{\text{REF}} = 100\mu$A, design the current source circuit to obtain an output current whose nominal value is 100μA. Find R if Q_1 and Q_2 are matched and have channel lengths of 1μm, channel widths of 10μm, $V_t = 0.7$ V, and $k'_n = 200\mu\text{A/V}^2$. What is the lowest possible value of V_o ? Assuming that for this process technology, the Early voltage $V'_A = 20$ V/μm, find the output resistance of the current source. Also, find the change in output current resulting from a $+1 - V$ change in V_o.**

1. A

$$I_{D1} = I_{\text{REF}} = \frac{1}{2} k'_n \left(\frac{W}{L}\right)_1 V_{OV}^2$$

$$100 = \frac{1}{2} \times 200 \times 10 V_{OV}^2$$

Thus,

$$V_{oV} = 0.316 \text{ V}$$

and

$$V_{GS} = V_t + V_{OV} = 0.7 + .316 \simeq 1 \text{ V}$$

$$R = \frac{V_{DD} - V_{GS}}{I_{\text{REF}}} = \frac{3 - 1}{0.1 \text{ mA}} = 20\text{k}\Omega$$

$$V_{O\,\min} = V_{OV} \simeq 0.3 \text{ V}$$

Now we can find the output resistance of the current source, r_{o2}. For the transistors

used, $L = 1\mu$m, thus,

$$V_A = 20 \times 1 = 20 \text{ V}$$

$$r_{o2} = \frac{20 \text{ V}}{100\mu\text{A}} = 0.2\text{M}\Omega$$

The output current will be 100μA at $V_O = V_{GS} = 1$ V. If V_O changes by +1 V, the corresponding change in I_O will be

$$\Delta I_O = \frac{\Delta V_O}{r_{o2}} = \frac{1 \text{ V}}{0.2\text{M}\Omega} = 5\mu\text{A}$$