EEC 332 - Analog Integrated Circuits

Lecture 2
Cascaded Amplifiers

Taha Ahmed

1 Cascaded Amplifier

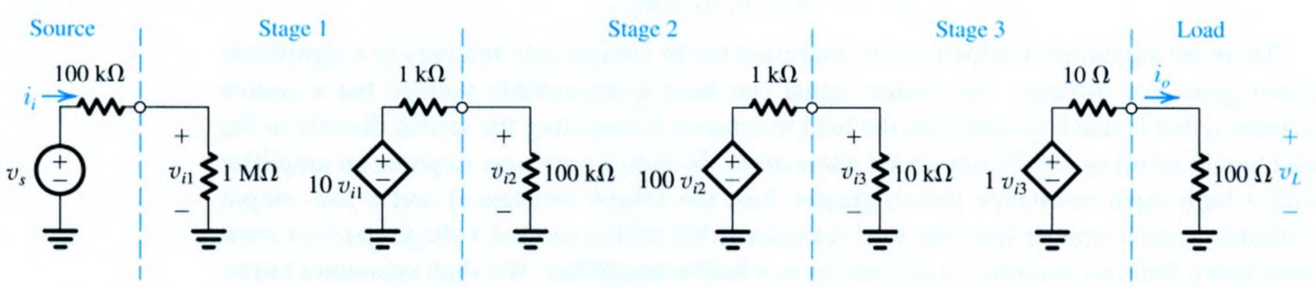
To meet given amplifier specifications, we often need to design the amplifier as a cascade of two or more stages. For instance, in order to provide the overall amplifier with a large input resistance, the first stage is usually required to have a large input resistance. Also, in order to equip the overall amplifier with a low output resistance, the final stage in the cascade is usually designed to have a low output resistance. To illustrate the analysis and design of cascaded amplifiers, we consider a practical example.

The overall gain of an amplifier of n stages :

$$A_v = A_{v1} \times A_{v2} \times A_{v3} \times \cdots \times A_{vn}$$

---

1. Q The following figure depicts an amplifier composed of a cascade of three stages. The amplifier is fed by a signal source with a source resistance of 100 kΩ and delivers its output into a load resistance of 100 Ω. The first stage has a relatively high input resistance and a modest gain factor of 10. The second stage has a higher gain factor but lower input resistance. Finally, the last, or output, stage has unity gain but a low output resistance.



Find:

1) the overall voltage gain i.e. $\dfrac{v_L}{v_s}$

2) the current gain

3) the power gain.

1. A

1) First gain A_{v1}

$$A_{v1} \equiv \frac{v_{i2}}{v_{i1}} = 10 \frac{100\text{k}\Omega}{100\text{k}\Omega + 1\text{k}\Omega} = 9.9 \text{ V/V}$$

Second gain A_{v2}

$$A_{v2} \equiv \frac{v_{i3}}{v_{i2}} = 100 \frac{10\text{k}\Omega}{10\text{k}\Omega + 1\text{k}\Omega} = 90.9 \text{ V/V}$$

Third gain A_{v3}

$$A_{v3} \equiv \frac{v_L}{v_{i3}} = 1 \frac{100\Omega}{100\Omega + 10\Omega} = 0.909 \text{ V/V}$$

The total gain of the three stages in cascade can now be found from

$$A_v \equiv \frac{v_L}{v_{i1}} = A_{v1} A_{v2} A_{v3} = 818 \text{ V/V}$$

In decibels

$$A_{v\,(\text{dB})} = 20 \log(818) = 58.26 \text{ dB}$$

To find the voltage gain from source to load, we multiply A_v by the factor representing the loss of gain at the input; that is,

$$\frac{v_L}{v_s} = \frac{v_L}{v_{i1}}\frac{v_{i1}}{v_s} = A_v\frac{v_{i1}}{v_s}$$
$$= 818 \times \frac{1\text{M}\Omega}{1\text{M}\Omega + 100\text{k}\Omega}$$
$$= 818 \times 0.909 = 743.6 \text{ V/V}$$

In decibels

$$\frac{v_L}{v_s} = 20\log(743.6) = 57.43 \text{ dB}$$

2) The current gain is found as follows:

$$A_i \equiv \frac{i_o}{i_i} = \frac{v_L/100\Omega}{v_{i1}/1\text{M}\Omega}$$
$$= 10^4 \times A_v = 8.18 \times 10^6 \text{ A/A}$$

In decibels

$$A_{i\,(\text{dB})} = 20\log(8.18 \times 10^6) = 138.25 \text{ dB}$$

3) The power gain

$$A_p \equiv \frac{P_L}{P_I} = \frac{v_L i_o}{v_{i1} i_i}$$
$$= A_v A_i = 818 \times 8.18 \times 10^6 = 6.69 \times 10^9 \text{ W/W}$$

$$A_{P\,(\mathrm{dB})} = 10\log(6.69\times 10^9) = 98.25\ \mathrm{dB}$$

---

2. Q Derive the relation between $A_p(\mathrm{~dB})$ and $A_v(\mathrm{~dB})$, $A_i(\mathrm{~dB})$

2. A

$$\begin{aligned} A_p(\mathrm{~dB}) &= 10\log(A_v A_i) \\ &= 10\log(A_v) + 10\log(A_i) \\ &= \frac{1}{2}\left(20\log(A_v) + 20\log(A_i)\right) \\ &\boxed{= \frac{1}{2}\left(A_v(\mathrm{~dB}) + A_i(\mathrm{~dB})\right)} \end{aligned}$$

---

3. Q Comment on the results of example 1.

3. A

- To avoid losing voltage signal strength at the amplifier input, the first stage is designed to have an input resistance much larger than the source resistance (1 MΩ >> 100 kΩ). The trade-off appears to be a moderate voltage gain (10 V/V).
- The second stage realizes the bulk of the required voltage gain.(90 V/V)
- The third and final, or output stage functions as a buffer amplifier, providing an output resistance much lower than R_L (10 Ω << 100 Ω).

---

4. Q What would the overall voltage gain of the cascade amplifier in Example 1.3 be without stage 3 (i.e., with the load resistance connected to the output of the second stage)?

4. A First gain A_{v1}

$$A_{v1} \equiv \frac{v_{i2}}{v_{i1}} = 10\frac{100\mathrm{k}\Omega}{100\mathrm{k}\Omega + 1\mathrm{k}\Omega} = 9.9\ \mathrm{V/V}$$

Second gain A_{v2} (Consider the load resistance instead of the initially 10 kΩ)

$$A_{v2} \equiv \frac{v_{i3}}{v_{i2}} = 100\frac{100\Omega}{100\Omega + 1\text{k}\Omega} = 9.09 \text{ V/V}$$

The total gain of the two stages in cascade can now be found from

$$A_v \equiv \frac{v_L}{v_{i1}} = A_{v1}A_{v2} = 90 \text{ V/V}$$

To find the voltage gain from source to load, we multiply A_v by the factor representing the loss of gain at the input; that is,

$$\begin{aligned}\frac{v_L}{v_s} = \frac{v_L}{v_{i1}}\frac{v_{i1}}{v_s} &= A_v\frac{v_{i1}}{v_s} \\ &= 90 \times \frac{1\text{M}\Omega}{1\text{M}\Omega + 100\text{k}\Omega} \\ &= 90 \times 0.909 = 81.81 \text{ V/V}\end{aligned}$$

a decrease by a factor of ≈ 9.

---

5. Q For the cascade amplifier of example 1, let v_s be 1mV. Find v_{i1}, v_{i2}, v_{i3}, and v_L.

5. A

$$v_{i1} = A_{v_1} \times \frac{1\text{M}\Omega}{1\text{M}\Omega + 100\text{k}\Omega} = 1\text{mV} \times 0.909 = 0.909\text{mV}$$

$$v_{i2} = A_{v1} \times v_{i1} = 9.9 \times 0.909\text{mV} \approx 9\text{mV}$$

$$v_{i3} = A_{v2} \times v_{i2} = 90.9 \times 9\text{mV} \approx 818\text{mV}$$

$$v_L = A_{v3} \times v_{i3} = 0.909 \times 818\text{mV} \approx 743.56\text{mV}$$

---

6. Q Model the three-stage amplifier of example 1 (without the source and load), using the voltage amplifier model of example 1. What are the values of R_i, A_{vo}, and R_o ?

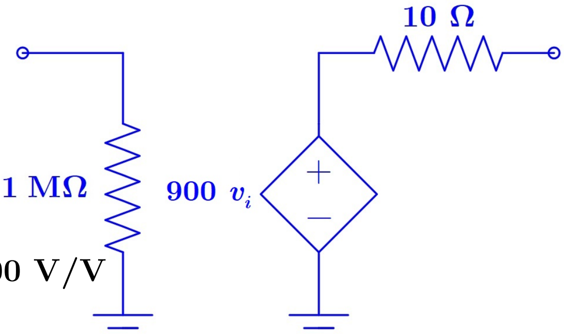
6. A

$R_i = 1$ MΩ, $R_o = 10\Omega$

because the load resistance is discarded

therefore $A_{v3} = \dfrac{v_L}{v_{i3}} = 1$ V/V

The total gain :



$$A_v \equiv \frac{v_L}{v_{i1}} = A_{v1}A_{v2}A_{v3} = 9.9 \times 90.9 \times 1 = 900 \text{ V/V}$$

Notice that this model can be used as an alternative to calculate and values associated to example 1 (except the power dissipated in internal resistors of the amplifier)

---

7. Q Voltage amplifier delivers 200 mV across a load resistance of 1 kΩ, it is found the the output voltage decreased by 5mV when the load resistance decreased to 780Ω , What the values of open circuit output voltage and output resistance of the amplifier

Consider an amplifier with the following values:

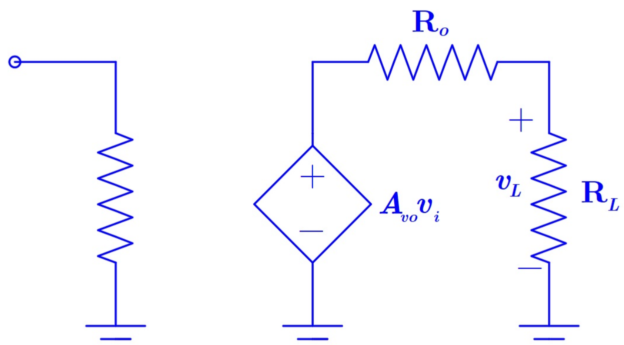
Find the output resistance and the voltage gain, then draw a graph represents the relation between R_L and V_L

7. A

Note that :

When $R_L = 780\Omega$, $V_L = 195$mV

When $R_L = 1000\Omega$, $V_L = 200$mV



$$\boldsymbol{v_L = A_{vo} v_i \frac{R_L}{R_L + R_o}}$$

open circuit voltage

$$\boldsymbol{v_{L\,oc} = A_{vo} v_i}$$

$$\therefore \boldsymbol{0.195 = A_{vo} v_i \frac{780}{780 + R_o}} \tag{1}$$

$$\therefore \boldsymbol{0.2 = A_{vo} v_i \frac{1000}{1000 + R_o}} \tag{2}$$

divide (1) by (2)

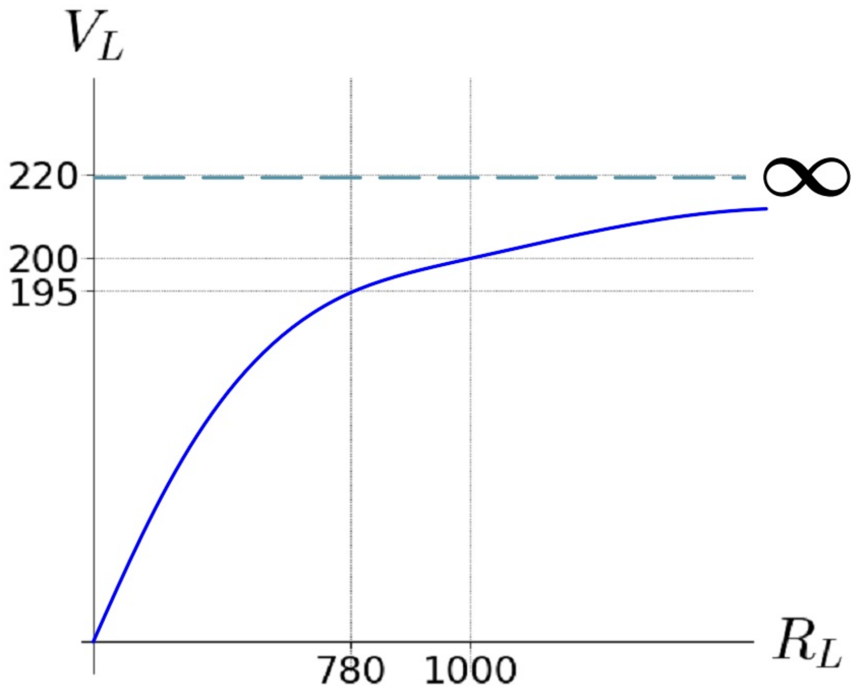
$$\therefore \boldsymbol{0.975 = \frac{780}{780 + R_o} \times \frac{1000 + R_o}{1000}}$$

$$\therefore \boldsymbol{R_o = 100\Omega}$$

Substitute in (1)

$$\therefore \boldsymbol{0.195 = A_{vo} v_i \frac{780}{780 + 100}}$$

$$\therefore \boldsymbol{A_{vo} v_i = 0.22 (\text{open circuit voltage gain (when } R_L \to \infty,\, v_L \to 220 \text{ mV))}}$$


V_L
220
200
195
∞
780
1000
R_L