EEC 221 - Circuits2 cheat sheet[1]

Transfer function

$$\mathbf{H}(\omega) = \frac{\mathbf{Y}(\omega)}{\mathbf{X}(\omega)}$$

$$\text{Voltage gain} = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$$
$$\text{Current gain} = \frac{\mathbf{I}_o(\omega)}{\mathbf{I}_i(\omega)}$$
$$\text{Transfer impedence} = \frac{\mathbf{V}_o(\omega)}{\mathbf{I}_i(\omega)}$$
$$\text{Transfer admittance} = \frac{\mathbf{I}_o(\omega)}{\mathbf{V}_i(\omega)}$$

For RC circuit: $\omega_0 = \dfrac{1}{RC}$

For RL circuit: $\omega_0 = \dfrac{R}{L}$

Resonance

Series resonance

$$\omega_0 = \frac{1}{\sqrt{LC}}$$
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

at resonance:

$$Z = R \qquad |\, I\,|_{max} = \frac{|\, V\,|_{max}}{R} \qquad X_L = X_C$$

The average power at RLC circuits:

$$P(\omega) = \frac{1}{2} I^2 R$$

The average power at resonance:

$$P(\omega) = \frac{1}{2} I_{max}^2 R = \frac{1}{2}\frac{V_{max}^2}{R}$$

Half power frequencies:

$$\omega_1, \omega_2 = \mp \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$
$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

Bandwidth:

$$B = \omega_2 - \omega_1 = \frac{R}{L}$$

Quality factor:

$$Q = \frac{\text{Peak energy}}{\text{Energy dissipated}} = \frac{\omega_0}{B}$$

When $Q > 10$:

$$\omega_1 = \omega_0 - \frac{B}{2}$$
$$\omega_2 = \omega_0 + \frac{B}{2}$$

Parallel resonance

$$\omega_0 = \frac{1}{\sqrt{LC}}$$
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

at resonance:

$$Z = R \qquad |\, I\,|_{max} = \frac{|\, V\,|_{max}}{R} \qquad X_L = X_C$$

The average power at RLC circuits:

$$P(\omega) = \frac{1}{2} I^2 R$$

The average power at resonance:

$$P(\omega) = \frac{1}{2} I_{max}^2 R = \frac{1}{2}\frac{V_{max}^2}{R}$$

Half power frequencies:

$$\omega_1, \omega_2 = \mp \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$
$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

Bandwidth:

$$B = \omega_2 - \omega_1 = \frac{1}{RC}$$

Quality factor:

$$Q = \frac{\text{Peak energy}}{\text{Energy dissipated}} = \frac{\omega_0}{B}$$

When $Q > 10$:

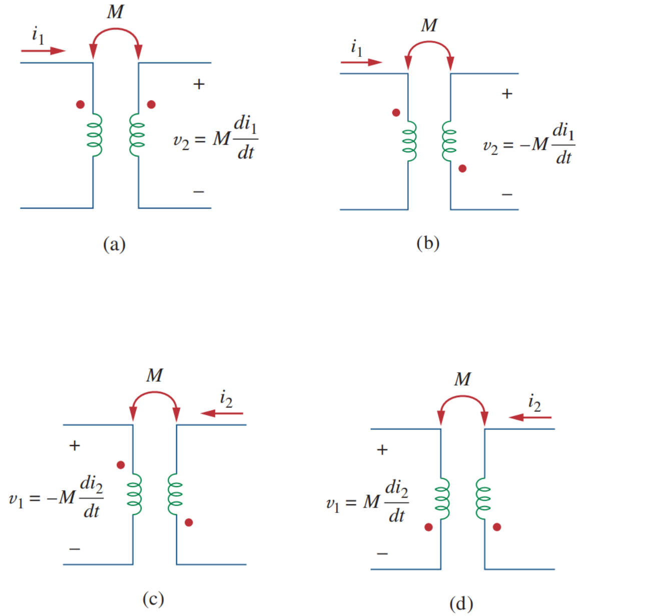
$$\omega_1 = \omega_0 - \frac{B}{2}$$
$$\omega_2 = \omega_0 + \frac{B}{2}$$

Magnetically Coupled Circuits

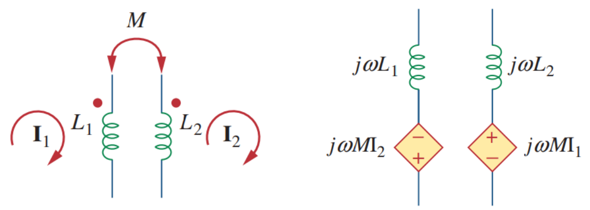
M (Mutual inductance):

$$M = K\sqrt{L_1 L_2} \qquad \text{(k: couppling coefficent)}$$

Dot convention:



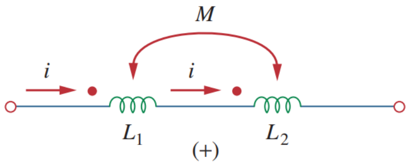
Dependent source representation:



Series-aiding connection:

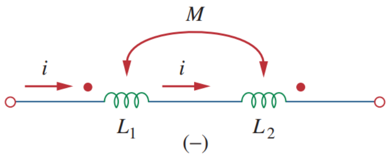
$$l_{eq} = L_1 + L_2 + 2M$$

[1]Taha Ahmed



Series-opposing connection:

$$l_{eq} = L_1 + L_2 - 2M$$



Energy stored:

$$W = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M I_1 I_2$$

(two currents enter the dots or leave)

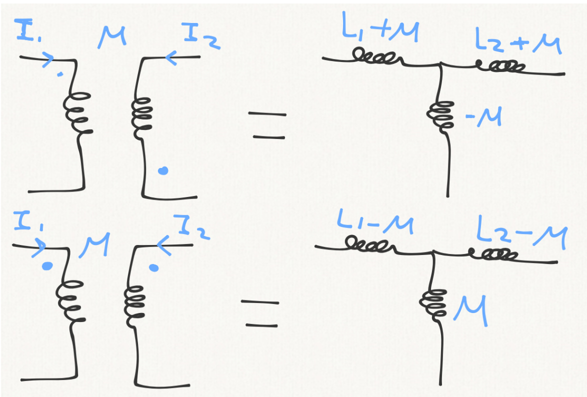
$$W = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 - M I_1 I_2 \qquad \text{(one enters, one leaves)}$$

Power dissipated in a resistor:

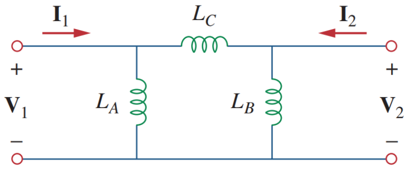
$$P = |I_{RMS}|^2 R \qquad (I_{RMS} = \frac{I_{peak}}{\sqrt{2}})$$

$$P = \frac{1}{2}|I_{peak}|^2 R$$

T model:



π model:

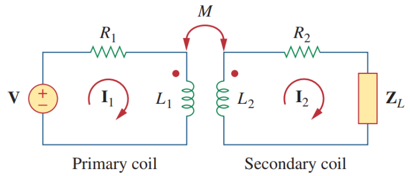


$$L_A = \frac{L_1 L_2 - M^2}{L_2 - M} \qquad L_B = \frac{L_1 L_2 - M^2}{L_1 - M}$$

$$L_C = \frac{L_1 L_2 - M^2}{M}$$

Transformer

Linear transformer $0 < k < 1$



$$Z_{in} = R_1 + j\omega L_1 + \underbrace{\frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}}_{\text{reflected impedence}} = \frac{V}{I_1}$$

$$Z_{th} = R_2 + j\omega L_2 + \underbrace{\frac{\omega^2 M^2}{R_1 + j\omega L_1}}_{\text{reflected impedence}}$$

for maximum power transfer:

$$Z_L = Z_{th}^*$$

for maximum power transfer if R_L is pure resistive:

$$R_L = |Z_{th}|$$

Ideal transformer k = 1

$$n = \frac{N_2}{N_1} \qquad \text{(turns ratio)}$$

$$\frac{V_2}{V_1} = n$$

$$\frac{I_2}{I_1} = \frac{1}{n}$$

Referring from secondary to primary:

$$V_1' = \frac{V_2}{n}$$

$$I_1' = n I_2$$

$$Z_1' = \frac{V_2}{n^2}$$

Referring from primary to secondary:

$$V_2' = n V_1$$

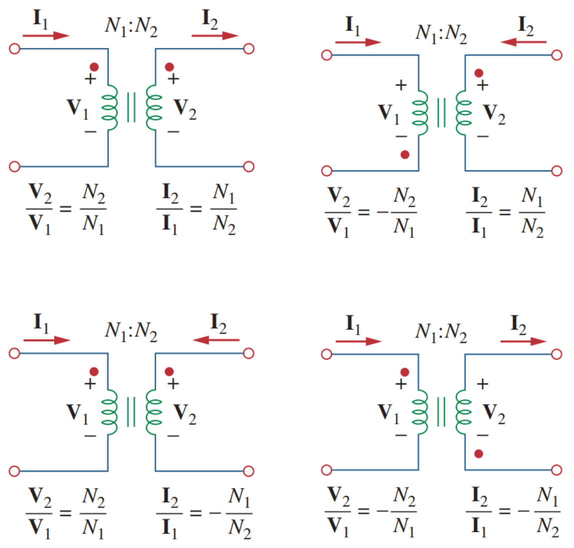
$$I_2' = \frac{I_1}{n}$$

$$Z_2' = n^2 Z_1$$

Sign roles:

if both voltages are +ve or -ve relative to the dotted terminals → use $+n$
otherwise use $-n$

if both currents enter or leave the dotted terminals → use $-n$
otherwise use $+n$



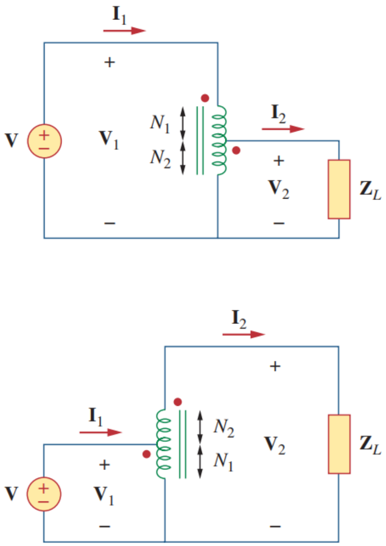
Complex power:

$$s_1 = V_1 I_1^*$$

$$s_2 = V_2 I_2^*$$

$$\boxed{s_1 = s_2}$$

Ideal autotransformer



(a) Step-down autotransformer, (b) step-up autotransformer.

Step down autotransformer:

$$\frac{V_1}{V_2} = \frac{N_1 + N_2}{N_2}$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1 + N_2}$$

Step down autotransformer:

$$\frac{V_1}{V_2} = \frac{N_1}{N_1 + N_2}$$

$$\frac{I_1}{I_2} = \frac{N_1 + N_2}{N_1}$$

Two port network

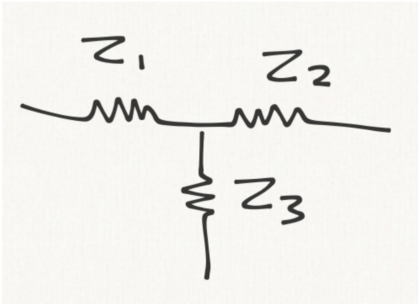
Impedance parameters [Z]

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$
$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

for any T network:



$$Z = \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z2 + Z_3 \end{bmatrix}$$

if $Z_{11} = Z_{22} \rightarrow$ symmetrical network
if $Z_{12} = Z_{21}$and has no dependent sources $\rightarrow$ reciprocal network

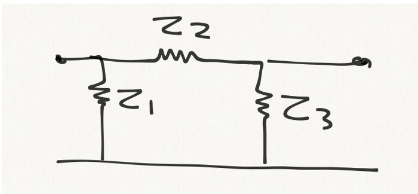
Admittance parameters [y]

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$
$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

for any T network:



$$y = \begin{bmatrix} \frac{1}{Z_1} + \frac{1}{Z_2} & \frac{-1}{Z_2} \\ \frac{-1}{Z_2} & \frac{1}{Z_2} + \frac{1}{Z_3} \end{bmatrix}$$

note that

$$[Z] = [y]^{-1}$$

remember $\rightarrow$ Let

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

be a full-rank 2×2 matrix. Then $\det A \equiv |A| = a_{11}a_{22} - a_{12}a_{21} \neq 0$ and

$$A^{-1} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} = \frac{1}{|A|} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}.$$

Hybrid parameters [h]

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$
$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

Inverse Hybrid parameters [g]

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$I_1 = g_{11}V_1 + g_{12}I_2$$
$$V_2 = g_{21}V_1 + g_{22}I_2$$

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \qquad g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$
$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} \qquad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

note that

$$[g] = [h]^{-1}$$

Transmission parameters [T]

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad B = \left.\frac{-V_1}{I_2}\right|_{V_2=0}$$
$$C = \left.\frac{I_2}{V_2}\right|_{I_2=0} \qquad D = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$

Inverse transmission parameters [t]

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

$$V_2 = aV_1 - bI_1$$
$$I_2 = cV_1 - dI_1$$

$$a = \left.\frac{V_2}{V_1}\right|_{I_1=0} \qquad b = \left.\frac{-V_2}{I_1}\right|_{V_1=0}$$
$$c = \left.\frac{I_2}{V_1}\right|_{I_1=0} \qquad d = \left.\frac{-I_2}{I_1}\right|_{V_2=0}$$

note that

$$[t] \neq [T]^{-1}$$

Network is reciprocal if $ad - bc = 1$

Interconnection of networks

Series-series

$$Z_T = Z_1 + Z_2$$

Parallel-parallel

$$y_T = y_1 + y_2$$

Series-parallel

$$h_T = h_1 + h_2$$

Parallel-series

$$g_T = g_1 + g_2$$

Cascaded

$$[T_T] = [T_1]\,[T_2] \qquad \text{(matrix multiplication)}$$