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## Chapter 2

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1. Q — Compare between electrons in isolated atoms and electrons in solids

A —

- Electrons in isolated atoms are restricted to sets of discrete energy levels
- Electrons in solids are restricted to bands of available energies and are not allowed at others

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2. Q — Illustrate the bonding forces in solids

A —

1. **The ionic bonding** : bonds between positive and negative ions.
2. **The metallic bonding** : The outer electron in each atom is contributed to the crystal as a whole so that the solid is made up ions with closed shells immersed in a sea of electrons.

The forces holding the lattice together arise from an interaction between positive ion cores and free electrons.

3. **The Covalent bonding** : each atom shares its valence electrons with its four neighbors.

Compound semiconductors such as GaAs have mixed bonding in which both ionic and covalent bonding forces participate.

At 0° K both ionic and covalent structures are insulators.

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3. Q — What is the importance of secondary energy levels

A —

- When two atoms are completely isolated from each other so that there are no interaction of electron wave functions between them, they can have identical electron structures.

- As the spacing between atoms get smaller, electron wave functions begin to overlap
- *Pauli Exclusion principle* states that no two electrons in a given interacting system may have the same quantum state, therefore there must be splitting of discrete energy levels of the atoms into new levels (the secondary levels).

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4. Q — "The variation of the characteristics of energy band structures is responsible for the wide range of electrical characteristics in various materials"

Explain how the characteristics of energy band structures affect electrical characteristics.  
or

Compare between characteristics of energy band structures of metals, insulators and semiconductors.

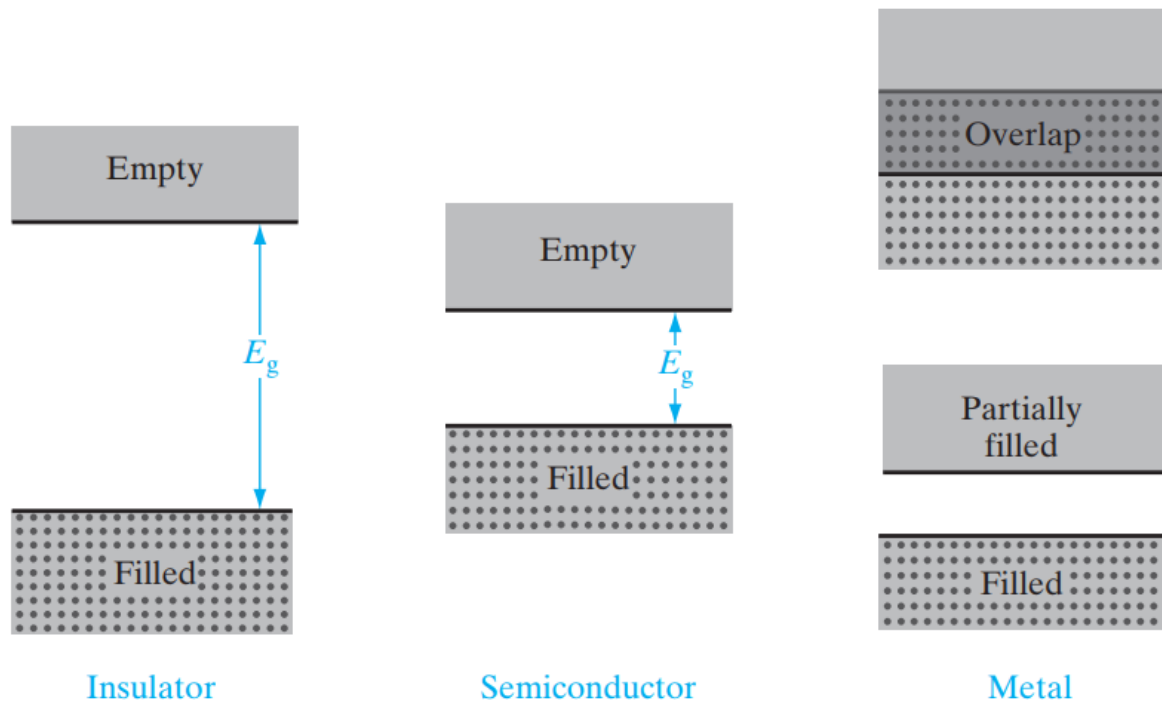
A —

For electrons to experience acceleration in an applied electric fields, they must be able to move into new energy states. this implies there must be empty states available for the electrons

1. **Insulators** : They have a filled conduction band and an empty valence band separated by a **high energy gab** , *no allowed energy states*
2. **Metals** : they have two cases of energy band structure
  - A filled valence band and partially filled conduction band
  - Overlapping valence and conduction bands
3. **Semiconductors** : At 0° the are like insulators They have a filled conduction band and an empty valence band separated by a **high energy gab** , *no allowed energy states*

At room temperature they have number of electrons excited thermally across the energy gab into the conduction band.

The difference lies in the size of energy gabs




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5. Q — Explain the difference in the recombination process between indirect and direct semiconductors. (final 2020)

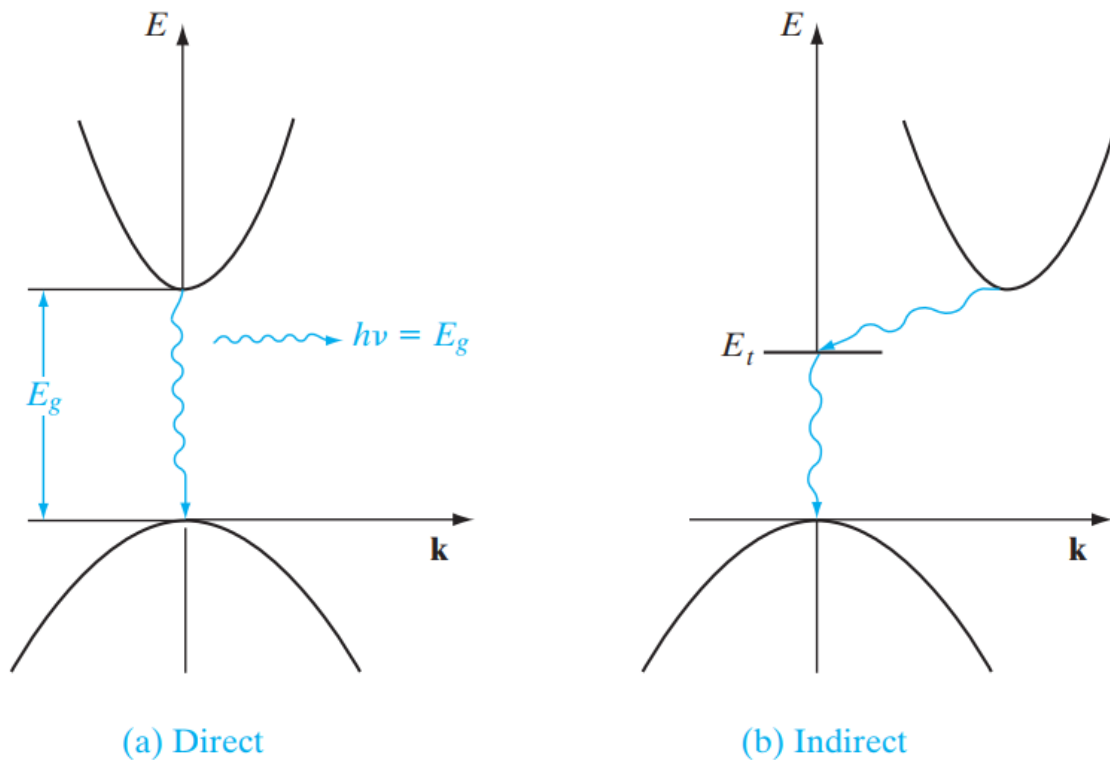
or

Explain why GaAs makes a better light emitting diodes while Si and Ge are better for transistors and IC applications. (midterm 2019)

or

Explain why indirect semiconductors are suitable for fabricating transistors and ic applications (midterm 2021)

| Direct semiconductors   | Indirect semiconductors  |
|---|--|
| its band structure has a minimum at conduction band and a maximum at the valence band for <i>the same k (momentum)</i>  | its band structure has a minimum at conduction band and a maximum at the valence band for <i>for different k (momentum)</i>  |
| Electrons at conduction band can fall directly to empty states in valence band, giving off the energy difference as a <i>photon</i> of light  | Electrons at conduction band can't fall directly to valence band, but they require a change in momentum, this can be done through some defect states within the band gap, it has first to lose energy through collisions with the crystals vibrating in the lattice, the quantization of the lattice vibration energy gives rise to the concept of <i>phonon</i> particles. The participation of phonons and photons are necessary in the indirect transistors |
| Used in LEDs and lasers because <ol style="list-style-type: none"> <li>1. the direct excitation is favored energetically</li> <li>2. the inverse reaction (<i>direct recombination</i>) is highly probable</li> </ol> | Used in transistor and IC application because current gain is high, since the carriers have to undergo a momentum change to recombine in transistor between emitter and collector, so they don't recombine easily which can reduce current gain.   |




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6. Q — What is the disadvantage for using indirect semiconductors in LEDs and Laser applications?

A — Most of the recombination energy goes into the vibrational energy of the crystal atoms which heats up the crystal.

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7. Q — Explain electron hole pair in semiconductors

A —

- As the temperature of a semiconductor increases from  $0^\circ \text{K}$  some electrons in valence band receive enough thermal energy to be ejected to the conduction band, thus an electron hole pair is created.
- After excitation to the conduction band, the electron is surrounded by a large number of unoccupied energy states, therefore they are free to move in many available empty states.
- in a filled band, all energy states are occupied, the net motion of electrons under

the influence of external electric field is zero.

- if an electron is removed, the net current will not be zero, this is equivalent to the motion of a positive charge in the opposite direction

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8. Q — Explain briefly the meaning of effective mass

A — Due to the interaction of the electrons in the periodic potential of the lattice their wave-particle motion is not the same as that for electrons in free space.

We account for the influence of the by altering the value of the particle mass by  $m^*$

$$m^* = \frac{\hbar^2}{\frac{d^2 E}{dk^2}}$$

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9. Q — Compare between **intrinsic** and **extrinsic** materials

A — **Intrinsic materials :**

- A perfect semiconductor lattice with no impurities or lattice defects.
- There are no charge carriers at  $0^\circ$
- At higher temperature electron-hole pairs are generated
- breaking of covalent bonds requires an energy equals to  $E_g$
- the position of free electrons and holes are not localized in the lattice, they are spread out over several lattice spacing and should be considered quantum mechanically by probability distribution for intrinsic materials.
- $n = p = n_i$   
 $n$  : conduction band electron concentration  
 $p$  : valence band hole concentration  
 $n_i$  : intrinsic concentration
- *Generation rate* of electron-hole pairs must be equal to *Recombination rate*  
 $r_i = g_i$  , each of them is temperature dependent.

- At any temperature rates of generation and recombination are proportional to the equilibrium concentration of  $n_0$  and  $p_0$  ( $r_i = g_i = \alpha_r n_0 p_0 = \alpha_r n_i^2$ )

### Extrinsic materials :

- Semiconductor with impurities, it has additional carriers beside the intrinsic carriers
- *n type* : an impurity from column V (P,As,Sb) introduces an energy level near the conduction band.  
At about  $50^\circ \text{ K}$  to  $100^\circ \text{ K}$  all the electrons in the impurities are donated to the conduction band ( $n_0 \gg n_i, p_0$ )
- *p type* : an impurity from column III (Al, Ga, In) introduces an energy level near the valence band.  
At about  $50^\circ \text{ K}$  to  $100^\circ \text{ K}$  all the electrons in the conduction band are accepted to the impurities from valence band ( $p_0 \gg n_i, n_0$ )
- in Ge (donor/acceptor) levels lies about 0.01 eV (below/above) (conduction/valence) bands
- in Si corresponding values are 0.03 & 0.06 eV

## 10. Q — Define Fermi Dirac distribution function

A —

- Fermi Dirac distribution function is a function that gives the probability that an available energy state at  $E$  will be occupied by an electron at absolute temperature  $T$

$$f(E) = \frac{1}{1 + e^{(E-E_f)/KT}}$$

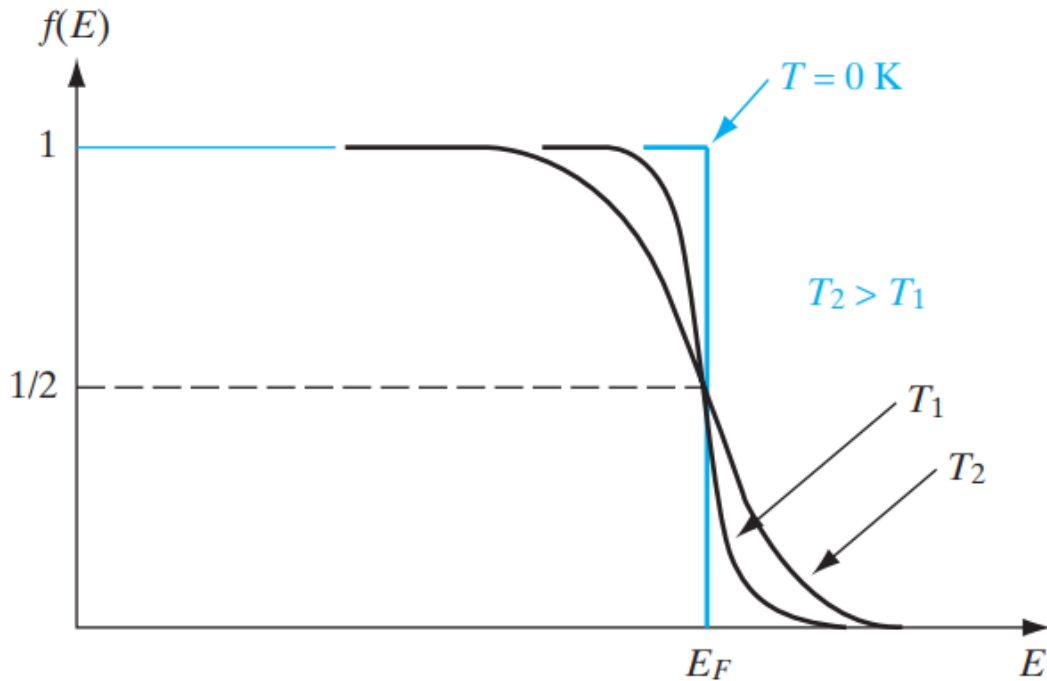
$E_f$  : Fermi level  $K$  : Boltzmann's constant =  $8.62 \times 10^{-23} \text{ J/K}$

- Probability  $f(E)$  means that above  $E_f$  the states are filled. (electrons in the C.B)
- Probability  $1 - f(E)$  means states below  $E_f$  are empty . (holes in the V.B)



- Due to the symmetrical nature of the distribution function about  $E_f$  for all  $T$ .

$$f(E_f + \Delta E) = 1 - f(E_f - \Delta E)$$

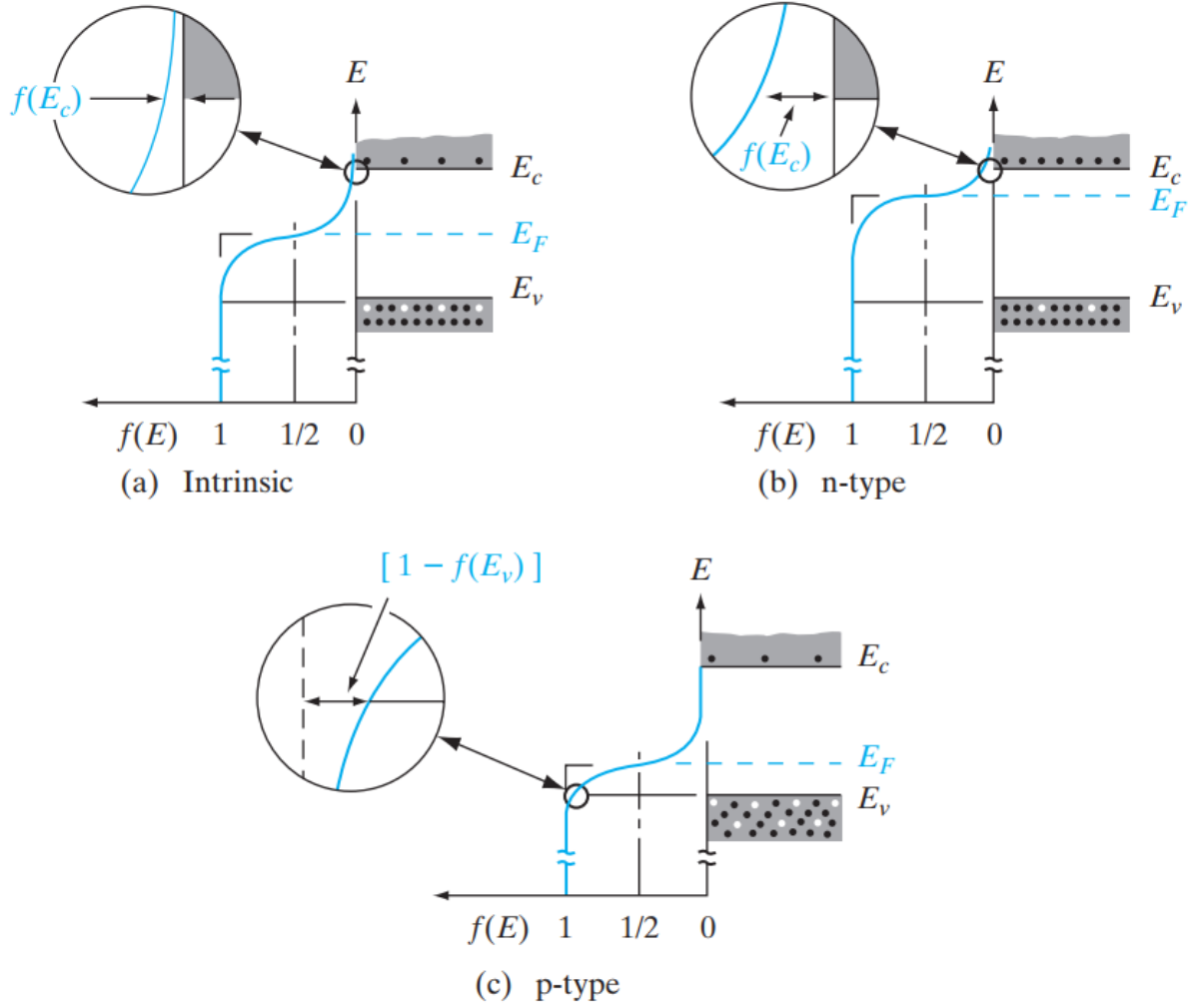



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11. Q — Explain how Fermi level is an indication of the type of semiconductor and the amount of doping

A —

1. **Intrinsic Materials:** Concentration of electrons in the conduction band is equal to concentration of holes in the valence band, therefore, the Fermi level is in the middle of the band gap.
2. **N type extrinsic Materials:** Concentration of electrons in the conduction band is higher than concentration of holes in the valence band, therefore, the Fermi level is near the conduction band.
3. **P type extrinsic Materials:** Concentration of electrons in the conduction band is less than concentration of holes in the valence band, therefore, the Fermi level is near the valence band.



12. Q — Prove that for any semiconductor material  $n_0 p_0 = n_i^2$ .

A — in the intrinsic semiconductor<sup>2</sup>

$$n_i = N_c e^{-(E_c - E_i)} \quad p_i = N_v e^{-(E_i - E_v)}$$

$$n_i = p_i$$

$$n_i p_i = N_c N_v e^{-(E_c - E_v)/KT} = N_c N_v e^{-(E_g)/KT}$$

in the extrinsic semiconductor:

$$n_0 = N_c e^{-(E_c - E_f)} \quad p_0 = N_v e^{-(E_f - E_v)}$$

$$n_0 = p_0$$

$$n_0 p_0 = N_c N_v e^{-(E_c - E_v)/KT} = N_c N_v e^{-(E_g)/KT}$$

<sup>2</sup>Note that in intrinsic materials  $E_f$  lies at intrinsic level  $E_i$ , near the middle of the band gap

so  $n_0 p_0 = n_i^2$  for any semiconductor.

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13. Q — In an n type material, draw the relation between majority carrier concentration and the temperature. Indicate the optimum temperature range for device operation and give reasons for this choice (*midterm 19*)(*midterm 20*)

A — At low temperature (*high  $\frac{1}{T}$* )

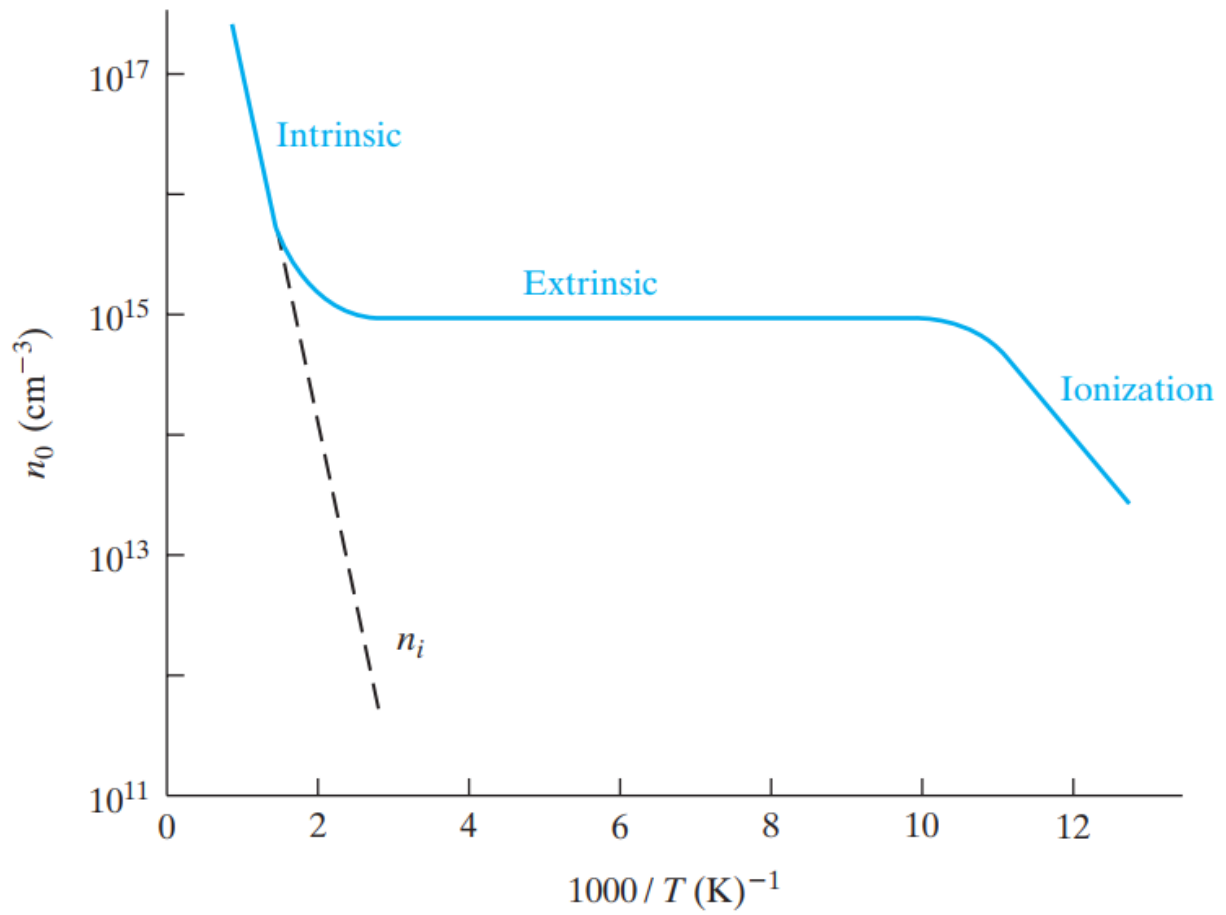
- Very small intrinsic electron-hole pairs
- Donor electrons bound are bounded to donor atoms.

At temperature increases ( *$\frac{1}{T}$  decreases*)

- Donor electrons are donated to the conduction band until all donors are ionized
- Electron concentration remains constants at  $n_0 = N_d$ , this is why this region is the optimum region for device operation.

At large temperature (*small  $\frac{1}{T}$* )

- more electrons are donated due to intrinsic electron-hole pair generation.  $n_o = n_i \gg N_d$




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14. Q — A si sample is doped with Ga atoms (trivalent material) at a concentration of  $10^6$  atoms/ $\text{cm}^3$ . Calculate both majority and minority carriers concentration at  $300^\circ \text{K}$  and energy difference between Fermi level and intrinsic level. (*midterm 19*)(*final 20*)

A — Given ( $T = 300^\circ K$ ,  $n_i = 1.5 \times 10^{10} \text{cm}^{-3}$ ,  $KT = 0.0259 \text{eV}$ )

$$\begin{aligned}
 p_0 &\approx N_a = 10^{16} \text{atoms/cm}^{-1} \\
 p_0 &= n_i e^{(E_i - E_f)/KT} \\
 \frac{p_0}{n_i} &= e^{(E_i - E_f)/KT} \\
 \ln \left( \frac{p_0}{n_i} \right) &= \frac{E_i - E_f}{KT} \\
 E_f - E_i &= -KT \ln \left( \frac{p_0}{n_i} \right) \\
 &= -0.0259 \ln \left( \frac{10^{16}}{1.5 \times 10^{10}} \right) \\
 &= -0.0347 \text{eV}
 \end{aligned}$$

15. Q — Illustrate compensation and space charge neutrality process.

A —

- if a semiconductor has donors and acceptors and  $N_d > N_a$   
the material is n type and Fermi level is near the conduction band
- The filling of the  $E_a$  states occurs at the expense of the donated conduction band electrons
- If the acceptors states are filled with valence band electrons the holes are then filled due to recombination with conduction band electrons, therefore the resultant concentration of conduction band electrons will be  $N_d - N_a$
- this process is called compensation
- if the materials remains electrostatically neutral

$$P_0 + N_d^+ = n_0 + N_a^-$$

For n type material  $n_0 \gg p_0$   $\therefore n_0 = N_d - N_a$

For p type material  $p_0 \gg n_0$   $\therefore p_0 = N_a - N_d$

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16. Q — A Ge sample is doped with  $5 \times 10^{13}$  Sb atoms/cm<sup>3</sup> using the requirements of charge neutrality calculate the electrons concentration  $n_0$  at 300° k.

Given ( $n_i$  for Ge =  $2.5 \times 10^{13}$  atoms/cm<sup>3</sup>)

A —

$$n_0 = p_0 + N_d \quad (\times n_0)$$

$$n_0^2 = n_i^2 + N_d n_0$$

$$n_0^2 - N_d n_0 - n_i^2 = 0 \quad (\text{Solve using quadratic formula})$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (a = 1, b = -N_d, c = -n_i^2)$$

$$n_0 = \frac{N_d \pm \sqrt{N_d^2 + 4n_i^2}}{2}$$

$$n_0 = \frac{5 * 10^{13} \pm \sqrt{(5 * 10^{13})^2 + 4(2.5 * 10^{13})^2}}{2}$$

$$= 3.036 * 10^{13} \text{ atoms/cm}^3 \quad (\text{The negative solution is refused})$$

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17. Q — How long does it take an average electrons to drift  $1\mu\text{m}$  in pure Si at electric field = 100 v/cm ( $\mu_m = 1350\text{cm}^2/\text{V.s}$ )

A —

$$v = \mu_m E = 1350 * 100 = 135 * 10^3 \text{ cm/st} = \frac{d}{v} = \frac{1 * 10^4 \text{ cm}}{135 * 10^3 \text{ cm/s}} = 7.41 \times 10^{-10} \text{ s.}$$

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18. Q — How does the higher values of electric field affect the carriers velocities in different types of semiconductor materials. (*final 2021*)(*midterm 2022*)

A —

- At low value of  $E$

$$J = \sigma E$$

- For  $E > 10^3 \text{ V/cm}$ ,  $\sigma$  becomes dependent on  $E$ .

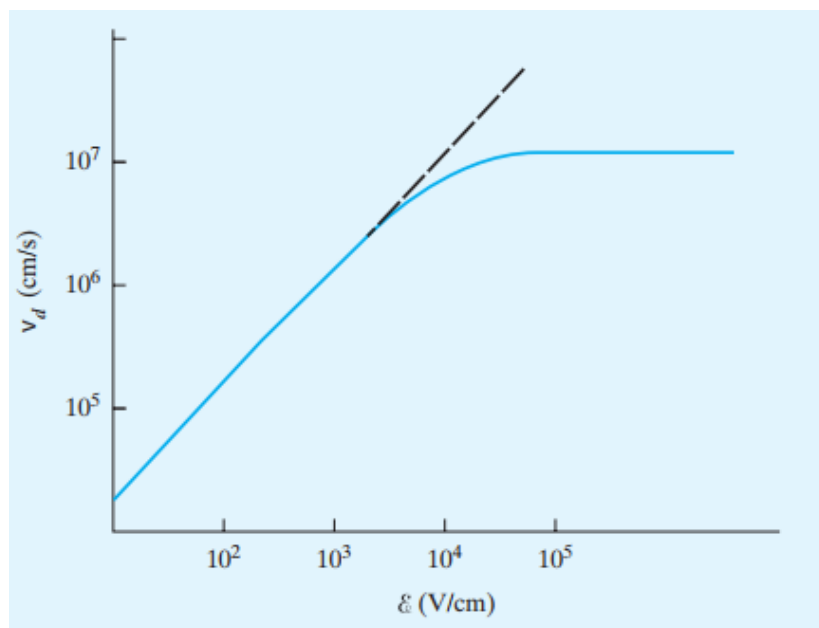
This is called hot carrier effect, in this case the carrier drift velocity  $V_d$  is comparable to the thermal velocity  $V_{th}$

$$\frac{1}{2}mv_{th}^2 = KT$$

$$v_{th} = \sqrt{\frac{2KT}{m}} = 10^7 \text{ cm/s}$$

An upper limit of drift velocity  $V_d$  is reached near  $V_d = V_{th}$

- At this point any added energy is transferred to the lattice rather than increasing the carrier velocity, this limit is called the scattering velocity.
- In GaAs, Electron velocity decreases at high fields giving negative conductivity.




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19. Q — Explain how does the carriers mobility in semiconductors varies with temperature, doping concentration and high field. (final 2020)(midterm 2022)

A —

- At low temperature, *Impurity scattering* is dominant, as slow electrons are likely to be scattered more strongly by the interaction with the charged ions

- By increasing the temperature, *impurity scattering* becomes less effective, therefore the mobility increases up to certain temperature.
- At certain temperature, *lattice scattering* becomes dominant, this type of scattering is caused by the thermal vibrations, therefore the mobility decreases

$$\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \dots$$

- Increasing doping increases *impurity scattering*, therefore the mobility decreased.

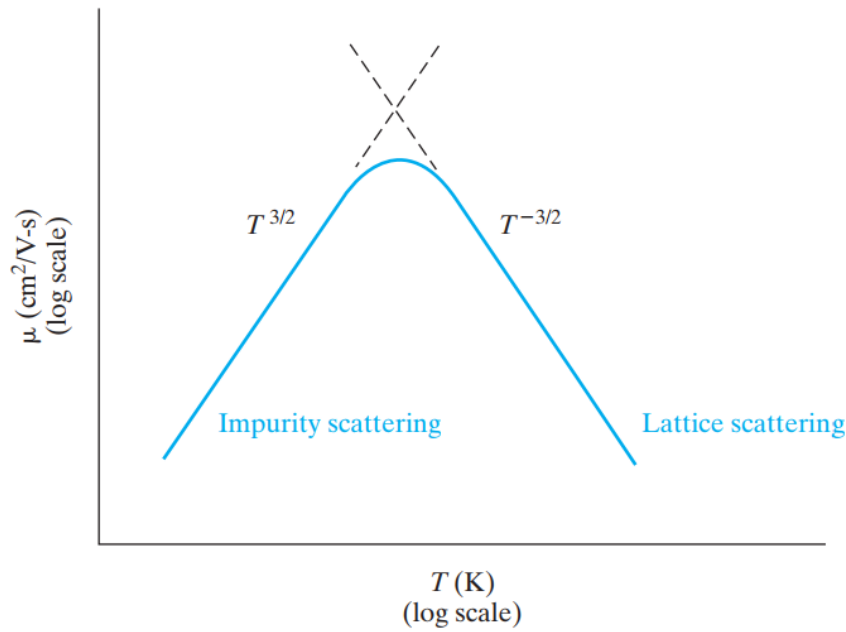


Figure 1: Approximate temperature dependence of mobility with both lattice and impurity scattering.

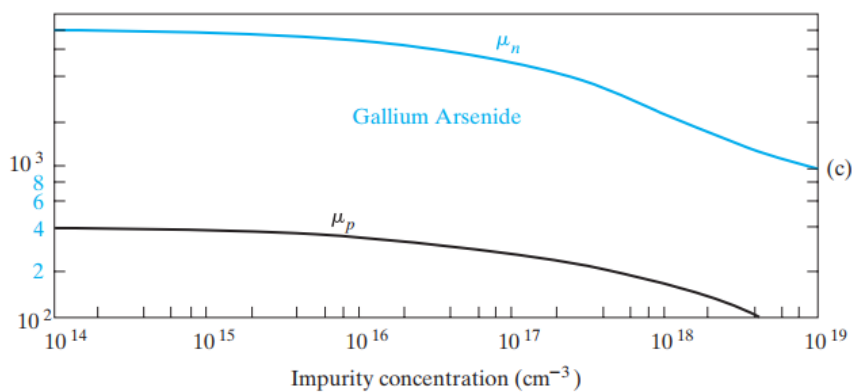


Figure 2: Variation of mobility with total doping impurity concentration

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20. Q — Based on the *Hall effect* derive expressions for the majority carriers concentration, resistivity and carrier mobility (*midterm 2020*)(*midterm 2021*)



A — If a magnetic field is applied perpendicular to the direction in which holes drift in a p type bar, a force in the -ve y direction results and equals to

$$F_y = qE_y - v_x B_Z$$

since the net force equals zero

$$qE_y = q v_x B_Z$$

$$E_y = v_x B_Z \quad \left(v_x = \frac{J_x}{qp_0}\right)$$

$$E_y = \frac{J_x}{qp_0} B_Z \quad \left(\text{let } R_H = \frac{1}{qp_0}\right)$$

$$E_y = R_H J_x B_Z \quad (R_H \text{ is the hall coefficient})$$

$$P_0 = \frac{1}{q R_H} = \frac{J_x B_z}{q E_y} = \frac{\frac{I_x}{wt} B_z}{q \frac{V_{AB}}{w}} = \frac{I_x B_z}{q t V_{AB}} \quad (1)$$

$$\rho = \frac{RA}{L} = \frac{Rwt}{L} = \frac{V_{CD}/I_x}{L/wt}$$

$$\mu_p = \frac{\rho}{qp_0} = \frac{1/\rho}{\rho/(1/\rho R_H)} = \frac{R_H}{\rho}$$

from (1) the sign of  $V_{AB}$  indicates the type of the semiconductor, if  $V_{AB} > 0 \Rightarrow$  p-type - if  $V_{AB} < 0 \Rightarrow$  n-type.

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21. Q — In n-type semiconductor derive expressions for the *drift current intensity* and the *electrons mobility* .

A —

- At room temperature, the thermal motion of an electron may be visualized as random scattering from lattice vibration, impurities, other electrons and defects.
- There are no net motion of the group of n electrons/cm<sup>3</sup> over any period of time .

- if an electric field  $E_x$  is applied, the force of the field on the  $n$  electrons/cm<sup>3</sup> is

$$-nqE_x = \left. \frac{dp_x}{dt} \right|_{\text{field}} \quad p_x = (mV_x) \times n$$

- $p_x$  is the total momentum of the group
- At steady state, the net acceleration is balanced by the deceleration of collision process.
- if the collisions are random, there will be a constant probability of collisions at any time for each electron.
- Rate of decrease of  $N(t)$  at any time is proportional to the number left unscattered.

$$-\frac{dN(t)}{dt} \propto N(t) \quad , \quad -\frac{dN(t)}{dt} = \frac{1}{\bar{t}}N(t) \quad , \quad N(t) = N_0 e^{-t/\bar{t}}$$

$N(t)$  : the number of electrons which have not undergo a collision by time  $t$ .

$N_0$  : group of electrons at  $t = 0$ .  $\bar{t}$  : mean time between scattering events.

- Probability that any electron has a collision in the time interval  $dt = \frac{dt}{\bar{t}}$
- Differential change in  $p_x$  due to collisions is

$$dp_x = -p_x \frac{dt}{\bar{t}}$$

- the rate of change of  $p_x$  due to the deceleration effect of collisions is

$$\frac{dp_x}{dt} = -\frac{p_x}{\bar{t}}$$

.

- The net acceleration is zero

$$\begin{aligned} \left. \frac{dp_x}{dt} \right|_{acc} + \left. \frac{dp_x}{dt} \right|_{decc} &= 0 \\ -nqE_x - \frac{p_x}{\bar{t}} &= 0 \\ p_x &= -nq\bar{t}E_x \end{aligned}$$

- The average momentum  $\langle p_x \rangle = \frac{p_x}{n} = -q\bar{t}E_x$ .
- The average velocity  $\langle v_x \rangle = \frac{\langle p_x \rangle}{m_n^*} = -\frac{q\bar{t}m_n^*}{E_x}$
- The current density

$$J_x = -qn \langle v_x \rangle = \frac{nq^2\bar{t}}{m_n^*}E_x$$

but  $J_x = \rho E_x$

$$\therefore \rho = \frac{nq^2\bar{t}}{m_n^*}$$

but  $\rho = qn\mu_n$

$$\therefore \mu_n = \frac{q\bar{t}m_n^*}{= E_x} \quad (\text{cm}^2/\text{V}\cdot\text{sec})$$

$$\therefore J_x = qn\mu_n E_x \text{ for electrons}$$

$$\therefore j_x = q(n\mu_n + p\mu_p)E_x = \rho E_x \text{ for both electrons and holes}$$