EEE 333 - Optical Communication Devices

Lecture 2
Laser Diodes

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1 Remember

$$\frac{\textbf{Stimulated emission rate}}{\textbf{Spontaneous emission rate}} = \frac{1}{\exp(hf/KT) - 1}$$

$$\underbrace{B_{12}N_1\rho_f}_{\text{absorption}} = \underbrace{N_2A_{21}}_{\text{spontaneous emission}} + \underbrace{B_{21}N_2\rho_f}_{\text{stimulated emission}}$$

$\boxed{1.\ Q}$ **How to increase stimulated emission compared to absorption ?**

$\boxed{1.\ A}$ **By Population inversion (make $N_2 > N_1$)**

2 Population inversion

Under the conditions of thermal equilibrium the lower energy level E_1 of the two-level atomic system contains more atoms than the upper energy level E_2 as illustrated in the following figure (a) .

However, to achieve optical amplification it is necessary to create a non-equilibrium distribution of atoms such that the population of the upper energy level is greater than that of the lower energy level (i.e. $N_2 > N_1$). This condition, which is known as population inversion, is illustrated in the following figure (b).

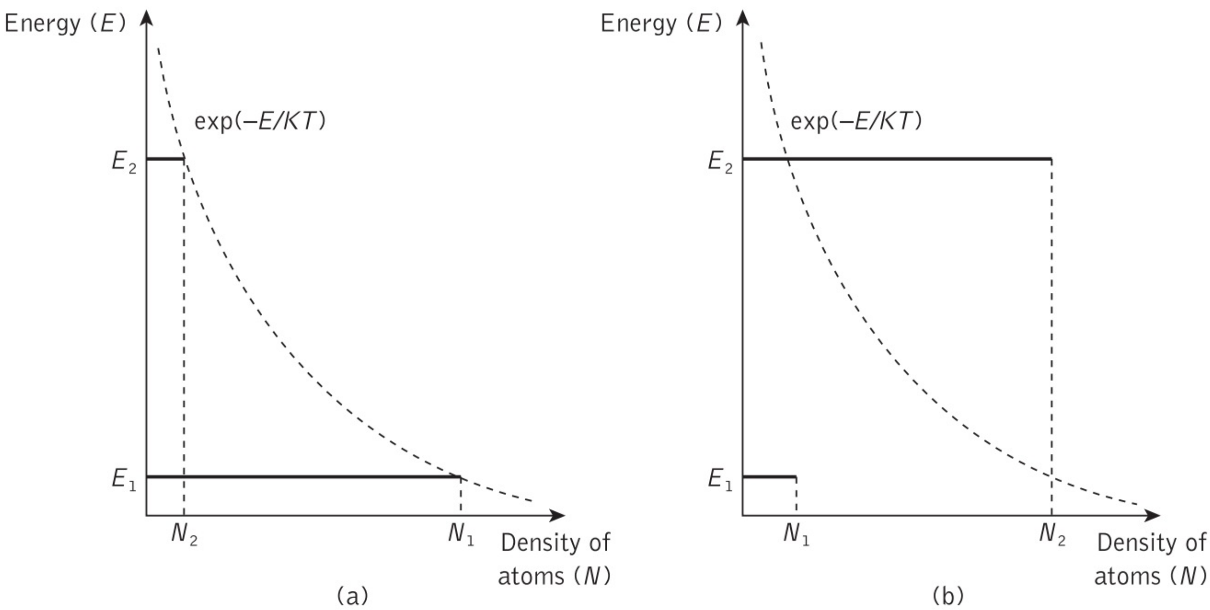

Figure 1: Populations in a two-energy-level system: (a) Boltzmann distribution for a system in thermal equilibrium; (b) a non-equilibrium distribution showing population inversion

This process is achieved using an external energy source (electric current) and is referred to as 'pumping'.

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2. Q How to increase stimulated emission compared to spontaneous emission ?

2. A Increase number of incoming photons (via optical feedback)

3 Optical feedback

Light amplification in the laser occurs when a photon colliding with an atom in the excited energy state causes the stimulated emission of a second photon and then both these photons release two more.

So increase the number of colliding photons we place or form mirrors (plane or curved) at either end of the amplifying medium

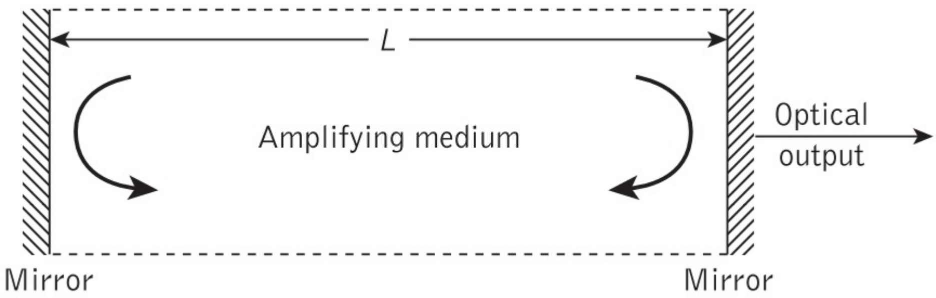


Figure 2: The basic laser structure incorporating plane mirrors

when sufficient population inversion exists in the amplifying medium the radiation builds up and becomes established as standing waves between the mirrors. These standing waves exist only at frequencies for which the distance between the mirrors is an integral number of half wavelengths. Thus when the optical spacing between the mirrors is L, the resonance condition along the axis of the cavity is given by :

$$L = \frac{\lambda q}{2n} \tag{1}$$

Where

λ : the emission wavelength

n : the refractive index of the amplifying medium

q : an integer

so we can get the wavelength λ :

$$\lambda = \frac{2nL}{q} \tag{2}$$

therefore the discrete emission frequencies f :

$$f = \frac{qc}{2nL} \tag{3}$$

The different frequencies of oscillation within the laser cavity are determined by the various integer values of q and each constitutes a resonance or mode.

from the Eq 3, it may be observed that these modes are separated by a frequency interval δf where :

$$\delta f = \frac{c}{2nL} \tag{4}$$

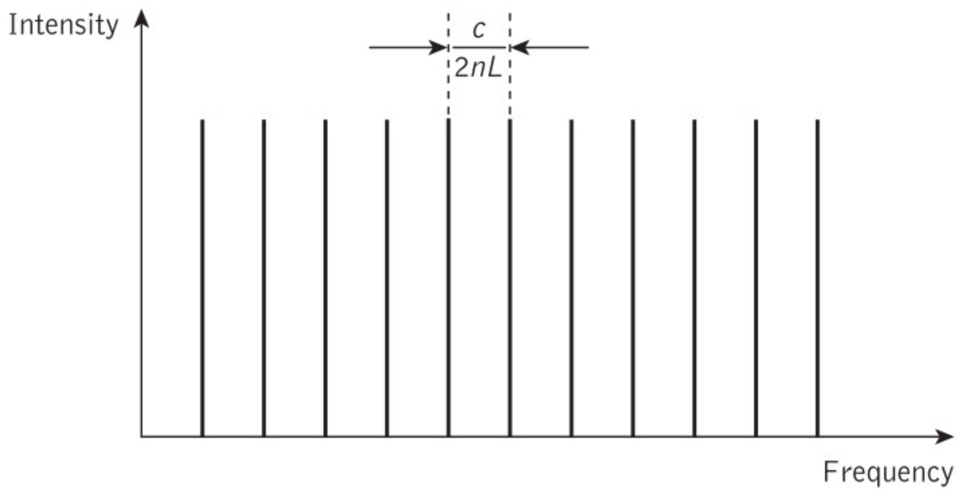


Figure 3: The modes in the laser cavity. Note how they are separated by a frequency interval δf

Although there are large number of modes may be generated within the laser cavity, the spectral output from the device is defined by the gain curve. Hence the laser emission will only include the longitudinal modes contained within the spectral width of the gain curve.

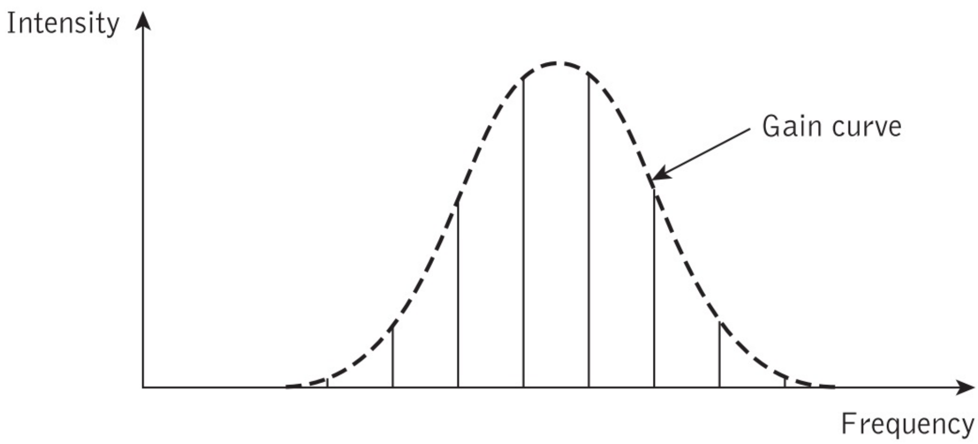


Figure 4: The longitudinal modes in the laser output (only contained within the spectral width of the gain curve)

Notice that $f = \frac{c}{\lambda}$, so the relation between f and λ is not linear, so $\delta\lambda \neq \delta f$
To get $\delta\lambda$:

$$\because \lambda = \frac{c}{f} \qquad \therefore \delta\lambda = -\frac{c}{f^2} \tag{5}$$

$$\therefore \delta\lambda = \frac{\lambda^2}{c}\delta f \tag{6}$$

To get the mode separation in terms of the free space wavelength from separation in terms of the frequency, assuming that $\delta f << f$, substitute with Eq 6 in Eq 4:

$$\delta\lambda = \frac{\lambda^2}{2nL} \tag{7}$$

$\boxed{\text{3. Q}}$ A ruby laser contains a crystal of length 4 cm with a refractive index of 1.78. The peak emission wavelength from the device is 0.55 μm. Determine the frequency separation of longitudinal modes.

$\boxed{\text{3. A}}$ Using Eq. 4 the frequency separation of the modes is:

$$\delta f = \frac{2.998 \times 10^8}{2 \times 1.78 \times 0.04} = 2.1\text{GHz}$$

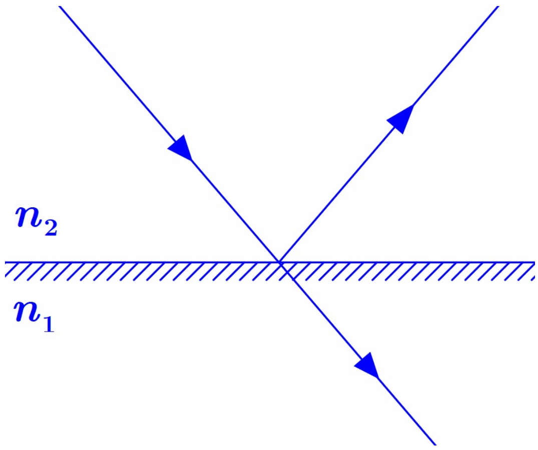
4 Reflectivity

For a light beam passes from one substance with refractive index n_1 into another with refractive index n_2, we define refraction coefficient as:

$$\Gamma = \frac{\text{reflected field}}{\text{incident field}} = \frac{n_1 - n_2}{n_1 + n_2} \tag{8}$$

therefore the reflectivity :

$$r = \frac{P_{\text{reflected}}}{P_{\text{incident}}} = \Gamma^2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \tag{9}$$



5 Threshold condition for laser oscillation

When the steady-state conditions for laser oscillation are achieved ? when the gain in the amplifying medium exactly balances the total losses

We want to define threshold gain per unit length $\bar{g}_{th}$, a high threshold gain per unit length is required in order to balance the losses from the cavity for laser action to be easily achieved

the fractional loss incurred by the light beam is

$$\text{Fractional loss } = r_1 r_2 \exp(-2\bar{\alpha}L) \tag{10}$$

Where

$\bar{a}$: loss coefficient per unit length (cm^{-1})

L : amplifying medium length (spacing between the mirrors)

r_1 : reflectivity of first mirror

r_2 : reflectivity of second mirror

the fractional gain of round trip

$$\text{Fractional gain } = \exp(2\bar{g}L) \tag{11}$$

Where

$\bar{g}$: gain coefficient per unit length (cm^{-1})

Hence :

$$\exp(2\bar{g}L) \times r_1 r_2 \exp(-2\bar{\alpha}L) = 1 \tag{12}$$

therefore :

$$r_1 r_2 \exp[2(\bar{g} - \bar{\alpha})L] = 1 \tag{13}$$

The threshold gain per unit length may be obtained by rearranging the above expression to give:

$$\bar{g}_{\text{th}} = \bar{\alpha} + \frac{1}{2L} \ln \frac{1}{r_1 r_2} \tag{14}$$

Special case : **When the two mirrors have the same reflectivity $r_1 = r_2 = r$**

$$\bar{g}_{\text{th}} = \bar{\alpha} + \frac{1}{L} \ln \frac{1}{r} \tag{15}$$

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4. Q **An injection laser has an active cavity with losses of 30 cm^{-1} and the reflectivity of the each cleaved laser facet is 30%. Determine the laser gain coefficient for the cavity when it has a length of 600 μm.**

4. A **The threshold gain per unit length when $r_1 = r_2 = r$ is given by Eq. 15 as**

$$\begin{aligned}\bar{g}_{\text{th}} &= \bar{\alpha} + \frac{1}{L} \ln \frac{1}{r} \\ &= 30 + \frac{1}{0.06} + \ln \frac{1}{0.3} \\ &= 50 \text{ cm}^{-1}\end{aligned}$$

The given in that type of examples can be

r_1, r_2 : **use Eq 14**

$r_1 = r_2 = r$: **use Eq 15**

Γ : **use Eq 9 to get r**

n_1, n_2 :**use Eq 9 to get r**

MCQ Questions

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1. Q **Pumbing is the process of**

ⓐ optical feedback

ⓑ population inversion

ⓒ photo detection

ⓓ stimulated emission

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2. Q **What determines the different frequencies of oscillation within the laser cavity?**

ⓐ The temperature of the laser medium

ⓑ The pumping power of the laser

ⓒ The different integer values of q

ⓓ The type of laser medium used

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3. Q **What is a resonance (or mode) in a laser cavity?**

ⓐ A specific frequency of oscillation within the laser cavity

ⓑ The type of laser medium used

ⓒ The amount of pumping power required to achieve population inversion

ⓓ The temperature of the laser medium

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4. Q **The relation between λ and f is linear, so $\delta\lambda = \delta f$**

ⓐ True

ⓑ False

5. Q **What determines the spectral output of a laser device?**

(a) The number of longitudinal modes generated within the laser cavity

(b) The gain curve of the laser medium

(c) The temperature of the laser medium

(d) The pumping power of the laser

6. Q **The relation between reflectivity r and refraction coefficient Γ is :**

(a) $r = \Gamma^2$

(b) $\Gamma = r^2$

(c) $\Gamma = \dfrac{r_1 - r_2}{r_1 + r_2}$

(d) $\Gamma = \dfrac{r_1 + r_2}{r_1 - r_2}$

7. Q **When are steady-state conditions for laser oscillation achieved?**

(a) When the pumping power of the laser is at its maximum

(b) When the gain in the amplifying medium exactly balances the total losses

(c) When the temperature of the laser medium is at its maximum

(d) When the number of photons produced by the laser is at its maximum

Answers :

$\boxed{\textbf{1. A}}$ (b)

$\boxed{\textbf{2. A}}$ (c)

$\boxed{\textbf{3. A}}$ (a)

$\boxed{\textbf{4. A}}$ (b)

$\boxed{\textbf{5. A}}$ (b)

$\boxed{\textbf{6. A}}$ (a)

$\boxed{\textbf{7. A}}$ (b)