EEE 333 - Optical Communication Devices

Lecture 4

Electron Density n and Photon Density ϕ Rate Equations

Effeciency

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1 Electron Density n and Photon Density ϕ Rate Equations

The two rate equations for electron density n, and photon density ϕ, are:

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{J}{ed} - \frac{n}{\tau_{\mathrm{sp}}} - Cn\phi \quad (\mathrm{m}^{-3}\ \mathrm{s}^{-1}) \tag{1}$$

Where

$\frac{J}{ed}$: increase in the electron concentration in the conduction band as the current flows into the junction diode.

$\frac{n}{\tau_{\mathrm{sp}}}$: rate of decrease due to spontaneous emission

$Cn\phi$: rate of decrease due to stimulated emission

J : Current density

e : charge on an electron

d : thickness of the recombination region

n : electron density

τ_{sp} : spontaneous emission lifetime

C : coefficient which incorporates the B (Einstein coefficients)

ϕ : photon density

and

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = Cn\phi + \delta\frac{n}{\tau_{\mathrm{sp}}} - \frac{\phi}{\tau_{\mathrm{ph}}} \quad (\mathrm{m}^{-3}\ \mathrm{s}^{-1}) \tag{2}$$

Where

$Cn\phi$: increase of photon density due to stimulated emission

$\delta\frac{n}{\tau_{\mathrm{sp}}}$: The fraction of photons produced by spontaneous emission which combine to the energy in the lasing mode

$\frac{\phi}{\tau_{ph}}$: the decay in the number of photons resulting from losses in the optical cavity

C : coefficient which incorporates the B (Einstein coefficients)

n : electron density

ϕ : photon density

δ : small fractional value (number of contribution spontaneous emission photons are very small)

τ_{ph} : photon lifetime.

Although these rate equations may be used to study both the transient and steady-state behavior of the semiconductor laser, we are particularly concerned with the steady-state solutions. The steady state is characterized by the left hand side of Eqs (1) and (2) being equal to zero, when n and ϕ have nonzero values. In addition, the fields in the optical cavity which are represented by ϕ must build up from small initial values, and hence $\frac{d\phi}{dt}$ must be positive when ϕ is small. Therefore, setting δ equal to zero in Eq. (2), it is clear that for any value of ϕ, $\frac{d\phi}{dt}$ will only be positive when:

$$Cn - \frac{1}{\tau_{ph}} \geq 0 \tag{3}$$

There is therefore a threshold value of n which satisfies the equality of Eq. (3). If n is larger than this threshold value, then ϕ can increase; however, when n is smaller it cannot. From Eq. (3) the threshold value for the electron density n_{th} is:

$$n_{th} = \frac{1}{C\tau_{ph}} \quad \left(\mathrm{m}^{-3}\right) \tag{4}$$

The threshold current written in terms of its current density J_{th}, required to maintain $n = n_{th}$ in the steady state when $\phi = 0$, may be obtained from Eq. (1) as:

$$\frac{J_{th}}{ed} = \frac{n_{th}}{\tau_{sp}} \quad \left(\mathrm{m}^{-3}\ \mathrm{s}^{-1}\right) \tag{5}$$

Hence Eq. (5) defines the current required to sustain an excess electron density in the laser when spontaneous emission provides the only decay mechanism. The steady-state photon density ϕ_s is provided by substituting Eq. (5) in Eq. (1) giving:

$$0 = \frac{(J - J_{\mathrm{th}})}{ed} - Cn_{\mathrm{th}}\phi_s \tag{6}$$

Rearranging we obtain:

$$\phi_s = \frac{1}{Cn_{\mathrm{th}}}\frac{(J - J_{\mathrm{th}})}{ed} \quad (\mathrm{m}^{-3}) \tag{7}$$

Substituting for Cn_{th} from Eq. (4) we can write Eq. (7) in the form:

$$\phi_s = \frac{\tau_{\mathrm{ph}}}{ed}\frac{(J - J_{\mathrm{th}})}{ed} \quad (\mathrm{m}^{-3}) \tag{8}$$

The photon density ϕ_s cannot be a negative quantity as this is meaningless, and for ϕ_s to be greater than zero the current must exceed its threshold value. Moreover, ϕ_s is proportional to the amount by which J exceeds its threshold value.

The threshold current density for stimulated emission J_{th} is to a fair approximation related to the threshold gain coefficient $\overline{g_{\mathrm{th}}}$ for the laser cavity through:

$$\overline{g_{\mathrm{th}}} = \overline{\beta} J_{\mathrm{th}} \tag{9}$$

Where

$\overline{\beta}$: gain factor (cm A^{-1})

J_{th} : threshold current density for stimulated emission (A cm^{-2})

$\overline{g_{\mathrm{th}}}$:threshold gain coefficient (cm^{-1})

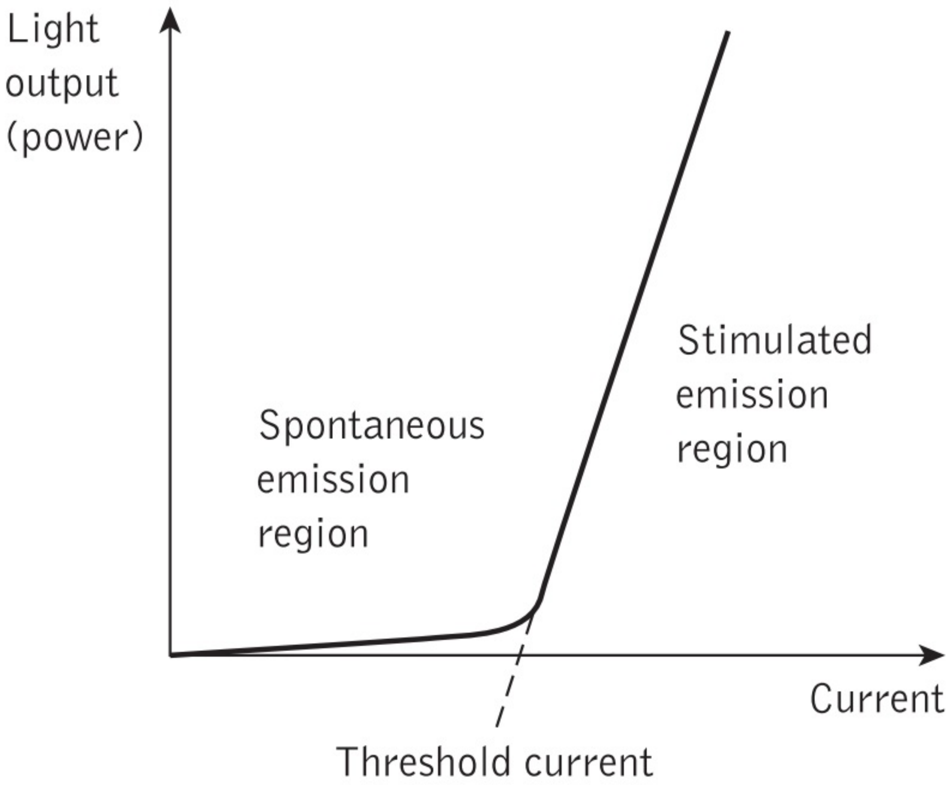

Figure 1: The ideal light output against current characteristic for an injection laser

remember from lecture 2 :

$$\bar{g}_{\mathrm{th}} = \bar{\alpha} + \frac{1}{2L} \ln \frac{1}{r_1 r_2} \tag{10}$$

Substituting for $\overline{g_{\mathrm{th}}}$ from Eq. (10) and rearranging we obtain:

$$J_{\mathrm{th}} = \frac{1}{\bar{\beta}} \left[\bar{\alpha} + \frac{1}{2L} \ln \frac{1}{r_1 r_2} \right] \tag{11}$$

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1. Q **A GaAs injection laser has an optical cavity of length 250 μm and width 100 μm. At normal operating temperature the gain factor $\overline{B}$ is 21×10^3 cm A^{-1} and the loss coefficient $\overline{\alpha}$ per cm is 10. Determine the threshold current density and hence the threshold current for the device. It may be assumed that the cleaved mirrors are uncoated and that the current is restricted to the optical cavity. The refractive index of GaAs may be taken as 3.6.**

1. A

The reflectivity for normal incidence of a plane wave on the GaAs–air interface may be obtained from the following figure where:

$$r = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = \left(\frac{3.6 - 1}{3.6 + 1}\right)^2 \simeq 0.32$$

The threshold current density may be obtained from Eq. (11) where:

$$\begin{aligned} J_{\text{th}} &= \frac{1}{\bar{\beta}}\left[\bar{\alpha}\frac{1}{L}\ln\frac{1}{r}\right] \\ &= \frac{1}{21 \times 10^{-3}}\left[10 + \frac{1}{250 \times 10^{-4}}\ln\frac{1}{0.32}\right] \\ &= 2.65 \times 10^3 \text{ A cm}^{-2} \end{aligned}$$

The threshold current I_{th} is given by:

$$\begin{aligned} I_{\text{th}} &= J_{\text{th}} \times \text{ area of the optical cavity} \\ &= 2.65 \times 10^3 \times 250 \times 100 \times 10^{-8} \\ &\simeq 663 \text{ mA} \end{aligned}$$

Therefore the threshold current for this device is 663 mA if the current flow is restricted to the optical cavity.

2 Efficiency

There are a number of ways in which the operational efficiency of the semiconductor laser may be defined.

2.1 Differential External Quantum Efficiency η_{D}

Definition : ratio of the increase in photon output rate for a given increase in the number of injected electrons.

$$\eta_{\text{D}} = \frac{\mathrm{d}P_{\text{e}}/hf}{\mathrm{d}I/e} \simeq \frac{\mathrm{d}P_{\text{c}}}{\mathrm{d}I\,(E_{\text{g}})} \tag{12}$$

Where

η_D : differential external quantum efficiency

P_e : optical power emitted from the device[1]

I :current

e : charge on an electron

hf : photon energy

E_{gg} : bandgap energy (eV)

It may be noted that η_D gives a measure of the rate of change of the optical output power with current and hence defines the slope of the output characteristic (figure 1) in the lasing region for a particular device Hence η_D is sometimes referred to as the slope quantum efficiency.

For a continuous wave semiconductor laser it usually has values in the range 40% to 60%

2.2 Internal Quantum Efficiency η_i

Definition : $\eta_i = \frac{\text{number of photons produced in the laser cavity}}{\text{number of injected electrons}}$.
may be quite high with values usually in the range 50% to 100%

$$\eta_D = \eta_i \left[\frac{1}{1 + (2\bar{\alpha}L/\ln(1/r_1 r_2))} \right] \tag{13}$$

Where

$\bar{\alpha}$: loss coefficient of the laser cavity,

L : length of the laser cavity

r_1, r_2 : cleaved mirror reflectivities.

2.3 Total Efficiency (External Quantum Efficiency) η_i

Definition : $\eta_T = \dfrac{\text{total number of output photons}}{\text{total number of injected electrons}}$.

$$\eta_T = \frac{P_e/hf}{I/e} \simeq \frac{P_e}{IE_g} \tag{14}$$

As the power emitted P_e changes linearly when the injection current I is greater than the threshold current I_{th}, then:

$$\frac{\eta_T}{\eta_D} = \frac{dI}{I} = \frac{I - I_{th}}{I} = 1 - \frac{I_{th}}{I}$$

$$\therefore \eta_T \simeq \eta_D \left(1 - \frac{I_{th}}{I}\right)$$

2.4 External Power Efficiency

Definition : $\eta_{ep} = \dfrac{\text{optical output power}}{\text{electrical input power}}$.

$$\eta_{ep} = \frac{P_c}{P} = \frac{P_c}{IV} \tag{15}$$

Where

$P = IV$: the d.c. electrical input power.

Using Eq. (14) for the total efficiency we find:

$$\eta_{ep} = \eta_T \left(\frac{E_g}{V}\right) \tag{16}$$

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2. Q The total efficiency of an injection laser with a GaAs active region is 18%. The voltage applied to the device is 2.5 V and the bandgap energy for GaAs is 1.43 eV. Calculate the external power efficiency of the device.

2. A Using Eq. (16), the external power efficiency is given by:

$$\eta_{ep} = 0.18 \left(\frac{1.43}{2.5}\right) \simeq 10\%$$

This result indicates the possibility of achieving high overall power efficiencies from

semiconductor lasers which are much larger than for other laser types.

3 Single mode laser

Remember from lecture two :

$$\delta f = \frac{c}{2nL} \tag{17}$$

in order to get a single mode laser, the modes must be separated by large δf, so that the gain curve covers only one mode , the FSR is the spacing between two successive modes

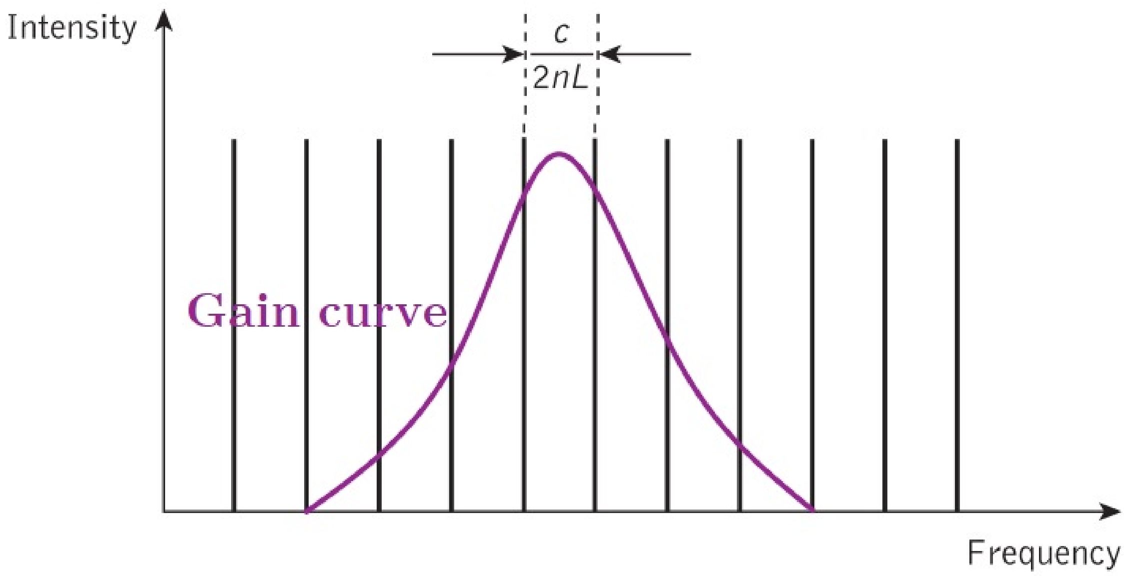

Figure 2: The modes in the laser cavity. Note how they are separated by a frequency interval δf, and the gain curve

note that nodes are not perfectly delta as shown in figure (lion), the FWHM is the width of a line shape at half of its maximum amplitude

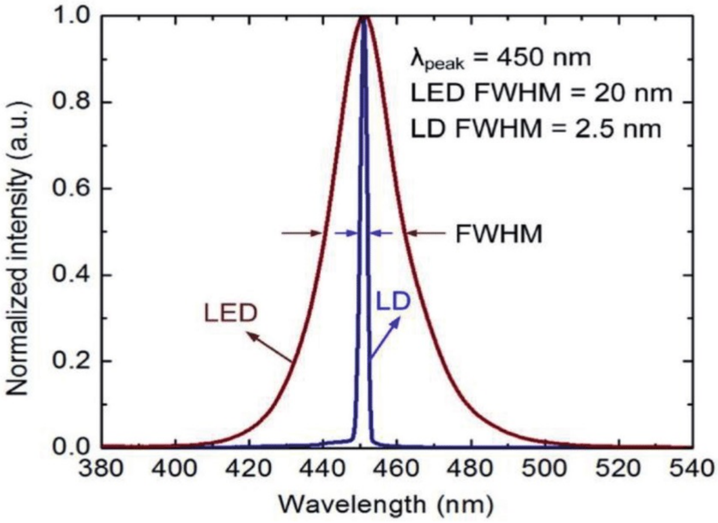

Figure 3: FWHM (Full Width at Half Maximum) of LED compared to Laser (LD)

In the next lecture, we will dicusss finesse, the finesse of an optical resonator is defined as the ratio of the free spectral range (FSR) of the cavity to the full width at half maximum (FWHM) of the resonances:

$$\text{finesse} = \frac{\mathbf{FSR}}{\mathbf{FWHM}}$$

a high finesse means sharp resonances and better laser source with less dispertion system

MCQ Questions

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1. Q What is the physical meaning of the term $Cn\phi$ in the rate equation for electron density?

A. The rate of electron recombination B. The rate of electron injection C. The rate of photon absorption D. The rate of photon emission

(a) rate of increase due to current flow

(b) rate of decrease due to absorption

(c) rate of decrease due to spontaneous emission

(d) rate of decrease due to stimulated emission

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2. Q What is the condition for the rate of field buildup in the optical cavity, $\frac{d\phi}{dt}$, to be positive when the field ϕ is small?

(a) $Cn < \frac{1}{\tau_{ph}}$

(b) $Cn > \frac{1}{\tau_{ph}}$

(c) $Cn = \frac{1}{\tau_{ph}}$

(d) $Cn \leq \frac{1}{\tau_{ph}}$

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3. Q What is the threshold gain coefficient for a laser cavity with a threshold current density of 100 A cm^{-2} and a gain factor of 0.2 cm A^{-1}?

(a) 5 cm^{-1}

(b) 20 cm^{-1}

(c) 50 cm^{-1}

(d) 200 cm^{-1}

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4. Q **The gain factor $\overline{\beta}$ is in units**

(a) (cm A^{-1})

(b) (A cm^{-1})

(c) (cm A^{-2})

(d) (A cm^{-2})

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5. Q **Which of the following best describes the differential external quantum efficiency?**

(a) The ratio of the current output rate to the number of incident photons.

(b) The ratio of the current output rate to the power of incident photons.

(c) The ratio of the increase in photon output power to the injected current.

(d) The ratio of the increase in photon output rate to the number of injected electrons.

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6. Q **What is the definition of internal quantum efficiency?**

(a) The ratio of the photon output rate to the injected current.

(b) The ratio of the photon output rate to the number of injected electrons.

(c) The ratio of the number of photons produced in the laser cavity to the number of injected electrons.

(d) The ratio of the number of photons produced in the laser cavity to the number of absorbed photons.

7. Q $\dfrac{\text{total number of output photons}}{\text{total number of injected electrons}}$ best describes ....

(a) Differential external quantum efficiency

(b) Internal quantum efficiency

(c) External quantum efficiency

(d) External power efficiency

8. Q What is the definition of external power efficiency?

(a) The ratio of the optical output power to the electrical input power

(b) The ratio of the increase in photon output rate to the increase in the number of injected electrons

(c) The ratio of the number of photons produced in the laser cavity to the number of injected electrons

(d) The ratio of the emitted radiation to the absorbed radiation

9. Q What does FWHM and FSR stand for?

(a) Full Width at Half Maximum / Fiber Spectral Ratio

(b) Full Width at Half Maximum / Free Spectral Range

(c) Finite Waveguide Hierarchy Model / Fiber Spectral Ratio

(d) Finite Waveguide Hierarchy Model / Free Spectral Range

$\boxed{\textbf{10. Q}}$ **finesse is ....**

ⓐ $\dfrac{\textbf{FSR}}{\textbf{FWHM}}$

ⓑ $\dfrac{\textbf{FWHM}}{\textbf{FSR}}$

ⓒ $\textbf{FWHM} \times \textbf{FSR}$

ⓓ $\textbf{FWHM} + \textbf{FSR}$

Answers :

$\boxed{\textbf{1. A}}$ (d)

$\boxed{\textbf{2. A}}$ (b)

$\boxed{\textbf{3. A}}$ (b)

$\boxed{\textbf{4. A}}$ (a)

$\boxed{\textbf{5. A}}$ (d)

$\boxed{\textbf{6. A}}$ (c)

$\boxed{\textbf{7. A}}$ (c)

$\boxed{\textbf{8. A}}$ (a)

$\boxed{\textbf{9. A}}$ (b)

$\boxed{\textbf{10. A}}$ (a)