Optics Cheat Sheet

[1]

1^{st} window : 850 nm

2^{nd} window : 1310 nm

3^{rd} window : 1550 nm

$$\frac{\text{Stimulated emission rate}}{\text{Spontaneous emission rate}} = \frac{1}{\exp(hf/KT) - 1} \tag{1}$$

The Einstein relations

Absorption

the upward transition rate R_{12} (indicating an electron transition from level 1 to level 2) may be written as:

$$R_{12} \propto N_1 \rho_f \tag{2}$$
$$R_{12} = B_{12} N_1 \rho_f \tag{3}$$

Where

R_{12} : electron transition from level 1 to level 2

ρ_f : spectral density

N_1 : represent the density of atoms in energy levels E_1

B_{12} : Einstein coefficient of absorption.

Spontaneous Emission

R_{21} (indicating an electron transition from level 2 to level 1) in case of spontaneous emission

$$R_{21} \propto N_2 \tag{4}$$
$$R_{21} = A_{21} N_2 \tag{5}$$

Where

R_{12} : electron transition from level 2 to level 1

N_2 : represent the density of atoms in energy levels E_2

A_{21} : Einstein coefficient of spontaneous emission

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Stimulated Emission

R_{21} (indicating an electron transition from level 2 to level 1) in case of stimulated emission

$$R_{21} \propto N_2 \rho_f \tag{6}$$
$$R_{21} = B_{21} N_2 \rho_f \tag{7}$$

Where

R_{21} : electron transition from level 2 to level 1

ρ_f : spectral density

N_2 : represent the density of atoms in energy levels E_2

B_{21} : Einstein coefficient of stimulated emmision.

Optical feedback

optical spacing between the mirrors is L, the resonance condition along the axis of the cavity is given by :

$$L = \frac{\lambda q}{2n} \tag{8}$$

Where

λ : the emission wavelength

n : the refractive index of the amplifying medium

q : an integer

Rearranging :

$$f = \frac{qc}{2nL} \tag{9}$$

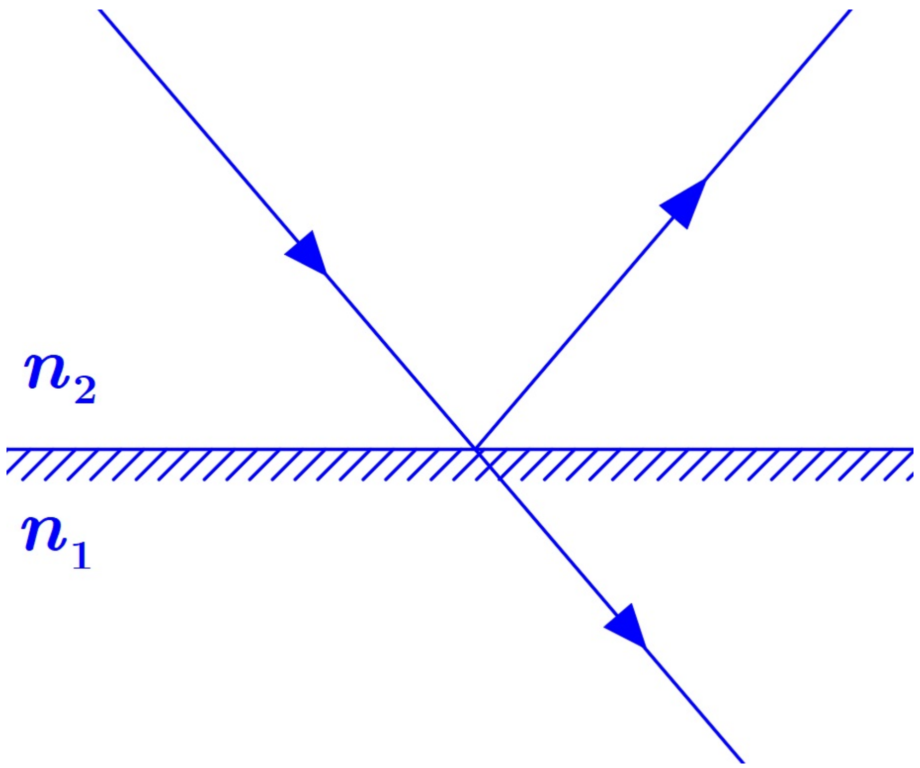
from the Eq 9, it may be observed that these modes are separated by a frequency interval δf where :

$$\delta f = \frac{c}{2nL} \tag{10}$$

mode separation in terms of the free space wavelength :

$$\delta\lambda = \frac{\lambda^2}{2nL} \tag{11}$$

Reflectivity



For a light beam passes from one substance with refractive index n_2 into another with refractive index n_1, we define refraction coefficient as:

$$\Gamma = \frac{\text{reflected field}}{\text{incident field}} = \frac{n_1 - n_2}{n_1 + n_2} \tag{12}$$

therefore the reflectivity :

$$r = \frac{P_{\text{reflected}}}{P_{\text{incident}}} = \Gamma^2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \tag{13}$$

Threshold condition for laser oscillation

the fractional loss incurred by the light beam is

$$\text{Fractional loss } = r_1 r_2 \exp(-2\bar{\alpha} L) \tag{14}$$

Where

$\bar{a}$: loss coefficient per unit length (cm^{-1})

L : amplifying medium length (spacing between the mirrors)

r_1 : reflectivity of first mirror

r_2 : reflectivity of second mirror

the fractional gain of round trip

$$\text{Fractional gain } = \exp(2\bar{g}L) \tag{15}$$

Where

$\bar{g}$: gain coefficient per unit length (cm^{-1})

Therefore :

$$\bar{g}_{\mathrm{th}} = \bar{\alpha} + \frac{1}{2L}\ln\frac{1}{r_1 r_2} \tag{16}$$

$$E_{\mathrm{g}} = hf = \frac{hc}{\lambda} \tag{17}$$

Where

c : velocity of light

λ : wavelength

Substituting the appropriate values for h and c and rearranging gives:
Where

$$\lambda = \frac{1.24}{E_{\mathrm{g}}} \tag{18}$$

E_g in (eV)

λ in (μm)

The radiative minority carrier lifetime τ_r

$$\tau_{\mathrm{r}} = [B_{\mathrm{r}}(N+P)]^{-1} \tag{19}$$

Where

N : majority carrier concentrations in the n-type

P : majority carrier concentrations in the p-type

B_r : recombination coefficient

Electron Density n and Photon Density ϕ Rate Equations

The two rate equations for electron density n, and photon density ϕ, are:

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{J}{ed} - \frac{n}{\tau_{\mathrm{sp}}} - Cn\phi \quad \left(\mathrm{m}^{-3}\ \mathrm{s}^{-1}\right) \tag{20}$$

Where

$\frac{J}{ed}$: increase in the electron concentration in the conduction band as the current flows into the junction diode.

$\frac{n}{\tau_{\mathrm{sp}}}$: rate of decrease due to spontaneous emission

$Cn\phi$: rate of decrease due to stimulated emission

J : Current density

e : charge on an electron

d : thickness of the recombination region

n : electron density

τ_{sp} : spontaneous emission lifetime

C : coefficient which incorporates the B (Einstein coefficients)

ϕ : photon density

and

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = Cn\phi + \delta\frac{n}{\tau_{\mathrm{sp}}} - \frac{\phi}{\tau_{\mathrm{ph}}} \quad \left(\mathrm{m}^{-3}\ \mathrm{s}^{-1}\right) \tag{21}$$

Where

$Cn\phi$: increase of photon density due to stimulated emission

$\delta\frac{n}{\tau_{\mathrm{sp}}}$: The fraction of photons produced by spontaneous emission which combine to the energy in the lasing mode

$\frac{\phi}{\tau_{\mathrm{ph}}}$: the decay in the number of photons resulting from losses in the optical cavity

C : coefficient which incorporates the B (Einstein coefficients)

n : electron density

ϕ : photon density

δ : small fractional value (number of contribution spontaneous emission photons are very small)

τ_{ph} : photon lifetime.

The photon density ϕ_s

$$\phi_s = \frac{\tau_{\mathrm{ph}}}{ed}(J - J_{\mathrm{th}}) \quad \left(\mathrm{m}^{-3}\right) \tag{22}$$

The threshold current density for stimulated emission J_{th} is to a fair approximation related to the threshold gain coefficient $\overline{g_{\mathrm{th}}}$ for the laser cavity through:

$$\overline{g_{\mathrm{th}}} = \overline{\beta}J_{\mathrm{th}} \tag{23}$$

Where

$\overline{\beta}$: gain factor (cm A^{-1})

J_{th} : threshold current density for stimulated emission (A cm^{-2})

$\overline{g_{\mathrm{th}}}$:threshold gain coefficient (cm^{-1})

Recall :

$$\overline{g}_{\mathrm{th}} = \bar{\alpha} + \frac{1}{2L}\ln\frac{1}{r_1 r_2} \tag{24}$$

Substituting for $\overline{g_{\mathrm{th}}}$ from Eq. (24) and rearranging we obtain:

$$J_{\mathrm{th}} = \frac{1}{\overline{\beta}}\left[\bar{\alpha} + \frac{1}{2L}\ln\frac{1}{r_1 r_2}\right] \tag{25}$$

Efficiency

Differential External Quantum Efficiency η_{D}

ratio of the increase in photon output rate for a given increase in the number of injected electrons.

$$\eta_{\mathrm{D}} = \frac{\mathrm{d}P_{\mathrm{e}}/hf}{\mathrm{d}I/e} \simeq \frac{\mathrm{d}P_{\mathrm{c}}}{\mathrm{d}I\left(E_{\mathrm{g}}\right)} \tag{26}$$

Where

η_{D} : differential external quantum efficiency

P_{e} : optical power emitted from the device[2]

I :current

e : charge on an electron

hf : photon energy

E_{gg} : bandgap energy (eV)

Internal Quantum Efficiency η_{i}

$\eta_{\mathrm{i}} = \frac{\text{number of photons produced in the laser cavity}}{\text{number of injected electrons}}$.

$$\eta_{\mathrm{D}} = \eta_{\mathrm{i}}\left[\frac{1}{1 + (2\bar{\alpha}L/\ln(1/r_1 r_2))}\right] \tag{27}$$

Where

$\bar{\alpha}$: loss coefficient of the laser cavity,

L : length of the laser cavity

r_1, r_2 : cleaved mirror reflectivities.

Total Efficiency (External Quantum Efficiency) η_{i}

$\eta_{\mathrm{T}} = \dfrac{\text{total number of output photons}}{\text{total number of injected electrons}}.$

$$\eta_{\mathrm{T}} = \frac{P_{\mathrm{e}}/hf}{I/e} \simeq \frac{P_{\mathrm{e}}}{IE_{\mathrm{g}}} \tag{28}$$

$$\eta_{\mathrm{T}} \simeq \eta_{\mathrm{D}} \left(1 - \frac{I_{\mathrm{th}}}{I}\right) \tag{29}$$

External Power Efficiency

$\eta_{\mathrm{ep}} = \dfrac{\text{optical output power}}{\text{electrical input power}}$

$$\eta_{\mathrm{ep}} = \frac{P_{\mathrm{c}}}{P} = \frac{P_{\mathrm{c}}}{IV} \tag{30}$$

Where

$P = IV$: the d.c. electrical input power.

Using Eq. (28) for the total efficiency we find:

$$\eta_{\mathrm{ep}} = \eta_{\mathrm{T}} \left(\frac{E_{\mathrm{g}}}{V}\right) \tag{31}$$

free spectral range (FSR) of the cavity to the full width at half maximum (FWHM) of the resonances:

$$\text{finesse} = \frac{\text{FSR}}{\text{FWHM}} \tag{32}$$

Where

FSR : free spectral range

FWHM : full width at half maximum

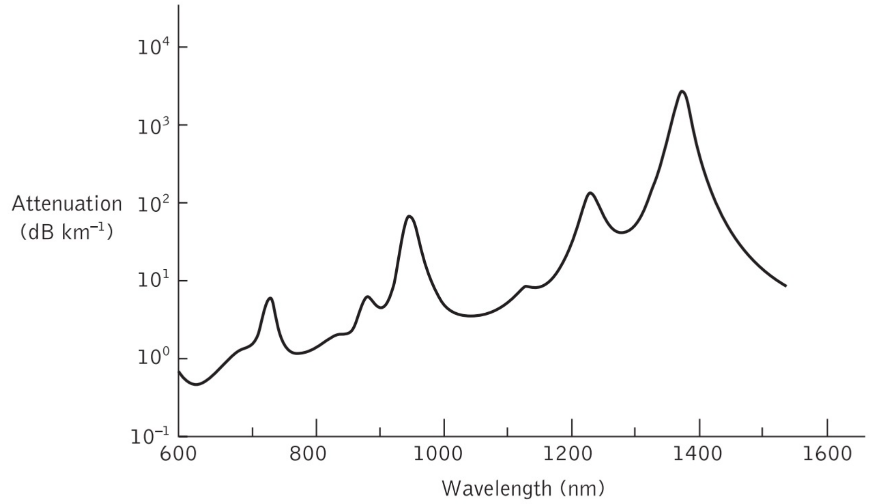


Figure 1: The absorption spectrum for the hydroxyl (OH) group in silica

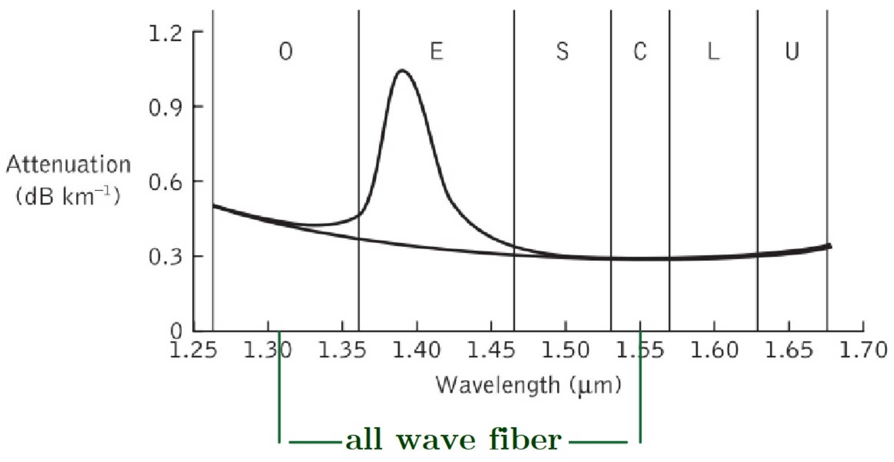


Figure 2: Fiber attenuation spectra, some technologies are use to eliminate the attentuation in the range of (1310 nm 1550 nm) (called the All wave fiber) leades to the use of the 2nd window and 3rd window

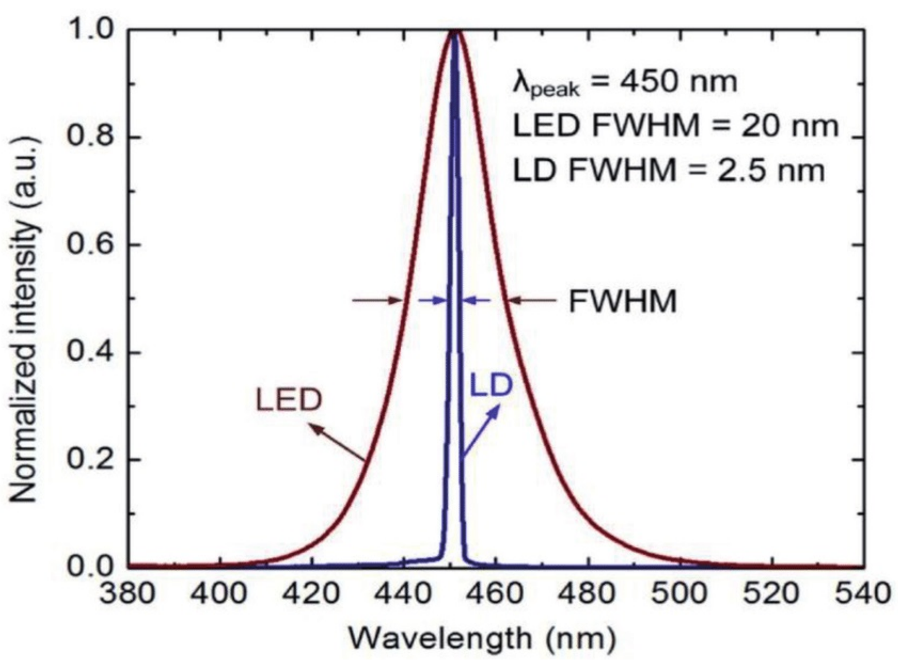


Figure 3: FWHM (Full Width at Half Maximum) of LED compared to Laser (LD)

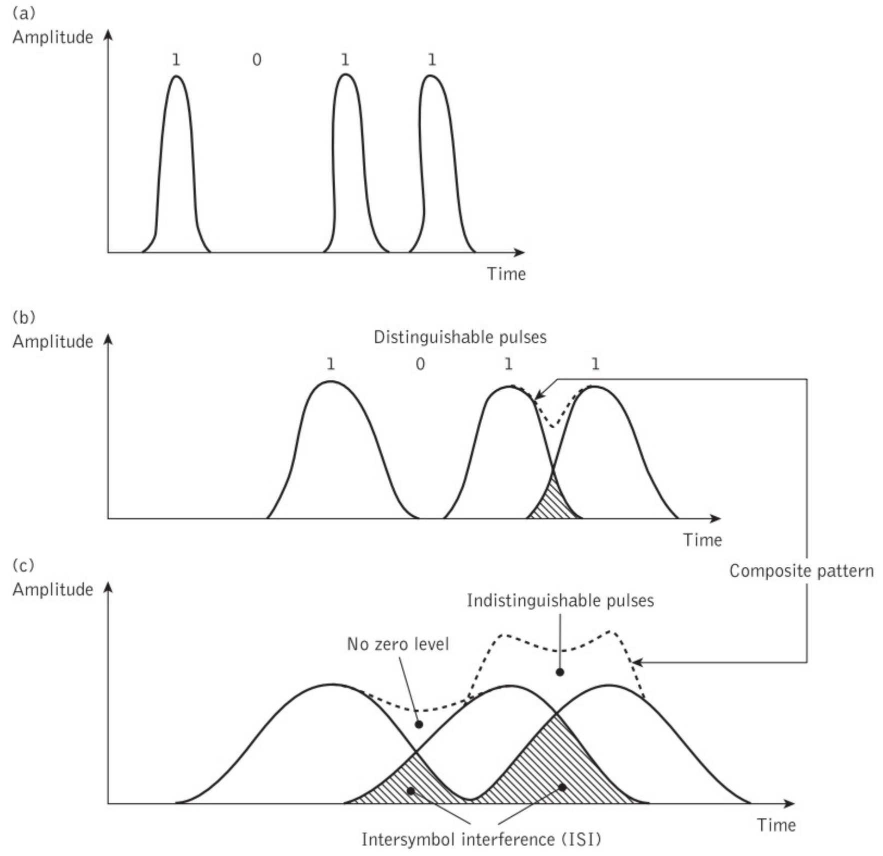


Figure 4: An illustration using the digital bit pattern 1011 of the broadening of light pulses as they are transmitted along a fiber: (a) fiber input; (b) fiber output at a distance L_1; (c) fiber output at a distance $L_2 > L_1$ (Note dispersion effect)

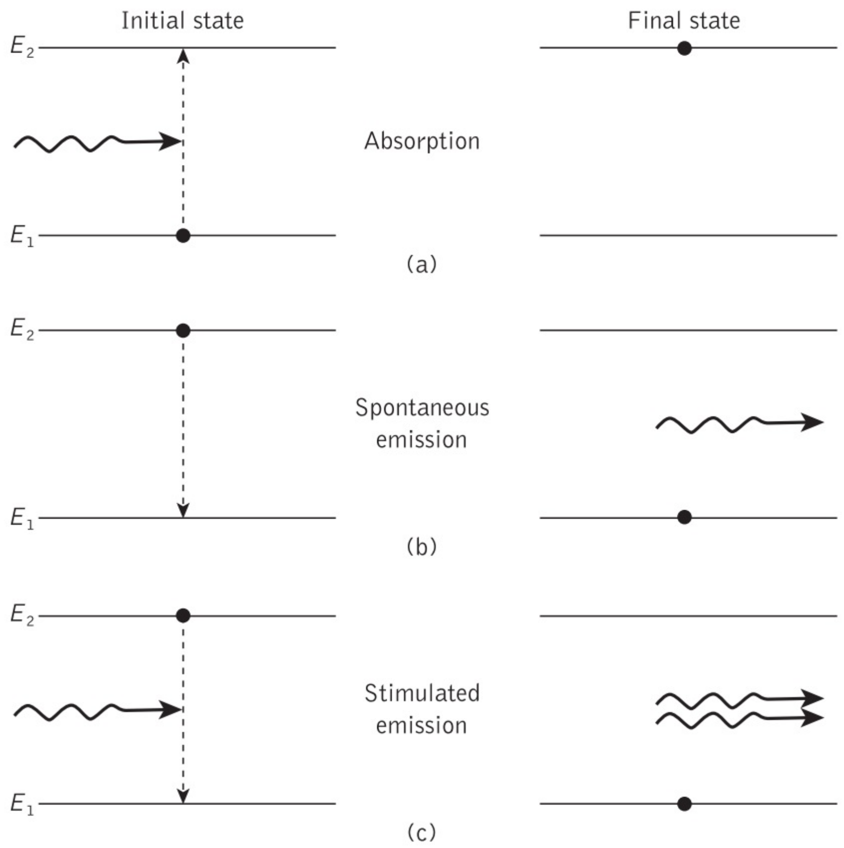


Figure 5: Energy state diagram showing: (a) absorption; (b) spontaneous emission; (c) stimulated emission. The black dot indicates the state of the atom before and after a transition takes place

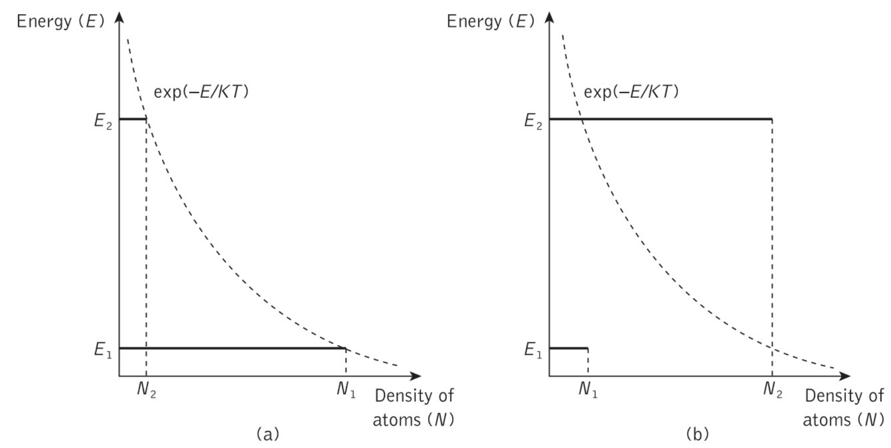


Figure 6: Populations in a two-energy-level system: (a) Boltzmann distribution for a system in thermal equilibrium; (b) a non-equilibrium distribution showing population inversion

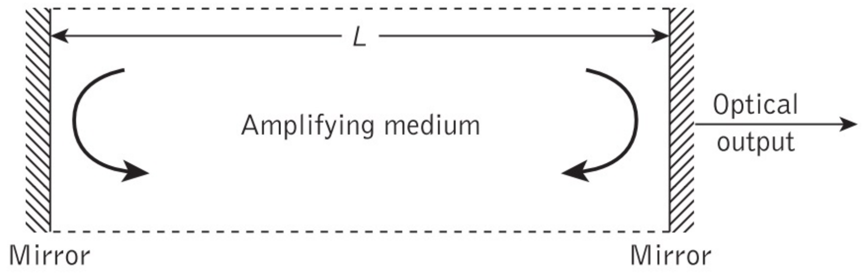


Figure 7: The basic laser structure incorporating plane mirrors (optical feedback)

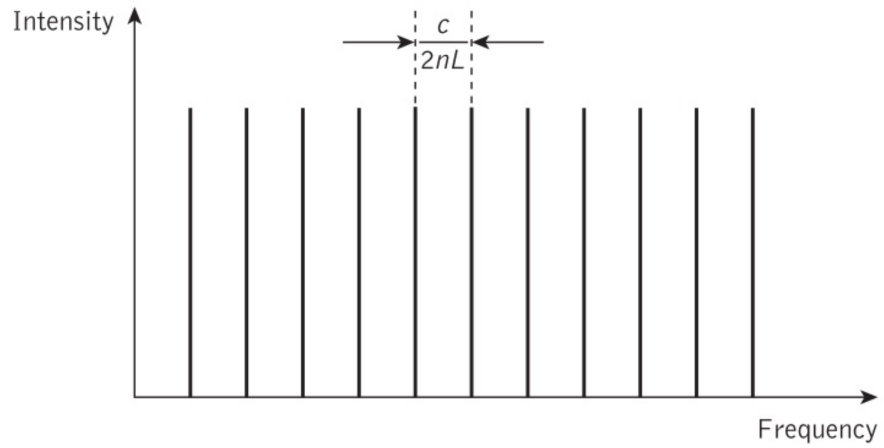


Figure 8: The modes in the laser cavity. Note how they are separated by a frequency interval $\delta f = \frac{c}{2nL}$

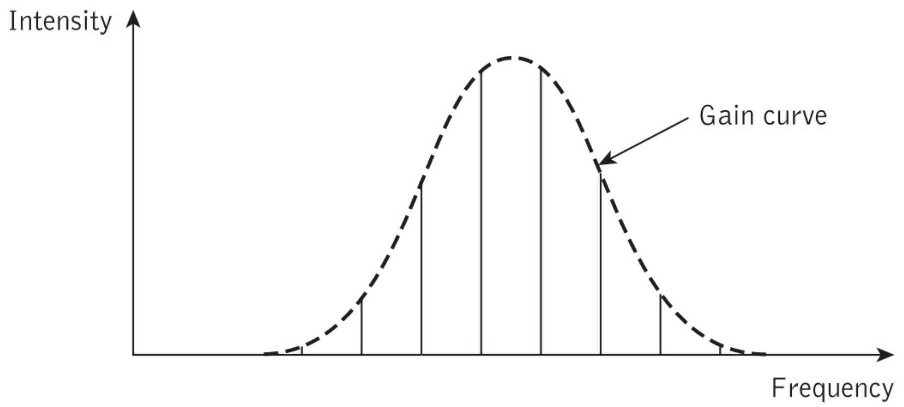


Figure 9: The longitudinal modes in the laser output (only contained within the spectral width of the gain curve)

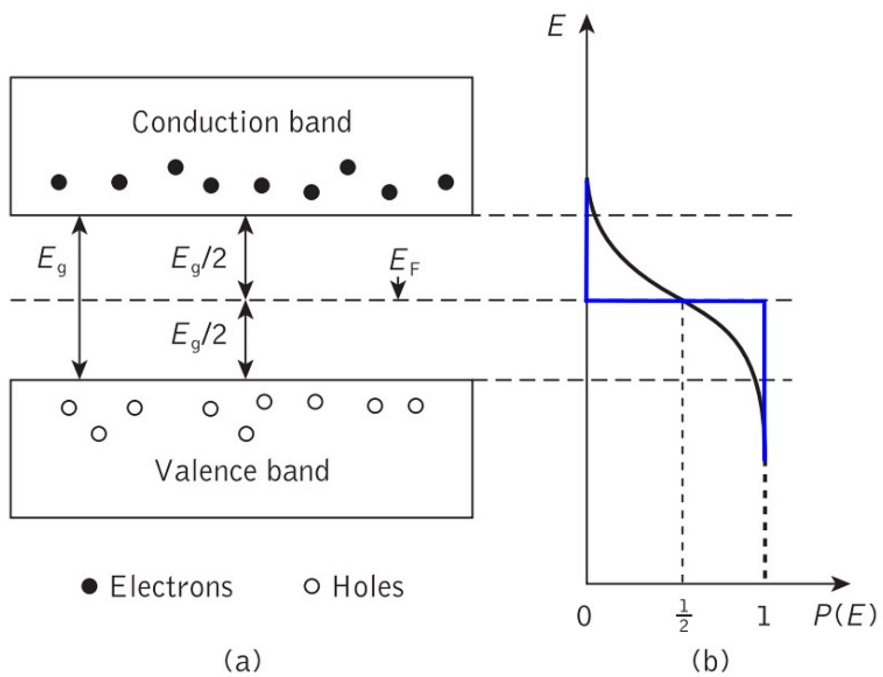


Figure 10: (a) The energy band structure of an intrinsic semiconductor at a temperature above absolute zero, showing an equal number of electrons and holes in the conduction band and the valence band respectively. (b) The Fermi–Dirac probability distribution corresponding to (a) (note that the blue distribution correspond to Fermi–Dirac probability distribution at 0K)

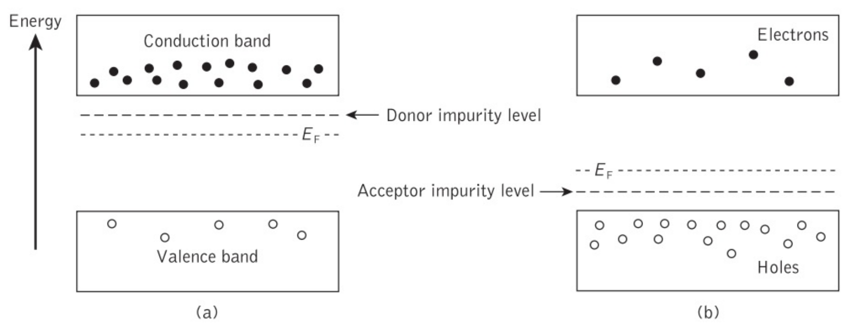


Figure 11: Energy band diagrams: (a) *n*-type semiconductor; (b) *p*-type semiconductor

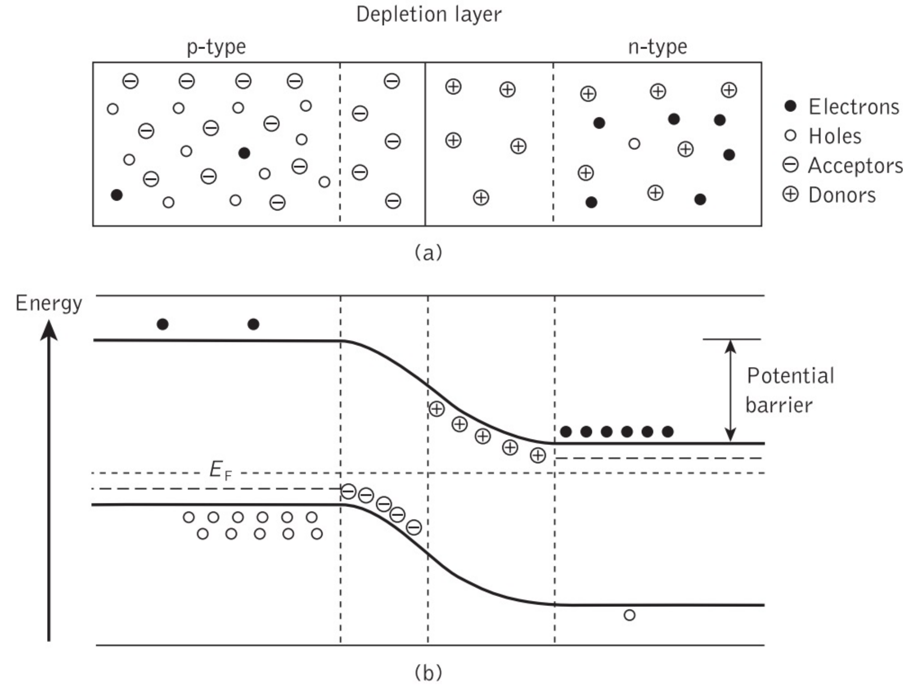


Figure 12: (a) The impurities and charge carriers at a *p*–*n* junction. (b) The energy band diagram corresponding to (a)

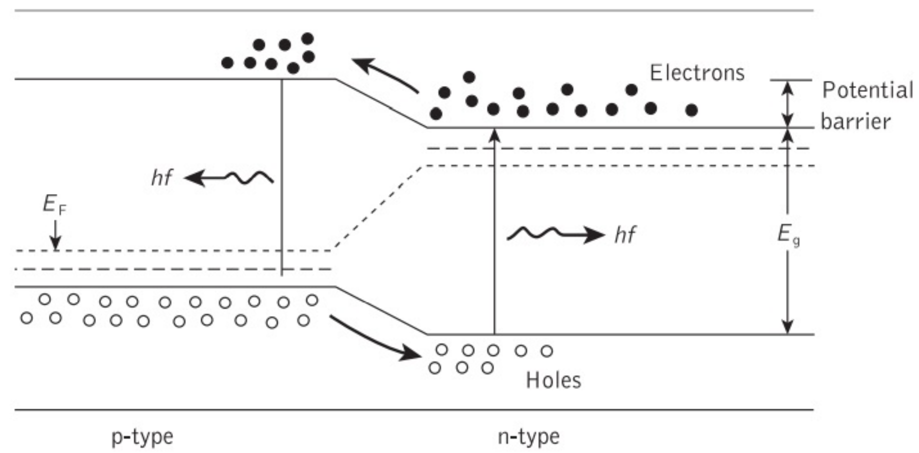


Figure 13: The *p*–*n* junction with forward bias giving spontaneous emission of photons

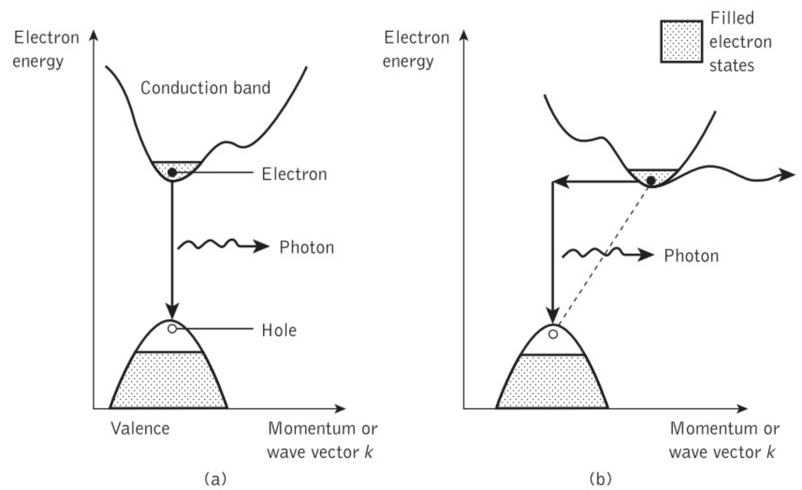


Figure 14: Energy–momentum diagrams showing the types of transition: (a) direct bandgap semiconductor; (b) indirect bandgap semiconductor

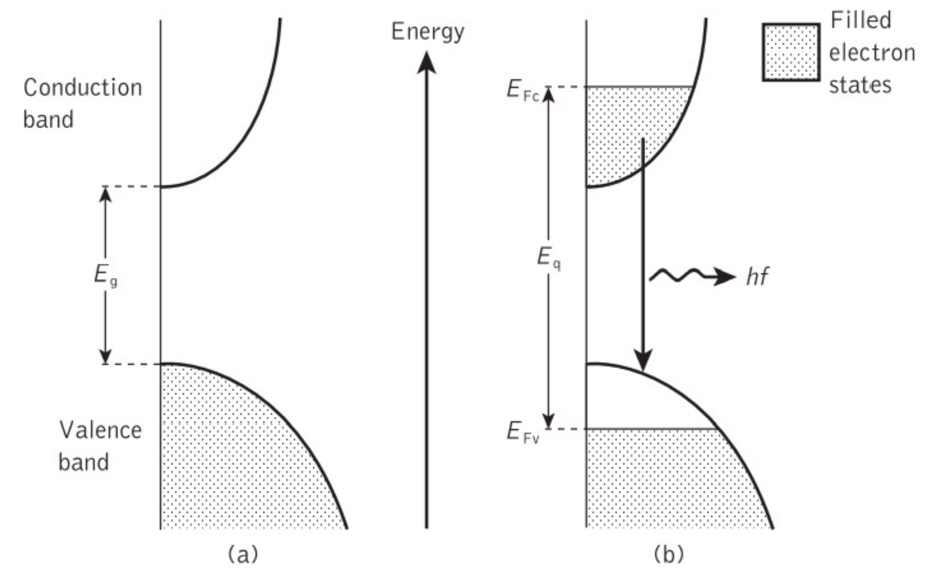


Figure 15: The filled electron states for an intrinsic direct bandgap semiconductor at absolute zero : (a) in equilibrium; (b) with high carrier injection

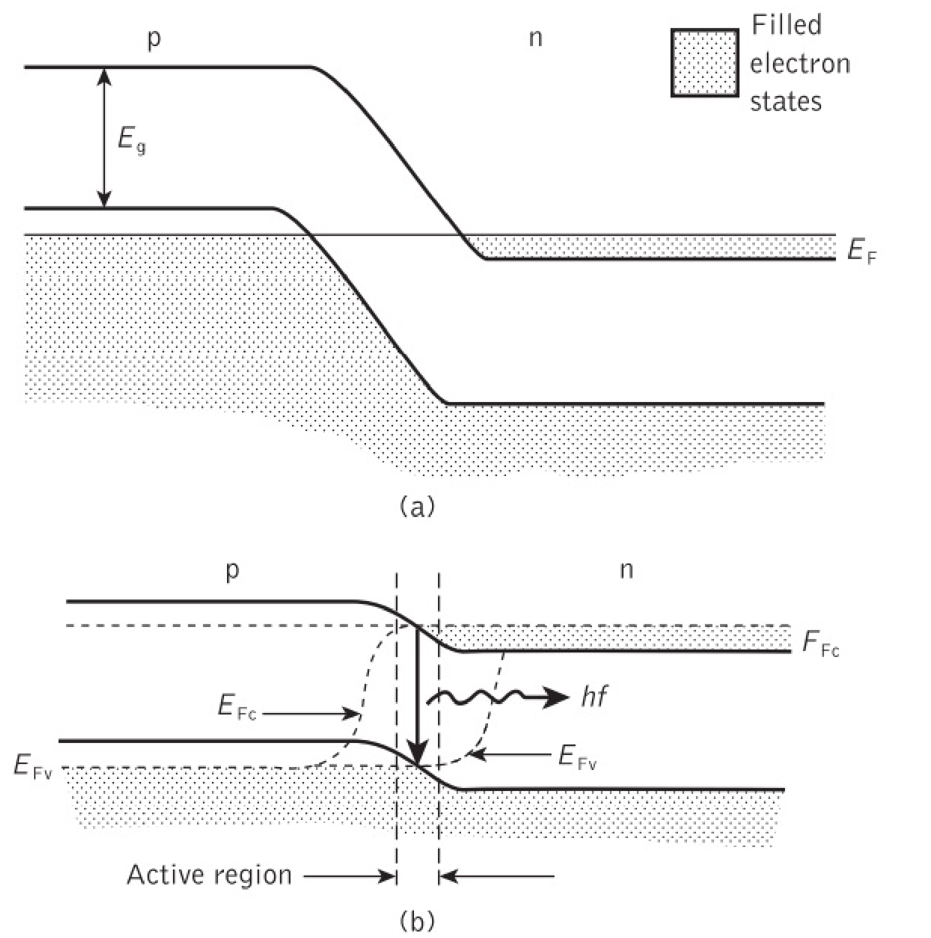


Figure 16: The degenerate *p*–*n* junction: (a) with no applied bias; (b) with strong forward bias such that the separation of the quasi-Fermi levels is higher than the electron–hole recombination energy hf in the narrow active region. Hence stimulated emission is obtained in this region

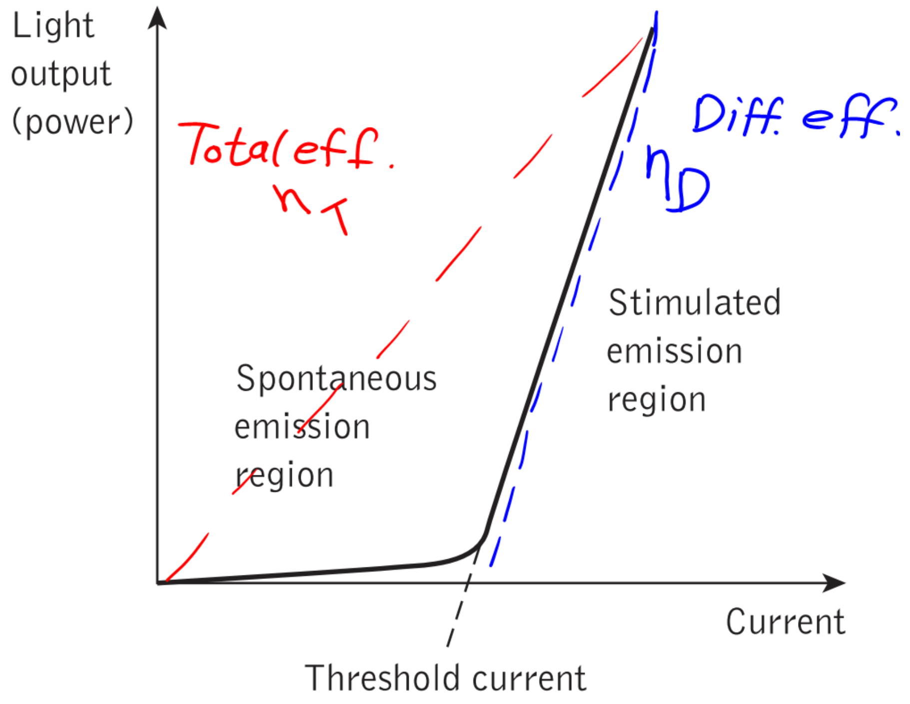


Figure 17: The ideal light output against current characteristic for an injection laser