Linear Algebra Questions[1]

Which of these statements are true?

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1. Q — A system of four equations and three unknowns can never have a unique solution

A — False

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2. Q — If the augmented matrix of a system of linear equations has a zero row at the bottom then it has an infinite number of solutions

A — False, zero row means that there are just an extra equation

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3. Q — A system of linear equations has no solution if the rank of the augmented matrix is less than that of the coefficient matrix

A — False, $\rho(\text{aug A}) < \rho(A)$ can't happen, the true statement is :
A system of linear equations has no solution if the rank of the augmented matrix is greater than that of the coefficient matrix (*by one*)

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4. Q — The vector space $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$

A — False, $\mathbb{R}^3$ is all 3×1 vectors, so it doesn't contain and 2×1 vectors

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5. Q — A straight line passing through the origin in $\mathbb{R}^3$ is a vector space

A — True

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6. Q — If the non-homogeneous system $Ax = b$ has an infinite number of solutions, then its solution set forms a subspace

A — False, if we substituted with $x = 0$, that would be invalid since $b \neq 0$, therefore it does not contain the zero element

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7. Q — Every vector space has only one basis set

A — False

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8. Q — If a homogeneous system has a unique solution, then the columns of the coefficient matrix are independent vectors

A — True

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9. Q — Any 4 column vectors with 3 elements form a spanning set of $\mathbb{R}^3$

A — False, to be true, three of them must be independent

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10. Q — If W is a subspace of V and dim V = 4 then dim W = 3

A — False, dim W $\leq$ dim V, so dim W ≤ 4

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11. Q — Any orthogonal set is an independent set

A — True

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12. Q — Any independent set is an orthogonal set

A — False

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13. Q — A vector space has only one orthonormal basis

A — False, infinite number of orthonormal bases

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14. Q — Any set of orthogonal vectors in $\mathbb{R}^3$ is a basis for $\mathbb{R}^3$

A — False, if there are a set of two orthogonal vectors, then they are not a basis of $\mathbb{R}^3$

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15. Q — Any set of three orthogonal vectors in $\mathbb{R}^3$ is a basis for $\mathbb{R}^3$

A — True

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16. Q — If $\{v1\ ,\ v2\ ,\ v3\ \}$ is a basis for V, and $\mathbf{x}$ is orthogonal to each of $v1$, $v2$ and $v3$, then $\mathbf{x}$ is orthogonal to all vectors in V

A — True
Any vector in V $\rightarrow \mathbf{w} = c_1\mathbf{v_1} + c_2\mathbf{v_2} + c_3\mathbf{v_3}$

$$\begin{aligned}\mathbf{x} \cdot \mathbf{w} &= \mathbf{x} \cdot (c_1\mathbf{v_1} + c_2\mathbf{v_2} + c_3\mathbf{v_3}) \\ &= c_1(\mathbf{x} \cdot \mathbf{v_1}) + c_2(\mathbf{x} \cdot \mathbf{v_2}) + c_3(\mathbf{x} \cdot \mathbf{v_3}) = 0\end{aligned}$$

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[1] Taha Ahmed