EEC 233 : Electronics II [1]
Part 1

1 Bipolar Junction Transistor (BJT)

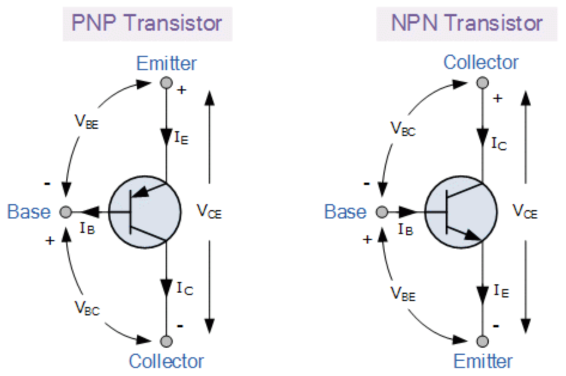

Figure 1

$$I_E = I_B + I_C \tag{1}$$

For active mode:

$$I_C = \beta I_B \tag{2}$$
$$I_C = \alpha I_E \approx I_E \tag{3}$$
$$I_B = \frac{I_E}{\beta + 1} \tag{4}$$
$$\beta = \frac{\alpha}{1-\alpha} \tag{5}$$
$$\alpha = \frac{\beta}{\beta+1} \tag{6}$$
$$r_o = \frac{V_A}{I_C} \tag{7}$$

r_o : Output resistance.

V_A : Early voltage.

[1] Taha Ahmed

2 Current Mirror

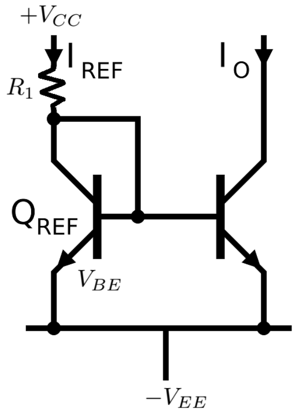

Figure 2

$$\frac{I_{\text{ref}}}{I_o} = \frac{1}{1 + \frac{2}{\beta}} \approx 1 \tag{8}$$

$$I_o = \frac{V_{EE} + V_{CC} - V_{BE}}{R_1} \tag{9}$$

3 Multiple Current Mirror

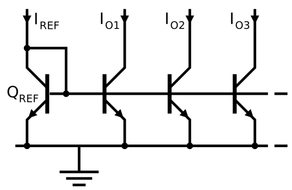

Figure 3

$$\frac{I_o}{I_{\text{ref}}} = \frac{1}{1 + \frac{(N+1)}{\beta}} \tag{10}$$

4 Modified Multiple Current Mirror

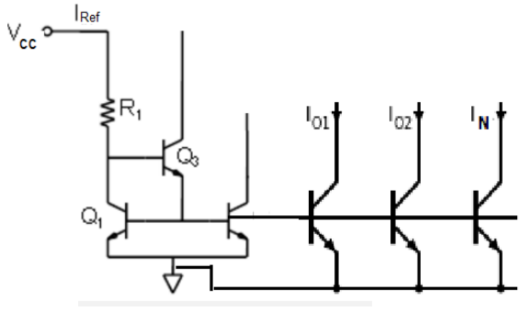

Figure 4

$$\frac{I_o}{I_{\text{ref}}} = \frac{1}{1 + \frac{(N+1)}{\beta^2}} \tag{11}$$

$$I_o = \frac{V_{EE} + V_{CC} - 2V_{BE}}{R_1} \tag{12}$$

5 Differential Amplifiers

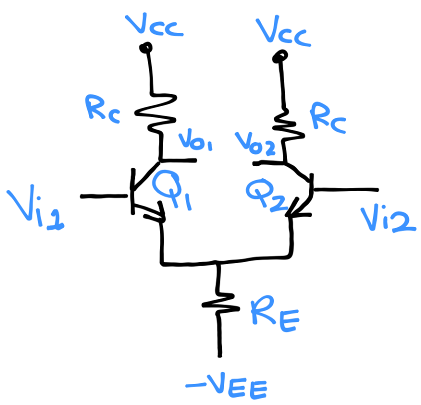

Figure 5

5.1 DC Analysis

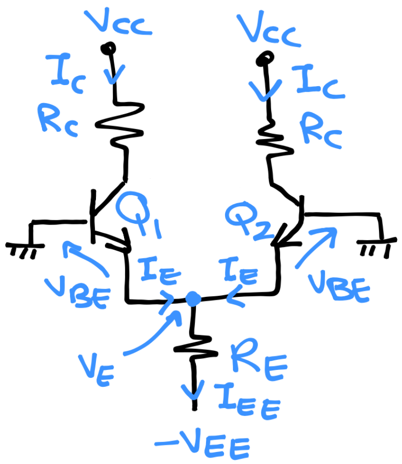

Figure 6

$$V_E = -V_{BE} \tag{13}$$

$$I_{EE} = \frac{V_{EE} - V_{BE}}{R_E} \tag{14}$$

$$I_E = \frac{1}{2} I_{EE} = \frac{V_{EE} - V_{BE}}{2R_E} \tag{15}$$

$$I_C = \alpha I_E \approx I_E \tag{16}$$

$$V_C = V_{CC} - I_C R_C \tag{17}$$

5.2 AC Analysis : Common mode

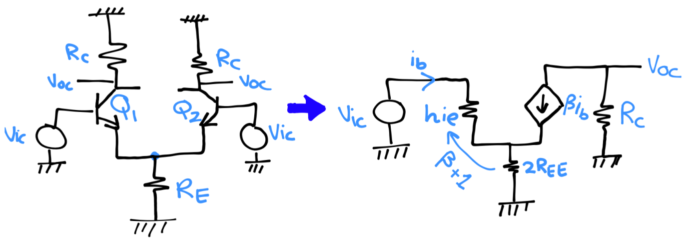

Figure 7

$$A_C = \frac{V_{oc}}{V_{ic}} = \frac{-R_c}{2R_{EE}} \tag{18}$$

$$R_{ic} = \frac{R_{in}}{2} = \frac{h_{ie} + (\beta+1)(2R_{EE})}{2} \tag{19}$$

$$h_{ie} = r_\pi = \beta r_e = \frac{(\beta+1) \times 25 \text{ mV}}{I_E} \tag{20}$$

5.3 AC Analysis : Difference Mode

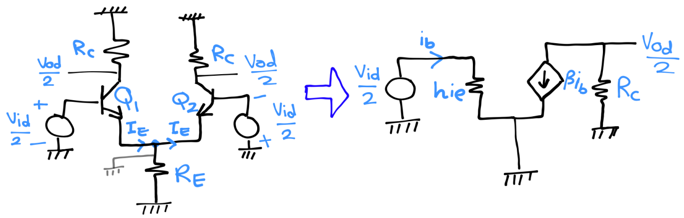

Figure 8

$$A_d = \frac{V_{od}}{V_{id}} = \frac{-\beta R_C}{h_{ie}} \tag{21}$$

$$R_{in} = h_{ie} \tag{22}$$

$$R_{id} = 2R_{in} = 2h_{ie} \tag{23}$$

Common mode rejection ratio:

$$\text{CMRR} = 20 \log_{10} \left| \frac{A_d/2}{A_c} \right| \text{ dB} \tag{24}$$

6 Multistage Differential Amplifier

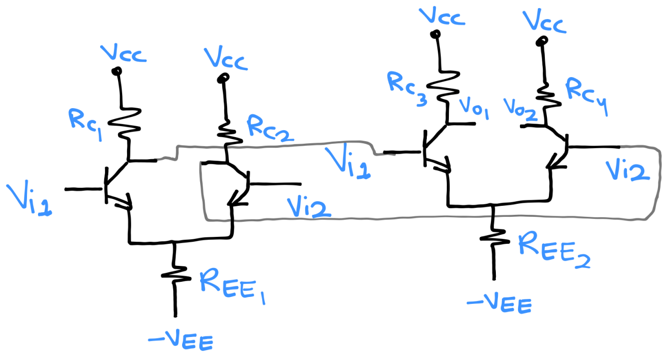

Figure 9

6.1 DC Analysis

$$I_{EE_1} = \frac{V_{EE} - V_{BE}}{R_{EE_1}} \tag{25}$$

$$I_{EE_2} = \frac{V_{EE} + V_{CC} - I_{C1}R_{C1} - V_{BE}}{R_{EE_1}} \tag{26}$$

6.2 AC Analysis : Common Mode

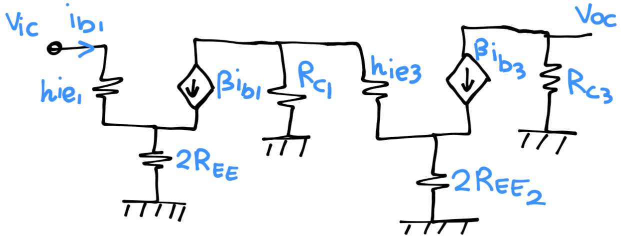

Figure 10

$$A_C = \frac{V_{oc}}{V_{ic}} = \frac{\beta^2 R_{C1} R_{C3}}{(h_{ie_1} + \beta \times 2R_{EE}) \times (R_{C1} + h_{ie_3} + \beta \times 2R_{EE})} \tag{27}$$

6.3 AC Analysis : Difference Mode

Substitute with $R_{EE_1} = 0$ and $R_{EE_2} = 0$

$$A_C = \frac{V_{od}}{V_{id}} = \frac{\beta^2 R_{C1} R_{C3}}{h_{ie_1} \times (R_{C1} + h_{ie_3})} \tag{28}$$

Common mode rejection ratio:

$$\text{CMRR} = 20 \log_{10} \left| \frac{A_d/2}{A_c} \right| \text{ dB}$$

7 MOSFET

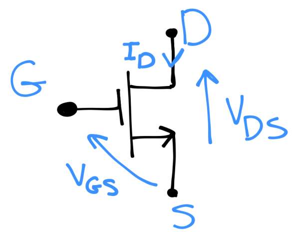

Figure 11: N-channel E-MOSFET

$$I_G = 0 \tag{29}$$

$$I_D = \frac{1}{2}K'_n \times \frac{W}{L} \times (V_{GS} - V_T)^2 \tag{30}$$

$$g_m = \frac{\partial I_d}{\partial V_{GS}} = K'_n \times \frac{W}{L} \times (V_{GS} - V_T) \tag{31}$$

Solving the quadratic equation, consider the solution such that $V_{GS} > V_T$

8 Current Mirror Using MOSFET

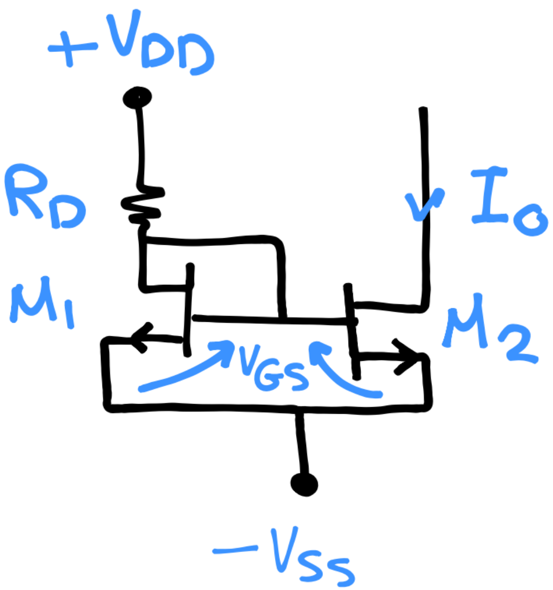

Figure 12

$$\frac{I_{\text{ref}}}{I_o} = \frac{(W/L)_1}{(W/L)_2} \tag{32}$$

Design equation:

$$I_{\text{ref}} = \frac{V_{SS} + V_{DD} - V_{GS}}{R_D} \tag{33}$$

9 Differential Amplifier Using MOSFET

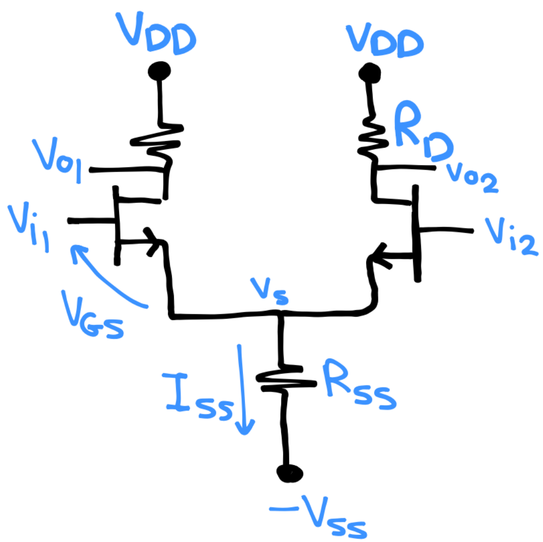

Figure 13

9.1 DC Analysis

$$I_{SS} = \frac{V_{SS} - V_{GS}}{R_{SS}} \tag{34}$$

$$I_D = \frac{I_{SS}}{2} = \frac{V_{SS} - V_{GS}}{2R_{SS}} \tag{35}$$

$$I_D = K_n(V_{GS} - V_T)^2 \tag{36}$$

Such that $K_n = \frac{1}{2}K'_n \times \frac{W}{L}$.
Solve 35 with 36 to get V_{GS}

$$g_m = K'_n \times \frac{W}{L} \times (V_{GS} - V_T)$$

9.2 AC Analysis : Common mode

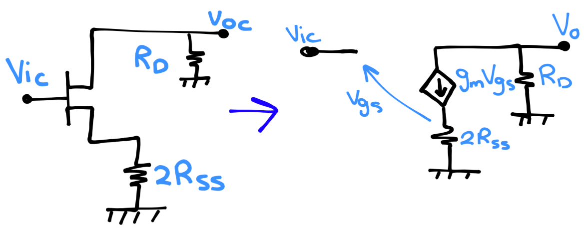

Figure 14

$$A_C = \frac{V_{OC}}{V_{IC}} = \frac{-g_m R_d}{1 + 2g_m R_{SS}} \tag{37}$$

9.3 AC Analysis : Difference Mode

Substitute with $R_{SS} = 0$

$$A_d = -g_m R_D \tag{38}$$

Common mode rejection ratio:

$$\text{CMRR} = 20\log_{10}\left|\frac{A_d/2}{A_c}\right| \text{ dB}$$

10 DC Level Shifting

When $v_i = 0$, v_o must be zero.
If $v_o \neq 0$, it is called DC offset.

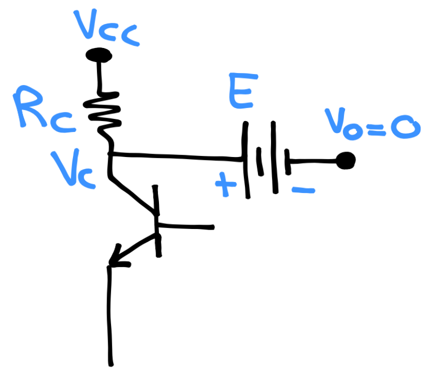

Figure 15: Put a DC source, $|v_c| = |E|$ in opposite directions

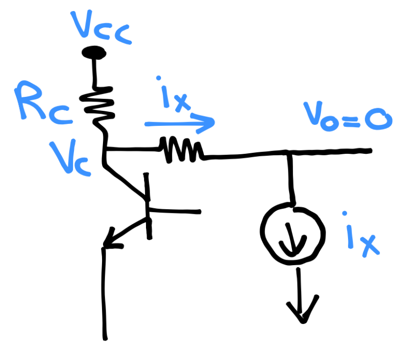

Figure 16: $v_o = v_c - i_x R_x$

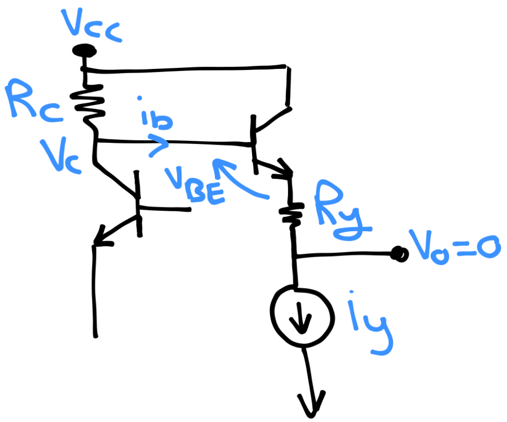

Figure 17: $v_o = v_c - V_{BE} - i_y R_y$

11 Operational Amplifiers

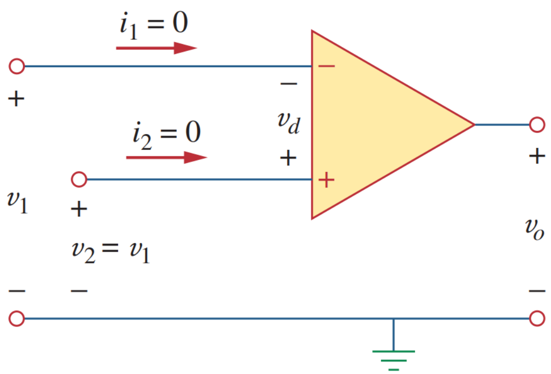

Figure 18: Ideal op amp model.

11.1 Non Inverting Amplifier

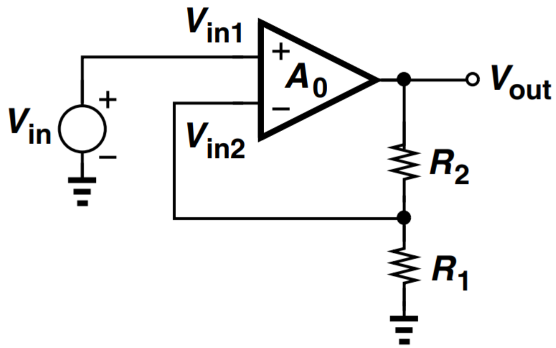

Figure 19

$$\frac{V_{\text{out}}}{V_{\text{in}}} = 1 + \frac{R_2}{R_1} \tag{39}$$

11.2 Inverting Amplifier

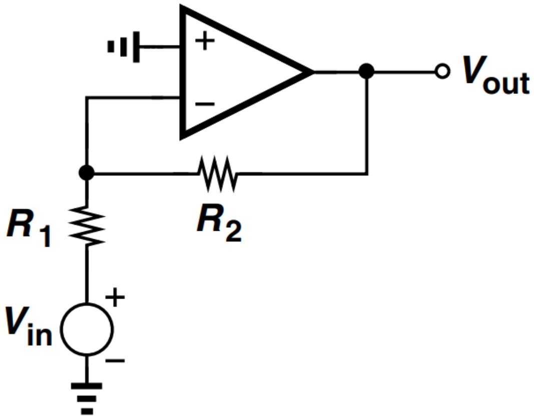

Figure 20

$$\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{R_2}{R_1} \tag{40}$$

11.3 Summing Amplifier

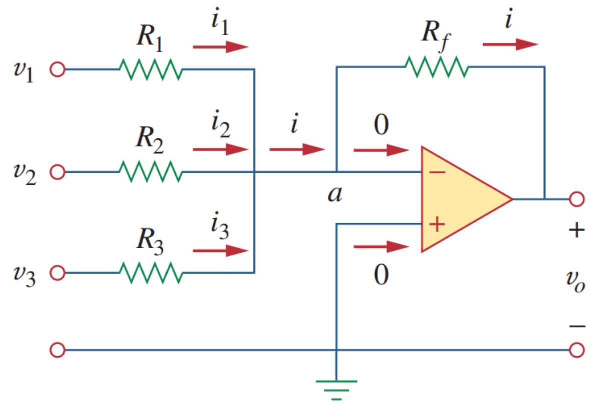

Figure 21

$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right) \tag{41}$$

11.4 Summing Non Inverting Amplifier

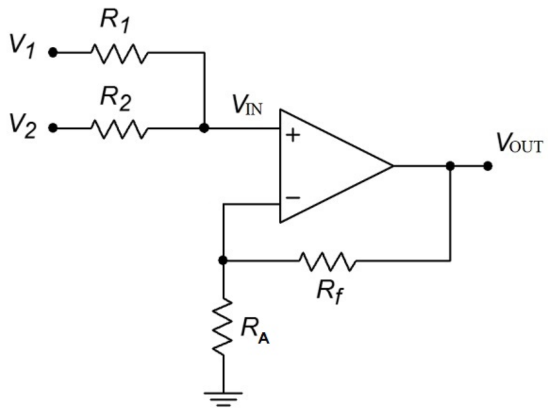

Figure 22

$$V_{\text{out}} = V_1 \frac{R_2}{R_1 + R_2}\left(1 + \frac{R_f}{R_a}\right) + V_2 \frac{R_1}{R_1 + R_2}\left(1 + \frac{R_f}{R_a}\right) \tag{42}$$

11.5 Difference Amplifier

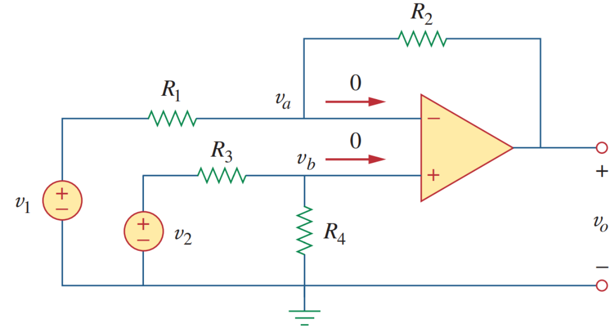

Figure 23

$$v_o = \frac{R_2(1 + R_1/R_2)}{R_1(1 + R_3/R_4)} v_2 - \frac{R_2}{R_1} v_1 \tag{43}$$

if $\dfrac{R_1}{R_2} = \dfrac{R_3}{R_4}$ op amp circuit is a difference amplifier, Equation 43 becomes

$$v_o = \frac{R_2}{R_1}(v_2 - v_1) \tag{44}$$

11.6 Instrumentation Amplifier

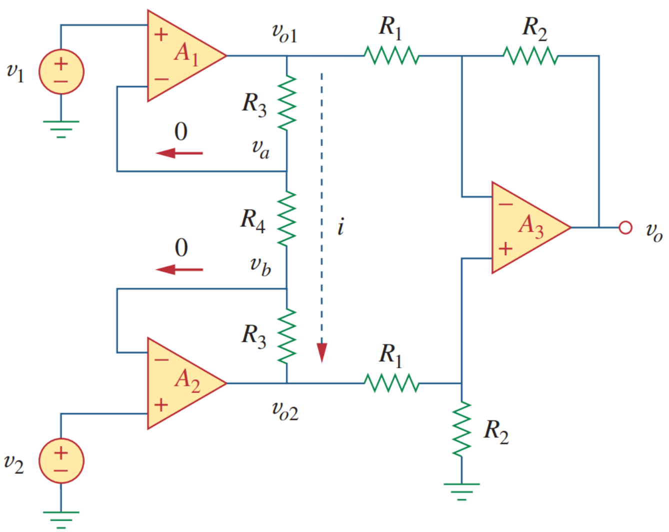

Figure 24

$$v_0 = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1) \tag{45}$$

11.7 Integrator

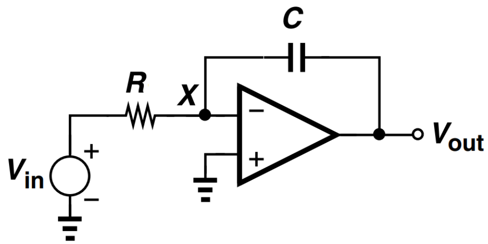

Figure 25

$$V_{\text{out}} = -\frac{1}{RC}\int V_{\text{in}} \mathrm{d}t \tag{46}$$

$$\therefore \frac{\Delta V_o}{\Delta t} = -\frac{E}{RC} \tag{47}$$

11.7.1 Triangular Wave generator

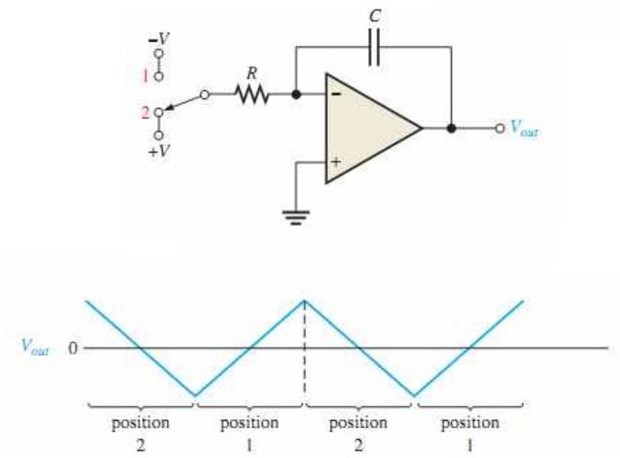

Figure 26: Triangular wave oscillator - Output voltage as the switch is thrown back and forth at regular intervals

Another way to generate triangular wave is discussed in Subsection 13.4.

11.7.2 Sawtooth Wave generator

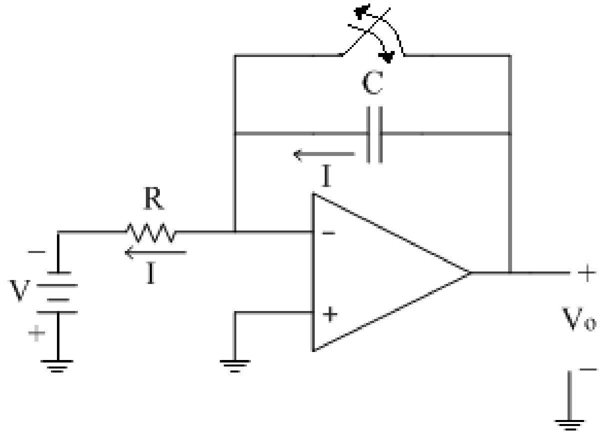

Figure 27: Sawtooth wave generator

11.8 Differentiator

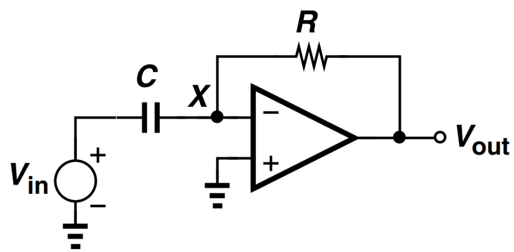

Figure 28

$$V_{\text{out}} = -RC\frac{\mathrm{d}V_{\text{in}}}{\mathrm{d}t} \tag{48}$$

12 Converters

12.1 Voltage to Current Converters

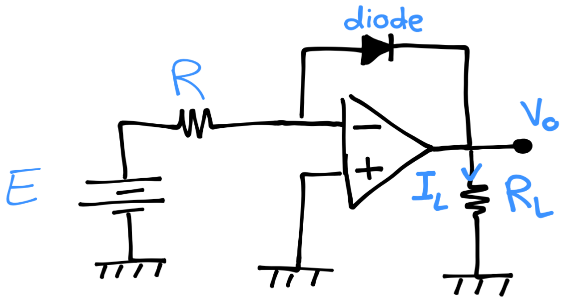

Figure 29

$$I_D = \frac{E}{R} \tag{49}$$

$$V_o = -0.7 \text{ Volt} \tag{50}$$

$$I_L = \frac{V_o}{R_L} \tag{51}$$

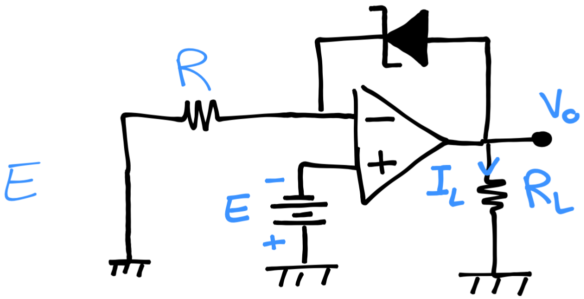

Figure 30

$$V_o = -E - V_z \tag{52}$$

$$I_L = \frac{V_o}{R_L} \tag{53}$$

V_z : Zener voltage.

We don't want I_L to be dependent of the load R_L

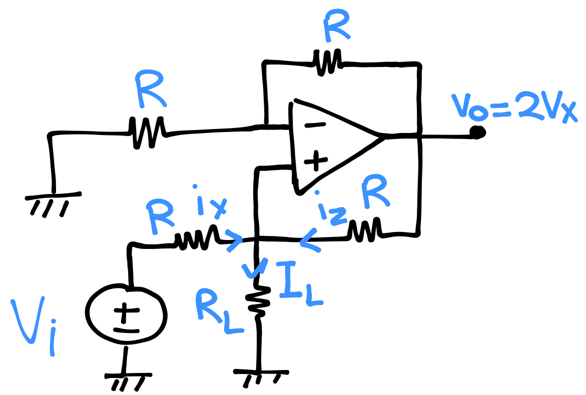

Figure 31: Constant current source

$$I_L = \frac{V_i}{R} \tag{54}$$

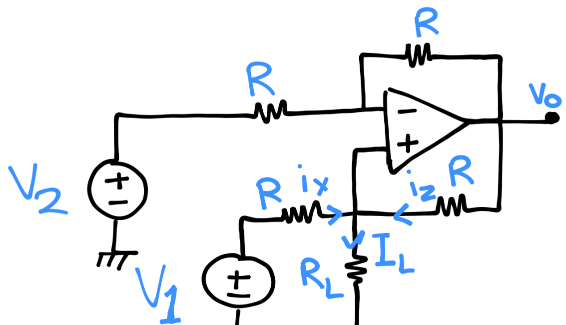

Figure 32: Constant current source - Voltage to current converter

$$I_L = \frac{V_1 - V_2}{R} \tag{55}$$

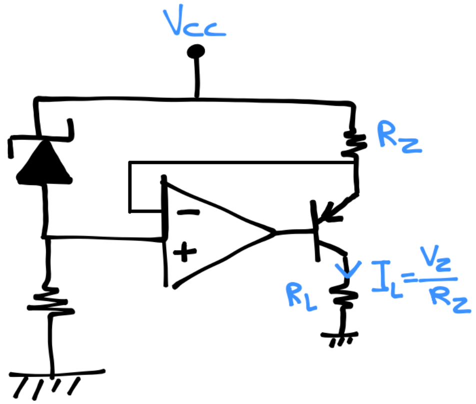

Figure 33: Constant high current source with grounded load

12.2 Current to Voltage Converters

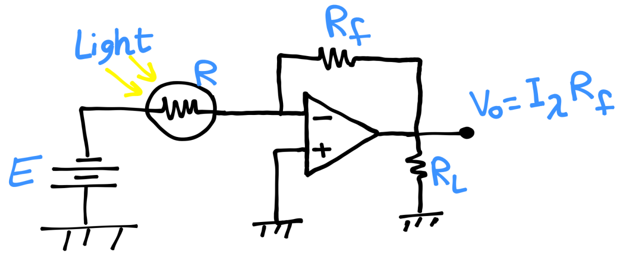

Figure 34

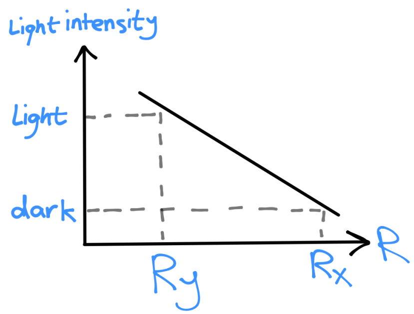

Figure 35

$$V_o = I_\lambda R_f \tag{56}$$

$$I_\lambda = \frac{E}{\text{photo resistance}} \tag{57}$$

12.3 Basic Bridge Circuit

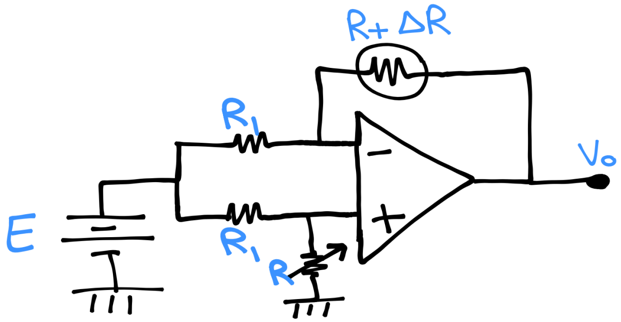

Figure 36

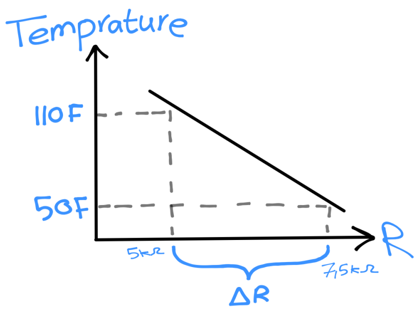

Figure 37

$$V_o = \frac{-\Delta R}{R_1 + R} E \tag{58}$$

12.4 Phase Shifter Circuit

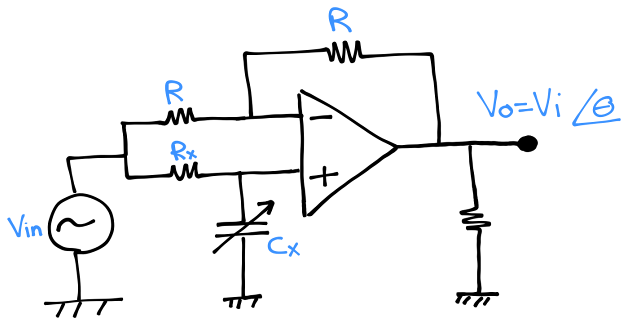

Figure 38

From superposition:

$$V_o = V_{o1} + V_{o2}$$

$$V_{o1} = -V_i$$

$$V_{o2} = 2V_x = 2V_i \frac{\frac{1}{j\omega C_x}}{R_x + \frac{1}{j\omega C_x}} = 2V_i \frac{1}{1 + j\omega C_x R_x}$$

$$\therefore V_o = \frac{2V_i}{1 + j\omega C_x R_x} - V_i = V_i \times \frac{2 - (1 + j\omega C_x R_x)}{1 + j\omega C_x R_x}$$

$$\therefore \frac{V_o}{V_i} = \frac{1 - j\omega C_x R_x}{1 + j\omega C_x R_x}$$

$$\left|\frac{V_o}{V_i}\right| = 1 \quad \text{(unity gain)} \tag{59}$$

$$\theta = \frac{\angle - \tan^{-1}(\omega C_x R_x)}{\angle + \tan^{-1}(\omega C_x R_x)} = -2\tan^{-1}(\omega C_x R_x) \tag{60}$$

$$\therefore -\frac{\theta}{2} = \tan^{-1}(\omega C_x R_x)$$

$$\therefore \omega C_x R_x = -\tan\left(\frac{\theta}{2}\right) \tag{61}$$

13 Operational Amplifier Applications

13.1 Negative Impedance Circuit

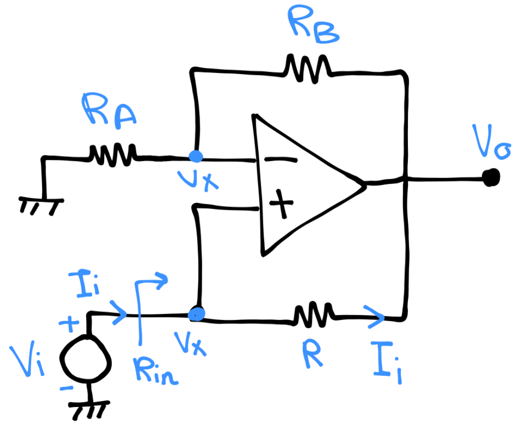

Figure 39

$$R_{\text{in}} = \frac{V_i}{I_i} = -\frac{R_A R}{R_B} \tag{62}$$

If R is replaced by Z, the circuit develops negative impedance.

(e.g. if replaced by a capacitor, it develops -ve × -ve = positive impedance)

13.2 Dependent Current Generator

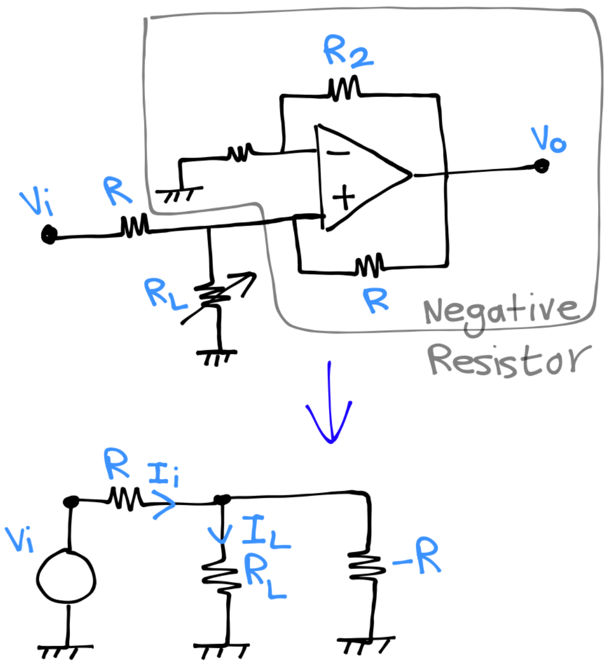

Figure 40

$$I_l = \frac{V_i}{R + R_L||-R} = \frac{V_i}{R + \dfrac{-R_L R}{R_L - R}} = \frac{V_i(R_L - R)}{-R^2}$$

From current divider :

$$I_L = I_i \frac{-R}{R_L - R} = \frac{V_i(R_L - R)}{-R^2} \times \frac{-R}{R_L - R}$$

$$\therefore I_L = \frac{V_i}{R} \tag{63}$$

Doesn't depend on the load, it is considered as voltage to current converter.

13.3 Non Inverting Miller Integrator

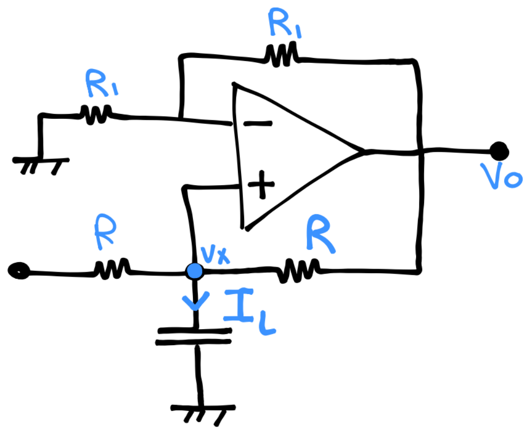

Figure 41

From Equation 63 :

$$I_L = \frac{V_i}{R}$$

$$V_o = V_x \left(1 + \frac{R}{R}\right) = 2V_x$$

$$V_x = V_c = \frac{1}{C}\int I_L \mathrm{d}t = \frac{1}{C}\int \frac{V_i}{R}\mathrm{d}t$$

$$\therefore V_o(t) = \frac{2}{RC}\int V_i \mathrm{d}t \tag{64}$$

Notice that there are no negative sign, unlike the integrator discussed in Subsection 11.7.

13.4 Triangular Wave Oscillator

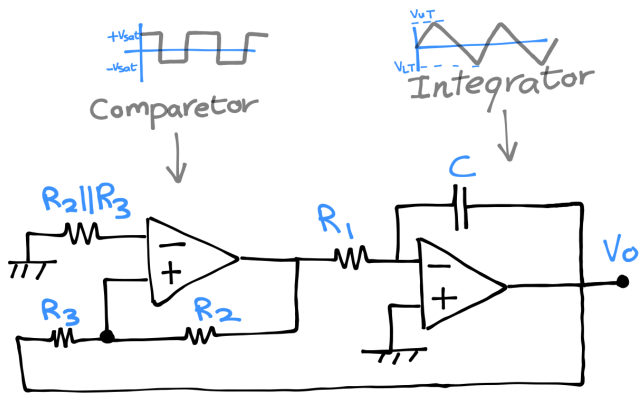

Figure 42

$$V_{\text{upper limit}} = +V_{\text{saturation}} \times \frac{R_3}{R_2} \tag{65}$$

$$V_{\text{lower limit}} = -V_{\text{saturation}} \times \frac{R_3}{R_2} \tag{66}$$

$$f = \frac{1}{4R_1C} \times \frac{R_2}{R_3} \tag{67}$$

f : frequency of oscillations.

Another way to generate triangular wave is discussed in Subsubsection 11.7.1.

13.5 Square Wave Oscillator

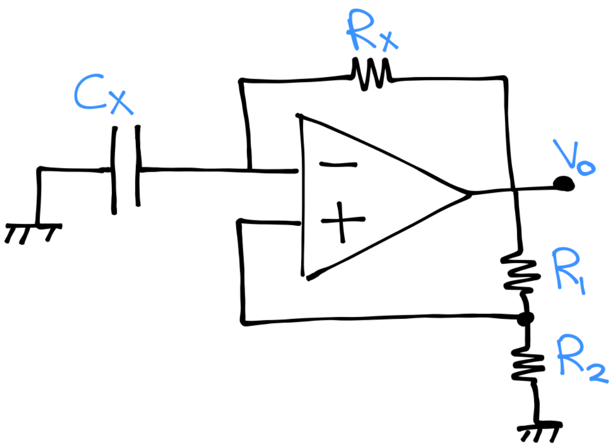

Figure 43

$$V_{\text{upper limit}} = +V_{\text{saturation}} \times \frac{R_2}{R_1 + R_2} \tag{68}$$

$$V_{\text{lower limit}} = -V_{\text{saturation}} \times \frac{R_2}{R_1 + R_2} \tag{69}$$

$$T = 2\tau \ln\left(\frac{1+\beta}{1-\beta}\right) \tag{70}$$

$$f = \frac{1}{T} \tag{71}$$

$$\tau = R_x C_x \tag{72}$$

$$\beta = \frac{R_2}{R_1 + R_2} \tag{73}$$

13.6 Voltage Comparetor (Saturation Comparetor)

We operate the op-amp in the open loop mode depending on the high open loop gain $A \approx \infty$ to drive the op-amp into saturation.

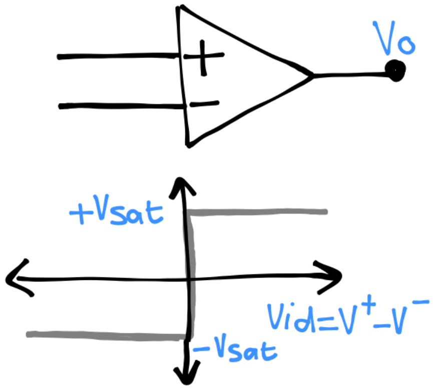

Figure 44

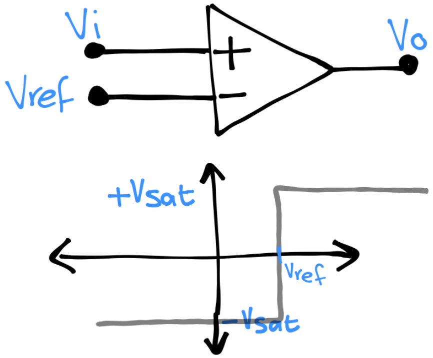

Figure 45

$$V_o = \begin{cases} +V_{\text{sat}} & V_i > V_{\text{ref}} \\ 0 & V_i = V_{\text{ref}} \\ -V_{\text{sat}} & V_i < V_{\text{ref}} \end{cases} \tag{74}$$

13.7 Zero Voltage Comparetor

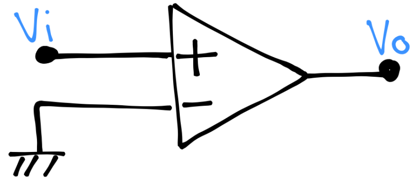

Figure 46

$$V_{\text{ref}} = 0$$

$$V_o = \begin{cases} +V_{\text{sat}} & V_i > 0 \\ 0 & V_i = 0 \\ -V_{\text{sat}} & V_i < 0 \end{cases} \tag{75}$$

The output toggles when the input crosses zero

13.8 Voltage (Saturation) Comparetor With Inverting Op-Amp

Connect V_i to the inverting terminal to invert the functionality of the comparetor

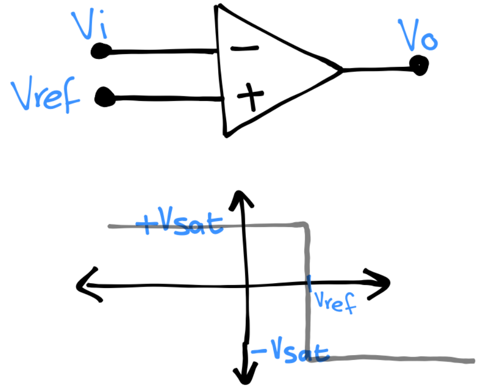

Figure 47

$$V_o = \begin{cases} -V_{\text{sat}} & V_i > V_{\text{ref}} \\ 0 & V_i = V_{\text{ref}} \\ +V_{\text{sat}} & V_i < V_{\text{ref}} \end{cases} \tag{76}$$

13.8.1 Control Street Light

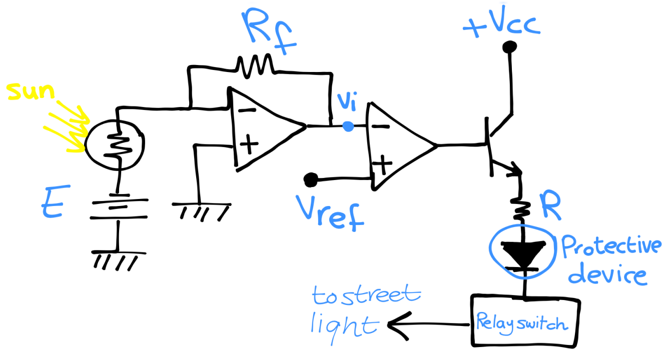

Figure 48

13.9 Comparator With Bounded Output

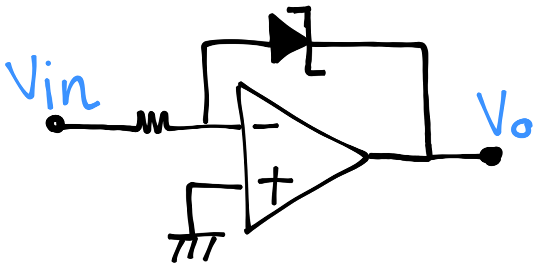

Figure 49

The operation is as follows, since the anode of the zener is connected to the inverting (−) input, it is a virtual ground (= 0 V). Therefore, when the output voltage reaches a positive value equals to the zener voltage, it limits at that value. When the output switches negative, the zener acts as regular diode and becomes forward biased at 0.7 V, limiting the negative output to that value . Turnung the zener around limits the output voltages in the opposite direction

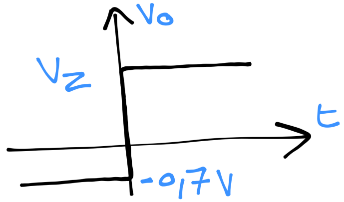

Figure 50

Two Zener diodes arranged as shown in Figure 51 limit the output voltage to the zener voltage plus the forward voltage drop (0.7 V) of the forward biased zener both positively and negatively

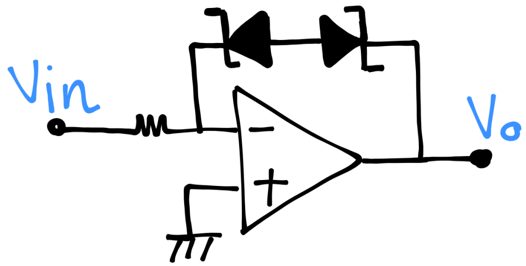

Figure 51

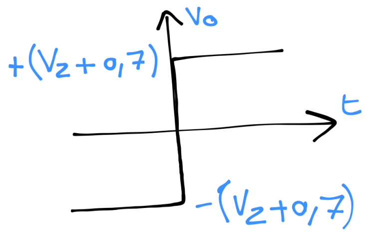

Figure 52

13.10 Schmitt Trigger

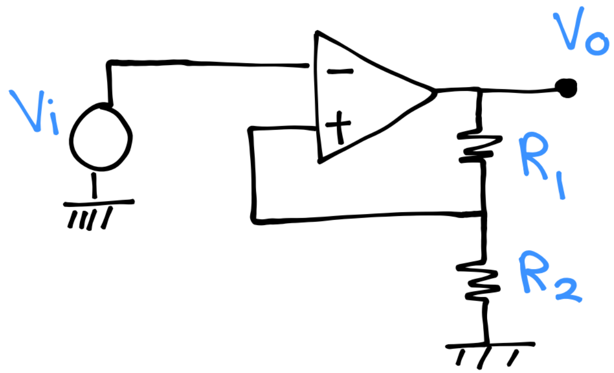

Figure 53

Made with positive feedback to eliminate the effect of noise.

Upper threshold voltage is the input voltage V_i when the output voltage V_o equals $V_{\text{saturation}}$.

Similarly, lower threshold voltage is the input voltage V_i when the output voltage V_o equals $-V_{\text{saturation}}$.

$$V_{\text{upper threshold}} = +V_{\text{sat}} \frac{R_2}{R_1 + R_2} \tag{77}$$

$$V_{\text{upper threshold}} = -V_{\text{sat}} \frac{R_2}{R_1 + R_2} \tag{78}$$

$$V_o = \begin{cases} -V_{\text{sat}} & V_i > V_{\text{upper threshold}} \\ \text{No change} & V_{\text{lower threshold}} < V_i < V_{\text{upper threshold}} \\ +V_{\text{sat}} & V_i < V_{\text{lower threshold}} \end{cases} \tag{79}$$

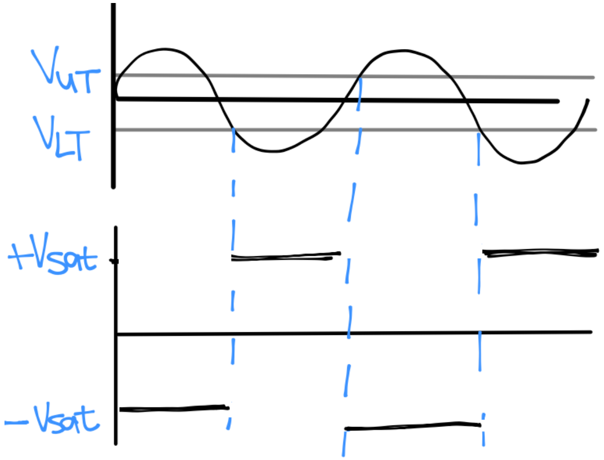

Figure 54

Another design of the schmitt trigger:

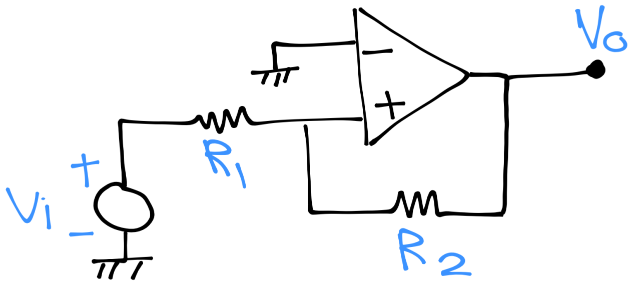

Figure 55

Note that :

$$R_2 > R_1$$

$$V_o = \begin{cases} +V_{\text{sat}} & V_i > \frac{R_1}{R_2}V_{\text{sat}} \\ \text{No change} & -\frac{R_1}{R_2}V_{\text{sat}} < V_i < \frac{R_1}{R_2}V_{\text{sat}} \\ -V_{\text{sat}} & V_i < -\frac{R_1}{R_2}V_{\text{sat}} \end{cases} \tag{80}$$

13.10.1 Schmitt Trigger With Reference

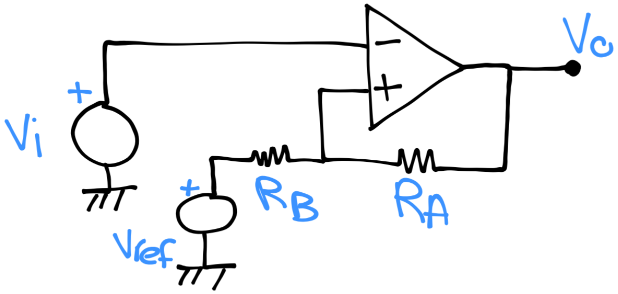

Figure 56

$$V_{\text{UT}} = V_{\text{ref}} + \frac{+V_{\text{sat}} - V_{\text{ref}}}{R_A + R_B} R_B \tag{81}$$

$$V_{\text{LT}} = V_{\text{ref}} + \frac{-V_{\text{sat}} - V_{\text{ref}}}{R_A + R_B} R_B \tag{82}$$

$$V_o = \begin{cases} -V_{\text{sat}} & V_i > V_{\text{UT}} \\ \text{No change} & V_{\text{LT}} < V_i < V_{\text{UT}} \\ +V_{\text{sat}} & V_i < V_{\text{LT}} \end{cases} \tag{83}$$

13.11 Window Comparetor

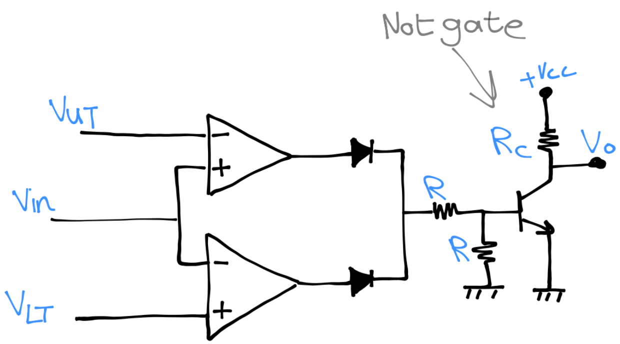

Figure 57

$$V_o \text{ (on) when } V_{\text{LT}} < V_i < V_{\text{UT}} \tag{84}$$

13.12 Logarithmic Amplifier

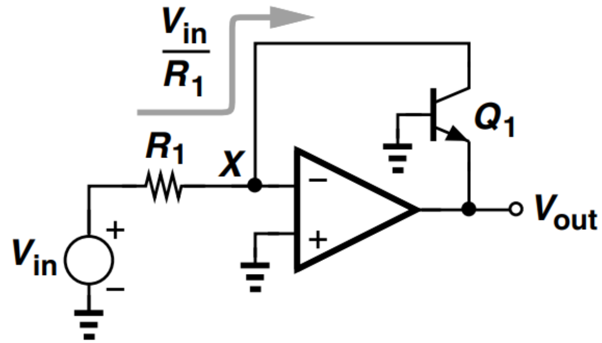

Figure 58

$$V_{\text{out}} = -V_{BE}$$

$$V_{\text{out}} = -V_T \ln\left(\frac{V_{\text{in}}}{R_1 I_S}\right) = -K \ln(V_{\text{in}}) \tag{85}$$

I_S : Saturation current.

K : constant.

The output is therefore proportional to the natural logarithm of V_{in}

Contents