

Linear Algebra Questions¹

Which of these statements are true?

1. Q — A system of four equations and three unknowns can never have a unique solution

A — False

2. Q — If the augmented matrix of a system of linear equations has a zero row at the bottom then it has an infinite number of solutions

A — False, zero row means that there are just an extra equation

3. Q — A system of linear equations has no solution if the rank of the augmented matrix is less than that of the coefficient matrix

A — False, $\rho(\text{aug } A) < \rho(A)$ can happen, the true statement is :

A system of linear equations has no solution if the rank of the augmented matrix is greater than that of the coefficient matrix (*by one*)

4. Q — The vector space \mathbb{R}^2 is a subspace of \mathbb{R}^3

A — False, \mathbb{R}^3 is all 3×1 vectors, so it doesn't contain and 2×1 vectors

5. Q — A straight line passing through the origin in \mathbb{R}^3 is a vector space

A — True

6. Q — If the non-homogeneous system $Ax = b$ has an infinite number of solutions, then its solution set forms a subspace

A — False, if we substituted with $x = 0$, that would be invalid since $b \neq 0$, therefore it does not contain the zero element

7. Q — Every vector space has only one basis set

A — False

8. Q — If a homogeneous system has a unique solution, then the columns of the coefficient matrix are independent vectors

A — True

9. Q — Any 4 column vectors with 3 elements form a spanning set of

A — False, to be true, three of them must be independent

10. Q — If W is a subspace of V and $\dim V = 4$ then $\dim W = 3$

A — False, $\dim W \leq \dim V$, so $\dim W \leq 4$

11. Q — Any orthogonal set is an independent set

A — True

12. Q — Any independent set is an orthogonal set

A — False

13. Q — A vector space has only one orthonormal basis

A — False, infinite number of orthonormal bases

14. Q — Any set of orthogonal vectors in \mathbb{R}^3 is a basis for \mathbb{R}^3

A — False, if there are a set of two orthogonal vectors, then they are not a basis of \mathbb{R}^3

15. Q — Any set of three orthogonal vectors in \mathbb{R}^3 is a basis for \mathbb{R}^3

A — True

16. Q — If $\{v_1, v_2, v_3\}$ is a basis for V , and \mathbf{x} is orthogonal to each of v_1, v_2 and v_3 , then \mathbf{x} is orthogonal to all vectors in V

A — True

Any vector in $V \rightarrow \mathbf{w} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3$

$$\begin{aligned}\mathbf{x} \cdot \mathbf{w} &= \mathbf{x} \cdot (c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3) \\ &= c_1(\mathbf{x} \cdot \mathbf{v}_1) + c_2(\mathbf{x} \cdot \mathbf{v}_2) + c_3(\mathbf{x} \cdot \mathbf{v}_3) = 0\end{aligned}$$

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