EMP 214 - Probability cheat sheet 2 [1]

1 Prerequisites

- $$\sin(\text{angle}_1) \times \sin(\text{angle}_2) = \frac{1}{2}(\cos(\text{difference}) - \cos(\text{sum}))$$
- $$\cos(\text{angle}_1) \times \cos(\text{angle}_2) = \frac{1}{2}(\cos(\text{difference}) + \cos(\text{sum}))$$

2 Moment generating function

- MGF:
$$M_X(t) = \mathbb{E}[e^{tX}]$$
- r^{th} moment:
$$\mathbb{E}[X^r] = \frac{d^r}{dt^r} M_X(t)\Big|_{t=0}$$

3 Joint Discrete Probability Distributions

- $$f_{XY}(x,\, y) = \mathbb{P}[X = x,\, Y = y]$$
- Properties :
$$\sum_X \sum_Y f_{XY}(x,\, y) = 1$$
- Marginal PMF :
$$f_X(x) = \sum_Y f_{XY}(x,\, y)$$
$$f_Y(x) = \sum_X f_{XY}(x,\, y)$$
- Independence : X and Y are independent if
$$\underbrace{f_{XY}(x,y) = f_X(x) \times f_Y(y)}_{\text{for all values of x and y}}$$

  also if :
$$f_{X|Y} = f_X$$
$$f_{Y|X} = f_Y$$
- Conditional probability:
$$f_{Y|X} = \frac{f_{XY}(x,\, y)}{f_X(x)}$$
$$f_{X|Y} = \frac{f_{XY}(x,\, y)}{f_Y(y)}$$

4 Joint Continuous Probability Distributions

- $$\mathbb{P}[\Lambda] = \iint_\Lambda f_{XY}\; dx\; dy$$
  for any event $\Lambda \subseteq \Omega_X \times \Omega_Y$
- Properties :
$$\iint_A f_{XY}(x,\, y)\; dA = 1$$
- Marginal PMF :
$$f_X(x) = \int_Y f_{XY}(x,\, y)\; dy$$
$$f_Y(x) = \int_X f_{XY}(x,\, y)\; dx$$
- Independence : X and Y are independent if
$$f_{XY}(x,y) = f_X(x) \times f_Y(y)$$

  also if :
$$f_{X|Y} = f_X$$
$$f_{Y|X} = f_Y$$
- Conditional probability:
$$f_{Y|X} = \frac{f_{XY}(x,\, y)}{f_X(x)}$$
$$f_{X|Y} = \frac{f_{XY}(x,\, y)}{f_Y(y)}$$

5 Expectation, Covariance and Correlation Coefficient

- Discrete:
$$\mathbb{E}[g(x,\, y)] = \sum_X \sum_Y g(x,\, y)\, f_{XY}(x,\, y)$$
- Continuous:
$$\mathbb{E}[g(x,\, y)] = \int_X \int_Y g(x,\, y)\, f_{XY}(x,\, y) dx\, dy$$
- Properties:
$$\mathbb{E}[x + y] = \mathbb{E}[x] + \mathbb{E}[y]$$
- if x and y are independent:
$$\mathbb{E}[xy] = \mathbb{E}[x]\mathbb{E}[y]$$
- Covariance:
$$\begin{aligned}\text{Cov}(X,\, Y) = \sigma_{XY} &= \mathbb{E}[(X - \mu_X)(Y - \mu_Y)] \\ &= \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]\end{aligned}$$
- Variance:
$$\mathbb{V}[X + Y] = \mathbb{V}[X] + \mathbb{V}[Y] + 2\text{Cov}(X,\, Y)$$
- if x and y are independent:
$$\mathbb{V}[X + Y] = \mathbb{V}[X] + \mathbb{V}[Y]$$
- Correlation Coefficient:
$$\rho_{X,Y} = \frac{\text{Cov}(X,\, Y)}{\sigma_X \sigma_Y}$$

6 Random Processes

- Expectation:
$$\mu_X(t) = \mathbb{E}[X(t, A)] = \sum_A X(t, A) f_A(a)$$
$$\mu_X(t) = \mathbb{E}[X(t, A)] = \int_A X(t, A) f_A(a)\, dA$$
- Auto-correlation function :
$$R_{XX}(t, t + \tau) = \mathbb{E}[X(t)X(t + \tau)]$$
- Auto-covariance function :
$$\begin{aligned}\text{Cov}_{XX}(t, t + \tau) &= R_{XX}(t, t + \tau) - \mu_X(t)\mu_X(t + \tau) \\ &= \mathbb{E}[X(t)X(t + \tau)] - \mathbb{E}[X(t)]\mathbb{E}[X(t + \tau)]\end{aligned}$$
- Wide Sense Stationary Process WSSP:
$$\text{Expectaion} = \text{Constant, Not depend on time}$$
$$R_{XX}(t, t + \tau) = \mathbb{E}[X(t)X(t + \tau)] = R_{XX}(\tau)$$
$$\text{depend on time difference only}$$
- Average power for WSSP:
$$R_{XX}(\tau = 0) = \mathbb{E}[X(t)X(t + 0)] = \mathbb{E}[X^2(t)]$$

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