Lecture 1, 2

Probability of error $\mathbb{P}_e$:

$$\mathbb{P}_e = \mathbb{P}(1S, 0D) + \mathbb{P}(0S, 1D) \quad (1)$$

$$= \mathbb{P}(0D|1S)\mathbb{P}(1S) + \mathbb{P}(1D|0S)\mathbb{P}(0S) \quad (2)$$

$$= \mathbb{P}(S_1(T) + N < V_{th})\mathbb{P}(1S) + \mathbb{P}(S_2(T) + N \geq V_{th})\mathbb{P}(0S) \quad (3)$$

we define : $x = S_1(T) + N$, $y = S_2(T) + N$

therefore :

x is a Gaussian with $S_1(T)$ mean and σ^2 variance

y is a Gaussian with $S_2(T)$ mean and σ^2 variance

$$\mathbb{P}_e = \mathbb{P}(x < V_{th})\mathbb{P}(1S) + \mathbb{P}(y \geq V_{th})\mathbb{P}(0S) \quad (4)$$

$$= \int_{-\infty}^{V_{th}} \mathbb{P}(x)\mathbb{P}(1S) + \int_{V_{th}}^{\infty} \mathbb{P}(y)\mathbb{P}(0S) \quad (5)$$

In case of Gaussian Noise, zero mean, variance σ_n^2 and $\mathbb{P}(0S) = p(1S) = \frac{1}{2}$

$$\mathbb{P}_e|_{\min} = \frac{1}{2}\,\text{erfc}\left[\frac{S_1(T) - S_2(T)}{2\sqrt{2}\sigma_n}\right] \quad (6)$$

$$V_{th}|_{\text{opt}} = \frac{S_1(T) + S_2(T)}{2} \quad (7)$$

How To minimize $\mathbb{P}_e$:

1) choose V_{th} at the intersection of the curves

2) choose sampling time T such that

$$|S_1(T) - S_2(T)| = \max$$

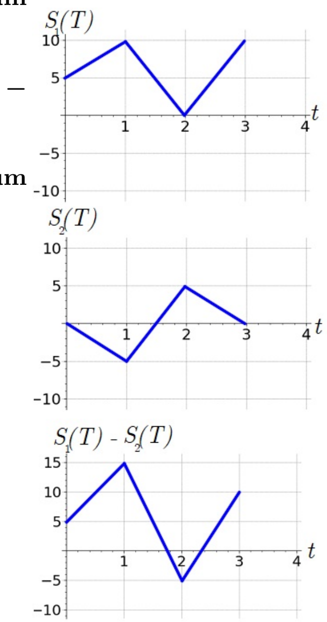
why ? as $|S_1(T) - S_2(T)| \upuparrows$, erfc $\left[\frac{S_1(T)-S_2(T)}{2\sqrt{2}\sigma_n}\right] \downdownarrows$ and $\mathbb{P}_e \downdownarrows$

$\boxed{1.\ \text{Q}}$ For $S_1(T)$, $S_2(T)$, find the optimum sampling time

$\boxed{1.\ \text{A}}$ Find $|S_1(T) - S_2(T)|$ and choose the maximum value

from the figure : $|S_1(1\mu s) - S_2(1\mu s)| = 15$ V max

Therefore the optimum sampling time $1\mu s$



How to calculate $\mathbb{P}_e$?

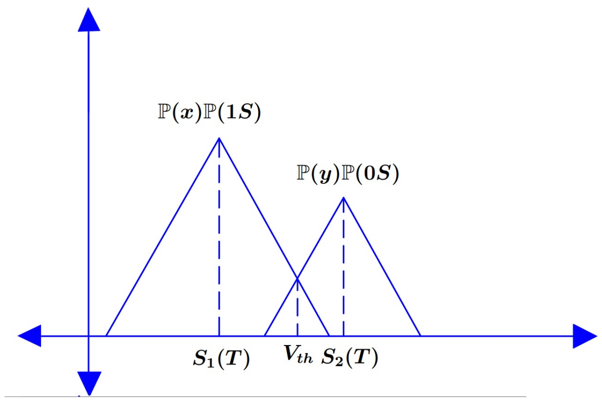
1) Choose optimum T at $|S_1(T) - S_2(T)| = \max$

2) find x, y

x is the same distribution of the noise with $S_1(T)$ mean and σ^2 variance

y is the same distribution of the noise with $S_2(T)$ mean and σ^2 variance

3) multiply $\mathbb{P}(x)\mathbb{P}(1S)$ and $\mathbb{P}(y)\mathbb{P}(0S)$ and draw new distribution



4) choose optimum V_{th} at the intersection (where $\mathbb{P}_e$ (area) is minimum)

$$\mathbb{P}(x)\mathbb{P}(1S) = \mathbb{P}(y)\mathbb{P}(0S)|_{V_{th}} \quad (8)$$

5) find $\mathbb{P}_e$ = area at intersection

$\boxed{\text{2. Q}}$ **Given $S_1(T)$, $S_2(T)$ and $\mathbb{P}_n$, $\mathbb{P}(0S) = \frac{1}{2}\mathbb{P}(1S)$, find minimum probability of error :**

$\boxed{\text{2. A}}$

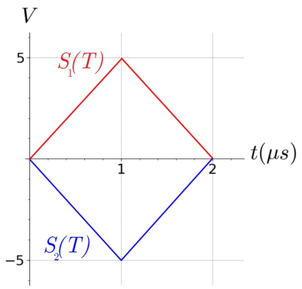
Noise is uniform, area $= 1 \therefore \mathbb{P}_n = \dfrac{1}{12}$

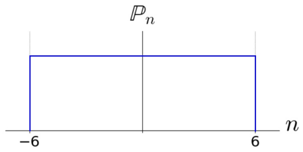
1) optimum sampling time (from the figure) : $|S_1(1\mu s) - S_2(1\mu s)| = 10$ V max, at $1\mu s$ $S_1(T) = 5V$, $S_2(T) = -5V$

To calculate the pdf :

2) get x, y

x is the same distribution of the noise (uniform) with $S_1(T)$ mean (5 V) and σ^2 variance





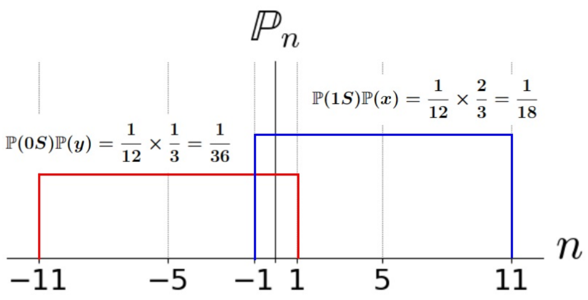
y is the same distribution of the noise with $S_2(T)$ mean (-5 V) and σ^2 variance

$$\because \mathbb{P}(0S) = \frac{1}{2}\mathbb{P}(1S) \Rightarrow \mathbb{P}(0S) = \frac{1}{3},\ \mathbb{P}(1S) = \frac{1}{3} \tag{9}$$

3) multiply $\mathbb{P}(x)\mathbb{P}(1S)$ and $\mathbb{P}(y)\mathbb{P}(0S)$ and draw new distribution

$$\mathbb{P}(0S)\mathbb{P}(y) = 2 \times \frac{3}{5}$$

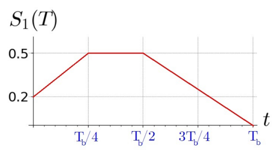
$$\mathbb{P}(1S)\mathbb{P}(x) = 2 \times \frac{2}{5}$$

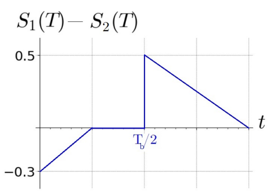


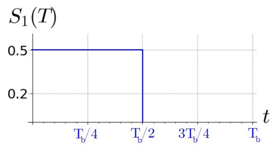
4) Choose V_{th} : at the intersection (at -1)

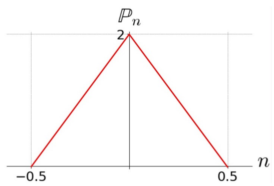
5) Get $\mathbb{P}_e$ = area of intersection $= \dfrac{1}{36} \times 2 = \dfrac{1}{18}$

$\boxed{\text{3. Q}}$ **Given $S_1(T)$, $S_2(T)$ and $\mathbb{P}_n$, $\mathbb{P}(1S) = \frac{2}{3}\mathbb{P}(0S)$, find minimum probability of error :**









$\boxed{\text{3. A}}$

$$\because \mathbb{P}(0S) = \frac{2}{3}\mathbb{P}(0S) \Rightarrow \mathbb{P}(0S) = \frac{3}{5},\ \mathbb{P}(1S) = \frac{2}{5} \tag{10}$$

Noise is triangle, area $= 1 \therefore \frac{1}{2} \times 1 \times \mathbb{P}_n(0) = 1$

$\therefore \mathbb{P}_n(0) = 2$

1) optimum sampling time (subtract $S_2(T)$ from $S_1(T)$) :

$|S_1(T_b/2) - S_2(T_b/2)| = 0.5$ V max,

at $T_b/2s$ $S_1(T) = 0.5V$, $S_2(T) = 0V$

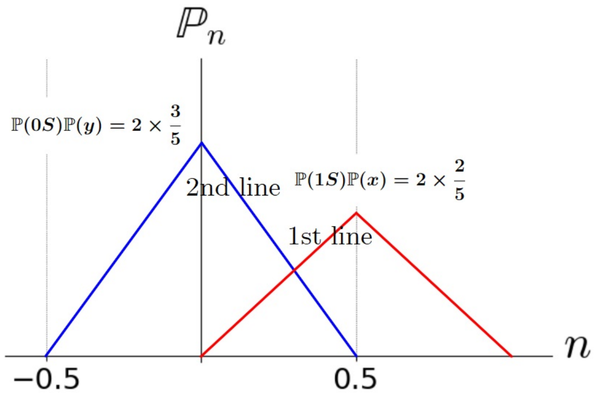
To calculate the pdf :

2) get x, y

x is the same distribution of the noise (triangle) with $S_1(T)$ mean (0.5 V) and σ^2 variance

y is the same distribution of the noise (triangle) with $S_2(T)$ mean (0 V) and σ^2 variance

3) multiply $\mathbb{P}(x)\mathbb{P}(1S)$ and $\mathbb{P}(y)\mathbb{P}(0S)$ and draw new distribution



4) Choose V_{th} :

from the figure :

equation of any line $y = mx + c$

equation of 1^st line :

$$m = \frac{\Delta y}{\Delta x} = \frac{2 \times \frac{2}{5}}{0.5} = \frac{8}{5}$$

$c = 0$

therefore $y_1 = \frac{8}{5}x_1$

equation of 2^nd line :

$$m = \frac{\Delta y}{\Delta x} = \frac{0 - \frac{6}{5}}{\frac{1}{2} - 0} = -\frac{12}{5}$$

$c = \frac{6}{5}$

therefore $y_2 = -\frac{12}{5}x_2 + \frac{6}{5}$

Solve 1^st line with 2^nd line

$\therefore V_{th} = 0.3V$

5) Get $\mathbb{P}_e$ = area of intersection $\frac{1}{2} \times \underbrace{0.5}_{\text{base}} \times \underbrace{\frac{8}{5} \times 0.3}_{\text{height}}$