Sorting in linear time

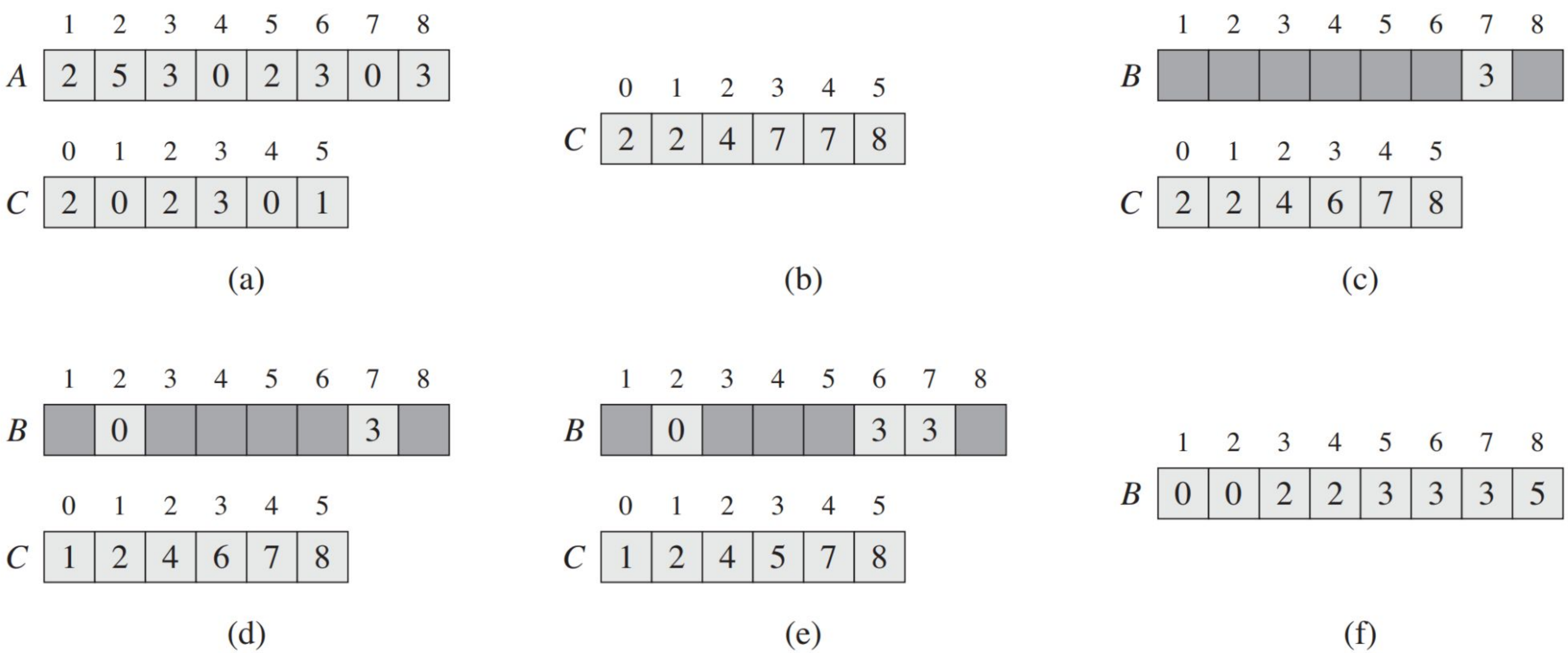
Data Structures & Algorithms

Counting Sort

Counting sort assumes that each of the **n** input elements is an integer in the range **0 to k,** for some integer **k.**

```
COUNTING-SORT(A, B, k)
let C[0..k] be a new array
for i = 0 to k
    C[i] = 0
for j = 1 to A.length
    C[A[j]] = C[A[j]] + 1
// C[i] now contains the number of elements equal to i.
for i = 1 to k
    C[i] = C[i] + C[i - 1]
// C[i] now contains the number of elements less than or equal to i.
for j = A.length downto 1
    B[C[A[j]]] = A[j]
    C[A[j]] = C[A[j]] - 1
```

Counting Sort



Stability

- numbers with the same value appear in the output array in the same order as they do in the input array.
- Normally, the property of stability is important only when satellite data are carried around with the element being sorted.
- Counting sort is stable.

Radix Sort

329	720	720	329
457	355	329	355
657	436	436	436
839	457	839	457
436	657	355	657
720	329	457	720
355	839	657	839

Radix sort

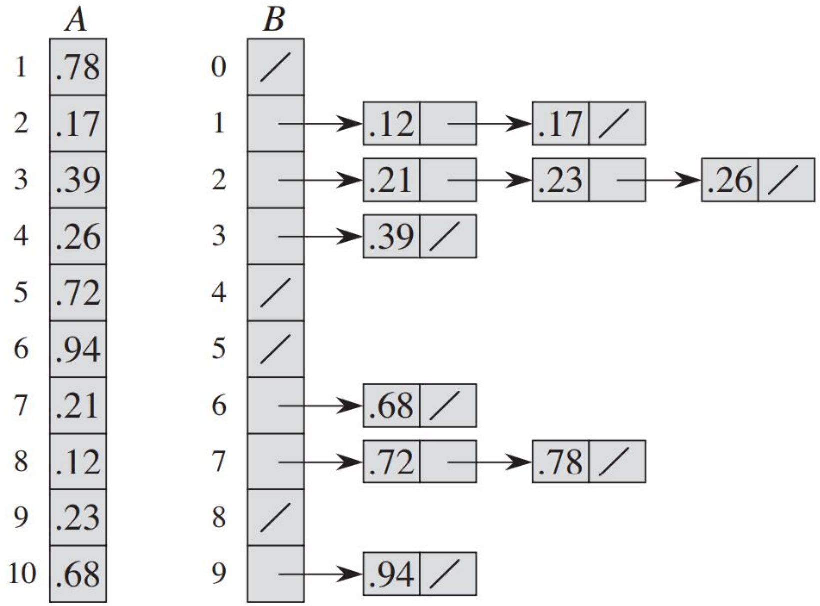
RADIX-SORT(A, d)

for $i = 1$ **to** d
    use a stable sort to sort array A on digit i

Given n d-digit numbers in which each digit can take on up to k possible values, RADIX-SORT correctly sorts these numbers in $\Theta(d(n+k))$ time if the stable sort it uses takes $\Theta(n+k)$ time.

Bucket sort

- Bucket sort assumes that the input is drawn from a uniform distribution and has an average-case running time of O(n)



(a) (b)

Bucket sort

BUCKET-SORT(A)

```
let B[0 .. n − 1] be a new array
n = A.length
for i = 0 to n − 1
    make B[i] an empty list
for i = 1 to n
    insert A[i] into list B[⌊nA[i]⌋]
for i = 0 to n − 1
    sort list B[i] with insertion sort
concatenate the lists B[0], B[1], . . . , B[n − 1] together in order
```

Bucket sort

$$T(n) = \Theta(n) + \sum_{i=0}^{n-1} O(n_i^2)$$

$$\begin{aligned} \mathrm{E}[T(n)] &= \mathrm{E}\left[\Theta(n) + \sum_{i=0}^{n-1} O(n_i^2)\right] \\ &= \Theta(n) + \sum_{i=0}^{n-1} \mathrm{E}\left[O(n_i^2)\right] \\ &= \Theta(n) + \sum_{i=0}^{n-1} O\left(\mathrm{E}\left[n_i^2\right]\right) \end{aligned}$$

Bucket sort

$X_{ij} = \mathrm{I}\{A[j] \text{ falls in bucket } i\}$

for $i = 0, 1, \ldots, n-1$ and $j = 1, 2, \ldots, n$. Thus,

$$n_i = \sum_{j=1}^{n} X_{ij} .$$

$$\begin{aligned}
\mathrm{E}\left[n_i^2\right] &= \mathrm{E}\left[\left(\sum_{j=1}^{n} X_{ij}\right)^2\right] \\
&= \mathrm{E}\left[\sum_{j=1}^{n}\sum_{k=1}^{n} X_{ij} X_{ik}\right] \\
&= \mathrm{E}\left[\sum_{j=1}^{n} X_{ij}^2 + \sum_{1 \le j \le n} \sum_{\substack{1 \le k \le n \\ k \ne j}} X_{ij} X_{ik}\right] \\
&= \sum_{j=1}^{n} \mathrm{E}\left[X_{ij}^2\right] + \sum_{1 \le j \le n} \sum_{\substack{1 \le k \le n \\ k \ne j}} \mathrm{E}\left[X_{ij} X_{ik}\right]
\end{aligned}$$

$$\begin{aligned}
\mathrm{E}\left[X_{ij}^2\right] &= 1^2 \cdot \frac{1}{n} + 0^2 \cdot \left(1 - \frac{1}{n}\right) \\
&= \frac{1}{n} .
\end{aligned}$$

$$\begin{aligned}
\mathrm{E}\left[X_{ij} X_{ik}\right] &= \mathrm{E}\left[X_{ij}\right] \mathrm{E}\left[X_{ik}\right] \\
&= \frac{1}{n} \cdot \frac{1}{n} \\
&= \frac{1}{n^2} .
\end{aligned}$$

$$\begin{aligned}
\mathrm{E}\left[n_i^2\right] &= \sum_{j=1}^{n} \frac{1}{n} + \sum_{1 \le j \le n} \sum_{\substack{1 \le k \le n \\ k \ne j}} \frac{1}{n^2} \\
&= n \cdot \frac{1}{n} + n(n-1) \cdot \frac{1}{n^2} \\
&= 1 + \frac{n-1}{n} \\
&= 2 - \frac{1}{n} ,
\end{aligned}$$