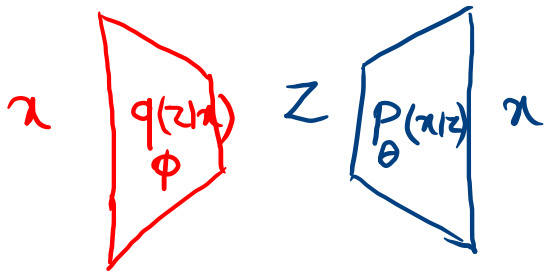


$$z \sim N(0, I)$$

$$p_\theta(x|z)$$

$$p_\theta(x, z) = p(z)\, p_\theta(x|z)$$

$$p_\theta(z|x) = \frac{p_\theta(x, z)}{p_\theta(x)} = \frac{p(z)\, p_\theta(x|z)}{\int_z p(z)\, p_\theta(x|z)\, dz}$$

$$\min_\phi \; KL\left(q_\phi(z|x) \,\|\, p_\theta(z|x) \right)$$

$$KL\left(q_\phi(z|x) \,\|\, p_\theta(z|x)\right) = KL\left(q_\phi(z|x) \,\|\, p_\theta(x,z)\right) + \log p_\theta(x)$$

$$\log p_\theta(x) = -KL\left(q_\phi(z|x) \,\|\, p_\theta(x,z)\right) + KL\left(q_\phi(z|x) \,\|\, p_\theta(z|x)\right)$$

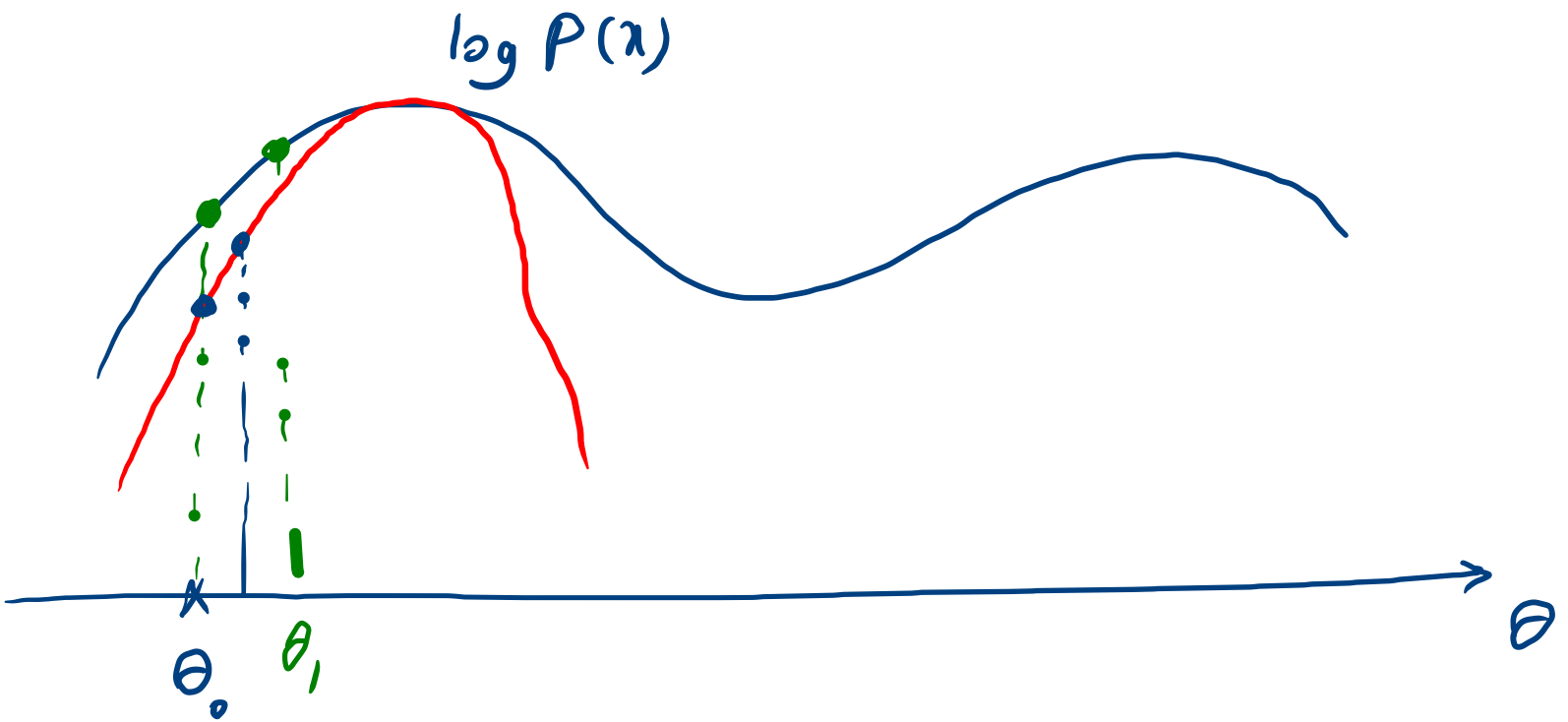
ELBO

$$\max_{\phi,\theta} \; -KL\left(q_\phi(z|x) \,\|\, p_\theta(x,z)\right)$$

$P(x;\theta)$

$D = \{x_1, x_2, \ldots, x_n\}$

$$\hat{\theta}_{ML} = \arg\max_{\theta} \sum_{i=1}^{n} \log P(x_i;\theta)$$



$$\min_{\theta,\phi} \; KL\left(q_\phi(z|x) \,\|\, p_\theta(x,z)\right)$$

$$KL(q\|p) = \int q(z)\left(\log \frac{q(z)}{p(z)}\right) dz = E_q\left[\log \frac{q(z)}{p(z)}\right]$$

$$= E_{q_\phi}\left[\log \frac{q_\phi(z|x)}{p_\theta(x,z)}\right] = E_{q_\phi}\left[\log \frac{q_\phi(z|x)}{p(z)} \cdot \frac{1}{p_\theta(x|z)}\right]$$

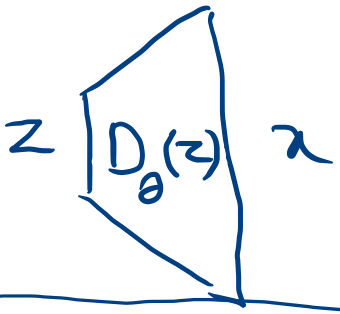
$p(z)\, p_\theta(x|z)$

$$= E_{q_\phi}\left[\log \frac{q_\phi(z|x)}{p(z)}\right] - E_{q_\phi}\left[\log p_\theta(x|z)\right]$$

$$= KL\left(q_\phi(z|x) \,\|\, p(z)\right) - E_{q_\phi}\left[\log p_\theta(x|z)\right]$$

Reconstruction error

$$-\mathbb{E}_{q_\phi}\left[\log p_\theta(x|z)\right]$$



$$p_\theta(x|z) = \mathcal{N}\left(D(z), I\right)$$

$$\log p_\theta(x|z) = \log \exp\left(-\frac{1}{2}\|x - D(z)\|^2\right) + \text{const}$$

$$= \frac{-1}{2}\|x - D(z)\|^2$$

$$\frac{1}{2}\mathbb{E}_q\left[\|x - D(z)\|^2\right]$$