

RL for LLM Alignment and Inference

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Research topics

- **Machine Learning:** reinforcement learning, uncertainty quantification, federated learning, inverse constraint learning
- **Natural Language Processing:** LLM alignment and inference, agentic LLMs, knowledge graphs, post-editing ASR error correction
- **Applications:** autonomous driving, sports analytics, material design for CO₂ recycling



Industry Partners



research
director



BOREALIS AI



TalkIQ
consultant



PRONAV
Technologies Ltd

consultant



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Outline

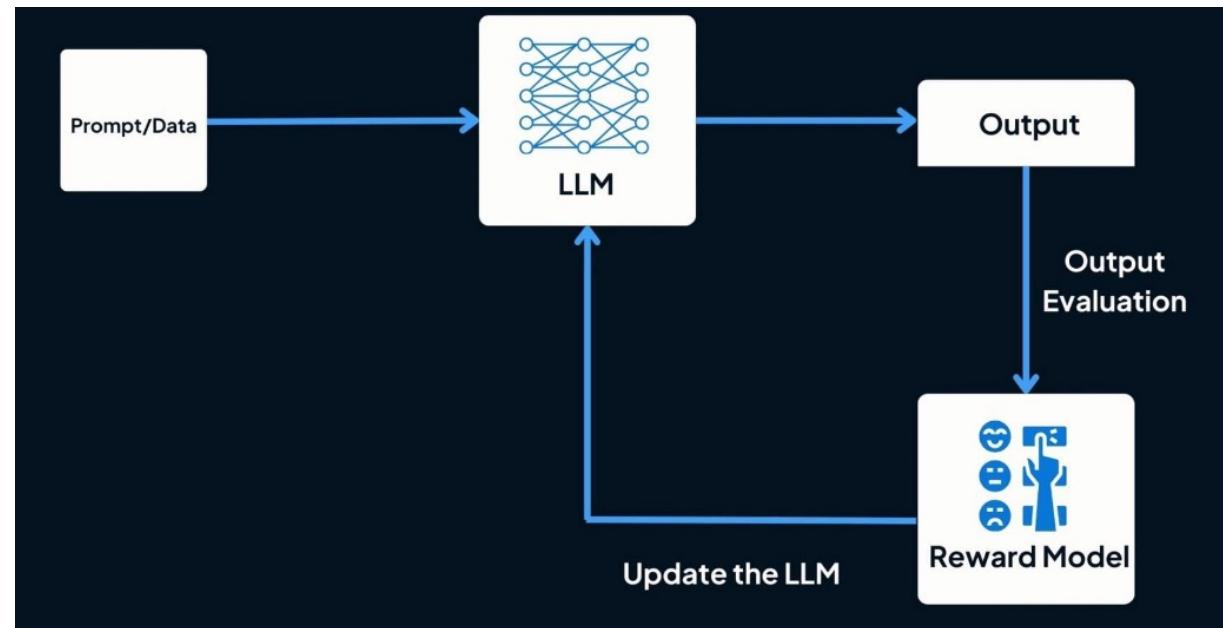
- LLM Alignment
 - Reinforcement Learning from Human Feedback
 - Direct Preference Optimization
 - Reward Guided Text Generation
- LLM Reasoning
 - Search and planning
 - Group Relative Policy Optimization (GRPO)
 - Reflection: Verbalized RL

Large Language Models

- **Agent:** system
- **Environment:** user
- **State:** history of past utterances
- **Action:** system utterance
- **Reward:** task completion, human feedback

"We posit that the superior writing abilities of LLMs, as manifested in surpassing human annotators in certain tasks, are fundamentally driven by RLHF, as documented in Gilardi et al. (2023) and Huang et al. (2023)."

Llama 2 Technical Report (Touvron et. al 2023)



Credit: <https://www.twine.net/blog/what-is-reinforcement-learning-from-human-feedback-rlhf-and-how-does-it-work/>

"This behavior (re-evaluation) is not only a testament to the model's growing reasoning abilities but also a captivating example of how reinforcement learning can lead to unexpected and sophisticated outcomes"
Deepseek-R1 Technical Report (Guo et. al 2025)

Recipe For Large Language Model Training

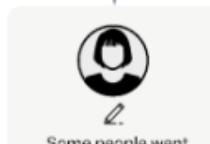
Step 1

Collect demonstration data, and train a supervised policy.

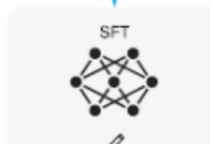
A prompt is sampled from our prompt dataset.



A labeler demonstrates the desired output behavior.



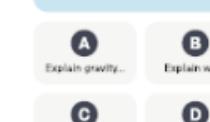
This data is used to fine-tune GPT-3 with supervised learning.



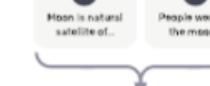
Step 2

Collect comparison data, and train a reward model.

A prompt and several model outputs are sampled.



A labeler ranks the outputs from best to worst.



This data is used to train our reward model.



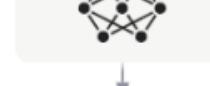
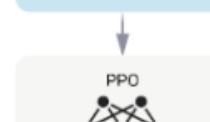
Step 3

Optimize a policy against the reward model using reinforcement learning.

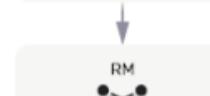
A new prompt is sampled from the dataset.



The policy generates an output.



The reward model calculates a reward for the output.



The reward is used to update the policy using PPO.

InstructGPT (Ouyang et. al 2022)

Response Ranking

PP

Explain reinforcement learning in three sentences.

< 1 / 2 >



Reinforcement learning is a type of machine learning where an agent learns to make decisions by receiving feedback in the form of rewards or punishments. The agent's goal is to maximize its cumulative reward over time by exploring different actions and learning which actions lead to the highest rewards. Reinforcement learning has been successfully applied to a variety of tasks, including game playing, robotics, and recommendation systems.



< 2 / 2 >



Reinforcement learning is a type of machine learning that focuses on training an agent to make decisions based on trial and error. The agent receives feedback in the form of rewards or penalties for each decision it makes. By learning from this feedback, the agent can improve its decision-making abilities over time.



Was this response better or worse? Better Worse Same

RL from Human Feedback (RLHF)

- Collect a preference data set:

$$D = \{(s, a_+, a_-)_k\}_{k=1}^K \text{ where } a_+ \succ a_-$$

- Train a reward model according to the Bradley Terry Model:

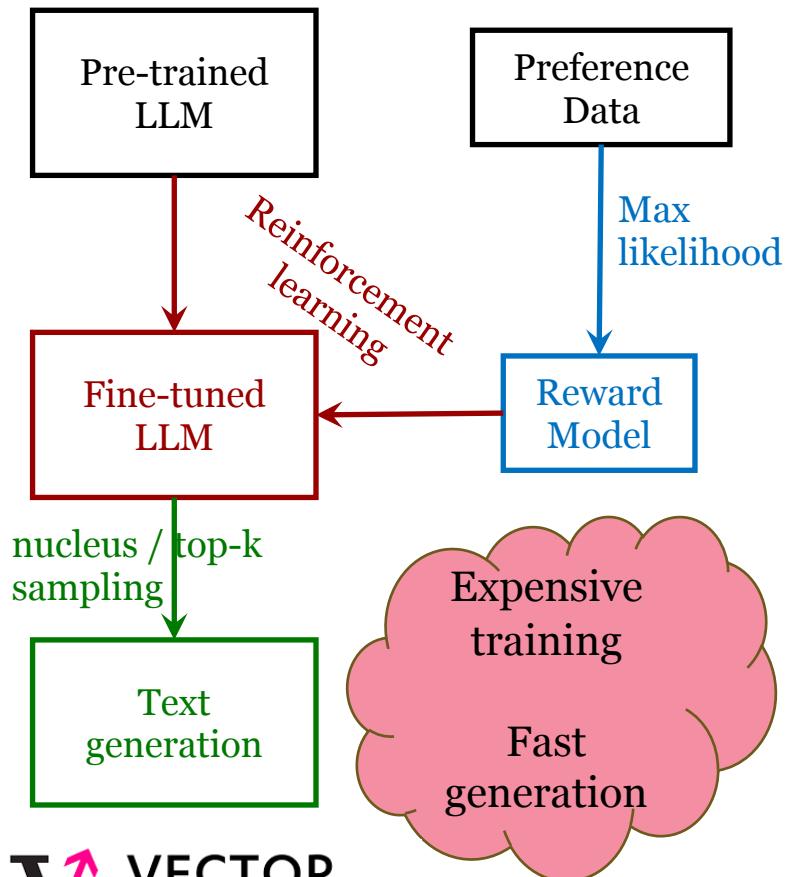
$$\max_{\theta} E_D [\log \sigma(r_{\theta}(s, a_+) - r_{\theta}(s, a_-))]$$

- Make a copy of the LLM and finetune it to maximize:

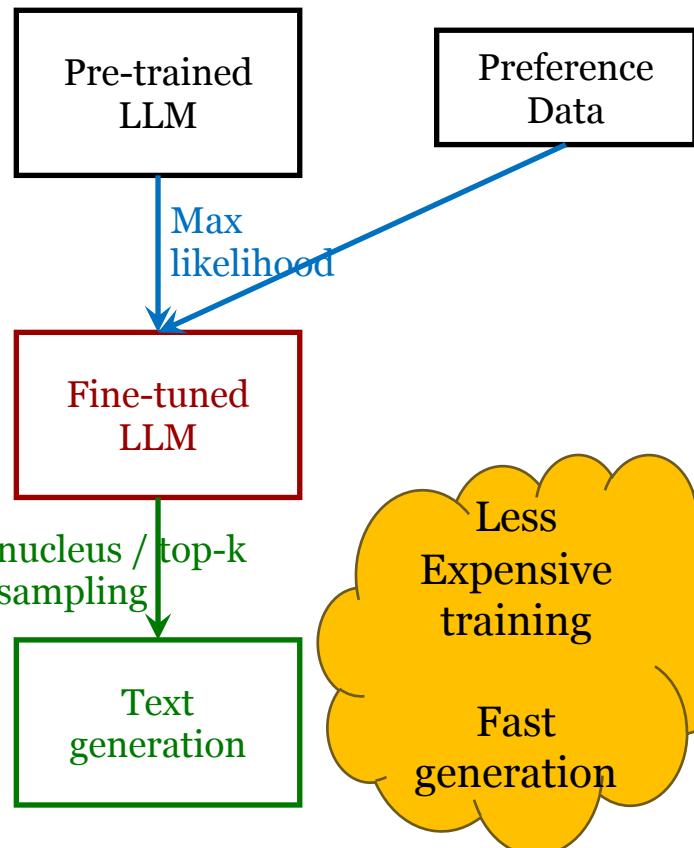
$$\max_{\phi} E_{D, \pi_{\phi}} [r_{\phi}(s, a)] - \beta KL[\pi_{\phi}(a|s) || \pi_{pretrained}(a|s)]$$

RLHF Improvements

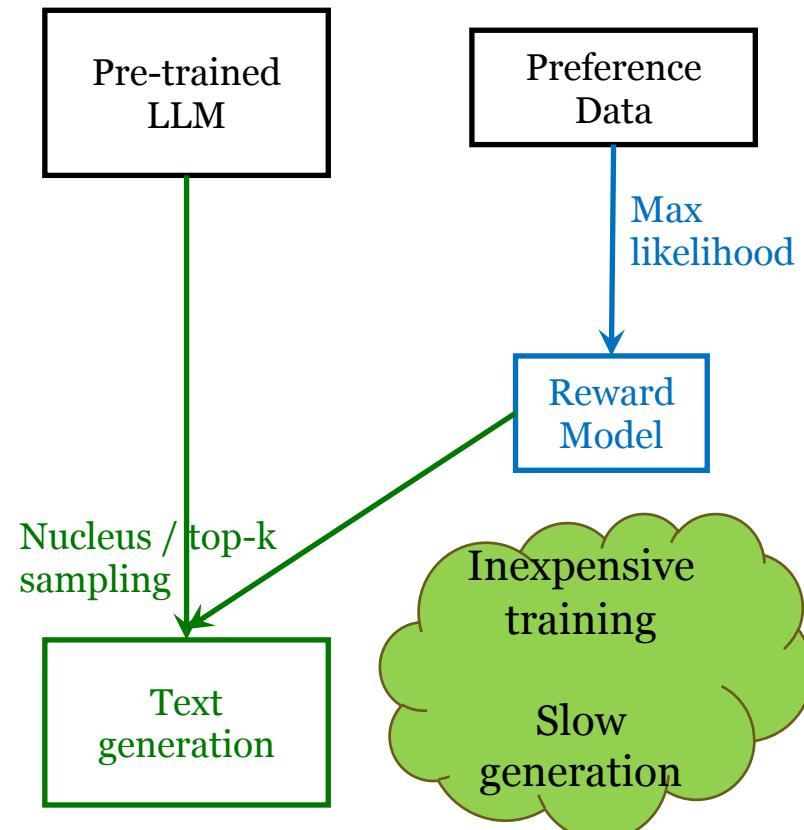
Proximal Policy Optimization (PPO)
Ouyang et al., 2022



Direct Preference Optimization (DPO)
Rafailov et al., 2023

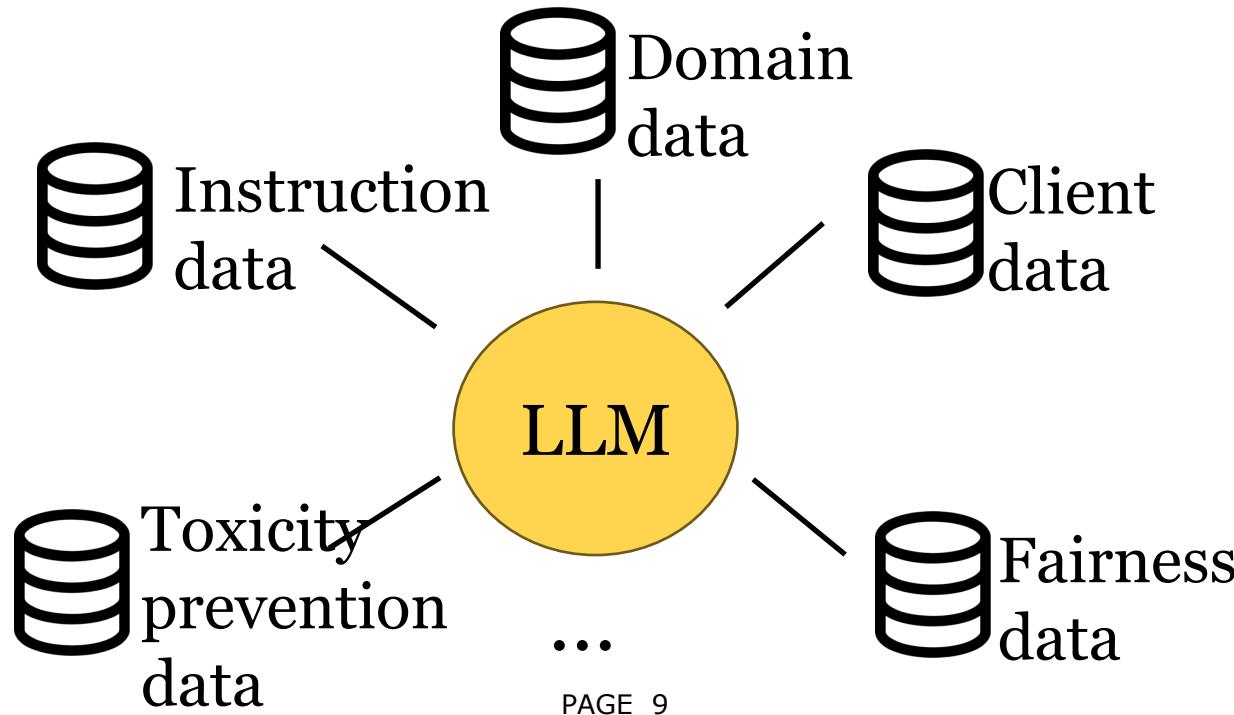


Reward Guided Text Generation (RGTG)
Khanov et al, 2024
Rashid et al., 2025



LLM Alignment with Preference Data

- Collect preference data: $D = \{(s, a_+, a_-)_k\}_{k=1}^K$
where s : user prompt a : system response
 a_+ is preferred to a_- (i.e., $a_+ > a_-$)



Reward Model

Stiennon, Ouyang, Wu, Ziegler, Lowe Voss, Radford, Amodei, Christiano
(2020) Learning to summarize from human feedback, NeurIPS.

- Reward function: $r_\theta(s, a) = \text{real number}$
- Consider several possible responses $a_1 \geq a_2 \geq \dots \geq a_k$ ranked by annotator
- Training reward function to be consistent with the ranking:

$$\text{Loss}(\theta) = -\frac{1}{\binom{k}{2}} E_{(s, a_i, a_j) \in \text{Dataset}} \log \sigma(r_\theta(s, a_i) - r_\theta(s, a_j))$$

Reinforcement Learning

Ouyang, Wu, Jiang, Wainwright, et al. (2022) **Training language models to follow instructions with human feedback**, *NeurIPS*.

- Pretrain language model (GPT-3)
- Fine-Tune GPT-3 by RL to obtain InstructGPT
 - Policy (language model): $\pi_\phi(a|s)$
 - Optimize $\pi_\phi(s)$ by Proximal Policy Iteration (PPO)

$$\max_{\phi} E_{s \in \text{Dataset}} \left[E_{a \sim \pi_\phi(a|s)} [r_\theta(s, a)] - \beta \text{KL}(\pi_\phi(\cdot|s) \mid \pi_{ref}(\cdot|s)) \right]$$

Policy Optimization

Stochastic policy $\pi_\phi(a|s) = \Pr(a|s; \phi)$ parametrized by ϕ .

	Supervised Fine-Tuning	Reinforcement Learning
Data	$\{(s_1, a_1^*), (s_2, a_2^*), \dots\}$ $(a^* \text{ denotes optimal action})$	$\{(s_1, a_1, r_1), (s_2, a_2, r_2), \dots\}$ $(r \text{ denotes reward for s,a pair})$
Objective	Maximum likelihood $\max_{\phi} \sum_n \log \pi_\phi(a_n^* s_n)$	Maximum expected rewards $\max_{\phi} \sum_n \gamma^n E_{\pi_\phi}[r_n s_n, a_n]$
Policy update	$\phi \leftarrow \phi + \alpha \nabla_{\phi} \log \pi_\phi(a_n^* s_n)$	$\phi \leftarrow \phi + \alpha \mathbf{G}_n \nabla_{\phi} \log \pi_\phi(a_n s_n)$ where $G_n = \sum_{t=n}^{\infty} \gamma^t r_t$

REINFORCE Algorithm

REINFORCE(s_0)

Initialize π_ϕ to anything

Loop forever (for each episode)

 Generate episode $s_0, a_0, r_0, s_1, a_1, r_1, \dots, s_T, a_T, r_T$ with π_ϕ

 Loop for each step of the episode $n = 0, 1, \dots, T$

$$G_n \leftarrow \sum_{t=n}^T \gamma^t r_t$$

 Update policy: $\phi \leftarrow \phi + \alpha G_n \nabla_\theta \log \pi_\phi(a_n | s_n)$

Return π_ϕ

Proximal Policy Optimization (PPO)

Initialize π_ϕ and V_w to anything

Loop forever (for each episode)

 Generate episode $s_0, a_0, r_0, s_1, a_1, r_1, \dots, s_{N-1}, a_{N-1}, r_{N-1}$ with π_ϕ

 Loop for each step of the episode $n = 0, 1, \dots, N - 1$

$$G_n \leftarrow \sum_{t=n}^N \gamma^t r_t$$

$$A(s_n, a_n) \leftarrow G_n - V_w(s_n)$$

 Update value function: $w \leftarrow w + \alpha_w A(s_n, a_n) \nabla_w V_w(s_n)$

 Update π :

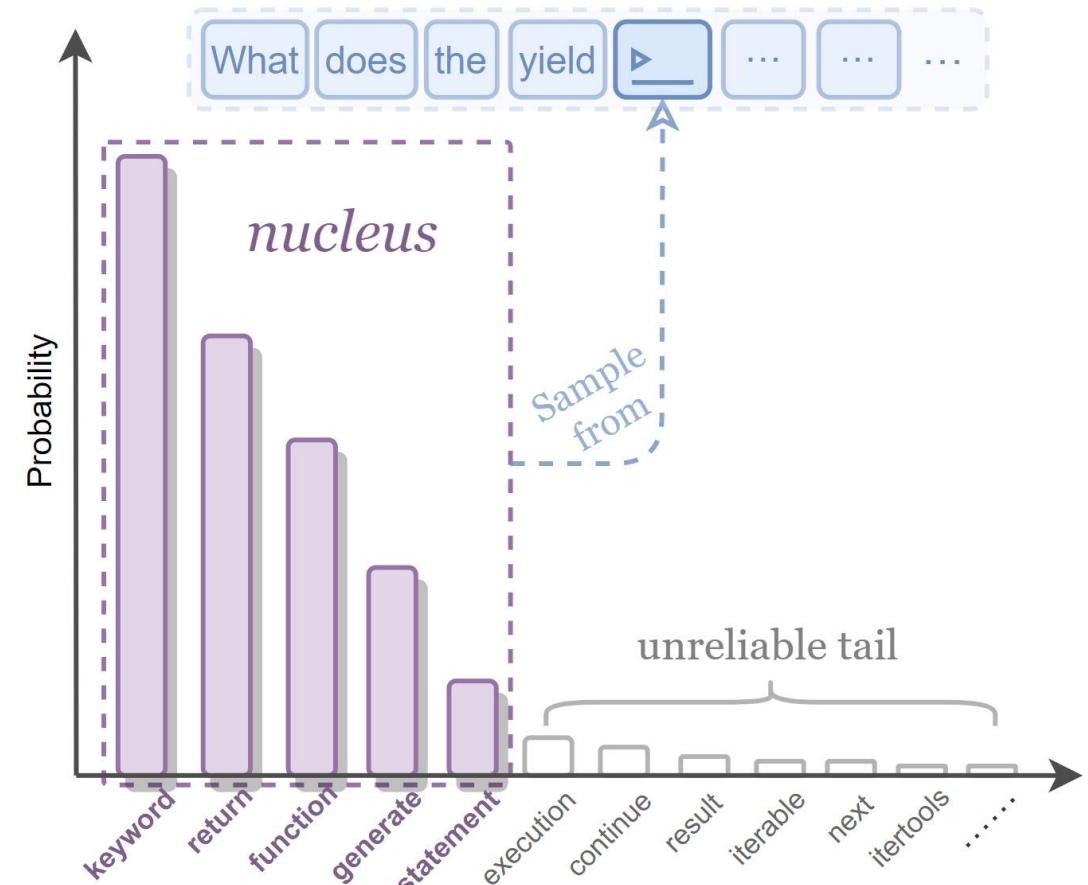
optimize by stochastic gradient descent

$$\phi \leftarrow \operatorname{argmax}_{\tilde{\phi}} \frac{1}{N} \sum_{n=0}^{N-1} \min \left\{ \begin{array}{l} \frac{\pi_{\tilde{\phi}}(a_n|s_n)}{\pi_\phi(a_n|s_n)} A(s_n, a_n), \\ \operatorname{clip} \left(\frac{\pi_{\tilde{\phi}}(a_n|s_n)}{\pi_\phi(a_n|s_n)}, 1 - \epsilon, 1 + \epsilon \right) A(s_n, a_n) \end{array} \right\}$$

Inference: Nucleus sampling

Sample from nucleus (top tokens only) to avoid unreliable responses while ensuring diversity

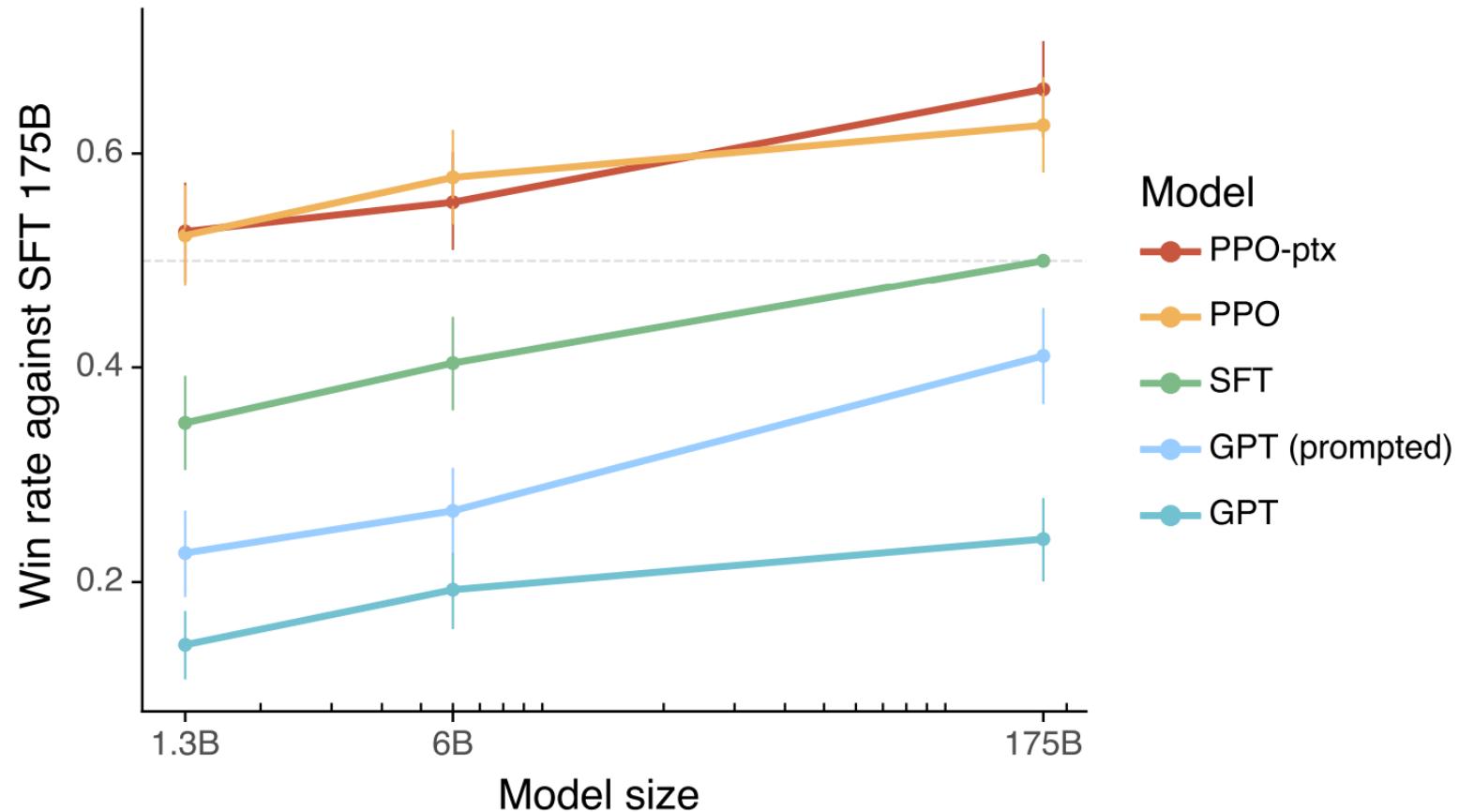
Holtzman, Ari; Buys, Jan; Du, Li;
Forbes, Maxwell; Choi, Yejin (2019).
The Curious Case of Neural Text Degeneration, arxiv.



Credit: <https://arxiv.labs.arxiv.org/html/2208.11523>

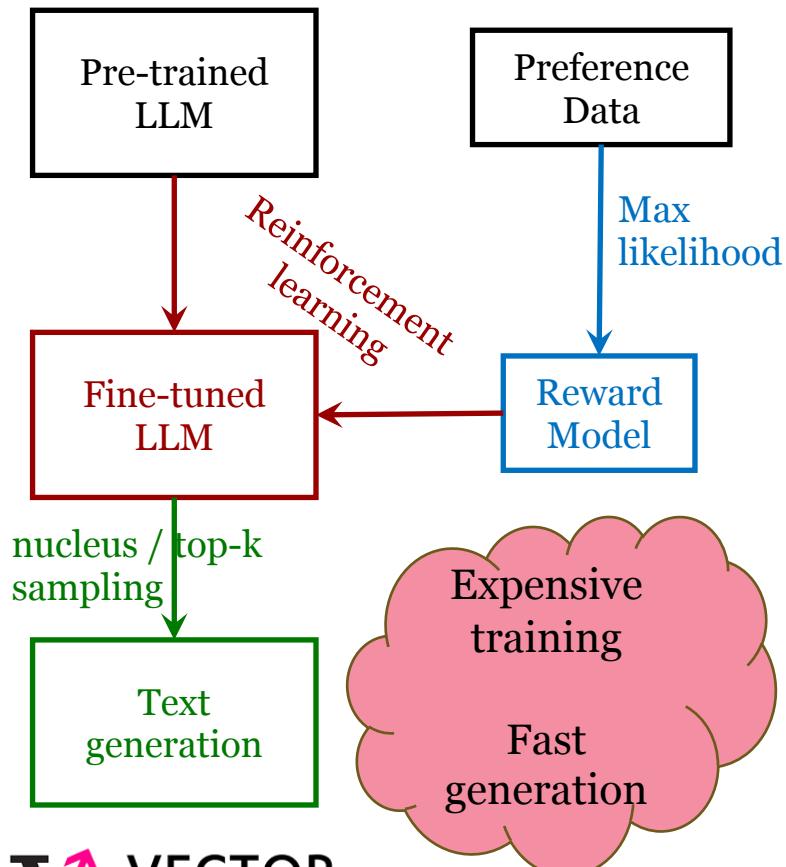
InstructGPT Results

Ouyang, Wu, Jiang, Wainwright, et al. (2022)

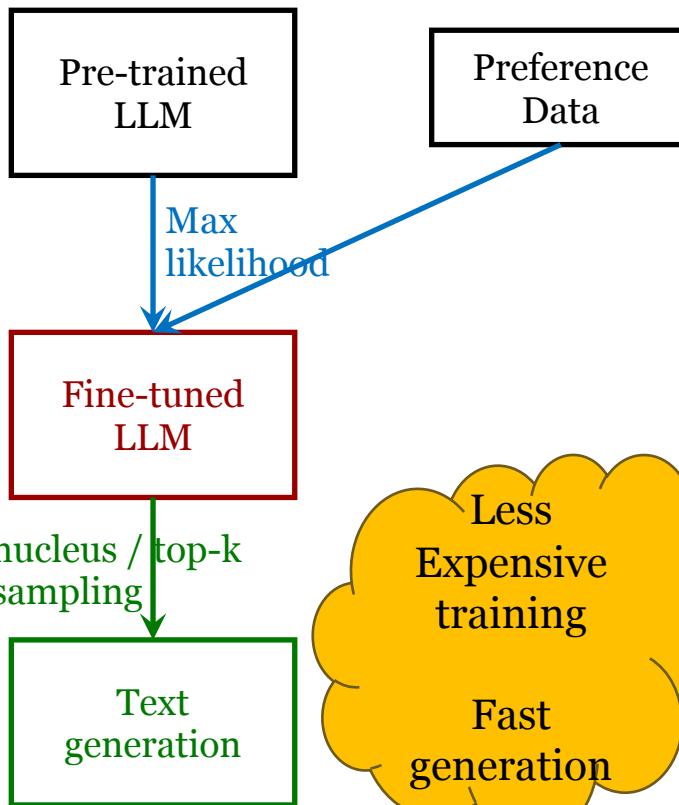


RLHF Improvements

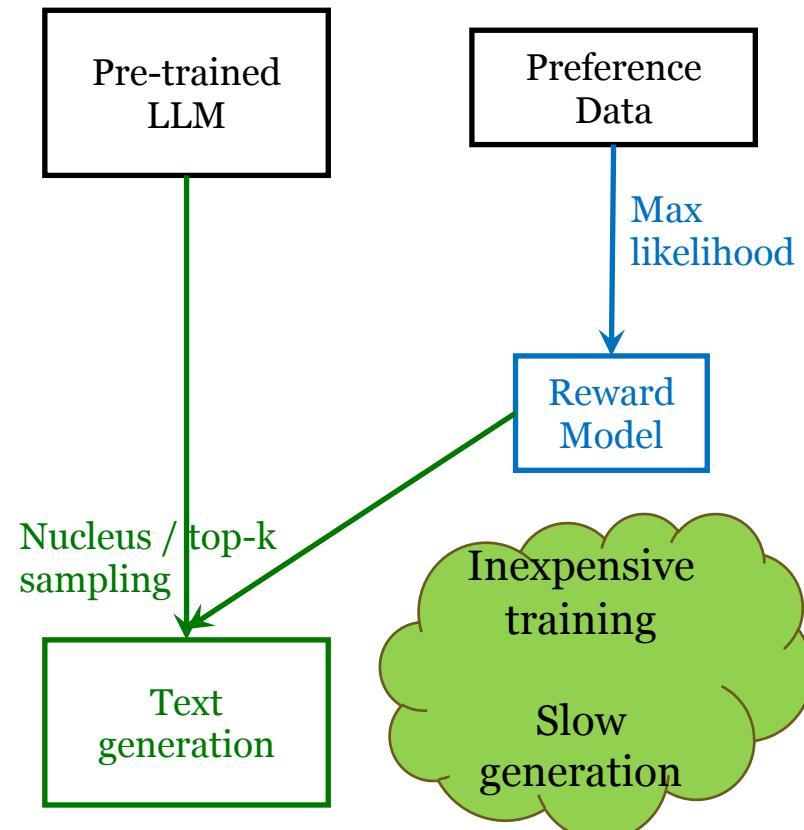
Proximal Policy Optimization (PPO)
Ouyang et al., 2022



Direct Preference Optimization (DPO)
Rafailov et al., 2023



Reward Guided Text Generation (RG TG)
Khanov et al, 2024
Rashid et al., 2025



Direct Preference Optimization

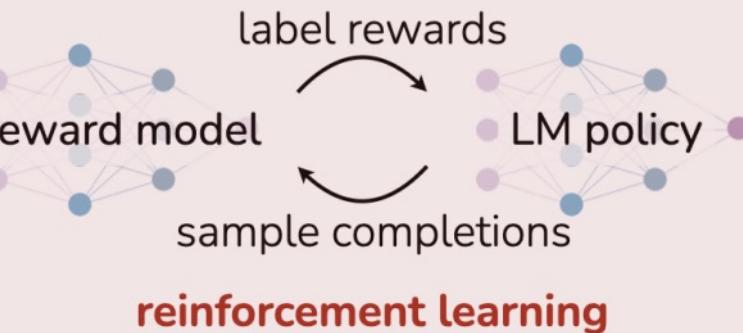
Rafailov, Sharma, Mitchell, Ermon, Manning, Finn (2023) **Direct Preference Optimization: Your Language Model is Secretly a Reward Model, NeurIPS.**

Reinforcement Learning from Human Feedback (RLHF)

x: "write me a poem about
the history of jazz"



maximum likelihood



Direct Preference Optimization (DPO)

x: "write me a poem about
the history of jazz"



maximum likelihood



Bypassing RL

- Recall RL objective:

$$\max_{\phi} E_{s \in \text{Dataset}} \left[E_{a \sim \pi_{\phi}(a|s)} [r_{\theta}(s, a)] - \beta KL(\pi_{\phi}(\cdot|s) \mid\mid \pi_{ref}(\cdot|s)) \right]$$

- Closed form solution (based on maximum entropy RL):

$$\pi_{\phi}(a|s) = \frac{1}{Z(s)} \pi_{ref}(a|s) \exp\left(\frac{r_{\theta}(s,a)}{\beta}\right)$$

- Isolate reward: $r_{\theta}(s, a) = \beta \log \frac{\pi_{\phi}(a|s)}{\pi_{ref}(a|s)} + \beta \log Z(s)$

- Plug into preference objective:

$$\begin{aligned} Loss(\theta) &= -\frac{1}{\binom{k}{2}} E_{(s, a_i, a_j) \in \text{Dataset}} \log \sigma(r_{\theta}(s, a_i) - r_{\theta}(s, a_j)) \\ &= -\frac{1}{\binom{k}{2}} E_{(s, a_i, a_j) \in \text{Dataset}} \log \sigma\left(\beta \log \frac{\pi_{\phi}(a_i|s)}{\pi_{ref}(a_i|s)} - \beta \log \frac{\pi_{\phi}(a_j|s)}{\pi_{ref}(a_j|s)}\right) \end{aligned}$$

Optimal Policy Derivation

$$\operatorname{argmax}_{\phi} E_{s \in \text{Dataset}} \left[E_{a \sim \pi_{\phi}(a|s)} [r_{\theta}(s, a)] - \beta \operatorname{KL}(\pi_{\phi}(\cdot|s) \mid \pi_{ref}(\cdot|s)) \right]$$

$$= \operatorname{argmax}_{\phi} E_{s \in \text{Dataset}} \left[E_{a \sim \pi_{\phi}(a|s)} \left[r_{\theta}(s, a) - \beta \log \frac{\pi_{\phi}(a|s)}{\pi_{ref}(a|s)} \right] \right]$$

$$= \operatorname{argmin}_{\phi} E_{s \in \text{Dataset}} \left[E_{a \sim \pi_{\phi}(a|s)} \left[\log \frac{\pi_{\phi}(a|s)}{\pi_{ref}(a|s)} - \frac{1}{\beta} r_{\theta}(s, a) \right] \right]$$

$$= \operatorname{argmin}_{\phi} E_{s \in \text{Dataset}} \left[E_{a \sim \pi_{\phi}(a|s)} \left[\log \frac{\pi_{\phi}(a|s)}{\frac{1}{Z(s)} \pi_{ref}(a|s) \exp\left(\frac{r_{\theta}(s,a)}{\beta}\right)} - \log Z(s) \right] \right]$$

$$= \operatorname{argmin}_{\phi} E_{s \in \text{Dataset}} \left[E_{a \sim \pi_{\phi}(a|s)} \left[\log \frac{\pi_{\phi}(a|s)}{\frac{1}{Z(s)} \pi_{ref}(a|s) \exp\left(\frac{r_{\theta}(s,a)}{\beta}\right)} \right] \right]$$

$$= \operatorname{argmin}_{\phi} E_{s \in \text{Dataset}} \left[E_{a \sim \pi_{\phi}(a|s)} \left[\log \frac{\pi_{\phi}(a|s)}{\pi_{\phi^*}(a|s)} \right] \right]$$

where $\pi_{\phi^*}(a|s) = \frac{1}{Z(s)} \pi_{ref}(a|s) \exp\left(\frac{r_{\theta}(s,a)}{\beta}\right)$

$$= \operatorname{argmin}_{\phi} E_{s \in \text{Dataset}} [\operatorname{KL}(\pi_{\phi}(\cdot|s) \mid \pi_{\phi^*}(\cdot|s))]$$

by KL definition

since KL is minimized when both arguments are equal

by KL definition

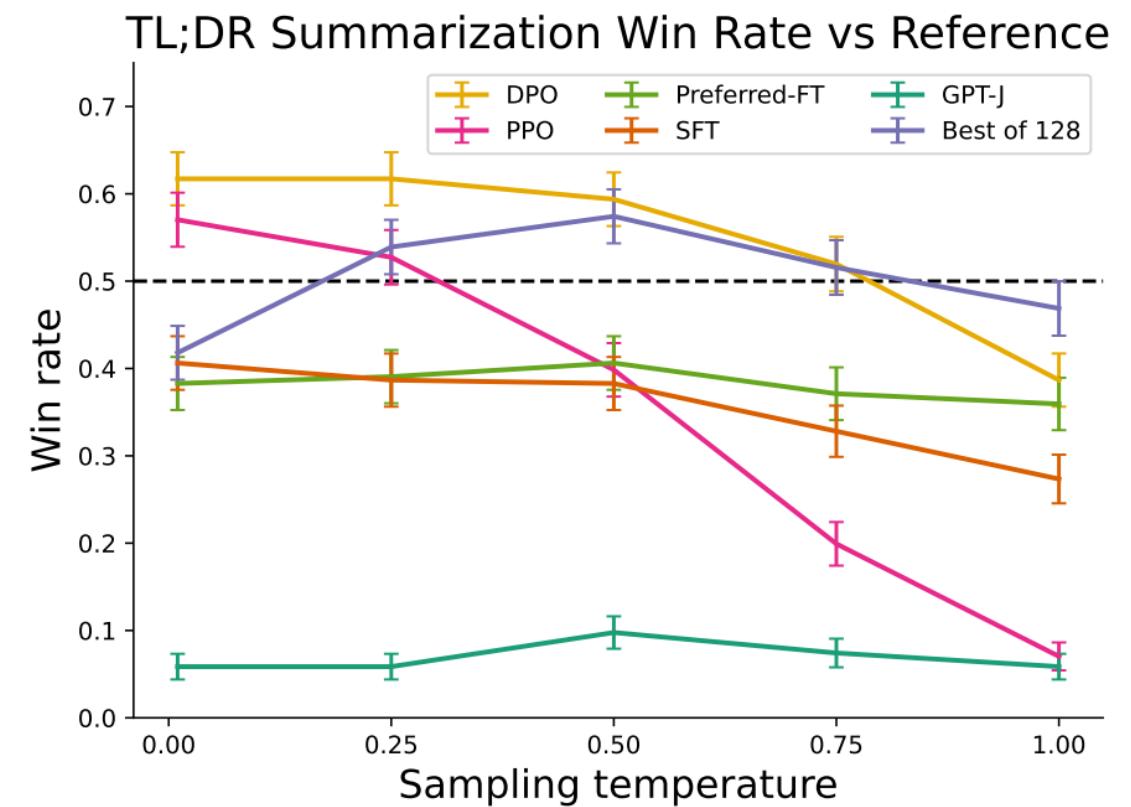
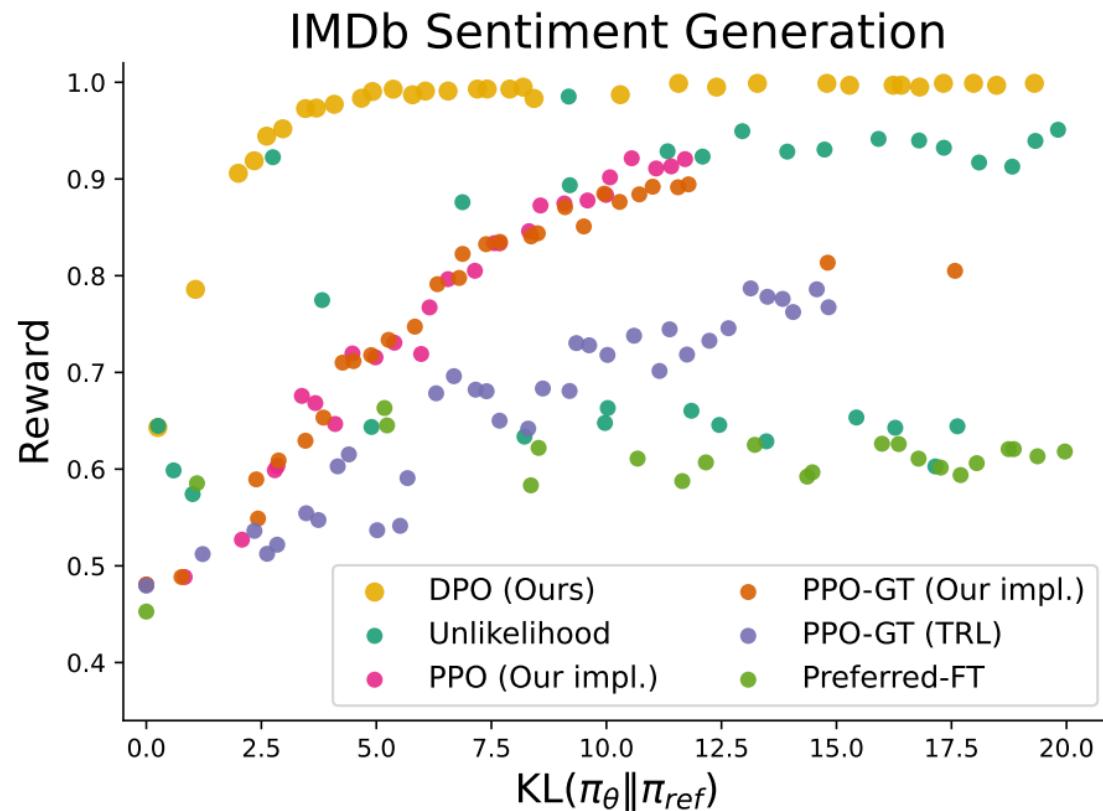
since $\max = -\min$

where $Z(s) = \sum_a \pi_{ref}(a|s) \exp\left(\frac{r_{\theta}(s,a)}{\beta}\right)$

since $\log Z(s)$ is independent of ϕ

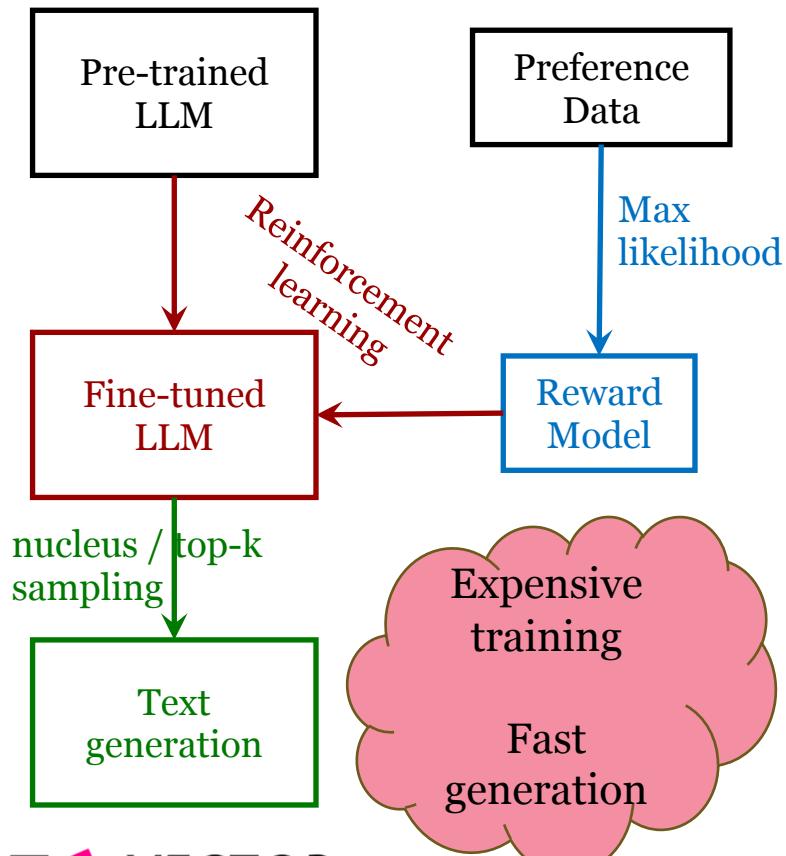
Empirical Results

Rafailov et al. 2023

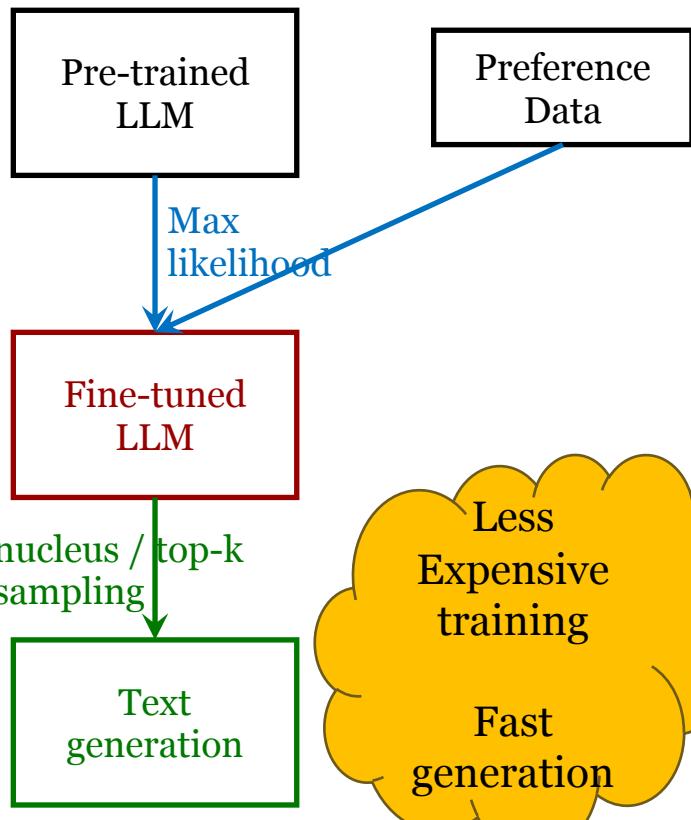


RLHF Improvements

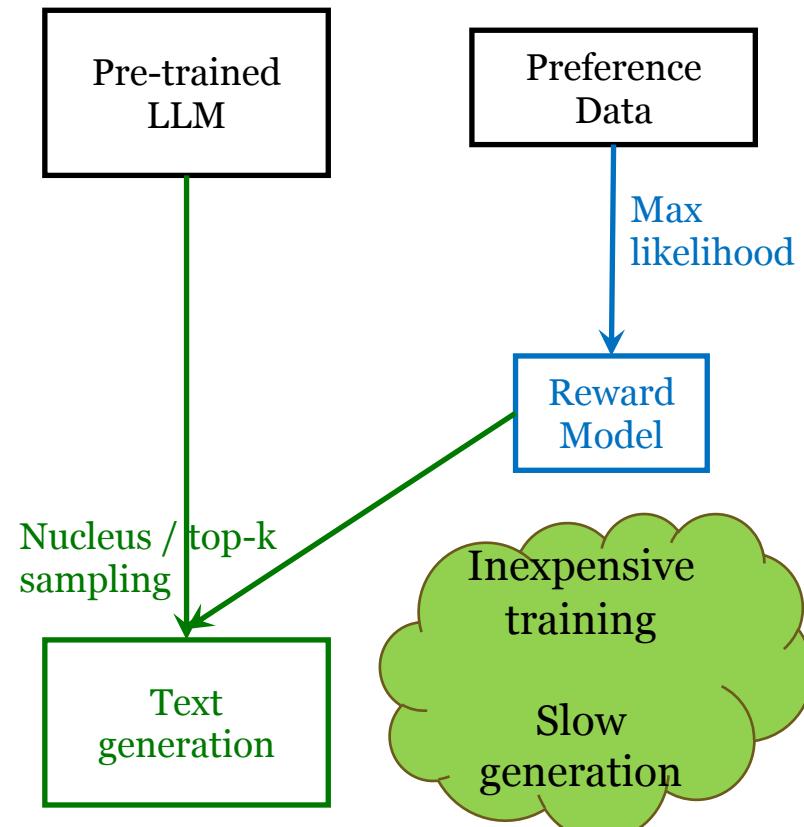
Proximal Policy Optimization (PPO)
Ouyang et al., 2022



Direct Preference Optimization (DPO)
Rafailov et al., 2023



Reward Guided Text Generation (RGTG)
Khanov et al, 2024
Rashid et al., 2025



Sequence Generation

- Recall closed form solution

$$\begin{aligned}\pi_{\phi}(a|s) &= \frac{1}{Z(s)} \pi_{ref}(a|s) \exp\left(\frac{r_{\theta}(s,a)}{\beta}\right) \\ &= \text{softmax}\left(\log \pi_{ref}(a|s) + \frac{r_{\theta}(s,a)}{\beta}\right)\end{aligned}$$

- Text generation:

$$a \sim \text{softmax} \left(\log \begin{pmatrix} \pi_{ref}(a_1|s) \\ \pi_{ref}(a_2|s) \\ \pi_{ref}(a_3|s) \\ \dots \\ \pi_{ref}(a_n|s) \end{pmatrix} + \begin{pmatrix} r_{\theta}(s,a_1) \\ r_{\theta}(s,a_2) \\ r_{\theta}(s,a_3) \\ \dots \\ r_{\theta}(s,a_n) \end{pmatrix} / \beta \right)$$

Token Generation

- Token-wise LLM modeling

$$\begin{aligned}\pi_{\phi}(\mathbf{a}^i | s, \mathbf{a}^{1:i-1}) &= \frac{1}{Z(s)} \pi_{ref}(\mathbf{a}^i | s, \mathbf{a}^{1:i-1}) \exp\left(\frac{r_{\theta}(s, \mathbf{a}^{1:i})}{\beta}\right) \\ &= \text{softmax}\left(\log \pi_{ref}(\mathbf{a}^i | s, \mathbf{a}^{1:i-1}) + \frac{r_{\theta}(s, \mathbf{a}^{1:i})}{\beta}\right)\end{aligned}$$

- Token generation:

$$a^i \sim \text{softmax} \left(\log \begin{pmatrix} \pi_{ref}(\mathbf{a}_1^i | s, \mathbf{a}^{1:i-1}) \\ \pi_{ref}(\mathbf{a}_2^i | s, \mathbf{a}^{1:i-1}) \\ \pi_{ref}(\mathbf{a}_3^i | s, \mathbf{a}^{1:i-1}) \\ \dots \\ \pi_{ref}(\mathbf{a}_n^i | s, \mathbf{a}^{1:i-1}) \end{pmatrix} + \begin{pmatrix} r_{\theta}(s, \mathbf{a}^{1:i-1}, \mathbf{a}_1^i) \\ r_{\theta}(s, \mathbf{a}^{1:i-1}, \mathbf{a}_2^i) \\ r_{\theta}(s, \mathbf{a}^{1:i-1}, \mathbf{a}_3^i) \\ \dots \\ r_{\theta}(s, \mathbf{a}^{1:i-1}, \mathbf{a}_n^i) \end{pmatrix} / \beta \right)$$

FaRMA: Faster Reward Model for Alignment

- Rashid, Wu, Fan, Li, Kristiadi, Poupart (2025) **Towards Cost-Effective Reward Guided Text Generation, ICML.**
- Optimization problem:

$$\max_{\theta} E_{(s, a_+, a_-) \in \text{Dataset}} \log \sigma(r_{\theta}(s, a_+) - r_{\theta}(s, a_-))$$

$$\text{Subject to } r_{\theta}(s, a^{1:i}) = \max_{a^{i+1:|a|}} r_{\theta}(s, [a^{1:i}, a^{i+1:|a|}]) \quad \forall s, a, i$$

- In practice: alternate between minimizing two loss functions

$$\bullet L_1(\theta) = -E_{(s, a_+, a_-) \in \text{Dataset}} \log \sigma(r_{\theta}(s, a_+) - r_{\theta}(s, a_-))$$

$$\bullet L_2(\theta) = \frac{1}{2} E_{(s, a) \in \text{Dataset}, i \leq |a|} \left(r_{\theta}(s, a^{1:i}) - \max_{a^{i+1:|a|}} r_{\theta}(s, [a^{1:i}, a^{i+1:|a|}]) \right)^2$$

FaRMA Pseudocode

Repeat

 Repeat for each $(s, \mathbf{a}_+, \mathbf{a}_-)$ in minibatch

$$L_1(\theta) = \log \sigma(r_\theta(s, \mathbf{a}_+) - r_\theta(s, \mathbf{a}_-))$$

$$\theta \leftarrow \theta - \alpha \nabla L_1(\theta)$$

 Repeat for each (s, \mathbf{a}, i) in minibatch

$$L_2(\theta) = \frac{1}{2} \left(r_\theta(s, \mathbf{a}^{1:i}) - \max_{\mathbf{a}^{i+1}} r_\theta(s, \mathbf{a}^{1:i+1}) \right)^2$$

$$\theta \leftarrow \theta - \alpha \nabla L_2(\theta)$$

Empirical Results

TL;DR Summarization			
Method	LLM	$r \pm \text{SE}$	Time(min)
π_{ref}	frozen	0.98 ± 0.18	2
ARGS	frozen	1.46 ± 0.16	32
PARGS	frozen	1.56 ± 0.19	31
CD	frozen	1.15 ± 0.16	29
FaRMA	frozen	2.05 ± 0.15	5
CARDS	frozen	1.73 ± 0.16	17
DPO	trained	2.08 ± 0.18	2
PPO	trained	2.05 ± 0.14	2

Table 2. Avg. reward (over 100 samples) \pm standard error total generation time for the TL;DR summarization task.

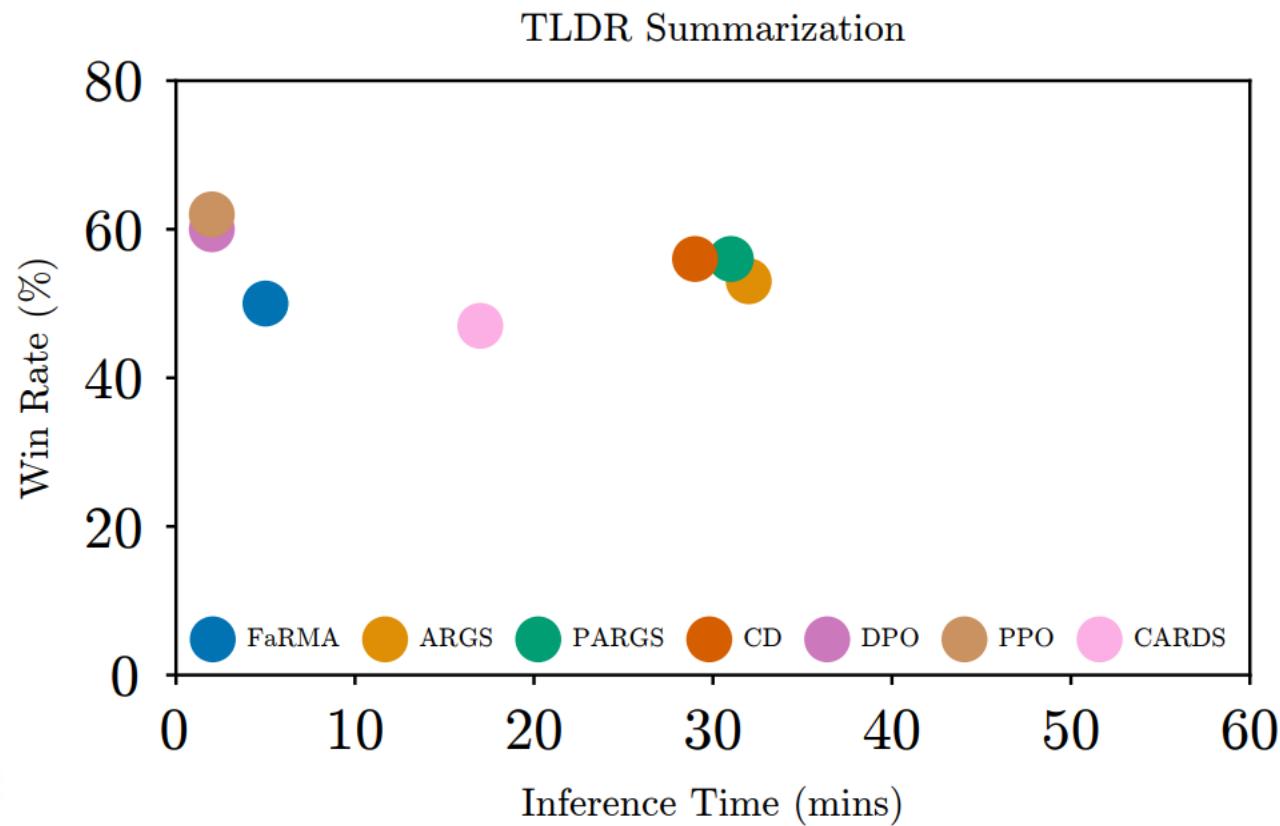
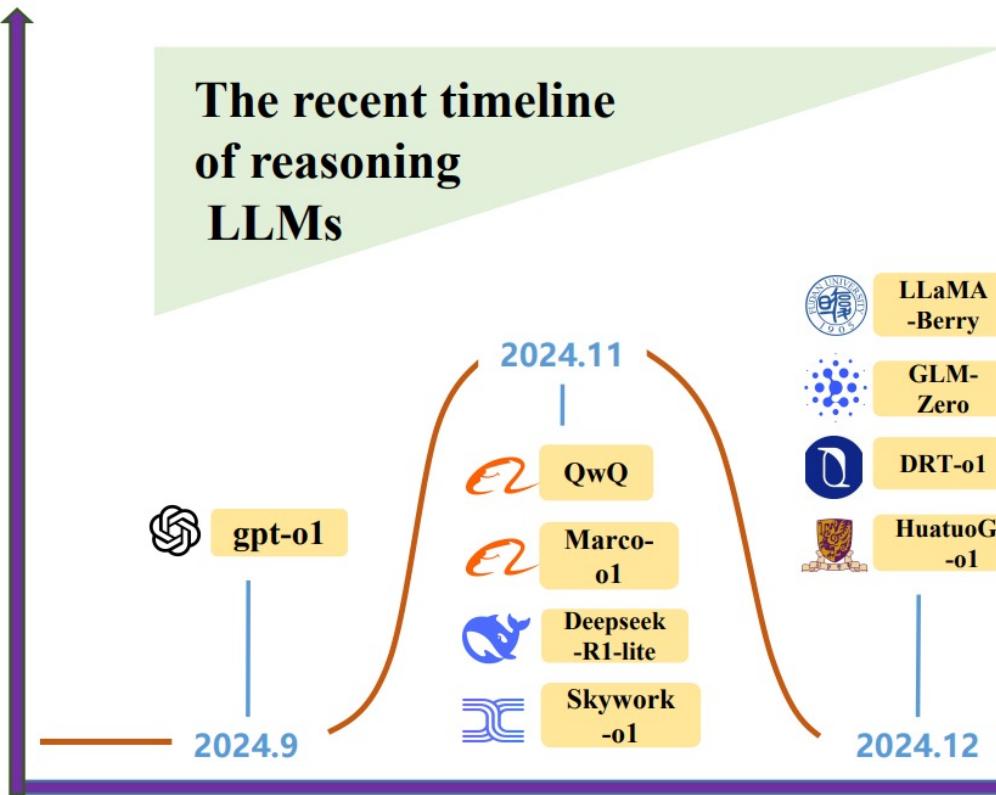


Figure 2. GPT4 evaluation on TLDR

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- LLM Reasoning
 - Search and planning
 - Group Relative Policy Optimization (GRPO)
 - Reflection: Verbalized RL

Reasoning LLMs



Source: Pan, Ji et al. (2025) A Survey of Slow Thinking-based Reasoning LLMs using Reinforced Learning and Inference-time Scaling Law

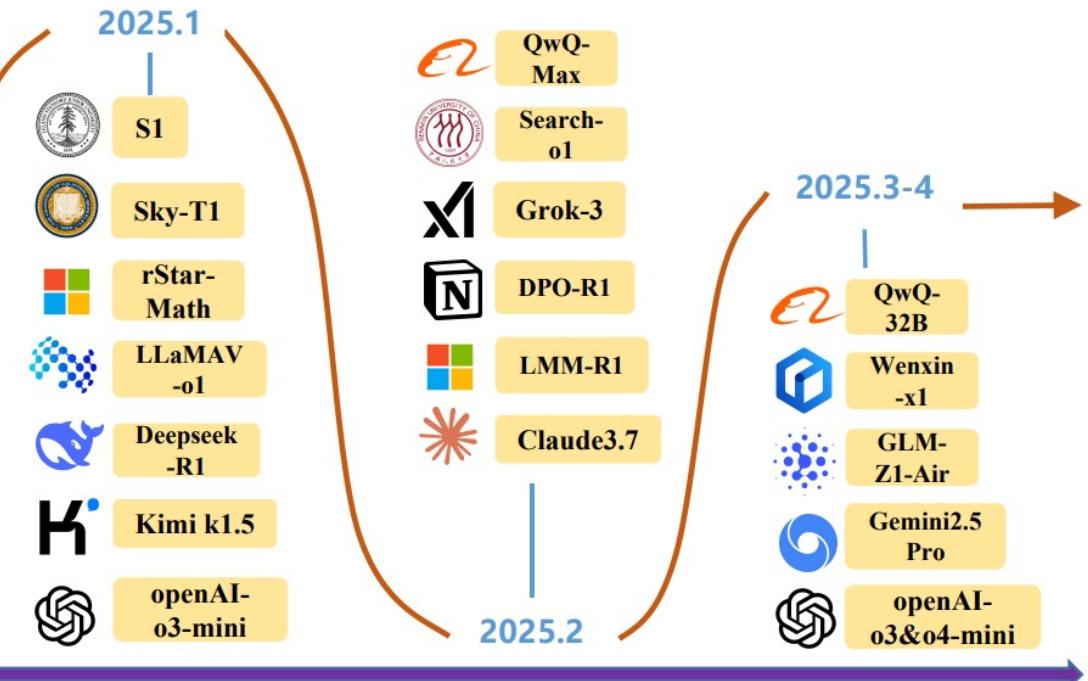
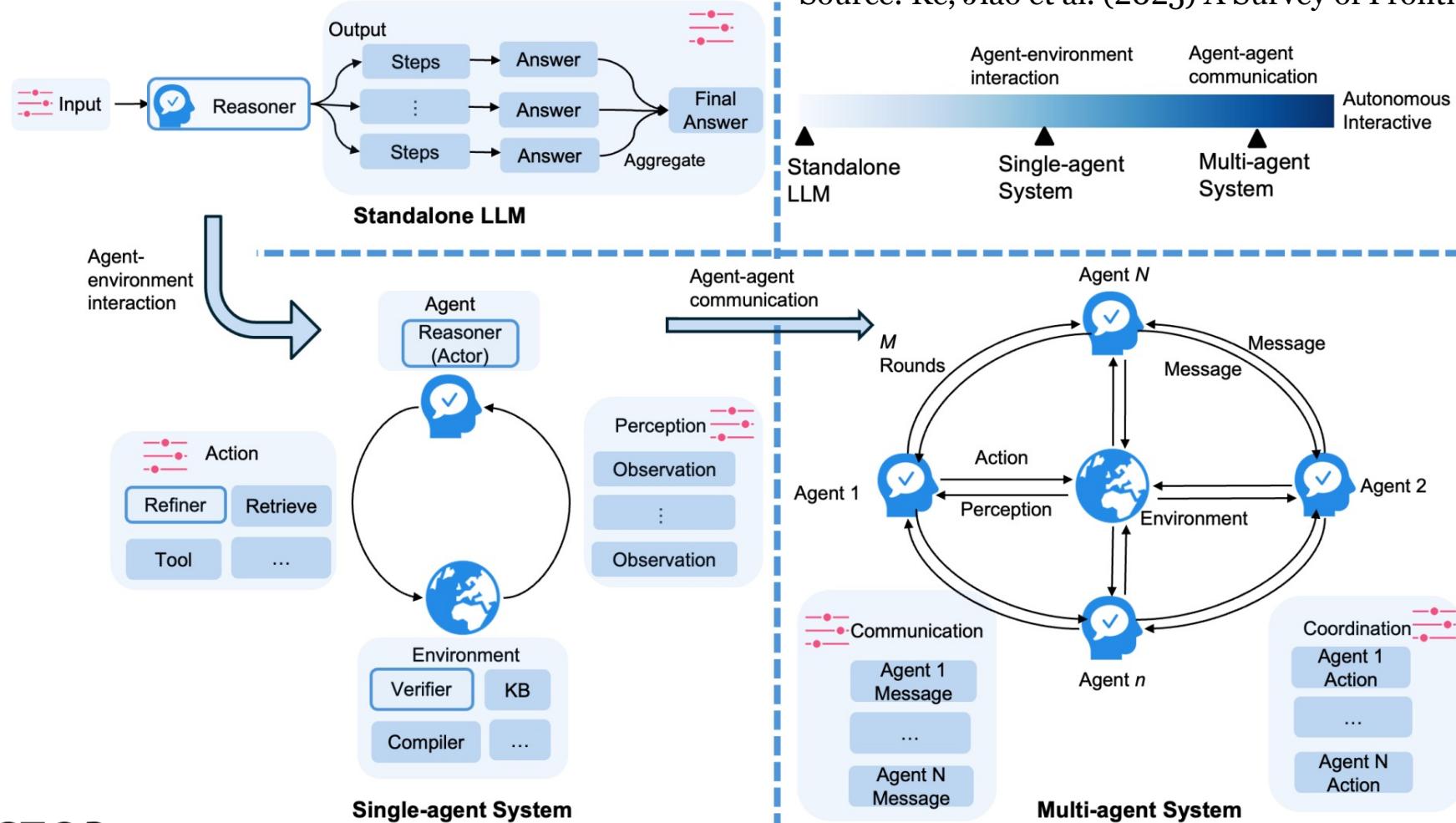


Fig. 1. The timeline of main reasoning LLMs.

Inference Time Reasoning

Source: Ke, Jiao et al. (2025) A Survey of Frontiers in LLM Reasoning



Reasoning by Searching

Source: Pan, Ji et al. (2025) A Survey of Slow Thinking-based Reasoning LLMs using Reinforced Learning and Inference-time Scaling Law

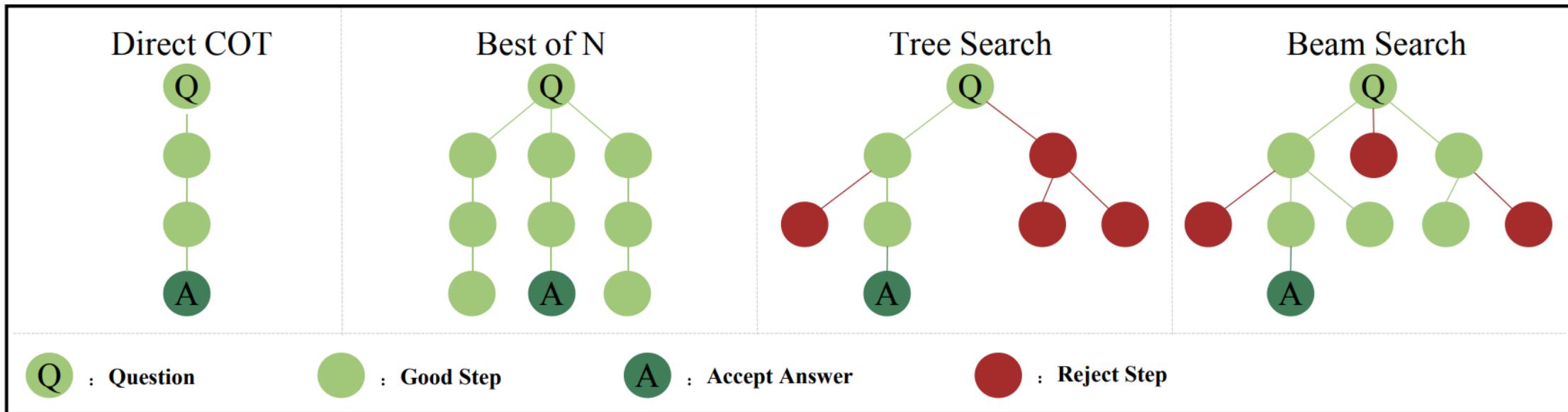
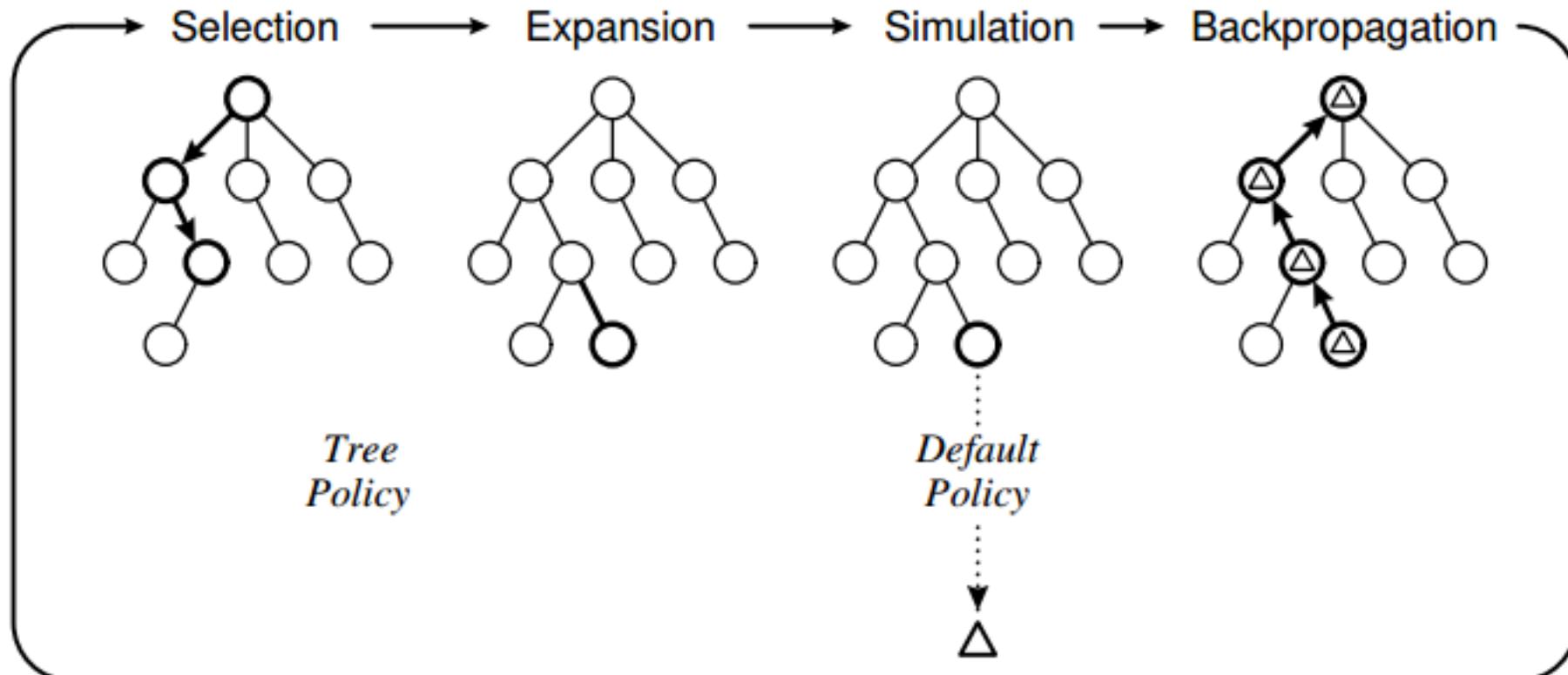


Fig. 3. The search algorithms for test-time scaling

Monte Carlo Tree Search



Learning to Reason

Source: Pan, Ji et al. (2025) A Survey of Slow Thinking-based Reasoning LLMs using Reinforced Learning and Inference-time Scaling Law

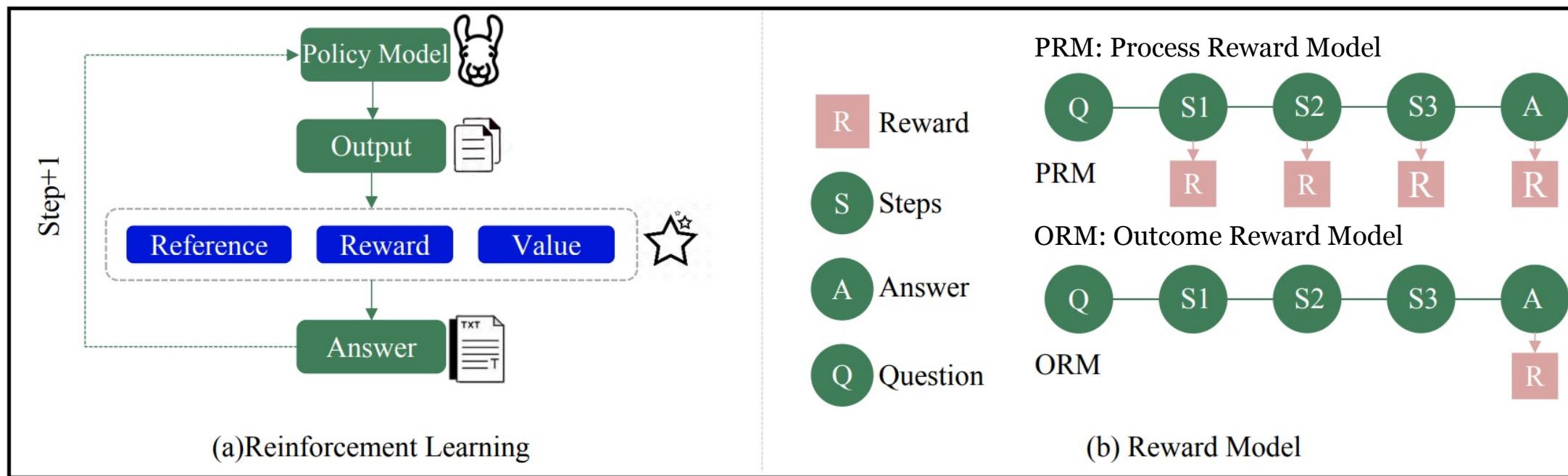


Fig. 4. The reinforcement learning framework and reward model

Simplifying PPO

Source: Shao, Wang et al. (2024) DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models

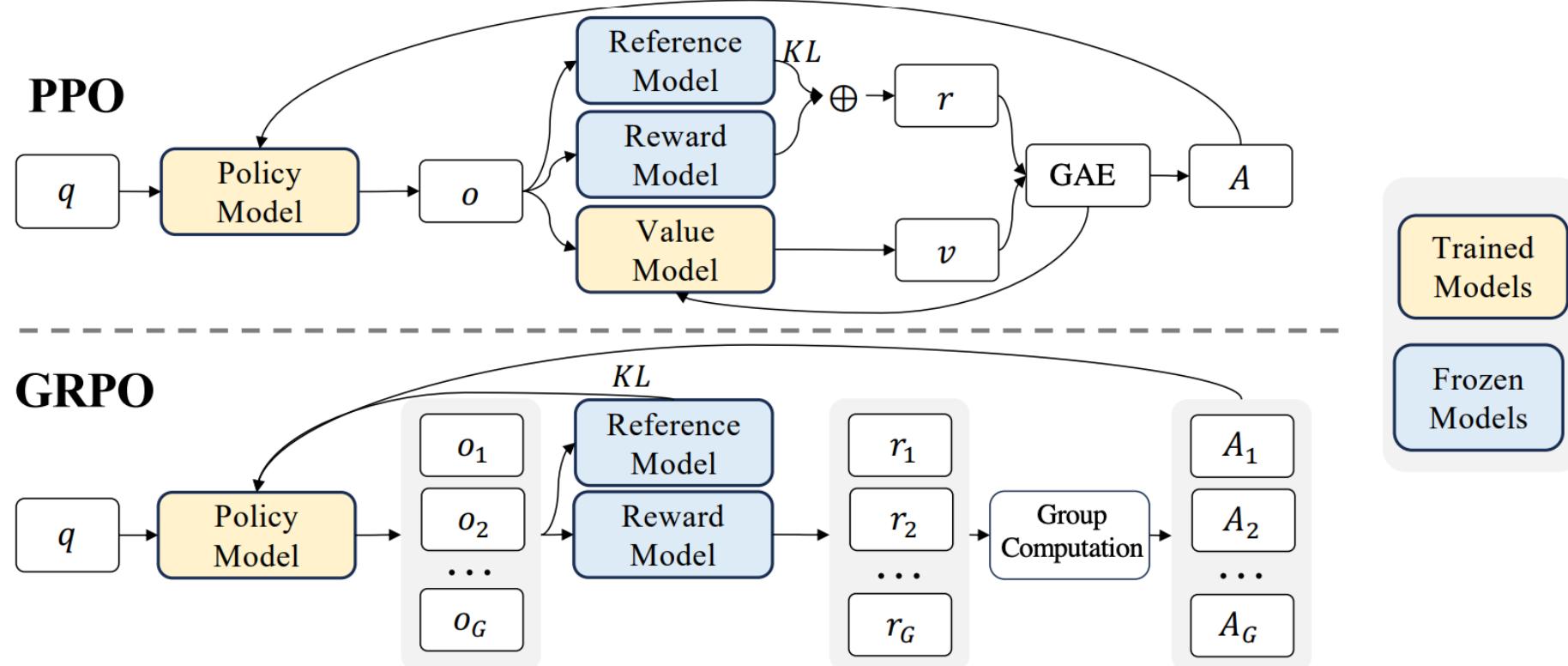


Figure 4 | Demonstration of PPO and our GRPO. GRPO foregoes the value model, instead estimating the baseline from group scores, significantly reducing training resources.

Group Relative Policy Optimization (GRPO)

Initialize π_ϕ and V_w to anything

Loop forever

Generate set of episodes $\{\tau_0, \dots, \tau_{G-1}\}$:

Sample $\tau_g = (s_0^g, a_0^g, r_0^g, s_1^g, a_1^g, r_1^g, \dots, s_{N-1}^g, a_{N-1}^g, r_{N-1}^g)$ with π_ϕ

Evaluate: $R_n^g \leftarrow \sum_{t=n}^N \gamma^t r(s_t^g, a_t^g) \forall n$

Loop for each episode g and step n

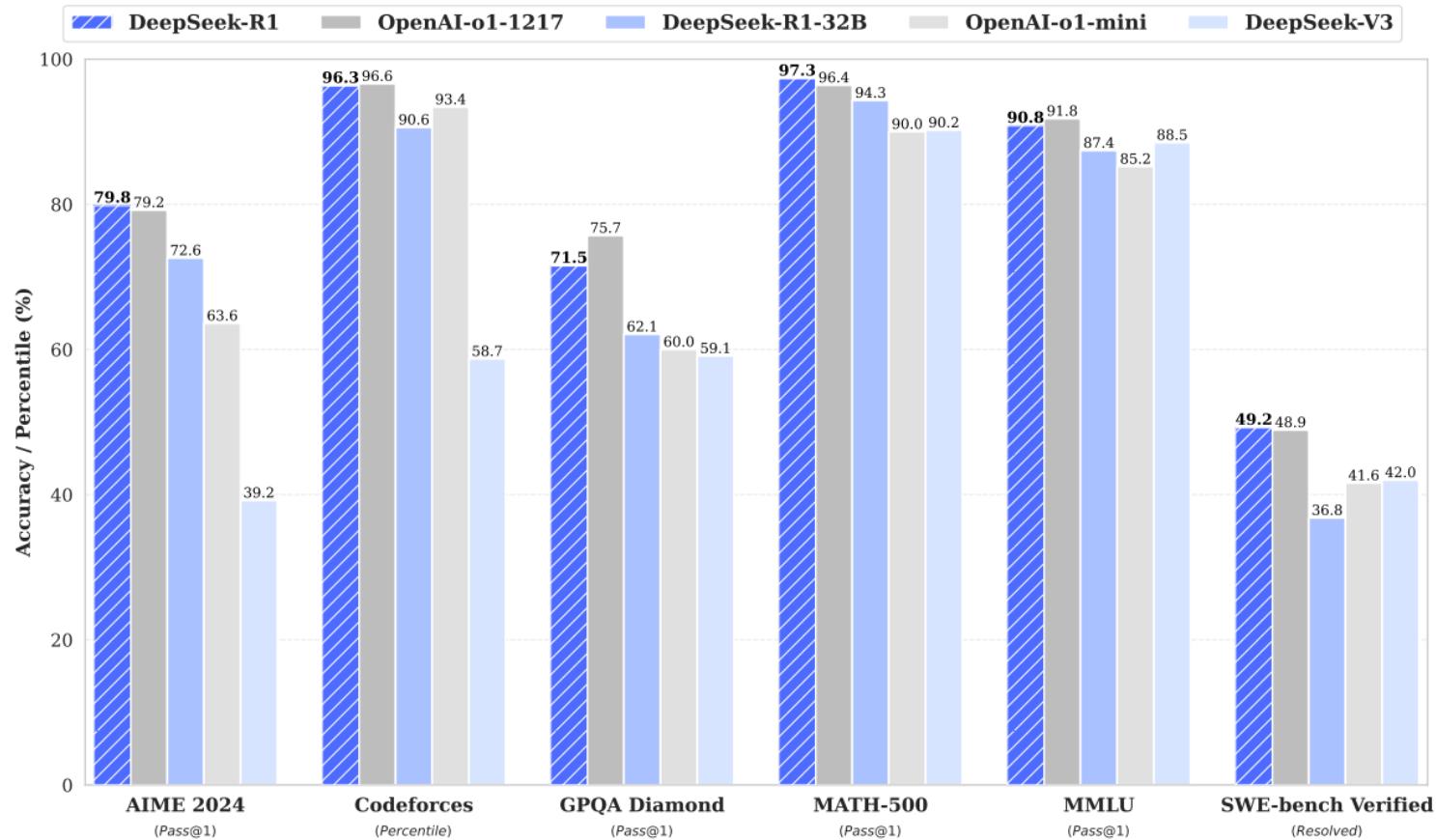
$$A_n^g \leftarrow (R_n^g - \text{mean}(\{R_n^0, \dots, R_n^{G-1}\})) / \text{std}(\{R_n^0, \dots, R_n^{G-1}\})$$

Update π :

$$\phi \leftarrow \underset{\tilde{\phi}}{\operatorname{argmax}} \frac{1}{G} \sum_{g=0}^{G-1} \frac{1}{N} \sum_{n=0}^{N-1} \min \left\{ \begin{array}{l} \frac{\pi_{\tilde{\phi}}(a_n^g | s_n^g)}{\pi_\phi(a_n^g | s_n^g)} A_n^g \\ \text{clip} \left(\frac{\pi_{\tilde{\phi}}(a_n^g | s_n^g)}{\pi_\phi(a_n^g | s_n^g)}, 1 - \epsilon, 1 + \epsilon \right) A_n^g \end{array} \right\}$$

DeepSeek-R1

Source: DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning (2025)



Reflexion: Verbalized Reinforcement Learning

Source: Shinn, Cassano et al. (2023) Reflexion: Language Agents with Verbal Reinforcement Learning

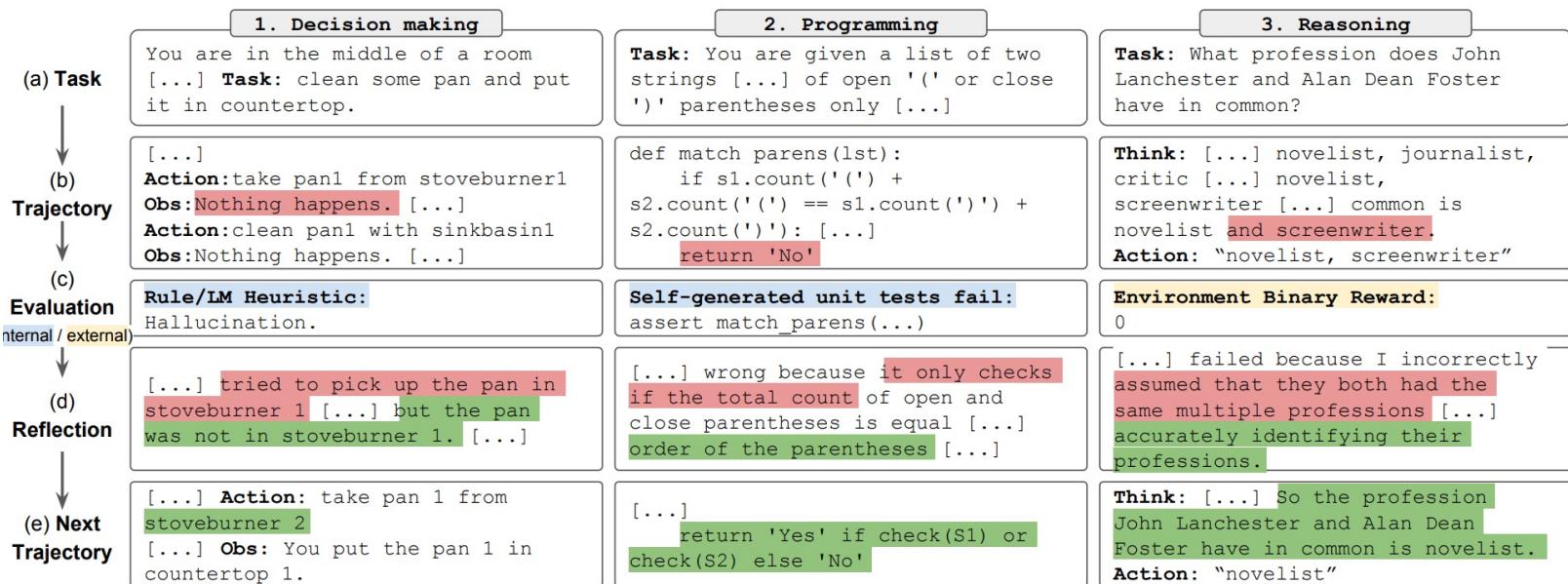
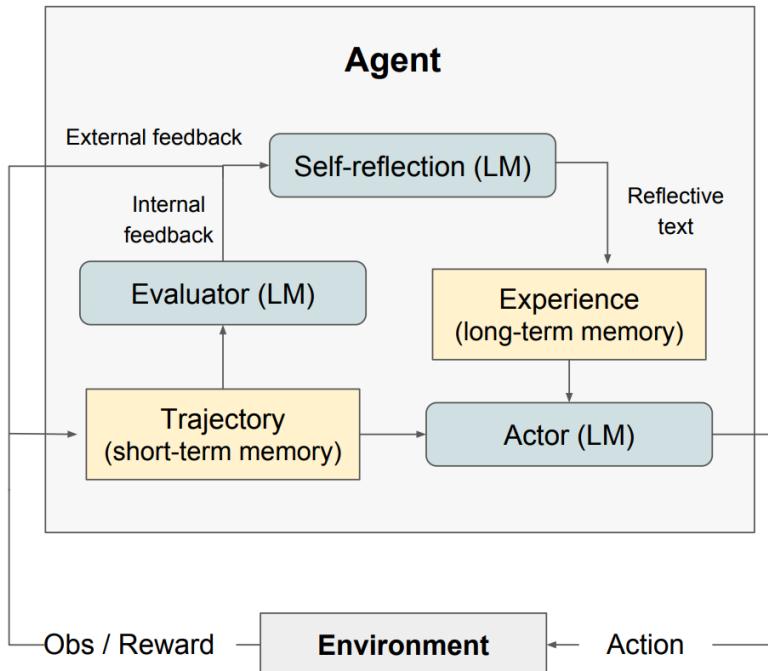


Figure 1: Reflexion works on decision-making 4.1, programming 4.3, and reasoning 4.2 tasks.

Improved Reasoning by Self-Reflection

Source: Shinn, Cassano et al. (2023) Reflexion: Language Agents with Verbal Reinforcement Learning

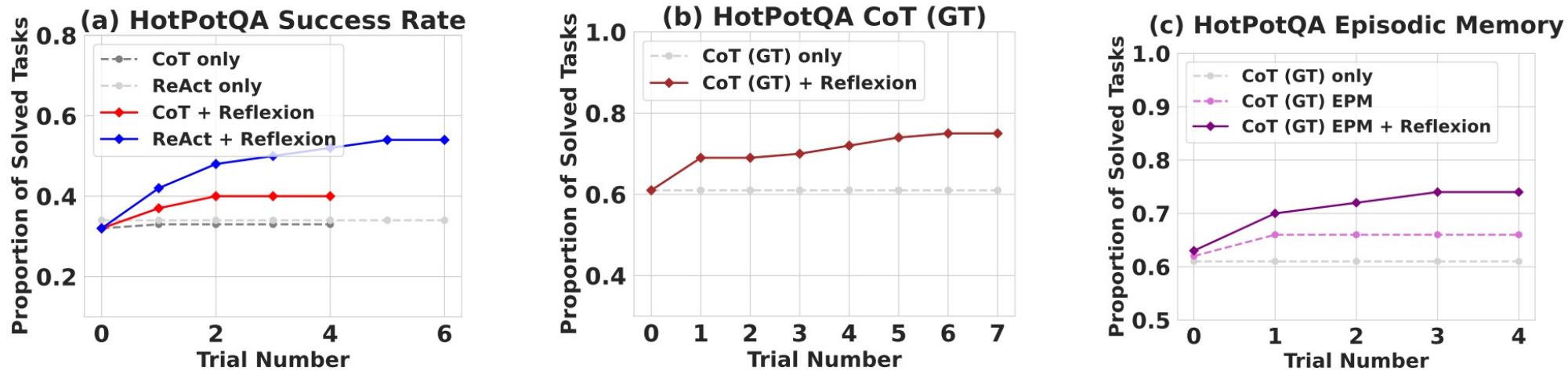


Figure 4: Chain-of-Thought (CoT) and ReAct. Reflexion improves search, information retrieval, and reasoning capabilities on 100 HotPotQA questions. (a) Reflexion ReAct vs Reflexion CoT (b) Reflexion CoT (GT) for reasoning only (c) Reflexion vs episodic memory ablation.

Conclusion

- RL key to
 - LLM Alignment
 - LLM Reasoning
- Current Frontier:
 - Multi-agent RL for agentic orchestration

Source: Taghizadeh (2024) How Multi Agent LLMs Are Revolutionizing Reporting

