

دانتگاه تهران- دانشگده مهندی برق و کامپوتر ریاضیات مهندسی-نیم سال اول سال ۱۴۰۰–۱۴۰۱ پاسخنامه آزمون میان ترم



مدت آزمون: ۱۸۰ دقیقه

	11.4	ازمون: ۱۸۰ دفیقه	مدت
$f(x) = \sinh(x), A = \sum_{n=1}^{\infty} \frac{1}{n^2 + \frac{1}{4}}, B = \sum_{n=1}^{\infty} \frac{1}{(n^2 + \frac{1}{4})^2}$ $a_0 = \frac{1}{\pi} \int_0^{\pi} \sinh(x) dx = \frac{\cosh(\pi) - 1}{2} = \frac{2 \sinh^2\left(\frac{\pi}{2}\right)}{\pi}$ $a_n = \frac{2}{\pi} \int_0^{\pi} \sinh(x) \cos(2nx) dx = \frac{1}{2\pi} \sinh(x) \sin(2nx) - \frac{1}{2\pi} \int \cosh(x) \sin(2nx) dx$ $= \frac{1}{2n} \sinh(x) \sin(2nx) - \frac{1}{2n} \left(-\frac{1}{2n} \cosh(x) \cos(2nx) + \frac{1}{2n} \int \sinh(x) \cos(2nx) dx \right)$ $\Rightarrow a_n = \frac{2}{\pi} \left[\frac{2n \sinh(x) \sin(2nx) + \cosh(x) \cos(2nx)}{4n^2 + 1} \right]_0^{\pi} = \frac{2(\cosh(\pi) - 1)}{(4n^2 + 1)\pi} = \frac{4 \sinh^2\left(\frac{\pi}{2}\right)}{(4n^2 + 1)\pi} = \frac{\sinh^2\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})\pi}$ $b_0 = \frac{2}{\pi} \int_0^{\pi} \sinh(x) \cos(2nx) dx$ $= \frac{1}{2n} \sinh(x) \cos(2nx) dx = -\frac{1}{2n} \sinh(x) \cos(2nx) dx$ $= \frac{1}{2n} \sinh(x) \cos(2nx) dx = -\frac{1}{2n} \sinh(x) \cos(2nx) dx$ $= \frac{1}{2n} \sinh(x) \cos(2nx) dx = -\frac{1}{2n} \sinh(x) \cos(2nx) + \frac{1}{2n} \int \cosh(x) \cos(2nx) dx$ $= \frac{1}{2n} \sinh(x) \cos(2nx) dx = -\frac{1}{2n} \sinh(x) \cos(2nx) + \frac{1}{2n} \int \sinh(x) \sin(2nx) dx$ $\Rightarrow b_n = \frac{2}{\pi} \left[\cosh(x) \sin(2nx) - 2n \sinh(x) \cos(2nx) \right]_0^{\pi} = -\frac{n \sinh(n)}{(n^2 + \frac{1}{4})\pi}$ $\Rightarrow \left[f(x) = \frac{2\sinh(n)}{\pi} \right] = \frac{2 \sinh^2\left(\frac{n}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \left(\frac{\sinh^2\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})\pi} \right) \cos(2nx) + \left(-\frac{n \sinh(n)}{(n^2 + \frac{1}{4})\pi} \right) \sin(2nx) \right]$ $f(\pi) = \frac{\sinh(n)}{\pi} = \frac{2 \sinh^2\left(\frac{n}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \left(\frac{\sinh^2\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})\pi} \right) \cos(2nx) + \left(-\frac{n \sinh(n)}{(n^2 + \frac{1}{4})\pi} \right) \sin(2nx)$ $f(\pi) = \frac{\sinh(n)}{\pi} = \frac{2 \sinh^2\left(\frac{n}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \frac{\sinh^2\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})^2} = \frac{n \sinh^2\left(\frac{n}{2}\right)}{\pi^2} + \sum_{n=1}^{\infty} \frac{\sinh^2\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})^2} + \sum_{n=1}^{\infty} \frac{\sinh^2\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})^2} + \frac{n^2 \sinh^2\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})^2} + \frac{n^2 \sinh^2\left(\frac{n}{2}\right)}{\pi^2} + \frac{n^2 \sinh^2\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})^2} = \frac{n \sinh^4\left(\frac{n}{2}\right)}{n^2} + \frac{n^2 \sinh^2\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})^2} = \frac{n^2 \sinh^4\left(\frac{n}{2}\right)}{n^2} + \frac{n^2 \sinh^4\left(\frac{n}{2}\right)}{n^2} + \frac{n^2 \sinh^2\left(\frac{n}{2}\right)}{n^2} + \frac{n^2 \sinh^2\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})^2} = \frac{n^2 \sinh^4\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})^2} = \frac{n^2 \sinh^4\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})^2} = \frac{n^2 \sinh^4\left(\frac{n}{2}\right)}{n^2} + \frac{n^2 \sinh^2\left(\frac{n}{2}\right)}{n^2} = \frac{n^2 \sinh^4\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})^2} = \frac{n^2 \sinh^2\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})^2} = n^2 $	نمره		شماره
$a_{0} = \frac{1}{\pi} \int_{0}^{\pi} \sinh(x) dx = \frac{\cosh(\pi) - 1}{2} = \frac{2 \sinh^{2} \left(\frac{\pi}{2}\right)}{\pi}$ $a_{n} = \frac{2}{\pi} \int_{0}^{\pi} \sinh(x) \cos(2\pi x) dx$ $= \frac{1}{2\pi} \sinh(x) \sin(2\pi x) - \frac{1}{2\pi} \left(-\frac{1}{2\pi} \cosh(x) \cos(2\pi x) + \frac{1}{2\pi}\right) \left[\cosh(x) \sin(2\pi x) dx\right]$ $= \frac{1}{2\pi} \sinh(x) \sin(2\pi x) - \frac{1}{2\pi} \left(-\frac{1}{2\pi} \cosh(x) \cos(2\pi x) + \frac{1}{2\pi}\right) \left[\sinh(x) \cos(2\pi x) dx\right]$ $\Rightarrow a_{n} = \frac{2}{\pi} \left[\frac{2\pi \sinh(x) \sin(2\pi x) + \cosh(x) \cos(2\pi x)}{4\pi^{2} + 1}\right]_{0}^{\pi} = \frac{2(\cosh(\pi) - 1)}{(4\pi^{2} + 1)\pi} = \frac{4 \sinh^{2} \left(\frac{\pi}{2}\right)}{(4\pi^{2} + 1)\pi} = \frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{(4\pi^{2} + 1)\pi}$ $\Rightarrow h_{n} = \frac{2}{\pi} \int_{0}^{\pi} \sinh(x) \cos(2\pi x) dx$ $= -\frac{1}{2\pi} \sinh(x) \cos(2\pi x) + \frac{1}{2\pi} \left(\frac{1}{2\pi} \cosh(x) \sin(2\pi x) - 2\pi \sinh(x) \cos(2\pi x) dx\right)$ $\Rightarrow h_{n} = \frac{2}{\pi} \left[\cosh(x) \sin(2\pi x) - 2\pi \sinh(x) \cos(2\pi x)\right]_{0}^{\pi} - \frac{n \sinh(\pi)}{(\pi^{2} + \frac{1}{4})\pi}$ $\Rightarrow h_{n} = \frac{2}{\pi} \left[\cosh(x) \sin(2\pi x) - 2\pi \sinh(x) \cos(2\pi x)\right]_{0}^{\pi} - \frac{n \sinh(\pi)}{(\pi^{2} + \frac{1}{4})\pi}$ $\Rightarrow h_{n} = \frac{2}{\pi} \left[\cosh(x) \sin(2\pi x) - 2\pi \sinh(x) \cos(2\pi x)\right]_{0}^{\pi} - \frac{n \sinh(\pi)}{(\pi^{2} + \frac{1}{4})\pi}$ $\Rightarrow h_{n} = \frac{2 \sinh^{2} \left(\frac{\pi}{2}\right)}{\pi^{2}} + \sum_{n=1}^{\infty} \left(\frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{(\pi^{2} + \frac{1}{4})\pi}\right) \cos(2\pi x) + \left(\frac{-n \sinh(\pi)}{(\pi^{2} + \frac{1}{4})\pi}\right) \sin(2\pi x)$ $f(\pi) = \frac{2 \sinh(\pi)}{2} = \frac{2 \sinh^{2} \left(\frac{\pi}{2}\right)}{\pi^{2}} + \sum_{n=1}^{\infty} \frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{(\pi^{2} + \frac{1}{4})\pi} = \frac{\pi \coth \left(\frac{\pi}{2}\right)}{\pi^{2}} - 2$ $\frac{2}{\pi} \int_{0}^{\pi} \sinh^{2}(x) dx = \frac{1}{\pi} \int_{0}^{\pi} (\cosh(2x) - 1) dx = \frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{2\pi} - 1 = \frac{8 \sinh^{4} \left(\frac{\pi}{2}\right)}{\pi^{2}} + \sum_{n=1}^{\infty} \frac{\sinh^{4} \left(\frac{\pi}{2}\right)}{(\pi^{2} + \frac{1}{4})\pi^{2}} + \sum_{n=1}^{\infty} \frac{\sinh^{4} \left(\frac{\pi}{2}\right)}{(\pi^{2} + \frac{1}{4})\pi^{2}} + \sum_{n=1}^{\infty} \frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{(\pi^{2} + \frac{1}{4})^{2} \pi^{2}} + \frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{(\pi^{2} + \frac{1}{4})^{2} \pi^{2}} + \frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{\pi^{2}} + $		سری فوریه تابع $f(x)$ را در بازه $[0,\pi]$ بدست آورید و با استفاده از آن حاصل سری A و B را بدست آورید.	١
$a_{0} = \frac{1}{\pi} \int_{0}^{\pi} \sinh(x) dx = \frac{\cosh(\pi) - 1}{2} = \frac{2 \sinh^{2} \left(\frac{\pi}{2}\right)}{\pi}$ $a_{n} = \frac{2}{\pi} \int_{0}^{\pi} \sinh(x) \cos(2\pi x) dx$ $= \frac{1}{2\pi} \sinh(x) \sin(2\pi x) - \frac{1}{2\pi} \left(-\frac{1}{2\pi} \cosh(x) \cos(2\pi x) + \frac{1}{2\pi}\right) \left[\cosh(x) \sin(2\pi x) dx\right]$ $= \frac{1}{2\pi} \sinh(x) \sin(2\pi x) - \frac{1}{2\pi} \left(-\frac{1}{2\pi} \cosh(x) \cos(2\pi x) + \frac{1}{2\pi}\right) \left[\sinh(x) \cos(2\pi x) dx\right]$ $\Rightarrow a_{n} = \frac{2}{\pi} \left[\frac{2\pi \sinh(x) \sin(2\pi x) + \cosh(x) \cos(2\pi x)}{4\pi^{2} + 1}\right]_{0}^{\pi} = \frac{2(\cosh(\pi) - 1)}{(4\pi^{2} + 1)\pi} = \frac{4 \sinh^{2} \left(\frac{\pi}{2}\right)}{(4\pi^{2} + 1)\pi} = \frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{(4\pi^{2} + 1)\pi}$ $\Rightarrow h_{n} = \frac{2}{\pi} \int_{0}^{\pi} \sinh(x) \cos(2\pi x) dx$ $= -\frac{1}{2\pi} \sinh(x) \cos(2\pi x) + \frac{1}{2\pi} \left(\frac{1}{2\pi} \cosh(x) \sin(2\pi x) - 2\pi \sinh(x) \cos(2\pi x) dx\right)$ $\Rightarrow h_{n} = \frac{2}{\pi} \left[\cosh(x) \sin(2\pi x) - 2\pi \sinh(x) \cos(2\pi x)\right]_{0}^{\pi} - \frac{n \sinh(\pi)}{(\pi^{2} + \frac{1}{4})\pi}$ $\Rightarrow h_{n} = \frac{2}{\pi} \left[\cosh(x) \sin(2\pi x) - 2\pi \sinh(x) \cos(2\pi x)\right]_{0}^{\pi} - \frac{n \sinh(\pi)}{(\pi^{2} + \frac{1}{4})\pi}$ $\Rightarrow h_{n} = \frac{2}{\pi} \left[\cosh(x) \sin(2\pi x) - 2\pi \sinh(x) \cos(2\pi x)\right]_{0}^{\pi} - \frac{n \sinh(\pi)}{(\pi^{2} + \frac{1}{4})\pi}$ $\Rightarrow h_{n} = \frac{2 \sinh^{2} \left(\frac{\pi}{2}\right)}{\pi^{2}} + \sum_{n=1}^{\infty} \left(\frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{(\pi^{2} + \frac{1}{4})\pi}\right) \cos(2\pi x) + \left(\frac{-n \sinh(\pi)}{(\pi^{2} + \frac{1}{4})\pi}\right) \sin(2\pi x)$ $f(\pi) = \frac{2 \sinh(\pi)}{2} = \frac{2 \sinh^{2} \left(\frac{\pi}{2}\right)}{\pi^{2}} + \sum_{n=1}^{\infty} \frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{(\pi^{2} + \frac{1}{4})\pi} = \frac{\pi \coth \left(\frac{\pi}{2}\right)}{\pi^{2}} - 2$ $\frac{2}{\pi} \int_{0}^{\pi} \sinh^{2}(x) dx = \frac{1}{\pi} \int_{0}^{\pi} (\cosh(2x) - 1) dx = \frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{2\pi} - 1 = \frac{8 \sinh^{4} \left(\frac{\pi}{2}\right)}{\pi^{2}} + \sum_{n=1}^{\infty} \frac{\sinh^{4} \left(\frac{\pi}{2}\right)}{(\pi^{2} + \frac{1}{4})\pi^{2}} + \sum_{n=1}^{\infty} \frac{\sinh^{4} \left(\frac{\pi}{2}\right)}{(\pi^{2} + \frac{1}{4})\pi^{2}} + \sum_{n=1}^{\infty} \frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{(\pi^{2} + \frac{1}{4})^{2} \pi^{2}} + \frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{(\pi^{2} + \frac{1}{4})^{2} \pi^{2}} + \frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{\pi^{2}} + $		$f(x) = \sinh(x)$, $A = \sum_{i=1}^{\infty} \frac{1}{i}$, $B = \sum_{i=1}^{\infty} \frac{1}{i}$	
$a_{n} = \frac{2}{\pi} \int_{0}^{\pi} \sinh(x) \cos(2\pi x) dx$ $\int \sinh(x) \cos(2\pi x) dx = \frac{1}{2n} \sinh(x) \sin(2\pi x) - \frac{1}{2n} \int \cosh(x) \sin(2\pi x) dx$ $= \frac{1}{2n} \sinh(x) \sin(2\pi x) - \frac{1}{2n} \left(-\frac{1}{2n} \cosh(x) \cos(2\pi x) + \frac{1}{2n} \int \sinh(x) \cos(2\pi x) dx \right)$ $\Rightarrow a_{n} = \frac{2}{\pi} \left[\frac{2n \sinh(x) \sin(2\pi x) + \cosh(x) \cos(2\pi x) + \frac{1}{2n} \int \sinh(x) \cos(2\pi x) dx}{4\pi^{2} + 1} \right]$ $\Rightarrow a_{n} = \frac{2}{\pi} \left[\frac{2n \sinh(x) \sin(2\pi x) + \cosh(x) \cos(2\pi x) dx}{4\pi^{2} + 1} \right]_{0}^{\pi} = \frac{2(\cosh(n) - 1)}{(4\pi^{2} + 1)\pi} = \frac{4 \sinh^{2} \left(\frac{\pi}{2}\right)}{(4\pi^{2} + 1)\pi} = \frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{(\pi^{2} + \frac{1}{4})\pi}$ $\Rightarrow h_{n} = \frac{2}{\pi} \int_{0}^{\pi} \sinh(x) \cos(2\pi x) dx$ $= -\frac{1}{2n} \sinh(x) \cos(2\pi x) dx$ $\Rightarrow h_{n} = \frac{2}{\pi} \left[\frac{\cosh(x) \sin(2\pi x) - 1}{(2\pi - \cos(x))} \sin(2\pi x) - \frac{1}{2n} \int \sinh(x) \sin(2\pi x) dx \right]$ $\Rightarrow h_{n} = \frac{2}{\pi} \left[\frac{\cosh(x) \sin(2\pi x) - 1}{(\pi^{2} + \frac{1}{4})\pi} \cos(2\pi x) dx \right]$ $\Rightarrow h_{n} = \frac{2}{\pi} \left[\frac{\cosh(x) \sin(2\pi x) - 2\pi \sinh(x) \cos(2\pi x)}{(\pi^{2} + \frac{1}{4})\pi} \cos(2\pi x) dx \right]$ $\Rightarrow h_{n} = \frac{2}{\pi} \left[\frac{\cosh(x) \sin(2\pi x) - 2\pi \sinh(x) \cos(2\pi x)}{(\pi^{2} + \frac{1}{4})\pi} \right] \cos(2\pi x) + \left(-\frac{n \sinh(\pi)}{(\pi^{2} + \frac{1}{4})\pi} \right) \sin(2\pi x)$ $f(\pi) = \frac{\sinh(\pi)}{2} = \frac{2 \sinh^{2} \left(\frac{\pi}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \left(\frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})\pi} \right) \cos(2\pi x) + \left(-\frac{n \sinh(\pi)}{(\pi^{2} + \frac{1}{4})\pi} \right) \sin(2\pi x)$ $f(\pi) = \frac{\sinh(\pi)}{2} = \frac{2 \sinh^{2} \left(\frac{\pi}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \left(\frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})\pi} \right) \cos(2\pi x) + \left(-\frac{n \sinh(\pi)}{(\pi^{2} + \frac{1}{4})\pi} \right) \sin(2\pi x)$ $f(\pi) = \frac{\sinh(\pi)}{2} = \frac{2 \sinh^{2} \left(\frac{\pi}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})\pi} \sin(2\pi x)$ $= \frac{1}{\pi} \int_{0}^{\pi} \sinh^{2} \left(\frac{\pi}{2}\right) + \sum_{n=1}^{\infty} \frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})\pi^{2}} + \sum_{n=1}^{\infty} \frac{1}{(n^{2} + \frac{1}{4})\pi^{2}} + \sum_{n=1}^{\infty} \frac{1}{(n^{2} + \frac{1}{4})\pi^{2}} \left(\frac{1}{n^{2} + \frac{1}{4}} \right) \frac{1}{n^{2}} + \sum_{n=1}^{\infty} \frac{1}{(n^{2} + \frac{1}{4})^{2}} \frac{1}{n^{2}} + \sum_{n=1}^{\infty} \frac{1}$			
$a_{n} = \frac{2}{\pi} \int_{0}^{\pi} \sinh(x) \cos(2\pi x) dx$ $\int \sinh(x) \cos(2\pi x) dx = \frac{1}{2n} \sinh(x) \sin(2\pi x) - \frac{1}{2n} \int \cosh(x) \sin(2\pi x) dx$ $= \frac{1}{2n} \sinh(x) \sin(2\pi x) - \frac{1}{2n} \left(-\frac{1}{2n} \cosh(x) \cos(2\pi x) + \frac{1}{2n} \int \sinh(x) \cos(2\pi x) dx \right)$ $\Rightarrow a_{n} = \frac{2}{\pi} \left[\frac{2n \sinh(x) \sin(2\pi x) + \cosh(x) \cos(2\pi x) + \frac{1}{2n} \int \sinh(x) \cos(2\pi x) dx}{4\pi^{2} + 1} \right]$ $\Rightarrow a_{n} = \frac{2}{\pi} \left[\frac{2n \sinh(x) \sin(2\pi x) + \cosh(x) \cos(2\pi x) dx}{4\pi^{2} + 1} \right]_{0}^{\pi} = \frac{2(\cosh(n) - 1)}{(4\pi^{2} + 1)\pi} = \frac{4 \sinh^{2} \left(\frac{\pi}{2}\right)}{(4\pi^{2} + 1)\pi} = \frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{(\pi^{2} + \frac{1}{4})\pi}$ $\Rightarrow h_{n} = \frac{2}{\pi} \int_{0}^{\pi} \sinh(x) \cos(2\pi x) dx$ $= -\frac{1}{2n} \sinh(x) \cos(2\pi x) dx$ $\Rightarrow h_{n} = \frac{2}{\pi} \left[\frac{\cosh(x) \sin(2\pi x) - 1}{(2\pi - \cos(x))} \sin(2\pi x) - \frac{1}{2n} \int \sinh(x) \sin(2\pi x) dx \right]$ $\Rightarrow h_{n} = \frac{2}{\pi} \left[\frac{\cosh(x) \sin(2\pi x) - 1}{(\pi^{2} + \frac{1}{4})\pi} \cos(2\pi x) dx \right]$ $\Rightarrow h_{n} = \frac{2}{\pi} \left[\frac{\cosh(x) \sin(2\pi x) - 2\pi \sinh(x) \cos(2\pi x)}{(\pi^{2} + \frac{1}{4})\pi} \cos(2\pi x) dx \right]$ $\Rightarrow h_{n} = \frac{2}{\pi} \left[\frac{\cosh(x) \sin(2\pi x) - 2\pi \sinh(x) \cos(2\pi x)}{(\pi^{2} + \frac{1}{4})\pi} \right] \cos(2\pi x) + \left(-\frac{n \sinh(\pi)}{(\pi^{2} + \frac{1}{4})\pi} \right) \sin(2\pi x)$ $f(\pi) = \frac{\sinh(\pi)}{2} = \frac{2 \sinh^{2} \left(\frac{\pi}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \left(\frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})\pi} \right) \cos(2\pi x) + \left(-\frac{n \sinh(\pi)}{(\pi^{2} + \frac{1}{4})\pi} \right) \sin(2\pi x)$ $f(\pi) = \frac{\sinh(\pi)}{2} = \frac{2 \sinh^{2} \left(\frac{\pi}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \left(\frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})\pi} \right) \cos(2\pi x) + \left(-\frac{n \sinh(\pi)}{(\pi^{2} + \frac{1}{4})\pi} \right) \sin(2\pi x)$ $f(\pi) = \frac{\sinh(\pi)}{2} = \frac{2 \sinh^{2} \left(\frac{\pi}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})\pi} \sin(2\pi x)$ $= \frac{1}{\pi} \int_{0}^{\pi} \sinh^{2} \left(\frac{\pi}{2}\right) + \sum_{n=1}^{\infty} \frac{\sinh^{2} \left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})\pi^{2}} + \sum_{n=1}^{\infty} \frac{1}{(n^{2} + \frac{1}{4})\pi^{2}} + \sum_{n=1}^{\infty} \frac{1}{(n^{2} + \frac{1}{4})\pi^{2}} \left(\frac{1}{n^{2} + \frac{1}{4}} \right) \frac{1}{n^{2}} + \sum_{n=1}^{\infty} \frac{1}{(n^{2} + \frac{1}{4})^{2}} \frac{1}{n^{2}} + \sum_{n=1}^{\infty} \frac{1}$		$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} \sinh(x) dx = \frac{\cosh(\pi) - 1}{\pi} = \frac{2 \sinh^2(\frac{\pi}{2})}{\pi}$	
$\int \sinh(x) \cos(2nx) dx = \frac{1}{2n} \sinh(x) \sin(2nx) - \frac{1}{2n} \left[\sinh(x) \sin(2nx) - \frac{1}{2n} \left[\cosh(x) \sin(2nx) dx \right] \right]$ $= \frac{1}{2n} \sinh(x) \sin(2nx) - \frac{1}{2n} \left[-\frac{1}{2n} \cosh(x) \cos(2nx) + \frac{1}{2n} \right] \sinh(x) \cos(2nx) dx$ $\Rightarrow a_n = \frac{2}{n} \left[\frac{2n \sinh(x) \sin(2nx) + \cosh(x) \cos(2nx)}{4n^2 + 1} \right]_0^n = \frac{2(\cosh(n) - 1)}{(4n^2 + 1)\pi} = \frac{4 \sinh^2\left(\frac{\pi}{2}\right)}{(4n^2 + 1)\pi} = \frac{\sinh^2\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})\pi}$ $b_n = \frac{2}{n} \int_0^\pi \sinh(x) \cos(2nx) dx$ $= -\frac{1}{2n} \sinh(x) \cos(2nx) dx - \frac{1}{2n} \int_0^\pi \sinh(x) \cos(2nx) dx$ $= -\frac{1}{2n} \sinh(x) \cos(2nx) + \frac{1}{2n} \left(\frac{1}{2n} \cosh(x) \sin(2nx) - 2n \sinh(x) \cos(2nx) dx\right)$ $\Rightarrow b_n = \frac{2}{n} \left[\frac{\cosh(x) \sin(2nx) - 2n \sinh(x) \cos(2nx)}{4n^2 + 1} \right]_0^n = \frac{n \sinh(n)}{(n^2 + \frac{1}{4})\pi}$ $\Rightarrow \int_0^\pi \left(\frac{2 \sinh^2\left(\frac{\pi}{2}\right)}{n} + \sum_{n=1}^\infty \left(\frac{\sinh^2\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})\pi} \right) \cos(2nx) + \left(-\frac{n \sinh(n)}{(n^2 + \frac{1}{4})\pi} \right) \sin(2nx) \right)$ $f(n) = \frac{\sinh(n)}{2n} = \frac{2 \sinh^2\left(\frac{\pi}{2}\right)}{n} + \sum_{n=1}^\infty \left(\frac{n^2 + \frac{1}{4}}{(n^2 + \frac{1}{4})^2} \right) \Rightarrow \sum_{n=1}^\infty \frac{1}{n^2 + \frac{1}{4}} = n \cot\left(\frac{\pi}{2}\right) - 2$ $\frac{2}{n} \int_0^\pi \sinh^2(x) dx = \frac{1}{n} \int_0^\pi \cosh(2x) - 1 dx = \frac{\sinh^4\left(\frac{\pi}{2}\right)}{2n} - 1 = \frac{8 \sinh^4\left(\frac{\pi}{2}\right)}{n^2} + \sum_{n=1}^\infty \frac{\sinh^4\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})^2 n^2} + \frac{n^2 \sinh^2\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})^2 n^2} - \frac{\sinh^2\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})^2 n^2} + \frac{\sinh^2\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})^2 n^2} - \frac{\sinh^2\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})^2 n^2} - \frac{\sinh^2\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})^2 n^2} - \frac{\sinh^4\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})^2 n^2} - \frac{\sinh^4\left(\frac{\pi}{2$			
$= \frac{1}{2n} \sinh(x) \sin(2nx) - \frac{1}{2n} \left(-\frac{1}{2n} \cosh(x) \cos(2nx) + \frac{1}{2n} \right) \sinh(x) \cos(2nx) dx$ $\Rightarrow a_n = \frac{2}{\pi} \left[\frac{2n \sinh(x) \sin(2nx) + \cosh(x) \cos(2nx)}{4n^2 + 1} \right]_0^{\pi} = \frac{2(\cosh(\pi) - 1)}{(4n^2 + 1)\pi} = \frac{4 \sinh^2\left(\frac{\pi}{2}\right)}{(4n^2 + 1)\pi} = \frac{\sinh^2\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)\pi}$ $b_n = \frac{2}{\pi} \int_0^{\pi} \sinh(x) \cos(2nx) dx$ $= -\frac{1}{2n} \sinh(x) \cos(2nx) dx - \frac{1}{2n} \sinh(x) \cos(2nx) + \frac{1}{2n} \int \cosh(x) \cos(2nx) dx$ $= -\frac{1}{2n} \sinh(x) \cos(2nx) + \frac{1}{2n} \left(\frac{1}{2n} \cosh(x) \sin(2nx) - \frac{1}{2n} \int \sinh(x) \sin(2nx) dx \right)$ $\Rightarrow b_n = \frac{2}{\pi} \left[\frac{\cosh(x) \sin(2nx) - 2n \sinh(x) \cos(2nx)}{4n^2 + 1} \right]_0^{\pi} = -\frac{n \sinh(\pi)}{(n^2 + \frac{1}{4})\pi}$ $\Rightarrow \left[f(x) = \frac{2 \sinh^2\left(\frac{\pi}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \left(\frac{\sinh^2\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)\pi}\right) \cos(2nx) + \left(-\frac{n \sinh(\pi)}{(n^2 + \frac{1}{4})\pi}\right) \sin(2nx) \right]$ $f(\pi) = \frac{\sinh(\pi)}{2} = \frac{2 \sinh^2\left(\frac{\pi}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \frac{\sinh^2\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)\pi} = \sum_{n=1}^{\infty} \frac{1}{n^2 + \frac{1}{4}} = \pi \coth\left(\frac{\pi}{2}\right) - 2$ $\frac{2}{\pi} \int_0^{\pi} \sinh^2(x) dx = \frac{1}{\pi} \int_0^{\pi} (\cosh(2x) - 1) dx = \frac{\sinh^4\left(\frac{\pi}{2}\right)}{2\pi} - 1 = \frac{8 \sinh^4\left(\frac{\pi}{2}\right)}{n^2} + \sum_{n=1}^{\infty} \frac{\sinh^4\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2} + \sum_{n=1}^{\infty} \frac{\sinh^4\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2} + \frac{1}{n^4 \sinh\left(\frac{\pi}{2}\right)} \cosh^2\left(\frac{\pi}{2}\right) + \frac{1}{n^2} \sinh^2\left(\pi\right) - \frac{1}{n^2} \sinh^2\left(\pi\right)$ $\Rightarrow \frac{\sinh^2\left(\frac{\pi}{2}\right)}{2\pi} - 1 - \frac{8 \sinh^4\left(\frac{\pi}{2}\right)}{n^2} - \frac{1}{n^4 \sinh\left(\frac{\pi}{2}\right)} \cosh^3\left(\frac{\pi}{2}\right) + \frac{2}{n^2} \sinh^2\left(\pi\right) - \frac{1}{n^2} \sinh^2\left(\pi\right)}{n^2} + \frac{1}{n^4 \sinh\left(\frac{\pi}{2}\right)} \cosh^3\left(\frac{\pi}{2}\right) - \frac{2}{n^2} \sinh^2\left(\pi\right) - \frac{\sinh(2\pi)}{2\pi}$ $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n^2 + \frac{1}{4})^2} = \pi^2 \cosh^2\left(\frac{\pi}{2}\right) + 8 \sinh^4\left(\frac{\pi}{2}\right) + 4\pi \coth\left(\frac{\pi}{2}\right) \cosh^2\left(\frac{\pi}{2}\right) - 8 \cosh^2\left(\frac{\pi}{2}\right) - 2\pi \left(2 \cosh^2\left(\frac{\pi}{2}\right) - 1\right) \coth\left(\frac{\pi}{2}\right)$ $= \pi^2 \cosh^2\left(\frac{\pi}{2}\right) + 4\pi \coth\left(\frac{\pi}{2}\right) \cosh^2\left(\frac{\pi}{2}\right) - 8 \cosh^2\left(\frac{\pi}{2}\right) - 2\pi \left(2 \cosh^2\left(\frac{\pi}{2}\right) - 1\right) \coth\left(\frac{\pi}{2}\right)$ $= \pi^2 \cosh^2\left(\frac{\pi}{2}\right) + 4\pi \coth\left(\frac{\pi}{2}\right) \cosh^2\left(\frac{\pi}{2}\right) - 8 \cosh^2\left(\frac{\pi}{2}\right) - 2\pi \left(2 \cosh^2\left(\frac{\pi}{2}\right) - 1\right) \coth\left(\frac{\pi}{2}\right)$ $= \pi^2 \cosh^2\left(\frac{\pi}{2}\right) + 4\pi \coth\left(\frac{\pi}{2}\right) \cosh^2\left(\frac{\pi}{2}\right) - 8 \cosh^2\left(\frac{\pi}{2}\right) - 2\pi \left(2 \cosh^2\left(\frac{\pi}{2}\right) - 1\right) \coth\left(\frac{\pi}{2}\right)$ $= \pi^2 \cosh^2\left(\frac{\pi}{2}\right) + 4\pi \coth\left(\frac{\pi}{2}\right) \cosh^2\left(\frac{\pi}{2}\right) - 3\pi \cosh^2\left(\frac{\pi}{2}\right) - 3\pi \cosh^2\left(\frac{\pi}{2}\right) + 3\pi \cosh^2\left(\frac$		1, 30	
$\Rightarrow a_n = \frac{2}{\pi} \left[\frac{2n \sinh(x) \sin(2nx) + \cosh(x) \cos(2nx)}{4n^2 + 1} \right]_0^{\pi} = \frac{2(\cosh(\pi) - 1)}{(4n^2 + 1)\pi} = \frac{4 \sinh^2\left(\frac{\pi}{2}\right)}{(4n^2 + 1)\pi} = \frac{\sinh^2\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})\pi}$ $b_n = \frac{2}{\pi} \int_0^{\pi} \sinh(x) \cos(2nx) dx$ $= \int \sinh(x) \sin(2nx) dx = -\frac{1}{2n} \sinh(x) \cos(2nx) + \frac{1}{2n} \int \cosh(x) \cos(2nx) dx$ $= -\frac{1}{2n} \sinh(x) \cos(2nx) + \frac{1}{2n} \left(\frac{1}{2n} \cosh(x) \sin(2nx) - \frac{1}{2n} \int \sinh(x) \sin(2nx) dx\right)$ $\Rightarrow b_n = \frac{2}{\pi} \left[\frac{\cosh(x) \sin(2nx) - 2n \sinh(x) \cos(2nx)}{4n^2 + 1} \right]_0^{\pi} = -\frac{n \sinh(n)}{(n^2 + \frac{1}{4})\pi}$ $\Rightarrow \left[f(x) = \frac{2 \sinh^2\left(\frac{\pi}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \left(\frac{\sinh^2\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})\pi}\right) \cos(2nx) + \left(-\frac{n \sinh(n)}{(n^2 + \frac{1}{4})\pi}\right) \sin(2nx) \right]$ $f(\pi) = \frac{\sinh(\pi)}{2} = \frac{2}{\pi} \frac{\sinh^2\left(\frac{\pi}{2}\right)}{\pi^2} + \sum_{n=1}^{\infty} \frac{\sinh^2\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})\pi^2} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2 + \frac{1}{4}} = \pi \cot\left(\frac{\pi}{2}\right) - 2$ $\frac{2}{\pi} \int_0^{\pi} \sinh^2(x) dx = \frac{1}{\pi} \int_0^{\pi} (\cosh(2x) - 1) dx = \frac{\sinh(2\pi)}{2\pi} - 1 = \frac{8 \sinh^4\left(\frac{\pi}{2}\right)}{\pi^2} + \sum_{n=1}^{\infty} \frac{\sinh^4\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})^2 \pi^2} + \frac{n^2 \sinh^2(\pi)}{(n^2 + \frac{1}{4})^2 \pi^2}$ $= \frac{8 \sinh^4\left(\frac{\pi}{2}\right)}{2\pi} + \sum_{n=1}^{\infty} \frac{\sinh^4\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})^2 \pi^2} + \frac{\sinh^2(\pi)}{(n^2 + \frac{1}{4})^2 \pi^2} - \frac{1}{4} \sinh^2\left(\frac{\pi}{2}\right)$ $= \frac{\sinh(2\pi)}{2\pi} - 1 - \frac{8 \sinh^4\left(\frac{\pi}{2}\right)}{\pi^2} - \frac{1}{4} \sinh\left(\frac{\pi}{2}\right) \cosh^3\left(\frac{\pi}{2}\right) - \frac{1}{\pi^2} \sinh^2(\pi) - \frac{1}{2\pi} \sinh^2(\pi)}$ $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n^2 + \frac{1}{4})^2} e^{-\frac{1}{2}} + \frac{1}{8 \sinh^4\left(\frac{\pi}{2}\right)} + \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^3\left(\frac{\pi}{2}\right) - \frac{2}{\pi^2} \sinh^2(\pi) - \frac{\sinh(2\pi)}{2\pi}$ $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n^2 + \frac{1}{4})^2} e^{-\frac{1}{2}} + \frac{1}{8 \sinh^4\left(\frac{\pi}{2}\right)} + \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^3\left(\frac{\pi}{2}\right) - \frac{2}{\pi^2} \sinh^2(\pi) - \frac{\sinh(2\pi)}{2\pi}$ $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n^2 + \frac{1}{4})^2} e^{-\frac{1}{2}} + \frac{1}{8 \sinh^4\left(\frac{\pi}{2}\right)} + \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^3\left(\frac{\pi}{2}\right) - \frac{2}{\pi^2} \sinh^2(\pi) - \frac{\sinh(2\pi)}{2\pi}$ $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n^2 + \frac{1}{4})^2} e^{-\frac{1}{2}} + \frac{1}{8 \sinh^4\left(\frac{\pi}{2}\right)} + \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^3\left(\frac{\pi}{2}\right) - \frac{2}{\pi^2} \sinh^2(\pi) - \frac{1}{\pi^2} \sinh^2(\pi)$ $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n^2 + \frac{1}{4})^2} e^{-\frac{1}{2}} + \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^3\left(\frac{\pi}{2}\right) - \frac{2}{\pi^2} \sinh^2(\pi) - \frac{1}{\pi^2} \sinh^2(\pi)$ $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n^2 + \frac{1}{4})^2} $		$\int \sinh(x)\cos(2nx)dx = \frac{1}{2n}\sinh(x)\sin(2nx) - \frac{1}{2n}\int \cosh(x)\sin(2nx)dx$	
$\Rightarrow a_n = \frac{2}{\pi} \left[\frac{2n \sinh(x) \sin(2nx) + \cosh(x) \cos(2nx)}{4n^2 + 1} \right]_0^{\pi} = \frac{2(\cosh(\pi) - 1)}{(4n^2 + 1)\pi} = \frac{4 \sinh^2\left(\frac{\pi}{2}\right)}{(4n^2 + 1)\pi} = \frac{\sinh^2\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})\pi}$ $b_n = \frac{2}{\pi} \int_0^{\pi} \sinh(x) \cos(2nx) dx$ $= \int \sinh(x) \sin(2nx) dx = -\frac{1}{2n} \sinh(x) \cos(2nx) + \frac{1}{2n} \int \cosh(x) \cos(2nx) dx$ $= -\frac{1}{2n} \sinh(x) \cos(2nx) + \frac{1}{2n} \left(\frac{1}{2n} \cosh(x) \sin(2nx) - \frac{1}{2n} \int \sinh(x) \sin(2nx) dx\right)$ $\Rightarrow b_n = \frac{2}{\pi} \left[\frac{\cosh(x) \sin(2nx) - 2n \sinh(x) \cos(2nx)}{4n^2 + 1} \right]_0^{\pi} = -\frac{n \sinh(n)}{(n^2 + \frac{1}{4})\pi}$ $\Rightarrow \left[f(x) = \frac{2 \sinh^2\left(\frac{\pi}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \left(\frac{\sinh^2\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})\pi}\right) \cos(2nx) + \left(-\frac{n \sinh(n)}{(n^2 + \frac{1}{4})\pi}\right) \sin(2nx) \right]$ $f(\pi) = \frac{\sinh(\pi)}{2} = \frac{2}{\pi} \frac{\sinh^2\left(\frac{\pi}{2}\right)}{\pi^2} + \sum_{n=1}^{\infty} \frac{\sinh^2\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})\pi^2} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2 + \frac{1}{4}} = \pi \cot\left(\frac{\pi}{2}\right) - 2$ $\frac{2}{\pi} \int_0^{\pi} \sinh^2(x) dx = \frac{1}{\pi} \int_0^{\pi} (\cosh(2x) - 1) dx = \frac{\sinh(2\pi)}{2\pi} - 1 = \frac{8 \sinh^4\left(\frac{\pi}{2}\right)}{\pi^2} + \sum_{n=1}^{\infty} \frac{\sinh^4\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})^2 \pi^2} + \frac{n^2 \sinh^2(\pi)}{(n^2 + \frac{1}{4})^2 \pi^2}$ $= \frac{8 \sinh^4\left(\frac{\pi}{2}\right)}{2\pi} + \sum_{n=1}^{\infty} \frac{\sinh^4\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})^2 \pi^2} + \frac{\sinh^2(\pi)}{(n^2 + \frac{1}{4})^2 \pi^2} - \frac{1}{4} \sinh^2\left(\frac{\pi}{2}\right)$ $= \frac{\sinh(2\pi)}{2\pi} - 1 - \frac{8 \sinh^4\left(\frac{\pi}{2}\right)}{\pi^2} - \frac{1}{4} \sinh\left(\frac{\pi}{2}\right) \cosh^3\left(\frac{\pi}{2}\right) - \frac{1}{\pi^2} \sinh^2(\pi) - \frac{1}{2\pi} \sinh^2(\pi)}$ $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n^2 + \frac{1}{4})^2} e^{-\frac{1}{2}} + \frac{1}{8 \sinh^4\left(\frac{\pi}{2}\right)} + \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^3\left(\frac{\pi}{2}\right) - \frac{2}{\pi^2} \sinh^2(\pi) - \frac{\sinh(2\pi)}{2\pi}$ $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n^2 + \frac{1}{4})^2} e^{-\frac{1}{2}} + \frac{1}{8 \sinh^4\left(\frac{\pi}{2}\right)} + \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^3\left(\frac{\pi}{2}\right) - \frac{2}{\pi^2} \sinh^2(\pi) - \frac{\sinh(2\pi)}{2\pi}$ $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n^2 + \frac{1}{4})^2} e^{-\frac{1}{2}} + \frac{1}{8 \sinh^4\left(\frac{\pi}{2}\right)} + \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^3\left(\frac{\pi}{2}\right) - \frac{2}{\pi^2} \sinh^2(\pi) - \frac{\sinh(2\pi)}{2\pi}$ $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n^2 + \frac{1}{4})^2} e^{-\frac{1}{2}} + \frac{1}{8 \sinh^4\left(\frac{\pi}{2}\right)} + \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^3\left(\frac{\pi}{2}\right) - \frac{2}{\pi^2} \sinh^2(\pi) - \frac{1}{\pi^2} \sinh^2(\pi)$ $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n^2 + \frac{1}{4})^2} e^{-\frac{1}{2}} + \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^3\left(\frac{\pi}{2}\right) - \frac{2}{\pi^2} \sinh^2(\pi) - \frac{1}{\pi^2} \sinh^2(\pi)$ $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n^2 + \frac{1}{4})^2} $		$= \frac{1}{2n} \sinh(x) \sin(2nx) - \frac{1}{2n} \left(-\frac{1}{2n} \cosh(x) \cos(2nx) + \frac{1}{2n} \int \sinh(x) \cos(2nx) dx \right)$	
$b_{n} = \frac{2}{\pi} \int_{0}^{\pi} \sinh(x) \cos(2nx) dx$ $\int \sinh(x) \sin(2nx) dx = -\frac{1}{2n} \sinh(x) \cos(2nx) + \frac{1}{2n} \int \cosh(x) \cos(2nx) dx$ $= -\frac{1}{2n} \sinh(x) \cos(2nx) + \frac{1}{2n} \left(\frac{1}{2n} \cosh(x) \sin(2nx) - \frac{1}{2n} \int \sinh(x) \sin(2nx) dx\right)$ $\Rightarrow b_{n} = \frac{2}{\pi} \left[\frac{\cosh(x) \sin(2nx) - 2n \sinh(x) \cos(2nx)}{4n^{2} + 1} \right]_{0}^{\pi} = -\frac{n \sinh(\pi)}{(n^{2} + \frac{1}{4})\pi}$ $\Rightarrow \int f(x) = \frac{2 \sinh^{2}\left(\frac{\pi}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \left(\frac{\sinh^{2}\left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})\pi}\right) \cos(2nx) + \left(-\frac{n \sinh(\pi)}{(n^{2} + \frac{1}{4})\pi}\right) \sin(2nx)$ $f(\pi) = \frac{\sinh(\pi)}{2} = \frac{2 \sinh^{2}\left(\frac{\pi}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \frac{\sinh^{2}\left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})\pi} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^{2} + \frac{1}{4}} = \pi \coth\left(\frac{\pi}{2}\right) - 2$ $\frac{2}{\pi} \int_{0}^{\pi} \sinh^{2}(x) dx = \frac{1}{\pi} \int_{0}^{\pi} (\cosh(2x) - 1) dx = \frac{\sinh(2\pi)}{2\pi} - 1 = \frac{8 \sinh^{4}\left(\frac{\pi}{2}\right)}{\pi^{2}} + \sum_{n=1}^{\infty} \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})^{2}\pi^{2}} + \sum_{n=1}^{\infty} \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})^{2}\pi^{2}} + \frac{\sinh^{2}\left(\pi\right)}{(n^{2} + \frac{1}{4})^{2}\pi^{2}} + \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})^{2}\pi^{2}} + \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{n^{2}} + \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{n^{2}} + \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})^{2}\pi^{2}} + \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{n^{2}} + \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})^{2}\pi^{2}} + \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})^{2}\pi^{2}} + \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})^{2}\pi^{2}} + \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{n^{2}} + \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{n^{2}} + \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})^{2}\pi^{2}} + \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})^{2}\pi^{2}} + \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{n^{2}} + \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{n^{2}} + \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})^{2}\pi^{2}} + \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{n^{2}} + \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})^{2}\pi^{2}} + \frac{\sinh^{4}\left$			
$\int \sinh(x) \sin(2nx) dx = -\frac{1}{2n} \sinh(x) \cos(2nx) + \frac{1}{2n} \int \cosh(x) \cos(2nx) + \frac{1}{2n} \int \cosh(x) \cos(2nx) dx$ $= -\frac{1}{2n} \sinh(x) \cos(2nx) + \frac{1}{2n} \left(\frac{1}{2n} \cosh(x) \sin(2nx) - \frac{1}{2n} \int \sinh(x) \sin(2nx) dx \right)$ $\Rightarrow b_n = \frac{2}{n} \left[\frac{\cosh(x) \sin(2nx) - 2n \sinh(x) \cos(2nx)}{4n^2 + 1} \right]_0^n = -\frac{n \sinh(n)}{(n^2 + \frac{1}{4})n}$ $\Rightarrow \left[f(x) = \frac{2 \sinh^2\left(\frac{n}{2}\right)}{n} + \sum_{n=1}^{\infty} \left(\frac{\sinh^2\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})n} \right) \cos(2nx) + \left(-\frac{n \sinh(n)}{(n^2 + \frac{1}{4})n} \right) \sin(2nx) \right]$ $f(\pi) = \frac{\sinh(\pi)}{2} = \frac{2 \sinh^2\left(\frac{n}{2}\right)}{n} + \sum_{n=1}^{\infty} \frac{\sinh^2\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})n} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2 + \frac{1}{4}} = \pi \coth\left(\frac{n}{2}\right) - 2$ $\frac{2}{n} \int_0^{\pi} \sinh^2(x) dx = \frac{1}{n} \int_0^{\pi} (\cosh(2x) - 1) dx = \frac{\sinh(2\pi)}{2\pi} - 1 = \frac{8 \sinh^4\left(\frac{n}{2}\right)}{n^2} + \sum_{n=1}^{\infty} \frac{\sinh^4\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})^2 n^2} + \frac{n^2 \sinh^2(n)}{(n^2 + \frac{1}{4})^2 n^2} + \frac{n^2 \sinh^4\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})^2 n^2} + \frac{\sinh^4\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})^2 n^2} + \frac{\sinh^4\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})^2 n^2} - \frac{1}{n^2 \sinh^2\left(\frac{n}{2}\right)} + \frac{\sinh^4\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})^2 n^2} + \frac{1}{n^2 \sinh^2\left(\frac{n}{2}\right)} + \frac{1}{n^2 \sinh^2\left(\frac{n}$		$\Rightarrow a_n = \frac{1}{\pi} \left[\frac{4n^2 + 1}{(4n^2 + 1)\pi} \right] = \frac{(2n^2 + 1)\pi}{(4n^2 + 1)\pi} = \frac{(2n^2 + 1)\pi}{(n^2 + \frac{1}{4})\pi}$	
$\int \sinh(x) \sin(2nx) dx = -\frac{1}{2n} \sinh(x) \cos(2nx) + \frac{1}{2n} \int \cosh(x) \cos(2nx) + \frac{1}{2n} \int \cosh(x) \cos(2nx) dx$ $= -\frac{1}{2n} \sinh(x) \cos(2nx) + \frac{1}{2n} \left(\frac{1}{2n} \cosh(x) \sin(2nx) - \frac{1}{2n} \int \sinh(x) \sin(2nx) dx \right)$ $\Rightarrow b_n = \frac{2}{n} \left[\frac{\cosh(x) \sin(2nx) - 2n \sinh(x) \cos(2nx)}{4n^2 + 1} \right]_0^n = -\frac{n \sinh(n)}{(n^2 + \frac{1}{4})n}$ $\Rightarrow \left[f(x) = \frac{2 \sinh^2\left(\frac{n}{2}\right)}{n} + \sum_{n=1}^{\infty} \left(\frac{\sinh^2\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})n} \right) \cos(2nx) + \left(-\frac{n \sinh(n)}{(n^2 + \frac{1}{4})n} \right) \sin(2nx) \right]$ $f(\pi) = \frac{\sinh(\pi)}{2} = \frac{2 \sinh^2\left(\frac{n}{2}\right)}{n} + \sum_{n=1}^{\infty} \frac{\sinh^2\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})n} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2 + \frac{1}{4}} = \pi \coth\left(\frac{n}{2}\right) - 2$ $\frac{2}{n} \int_0^{\pi} \sinh^2(x) dx = \frac{1}{n} \int_0^{\pi} (\cosh(2x) - 1) dx = \frac{\sinh(2\pi)}{2\pi} - 1 = \frac{8 \sinh^4\left(\frac{n}{2}\right)}{n^2} + \sum_{n=1}^{\infty} \frac{\sinh^4\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})^2 n^2} + \frac{n^2 \sinh^2(n)}{(n^2 + \frac{1}{4})^2 n^2} + \frac{n^2 \sinh^4\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})^2 n^2} + \frac{\sinh^4\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})^2 n^2} + \frac{\sinh^4\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})^2 n^2} - \frac{1}{n^2 \sinh^2\left(\frac{n}{2}\right)} + \frac{\sinh^4\left(\frac{n}{2}\right)}{(n^2 + \frac{1}{4})^2 n^2} + \frac{1}{n^2 \sinh^2\left(\frac{n}{2}\right)} + \frac{1}{n^2 \sinh^2\left(\frac{n}$		$b_n = \frac{2}{\pi} \int_0^{\pi} \sinh(x) \cos(2nx) dx$	
$ = -\frac{1}{2n} \sinh(x) \cos(2nx) + \frac{1}{2n} \left(\frac{1}{2n} \cosh(x) \sin(2nx) - \frac{1}{2n} \int \sinh(x) \sin(2nx) dx \right) $ $ \Rightarrow b_n = \frac{2}{\pi} \left[\frac{\cosh(x) \sin(2nx) - 2n \sinh(x) \cos(2nx)}{4n^2 + 1} \right]_0^{\pi} = -\frac{n \sinh(\pi)}{(n^2 + \frac{1}{4})\pi} $ $ \Rightarrow \left[f(x) = \frac{2 \sinh^2\left(\frac{\pi}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \left(\frac{\sinh^2\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})\pi} \right) \cos(2nx) + \left(-\frac{n \sinh(\pi)}{(n^2 + \frac{1}{4})\pi} \right) \sin(2nx) \right] $ $ f(n) = \frac{\sinh(\pi)}{2} = \frac{2 \sinh^2\left(\frac{\pi}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \frac{\sinh^2\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})\pi} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2 + \frac{1}{4}} = \pi \coth\left(\frac{\pi}{2}\right) - 2 $ $ \frac{2}{\pi} \int_0^{\pi} \sinh^2(x) dx = \frac{1}{\pi} \int_0^{\pi} (\cosh(2x) - 1) dx = \frac{\sinh(2\pi)}{2\pi} - 1 = \frac{8 \sinh^4\left(\frac{\pi}{2}\right)}{\pi^2} + \sum_{n=1}^{\infty} \frac{\sinh^4\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})^2 \pi^2} + \frac{n^2 \sinh^2(\pi)}{(n^2 + \frac{1}{4})^2 \pi^2} $ $ = \frac{8 \sinh^4\left(\frac{\pi}{2}\right)}{\pi^2} + \sum_{n=1}^{\infty} \frac{\sinh^4\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})^2 \pi^2} + \frac{\sinh^2(\pi)}{(n^2 + \frac{1}{4})^2 \pi^2} $ $ \frac{\sinh(2\pi)}{2\pi} - 1 - \frac{8 \sinh^4\left(\frac{\pi}{2}\right)}{\pi^2} - \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^3\left(\frac{\pi}{2}\right) + \frac{2}{\pi^2} \sinh^2(\pi) - \frac{1}{\pi^2} \sinh^2\left(\frac{\pi}{2}\right) $ $ \frac{\sinh^2\left(\frac{\pi}{2}\right)}{(n^2 + \frac{1}{4})^2 \pi^2} = 1 + \frac{8 \sinh^4\left(\frac{\pi}{2}\right)}{\pi^2} + \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^3\left(\frac{\pi}{2}\right) - \frac{2}{\pi^2} \sinh^2(\pi) - \frac{\sinh(2\pi)}{2\pi} $ $ \Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n^2 + \frac{1}{4})^2} = \pi^2 \cosh^2\left(\frac{\pi}{2}\right) + 8 \sinh^2\left(\frac{\pi}{2}\right) + 4\pi \coth\left(\frac{\pi}{2}\right) \cosh^2\left(\frac{\pi}{2}\right) - 8 \cosh^2\left(\frac{\pi}{2}\right) - 2\pi \left(2 \cosh^2\left(\frac{\pi}{2}\right) - 1\right) \coth\left(\frac{\pi}{2}\right) $ $ = \pi^2 \cosh^2\left(\frac{\pi}{2}\right) - 8 + 2\pi \coth\left(\frac{\pi}{2}\right)$			
$ \Rightarrow b_{n} = \frac{2}{\pi} \left[\frac{\cosh(x) \sin(2nx) - 2n \sinh(x) \cos(2nx)}{4n^{2} + 1} \right]_{0}^{\pi} = -\frac{n \sinh(\pi)}{(n^{2} + \frac{1}{4})\pi} $ $ \Rightarrow \int f(x) = \frac{2 \sinh^{2}\left(\frac{\pi}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \left(\frac{\sinh^{2}\left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})\pi} \right) \cos(2nx) + \left(-\frac{n \sinh(\pi)}{(n^{2} + \frac{1}{4})\pi} \right) \sin(2nx) $ $ f(\pi) = \frac{\sinh(\pi)}{2} = \frac{2 \sinh^{2}\left(\frac{\pi}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \frac{\sinh^{2}\left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})\pi} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^{2} + \frac{1}{4}} = \pi \cot\left(\frac{\pi}{2}\right) - 2 $ $ \frac{2}{\pi} \int_{0}^{\pi} \sinh^{2}(x) dx = \frac{1}{\pi} \int_{0}^{\pi} (\cosh(2x) - 1) dx = \frac{\sinh(2\pi)}{2\pi} - 1 = \frac{8 \sinh^{4}\left(\frac{\pi}{2}\right)}{\pi^{2}} + \sum_{n=1}^{\infty} \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})^{2}\pi^{2}} + \sum_{n=1}^{\infty} \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})^{2}\pi^{2}} + \frac{n^{2} \sinh^{2}(\pi)}{(n^{2} + \frac{1}{4})^{2}\pi^{2}} $ $ = \frac{8 \sinh^{4}\left(\frac{\pi}{2}\right)}{\pi^{2}} + \sum_{n=1}^{\infty} \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})^{2}\pi^{2}} + \frac{\sinh^{2}(\pi)}{(n^{2} + \frac{1}{4})^{2}\pi^{2}} - \frac{\frac{1}{4} \sinh^{2}(\pi)}{(n^{2} + \frac{1}{4})^{2}\pi^{2}} - \frac{\frac{1}{4} \sinh^{2}(\pi)}{(n^{2} + \frac{1}{4})^{2}\pi^{2}} $ $ = \frac{\sinh^{2}\left(\frac{\pi}{2}\right)}{2\pi} - 1 - \frac{8 \sinh^{4}\left(\frac{\pi}{2}\right)}{\pi^{2}} - \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^{3}\left(\frac{\pi}{2}\right) + \frac{2}{\pi^{2}} \sinh^{2}(\pi) - \sum_{n=1}^{\infty} \frac{\sinh^{2}\left(\frac{\pi}{2}\right)}{(n^{2} + \frac{1}{4})^{2}\pi^{2}} $ $ = \sum_{n=1}^{\infty} \frac{1}{(n^{2} + \frac{1}{4})^{2}} = 1 + \frac{8 \sinh^{4}\left(\frac{\pi}{2}\right)}{\pi^{2}} + \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^{3}\left(\frac{\pi}{2}\right) - \frac{2}{\pi^{2}} \sinh^{2}(\pi) - \frac{\sinh(2\pi)}{2\pi} $ $ \Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n^{2} + \frac{1}{4})^{2}} = \pi^{2} \operatorname{csch}^{2}\left(\frac{\pi}{2}\right) + 8 \sinh^{2}\left(\frac{\pi}{2}\right) + 4\pi \coth\left(\frac{\pi}{2}\right) \cosh^{2}\left(\frac{\pi}{2}\right) - 8 + 2\pi \coth\left(\frac{\pi}{2}\right) $ $ = \pi^{2} \operatorname{csch}^{2}\left(\frac{\pi}{2}\right) - 1 \coth\left(\frac{\pi}{2}\right) $			
$f(x) = \frac{2\sinh^2\left(\frac{\pi}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \left(\frac{\sinh^2\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)\pi}\right) \cos(2nx) + \left(-\frac{n\sinh(\pi)}{\left(n^2 + \frac{1}{4}\right)\pi}\right) \sin(2nx)$ $f(\pi) = \frac{\sinh(\pi)}{2} = \frac{2\sinh^2\left(\frac{\pi}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \frac{\sinh^2\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)\pi} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2 + \frac{1}{4}} = \pi \coth\left(\frac{\pi}{2}\right) - 2$ $\frac{2}{\pi} \int_0^{\pi} \sinh^2(x) dx = \frac{1}{\pi} \int_0^{\pi} (\cosh(2x) - 1) dx = \frac{\sinh(2\pi)}{2\pi} - 1 = \frac{8\sinh^4\left(\frac{\pi}{2}\right)}{\pi^2} + \sum_{n=1}^{\infty} \frac{\sinh^4\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2} + \frac{\sinh^2\left(\pi\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2} + \frac{\sinh^2\left(\pi\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2}$ $= \frac{8\sinh^4\left(\frac{\pi}{2}\right)}{\pi^2} + \sum_{n=1}^{\infty} \frac{\sinh^4\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2} + \frac{\sinh^2\left(\pi\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2} + \frac{1}{n^2} \sinh^2\left(\pi\right) = -\sum_{n=1}^{\infty} \frac{\sinh^2\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2}$ $= \frac{\sinh^2\left(\frac{\pi}{2}\right)}{\pi^2} - \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^3\left(\frac{\pi}{2}\right) + \frac{2}{\pi^2} \sinh^2\left(\pi\right) = -\sum_{n=1}^{\infty} \frac{\sinh^2\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2}$ $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{\left(n^2 + \frac{1}{4}\right)^2} = 1 + \frac{8\sinh^4\left(\frac{\pi}{2}\right)}{\pi^2} + 4\pi \coth\left(\frac{\pi}{2}\right) \cosh^2\left(\frac{\pi}{2}\right) - 8\cosh^2\left(\frac{\pi}{2}\right) - 2\pi \left(2\cosh^2\left(\frac{\pi}{2}\right) - 1\right) \coth\left(\frac{\pi}{2}\right)$ $= \pi^2 \operatorname{csch}^2\left(\frac{\pi}{2}\right) - 8 + 2\pi \coth\left(\frac{\pi}{2}\right)$			
$f(x) = \frac{2\sinh^2\left(\frac{\pi}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \left(\frac{\sinh^2\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)\pi}\right) \cos(2nx) + \left(-\frac{n\sinh(\pi)}{\left(n^2 + \frac{1}{4}\right)\pi}\right) \sin(2nx)$ $f(\pi) = \frac{\sinh(\pi)}{2} = \frac{2\sinh^2\left(\frac{\pi}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \frac{\sinh^2\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)\pi} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2 + \frac{1}{4}} = \pi \coth\left(\frac{\pi}{2}\right) - 2$ $\frac{2}{\pi} \int_0^{\pi} \sinh^2(x) dx = \frac{1}{\pi} \int_0^{\pi} (\cosh(2x) - 1) dx = \frac{\sinh(2\pi)}{2\pi} - 1 = \frac{8\sinh^4\left(\frac{\pi}{2}\right)}{\pi^2} + \sum_{n=1}^{\infty} \frac{\sinh^4\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2} + \frac{\sinh^2\left(\pi\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2} + \frac{\sinh^2\left(\pi\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2}$ $= \frac{8\sinh^4\left(\frac{\pi}{2}\right)}{\pi^2} + \sum_{n=1}^{\infty} \frac{\sinh^4\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2} + \frac{\sinh^2\left(\pi\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2} + \frac{1}{n^2} \sinh^2\left(\pi\right) = -\sum_{n=1}^{\infty} \frac{\sinh^2\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2}$ $= \frac{\sinh^2\left(\frac{\pi}{2}\right)}{\pi^2} - \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^3\left(\frac{\pi}{2}\right) + \frac{2}{\pi^2} \sinh^2\left(\pi\right) = -\sum_{n=1}^{\infty} \frac{\sinh^2\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2}$ $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{\left(n^2 + \frac{1}{4}\right)^2} = 1 + \frac{8\sinh^4\left(\frac{\pi}{2}\right)}{\pi^2} + 4\pi \coth\left(\frac{\pi}{2}\right) \cosh^2\left(\frac{\pi}{2}\right) - 8\cosh^2\left(\frac{\pi}{2}\right) - 2\pi \left(2\cosh^2\left(\frac{\pi}{2}\right) - 1\right) \coth\left(\frac{\pi}{2}\right)$ $= \pi^2 \operatorname{csch}^2\left(\frac{\pi}{2}\right) - 8 + 2\pi \coth\left(\frac{\pi}{2}\right)$		$\Rightarrow b_n = \frac{2}{\pi} \left \frac{\cosh(x) \sin(2nx) - 2n \sin(x) \cos(2nx)}{4n^2 + 1} \right _{x=0} = -\frac{n \sin(\pi)}{\left(n^2 + \frac{1}{2}\right)\pi}$	
$f(\pi) = \frac{\sinh(\pi)}{2} = \frac{2\sinh^2(\frac{\pi}{2})}{\pi} + \sum_{n=1}^{\infty} \frac{\sinh^2(\frac{\pi}{2})}{(n^2 + \frac{1}{4})\pi} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2 + \frac{1}{4}} = \pi \coth(\frac{\pi}{2}) - 2$ $\frac{2}{\pi} \int_0^{\pi} \sinh^2(x) dx = \frac{1}{\pi} \int_0^{\pi} (\cosh(2x) - 1) dx = \frac{\sinh(2\pi)}{2\pi} - 1 = \frac{8\sinh^4(\frac{\pi}{2})}{\pi^2} + \sum_{n=1}^{\infty} \frac{\sinh^4(\frac{\pi}{2})}{(n^2 + \frac{1}{4})^2 \pi^2} + \frac{n^2 \sinh^2(\pi)}{(n^2 + \frac{1}{4})^2 \pi^2}$ $= \frac{8\sinh^4(\frac{\pi}{2})}{\pi^2} + \sum_{n=1}^{\infty} \frac{\sinh^4(\frac{\pi}{2})}{(n^2 + \frac{1}{4})^2 \pi^2} + \frac{\sinh^2(\pi)}{(n^2 + \frac{1}{4})^2 \pi^2} - \frac{1}{4} \sinh^2(\pi)$ $\frac{\sinh(2\pi)}{2\pi} - 1 - \frac{8\sinh^4(\frac{\pi}{2})}{\pi^2} - \frac{1}{\pi} 4 \sinh(\frac{\pi}{2}) \cosh^2(\frac{\pi}{2}) + \frac{2}{\pi^2} \sinh^2(\pi) = -\sum_{n=1}^{\infty} \frac{\sinh^2(\frac{\pi}{2})}{(n^2 + \frac{1}{4})^2 \pi^2}$ $\sum_{n=1}^{\infty} \frac{\sinh^2(\frac{\pi}{2})}{(n^2 + \frac{1}{4})^2 \pi^2} = 1 + \frac{8\sinh^4(\frac{\pi}{2})}{\pi^2} + \frac{1}{\pi} 4 \sinh(\frac{\pi}{2}) \cosh^3(\frac{\pi}{2}) - \frac{2}{\pi^2} \sinh^2(\pi) - \frac{\sinh(2\pi)}{2\pi}$ $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n^2 + \frac{1}{4})^2} = \pi^2 \operatorname{csch}^2(\frac{\pi}{2}) + 8 \sinh^2(\frac{\pi}{2}) + 4\pi \coth(\frac{\pi}{2}) \cosh^2(\frac{\pi}{2}) - 8 \cosh^2(\frac{\pi}{2}) - 2\pi \left(2 \cosh^2(\frac{\pi}{2}) - 1\right) \coth(\frac{\pi}{2})$ $= \pi^2 \operatorname{csch}^2(\frac{\pi}{2}) - 8 + 2\pi \coth(\frac{\pi}{2})$		47	
$\frac{2}{\pi} \int_{0}^{\pi} \sinh^{2}(x) dx = \frac{1}{\pi} \int_{0}^{\pi} (\cosh(2x) - 1) dx = \frac{\sinh(2\pi)}{2\pi} - 1 = \frac{8 \sinh^{4}\left(\frac{\pi}{2}\right)}{\pi^{2}} + \sum_{n=1}^{\infty} \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}} + \frac{n^{2} \sinh^{2}(\pi)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}}$ $= \frac{8 \sinh^{4}\left(\frac{\pi}{2}\right)}{\pi^{2}} + \sum_{n=1}^{\infty} \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}} + \frac{\sinh^{2}(\pi)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}} - \frac{\frac{1}{4} \sinh^{2}(\pi)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}}$ $= \frac{\sinh(2\pi)}{2\pi} - 1 - \frac{8 \sinh^{4}\left(\frac{\pi}{2}\right)}{\pi^{2}} - \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^{3}\left(\frac{\pi}{2}\right) + \frac{2}{\pi^{2}} \sinh^{2}(\pi) = -\sum_{n=1}^{\infty} \frac{\sinh^{2}\left(\frac{\pi}{2}\right)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}}$ $= \sum_{n=1}^{\infty} \frac{\sinh^{2}\left(\frac{\pi}{2}\right)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}} = 1 + \frac{8 \sinh^{4}\left(\frac{\pi}{2}\right)}{\pi^{2}} + \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^{3}\left(\frac{\pi}{2}\right) - \frac{2}{\pi^{2}} \sinh^{2}(\pi) - \frac{\sinh(2\pi)}{2\pi}$ $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{\left(n^{2} + \frac{1}{4}\right)^{2}} = \pi^{2} \operatorname{csch}^{2}\left(\frac{\pi}{2}\right) + 8 \sinh^{2}\left(\frac{\pi}{2}\right) + 4\pi \coth\left(\frac{\pi}{2}\right) \cosh^{2}\left(\frac{\pi}{2}\right) - 8 \cosh^{2}\left(\frac{\pi}{2}\right) - 2\pi \left(2 \cosh^{2}\left(\frac{\pi}{2}\right) - 1\right) \coth\left(\frac{\pi}{2}\right)$ $= \pi^{2} \operatorname{csch}^{2}\left(\frac{\pi}{2}\right) - 8 + 2\pi \coth\left(\frac{\pi}{2}\right)$	۴	$\Rightarrow f(x) = \frac{2\sinh\left(\frac{1}{2}\right)}{\pi} + \sum_{n=1}^{\infty} \left(\frac{\sinh\left(\frac{1}{2}\right)}{\left(n^2 + \frac{1}{4}\right)\pi}\right) \cos(2nx) + \left(-\frac{n\sinh(\pi)}{\left(n^2 + \frac{1}{4}\right)\pi}\right) \sin(2nx)$	
$\frac{2}{\pi} \int_{0}^{\pi} \sinh^{2}(x) dx = \frac{1}{\pi} \int_{0}^{\pi} (\cosh(2x) - 1) dx = \frac{\sinh(2\pi)}{2\pi} - 1 = \frac{8 \sinh^{4}\left(\frac{\pi}{2}\right)}{\pi^{2}} + \sum_{n=1}^{\infty} \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}} + \frac{n^{2} \sinh^{2}(\pi)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}}$ $= \frac{8 \sinh^{4}\left(\frac{\pi}{2}\right)}{\pi^{2}} + \sum_{n=1}^{\infty} \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}} + \frac{\sinh^{2}(\pi)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}} - \frac{\frac{1}{4} \sinh^{2}(\pi)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}}$ $= \frac{\sinh(2\pi)}{2\pi} - 1 - \frac{8 \sinh^{4}\left(\frac{\pi}{2}\right)}{\pi^{2}} - \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^{3}\left(\frac{\pi}{2}\right) + \frac{2}{\pi^{2}} \sinh^{2}(\pi) = -\sum_{n=1}^{\infty} \frac{\sinh^{2}\left(\frac{\pi}{2}\right)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}}$ $= \sum_{n=1}^{\infty} \frac{\sinh^{2}\left(\frac{\pi}{2}\right)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}} = 1 + \frac{8 \sinh^{4}\left(\frac{\pi}{2}\right)}{\pi^{2}} + \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^{3}\left(\frac{\pi}{2}\right) - \frac{2}{\pi^{2}} \sinh^{2}(\pi) - \frac{\sinh(2\pi)}{2\pi}$ $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{\left(n^{2} + \frac{1}{4}\right)^{2}} = \pi^{2} \operatorname{csch}^{2}\left(\frac{\pi}{2}\right) + 8 \sinh^{2}\left(\frac{\pi}{2}\right) + 4\pi \coth\left(\frac{\pi}{2}\right) \cosh^{2}\left(\frac{\pi}{2}\right) - 8 \cosh^{2}\left(\frac{\pi}{2}\right) - 2\pi \left(2 \cosh^{2}\left(\frac{\pi}{2}\right) - 1\right) \coth\left(\frac{\pi}{2}\right)$ $= \pi^{2} \operatorname{csch}^{2}\left(\frac{\pi}{2}\right) - 8 + 2\pi \coth\left(\frac{\pi}{2}\right)$		$\sinh(\pi) = 2\sinh^2\left(\frac{\pi}{2}\right) + \sum_{n=1}^{\infty} \sinh^2\left(\frac{\pi}{2}\right) = \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{\pi}{n!}\right)$	
$= \frac{8 \sinh^{4}\left(\frac{\pi}{2}\right)}{\pi^{2}} + \sum_{n=1}^{\infty} \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}} + \frac{\sinh^{2}(\pi)}{\left(n^{2} + \frac{1}{4}\right) \pi^{2}} - \frac{\frac{1}{4} \sinh^{2}(\pi)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}}$ $\frac{\sinh(2\pi)}{2\pi} - 1 - \frac{8 \sinh^{4}\left(\frac{\pi}{2}\right)}{\pi^{2}} - \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^{3}\left(\frac{\pi}{2}\right) + \frac{2}{\pi^{2}} \sinh^{2}(\pi) = -\sum_{n}^{\infty} \frac{\sinh^{2}\left(\frac{\pi}{2}\right)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}}$ $\sum_{n}^{\infty} \frac{\sinh^{2}\left(\frac{\pi}{2}\right)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}} = 1 + \frac{8 \sinh^{4}\left(\frac{\pi}{2}\right)}{\pi^{2}} + \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^{3}\left(\frac{\pi}{2}\right) - \frac{2}{\pi^{2}} \sinh^{2}(\pi) - \frac{\sinh(2\pi)}{2\pi}$ $\Rightarrow \sum_{n}^{\infty} \frac{1}{\left(n^{2} + \frac{1}{4}\right)^{2}} = \pi^{2} \operatorname{csch}^{2}\left(\frac{\pi}{2}\right) + 8 \sinh^{2}\left(\frac{\pi}{2}\right) + 4\pi \coth\left(\frac{\pi}{2}\right) \cosh^{2}\left(\frac{\pi}{2}\right) - 8 \cosh^{2}\left(\frac{\pi}{2}\right) - 2\pi \left(2 \cosh^{2}\left(\frac{\pi}{2}\right) - 1\right) \coth\left(\frac{\pi}{2}\right)$ $= \pi^{2} \operatorname{csch}^{2}\left(\frac{\pi}{2}\right) - 8 + 2\pi \coth\left(\frac{\pi}{2}\right)$			
$= \frac{8 \sinh^{4}\left(\frac{\pi}{2}\right)}{\pi^{2}} + \sum_{n=1}^{\infty} \frac{\sinh^{4}\left(\frac{\pi}{2}\right)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}} + \frac{\sinh^{2}(\pi)}{\left(n^{2} + \frac{1}{4}\right) \pi^{2}} - \frac{\frac{1}{4} \sinh^{2}(\pi)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}}$ $\frac{\sinh(2\pi)}{2\pi} - 1 - \frac{8 \sinh^{4}\left(\frac{\pi}{2}\right)}{\pi^{2}} - \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^{3}\left(\frac{\pi}{2}\right) + \frac{2}{\pi^{2}} \sinh^{2}(\pi) = -\sum_{n}^{\infty} \frac{\sinh^{2}\left(\frac{\pi}{2}\right)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}}$ $\sum_{n}^{\infty} \frac{\sinh^{2}\left(\frac{\pi}{2}\right)}{\left(n^{2} + \frac{1}{4}\right)^{2} \pi^{2}} = 1 + \frac{8 \sinh^{4}\left(\frac{\pi}{2}\right)}{\pi^{2}} + \frac{1}{\pi} 4 \sinh\left(\frac{\pi}{2}\right) \cosh^{3}\left(\frac{\pi}{2}\right) - \frac{2}{\pi^{2}} \sinh^{2}(\pi) - \frac{\sinh(2\pi)}{2\pi}$ $\Rightarrow \sum_{n}^{\infty} \frac{1}{\left(n^{2} + \frac{1}{4}\right)^{2}} = \pi^{2} \operatorname{csch}^{2}\left(\frac{\pi}{2}\right) + 8 \sinh^{2}\left(\frac{\pi}{2}\right) + 4\pi \coth\left(\frac{\pi}{2}\right) \cosh^{2}\left(\frac{\pi}{2}\right) - 8 \cosh^{2}\left(\frac{\pi}{2}\right) - 2\pi \left(2 \cosh^{2}\left(\frac{\pi}{2}\right) - 1\right) \coth\left(\frac{\pi}{2}\right)$ $= \pi^{2} \operatorname{csch}^{2}\left(\frac{\pi}{2}\right) - 8 + 2\pi \coth\left(\frac{\pi}{2}\right)$		$\frac{2}{2} \int_{-\sin h^{2}(x)}^{\pi} dx = \frac{1}{2} \int_{-\cos h(2x)}^{\pi} (\cosh(2x) - 1) dx = \frac{\sinh(2\pi)}{1 - \frac{8 \sinh^{4}\left(\frac{\pi}{2}\right)}{2}} + \sum_{-\cos h^{4}\left(\frac{\pi}{2}\right)}^{\infty} \sinh^{4}\left(\frac{\pi}{2}\right) = n^{2} \sinh^{2}(\pi)$	
$\frac{\sinh(2\pi)}{2\pi} - 1 - \frac{8\sinh^4\left(\frac{\pi}{2}\right)}{\pi^2} - \frac{1}{\pi} 4\sinh\left(\frac{\pi}{2}\right)\cosh^3\left(\frac{\pi}{2}\right) + \frac{2}{\pi^2}\sinh^2(\pi) = -\sum_{n}^{\infty} \frac{\sinh^2\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2}$ $\sum_{n}^{\infty} \frac{\sinh^2\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2} = 1 + \frac{8\sinh^4\left(\frac{\pi}{2}\right)}{\pi^2} + \frac{1}{\pi} 4\sinh\left(\frac{\pi}{2}\right)\cosh^3\left(\frac{\pi}{2}\right) - \frac{2}{\pi^2}\sinh^2(\pi) - \frac{\sinh(2\pi)}{2\pi}$ $\Rightarrow \sum_{n}^{\infty} \frac{1}{\left(n^2 + \frac{1}{4}\right)^2} = \pi^2 \operatorname{csch}^2\left(\frac{\pi}{2}\right) + 8\sinh^2\left(\frac{\pi}{2}\right) + 4\pi \coth\left(\frac{\pi}{2}\right)\cosh^2\left(\frac{\pi}{2}\right) - 8\cosh^2\left(\frac{\pi}{2}\right) - 2\pi\left(2\cosh^2\left(\frac{\pi}{2}\right) - 1\right)\coth\left(\frac{\pi}{2}\right)$ $= \pi^2 \operatorname{csch}^2\left(\frac{\pi}{2}\right) - 8 + 2\pi \coth\left(\frac{\pi}{2}\right)$		$\pi \int_0^{\pi} \int_0^{\pi} (\cos(2x)^2 - 1) dx = 2\pi$ $\pi^2 \int_0^{\pi} \left(n^2 + \frac{1}{4} \right)^2 \pi^2 \left(n^2 + \frac{1}{4} \right)^2 \pi^2$	
$\frac{\sinh(2\pi)}{2\pi} - 1 - \frac{8\sinh^4\left(\frac{\pi}{2}\right)}{\pi^2} - \frac{1}{\pi} 4\sinh\left(\frac{\pi}{2}\right)\cosh^3\left(\frac{\pi}{2}\right) + \frac{2}{\pi^2}\sinh^2(\pi) = -\sum_{n}^{\infty} \frac{\sinh^2\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2}$ $\sum_{n}^{\infty} \frac{\sinh^2\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2} = 1 + \frac{8\sinh^4\left(\frac{\pi}{2}\right)}{\pi^2} + \frac{1}{\pi} 4\sinh\left(\frac{\pi}{2}\right)\cosh^3\left(\frac{\pi}{2}\right) - \frac{2}{\pi^2}\sinh^2(\pi) - \frac{\sinh(2\pi)}{2\pi}$ $\Rightarrow \sum_{n}^{\infty} \frac{1}{\left(n^2 + \frac{1}{4}\right)^2} = \pi^2 \operatorname{csch}^2\left(\frac{\pi}{2}\right) + 8\sinh^2\left(\frac{\pi}{2}\right) + 4\pi \coth\left(\frac{\pi}{2}\right)\cosh^2\left(\frac{\pi}{2}\right) - 8\cosh^2\left(\frac{\pi}{2}\right) - 2\pi\left(2\cosh^2\left(\frac{\pi}{2}\right) - 1\right)\coth\left(\frac{\pi}{2}\right)$ $= \pi^2 \operatorname{csch}^2\left(\frac{\pi}{2}\right) - 8 + 2\pi \coth\left(\frac{\pi}{2}\right)$		$-\frac{8\sinh^4\left(\frac{\pi}{2}\right)}{2} + \sum_{n=1}^{\infty} \frac{\sinh^4\left(\frac{\pi}{2}\right)}{2} + \frac{\sinh^2(\pi)}{4} \sinh^2(\pi)$	
$\sum_{n}^{\infty} \frac{\sinh^2\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2} = 1 + \frac{8\sinh^4\left(\frac{\pi}{2}\right)}{\pi^2} + \frac{1}{\pi} 4\sinh\left(\frac{\pi}{2}\right)\cosh^3\left(\frac{\pi}{2}\right) - \frac{2}{\pi^2}\sinh^2(\pi) - \frac{\sinh(2\pi)}{2\pi}$ $\Rightarrow \sum_{n}^{\infty} \frac{1}{\left(n^2 + \frac{1}{4}\right)^2} = \pi^2 \operatorname{csch}^2\left(\frac{\pi}{2}\right) + 8\sinh^2\left(\frac{\pi}{2}\right) + 4\pi \coth\left(\frac{\pi}{2}\right)\cosh^2\left(\frac{\pi}{2}\right) - 8\cosh^2\left(\frac{\pi}{2}\right) - 2\pi \left(2\cosh^2\left(\frac{\pi}{2}\right) - 1\right)\coth\left(\frac{\pi}{2}\right)$ $= \pi^2 \operatorname{csch}^2\left(\frac{\pi}{2}\right) - 8 + 2\pi \coth\left(\frac{\pi}{2}\right)$		$-\frac{1}{n-1}\left(n^2+\frac{1}{4}\right)^2\pi^2 \left(n^2+\frac{1}{4}\right)\pi^2 \left(n^2+\frac{1}{4}\right)^2\pi^2$	
$\sum_{n}^{\infty} \frac{\sinh^2\left(\frac{\pi}{2}\right)}{\left(n^2 + \frac{1}{4}\right)^2 \pi^2} = 1 + \frac{8\sinh^4\left(\frac{\pi}{2}\right)}{\pi^2} + \frac{1}{\pi} 4\sinh\left(\frac{\pi}{2}\right)\cosh^3\left(\frac{\pi}{2}\right) - \frac{2}{\pi^2}\sinh^2(\pi) - \frac{\sinh(2\pi)}{2\pi}$ $\Rightarrow \sum_{n}^{\infty} \frac{1}{\left(n^2 + \frac{1}{4}\right)^2} = \pi^2 \operatorname{csch}^2\left(\frac{\pi}{2}\right) + 8\sinh^2\left(\frac{\pi}{2}\right) + 4\pi \coth\left(\frac{\pi}{2}\right)\cosh^2\left(\frac{\pi}{2}\right) - 8\cosh^2\left(\frac{\pi}{2}\right) - 2\pi \left(2\cosh^2\left(\frac{\pi}{2}\right) - 1\right)\coth\left(\frac{\pi}{2}\right)$ $= \pi^2 \operatorname{csch}^2\left(\frac{\pi}{2}\right) - 8 + 2\pi \coth\left(\frac{\pi}{2}\right)$		$\frac{\sinh(2\pi)}{1 - 1} = \frac{8\sinh^4\left(\frac{\pi}{2}\right)}{1 - 4\sinh^4\left(\frac{\pi}{2}\right)} = \frac{1}{1 - 4\sinh^4\left(\frac{\pi}{2}\right)} = \frac{1}{1 - 4\sinh^4\left(\frac{\pi}{2}\right)} = \frac{1}{1 - 2\sinh^4\left(\frac{\pi}{2}\right)} = \frac{1}{1 - 2\sinh^4\left(\frac{\pi}{2}\right)$	
$\Rightarrow \sum_{n}^{\infty} \frac{1}{\left(n^2 + \frac{1}{4}\right)^2} = \pi^2 \operatorname{csch}^2\left(\frac{\pi}{2}\right) + 8 \sinh^2\left(\frac{\pi}{2}\right) + 4\pi \coth\left(\frac{\pi}{2}\right) \cosh^2\left(\frac{\pi}{2}\right) - 8 \cosh^2\left(\frac{\pi}{2}\right) - 2\pi \left(2 \cosh^2\left(\frac{\pi}{2}\right) - 1\right) \coth\left(\frac{\pi}{2}\right)$ $= \pi^2 \operatorname{csch}^2\left(\frac{\pi}{2}\right) - 8 + 2\pi \coth\left(\frac{\pi}{2}\right)$		2π π^2 $\pi^{15\text{IM}}(2)^{25\text{SM}}(2) + \pi^2^{25\text{IM}}(n) = \sum_{n} \left(n^2 + \frac{1}{4}\right)^2 \pi^2$	
$\Rightarrow \sum_{n}^{\infty} \frac{1}{\left(n^2 + \frac{1}{4}\right)^2} = \pi^2 \operatorname{csch}^2\left(\frac{\pi}{2}\right) + 8 \sinh^2\left(\frac{\pi}{2}\right) + 4\pi \coth\left(\frac{\pi}{2}\right) \cosh^2\left(\frac{\pi}{2}\right) - 8 \cosh^2\left(\frac{\pi}{2}\right) - 2\pi \left(2 \cosh^2\left(\frac{\pi}{2}\right) - 1\right) \coth\left(\frac{\pi}{2}\right)$ $= \pi^2 \operatorname{csch}^2\left(\frac{\pi}{2}\right) - 8 + 2\pi \coth\left(\frac{\pi}{2}\right)$		$\sum_{n=0}^{\infty} \frac{\sinh^2(\frac{\pi}{2})}{\sinh^2(\frac{\pi}{2})} = 1 + \frac{8\sinh^4(\frac{\pi}{2})}{\sinh^4(\frac{\pi}{2})} + \frac{1}{4}\sinh(\frac{\pi}{2})\cosh^3(\frac{\pi}{2}) - \frac{2}{\sinh^2(\pi)} + \frac{\sinh(2\pi)}{\sinh(2\pi)}$	
$= \pi^2 \operatorname{csch}^2\left(\frac{\pi}{2}\right) - 8 + 2\pi \operatorname{coth}\left(\frac{\pi}{2}\right)$		$\sum_{n} \left(n^2 + \frac{1}{4}\right)^2 \pi^2 \qquad \pi^2 \qquad \pi^{13444} (2)^{13344} (2)^$	
$= \pi^2 \operatorname{csch}^2\left(\frac{\pi}{2}\right) - 8 + 2\pi \operatorname{coth}\left(\frac{\pi}{2}\right)$		$\Rightarrow \sum_{n=0}^{\infty} \frac{1}{1+2n^2} = \pi^2 \operatorname{csch}^2\left(\frac{\pi}{2}\right) + 8 \sinh^2\left(\frac{\pi}{2}\right) + 4\pi \coth\left(\frac{\pi}{2}\right) \cosh^2\left(\frac{\pi}{2}\right) - 8 \cosh^2\left(\frac{\pi}{2}\right) - 2\pi \left(2 \cosh^2\left(\frac{\pi}{2}\right) - 1\right) \coth\left(\frac{\pi}{2}\right)$	
		\setminus 4/	
$\Rightarrow \left[\sum_{n=1}^{\infty} \frac{1}{\left(n^2 + \frac{1}{4}\right)^2} = \pi^2 \operatorname{csch}^2\left(\frac{\pi}{2}\right) + 2\pi \operatorname{coth}\left(\frac{\pi}{2}\right) - 8 \right]$			
$\frac{1}{n}\left(n^2+\frac{1}{4}\right)$		$\Rightarrow \left \sum_{n=1}^{\infty} \frac{1}{\left(\frac{1}{2} - 1 \right)^2} = \pi^2 \operatorname{csch}^2\left(\frac{\pi}{2}\right) + 2\pi \operatorname{coth}\left(\frac{\pi}{2}\right) - 8 \right $	
		$\frac{1}{n}\left(n^2+\frac{1}{4}\right)$	

سری فوریه تابع f(x) را در بازه $[0,2\pi]$ بدست آورید و با استفاده از آن حاصل سری A و B را بدس $f(x) = \cosh(x)$, $A = \sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$, $B = \sum_{n=1}^{\infty} \frac{1}{(n^2 + 1)^2}$ $a_0 = \frac{1}{2\pi} \int_0^{2\pi} \cosh(x) \, dx = \frac{\sinh(2\pi)}{2\pi}$ $a_n = \frac{2}{2\pi} \int_0^{2\pi} \cosh(x) \cos(nx) \, dx$ $\int \cosh(x)\cos(nx) dx = \frac{1}{n}\cosh(x)\sin(nx) - \frac{1}{n}\int \sinh(x)\sin(nx) dx$ $= \frac{1}{n}\cosh(x)\sin(nx) - \frac{1}{n}\left(-\frac{1}{n}\sinh(x)\cos(nx) + \frac{1}{n}\int\cosh(x)\cos(nx)\,dx\right)$ $\Rightarrow a_n = \frac{1}{\pi} \left[\frac{n \cosh(x) \sin(nx) + \sinh(x) \cos(nx)}{n^2 + 1} \right]_0^{2\pi} = \frac{\sinh(2\pi)}{\pi (n^2 + 1)}$ $b_n = \frac{2}{2\pi} \int_0^{2\pi} \cosh(x) \sin(nx) dx$ $\int \cosh(x)\sin(nx)\,dx = -\frac{1}{n}\cosh(x)\cos(nx) + \frac{1}{n}\int \sinh(x)\cos(nx)\,dx$ $= -\frac{1}{n}\cosh(x)\cos(nx) + \frac{1}{n}\left(\frac{1}{n}\sinh(x)\sin(nx) - \frac{1}{n}\int\cosh(x)\sin(nx)\,dx\right)$ $\Rightarrow \int \cosh(x) \sin(nx) \, dx = \frac{-n \cosh(x) \cos(nx) + \sinh(x) \sin(nx)}{n^2 + 1}$ $\Rightarrow b_n = \frac{1}{\pi} \left[\frac{-n \cosh(x) \cos(nx) + \sinh(x) \sin(nx)}{n^2 + 1} \right]_0^{2\pi} = \frac{-n(\cosh(2\pi) - 1)}{\pi(n^2 + 1)} = \frac{-2n \sinh^2(\pi)}{\pi(n^2 + 1)}$ $\Rightarrow f(x) = \frac{\sinh(2\pi)}{2\pi} + \sum_{n=1}^{\infty} \left(\frac{\sinh(2\pi)}{\pi(n^2+1)}\right) \cos(nx) + \left(-\frac{2n\sinh^2(\pi)}{\pi(n^2+1)}\right) \sin(nx)$ $f(2\pi) = \frac{\cosh(2\pi) + 1}{2} = \cosh^2(\pi) = \frac{\sinh(2\pi)}{2\pi} + \sum_{n=1}^{\infty} \frac{\sinh(2\pi)}{\pi(n^2 + 1)} \Longrightarrow \left| \sum_{n=1}^{\infty} \frac{1}{n^2 + 1} = \frac{\pi}{2} \coth(\pi) - \frac{1}{2} \right|$ $\frac{1}{\pi} \int_0^{2\pi} \cosh^2(x) \, dx = 1 + \frac{\sinh(4\pi)}{4\pi} = \frac{\sinh^2(2\pi)}{2\pi^2} + \sum_{n=1}^{\infty} \frac{\sinh^2(2\pi)}{\pi^2(n^2+1)^2} + \frac{4n^2\sinh^4(\pi)}{\pi^2(n^2+1)^2}$ $\Rightarrow \sum_{n=1}^{\infty} \frac{4 \sinh^2(\pi) \cosh^2(\pi) + 4(n^2 + 1) \sinh^4(\pi) - 4 \sinh^4(\pi)}{\pi^2 (n^2 + 1)^2} = \sum_{n=1}^{\infty} \frac{4 \sinh^2(\pi)}{\pi^2 (n^2 + 1)^2} + \sum_{n=1}^{\infty} \frac{4 \sinh^4(\pi)}{\pi^2 (n^2 + 1)} = 1 + \frac{\sinh(4\pi)}{4\pi} - \frac{\sinh^2(2\pi)}{2\pi^2} + \frac{\sinh^2(\pi)}{4\pi} = \frac{\sinh^2(\pi)}{4\pi} + \frac{\sinh^2(\pi)}{4\pi} +$ $\Rightarrow \sum_{n=1}^{\infty} \frac{4 \sinh^2(\pi)}{\pi^2 (n^2 + 1)^2} = 1 + \frac{\sinh(4\pi)}{4\pi} - \frac{\sinh^2(2\pi)}{2\pi^2} - \frac{2 \sinh^3(\pi) \cosh(\pi)}{\pi} + \frac{2 \sinh^4(\pi)}{\pi^2}$ $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n^2+1)^2} = \frac{\pi^2}{4} \operatorname{csch}^2(\pi) + \frac{\pi}{4} \operatorname{cosh}(2\pi) \operatorname{coth}(\pi) - \frac{1}{2} \operatorname{cosh}^2(\pi) - \frac{\pi}{4} \sinh(2\pi) + \frac{1}{2} \sinh^2(\pi)$ $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n^2+1)^2} = \frac{\pi^2}{4} \operatorname{csch}^2(\pi) + \frac{\pi}{4} \coth(\pi) - \frac{1}{2}$

 $f(x) = \cos(x), \quad A = \sum_{n=1}^{\infty} \frac{1}{n^2 - \frac{1}{16}}, \quad B = \sum_{n=1}^{\infty} \frac{1}{(n^2 - \frac{1}{16})^2}$ $a_0 = \frac{1}{\pi} \int_{0}^{\frac{\pi}{2}} \cos(x) \, dx = \frac{2}{\pi} \sin\left(\frac{\pi}{2}\right) = \frac{2}{\pi}$ $a_1 = \frac{2}{\pi} \int_{0}^{\frac{\pi}{2}} \cos(x) \, dx = \frac{2}{\pi} \sin\left(\frac{\pi}{2}\right) = \frac{2}{\pi}$ $a_2 = \frac{2}{\pi} \int_{0}^{\frac{\pi}{2}} \cos(x) \cos(4nx) \, dx = \frac{2}{\pi} \int_{0}^{\frac{\pi}{2}} (\cos((4n+1)x) + \cos((4n-1)x)) \, dx = -\frac{4}{(16n^2 - 1)\pi}$ $b_1 = \frac{2}{\pi} \int_{0}^{\frac{\pi}{2}} \cos(x) \sin(4nx) \, dx = \frac{2}{\pi} \int_{0}^{\frac{\pi}{2}} (\sin((4n+1)x) + \sin((4n-1)x)) \, dx = \frac{16n}{(16n^2 - 1)\pi}$ $\Rightarrow \int_{0}^{\frac{\pi}{2}} \cos(x) \sin(4nx) \, dx = \frac{2}{\pi} \int_{0}^{\frac{\pi}{2}} (\sin((4n+1)x) + \sin((4n-1)x)) \, dx = \frac{16n}{(16n^2 - 1)\pi}$ $\Rightarrow \int_{0}^{\frac{\pi}{2}} \cos(x) \sin(4nx) \, dx = \frac{2}{\pi} \int_{0}^{\frac{\pi}{2}} (\sin((4n+1)x) + \sin((4n-1)x)) \, dx = \frac{16n}{(16n^2 - 1)\pi}$ $\Rightarrow \int_{0}^{\frac{\pi}{2}} \cos^2(x) \, dx = 1 = \frac{8}{\pi^2} + \sum_{n=1}^{\infty} \frac{1}{16n^2 (n^2 - \frac{1}{16})^2} + \frac{n^2}{\pi^2 (n^2 - \frac{1}{16})^2} = \frac{8}{\pi^2} + \sum_{n=1}^{\infty} \frac{1}{16n^2 (n^2 - \frac{1}{16})^2} + \frac{1}{\pi^2 (n^2 - \frac{1}{16})^2} + \frac{1}{$

$$B(\omega) = 0, \qquad A(\omega) = \frac{2}{\pi} \int_0^\infty \frac{\pi}{2} e^{-k|x|} \cos(\omega x) \, dx = \mathcal{L}[\cos(\omega x)]_{s=k} = \frac{k}{k^2 + \omega^2}$$

$$\Rightarrow f(x) = \int_0^\infty \frac{k}{k^2 + \omega^2} \cos(\omega x) d\omega$$

$$\frac{1}{\omega^2 + 1024} = \frac{1}{i64} \left(\frac{1}{\omega^2 - i32} - \frac{1}{\omega^2 + i32} \right) = \frac{1}{i64} \left(\frac{1}{\omega^2 + (4 - i4)^2} - \frac{1}{\omega^2 + (4 + i4)^2} \right)$$

$$g(x) = \int_0^\infty \frac{1}{i64} \left(\frac{1}{\omega^2 + (4 - i4)^2} - \frac{1}{\omega^2 + (4 + i4)^2} \right) \cos(\omega x) \, dx = \frac{1}{i64} \left(\frac{\pi}{8(1 - i)} e^{-4(1 - i)|x|} - \frac{\pi}{8(1 + i)} e^{-4(1 + i)|x|} \right)$$

$$= \frac{\pi}{i1024} \left((1 + i)e^{-4|x|} e^{i4|x|} - (1 - i)e^{-4|x|} e^{-i4|x|} \right) = \frac{\pi e^{-4|x|}}{i1024} (i2 \sin(4|x|) + i2 \cos(4|x|))$$

$$= \frac{\pi e^{-4|x|}}{512} (\sin(4|x|) + \cos(4|x|))$$

انتگرال فور به تابع f(x) را محاسبه کنید (۱–۲

$$f(x) = e^{-2|x|} \cos(x)$$

$$B(\omega) = 0, \qquad A(\omega) = \frac{2}{\pi} \int_0^\infty e^{-2|x|} \cos(x) \cos(\omega x) dx = \frac{1}{\pi} \int_0^\infty e^{-2x} \left(\cos((\omega + 1)x) + \cos((\omega - 1)x)\right) dx$$

$$= \frac{1}{\pi} \left(\mathcal{L} \left[\cos((\omega + 1)x)\right]_{s=2} + \mathcal{L} \left[\cos((\omega - 1)x)\right]_{s=2} \right) = \frac{1}{\pi} \left(\frac{2}{4 + (\omega + 1)^2} + \frac{2}{4 + (\omega - 1)^2} \right) = \frac{4(5 + \omega^2)}{\pi (25 + 6\omega^2 + \omega^4)}$$

$$\Rightarrow f(x) = \int_0^\infty \frac{4(5 + \omega^2)}{\pi (25 + 6\omega^2 + \omega^4)} \cos(\omega x) d\omega$$

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$$f(x) = e^{-2|x|}\sin(x)$$

$$A(\omega) = 0, \qquad B(\omega) = \frac{2}{\pi} \int_0^{\infty} e^{-2|x|} \sin(x) \sin(\omega x) \, dx = \frac{1}{\pi} \int_0^{\infty} e^{-2x} \left(\cos((\omega - 1)x) - \cos((\omega + 1)x) \right) dx$$

$$= \frac{1}{\pi} \left(\mathcal{L} \left[\cos((\omega - 1)x) \right]_{s=2} - \mathcal{L} \left[\cos((\omega + 1)x) \right]_{s=2} \right) = \frac{1}{\pi} \left(\frac{2}{4 + (\omega - 1)^2} - \frac{2}{4 + (\omega + 1)^2} \right) = \frac{8\omega}{\pi (25 + 6\omega^2 + \omega^4)}$$

$$\Rightarrow f(x) = \int_0^{\infty} \frac{8\omega}{\pi (25 + 6\omega^2 + \omega^4)} \sin(\omega x) \, d\omega$$

۰-۳) معادله دیفرانسیل زیر را به کمک تبدیل فوریه حل کنید.

$$y''' + 2y'' - 1y' - 2y = -e^{2x}u(-x)$$

$$-i\omega^{3}Y(\omega) - 2\omega^{2}Y(\omega) - i\omega Y(\omega) - 2Y(\omega) = -\frac{1}{2 - i\omega}$$

$$\Rightarrow Y(\omega) = \frac{1}{(2 + i\omega + 2\omega^{2} + i\omega^{3})(2 - i\omega)} = \frac{1}{(1 + \omega^{2})(2 + i\omega)(2 - i\omega)} = \frac{1}{(1 + \omega^{2})(4 + \omega^{2})} = \frac{\frac{1}{3}}{1 + \omega^{2}} - \frac{\frac{1}{3}}{4 + \omega^{2}}$$

$$= \frac{1}{61 + \omega^{2}} - \frac{1}{12} \frac{4}{4 + \omega^{2}}$$

$$\Rightarrow y(x) = \frac{1}{6}e^{-|x|} - \frac{1}{12}e^{-2|x|}$$

$$e^{-2|x|}$$

$$e^{-2|x|}$$

$$e^{-2|x|}$$

$$e^{-2|x|}$$

$$e^{-2|x|}$$

$$e^{-2|x|}$$

$$y''' + y'' - 9y' - 9y = -e^{x}u(-x)$$

$$-i\omega^{3}Y(\omega) - \omega^{2}Y(\omega) - i9\omega Y(\omega) - 9Y(\omega) = -\frac{1}{1 - i\omega}$$

$$\Rightarrow Y(\omega) = \frac{1}{(9 + i9\omega + \omega^{2} + i\omega^{3})(2 - i\omega)} = \frac{1}{(9 + \omega^{2})(1 + i\omega)(1 - i\omega)} = \frac{1}{(9 + \omega^{2})(1 + \omega^{2})} = \frac{\frac{1}{8}}{1 + \omega^{2}} - \frac{\frac{1}{8}}{9 + \omega^{2}}$$

$$\Rightarrow y(x) = \frac{1}{16}e^{-|x|} - \frac{1}{48}e^{-3|x|}$$

۲-۲) معادله دیفرانسیل زیر را به کمک تبدیل فوریه حل کنید.

$$y''' + 3y'' - 4y' - 12y = -e^{3x}u(-x)$$

$$-i\omega^{3}Y(\omega) - 3\omega^{2}Y(\omega) - i4\omega Y(\omega) - 12Y(\omega) = -\frac{1}{3 - i\omega}$$

$$\Rightarrow Y(\omega) = \frac{1}{(12 + i4\omega + 3\omega^{2} + i\omega^{3})(2 - i\omega)} = \frac{1}{(4 + \omega^{2})(3 + i\omega)(3 - i\omega)} = \frac{1}{(4 + \omega^{2})(9 + \omega^{2})} = \frac{\frac{1}{5}}{4 + \omega^{2}} - \frac{\frac{1}{5}}{9 + \omega^{2}}$$

$$= \frac{1}{20} \frac{4}{4 + \omega^{2}} - \frac{1}{30} \frac{6}{9 + \omega^{2}}$$

$$\Rightarrow y(x) = \frac{1}{20} e^{-2|x|} - \frac{1}{30} e^{-3|x|}$$

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$$\begin{array}{ll} u_{tt} = 4u_{xx} - x\cos(t)\,, & 0 \leq x \leq 1, & 0 \leq \\ u(0,t) = \sin(t)\,, & u(1,t) = \cos(t) \\ u(x,0) = x^2 + x, & u_t(x,0) = 1 - 3x \end{array}$$

$$u(x,t) = w(x,t) + v(x,t)$$

$$w(x,t) = (1-x)\sin(t) + x\cos(t)$$

$$\Rightarrow v_{tt} = 4u_{xx} + (1-x)\sin(t)$$

$$v(0,t) = 0, \quad v(1,t) = 0$$

$$v(x,0) = x^{2}, \quad u_{t}(x,0) = -2x$$

$$\Rightarrow v(x,t) = \sum_{n=1}^{\infty} T_{n}(t)\sin(n\pi x) \Rightarrow \sum_{n=1}^{\infty} \left(\ddot{T}_{n}(t) + 4n^{2}\pi^{2}T_{n}(t)\right)\sin(n\pi x) = (1-x)\sin(t)$$

$$\Rightarrow \ddot{T}_{n}(t) + 4n^{2}\pi^{2}T_{n}(t) = 2\sin(t)\int_{0}^{1} (1-x)\sin(n\pi x) dx = \frac{2}{n\pi}\sin(t)$$

$$\Rightarrow T_{n}(t) = a_{n}\cos(2n\pi t) + b_{n}\sin(2n\pi t) + \frac{\frac{2}{n\pi}}{4n^{2}\pi^{2} - 1}\sin(t)$$

$$v(x,t) = \sum_{n=1}^{\infty} \left(a_{n}\cos(2n\pi t) + b_{n}\sin(2n\pi t) + \frac{\frac{2}{n\pi}}{4n^{2}\pi^{2} - 1}\sin(t)\right)\sin(n\pi x)$$

$$v(x,t) = \sum_{n=1}^{\infty} \left(a_{n}\cos(2n\pi t) + b_{n}\sin(2n\pi t) + \frac{2}{n\pi}\sin(t)\right)\sin(n\pi x)$$

$$v(x,0) = \sum_{n=1}^{\infty} a_n \sin(n\pi x) = x^2$$

$$\Rightarrow a_n = 2 \int_0^1 x^2 \sin(n\pi x) \, dx = \frac{2(-1)^{n+1}}{n\pi} - \frac{4((-1)^{n+1} + 1)}{n^3 \pi^3}$$

$$v_t(x,0) = \sum_{n=1}^{\infty} \left(2b_n n\pi + \frac{\frac{2}{n\pi}}{4n^2 \pi^2 - 1} \right) \sin(n\pi x) = -2x$$

$$\Rightarrow 2b_n n\pi + \frac{\frac{2}{n\pi}}{4n^2 \pi^2 - 1} = -4 \int_0^1 x \sin(n\pi x) \, dx = \frac{4(-1)^n}{n\pi}$$

$$\Rightarrow b_n = \frac{2(-1)^n}{n^2 \pi^2} - \frac{\frac{1}{n^2 \pi^2}}{4n^2 \pi^2 - 1}$$

$$\Rightarrow u(x,t) = (1-x)\sin(t) + x\cos(t) + \sum_{n=1}^{\infty} \left(\frac{2(-1)^{n+1}}{n\pi} - \frac{4((-1)^{n+1}+1)}{n^3\pi^3} \right) \cos(2n\pi t) + \left(\frac{2(-1)^n}{n^2\pi^2} - \frac{\frac{1}{n^2\pi^2}}{4n^2\pi^2 - 1} \right) \sin(2n\pi t) + \frac{\frac{2}{n\pi}}{4n^2\pi^2 - 1} \sin(t) \sin(n\pi x)$$

۱-۴) معادله موج زیر را حل کنید.

$$u_{tt} = 4u_{xx} - \cos(t), \quad 0 \le x \le 1, \quad 0 \le t$$

 $u_x(0,t) = \sin(t), \quad u(1,t) = \cos(t)$
 $u(x,0) = x^2 + 1, \quad u_t(x,0) = -1$

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$$u(x,t) = w(x,t) + v(x,t)$$

$$w(x,t) = (x-1)\sin(t) + \cos(t)$$

$$\Rightarrow v_{tt} = 4u_{xx} + (x-1)\sin(t)$$

$$v_{x}(0,t) = 0, \quad v(1,t) = 0$$

$$v(x,0) = x^{2}, \quad u_{t}(x,0) = -x$$

$$\Rightarrow v(x,t) = \sum_{n=1}^{\infty} T_{n}(t)\cos\left(\frac{(2n-1)\pi}{2}x\right) \Rightarrow \sum_{n=1}^{\infty} \left(\ddot{T}_{n}(t) + (2n-1)^{2}\pi^{2}T_{n}(t)\right)\cos\left(\frac{(2n-1)\pi}{2}x\right) = (x-1)\sin(t)$$

$$\Rightarrow \ddot{T}_{n}(t) + (2n-1)^{2}\pi^{2}T_{n}(t) = 2\sin(t)\int_{0}^{1} (x-1)\cos\left(\frac{(2n-1)\pi}{2}x\right)dx = -\frac{8}{(2n-1)^{2}\pi^{2}}\sin(t)$$

$$\Rightarrow T_n(t) = a_n \cos((2n-1)\pi t) + b_n \sin((2n-1)\pi t) - \frac{\frac{\alpha}{(2n-1)^2\pi^2}}{(2n-1)^2\pi^2 - 1} \sin(t)$$

$$v(x,t) = \sum_{n=1}^{\infty} \left(a_n \cos((2n-1)\pi t) + b_n \sin((2n-1)\pi t) - \frac{\frac{\alpha}{(2n-1)^2\pi^2}}{(2n-1)^2\pi^2 - 1} \sin(t) \right) \cos\left(\frac{(2n-1)\pi}{2}x\right)$$

$$v(x,0) = \sum_{n=1}^{\infty} a_n \cos\left(\frac{(2n-1)\pi}{2}x\right) = x^2$$

$$\Rightarrow a_n = 2 \int_0^1 x^2 \cos\left(\frac{(2n-1)\pi}{2}x\right) dx = \frac{32(-1)^n}{(2n-1)^3\pi^3} + \frac{4(-1)^{n+1}}{(2n-1)\pi}$$

$$v_t(x,0) = \sum_{n=1}^{\infty} \left(b_n(2n-1)\pi - \frac{\frac{8}{(2n-1)^2\pi^2}}{(2n-1)^2\pi^2 - 1} \right) \cos\left(\frac{(2n-1)\pi}{2}x\right) = -x$$

 $\Rightarrow v(x,t) = \sum_{n=0}^{\infty} c_n e^{-4(2n-1)^2 t} \cos((2n-1)x) + \left(\frac{1}{4}t - \frac{1}{16}\right) \cos(x)$

$$v(x,0) = \sum_{n=1}^{\infty} c_n \cos((2n-1)x) - \frac{1}{16} \cos(x) = \cos(3x) \Rightarrow c_n = \delta[n-2] + \frac{1}{16} \delta[n-1] = \begin{cases} \frac{1}{16} & n=1\\ 1 & n=2\\ 0 & n \notin \{1,2\} \end{cases}$$

$$\Rightarrow v(x,t) = e^{-3\kappa t} \cos(3x) + \frac{1}{16} e^{-4t} \cos(x) + \left(\frac{1}{4}t - \frac{1}{16}\right) \cos(x)$$

$$\Rightarrow \frac{1}{2} (x,t) = \left(x - \frac{\pi}{2}\right) t + 1 + e^{-24\kappa t} \cos(3x) + \frac{1}{16} e^{-4t} \cos(x) + \left(\frac{1}{4}t - \frac{1}{16}\right) \cos(x)$$

$$u_t = u_{xx} + 2xt + t \sin(x), \quad 0 \le x \le \frac{\pi}{2}, \quad 0 \le t$$

$$u(0,t) = 1, \quad u_x \left(\frac{\pi}{2}, t\right) = t^2$$

$$u(x,0) = \sin(3x) + 1$$

$$u(x,t) = \psi(x,t) + \psi(x,t)$$

$$\psi(0,t) = 1 + xt^2$$

$$\Rightarrow v_x = t + t \sin(x)$$

$$v(0,0) = 0, \quad v(\frac{\pi}{2},t) = 0$$

$$v(x,0) = \sin(3x)$$

$$\Rightarrow v(x,t) = \sum_{n=1}^{\infty} T_n(t) \sin((2n-1)x) \Rightarrow \sum_{n=1}^{\infty} \left(T_n(t) + (2n-1)^2 T_n(t)\right) \sin((2n-1)x) - t \sin(x)$$

$$\Rightarrow \tilde{T}_n(t) + (2n-1)^2 T_n(t) = t \delta[n-1] = \begin{cases} t & n=1\\ 0 & n=1 \end{cases} \Rightarrow v(x,t) = \sum_{n=1}^{\infty} (t_n(t) + (2n-1)^2 T_n(t)) \sin((2n-1)x) + t \sin(x)$$

$$\Rightarrow v(x,t) = \sum_{n=1}^{\infty} c_n \sin((2n-1)x) - \sin(x) = \sin(3x) \Rightarrow c_n = \delta[n-2] + \delta[n-1] = \begin{cases} 1 & n=1\\ 1 & n=2\\ 0 & n \ne 1 \end{cases}$$

$$\Rightarrow v(x,t) = e^{-3t} \sin(3x) + e^{-t} \sin(x) + (t-1) \sin(x)$$

$$\Rightarrow v(x,t) = e^{-3t} \sin(3x) + e^{-t} \sin(x) + (t-1) \sin(x)$$

$$\Rightarrow u(x,t) = u_x + t \sin(2x), \quad 0 \le x \le \pi, \quad 0 \le t$$

$$u(x,t) = u_x + t \sin(2x), \quad 0 \le x \le \pi, \quad 0 \le t$$

$$u(x,t) = u_x + t \sin(2x), \quad 0 \le x \le \pi, \quad 0 \le t$$

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$$u(x,t) = u_x + t \sin(2x), \quad 0 \le x \le \pi, \quad 0 \le t$$

$$u(x,t) = u_x + t \sin(2x), \quad 0 \le x \le \pi, \quad 0 \le t$$

$$u(x,t) = u_x + t \sin(2x), \quad 0 \le x \le \pi, \quad 0 \le t$$

$$u(x,t) = u_x + t \sin(2x), \quad 0 \le x \le \pi, \quad 0 \le t$$

$$u(x,t) = u_x + t \sin(2x), \quad 0 \le x \le \pi, \quad 0 \le t$$

$$u(x,t) = u_x + t \sin(2x), \quad 0 \le x \le \pi, \quad 0 \le t$$

$$u(x,t) = v_x + t \sin(2x), \quad 0 \le x \le \pi, \quad 0 \le t$$

$$u(x,t) = u_x + t \sin(2x), \quad 0 \le x \le \pi, \quad 0 \le t$$

$$u(x,t) = u_x + t \sin(2x), \quad 0 \le t$$

$$u(x,t) = u_x + t \sin(2x), \quad 0 \le t$$

$$u(x,t) = u_x + t \sin(2x), \quad 0 \le t$$

$$u(x,t) = u_x + t \sin(2x), \quad 0 \le t$$

$$u(x,t) = u_x + t \sin(2x), \quad 0 \le t$$

$$u(x,t) = u_x + t \sin(2x), \quad 0 \le t$$

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