# درس ریاضی مهندسی

| پاسخ کوئیز ۵ | نیم‌سال دوم<br>۱۴۰۰-۱۴۰۱ |
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**پاسخ سوال ۱ :**

$x=0$

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} = \dfrac{1}{c^2}\dfrac{\partial u}{\partial t} \\ \left.\dfrac{\partial u}{\partial x}\right|_{x=0} = 0 \\ u(x,0) = e^{-\alpha x^2} \end{cases}$$

$$u(x,t) = X(x)T(t) \rightarrow X''T = \frac{1}{c^2}T'X \rightarrow \frac{X''}{X} = \frac{1}{c^2}\frac{T'}{T} = -k^2$$

$$\Rightarrow X''(x) + k^2 X(x) = 0 \Rightarrow X(x) = A\sin kx + B\cos kx$$

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = 0 \rightarrow A = 0 \Rightarrow X(x) = B\cos kx$$

$$T' + k^2c^2T = 0 \Rightarrow T(t) = C e^{-k^2c^2t}$$

$$\Rightarrow u(x,t) = \int_0^\infty D(k)\cos kx\, e^{-k^2c^2t}\,dk$$

$$u(x,0) = \int_0^\infty D(k)\cos kx\,dk = e^{-\alpha x^2} \Rightarrow D(k) = \frac{2}{\pi}\int_0^\infty e^{-\alpha x^2}\cos kx\,dx \underset{*}{=} \frac{1}{\sqrt{\pi\alpha}}e^{-k^2/4\alpha}$$

$$\Rightarrow u(x,t) = \int_0^\infty \frac{1}{\sqrt{\pi\alpha}}e^{-k^2/4\alpha}e^{-k^2c^2t}\cos kx\,dk = \frac{1}{\sqrt{1+4\alpha c^2t}}e^{-\left(\frac{\alpha x^2}{1+4\alpha c^2t}\right)}$$

$$* \int_0^\infty e^{-\alpha x^2}\cos kx\,dx \xlongequal[\frac{k}{\sqrt{\alpha}}=t]{\alpha x^2=u^2} \int_0^\infty e^{-u^2}\cos(tu)\frac{du}{\sqrt{\alpha}} = \frac{1}{\sqrt{\alpha}}\underbrace{\int_0^\infty e^{-u^2}\cos(tu)\,du}_{f(t)}$$

$$\cdot\ f'(t) = \frac{\partial}{\partial t}f(t) = \frac{-1}{\sqrt{\alpha}}\int_0^\infty u e^{-u^2}\sin(tu)\,du \overset{\text{ج.ب.ج}}{=} -\frac{t}{2}f(t) \Rightarrow f(t) = Ce^{-t^2/4} \xrightarrow{f(0)=\frac{1}{\sqrt{\alpha}}\int_0^\infty e^{-u^2}du=\frac{\sqrt{\pi}}{2\sqrt{\alpha}}} f(t) = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}e^{-t^2/4}$$

$$f'(t) = \frac{-1}{\sqrt{\alpha}}\int_0^\infty u e^{-u^2}\sin tu\,du = \frac{-1}{\sqrt{\alpha}}\left(\left.-\frac{e^{-u^2}}{2}\sin tu\right|_0^\infty + \int_0^\infty \frac{e^{-u^2}}{2}t\cos tu\,du\right) = \frac{-t}{2\sqrt{\alpha}}\int_0^\infty e^{-u^2}\cos tu\,du \Rightarrow f'(t) = -\frac{t}{2}f(t)$$

موفق باشید - خان چرلی