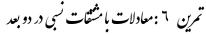


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_1

$$\frac{\partial^{\mathsf{T}} u}{\partial x^{\mathsf{T}}} + \frac{\partial^{\mathsf{T}} u}{\partial y^{\mathsf{T}}} = x + \mathsf{T} y$$

$$u(\cdot, y) = y; u(\cdot, y) = \cdot$$

$$u_{\nu}(x,\cdot) = x ; u_{\nu}(x,\cdot) = x + \cdot$$

همگن سازی :

$$u(x,y) = v(x,y) + ax + b \rightarrow u(x,y) = v(x,y) + x - xy - y^{\mathsf{T}}$$

$$v_{xx} + v_{yy} = x + \Upsilon y \rightarrow v(\cdot, y) = v(\cdot, y) = \cdot;$$

$$v_y(x, \cdot) = \Upsilon x ; v_y(x, \cdot) = \Upsilon x + \Upsilon$$

جواب حدسی

$$v(x,y) = \sum_{n=1}^{\infty} G(y) \sin(n\pi x) ; v_{xx} = \sum_{n=1}^{\infty} G(y) (-n^{\tau} \pi^{\tau}) \sin(n\pi x)$$

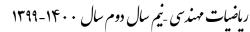
$$v_{yy} = \sum_{n=1}^{\infty} \ddot{G}(y) \sin(n\pi x)$$

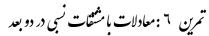
$$\sum_{n=1}^{\infty} (\ddot{G}(y) - n^{\mathsf{T}} \pi^{\mathsf{T}} G(y)) \sin(n\pi x) = x + \mathsf{T} y$$

$$\ddot{G}(y) - n^{\mathsf{T}} \pi^{\mathsf{T}} G(y) = \mathsf{T} \int_{0}^{\mathsf{T}} (x + \mathsf{T} y) \sin(n\pi x) dx$$

$$G(y) = A \sinh(n\pi y) + B \cosh(n\pi y) + \frac{-7}{(n\pi)^r} [(-1)^{n+1} + 7y(1 - (-1)^n)]$$











$$v(x,y) = \sum_{n=1}^{\infty} \{A \sinh(n\pi y) + B \cosh(n\pi y) + \frac{-7}{(n\pi)^{r}} ((-1)^{n+1} + 7y(1)^{n+1} + (-1)^{n})\} \sin(n\pi x)$$

$$v_{y}(x,\cdot) = \sum_{n=1}^{\infty} (An\pi - \frac{\mathfrak{f}}{(n\pi)^{\mathfrak{f}}}(1 - (-1)^{n})) \sin(n\pi x) = \mathfrak{f}x$$

$$A = \frac{1}{n\pi} \left\{ \frac{\mathfrak{f}}{(n\pi)^{\mathfrak{f}}} \left(1 - (-1)^{n} \right) + \mathfrak{f} \int_{-1}^{1} \mathsf{f} x \sin(n\pi x) dx \right\}$$
$$= \frac{1}{n\pi} \left\{ \frac{\mathfrak{f}}{(n\pi)^{\mathfrak{f}}} \left(1 - (-1)^{n} \right) + \frac{\mathfrak{f}}{n\pi} (-1)^{n+1} \right\}$$

$$v_{y}(x, 1) = \sum_{n=1}^{\infty} (An\pi \cosh(n\pi) + Bn\pi \sinh(n\pi) - \frac{\mathfrak{f}}{(n\pi)^{\mathfrak{r}}} (1)$$
$$-(-1)^{n}) \sin(n\pi x) = \mathfrak{f} \int_{-\infty}^{\infty} (\mathfrak{f}(x+\mathfrak{r}) \sin(n\pi x) dx$$

با حل انتگرال بالا و جاگذاری مقدار A، ضریب B نیز به دست می آید.

_٢

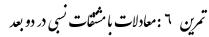
$$\frac{\partial^{\mathsf{T}} u}{\partial x^{\mathsf{T}}} + \frac{\partial^{\mathsf{T}} u}{\partial y^{\mathsf{T}}} = \cdot$$

$$X(x) = cos\left(\frac{n\pi}{L}x\right); n = \cdot, 1, \gamma, \dots$$

$$Y(y) = A \sinh\left(\frac{n\pi}{H}y\right) + B \cosh\left(\frac{n\pi}{H}y\right); \rightarrow B = \cdot;$$



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$$U(x,y) = Cy + \sum_{n=1}^{\infty} A \sinh(\frac{n\pi}{H}y) \cos(\frac{n\pi}{L}x)$$

$$U(x,h) = CH + \sum_{n=1}^{\infty} A \sinh(\frac{n\pi}{H}H) \cos(\frac{n\pi}{L}x) = f(x)$$

$$C = \frac{1}{LH} \int_{-L}^{L} f(x) dx$$

$$Asinh(n\pi) = \frac{7}{L} \int_{L}^{L} f(x) cos\left(\frac{n\pi}{L}x\right) dx \to$$

$$A = \frac{\tau}{L \sinh(n\pi)} \int_{-L}^{L} f(x) \cos\left(\frac{n\pi}{L}x\right) dx$$

_٣

$$ODE(\theta)$$
: $Y_K(\theta) = A_K cos(k\theta) + B_K sin(k\theta)$

$$ODE(r)$$
: $R_{\cdot}(r) = C_{\cdot} \ln 0 + D_{\cdot}$

$$R_K(r) = C_k r^k + D_k r^{-k}$$

 $C_{\cdot} = D_k = \cdot$ داريم، $r = \cdot$ چون $r = \cdot$

$$u(r,\cdot)=\cdot o Y_k(\cdot)=\cdot o A_K=\cdot$$
 شرایط مرزی:

همچنین نتیجه میگیریم: $Y_{\cdot}(\theta) = \cdot \rightarrow u_{\cdot} = \cdot$

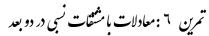
$$u_{ heta}\left(r,\frac{\pi}{r}\right)=\cdot \to -kB_k\cos(k heta)=\cdot \to k=r-1$$
 شرایط مرزی:

$$u(r,\theta) = R(r)Y(\theta)$$

$$= \sum_{n=1}^{\infty} R_n(r) Y_n(\theta) = \sum_{n=1}^{\infty} C_n r^{(\forall n-1)} B_n sin((\forall n-1)\theta)$$



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$$=\sum_{n=1}^{\infty}\widetilde{C_n}\,r^{(\mathsf{Y}n-1)}sin((\mathsf{Y}n-1)\theta)$$

شرایط مرزی آخر:
$$u(exttt{\dagger}, heta) = exttt{\dagger} = \sum_{n=1}^{\infty} \widetilde{C_n} sin((exttt{\dagger}n - exttt{\dagger}) heta)$$

$$\widetilde{C_n} = \frac{\tau}{\frac{\pi}{\tau}} \int_{\cdot}^{\frac{\pi}{\tau}} \sin((\tau n - \tau)\theta) d\theta = \frac{\tau}{\pi(\tau n - \tau)}$$

پس:
$$u(r,\theta) = \sum_{n=1}^{\infty} \frac{\mathfrak{r}}{\pi(\mathfrak{r} n-1)} r^{(\mathfrak{r} n-1)} sin((\mathfrak{r} n-1)\theta)$$

_۴

$$ODE(\theta)$$
: $Y_K(\theta) = A_K cos(k\theta) + B_K sin(k\theta)$

$$ODE(r)$$
: $R_{\cdot}(r) = C_{\cdot} \ln 0 + D_{\cdot}$

$$R_{\kappa}(r) = C_{k}r^{k} + D_{k}r^{-k}$$

$$C_{\cdot}=D_{k}=\cdot$$
 داريم، $r=\cdot$ چون

$$u(r,\cdot)=\cdot o Y_k(\cdot)=\cdot o A_K=\cdot$$
 شرایط مرزی:

همچنین نتیجه میگیریم: $Y_{\cdot}(\theta) = \cdot \rightarrow u_{\cdot} = \cdot$

$$u_{ heta}(r,\pi)=\cdot \to -kB_k\cos(k heta)=\cdot \to k=rac{rn-1}{r}$$
شرایط مرزی:

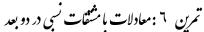
$$u(r,\theta) = R(r)Y(\theta)$$

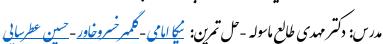
$$= \sum_{n=1}^{\infty} R_n(r) Y_n(\theta) = \sum_{n=1}^{\infty} C_n r^{\frac{r(n-1)}{r}} B_n sin\left(\left(\frac{r(n-1)}{r}\right)\theta\right)$$

$$=\sum_{n=0}^{\infty} \widetilde{C_n} r^{\frac{rn-1}{r}} sin\left(\frac{rn-1}{r}\theta\right)$$



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شرایط مرزی آخر:
$$u(a,\theta) = f(\theta) = \sum_{n=1}^{\infty} \widetilde{C_n} \ a^{\frac{\tau n-1}{\tau}} sin\left((\frac{\tau n-1}{\tau})\theta\right)$$

$$\widetilde{C_n} = \frac{\Upsilon}{\pi a^{\frac{\Upsilon n - 1}{\Upsilon}}} \int_{\cdot}^{\pi} f(\theta) \sin\left(\left(\frac{\Upsilon n - 1}{\Upsilon}\right)\theta\right) d\theta$$

_۵

$$\frac{\partial^{\mathsf{r}} u}{\partial x^{\mathsf{r}}} - \frac{\partial^{\mathsf{r}} u}{\partial y^{\mathsf{r}}} = \cdot$$

تبديل لاپلاس مي گيريم:

$$\frac{d^{\mathsf{Y}}U}{dx^{\mathsf{Y}}} = S^{\mathsf{Y}}U(x,S) - S\sin\left(\frac{\pi}{l}x\right) + \sin\left(\frac{\pi}{l}x\right) \to$$

$$\frac{d^{\mathsf{Y}}U}{dx^{\mathsf{Y}}} - S^{\mathsf{Y}}U(x,S) = (\mathsf{Y} - S)\sin(\frac{\pi}{l}x)$$

تبدیل لاپلاس شرایط مرزی:

$$U(\cdot,S) = U(l,S) = \cdot$$

$$U(x,S) = Ae^{Sx} + Be^{-Sx} + C\sin\left(\frac{\pi}{l}x\right) + D\cos\left(\frac{\pi}{l}x\right)$$

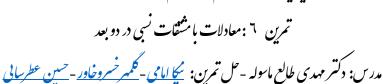
$$D = \cdot ; C = \frac{S - 1}{\left(\frac{\pi}{I}\right)^{r} + S^{r}}$$

$$U(\cdot,S) = \cdot \rightarrow A + B = \cdot;$$

$$U(l,S) = \cdot \rightarrow Ae^{Sl} + Be^{-Sl} = \cdot \rightarrow A = B = \cdot;$$



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$$U(x,S) = \frac{S - 1}{\left(\frac{\pi}{l}\right)^{r} + S^{r}} \sin\left(\frac{\pi}{l}x\right)$$

تبديل لاپلاس معكوس مي گيريم:

$$u(x,t) = \{\cos(\frac{\pi}{l}t) - \frac{l}{\pi}\sin(\frac{\pi}{l}t)\}\sin(\frac{\pi}{l}x)$$

$$\frac{d^{\mathsf{Y}}U}{dx^{\mathsf{Y}}} - s^{\mathsf{Y}}U = -se^{x} \to \begin{cases} U_{h} = Ae^{xs} + Be^{-xs} \\ U_{p} = (s/(s^{\mathsf{Y}} - \mathsf{Y}))e^{x} \end{cases}$$

$$u(\cdot,t) = 1 \stackrel{L}{\rightarrow} U(\cdot,s) = \frac{1}{s}$$

$$\lim_{x \to -\infty} u(x,t) = \cdot \xrightarrow{L} \lim_{x \to -\infty} U(x,s) = \cdot \Rightarrow B = \cdot \Rightarrow A = \frac{1}{s} - \frac{s}{s^{r} - 1}$$

$$U(x,s) = \frac{1}{s}e^{xs} - \frac{1}{r}\frac{1}{s-1}e^{xs} - \frac{1}{r}\frac{1}{s+1}e^{x} + \frac{1}{r}\frac{1}{s-1}e^{x} + \frac{1}{r}\frac{1}{s+1}e^{x}$$

$$u(x,t) = L^{-1}\{U(x,s)\} = (1 - \cosh(t+x))u(t+x) + e^x \cosh(t) u(t)$$

_Y

$$F(u_t) = c^{\mathsf{T}} F(u_{xx}) \Rightarrow \widehat{u_t} + c^{\mathsf{T}} \omega^{\mathsf{T}} \widehat{u} = \cdot$$

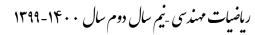
$$\hat{u}(w,t) = A(w)e^{-c^{\dagger}\omega^{\dagger}t}$$

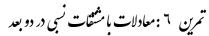
$$u(x, \cdot) = f(x) \Rightarrow \hat{u}(w, \cdot) = \hat{f}(w) \Rightarrow A(w) = \hat{f}(w)$$

$$\Rightarrow \hat{u}(w,t) = \hat{f}(w)e^{-c^{\dagger}\omega^{\dagger}t}$$

$$\Rightarrow u(x,t) = f(x) * \frac{e^{\frac{-x^{\tau}}{\tau c^{\tau}t}}}{\tau c\sqrt{\pi t}}$$











$$U_t - \Upsilon U_{\Upsilon \Upsilon} = e^{-\Delta |X|}$$

$$F\{U_t\} - \mathsf{Y}F\{U_{xx}\} = F\{e^{-\Delta|x|}\}$$

$$\widehat{U_t} - \mathsf{r} \left(-\omega^\mathsf{r} \widehat{U} \right) = \frac{\mathsf{r} \cdot \mathsf{r}}{\mathsf{r} \Delta + \omega^\mathsf{r}} \to \widehat{U_t} + \mathsf{r} \left(\omega^\mathsf{r} \widehat{U} \right) = \frac{\mathsf{r} \cdot \mathsf{r}}{\mathsf{r} \Delta + \omega^\mathsf{r}}$$

حل معادله ODE مرتبه ۱ برای \widehat{U} :

$$\widehat{U}(\omega, t) = C \exp\left\{\frac{-\Delta \cdot t\omega - rt\omega^{\mathsf{f}}}{r\Delta + \omega^{\mathsf{f}}}\right\} + \frac{\Delta}{r\Delta\omega^{\mathsf{f}} + \omega^{\mathsf{f}}}$$

شرایط مرزی:

$$U(x,\cdot) = f(x) \rightarrow fourier\ transform: \widehat{U}(\omega,\cdot) = \widehat{f}(\omega)$$

$$C = \hat{f}(\omega) - \frac{\Delta}{\Delta \omega^{\tau} + \omega^{\tau}}$$

$$\widehat{U}(\omega, t) = \left(\widehat{f}(\omega) - \frac{\Delta}{\Delta \tau \Delta \omega^{\tau} + \omega^{\tau}}\right) \exp\left\{\frac{-\Delta \cdot t\omega - \Delta \tau}{\Delta \tau \Delta \omega^{\tau}}\right\} + \frac{\Delta}{\Delta \tau \Delta \omega^{\tau} + \omega^{\tau}}$$

$$\to U(x,t) = F^{-1} \{ \widehat{U}(\omega,t) \} = \frac{1}{7\pi} \int_{-\pi}^{\pi} \widehat{U}(\omega,t) e^{iwx} d\omega$$