



المتكاول فوريه تايع
$$f(x)$$
 المتكاول وريد و با استفاده او آن حاصل التكاول $f(x) = \prod \left(\frac{x}{2}\right)$, $\Pi(x) = \begin{cases} 1, & |x| \leq \frac{1}{2}, & |I_1| = \int_{-\infty}^{+\infty} \frac{\sin(\omega)}{\omega} d\omega, & |I_2| = \int_{-\infty}^{+\infty} \frac{\sin^2(\omega)}{\omega^2} d\omega \end{cases}$

$$f(x) = f(-x) \Rightarrow f(x) \text{ is even} \Rightarrow B(\omega) = 0$$

$$A(\omega) = \frac{1}{\pi} \int_{-\infty}^{+\infty} f(x) \cos(\omega x) dx = \frac{2}{\pi} \int_{0}^{+\infty} f(x) \cos(\omega x) dx = \frac{2}{\pi} \int_{0}^{+\infty} \pi \left(\frac{x}{2}\right) \cos(\omega x) dx = \frac{2}{\pi} \int_{0}^{+\infty} \frac{\sin(\omega)}{\pi \omega} \cos(\omega x) d\omega$$

$$f(1) = \frac{f(1^+) + f(1^-)}{2} = \frac{1}{2} = \int_{0}^{\infty} \frac{2 \sin(\omega)}{\pi \omega} \cos(\omega x) d\omega = \int_{0}^{\infty} \frac{\sin(2\omega)}{\pi \omega} d\omega = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{\sin(2\omega)}{\omega} d\omega = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{\sin(v)}{v} dv$$

$$\Rightarrow I_1 = \int_{-\infty}^{+\infty} \frac{\sin(\omega)}{\omega} d\omega = \pi$$

$$\frac{1}{\pi} \int_{-\infty}^{+\infty} |f(x)|^2 dx = \int_{0}^{+\infty} (A^2(\omega) + B^2(\omega)) d\omega$$

$$\Rightarrow I_2 = \int_{-\infty}^{+\infty} \frac{\sin^2(\omega)}{\omega^2} d\omega = \pi$$

$$\Rightarrow I_3 = \int_{-\infty}^{+\infty} \frac{\sin^3(\omega)}{\omega^2} d\omega = \pi$$

$$\Rightarrow I_4 = \int_{-\infty}^{+\infty} \frac{\sin^2(\omega)}{\omega^2} d\omega = \pi$$

$$\Rightarrow I_5 = \int_{-\infty}^{+\infty} \frac{\sin^2(\omega)}{\omega^2} d\omega = \pi$$

$$\Rightarrow I_6 = \int_{-\infty}^{+\infty} \frac{\sin^2(\omega)}{\omega^2} d\omega = \pi$$

$$\Rightarrow I_7 = \int_{-\infty}^{+\infty} \frac{\sin^3(\omega)}{\omega^2} d\omega = \pi$$

$$\Rightarrow I_8 = \int_{-\infty}^{+\infty} \frac{\sin^4(\omega)}{\omega^2} d\omega = \pi$$

$$\Rightarrow I_8$$

$$B(\omega) = \frac{1}{\pi} \int_{-\infty}^{+\infty} f(x) \sin(\omega x) \, dx = \frac{2}{\pi} \int_{0}^{+\infty} f(x) \sin(\omega x) \, dx = \frac{2}{\pi} \int_{0}^{+\infty} \Pi\left(\frac{x}{2}\right) \sin(\omega x) \, dx = \frac{2}{\pi} \int_{0}^{+1} \sin(\omega x) \, dx = \frac{4 \sin^{2}\left(\frac{\omega}{2}\right)}{\pi \omega}$$

$$\Rightarrow \boxed{f(x) = \int_{0}^{\infty} \frac{4 \sin^{2}\left(\frac{\omega}{2}\right)}{\pi \omega} \sin(\omega x) \, d\omega}$$

$$f(1) = \frac{f(1^{+}) + f(1^{-})}{2} = \frac{1}{2} = \int_{0}^{\infty} \frac{4 \sin^{2}\left(\frac{\omega}{2}\right)}{\pi \omega} \sin(\omega) d\omega = \int_{0}^{\infty} \frac{8 \sin^{3}\left(\frac{\omega}{2}\right) \cos\left(\frac{\omega}{2}\right)}{\pi \omega} d\omega = \frac{4}{\pi} \int_{-\infty}^{+\infty} \frac{\sin^{3}\left(\frac{\omega}{2}\right) \cos\left(\frac{\omega}{2}\right)}{\omega} d\omega = \frac{4}{\pi} \int_{-\infty}^{+\infty} \frac{\sin^{3}(v) \cos(v)}{v} dv$$

$$\Rightarrow \boxed{I_{1} = \int_{-\infty}^{+\infty} \frac{\sin^{3}(\omega) \cos(\omega)}{\omega} d\omega = \frac{\pi}{8}}$$

Parseval:
$$\frac{1}{\pi} \int_{-\infty}^{+\infty} |f(x)|^2 dx = \int_0^{+\infty} \left(A^2(\omega) + B^2(\omega) \right) d\omega \Rightarrow \frac{1}{\pi} \int_{-\infty}^{+\infty} \left| \operatorname{sign}(x) \Pi\left(\frac{x}{2}\right) \right|^2 dx = \frac{2}{\pi} \int_0^{+1} dx = \frac{2}{\pi} = \int_0^{+\infty} \frac{16 \sin^4\left(\frac{\omega}{2}\right)}{\pi^2 \omega^2} d\omega$$
$$= \frac{8}{\pi^2} \int_{-\infty}^{+\infty} \frac{\sin^4\left(\frac{\omega}{2}\right)}{\omega^2} d\omega = \frac{4}{\pi^2} \int_{-\infty}^{+\infty} \frac{\sin^4(v)}{v^2} dv$$
$$\Rightarrow I_2 = \int_{-\infty}^{+\infty} \frac{\sin^4(\omega)}{\omega^2} d\omega = \frac{\pi}{2}$$



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انتگرال فوریه تابع $f(\mathfrak{x})$ را بدست آورید و با استفاده از آن حاصل انتگرال I_2 و I_1 را محاسبه کنید.

$$f(x) = \cos(\pi x) \, \Pi\left(\frac{x}{2}\right), \qquad \Pi(x) = \begin{cases} 1, & |x| \leq \frac{1}{2}, \\ 0, & \text{otherwise} \end{cases}, \qquad I_1 = \int_{-\infty}^{+\infty} \frac{\omega \sin(2\omega)}{\omega^2 - \pi^2} d\omega, \qquad I_2 = \int_{-\infty}^{+\infty} \frac{\omega^2 \sin^2(\omega)}{(\omega^2 - \pi^2)^2} d\omega$$

$$f(x) = f(-x) \Rightarrow f(x) \text{ is even} \Rightarrow B(\omega) = 0$$

$$A(\omega) = \frac{1}{\pi} \int_{-\infty}^{+\infty} f(x) \cos(\omega x) \, dx = \frac{2}{\pi} \int_{0}^{+\infty} f(x) \cos(\omega x) \, dx = \frac{2}{\pi} \int_{0}^{+\infty} \cos(\pi x) \, \Pi\left(\frac{x}{2}\right) \cos(\omega x) \, dx = \frac{2}{\pi} \int_{0}^{1} \cos(\pi x) \cos(\omega x) \, dx$$

$$= \frac{1}{\pi} \left[\frac{\sin((\omega + \pi)x)}{\omega + \pi} + \frac{\sin((\omega - \pi)x)}{\omega - \pi} \right]_{0}^{1} = \frac{2\omega \sin(\omega)}{\pi^3 - \pi\omega^2}$$

$$\Rightarrow \left[f(x) = \int_{0}^{\infty} \frac{2\omega \sin(\omega)}{\pi^3 - \pi\omega^2} \cos(\omega x) \, d\omega \right]$$

$$f(1) = \frac{f(1^+) + f(1^-)}{2} = -\frac{1}{2} = \int_{0}^{\infty} \frac{2\omega \sin(\omega)}{\pi^3 - \pi\omega^2} \cos(\omega) \, d\omega = \frac{1}{\pi} \int_{0}^{\infty} \frac{\omega \sin(2\omega)}{\pi^2 - \omega^2} d\omega = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{\omega \sin(2\omega)}{\pi^2 - \omega^2} d\omega$$

$$\Rightarrow \left[I_1 = \int_{-\infty}^{+\infty} \frac{\omega \sin(2\omega)}{\omega^2 - \pi^2} d\omega = \pi \right]$$

$$\text{Parseval: } \frac{1}{\pi} \int_{-\infty}^{+\infty} \left[f(x) \right]^2 dx = \int_{0}^{+\infty} (A^2(\omega) + B^2(\omega)) d\omega \Rightarrow \frac{1}{\pi} \int_{-\infty}^{+\infty} \left| \cos(\pi x) \, \Pi\left(\frac{x}{2}\right) \right|^2 dx = \frac{2}{\pi} \int_{0}^{+1} \left| \cos(\pi x) \right|^2 dx = \frac{1}{\pi} = \int_{0}^{+\infty} \frac{4\omega^2 \sin^2(\omega)}{(\pi^3 - \pi\omega^2)^2} d\omega$$

$$\Rightarrow \left[I_2 = \int_{-\infty}^{+\infty} \frac{\omega^2 \sin^2(\omega)}{(\omega^2 - \pi^2)^2} d\omega = \frac{\pi}{2} \right]$$

انتگرال فوریه تابع f(x) را بدست آورید و با استفاده از آن حاصل انتگرال I_1 و I_2 را محاسبه کنید. f(x)

$$f(x) = \sin(\pi x) \Pi(x), \qquad \Pi(x) = \begin{cases} 1, & |x| \le \frac{1}{2}, \\ 0, & \text{otherwise} \end{cases}, \qquad I_1 = \int_{-\infty}^{+\infty} \frac{\omega \sin(\omega)}{\pi^2 - \omega^2} d\omega, \qquad I_2 = \int_{-\infty}^{+\infty} \frac{\omega^2 \cos^2\left(\frac{\omega}{2}\right)}{(\pi^2 - \omega^2)^2} d\omega$$

$$f(x) = -f(-x) \Rightarrow f(x) \text{ is odd} \Rightarrow A(\omega) = 0$$

$$B(\omega) = \frac{1}{\pi} \int_{-\infty}^{+\infty} f(x) \sin(\omega x) dx = \frac{2}{\pi} \int_{0}^{+\infty} f(x) \sin(\omega x) dx = \frac{2}{\pi} \int_{0}^{+\infty} \sin(\pi x) \Pi(x) \sin(\omega x) dx = \frac{2}{\pi} \int_{0}^{+\frac{1}{2}} \sin(\pi x) \sin(\omega x) dx$$

$$= \frac{1}{\pi} \left[\frac{\sin((\omega - \pi)x)}{\omega - \pi} - \frac{\sin((\omega + \pi)x)}{\omega + \pi} \right]_{0}^{+\frac{1}{2}} = \frac{2\omega \cos\left(\frac{\omega}{2}\right)}{\pi^3 - \pi\omega^2}$$

$$\Rightarrow f(x) = \int_{0}^{\infty} \frac{2\omega \cos\left(\frac{\omega}{2}\right)}{\pi^3 - \pi\omega^2} \sin(\omega x) d\omega$$

$$f\left(\frac{1}{2}\right) = \frac{f\left(\frac{1}{2}\right) + f\left(\frac{1}{2}\right)}{2} = \frac{1}{2} = \int_{0}^{\infty} \frac{2\omega \cos\left(\frac{\omega}{2}\right)}{\pi^3 - \pi\omega^2} \sin\left(\frac{\omega}{2}\right) d\omega = \int_{0}^{\infty} \frac{\omega \sin(\omega)}{\pi^3 - \pi\omega^2} d\omega = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{\omega \sin(\omega)}{\pi^2 - \omega^2} d\omega$$

$$\Rightarrow I_1 = \int_{-\infty}^{+\infty} \frac{\omega \sin(\omega)}{\pi^2 - \omega^2} d\omega = \pi$$

Parseval:
$$\frac{1}{\pi} \int_{-\infty}^{+\infty} |f(x)|^2 dx = \int_{0}^{+\infty} (A^2(\omega) + B^2(\omega)) d\omega \Rightarrow \frac{1}{\pi} \int_{-\infty}^{+\infty} |\sin(\pi x) \Pi(x)|^2 dx = \frac{2}{\pi} \int_{0}^{+\frac{1}{2}} |\sin(\pi x)|^2 dx = \frac{1}{2\pi} = \int_{0}^{+\infty} \frac{4\omega^2 \cos^2\left(\frac{\omega}{2}\right)}{(\pi^3 - \pi\omega^2)^2} d\omega$$

$$= \frac{2}{\pi^2} \int_{-\infty}^{+\infty} \frac{\omega^2 \cos^2\left(\frac{\omega}{2}\right)}{(\pi^2 - \omega^2)^2} d\omega$$

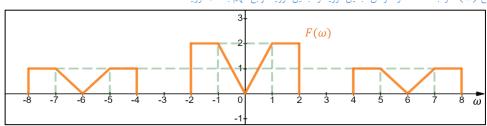
$$\Rightarrow I_2 = \int_{-\infty}^{+\infty} \frac{\omega^2 \cos^2\left(\frac{\omega}{2}\right)}{(\pi^2 - \omega^2)^2} d\omega = \frac{\pi}{4}$$



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را با استفاده از خواص تبدیل فوریه و تبدیل فوریه توابع مهم بدست آورید. $F(\omega)$ تبدیل فوریه معکوس تابع (۲-۱)



$$\mathcal{F}^{-1}{F(\omega)} = f(x), \qquad F(\omega) = G(\omega - 6) + G(\omega + 6) + 2G(\omega) = G(\omega - 3 - 3) + G(\omega - 3 + 3) + G(\omega + 3 - 3) + G(\omega + 3 + 3)$$

$$\Rightarrow f(x) = 4\cos^2(3x) g(x)$$

$$G(\omega) = \begin{cases} |\omega| & |\omega| \le 1 \\ 1 & 1 \le |\omega| \le 2 = \Pi\left(\frac{\omega}{4}\right) - \Lambda(\omega) \\ 0 & \text{otherwise} \end{cases}$$

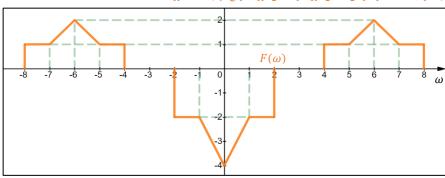
$$\mathcal{F}{\sin(\alpha x)} = \frac{1}{|\alpha|} \Pi\left(\frac{\omega}{2\pi a}\right) \Rightarrow \mathcal{F}^{-1}\left{\Pi\left(\frac{\omega}{4}\right)\right} = \frac{2}{\pi} \operatorname{sinc}\left(\frac{2x}{\pi}\right)$$

$$\mathcal{F}{\sin(\alpha x)} = \frac{1}{|\alpha|} \Lambda\left(\frac{\omega}{2\pi a}\right) \Rightarrow \mathcal{F}^{-1}\left{\Lambda(\omega)\right} = \frac{1}{2\pi} \operatorname{sinc}^2\left(\frac{x}{2\pi}\right)$$

$$\Rightarrow g(x) = \frac{2}{\pi} \operatorname{sinc}\left(\frac{2x}{\pi}\right) - \frac{1}{2\pi} \operatorname{sinc}^2\left(\frac{x}{2\pi}\right)$$

$$\Rightarrow f(x) = \left(\frac{8}{\pi} \operatorname{sinc}\left(\frac{2x}{\pi}\right) - \frac{2}{\pi} \operatorname{sinc}^2\left(\frac{x}{2\pi}\right)\right) \cos^2(3x)$$

تبدیل فوریه معکوس تابع $F(\omega)$ را با استفاده از خواص تبدیل فوریه و تبدیل فوریه توابع مهم بدست آورید.



$$\mathcal{F}^{-1}\{F(\omega)\} = f(x), \qquad F(\omega) = G(\omega - 6) + G(\omega + 6) - 2G(\omega) = \left(G(\omega - 3 - 3) - G(\omega - 3 + 3)\right) - \left(G(\omega + 3 - 3) - G(\omega + 3 + 3)\right)$$

$$\Rightarrow f(x) = 4i^{2} \sin^{2}(3x) g(x) = -4 \sin^{2}(3x) g(x)$$

$$G(\omega) = \begin{cases} 2 - |\omega| & |\omega| \le 1 \\ 1 & 1 \le |\omega| \le 2 = \Pi\left(\frac{\omega}{4}\right) + \Lambda(\omega) \\ 0 & \text{otherwise} \end{cases}$$

$$\mathcal{F}\{\operatorname{sinc}(ax)\} = \frac{1}{|a|} \Pi\left(\frac{\omega}{2\pi a}\right) \Rightarrow \mathcal{F}^{-1}\left\{\Pi\left(\frac{\omega}{4}\right)\right\} = \frac{2}{\pi} \operatorname{sinc}\left(\frac{2x}{\pi}\right)$$

$$\mathcal{F}\{\operatorname{sinc}^{2}(ax)\} = \frac{1}{|a|} \Lambda\left(\frac{\omega}{2\pi a}\right) \Rightarrow \mathcal{F}^{-1}\{\Lambda(\omega)\} = \frac{1}{2\pi} \operatorname{sinc}^{2}\left(\frac{x}{2\pi}\right)$$

$$\Rightarrow g(x) = \frac{2}{\pi} \operatorname{sinc}\left(\frac{2x}{\pi}\right) + \frac{1}{2\pi} \operatorname{sinc}^{2}\left(\frac{x}{2\pi}\right)$$

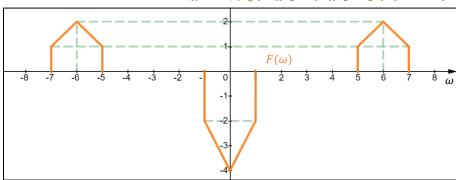
$$\Rightarrow f(x) = \left(-\frac{2}{\pi} \operatorname{sinc}^{2}\left(\frac{x}{2\pi}\right) - \frac{8}{\pi} \operatorname{sinc}\left(\frac{2x}{\pi}\right)\right) \sin^{2}(3x)$$



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۳-۳ تبدیل فوریه معکوس تابع F(w) را با استفاده از خواص تبدیل فوریه و تبدیل فوریه توابع مهم بدست آورید.



$$\mathcal{F}^{-1}\{F(\omega)\} = f(x), \qquad F(\omega) = G(\omega - 6) + G(\omega + 6) - 2G(\omega) = \left(G(\omega - 3 - 3) - G(\omega - 3 + 3)\right) - \left(G(\omega + 3 - 3) - G(\omega + 3 + 3)\right)$$

$$\Rightarrow f(x) = 4i^2 \sin^2(3x) g(x) = -4 \sin^2(3x) g(x)$$

$$G(\omega) = \begin{cases} 2 - |\omega| & |\omega| \le 1 \\ 0 & \text{otherwise} \end{cases} = \Pi\left(\frac{\omega}{2}\right) + \Lambda(\omega)$$

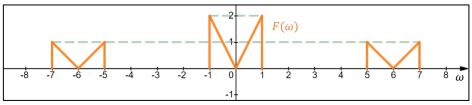
$$\mathcal{F}\{\sin(\alpha x)\} = \frac{1}{|\alpha|} \Pi\left(\frac{\omega}{2\pi a}\right) \Rightarrow \mathcal{F}^{-1}\left\{\Pi\left(\frac{\omega}{2}\right)\right\} = \frac{1}{\pi} \operatorname{sinc}\left(\frac{x}{\pi}\right)$$

$$\mathcal{F}\{\sin(\alpha x)\} = \frac{1}{|\alpha|} \Lambda\left(\frac{\omega}{2\pi a}\right) \Rightarrow \mathcal{F}^{-1}\{\Lambda(\omega)\} = \frac{1}{2\pi} \operatorname{sinc}^2\left(\frac{x}{2\pi}\right)$$

$$\Rightarrow g(x) = \frac{1}{\pi} \operatorname{sinc}\left(\frac{x}{\pi}\right) + \frac{1}{2\pi} \operatorname{sinc}^2\left(\frac{x}{2\pi}\right)$$

$$\Rightarrow f(x) = \left(-\frac{2}{\pi} \operatorname{sinc}^2\left(\frac{x}{2\pi}\right) - \frac{4}{\pi} \operatorname{sinc}\left(\frac{x}{\pi}\right)\right) \sin^2(3x)$$

تبدیل فوریه معکوس تابع $F(\omega)$ را با استفاده از خواص تبدیل فوریه و تبدیل فوریه توابع مهم بدست آورید.



$$\mathcal{F}^{-1}{F(\omega)} = f(x), \qquad F(\omega) = G(\omega - 6) + G(\omega + 6) + 2G(\omega) = \left(G(\omega - 3 - 3) + G(\omega - 3 + 3)\right) + \left(G(\omega + 3 - 3) + G(\omega + 3 + 3)\right)$$

$$\Rightarrow f(x) = 4\cos^2(3x) g(x) = 4\cos^2(3x) g(x)$$

$$G(\omega) = \begin{cases} |\omega| & |\omega| \le 1 \\ 0 & \text{otherwise} \end{cases} = \Pi\left(\frac{\omega}{2}\right) - \Lambda(\omega)$$

$$\mathcal{F}\{\sin(\alpha x)\} = \frac{1}{|\alpha|} \Pi\left(\frac{\omega}{2\pi a}\right) \Rightarrow \mathcal{F}^{-1}\left\{\Pi\left(\frac{\omega}{2}\right)\right\} = \frac{1}{\pi} \operatorname{sinc}\left(\frac{x}{\pi}\right)$$

$$\mathcal{F}\{\sin^2(\alpha x)\} = \frac{1}{|\alpha|} \Lambda\left(\frac{\omega}{2\pi a}\right) \Rightarrow \mathcal{F}^{-1}\{\Lambda(\omega)\} = \frac{1}{2\pi} \operatorname{sinc}^2\left(\frac{x}{2\pi}\right)$$

$$\Rightarrow g(x) = \frac{1}{\pi} \operatorname{sinc}\left(\frac{x}{\pi}\right) - \frac{1}{2\pi} \operatorname{sinc}^2\left(\frac{x}{2\pi}\right)$$

$$\Rightarrow f(x) = \left(\frac{4}{\pi} \operatorname{sinc}\left(\frac{x}{\pi}\right) - \frac{2}{\pi} \operatorname{sinc}^2\left(\frac{x}{2\pi}\right)\right) \cos^2(3x)$$