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ریاضی مهندسی

نیم‌سال دوم ۱۴۰۱-۱۴۰۰	پاسخ تکلیف شماره ۱	

سری فوریه

پاسخ سوال ۱ قسمت (الف): (۱۰ نمره)

$$< f_1, f_2 > = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2xsinx\,dx = (-2xcosx + 2sinx)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 4$$

$$\rightarrow\ f_1 \text{ and } f_2 \text{ are not orthogonal.}$$

پاسخ سوال ۱ قسمت (ب): (۱۰ نمره)

$$< g_1, g_2 > = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} coshx\,cosx\,dx = \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (e^x + e^{-x})cosx\,dx$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^x cosxdx = \frac{1}{2}e^x(sinx + cosx)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = cosh\frac{\pi}{2}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{-x} cosxdx = \frac{1}{2}e^{-x}(sinx - cosx)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = cosh\frac{\pi}{2}$$

$$< g_1, g_2 > = \frac{1}{2}\left(cosh\frac{\pi}{2} + cosh\frac{\pi}{2}\right) = cosh\frac{\pi}{2}$$

$$\rightarrow g_1 \text{and } g_2 \text{ are not orthogonal.}$$

پاسخ سوال ۲: (۱۰ نمره)

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi}{l}x\right) + b_n \sin\left(\frac{n\pi}{l}x\right)\right] \quad ; a_0 = \frac{2}{T}\int_{T} f(x)\,dx$$

$$f^2(x) = \frac{a_0^2}{4} + a_0 \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi}{l}x\right) + b_n \sin\left(\frac{n\pi}{l}x\right)\right] + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \left[a_n a_m \cos\left(\frac{n\pi}{l}x\right)\cos\left(\frac{m\pi}{l}x\right) + a_n b_m \cos\left(\frac{n\pi}{l}x\right)\sin\left(\frac{m\pi}{l}x\right) + a_m b_n \cos\left(\frac{m\pi}{l}x\right)\sin\left(\frac{n\pi}{l}x\right)\right.$$

$$\left. + b_n b_m \sin\left(\frac{n\pi}{l}x\right)\sin\left(\frac{m\pi}{l}x\right)\right]$$

$$\Rightarrow \int_{-l}^{l} f^2(x)\,dx = \int_{-l}^{l} \frac{a_0^2}{4}\,dx + a_0 \int_{-l}^{l} \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{n\pi}{l}x\right) + b_n \sin\left(\frac{n\pi}{l}x\right)\right]dx + \int_{-l}^{l} \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}[\cdot]\,dx = \frac{a_0^2}{4}(2l) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\int_{-l}^{l}[\cdot]\,dx$$

$$= \frac{a_0^2}{2}l + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[a_n a_m l\,\delta_{nm} + 0 + 0 + b_n b_m l\,\delta_{nm}\right] = \frac{a_0^2}{2}l + l\sum_{n=1}^{\infty}\left[a_n^2 + b_n^2\right]$$

$$\Rightarrow \frac{1}{l}\int_{-l}^{l} f^2(x)\,dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}\left[a_n^2 + b_n^2\right]$$

قضیه پارسوال

پاسخ سوال ۳ قسمت (الف): (۱۵ نمره)

$$even\ function\ \Rightarrow\ b_n = 0$$

$$a_0 = \frac{1}{l}\int_{(T)} f(x)\,dx = \frac{1}{\pi}\int_0^{\pi} xsinx\,dx = \frac{1}{\pi}\int_0^{\pi} xsinx\,dx = \frac{1}{\pi}(-xcosx + sinx)\Big|_0^{\pi} = 1$$

$$a_n = \frac{2}{l}\int_{(T)} f(x)\,cos(n\omega_0 x)\,dx = \frac{2}{\pi}\int_0^{\pi} xsinx\,cos(nx)\,dx = \frac{2}{\pi}\int_0^{\pi} xsinx\ \ cos(nx)\,dx$$

$$I = \int_0^{\pi} xsinx\ \ cos(nx)\,dx = \frac{1}{2}\int_0^{\pi} x(\sin(1+n)\,x + \sin(1-n)\,x)dx$$

$$= \frac{1}{2}\left(\frac{-1}{1+n}\,x\cos(1+n)\,x + \frac{1}{(1+n)^2}\,\sin(1+n)\,x + \frac{-1}{1-n}\,x\cos(1-n)\,x + \frac{1}{(1-n)^2}\,\sin(1-n)\,x\,\right)\Big|_0^{\pi}$$

$$= -\pi\,\frac{cosn\pi}{(1-n^2)}\ \ \rightarrow\ \ a_n = -2\,\frac{cosn\pi}{(1-n^2)}$$

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پاسخ سوال ۳ قسمت (ب): (۱۵ نمره)

$$g(x) = sin2x - cosx + x^2 + xcos^2x = sin2x - cosx + x^2 + x(\frac{Cos2x+1}{2})$$

$$= sin2x - cosx + x^2 + \frac{x}{2} + \frac{x}{2}cos2x$$

$$g_1(x) = g(x) - sin2x + cosx - \frac{x}{2}cos2x = x^2 + \frac{x}{2} \;\; ;$$

$$a_0 = \frac{1}{\pi}\int_0^{\pi}(x^2 + \frac{x}{2})dx = \frac{\pi^2}{3} + \frac{\pi}{4}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi}\left(x^2 + \frac{x}{2}\right)cos(nx)dx = \frac{2}{\pi}\int_0^{\pi}x^2cos(nx)dx + \frac{2}{\pi}\int_0^{\pi}\frac{x}{2}cos(nx)dx$$

$$= \frac{2}{\pi}(x^2\frac{sin(nx)}{n} - 2\frac{sin(nx)}{n^3} + 2x\frac{cos(nx)}{n^2} + x\frac{sin(nx)}{2n} + \frac{cos(nx)}{2n^2})|_0^{\pi}$$

$$a_n = \frac{1}{\pi}(\frac{-1}{n^2} + (1 + 4\pi)\frac{cos(n\pi)}{n^2})$$

$$b_n = \frac{2}{\pi}\int_0^{\pi}\left(x^2 + \frac{x}{2}\right)sin(nx)dx = \frac{2}{\pi}\int_0^{\pi}x^2sin(nx)dx + \frac{2}{\pi}\int_0^{\pi}\frac{x}{2}sin(nx)dx$$

$$= \frac{2}{\pi}(-x^2\frac{cos(nx)}{n} + 2\frac{cos(nx)}{n^3} + 2x\frac{sin(nx)}{n^2} - x\frac{cos(nx)}{2n} + \frac{sin(nx)}{2n^2})|_0^{\pi}$$

$$b_n = \frac{1}{\pi}(\frac{2}{n^3}(cosn\pi - 1) - \frac{(1+2\pi)}{n})cosn\pi)$$

$$g_2(x) = g(x) - sin2x + cosx - x^2 - \frac{x}{2} = \frac{x}{2}cos2x \, ;$$

$$odd\ function\ \rightarrow a_0 = a_n = 0$$

$$b_n = \frac{2}{\pi}\int_0^{\pi}\left(\frac{x}{2}cos2x\right)sin(nx)dx = \frac{n}{(4-n)^2}cosn\pi$$

پاسخ سوال ۳ قسمت (ج): (۱۰ نمره)

$h(x) = \sum_{k=-\infty}^{\infty}(-1)^{k+1}\delta(x-kL);$ $T=2L;$

$$a_0 = \frac{1}{2L}\int_{\varepsilon}^{2L+\varepsilon}\big(\delta(x-L)-\delta(x-2L)\big)dx = 0$$

$$a_n = \frac{1}{L}\int_{\varepsilon}^{2L+\varepsilon}\big(\delta(x-L)-\delta(x-2L)\big)Cos\left(\frac{\pi n}{L}x\right)dx = \frac{(-1)^n-1}{L}$$

$$b_n = \frac{1}{L}\int_{\varepsilon}^{2L+\varepsilon}\big(\delta(x-L)-\delta(x-2L)\big)Sin\left(\frac{\pi n}{L}x\right)dx = 0$$

$$h(x) = \sum_{n=1}^{\infty}\frac{(-1)^n-1}{L}Cos\left(\frac{n\pi}{L}x\right) = \sum_{k=1}^{\infty}-\frac{2}{L}Cos\left(\frac{(2k-1)\pi}{L}x\right)$$

پاسخ سوال ۴: (۱۵ نمره)

$$odd\ function \Rightarrow a_n = a_0 = 0$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} x\ sinnx\ dx = \frac{2}{\pi}\left[\frac{-x}{n}cosnx - \frac{1}{n^2}sinnx\right]_0^{\pi} = \frac{-2}{n}cosn\pi$$

$$\Rightarrow f(x) = \sum_{n=1}^{\infty}\frac{2(-1)^{n+1}}{n}sinnx$$

$$integrate\ twice\ :\ \frac{x^2}{2} = \sum_{n=1}^{\infty}\frac{2(-1)^n}{n^2}cosnx + C\ ,\ C = \frac{1}{\pi}\int_0^{\pi}\frac{x^2}{2}dx = \frac{\pi^2}{6}$$

$$\frac{x^3}{6} = \sum_{n=1}^{\infty}\frac{2(-1)^n}{n^3}sinnx + \frac{\pi^2}{6}x + c\ :\ odd\ function \rightarrow c = 0 \rightarrow \frac{x^3}{6} - \frac{\pi^2}{6}x = \sum_{n=1}^{\infty}\frac{2(-1)^n}{n^3}sinnx$$

$$Parseval: \frac{2}{\pi}\int_0^{\pi}\left[\frac{x^3}{6} - \frac{\pi^2}{6}x\right]^2 dx = 4\sum_{n=1}^{\infty}\frac{1}{n^6} \rightarrow \sum_{n=1}^{\infty}\frac{1}{n^6} = \frac{\pi^4}{945}$$

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پاسخ سوال ۵: (۱۵ نمره)

Since the argument of sine and cosine is nx $\rightarrow T = 2\pi;$

$$A = \int_{-\pi}^{\pi} f(x)\left(\cos^3 x - 2\sin^2\frac{x}{2}\right)dx = \int_{-\pi}^{\pi} f(x)\left(\frac{3\cos x + \cos 3x}{4} + \cos x - 1\right)dx$$

$$= \frac{3}{4}\int_{-\pi}^{\pi} f(x)\cos x dx + \frac{1}{4}\int_{-\pi}^{\pi} f(x)\cos 3x dx + \int_{-\pi}^{\pi} f(x)\cos x dx - \int_{-\pi}^{\pi} f(x)dx$$

$$= \frac{3\pi}{4}a_1 + \frac{\pi}{4}a_3 + \pi a_1 - 2\pi a_0 = \frac{3\pi}{4}(3) + \frac{\pi}{4}\left(\frac{3}{9}\right) + \pi(3) - 2\pi\left(\frac{\pi}{4}\right) = \frac{9\pi^2 + 96\pi}{18}$$