



## درس ریاضی مهندسی

### پاسخ کوئیز ۷

نیم سال دوم

۱۴۰۰-۱۴۰۱

### پاسخ سوال ۱:

$$u(x, y) = X(x) \cdot Y(y) \Rightarrow Y \frac{dX}{dx} + X \frac{dY}{dy} = \gamma(x+y)XY \Rightarrow \frac{1}{X} \frac{dX}{dx} - \gamma x = \gamma y - \frac{1}{Y} \frac{dY}{dy} = \lambda$$

$$\begin{cases} \frac{1}{X} \frac{dX}{dx} - \gamma x = \lambda \Rightarrow \frac{dX}{X} = (\lambda + \gamma x) dx \Rightarrow X = e^{x^{\gamma} + \lambda x + c_1} \\ \gamma y - \frac{1}{Y} \frac{dY}{dy} = \lambda \Rightarrow \frac{dY}{Y} = (\gamma y - \lambda) dy \Rightarrow Y = e^{y^{\gamma} + \lambda y + c_2} \end{cases}$$

$$\Rightarrow u(x, y) = XY = e^{x^{\gamma} + y^{\gamma} + \lambda(x+y) + c_1 + c_2}$$

### پاسخ سوال ۲:

۲۵) (الف)  $\frac{X''}{X} + \frac{Y''}{Y} = 0 \xrightarrow{\text{شرایط متری}} \begin{cases} X'' = -k^2 X \\ Y'' = \frac{n^2 \pi^2}{l^2} Y \end{cases} \Rightarrow \begin{cases} X = B \sin(\frac{n\pi}{l} x) \\ Y = B' e^{\frac{n\pi}{l} y} \end{cases}$

$$\Rightarrow u(x, y) = \sum_{n=1}^{\infty} D_n \sin(\frac{n\pi}{l} x) e^{\frac{n\pi}{l} y} \Rightarrow u(x, 0) = \sum_{n=1}^{\infty} D_n \sin(\frac{n\pi}{l} x)$$

$$\Rightarrow D_n = \frac{f}{r l} \int_0^l g(x) \sin(\frac{n\pi}{l} x) dx$$

۲۶)  $u(x, y) = W(x, y) + V(x) \Rightarrow \nabla^2 W(x, y) + \frac{V''}{0} = 0$

$$\begin{cases} V''(x) = 0 \Rightarrow V(x) = Ax + B \\ V(0) = c \\ V(l) = d \end{cases} \Rightarrow V(x) = \frac{d-c}{l} x + c$$

$$\begin{cases} \nabla^2 W(x, y) = 0 \\ W(0, y) = W(l, y) = 0 \\ W(x, 0) = g(x) - u(x) \end{cases} \Rightarrow \begin{cases} W(x, y) = \sum_{n=1}^{\infty} F_n \sin(\frac{n\pi}{l} x) e^{\frac{n\pi}{l} y} \\ F_n = \frac{f}{r l} \int_0^l (g(x) - V(x)) \sin(\frac{n\pi}{l} x) dx \end{cases}$$