سوال ۱: (۴ نمره)

$$R_e\{\hat{h}(\omega)\} = cos^2(\omega) + \frac{1}{1+\omega^2}$$

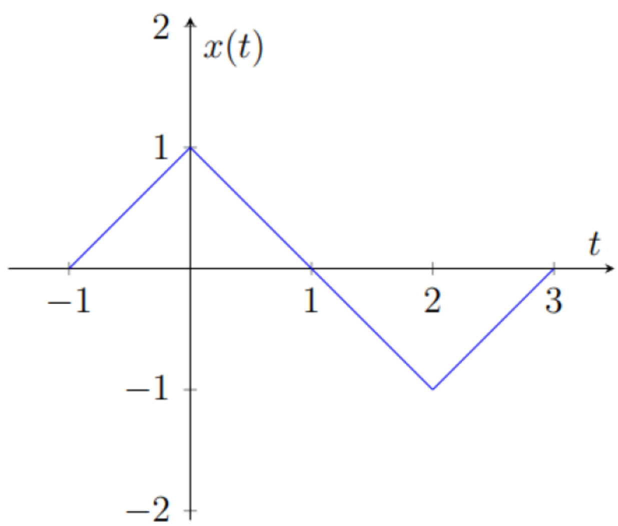
$$\rightarrow \quad F^{-1}\{Re\{\hat{h}(\omega)\}\} = F^{-1}\left\{\frac{1}{2} + \frac{e^{j2\omega}}{4} + \frac{e^{-j2\omega}}{4} + \frac{1}{1+\omega^2}\right\}$$

$$\rightarrow \quad h_e(t) = \frac{1}{2}\delta(t) + \frac{1}{4}\delta(t-2) + \frac{1}{4}\delta(t+2) + \frac{e^{-|t|}}{2}$$

$$h(t) = 2h_e(t)u(t) \quad \rightarrow \quad \boxed{\boldsymbol{h(t) = e^{-t}u(t) + \frac{1}{2}\delta(t-2) + \delta(t)}}$$ (۳ نمره)

$$\rightarrow \quad \boxed{\boldsymbol{\hat{h}(\omega) = \frac{1}{j\omega+1} + \frac{e^{-j2\omega}}{2} + 1}}$$ (۱ نمره)

سوال ۲: (۶ نمره)



$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{x}(\omega)\, e^{j\omega t} d\omega$$

الف)

$$F^{-1}\left\{\frac{2sin(\omega)}{\omega}\right\} = \prod\left(\frac{t}{2}\right)$$

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty}\hat{x}(\omega)\,\frac{2sin(\omega)}{\omega}\,e^{j\omega t}d\omega = x(t) * \prod\left(\frac{t}{2}\right)$$

$$\rightarrow \quad \int_{-\infty}^{+\infty}\hat{x}(\omega)\,\frac{2sin(\omega)}{\omega}\,e^{j2\omega}d\omega = \ 2\pi\int_{-1}^{1}x(2-\alpha)\,d\alpha = \ -2\pi$$

$$\Rightarrow \boxed{\int_{-\infty}^{+\infty}\hat{\boldsymbol{x}}(\boldsymbol{\omega})\,\frac{\boldsymbol{2sin(\omega)}}{\boldsymbol{\omega}}\,\boldsymbol{e^{j2\omega}d\omega = \ -2\pi}}$$

(۱ نمره)

$$x(0) \ = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\hat{x}(\omega)\,d\omega \quad \rightarrow \quad \boxed{\int_{-\infty}^{+\infty}\hat{\boldsymbol{x}}(\boldsymbol{\omega})\,\boldsymbol{d\omega = \ 2\pi}}$$

(۱ نمره)

$x(t+1)$ تابع فرد حقیقی است پس تبدیل فوریه آن فرد و موهومی خالص است

$$F\{x(t+1)\} = \ \hat{x}(\omega)\,e^{j\omega} \quad \rightarrow \quad \measuredangle\hat{x}(\omega) + \ \omega \ = \frac{\pi}{2}$$

$$\rightarrow \quad \boxed{\measuredangle\hat{\boldsymbol{x}}(\boldsymbol{\omega}) \ = \frac{\boldsymbol{\pi}}{\boldsymbol{2}} - \ \boldsymbol{\omega}}$$

(۱ نمره)

$$\overbrace{\frac{dx(t)}{dt}}^{-1}\Big|_{t=1} = \ \frac{1}{2\pi}\int_{-\infty}^{+\infty}(j\omega)\,\hat{x}(\omega)\,e^{j\omega t}d\omega$$

$$\rightarrow \quad \boxed{\int_{-\infty}^{+\infty}\boldsymbol{\omega}\,\hat{\boldsymbol{x}}(\boldsymbol{\omega})\,\boldsymbol{e^{j\omega t}d\omega = j2\,\pi}}$$

(۱ نمره)

پارسوال $\rightarrow \int_{-\infty}^{+\infty} |x(t)|^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |\hat{x}(\omega)|^2\, d\omega$

$$= \int_{-1}^{0}(t+1)^2\, dt + \int_{0}^{2}(1-t)^2\, dt + \int_{2}^{3}(3-t)^2\, dt = \frac{1}{3}+\frac{2}{3}+\frac{1}{3}$$

$$\Rightarrow \boxed{\int_{-\infty}^{+\infty} |\hat{\boldsymbol{x}}(\boldsymbol{\omega})|^2\, \boldsymbol{d\omega} = \frac{\mathbf{8\pi}}{\mathbf{3}}}$$

(۱ نمره)

$$\hat{x}(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\, dt \quad \rightarrow \quad \hat{x}(0) = \int_{-\infty}^{+\infty} x(t)\, dt$$

مساحت کل زیر نمودار

$$\Rightarrow \boxed{\hat{\boldsymbol{x}}(\mathbf{0}) = \mathbf{0}}$$

(۱ نمره)