Quiz 7 Solution

1) [10]

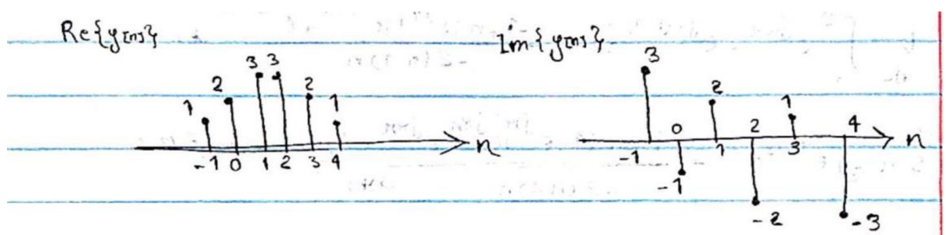
$$X(e^{j\omega}) = \sum_n x[n]e^{-j\omega n} \qquad\qquad x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

الف) $X(e^{j\omega})\big|_{\omega=0} = \sum_n x[n]e^{-j.0.n} = \sum_n x[n] = \sum_n Re\{x[n]\} + \sum_n Im\{x[n]\} = 12$ [2]

ب) $X(e^{j\omega})\big|_{\omega=\pi} = \sum_n x[n](-1)^n = -j12$ [2]

ج) $\int_{-\pi}^{\pi} X(e^{j\omega})d\omega = 2\pi\, x[0] = 2\pi(2-j) = 4\pi - j2\pi$ [2]

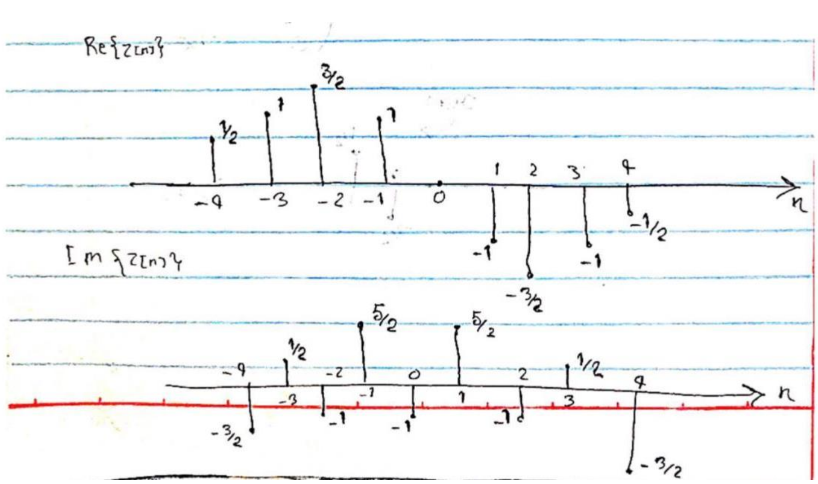
د) $Y(e^{j\omega}) = X(e^{-j\omega}) \rightarrow y[n] = x[-n]$ [2]



ه) $Z(e^{j\omega}) = jIm\{X(e^{j\omega}) = \frac{X(e^{j\omega}) - X^*(e^{j\omega})}{2}$ [2]

$$z[n] = \frac{x[n] - x^*[-n]}{2}$$

$$Re\{z[n]\} = \frac{Re\{x[n]\} - Re\{x[-n]\}}{2} \qquad Im\{z[n]\} = \frac{Im\{x[n]\} + Im\{x[-n]\}}{2}$$



2) [10]

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega = \frac{1}{2\pi}\int_{-\pi}^{0} \sin\left(\frac{\omega}{2}\right) e^{\frac{j\pi}{2}} e^{-\frac{j\omega}{2}} e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{0}^{\pi} \sin\left(\frac{\omega}{2}\right) e^{-\frac{j\pi}{2}} e^{-\frac{j\omega}{2}} e^{j\omega n}\, d\omega$$ [2]

$$I_{*} = \int_{-\pi}^{0} \sin\left(\frac{\omega}{2}\right)\, e^{-\frac{j\omega}{2}} e^{j\omega n}\, d\omega =$$

$$= \frac{1}{j2}\int_{-\pi}^{0} \left(e^{\frac{j\omega}{2}} - e^{-\frac{j\omega}{2}}\right) e^{\frac{j\pi}{2}}\, e^{-\frac{j\omega}{2}} e^{j\omega n}\, d\omega$$

$$= \frac{1}{j2}\int_{-\pi}^{0} e^{j\omega n} - e^{j\omega(n-1)} d\omega = \frac{-1 + n\left(e^{j\pi} - 1\right)e^{-j\pi n} + e^{-j\pi n}}{-2(n-1)n}$$

[3]

$$I_{**} = \int_{0}^{\pi} \sin\left(\frac{\omega}{2}\right)\, e^{-\frac{j\omega}{2}} e^{j\omega n}\, d\omega = \frac{1 + n\left(1 - e^{j\pi}\right)e^{-j\pi n} - e^{-j\pi n}}{-2(n-1)n}$$ [3]

$$x[n] = \frac{1}{2\pi} e^{\frac{j\pi}{2}} I_{*} + \frac{1}{2\pi} e^{-\frac{j\pi}{2}} I_{**} = -\frac{1}{2\pi}\left(\frac{\sin(n\pi)}{n} - \frac{\sin((n-1)\pi)}{n-1}\right)$$

$$= -\frac{1}{2}\delta[n] + \frac{1}{2}\delta[n-1]$$ [2]