به نام خدا

پاسخ تمرین سری ششم

درس سیگنال‌ها و سیستم‌ها - دکتر اخوان

1ـ الف)

$$x_1(t) = te^{-\alpha|t|}\cos(\beta t) \qquad\qquad \alpha > 0$$

$$x_{1\text{-}1}(t) = te^{-\alpha|t|} \quad\rightarrow\quad \hat{x}_{1\text{-}1}(\omega) = F\{te^{-\alpha|t|}\} = j\frac{dF\{e^{-\alpha|t|}\}}{d\omega} = j\frac{d}{d\omega}\left\{\frac{2\alpha}{\alpha^2+\omega^2}\right\} = j\frac{-4\alpha\omega}{(\alpha^2+\omega^2)^2}$$

$$x_{1\text{-}2}(t) = \cos(\beta t) \quad\rightarrow\quad \hat{x}_{1\text{-}2}(\omega) = F\{\cos(\beta t)\} = \pi[\delta(\omega-\beta) + \delta(\omega+\beta)]$$

$$x_1(t) = x_{1\text{-}1}(t)\, x_{1\text{-}2}(t) \quad\xrightarrow{F}\quad \hat{x}_1(\omega) = \frac{1}{2\pi}(\hat{x}_{1\text{-}1}(\omega) * \hat{x}_{1\text{-}2}(\omega))$$

$$=> \hat{x}_1(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} x_{1\text{-}1}(\gamma)\, x_{2\text{-}1}(\omega-\alpha)\, d\gamma$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{-j4\alpha\gamma}{(\alpha^2+\gamma^2)^2}\times\pi[\delta(\omega-\gamma-\beta) + \delta(\omega-\gamma+\beta)]d\gamma$$

$$= \boxed{-j2\alpha\left[\frac{\omega-\beta}{(\alpha^2+(\omega-\beta)^2)^2} + \frac{\omega+\beta}{(\alpha^2+(\omega+\beta)^2)^2}\right]}$$

$$x_2(t) = \left(\frac{\sin(\pi t)}{\pi t}\right)\left(\frac{\sin(2\pi(t-1))}{\pi(t-1)}\right)$$

$$x_{2\text{-}1}(t) = \left(\frac{\sin(\pi t)}{\pi t}\right) = sinc(t) \quad\rightarrow\quad \hat{x}_{2\text{-}1}(\omega) = F\{\text{sinc}(t)\} = \prod\left(\frac{\omega}{2\pi}\right)$$

$$x_{2\text{-}2}(t) = \left(\frac{\sin(2\pi(t-1))}{\pi(t-1)}\right) = 2sinc(2(t-1))$$

$$\rightarrow\quad \hat{x}_{2\text{-}2}(\omega) = F\{2sinc(2(t-1))\} = F\{2sinc(2t)\}e^{-j\omega} = \prod\left(\frac{\omega}{4\pi}\right)e^{-j\omega}$$

$$x_2(t) = x_{2\text{-}1}(t)\, x_{2\text{-}2}(t) \quad\xrightarrow{F}\quad \hat{x}_2(\omega) = \frac{1}{2\pi}(\hat{x}_{2\text{-}1}(\omega) * \hat{x}_{2\text{-}2}(\omega))$$

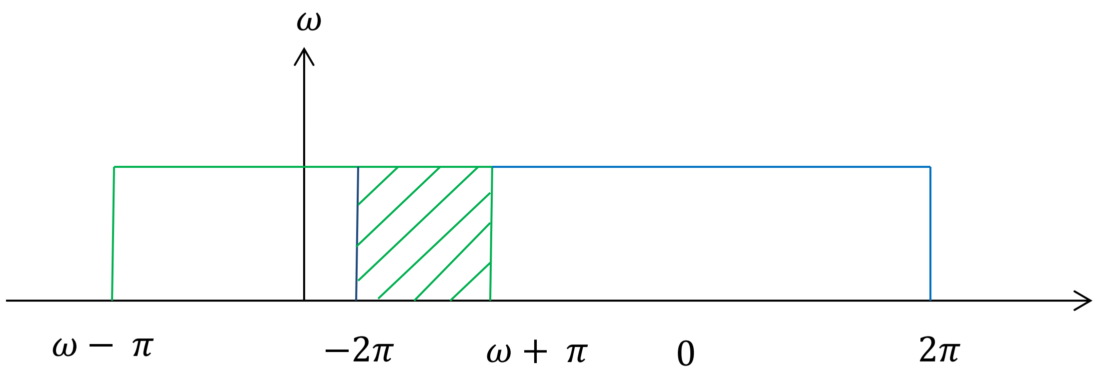
$$\rightarrow\quad \hat{x}_2(\omega) = \frac{1}{2\pi}\left[\prod\left(\frac{\omega}{2\pi}\right) * \prod\left(\frac{\omega}{4\pi}\right)e^{-j\omega}\right]$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\prod\left(\frac{\omega-\gamma}{2\pi}\right)\prod\left(\frac{\gamma}{4\pi}\right)e^{-j\gamma}\, d\gamma$$

$$= \frac{1}{2\pi}\int_{-2\pi}^{2\pi}\prod\left(\frac{\omega-\gamma}{2\pi}\right)e^{-j\gamma}\, d\gamma$$

در تعیین حدود انتگرال باید به این نکته توجه شود که تابع $\prod(\frac{\omega}{2\pi})$ تنها در بازه $[-\pi,\ \pi]$ مقدار غیر صفر دارد. برای سادگی فهم از تحلیل گرافیکی کانولوشن نیز استفاده می کنیم.

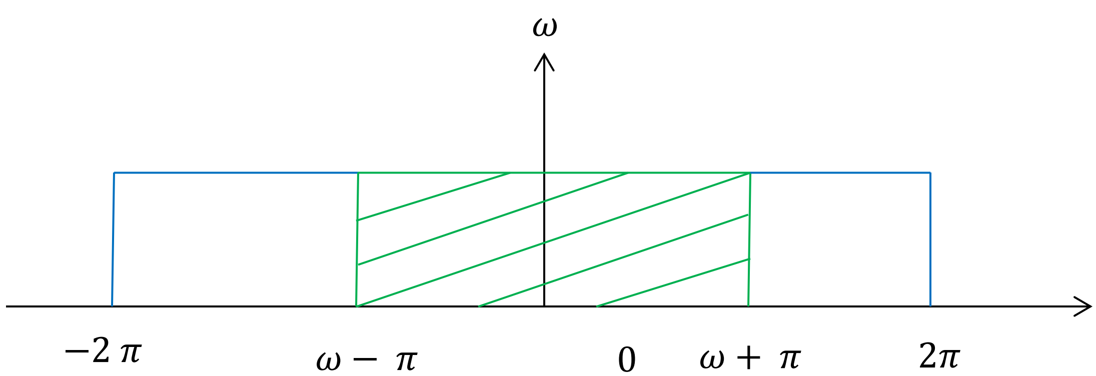
if $-3\pi < \omega < -\pi$



$$\hat{x}_2(\omega) = \frac{1}{2\pi}\int_{-2\pi}^{\omega+\pi} e^{-j\gamma}\, d\gamma = \frac{1}{-j2\pi}\left[\, e^{-j(\omega+\pi)} - e^{j2\pi} \,\right]$$

$$\rightarrow \quad \hat{x}_2(\omega) = \frac{-e^{-j\omega} - 1}{-j2\pi} = \frac{e^{\frac{-j}{2}(\omega+\pi)}}{\pi}\cos\left(\frac{\omega}{2}\right)$$

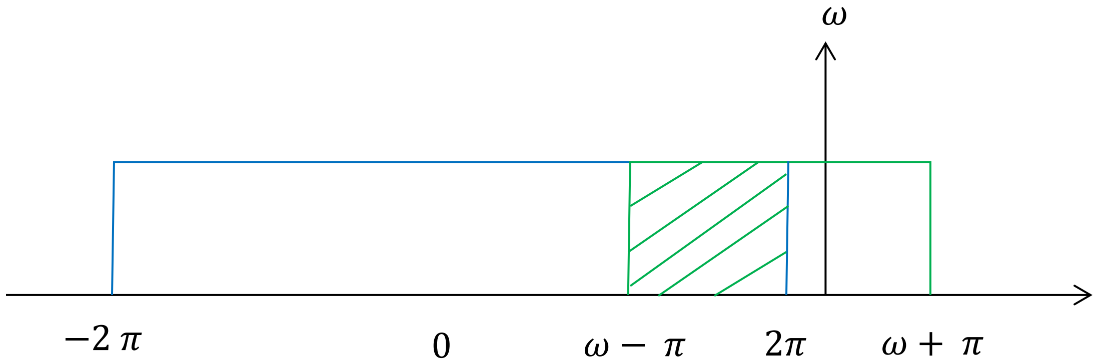
if $-\pi < \omega < \pi$



$$\hat{x}_2(\omega) = \frac{1}{2\pi}\int_{\omega-\pi}^{\omega+\pi} e^{-j\gamma}\, d\gamma = \frac{1}{-j2\pi}\left[\, e^{-j(\omega+\pi)} - e^{-j(\omega-\pi)} \,\right]$$

$$= 0$$

if $\pi < \omega < 3\pi$



$$\hat{x}_2(\omega) = \frac{1}{2\pi}\int_{\omega-\pi}^{2\pi} e^{-j\gamma}\, d\gamma = \frac{1}{-j2\pi}\left[\, e^{-j2\pi} - \; e^{-j(\omega-\pi)}\,\right]$$

$$= \frac{1+e^{-j\omega}}{-j2\pi} = \frac{e^{-j(\omega-\frac{\pi}{2})}}{\pi}\cos(\omega)$$

$$\Rightarrow \boxed{\boldsymbol{\hat{x}_2(\omega)} = \begin{cases} \dfrac{\mathbf{1+e^{-j\omega}}}{\mathbf{-j2\pi}} & ; \quad \boldsymbol{\pi < \omega < 3\pi} \\ \dfrac{\mathbf{1+e^{-j\omega}}}{\mathbf{j2\pi}} & ; \quad \boldsymbol{-3\pi < \omega < -\pi} \end{cases}}$$

ب)

$$\int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{x}(\omega)\, \hat{y}(\omega)\, d\omega$$

$$z(t) = \; x(t)\, y^*(t) \quad \rightarrow \quad \hat{z}(\omega) = \int_{-\infty}^{+\infty} z(t)e^{-j\omega t}dt = \int_{-\infty}^{+\infty} x(t)y^*(t)e^{-j\omega t}dt$$

$$\rightarrow \quad \hat{z}(0) = \int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt \qquad \textbf{(I)}$$

از بخش قبل می دانیم $\rightarrow \quad \hat{z}(\omega) = \frac{1}{2\pi}(\hat{x}(\omega) * \hat{y}^*(-\omega))$

$$\rightarrow \quad \hat{z}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{x}(\gamma)\, \hat{y}^*(-(\omega-\gamma))\, d\gamma$$

$$\rightarrow \quad \hat{z}(0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{x}(\gamma)\, \hat{y}^*(\gamma)\, d\gamma \quad \textbf{(II)}$$

$$\overset{\text{I,II}}{\Rightarrow} \boxed{\int_{-\infty}^{+\infty} \boldsymbol{x(t)\, y^*(t)\, dt} = \frac{\mathbf{1}}{\mathbf{2\pi}} \int_{-\infty}^{+\infty} \boldsymbol{\hat{x}(\omega)\, \hat{y}^*(\omega)\, d\omega}}$$

ج)

$$\mathrm{I}_1 = \int_0^{+\infty} \frac{dx}{(a^2+x^2)^2} = \frac{1}{2}\int_{-\infty}^{+\infty} \frac{dx}{(a^2+x^2)^2}$$

از قضیه بخش ب و استفاده از $: F\{e^{-\alpha|t|}\} = \frac{2\alpha}{\alpha^2+\omega^2}$

$$\int_{-\infty}^{+\infty} e^{-a|t|} \,.\, e^{-a|t|}\, dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{4a^2}{(a^2+\omega^2)^2}\, d\omega = \frac{4a^2}{\pi}\, \mathrm{I}_1$$

$$=> \quad \int_{-\infty}^{+\infty} e^{-2a|t|}\, dt = 2\int_{\mathbf{0}}^{+\infty} e^{-2a|t|}\, dt = \frac{1}{a}$$

$$\rightarrow \quad \boxed{\mathbf{I}_1 = \frac{\boldsymbol{\pi}}{\mathbf{4}\boldsymbol{a}^3}}$$

$$I_2 = \int_{\mathbf{0}}^{+\infty} \frac{sin^4(t)}{t^4}\, dt = \frac{1}{2}\int_{-\infty}^{+\infty} \frac{sin^4(t)}{t^4}\, dt$$

از رابطه بخش ب و $:F\left\{\Lambda(\frac{t}{2})\right\} = 2sinc^2(\frac{\omega}{\pi}) = \frac{2sin^2(\omega)}{\omega^2}$

$$\int_{-\infty}^{+\infty} \Lambda(\frac{t}{2})\ \Lambda(\frac{t}{2})\, dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{4sin^4(\omega)}{\omega^4}\, d\omega = \frac{4}{\pi}\, I_2$$

$$\int_{-\mathbf{2}}^{0} (1+\frac{t}{2})^2\, dt + \frac{1}{2}\int_{0}^{2} (1-\frac{t}{2})^2\, dt = \frac{4}{3}$$

$$\rightarrow \boxed{\mathbf{I_2 = \frac{\pi}{3}}}$$

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2 - الف)

$$x_1(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{j\omega t}\, d\omega \qquad\qquad X(j\omega) = |X(j\omega)|\, e^{j\measuredangle X(j\omega)}$$

$$= \frac{1}{2\pi}\int_{-3\pi}^{0} -3\omega e^{-j\frac{\pi}{2}}\, e^{j\omega t} d\omega + \frac{1}{2\pi}\int_{0}^{3\pi} 3\omega e^{j\frac{\pi}{2}}\, e^{j\omega t} d\omega$$

$$= \frac{j3}{2\pi}\int_{-3\pi}^{0} \omega e^{j\omega t} d\omega + \frac{j3}{2\pi}\int_{0}^{3\pi} \omega e^{j\omega t} d\omega = \frac{j3}{2\pi}\int_{-3\pi}^{3\pi} \omega e^{j\omega t} d\omega$$

$$= \frac{e^{-j3\pi t}.\left((j9\pi t-3)e^{j6\pi t}+ j9\pi t+3\right)}{2\pi\times jt^2}$$

$$= \boxed{\boldsymbol{\frac{9}{t}cos(3\pi t) - \frac{3}{\pi t^2}sin(3\pi t)}}$$

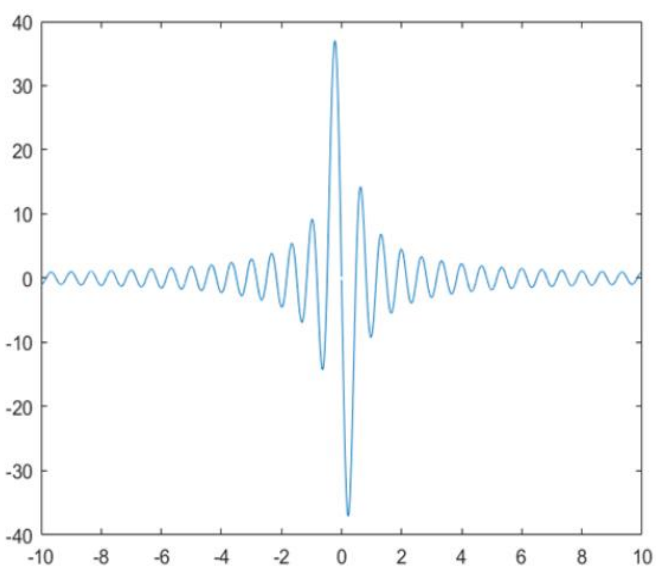
ب)

$$x_2(t) = \frac{3}{2\pi}\int_{-3\pi}^{0} -\omega e^{j\omega t}d\omega + \frac{3}{2\pi}\int_{0}^{3\pi} \omega e^{j\omega t}d\omega$$

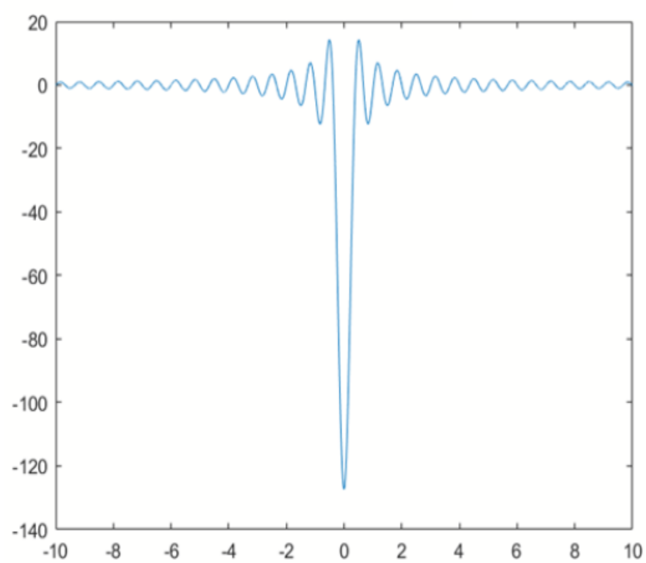
$$= \frac{1}{2\pi}\left[\frac{e^{-j3\pi t}\left(3e^{j3\pi t} - j9\pi t - 3\right)}{j^2t^2} + \frac{e^{j3\pi t}\left(j9\pi t - 3\right) + 3}{j^2t^2}\right]$$

$$= \frac{9sin(3\pi t)}{t} - \frac{3cos(3\pi t)}{\pi t^2} - \frac{3}{\pi t^2}$$

ج) با رسم در متلب:



شکل بخش اول



شکل بخش دوم

$x_2(t)$ حقیقی و زوج $\leftarrow$ $X_2(j\omega)$ $\leftarrow$ حقیقی و زوج

$x_1(t)$ حقیقی و فرد $\leftarrow$ $X_1(j\omega)$ $\leftarrow$ موهومی خالص و فرد

3

$$h_e(t) = \frac{h(t) + h(-t)}{2} \quad \xrightarrow{F} \quad \hat{h}_e(\omega) = \frac{\hat{h}(\omega) + \hat{h}(-\omega)}{2}$$

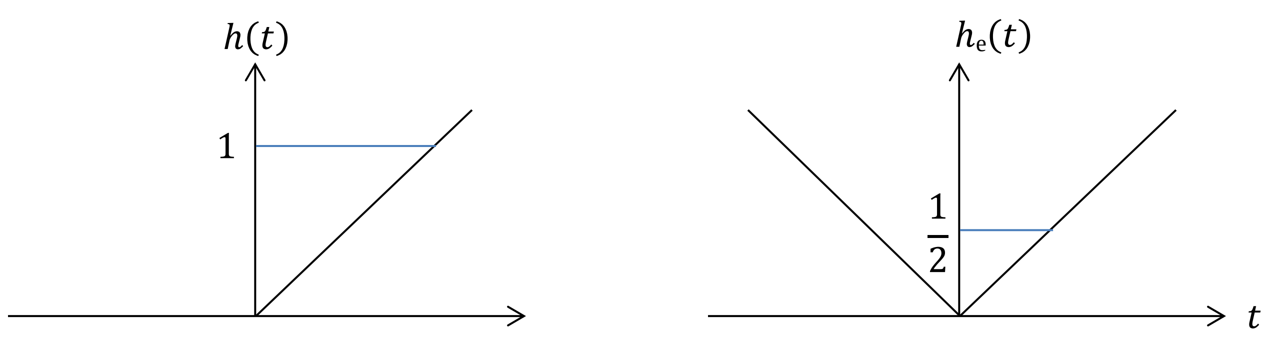
$h(t)$ حقیقی است $\Rightarrow \quad h(t) = h^*(t) \quad \xrightarrow{F} \quad \hat{h}(\omega) = \hat{h}^*(-\omega) \quad \rightarrow \quad \hat{h}(-\omega) = \hat{h}^*(\omega)$

$$\Rightarrow \quad \hat{h}_e(\omega) = \frac{\hat{h}(\omega) + \hat{h}^*(\omega)}{2} = Re\{\hat{h}(\omega)\}$$

$h(t)$ علی است $\Leftarrow \quad h(t) = 0 \quad : \quad \forall\, t < 0$

$h(t) = 2h_e(t)u(t)$ $\dashrightarrow$

پاورقی: برای درک بهتر این فرمول مثال زیر را بگیرید:



$$h_e(t) = \frac{h(t) + h(-t)}{2}$$

از آنجایی که $F\{h_e(t)\} = R_e\{\hat{h}(\omega)\}$ و $h(t) = 2h_e(t)u(t)$ می توانیم تنها با داشتن بخش حقیقی $\hat{h}(\omega)$ و $h_e(t)$ را محاسبه و سپس $h(t)$ را بدست آوریم

ب)

$$R_e\{\hat{h}(\omega)\} = cos^2(\omega) + \frac{1}{1+\omega^2}$$

$$\rightarrow \quad F^{-1}\{Re\{\hat{h}(\omega)\}\} = F^{-1}\left\{\frac{1}{2} + \frac{e^{j2\omega}}{4} + \frac{e^{-j2\omega}}{4} + \frac{1}{1+\omega^2}\right\}$$

$$\rightarrow \quad h_e(t) = \frac{1}{2}\delta(t) + \frac{1}{4}\delta(t-2) + \frac{1}{4}\delta(t+2) + \frac{e^{-|t|}}{2}$$

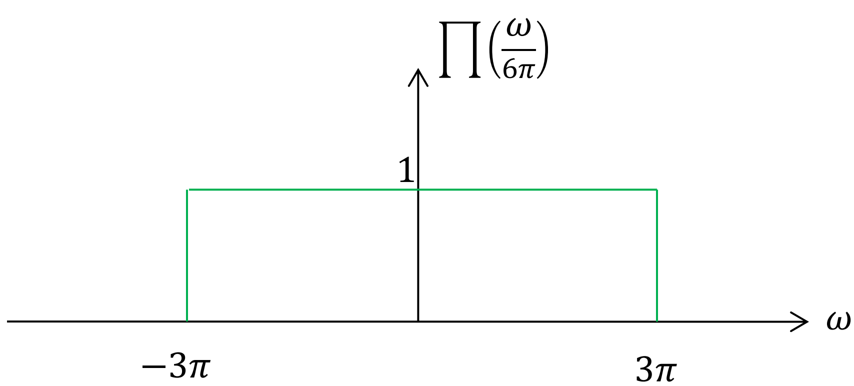
$$h(t) = 2h_e(t)u(t) \quad \rightarrow \quad \boxed{\boldsymbol{h(t) = e^{-t}u(t) + \frac{1}{2}\delta(t-2) + \delta(t)}}$$

4

$$h(t) = \frac{sin(3\pi(t-2))}{\pi(t-2)} = 3sinc(3(t-2))$$

$$\rightarrow \quad \hat{h}(\omega) = F\{3sinc(3(t-2))\} = F\{3sinc(3t)\}\, e^{-j2\omega}$$

$$= \prod\left(\frac{\omega}{6\pi}\right) e^{-j2\omega}$$



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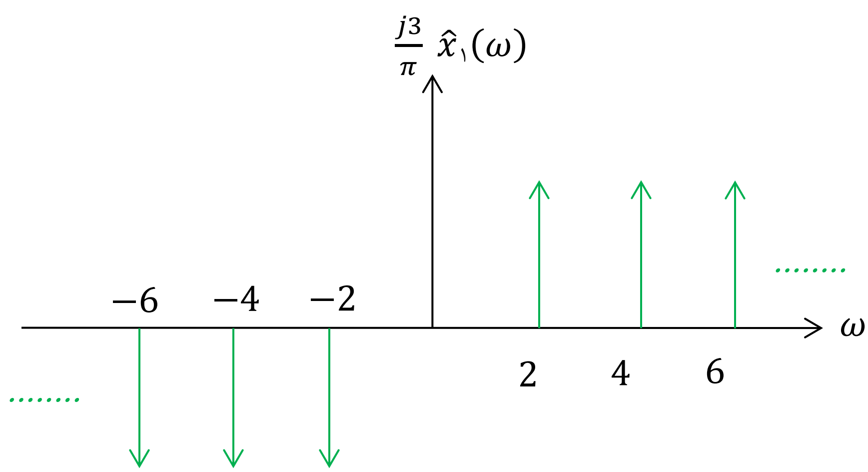
$$x_1(t) = \sum_{k=0}^{\infty} \left(\frac{1}{3}\right) sin(2kt) = \sum_{k=0}^{\infty} \left(\frac{1}{3}\right) \left[\frac{e^{j2kt} - e^{-j2kt}}{2j}\right]$$

$$\xrightarrow{F} \quad \hat{x}_1(\omega) = \sum_{k=0}^{\infty} \pi \left(\frac{1}{3}\right) \left[\frac{\delta(\omega-2k)-\delta(\omega+2k)}{j}\right]$$



$\leftarrow$ می توان دید $\hat{h}(\omega)$ مانند فیلتر، تنها بعضی از ضربه ها را عبور می دهد

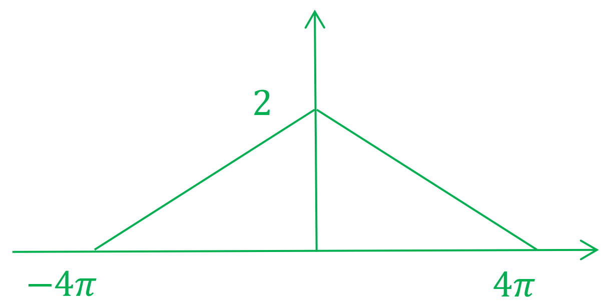
$$\rightarrow \quad \hat{y}_1(\omega) = \hat{x}_1(\omega)\hat{h}(\omega) = \prod\left(\frac{\omega}{6\pi}\right) e^{-j2\omega} \sum_{k=0}^{\infty} \left(\frac{\pi}{3}\right) \left[\frac{\delta(\omega-2k)-\delta(\omega+2k)}{j}\right]$$

$$\rightarrow \quad \hat{y}_1(\omega) = e^{-j2\omega} \sum_{k=0}^{4} \left(\frac{\pi}{3}\right) \left[\frac{\delta(\omega-2k)-\delta(\omega+2k)}{j}\right]$$

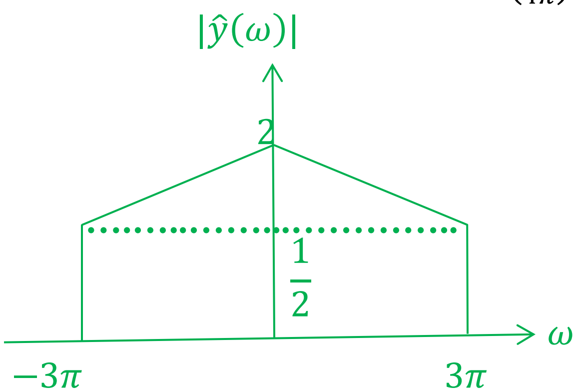
$$\overset{F^{-1}}{\Longrightarrow} \quad \boxed{\boldsymbol{y_1(t) = \sum_{k=0}^{4} \left(\frac{1}{3}\right) sin(2k(t-2))}}$$

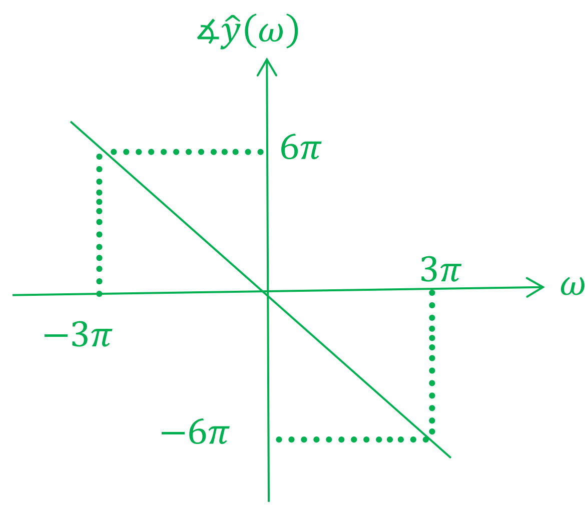
$$x_2(t) = \left(\frac{\sin(2\pi t)}{\pi t}\right)^2 = 4\, sinc^2(2t)$$

$$\hat{x}_2(\omega) = F\{4\, sinc^2(2t)\} = 2\Lambda(\frac{\omega}{4\pi})$$



$$\rightarrow \quad \hat{y}_2(\omega) = \hat{x}_2(\omega)\, \hat{h}(\omega) = 2\Lambda\left(\frac{\omega}{4\pi}\right) \prod\left(\frac{\omega}{6\pi}\right) e^{-j2\omega}$$





$$y_2(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{y}_2(\omega)\, e^{j\omega t} d\omega = \frac{1}{2\pi}\int_{-3\pi}^{3\pi} |\hat{y}\,(\omega)|\, e^{-j2\omega}\, e^{j\omega t} d\omega$$

$$= \frac{1}{2\pi}\int_{-3\pi}^{0} (\frac{\omega}{2\pi} + 2)\, e^{j\omega(t-2)}\, d\omega + \frac{1}{2\pi}\int_{0}^{3\pi} (\frac{-\omega}{2\pi} + 2)\, e^{j\omega(t-2)}\, d\omega$$

$$= \frac{1}{4\pi^2}\int_{-3\pi}^{0} \omega e^{j\omega(t-2)}\, d\omega + \frac{4}{2\pi}\int_{-3\pi}^{3\pi} e^{j\omega(t-2)} d\omega + \frac{1}{4\pi^2}\int_{0}^{3\pi} -\omega e^{j\omega(t-2)}\, d\omega$$

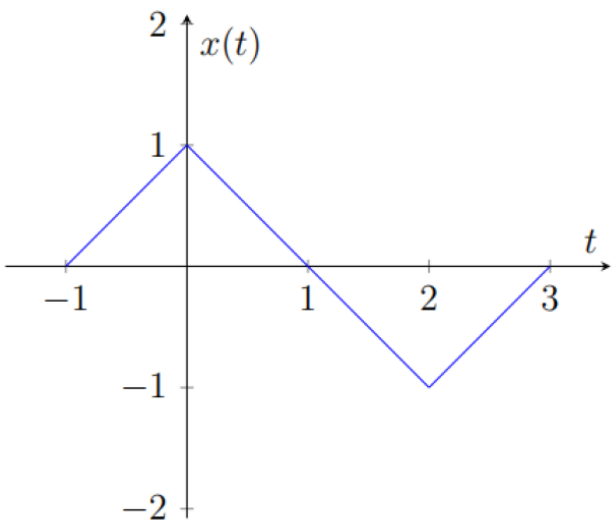
$$= \frac{1}{4\pi^2}\left[\frac{3\pi}{j(t-2)} e^{-j3\pi(t-2)} + \frac{e^{-j3\pi(t-2)}}{j^2(t-2)^2} - \frac{1}{j^2(t-2)^2}\right] + \frac{4\left(e^{j3\pi(t-2)} - e^{j3\pi(t-2)}\right)}{j(t-2)2\pi}$$

$$+ \frac{-1}{4\pi^2}\left[\frac{3\pi}{j(t-2)} e^{j3\pi(t-2)} - \frac{e^{j3\pi(t-2)}}{j^2(t-2)^2} + \frac{1}{j^2(t-2)^2}\right]$$

$$= \frac{-3sin\,(3\pi(t-2))}{2\pi(t-2)} - \frac{\cos\big(3\pi(t-2)\big)}{4\pi^2(t-2)^2} + \frac{1}{4\pi^2(t-2)^2} + \frac{4sin\,(3\pi(t-2))}{\pi(t-2)}$$

$$\rightarrow \boxed{\boldsymbol{y_2(t) = \frac{5}{2} \times \frac{sin\,(3\pi(t-2))}{\pi(t-2)} - \frac{\cos\big(3\pi(t-2)\big)}{4\pi^2(t-2)^2} + \frac{1}{4\pi^2(t-2)^2}}}$$

5



$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{x}(\omega)\, e^{j\omega t} d\omega$$

الف)

$$F^{-1}\left\{\frac{2sin\,(\omega)}{\omega}\right\} = \prod\left(\frac{t}{2}\right)$$

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{x}(\omega)\, \frac{2sin\,(\omega)}{\omega}\, e^{j\omega t} d\omega = x(t) * \prod\left(\frac{t}{2}\right)$$

$$\rightarrow \quad \int_{-\infty}^{+\infty} \hat{x}(\omega)\, \frac{2sin\,(\omega)}{\omega}\, e^{j2\omega} d\omega = \ 2\pi \int_{-1}^{1} x(2-\alpha)\, d\alpha = \ -2\pi$$

$$\Rightarrow \boxed{\int_{-\infty}^{+\infty} \hat{x}(\omega)\, \frac{2\sin(\omega)}{\omega}\, e^{j2\omega} d\omega = -2\pi}$$

$$x(0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{x}(\omega)\, d\omega \quad \rightarrow \quad \boxed{\int_{-\infty}^{+\infty} \hat{x}(\omega)\, d\omega = 2\pi}$$

$x(t+1)$ تابع فرد حقیقی است پس تبدیل فوریه آن فرد و موهومی خالص است

$$F\{x(t+1)\} = \hat{x}(\omega)\, e^{j\omega} \quad \rightarrow \quad \measuredangle\hat{x}(\omega) + \omega = \frac{\pi}{2}$$

$$\rightarrow \boxed{\measuredangle\hat{x}(\omega) = \frac{\pi}{2} - \omega}$$

-1

$$\frac{dx(t)}{dt}\Big|_{t=1} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} (j\omega)\, \hat{x}(\omega)\, e^{j\omega t} d\omega$$

$$\rightarrow \boxed{\int_{-\infty}^{+\infty} \omega\, \hat{x}(\omega)\, e^{j\omega t} d\omega = j2\pi}$$

پارسوال $\rightarrow \int_{-\infty}^{+\infty} |x(t)|^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |\hat{x}(\omega)|^2\, d\omega$

$$= \int_{-1}^{0} (t+1)^2\, dt + \int_{0}^{2} (1-t)^2\, dt + \int_{2}^{3} (3-t)^2\, dt = \frac{1}{3} + \frac{2}{3} + \frac{1}{3}$$

$$\Rightarrow \boxed{\int_{-\infty}^{+\infty} |\hat{\boldsymbol{x}}(\boldsymbol{\omega})|^2 \, \boldsymbol{d\omega} = \frac{\mathbf{8\pi}}{\mathbf{3}}}$$

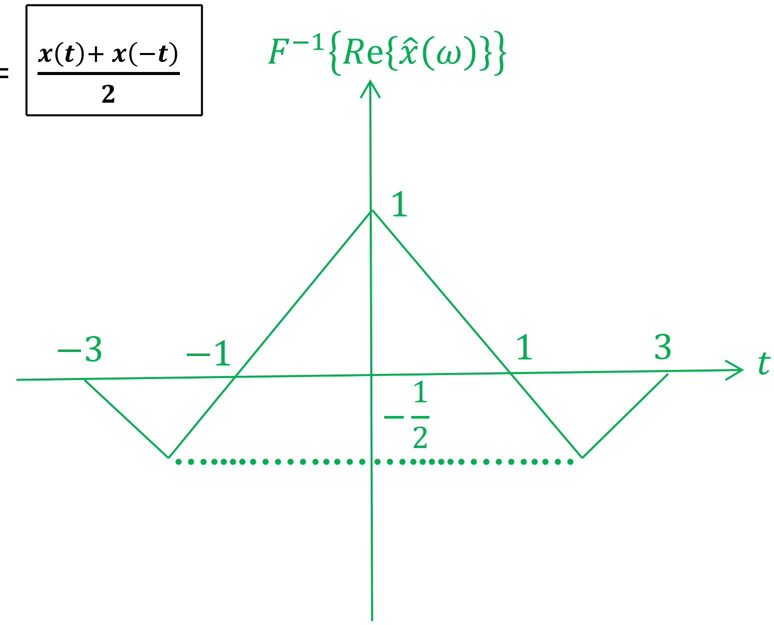
$$\hat{x}(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\, dt \quad \rightarrow \quad \hat{x}(0) = \int_{-\infty}^{+\infty} x(t)\, dt$$

مساحت کل زیر نمودار

$$\Rightarrow \boxed{\hat{\boldsymbol{x}}(\boldsymbol{\omega}) = \mathbf{0}}$$

ب)

$$\boldsymbol{R_e}\{\hat{\boldsymbol{x}}(\boldsymbol{\omega})\} = \frac{\hat{\boldsymbol{x}}(\boldsymbol{\omega}) + \hat{\boldsymbol{x}}^*(\boldsymbol{\omega})}{\mathbf{2}} \xrightarrow{\boldsymbol{F^{-1}}} \frac{\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{x}^*(-\boldsymbol{t})}{\mathbf{2}} = \boxed{\frac{\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{x}(-\boldsymbol{t})}{\mathbf{2}}}$$



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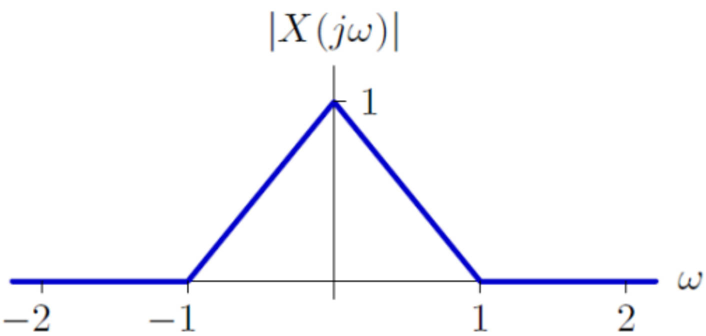

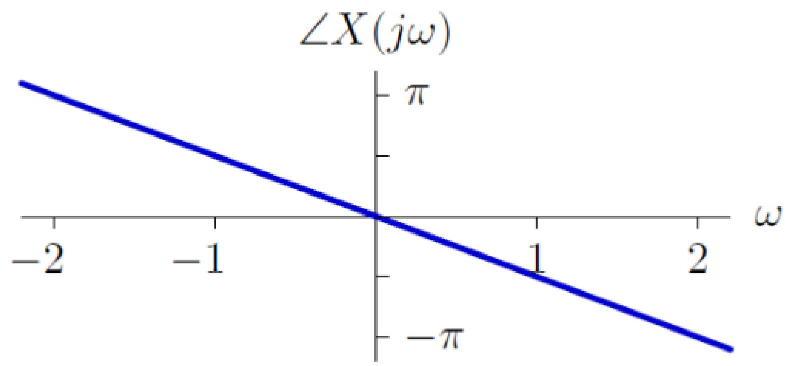

$$|X(j\omega)|\, e^{j\measuredangle X(j\omega)} = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\, dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(j\omega)|\, e^{j\measuredangle X(j\omega)} e^{j\omega t} dt$$

$$x_1(t) = \frac{dx(t)}{dt} \quad \rightarrow \quad |X_1(j\omega)| = |\omega X(j\omega)|$$

$$\measuredangle X_1(j\omega) = \begin{cases} \measuredangle X(j\omega) + \frac{\pi}{2} & ; \quad \omega > 0 \\ \measuredangle X(j\omega) - \frac{\pi}{2} & ; \quad \omega < 0 \end{cases}$$

هنگامی که ω منفی می شود، اندازه منفی شده که برای جبران آن π از فاز کم می شود.

$\Rightarrow$ $\boxed{\mathbf{M_5 \ \& \ A_4}}$

$$x_2(t) = (x * x)_{(t)} \quad \rightarrow \quad X_2(j\omega) = |X(j\omega)|^2$$

$$\measuredangle X_2(j\omega) = 2\measuredangle X(j\omega)$$

$\rightarrow$ $\boxed{\mathbf{M_3\ \&\ A_2}}$

$$x_3(t) = x\left(t - \frac{\pi}{2}\right) \quad \rightarrow \quad |X_3(j\omega)| = |X(j\omega)|$$

$$\measuredangle X_3(j\omega) = \measuredangle X(j\omega) - \frac{\pi}{2}\omega$$

$\rightarrow$ $\boxed{\mathbf{M_1\ \&\ A_2}}$

$$x_4(t) = x(2t) \quad \rightarrow \quad |X_4(j\omega)| = \frac{1}{2}\left|X\left(j\frac{\omega}{2}\right)\right|$$

$$\measuredangle X_4(j\omega) = \measuredangle X\left(j\frac{\omega}{2}\right)$$

$\rightarrow$ $\boxed{\mathbf{M_4\ \&\ A_3}}$

$$x_5(t) = x^2(t) \quad \rightarrow \quad X_5(j\omega) = \frac{1}{2\pi}\big(X(j\omega) * X(j\omega)\big)$$

$$\rightarrow \quad X_5(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(j\alpha)|\, e^{j\measuredangle X(j\alpha)} |X(j(\omega-\alpha))|\, e^{j\measuredangle X(j(\omega-\alpha))}\, d\alpha$$

$$\rightarrow \quad |X_5(j\omega)| = |X(j\omega)| * |X(j\omega)|\ \frac{1}{2\pi}$$

چون $\measuredangle X(j\omega)$ دارای شکل خطی است

$$\measuredangle X_5(j\omega) = \measuredangle X(j\omega)$$

$\rightarrow$ $\boxed{\mathbf{M_6\ \&\ A_1}}$

Signal	magnitude	Angle
$\frac{dx(t)}{dt}$	M_5	A_4
$(x * x)(t)$	M_3	A_2
$x\left(t-\frac{\pi}{2}\right)$	M_1	A_2
$x(2t)$	M_4	A_3
$x^2(t)$	M_6	A_1

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$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$y(t) = x(t-\mathrm{T}_1) + \epsilon\, x(t-\mathrm{T}_2) = x(t) * [\,\delta(t-\mathrm{T}_1) + \epsilon\, \delta(t-\mathrm{T}_2)\,]$$

$$\rightarrow \quad h(t) = \delta(t-\mathrm{T}_1) + \epsilon\, \delta(t-\mathrm{T}_2) \quad \rightarrow \quad \hat{h}(\omega) = e^{-j\omega \mathrm{T1}} + \epsilon\, e^{-j\omega \mathrm{T2}}$$

$$\hat{y}(\omega) = \hat{x}(\omega)\left[e^{-j\omega \mathrm{T1}} + \epsilon\, e^{-j\omega \mathrm{T2}}\right]$$

$$\left|\hat{h}(\omega)\right| = \sqrt{\hat{h}(\omega)\, \hat{h}^{*}(\omega)} = \sqrt{\left(e^{-j\omega \mathrm{T1}} + \epsilon\, e^{-j\omega \mathrm{T2}}\right)\left(e^{j\omega \mathrm{T1}} + \epsilon\, e^{j\omega \mathrm{T2}}\right)}$$

$$= \sqrt{1 + \epsilon\left(e^{j\omega(\mathrm{T2}-\mathrm{T1})} + e^{-j\omega(\mathrm{T2}-\mathrm{T1})}\right) + \epsilon^2} = \sqrt{1 + 2\epsilon\cos\big(\omega(\mathrm{T2}-\mathrm{T1})\big) + \epsilon^2}$$

$$\left|\hat{h}(\omega)\right|_{\omega=0} = 1.2 \quad \rightarrow \quad 1.44 = 1 + 2\epsilon + \epsilon^2 = (1+\epsilon)^2$$

$$\rightarrow \quad \begin{cases} 1+\epsilon = 1.2 & \rightarrow \quad \epsilon = 0.2 \ \checkmark \\ 1+\epsilon = -1.2 & \rightarrow \quad \epsilon = -2.2 \ . \text{※} \end{cases}$$

$$\text{دوره تناوب} = \frac{2\pi}{T2-T1} = 750 \quad \rightarrow \quad T_2 - T_1 = \frac{2\pi}{750}$$

$$\measuredangle\hat{h}(\omega)|_{\omega=1500} = -\pi \quad , \quad \hat{h}(\omega) = e^{-j\omega T1}\left[1 + \epsilon\, e^{-j\omega(T2-T1)}\right]$$

$$\rightarrow \quad \measuredangle\hat{h}(\omega) = -\omega T_1 + tan^{-1}\left(\frac{\epsilon\, sin(\omega(T2-T1))}{1+\epsilon\, cos(\omega(T2-T1))}\right) \quad , \quad T_2 - T_1 = \frac{2\pi}{750}$$

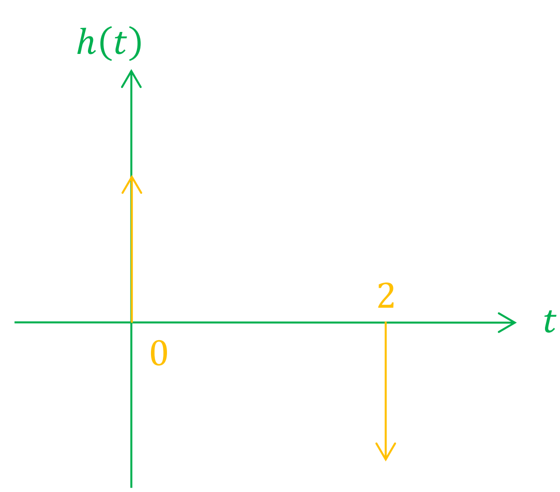
$$\rightarrow \quad \measuredangle\hat{h}(\omega)|_{\omega=1500} = -1500T_1 = -\pi \quad \rightarrow \quad T_1 = \frac{\pi}{1500}$$

$$\Rightarrow \quad \boxed{\begin{cases} \mathbf{T1} = \dfrac{\boldsymbol{\pi}}{\mathbf{1500}} \\ \mathbf{T2} = \dfrac{\mathbf{5}\boldsymbol{\pi}}{\mathbf{1500}} \\ \boldsymbol{\epsilon} = \mathbf{0.2} \end{cases}}$$

8 می توانیم ابتدا پاسخ ضربه کلی سیستم را بست آوریم.

$$\hat{h}_2(\omega) = j\omega \quad \xrightarrow{F^{-1}} \quad h_2(t) = \delta'(t)$$

$$h(t) = h_1(t) * \delta'(t) = h_1{}'(t)$$



به نام خدا

پاسخ تمرین سری ششم

درس سیگنال‌ها و سیستم‌ها - دکتر اخوان

$$\rightarrow \quad h(t) = \delta(t) - \delta(t-2) \quad \rightarrow \quad \hat{h}(\omega) = 1 - e^{-j2\omega}$$

الف)

$$x(t) = 2\prod\left(\frac{t-2}{4}\right) - 2\prod\left(\frac{t-6}{4}\right)$$

$$\xrightarrow{F} \quad \hat{x}(\omega) = 8\,sinc\left(\frac{2\omega}{\pi}\right)e^{-j2\omega} - 8\,sinc\left(\frac{2\omega}{\pi}\right)e^{-j6\omega}$$

$$\rightarrow \quad \hat{x}(\omega) = 8\,sinc\left(\frac{2\omega}{\pi}\right)\left[e^{-j2\omega} - e^{-j6\omega}\right]$$

$$\rightarrow \quad \boxed{\boldsymbol{\hat{y}(\omega) = \hat{x}(\omega)\hat{h}(\omega) = 8\,sinc\left(\frac{2\omega}{\pi}\right)\left[e^{-j2\omega} - e^{-j6\omega} - e^{-j4\omega} + e^{-j8\omega}\right]}}$$

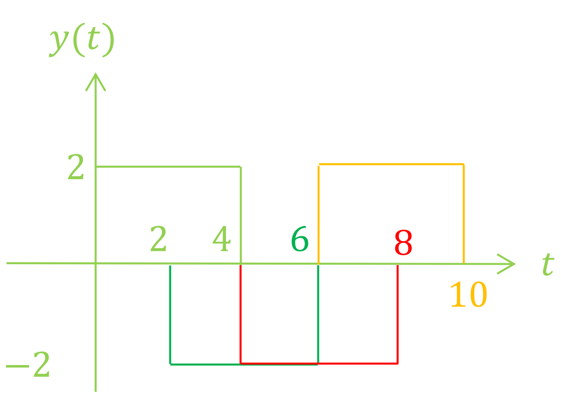
ب) همان طور که بدست آوردیم

$$h(t) = \delta(t) - \delta(t-2)$$

ج)

$$\hat{y}(\omega) = 8\,sinc\left(\frac{2\omega}{\pi}\right)\left[e^{-j2\omega} - e^{-j6\omega} - e^{-j4\omega} + e^{-j8\omega}\right]$$

$$\xrightarrow{F^{-1}} \quad \boxed{\boldsymbol{y(t) = 2\prod\left(\frac{t-2}{4}\right) - 2\prod\left(\frac{t-6}{4}\right) - 2\prod\left(\frac{t-4}{4}\right) + 2\prod\left(\frac{t-8}{4}\right)}}$$



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