به نام خدا

تمرین سری سوم

درس سیگنال‌ها و سیستم‌ها - دکتر اخوان

1ـ شناسایی سیستم

اگر $h[n]$ پاسخ ضربه ی سیستم باشد، $y[n] = x[n] * h[n]$ درنتیجه کافیست $h[n]$ را برحسب $g[n]$ پیدا کنیم.

تعریف میکنیم: $w[n] = (\frac{1}{3})^n u[n]$

$$T(w[n]) = g[n]\,,\;\; T(w[n-1]) = g[n-1]$$

از طرفی $\delta[n] = u[n] - u[n-1]$

$$w[n] - \frac{1}{3}w[n-1] = \left(\frac{1}{3}\right)^n u[n] - \left(\frac{1}{3}\right)^n u[n-1] = \left(\frac{1}{3}\right)^n \delta[n] = \delta[n]$$

واضح است $a^n\delta[n] = \delta[n]$

در نتیجه:

$$h[n] = T(\delta[n]) = T\left(w[n] - \frac{1}{3}w[n-1]\right) = T(w[n]) - \frac{1}{3}T(w[n-1]) = g[n] - \frac{1}{3}g[n-1]$$

$$y[n] = x[n] * h[n] \Longrightarrow y[n] = x[n] * (g[n] - \frac{1}{3}g[n-1])$$

2ـ شناسایی سیستم

اگر $h[n]$ پاسخ ضربه ی سیستم باشد،

$$\left\{\begin{array}{c} \forall n \notin [-1,1]: x[n] = 0 \\ \forall n \notin [-1,4]: y[n] = 0 \\ y[n] = \displaystyle\sum_{k=-\infty}^{+\infty} x[n-k]\,h[k] \end{array}\right\} \Longrightarrow \forall n \notin [0,3]:\; h[n] = 0$$

$n=-1:\quad y[-1]=x[-1]\ h[0]+x[0]\ h[-1]+x[1]\ h[-2]=2\rightarrow h[0]=2$

$n=0:\quad y[0]=x[-1]h[1]+x[0]h[0]+x[1]h[-1]=5\rightarrow h[1]=1$

$n=4:\quad y[4]=x[-1]h[5]+x[0]h[4]+x[1]h[3]=6\rightarrow h[3]=2$

$n=3:\quad y[3]=x[-1]h[4]+x[0]h[3]+x[1]h[2]=7\rightarrow h[2]=1$

$n=2:\quad y[2]=x[-1]h[3]+x[0]h[2]+x[1]h[1]=7\rightarrow\ b=7$

$n=1:\quad y[1]=x[-1]h[2]+x[0]h[1]+x[1]h[0]=9\rightarrow a=9$

3ـ خواص کانولوشن

بلوک مشتق گیر: $\delta'(t)$

$$x(t)=\int_{-\infty}^{+\infty}x(\tau)\,\delta(t-\tau)d\tau\rightarrow x'(t)=\frac{d}{dt}\int_{-\infty}^{+\infty}x(\tau)\,\delta(t-\tau)d\tau=\int_{-\infty}^{+\infty}x(\tau)\,(\frac{d}{dt}\delta(t-\tau))d\tau$$
$$=\int_{-\infty}^{+\infty}x(\tau)\,\delta'(t-\tau)d\tau=x(t)*\delta'(t)$$

بلوک انتگرال گیر: u(t)

$$x(t)*u(t)=\int_{-\infty}^{+\infty}x(\tau)\,u(t-\tau)d\tau=\int_{-\infty}^{t}x(\tau)\,d\tau$$

$$u(t-\tau)=\begin{cases}0 & \tau<0\\ 1 & o.w\end{cases}$$

ب)

$i)$

استفاده از مشتق گیر

$$y'(t)=y(t)*\delta'(t)=\big(x(t)*h(t)\big)*\delta'(t)=x(t)*\big(h(t)*\delta'(t)\big)=x(t)*h'(t)$$

$$y'(t)=\delta'(t)*\big(x(t)*h(t)\big)=\big(\delta'(t)*x(t)\big)*h(t)=x'(t)*h(t)$$

ii)

استفاده از انتگرال گیر

$$y(t) = x(t) * u(t) * \delta'(t) * h(t) = \big(x(t) * u(t)\big) * \big(\delta'(t) * h(t)\big) = \left(\int_{-\infty}^{t} x(\tau)\, d\tau\right) * h'(t)$$

$$y(t) = u(t) * \Big(\big(\, x(t) * \delta'(t)\big) * h(t)\Big) = u(t) * [x'(t) * h(t)] = \int_{-\infty}^{t} [x'(\tau) * h(\tau)] d\tau$$

$$y(t) = x(t) * \delta'(t) * u(t) * h(t) = \big(x(t) * \delta'(t)\big) * \big(u(t) * h(t)\big) = x'(t) * \left(\int_{-\infty}^{t} h(\tau)\, d\tau\right)$$

iii)

$$y(t) = x(t) * h(t) = x(t + kT) * h(t) = \int_{-\infty}^{+\infty} x(\tau + kT) h(t - \tau)\, d\tau \xrightarrow{\tau + kT = w} \; y(t)$$
$$= \int_{-\infty}^{+\infty} x(w) h(t + kT - w) dw = y(t + kT)$$

ج)

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$$[h(t+3) * x(t-5) * x(t)] * h(t) = [h(t+3) * x(t-5)] * [x(t) * h(t)] = y(t-2) * y(t)$$

$$x(t-5) * h(t+3) = \int_{-\infty}^{+\infty} x(\tau - 5) h(t - \tau + 3) d\tau \xrightarrow{\tau - 5 = w} \int_{-\infty}^{+\infty} x(w) h(t - 2 - w) dw = y(t-2)$$

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$$[x'(t) * h'(t) * x'(t)] * h(t) = \big[\big(x'(t) * h(t)\big) * \delta'(t)\big] * [x'(t) * h(t)] = \big(y'(t) * \delta'(t)\big) * y'(t)$$
$$= y''(t) * y'(t)$$

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$$\left[x'(t+3)*\left(\int_{-\infty}^{t-5} h(\tau)d\tau\right)*x(t)\right]*h(t)=[x(t+3)*\delta'(t)]*[h(t-5)*u(t)]*[x(t)*h(t)]$$
$$=[x(t+3)*h(t-5)]*[\delta'(t)*u(t)]*y(t)=y(t-2)*y(t)$$

4ـ بررسی مستقل از زمان بودن یک سیستم خطی

آ)

$$x_1[n]=\delta[n]-2\delta[n+1]-2\delta[n-1]\rightarrow y_1[n]=-\delta[n+1]+3\delta[n]+3\delta[n-1]+\delta[n-3]$$

$$x_2[n]=\delta[n]-2\delta[n+1]\rightarrow y_2[n]=-\delta[n+1]+\delta[n]-3\delta[n-1]-\delta[n-3]$$

$$x_3[n]=\delta[n]+\delta[n-1]\rightarrow y_3[n]=2\delta[n+2]+\delta[n+1]-3\delta[n]+2\delta[n-2]$$

$$L(x_1[n]-x_2[n]+2x_3[n])=L(2\delta[n])=y_1[n]-y_2[n]+2y_3[n]$$
$$=2\delta[n+1]-4\delta[n]+6\delta[n-1]+2\delta[n-3]+4\delta[n-2]+4\delta[n+2]=f_1[n]$$

$$L(-x_1[n]+x_2[n])=L(2\delta[n-1])=-y_1[n]+y_2[n]=-2\delta[n]-6\delta[n-1]-2\delta[n-3]=f_2[n]$$

$$\begin{array}{c} f_1[n]=L(2\delta[n]) \\ f_2[n]=L(2\delta[n-1]) \Longrightarrow \\ f_1[n-1]\neq f_2[n] \end{array}$$ سیستم مستقل از زمان نیست

ب)

$$f_1[n]=L(2\delta[n])=2L(\delta[n])\longrightarrow L(\delta[n])=\frac{1}{2}f_1[n]$$

$$y[n]=\frac{1}{2}f_1[n]=\delta[n+1]-2\delta[n]+3\delta[n-1]+\delta[n-3]+2\delta[n-2]+2\delta[n+2]$$

5ـ کانولوشن سیگنال های پیوسته در زمان

- ***a*a***

$$a * a = \int_{-\infty}^{+\infty} a(\tau)\ a(t-\tau)d\tau \quad =$$

$$0 < t \leq 1: \int_{0}^{t} \tau(t-\tau)d\tau = \left[t\frac{\tau^2}{2} - \frac{\tau^3}{3}\right]_0^t = \frac{t^3}{6}$$

$$1 < t \leq 2: \int_{t-1}^{1} \tau(t-\tau)d\tau = \left[t\frac{\tau^2}{2} - \frac{\tau^3}{3}\right]_{t-1}^1 = \frac{-t^3}{6} + t - \frac{2}{3}$$

$$o.w: \quad 0$$

- ***b*b***

$$b * b = \int_{-\infty}^{+\infty} b(\tau)\ b(t-\tau)d\tau \quad =$$

$$0 < t \leq 1: \int_{0}^{t} (1-\tau)(1-t+\tau)d\tau = \left[-\frac{\tau^3}{3} + t\frac{\tau^2}{2} - t\tau + \tau\right]_0^t = \frac{t^3}{6} - t^2 + t$$

$$1 < t \leq 2: \int_{t-1}^{1} (1-\tau)(1-t+\tau)d\tau = \left[-\frac{\tau^3}{3} + t\frac{\tau^2}{2} - t\tau + \tau\right]_{t-1}^1 = \frac{-(t-2)^3}{6}$$

$$o.w: \quad 0$$

- ***c*c***

$$c * c = \int_{-\infty}^{+\infty} c(\tau)\ \ c(t-\tau)d\tau \quad =$$

$$0 < t \leq 1: \int_{0}^{t} \frac{1}{4} d\tau = \left[\ \frac{\tau}{4}\ \right]_0^t = \frac{t}{4}$$

$$1 < t \leq 2: \int_{t-1}^{1} \frac{1}{4} d\tau = \left[\frac{\tau}{4}\right]_{t-1}^1 = \frac{2-t}{4}$$

$$o.w: \quad 0$$

- ***b*c***

$$b * c = \int_{-\infty}^{+\infty} b(\tau)\ \ c(t-\tau)d\tau \quad =$$

$$0 < t \leq 1: \int_0^t \frac{1-\tau}{2} d\tau = \left[\ \frac{\tau}{2} - \frac{\tau^2}{4}\ \right]_0^t = \frac{t}{2} - \frac{t^2}{4}$$

$$1 < t \leq 2: \int_{t-1}^1 \frac{1-\tau}{2} d\tau = \left[\frac{\tau}{2} - \frac{\tau^2}{4}\right]_{t-1}^1 = \frac{t^2}{4} - t + 1$$

$$o.w: \quad 0$$

- ***a*b***

$$b * a = \int_{-\infty}^{+\infty} a(\tau)\ \ b(t-\tau)d\tau \quad =$$

$$0 < t \leq 1: \int_0^t \tau(1-t+\tau)d\tau = \left[\ \frac{\tau^2}{2} - \frac{t\tau^2}{2} + \frac{\tau^3}{3}\right]_0^t = \frac{t^2}{2} - \frac{t^3}{6}$$

$$1 < t \leq 2: \int_{t-1}^1 \tau(1-t+\tau)d\tau = \left[\frac{\tau^2}{2} - \frac{t\tau^2}{2} + \frac{\tau^3}{3}\right]_{t-1}^1 = \frac{t^3}{6} - \frac{t^2}{2} + \frac{2}{3}$$

$$o.w: \quad 0$$

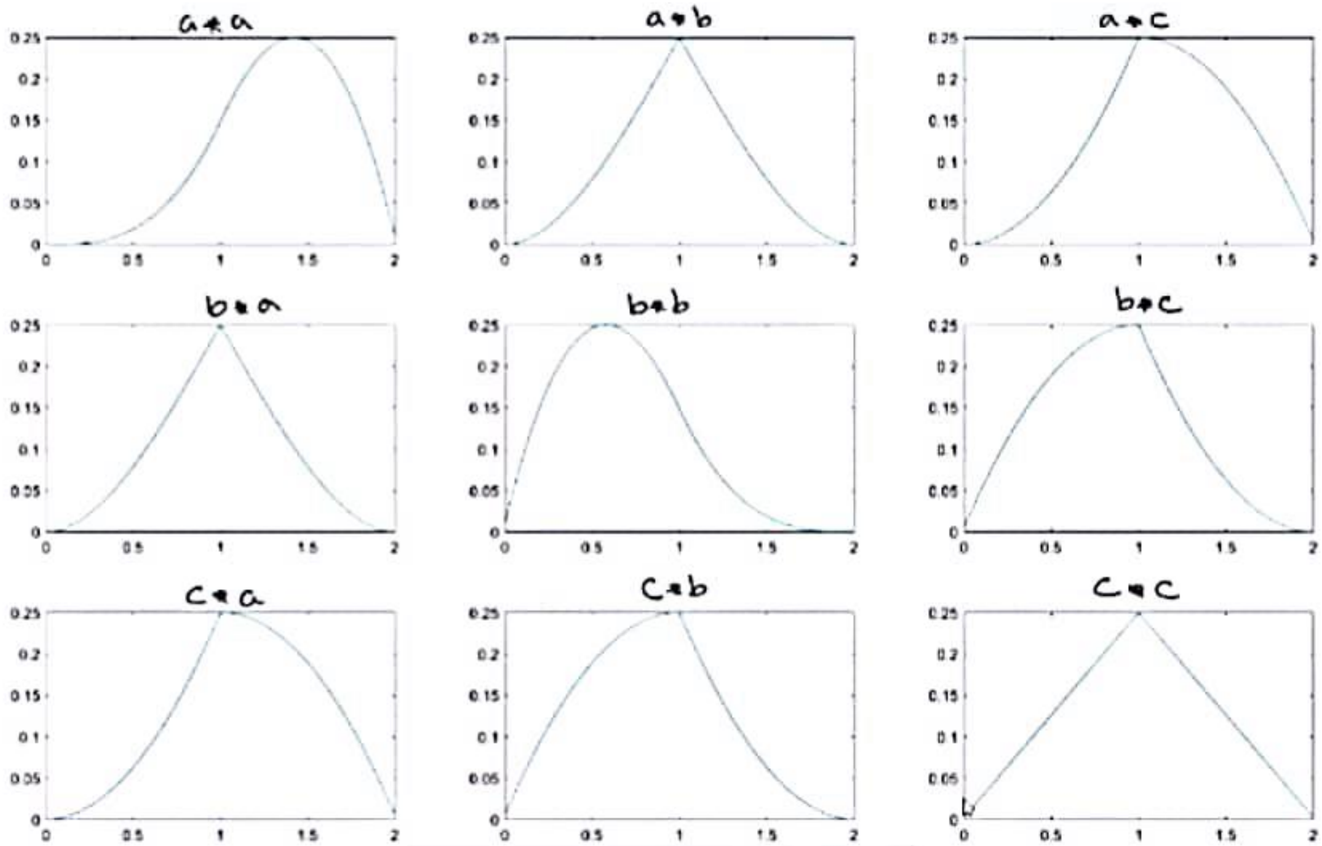
- ***c*a***

$$c * a = \int_{-\infty}^{+\infty} a(\tau)\ \ c(t-\tau)d\tau \quad =$$

$$0 < t \leq 1: \int_0^t \frac{\tau}{2} d\tau = \left[\ \frac{\tau^2}{4}\right]_0^t = \frac{t^2}{4}$$

$$1 < t \leq 2: \int_{t-1}^1 \frac{\tau}{2} d\tau = \left[\frac{\tau^2}{4}\right]_{t-1}^1 = \frac{t^2-2t}{4}$$

$$o.w: \quad 0$$



a*a
a*b
a*c
b*a
b*b
b*c
c*a
c*b
c*c