

Macroeconomic Modeling

Lecture 2: Solow Model and applications

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Outline

1. A snapshot to income differences
2. Solow Growth Model
3. A Basic Deterministic Solow Model in Dynare

Cross-Country Income Differences - 2021

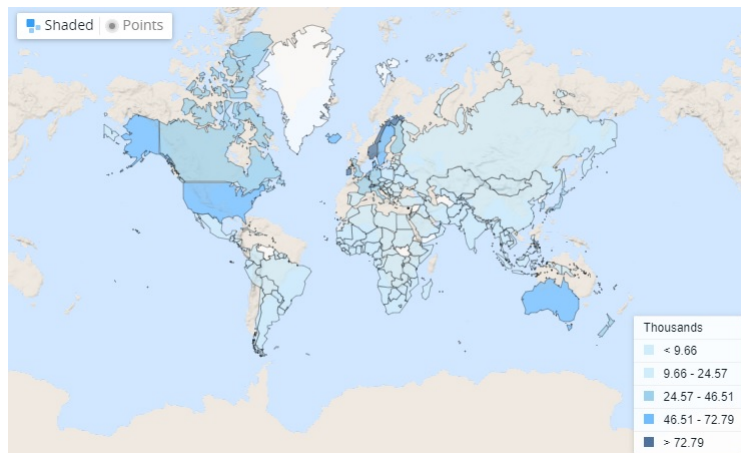


Figure: World Mapping for GDPPC across countries, US\$ constant

Source: World Bank (2023)

Cross-Country Income Differences - over time

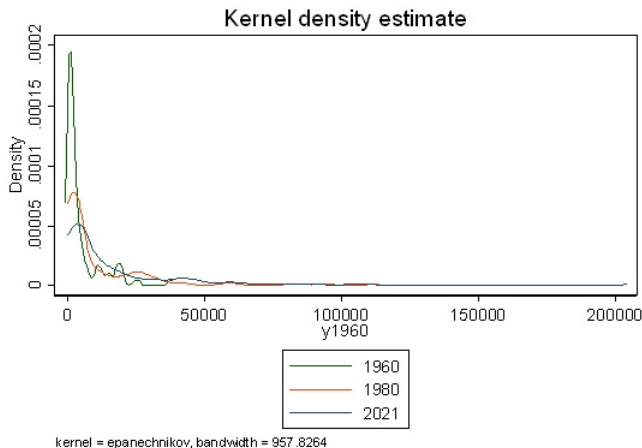


Figure: Kernel Density for GDPPC, US\$ constant

Cross-Country Income Differences - over time - natural logarithm

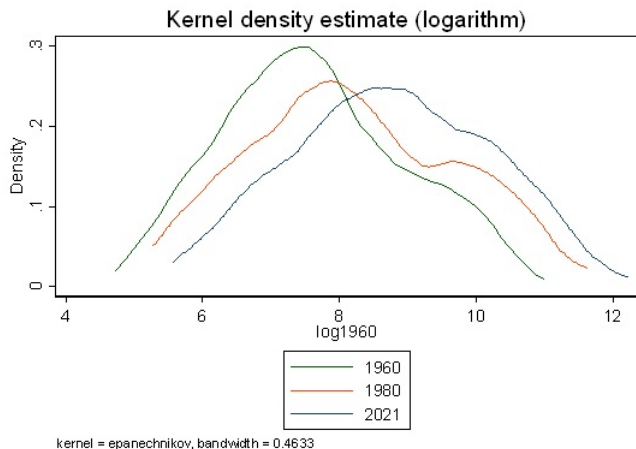


Figure: Kernel Density for log of GDPPC, US\$ constant

Economic Growth and Income Differences

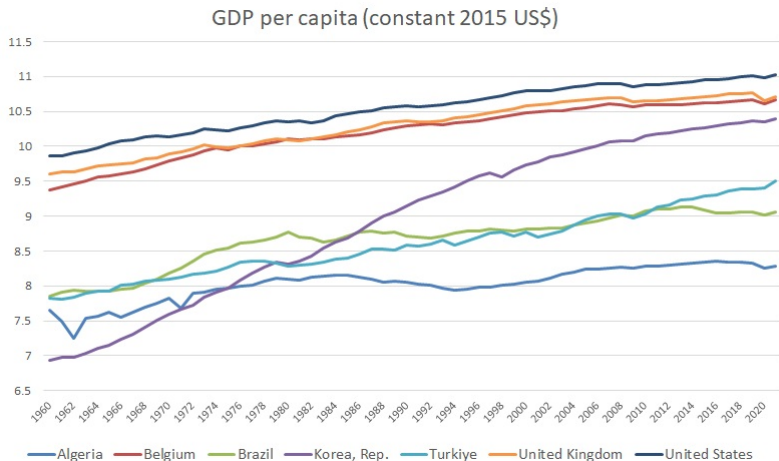


Figure: The evolution of income per capita 1960-2021

Solow Growth Model - Assumptions

- ▶ continuous time
- ▶ single product with continuous technology
- ▶ closed economy with no public side
- ▶ all factors are fully employed
- ▶ labor force grows at a constant rate of $\frac{\dot{L}}{L} = n$
- ▶ initial values for capital, labor are given, (K_0, L_0)
- ▶ price of the final goods are normalized to 1 in all periods

Solow Growth Model

Neoclassical Cobb-Douglas Prod.: $Y_t = A_t K_t^\alpha L_t^{1-\alpha}$ — > CRTS

Per worker terms: $y = Ak^\alpha$ — > DRTS

► Capital Accumulation

$\dot{K} = sy - \delta K \rightarrow$ dividing each side by K

$\Rightarrow \dot{k} = sAk^\alpha - (\delta + n)k$

\hookrightarrow Fundamental differential equation of Solow Model

► Evaluating Basic Solow Model

If countries are at their SS, rich countries have higher saving rates and lower population growth rates than poor countries.

► Introducing Technological Progress

Labor augmenting production technology $\rightarrow Y = K^\alpha (AL)^{1-\alpha}$

$\frac{\dot{A}}{A} = g > 0 \rightarrow$ growth is exogenous

Solow Growth Model

$$\dot{k} = sAk^\alpha - (n + \delta)k$$

- ▶ no growth in the SS
- ▶ positive/negative growth along the transition path

$$g_k = \frac{\dot{k}}{k} = sAk^{\alpha-1} - (n + \delta)$$

- ▶ SS is a balanced growth path with zero growth rate
- ▶ BGP growth rate equals to the rate of technological progress, variables grow by $g = \frac{\dot{A}}{A}$, no TFP
- ▶ source of growth is left unexplained
- ▶ differences in income levels across countries are explained by differences in s, n, δ

Profit Maximization of Firms

Or cost minimization

$$\max K^\alpha (AL)^{1-\alpha} - wL - rK$$

$$\text{FOC wrt } K: \alpha k^{\alpha-1} = r \quad \text{FOC wrt } L: (1-\alpha)Ak^\alpha = w$$

$$\text{, where } \frac{K}{AL} = k$$

$$\text{Capital share: } \frac{rK}{Y} = \alpha \quad \text{Labor share: } \frac{wL}{Y} = 1 - \alpha$$

- ▶ in BGP,
 - ▶ $k = k^*$, constant over time. Hence, r is constant over time
 - ▶ w grows constant over time since A grows at g over time
- ▶ output is produced by price taking firms
- ▶ real interest rate $\rightarrow r - \delta$

Applications on Dynare

- ▶ Solow Model - Transition to Steady State
Solow_SS.mod file
- ▶ Solow Model - Technological Shock
basic_solow.mod file

A deterministic Solow Model in Dynare

basic_solow.mod file

↪ 3 variable model (K, C, Y)

$$y_t = A_t k_{t-1}^\alpha$$

$$c_t = (1 - s)y_t$$

$$(1 + n)k_t = sy_{t-1} + (1 - \delta)k_{t-1}$$

References

Acemoglu, D. (2008). Introduction to modern economic growth.
Princeton university press.

Solow, R. M. (1956). A contribution to the theory of economic growth.
The quarterly journal of economics, 70(1), 65-94.