Lecture 1: Taylor Series expansion. → is an infinite sum of terms that are expressed in terms of the function's derivatives at a single point.

1^{st} order → $f(x) = f(x^{ss}) + \frac{f'(x^{ss})}{1!}(x - x^{ss})$

2^{nd} order → $f(x) = f(x^{ss}) + \frac{f'(x^{ss})}{1!}(x - x^{ss}) + \frac{f''(x^{ss})}{2!}(x - x^{ss})^2$

Ex: Cobb Douglass prod. function → $y_t = a_t k_t^{\alpha} l_t^{1-\alpha}$

1^{st} step: take log ⇒ $\ln y_t = \ln a_t + \alpha \ln k_t + (1-\alpha)\ln l_t$

2^{nd} step: approximation ⇒ $\ln y^* + \frac{1}{y^*}(y - y^*) = \ln a^* + \frac{1}{a^*}(a - a^*) + \alpha \ln k^* + \frac{\alpha}{k^*}(k - k^*) + (1-\alpha)\ln l^* + \frac{(1-\alpha)}{l^*}(l - l$

3^{rd} step: adjustment ⇒ since in SS → $\ln y^* = \ln a^* + \alpha \ln k^* + (1-\alpha)\ln l^*$

$$\Rightarrow \frac{\hat{y}_t}{y^*} = \frac{\hat{a}_t}{a^*} + \frac{\alpha \hat{k}_t}{k^*} + \frac{(1-\alpha)\hat{l}_t}{l^*} \quad , \quad \hat{y}_t = y_t - y^* \quad , \quad \tilde{y}_t = \frac{y_t - y^*}{y^*}$$

$$\Rightarrow \underline{\tilde{y}_t = \tilde{a}_t + \alpha \tilde{k}_t + (1-\alpha)\tilde{l}_t} \, //$$

Ex: $y_t = c_t + i_t$ national income accounting identity.

$\ln y_t = \ln(c_t + i_t)$

$\ln y^* + \frac{1}{y^*}\hat{y}_t = \ln(c^* + i^*) + \frac{1}{c^* + i^*}\hat{c}_t + \frac{1}{c^* + i^*}\hat{i}_t$, since ss ⇒ $\ln y^* = \ln(c^* + i^*)$

$\tilde{y}_t = \frac{\hat{c}_t}{y^*} + \frac{\hat{i}_t}{y^*}$ → multiply & divide each by c^* & i^* ⇒ $\tilde{y}_t = \frac{c^* \cdot \hat{c}_t}{c^* y^*} + \frac{i^* \hat{i}_t}{i^* y^*}$

$$\Rightarrow \underline{\tilde{y}_t = \frac{c^*}{y^*}\tilde{c}_t + \frac{i^*}{y^*}\tilde{i}_t} \, // , \quad \text{since } \frac{\hat{c}_t}{c^*} = \tilde{c}$$

Above method, which is conversion based on the definition can complicate the conversion process when eq. involve mainly addition or subtraction. Hence, we can use another method called conversion by substitution: $x_t = e^{\ln x_t}$, $\ln x_t = \ln x^* + \tilde{x}_t \Rightarrow x_t = e^{\ln x^* + \tilde{x}_t} \Rightarrow \boxed{x_t = x^* \cdot e^{\tilde{x}_t}}$

Apply Taylor series appr. to $e^{\tilde{x}_t}$ ⇒ $e^{\tilde{x}_t} \approx e^0 + e^0(\tilde{x}_t - 0) \Rightarrow \boxed{e^{\tilde{x}_t} \approx 1 + \tilde{x}_t}$ $\qquad \boxed{x_t = x^*(1 + \tilde{x}_t)}$

since $\tilde{x}_t = 0$ in the ss.

since no change!

Ex to national income model.

$y_t = c_t + i_t$

$y^*(1 + \tilde{y}_t) = c^*(1 + \tilde{c}_t) + i^*(1 + \tilde{i}_t)$

$$\underline{\tilde{y}_t = \frac{c^*}{y^*}\tilde{c}_t + \frac{i^*}{y^*}\tilde{i}_t} \, //$$

Ex: Capital accumulation

s → saving rate

δ → depreciation rate

α → production elasticity of capital.

$k_{t+1} = s k_t^{\alpha} + (1-\delta)k_t$

$k^*(1 + \tilde{k}_{t+1}) = s k^{\alpha}(1 + \alpha \tilde{k}_t) + (1-\delta)k(1 + \tilde{k}_t)$

$\tilde{k}_{t+1} = s k^{\alpha-1}\alpha \tilde{k}_t + (1-\delta)\tilde{k}_t$

$$\underline{\tilde{k}_{t+1} = \tilde{k}_t\left[\alpha s k^{\alpha-1} + (1-\delta)\right]} \, //$$

★ Do not forget: variables are percentage deviations from SS, and their associated coefficients are elasticities!

Ex:

CRRA utility : $\left(\frac{C_{t+1}}{C_t}\right)^{\sigma} = \beta(1+r_t)$

$\sigma > 0$
↳ coefficient of relative risk aversion.
$\beta \rightarrow$ time discount
$r \rightarrow$ real int. rate

$$\sigma \ln C_{t+1} - \sigma \ln C_t = \ln \beta + \ln(1+r_t)$$

$$\sigma \ln c^* + \frac{\sigma}{c} \cdot \hat{C}_{t+1} - \sigma \ln c - \frac{\sigma}{c}\hat{C}_t = \ln \beta + \ln(1+r^*) + \frac{1}{1+r^*} \cdot \hat{r}_t$$

$$SS \Rightarrow \sigma \ln c^* - \sigma \ln c^* = \ln \beta + \ln(1+r^*)$$

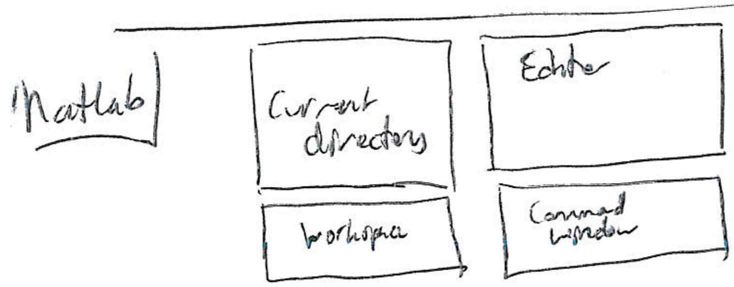
$$\sigma \tilde{C}_{t+1} - \sigma \tilde{C}_t = \frac{\hat{r}_t}{1+r^*}$$

$$\Rightarrow \underline{\underline{\tilde{C}_{t+1} - \tilde{C}_t = \frac{1}{\sigma} \tilde{r}_t}} \quad //$$

$\Leftarrow$ Important : As r is already a percent, it's common to leave it in absolute deviations as opposed to percentage deviations. Hence we can define $\tilde{r}_t = (r_t - r^*)$

Important : we approximate the term $\frac{1}{1+r^*} = 1$

↳ interpretation: gr. rate of consumption is approximately proportional to the deviation of the real int. rate from SS, with $1/\sigma$ interpreted as the elasticity of intertemporal substitution.



Dyrare (. mod file) $\rightarrow$ 4 distinct blocks

- $\rightarrow$ preamble : lists variables & parameters
- $\rightarrow$ model : spells out the model
- $\rightarrow$ steady state or initial values
- $\rightarrow$ shocks : defines the shocks to the system
- $\rightarrow$ computation $\rightarrow$ instructs Dynare to undertake operations (e.g forecasting, estimating IRFs

PREAMBLE

var .... $\rightarrow$ endo vars

varexo .... $\rightarrow$ exo vars.

parameters ... $\rightarrow$ lists & assigns values to parameters

Ⓐ we put (end;) at the end of model, initval, SS values, shocks.

Ⓐ we put ; (semicolon) at the end of each line.

Ⓐ If model is linear we need to write Model(linear);

Otherwise, you can have Dynare take Taylor series expansion in logs rather than levels, we insert variables as exp(xx) rather than ~~x~~,
xx is the log of x

Ⓐ X_{t-k} is written as $x(-k)$
X_{t+k} " " " $x(+k)$

Ⓐ if k_t is predetermined ; for instance $k_{t+1} = i_t + (1-\delta)k_t$
then we will translate this equation into Dynare as
$k = i + (1-\delta)*k(-1)$