# Macroconomic Modeling
# Lecture 1: Introduction

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# Outline

# Lagrange Multiplier

$$maximization \qquad z = f(x, y)$$

$$subject\ to \qquad x + y < 100$$

$$\mathcal{L} = f(x, y) + \lambda(100 - x - y)$$

or

$$\mathcal{L} = f(x, y) + \lambda[Zero]$$

$$\frac{\delta \mathcal{L}}{\delta x} = f_x - \lambda = 0$$

$$\frac{\delta \mathcal{L}}{\delta y} = f_y - \lambda = 0$$

$$f_x = \lambda$$
$$f_y = \lambda$$
$$f_x = f_y$$

# Example:Utility Maximixation

$$max \qquad u = 4x^2 + 3xy + 6y^2$$

$$st \qquad x + y = 56$$

Lagrangian equation:

$$\mathcal{L} = 4x^2 + 3xy + 6y^2 + \lambda(65 - x - y)$$

FOCs

$$\mathcal{L}_x = 8x + 3y - \lambda = 0$$
$$\mathcal{L}_y = 3x + 12y - \lambda = 0$$

We get $x = 36$ and $y = 348$

# Compound Interest

$$y = x + rx = x(1 + r)$$

In t years, it will be

$$y = x(1 + r)^t$$

In the same manner, the present value of y in t years will be

$$x = \frac{y}{(1 + r)^t}$$

# Compound Interest within a year

Semi-annual compounding

$$y = x(1 + \frac{r}{2}) \qquad \textit{at 6-months}$$

$$y = x(1 + \frac{r}{2})^2 \qquad \textit{at 1 year}$$

Monthly compounding

$$y = x(1 + \frac{r}{12})^{12} \qquad \textit{at 1 year}$$

$$y = x(1 + \frac{r}{12})^{12*t} \qquad \textit{at t year}$$

# Continuous Compounding

Daily interest for one year

$$y = x(1 + \frac{r}{365})^{365}$$

For higher frequency (m)

$$y = x(1 + \frac{r}{m})^{m}$$

Hence, for any interest $r$ as the frequency (m) is increasing to infinity (m $\rightarrow \infty$)

$$y = xe^{r}$$

Hence, continuous compounding at r per cent for $t$ years of a principle $x$ is

$$y = xe^{rt}$$

# Growth rate

$$y = xe^{rt}$$

The change in y wrt time is

$$\frac{dy}{dt} = rxe^{rt} = ry$$

The growth rate (percentage change) is

$$\frac{\dot{y}}{y} = \frac{dy}{y} = \frac{rxe^{rt}}{xe^{rt}} = r$$

# Present Value

If the current value is represented with $V(t)$, Present value ($PV(t)$) is calculated as

$$PV(t) = V(t)e^{-rt}$$

# Taylor series expansion

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$= f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots$$

$f(x)$ is called Taylor series of the function $f$ at a

# Log-linearization

Solutions to many discrete time dynamic economic problems take the form of a system of non-linear difference equations. Regarding the solution for such problems, we cannot have an analytical solution, hence, use numerical and/or approximation techniques, one of which is log-linearization that has the following steps:

- take natural logarithm of non-linear equations
- linearize by using Taylor series expansion at the steady state (SS)
- do adjustments so that everything will be expressed in percentage deviations from the SS

# Linearization for a single and multivariable function using Taylor rule

$$f(x) = f(x^*) + f'(x^*)(x - x^*)$$

, where $x^*$ is the SS value of $x$.

$$f(x, y) \approx f(x^*, y^*) + f_x(x^*, y^*)(x - x^*) + f_y(x^*, y^*)(y - y^*)$$

, where $x^*$ and $y^*$ are the SS values of $x$ and $y$.

# Log-linearization

Assume the below nonlinear function: $f(x) = \dfrac{g(x)}{h(x)}$

**Step 1:** Take natural logarithm: ln $f(x) =$ ln $g(x)$ - ln $h(x)$

**Step 2:** Use 1st order Taylor series expansion:

$$f(x) \approx f(x^*) + \frac{f'(x^*)}{f(x^*)}(x - x^*)$$

$$g(x) \approx g(x^*) + \frac{g'(x^*)}{g(x^*)}(x - x^*)$$

$$h(x) \approx h(x^*) + \frac{h'(x^*)}{h(x^*)}(x - x^*)$$

All 3 equations together:

$$\ln f(x^*) + \frac{f'(x^*)}{f(x^*)}(x - x^*) = \ln g(x^*) + \frac{g'(x^*)}{g(x^*)}(x - x^*) - \ln h(x^*) - \frac{h'(x^*)}{h(x^*)}(x - x^*)$$

# Log-linearization cont'd

Since $\ln f(x^*) = \ln g(x^*) - \ln h(x^*)$

**Step 3:** Do adjustments

$$\frac{f'(x^*)}{f(x^*)}(x - x^*) = \frac{g'(x^*)}{g(x^*)}(x - x^*) - \frac{h'(x^*)}{h(x^*)}(x - x^*)$$

Divide everything by $x^*$ and show all in percentage terms:

$$\frac{f'(x^*)}{f(x^*)}\tilde{x} = \frac{g'(x^*)}{g(x^*)}\tilde{x} - \frac{h'(x^*)}{h(x^*)}\tilde{x}\ \text{, where } \tilde{x} = (x - x^*)/x^*$$

# Log-linearization Example: Cobb-Douglass Production Function

Assume $y_t = a_t k_t^{\alpha} l_t^{(1-\alpha)}$

**Step 1:** $\ln y_t = \ln a_t + \alpha \ln k_t + (1-\alpha)\ln l_t$

**Step 2:**

$$\ln y^* + \frac{1}{y^*}(y_t - y^*) =$$

$$\ln a^* + \frac{1}{a^*}(a_t - a^*) + \alpha \ln k^* + \frac{\alpha}{k^*}(k_t - k^*) + (1-\alpha)\ln l^* + \frac{(1-\alpha)}{l^*}(l_t - l^*)$$

**Step 3:**

$\tilde{y}_t = \tilde{a}_t + \alpha \tilde{k}_t + (1-\alpha)\tilde{l}_t$ , since $\ln y^* = \ln a^* + \alpha \ln k^* + (1-\alpha)\ln l^*$

# Introducing Matlab

MATLAB is a programming and numeric computing platform used by millions of engineers and scientists to analyze data, develop algorithms, and create models.

Check the following Google Drive link

Lecture 1: Introduction to MATLAB
Lecture 2: Visualization and Programming
Lecture 3: Solving Equations, Curve Fitting, and Numerical Techniques
Lecture 4: Advanced Methods
Lecture 5: Various Functions and Toolboxes

Source: MIT Open Course Ware

# Introducing Dynare

Dynare is an interface to Matlab, to solve, simulate and estimate DSGE models.
1- Download and install Dynare: https://www.dynare.org/download/
2- Open MATLAB/Octave
3- Configure for Dynare

**For MATLAB**
3.1- On the MATLAB Home tab, in the Environment section, click on Set Path
3.2- Click Add Folder... (DO NOT select Add with Subfolders)
3.3- Select the matlab subdirectory of your Dynare installation, e.g., `C:\dynare\4.6.1\matlab`
3.4- Apply the setting by clicking Save button
**For OCTAVE**
3.1. Put it in a file called .octaverc in your home directory. This file will usually be called `C:\Users\USERNAME\.octaverc`

Check the following Google Drive link
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