Macroconomic Modeling
Lecture 1.1: Marshall and Hicks

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Outline

Assume $U(x, y) = xy$ wrt. $B = P_x x + P_y y$

Hence, $\mathcal{L} : xy + \lambda(B - P_x x - P_y y)$

FOCs:

$$\frac{\delta \mathcal{L}}{\delta x} : y - \lambda P_x = 0$$

$$\frac{\delta \mathcal{L}}{\delta y} : x - \lambda P_y = 0$$

$$\frac{\delta \mathcal{L}}{\delta \lambda} : B - P_x x - P_y y = 0$$

Marshallian Demand functions

equations after FOCs:

$$y = \lambda - P_x \qquad x = \lambda - P_y \tag{1}$$

$$B = P_x x + P_y y \tag{2}$$

From (1), we get:

$$y = \frac{P_x x}{P_y} \tag{3}$$

Plugging (3) into (2), we get

$x = \dfrac{B}{2P_x}$ and $y = \dfrac{B}{2P_y}$, which are the consumer's Marshallian demand functions (x^M and y^M)

Direct and Indirect Utility Function

$U(x, y) = xy$ is the **Direct Utility Function** since utility is directly depends on the quantities of xy the consumer chooses.

From the Lagrangian, we derived Marshallian demand functions (x^M and y^M) where the optimal quantities are determined by prices and income.

If we substitute the functions, derived from the solutions of the Lagrangian problem, into the utility function:

$$U(x, y) = xy = \frac{B}{P_x}\frac{B}{P_y}$$

$$\Rightarrow U(x, y) = \frac{B^2}{4P_xP_y}$$

, which is **Indirect Utility Function** since utility indirectly depends on prices and income.

Compensated Demand (Hicksian Demand)

Since $U(x, y) = \dfrac{B^2}{4P_xP_y} \Rightarrow \text{B} = 2\ \text{P}_x^{1/2}P_y^{1/2}U^{1/2}$

Hence, the expenditure function is written as:

$E(P_x, P_y, U) = 2P_x^{1/2}P_y^{1/2}U^{1/2} \rightarrow$ which tells us the minimum budget needed for a consumer to achieve a certain utility level.

The compensated demand (Hicksian demand) can be derived from the expenditure function.

$E(P_x, P_y, U)$ is the expenditure function. Then, the compensated demand for x is found by taking the derivation wrt P_x, and vice versa for y.

$$\frac{\delta E}{\delta P_x} \Rightarrow \text{ Hicksian demand } (x^H) \Rightarrow 2\frac{1}{2}P_x^{-\frac{1}{2}}P_y^{\frac{1}{2}}U^{\frac{1}{2}} \Rightarrow x^H = \frac{P_y^{\frac{1}{2}}U^{\frac{1}{2}}}{P_x^{\frac{1}{2}}}$$

$$\frac{\delta E}{\delta P_y} \Rightarrow \text{ Hicksian demand } (y^H) \Rightarrow 2\frac{1}{2}P_x^{\frac{1}{2}}P_y^{-\frac{1}{2}}U^{\frac{1}{2}} \Rightarrow y^H = \frac{P_x^{\frac{1}{2}}U^{\frac{1}{2}}}{P_y^{\frac{1}{2}}}$$

Marshallian Demand VS Hicksian Demand

Marshallian Demand Function

$$x^M = \frac{B}{2P_x} \qquad y^M = \frac{B}{2P_y}$$

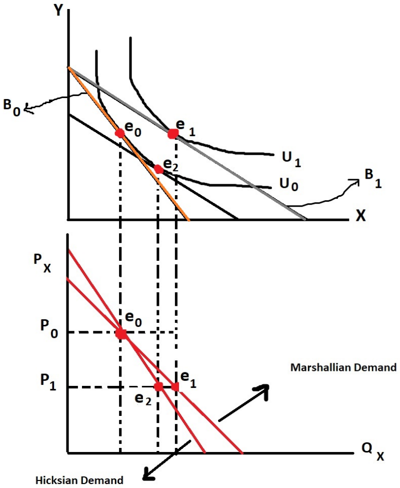
- function of budget and prices
- measures changes in demand when budget is held contant
- measures total effect (incomesubstitution effect)

Hicksian Demand Function

$$x^H = \frac{P_y^{\frac{1}{2}} U^{\frac{1}{2}}}{P_x^{\frac{1}{2}}} \qquad y^H = \frac{P_x^{\frac{1}{2}} U^{\frac{1}{2}}}{P_y^{\frac{1}{2}}}$$

- function of prices and utility
- measures changes in demand when utility is held contant
- measures substitution effect

Visualization for Marshallian and Hicksian Demand



B_0 is the initial budget constraint

B_1 is the budget constraint after the decline of P_x

E_0 is the equilibrium before the price fall

E_1 is the equilibrium after the price fall (Marshall)

E_2 is the equilibrium after the price fall (Hicks)

Figure: The Effect of a Price Fall in Good X