Introduction to Quantum Computing

Week 1: Quantum World; Superposition and Entanglement

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Quantum Talent and Learning Center
Amsterdam University of Applied Sciences

Week 1
April 25th, 2025

Agenda for today:

- Setting up the scene: install Python and Qiskit
- Introduction to Quantum World
- Why quantum systems are suitable for fast computing?

Meet the other TLC hubs!

- We are here at Hogeschool van Amsterdam (HvA)- Amsterdam University of Applied Sciences. Femke Verheijen, (TLC Amsterdam). Dr. Taha Selim, (TLC Amsterdam). Dr. Bernardo Villalba Frias (ICT/Amsterdam).

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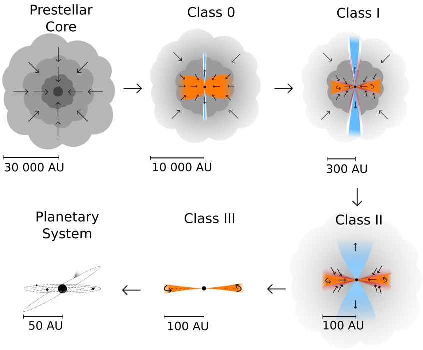
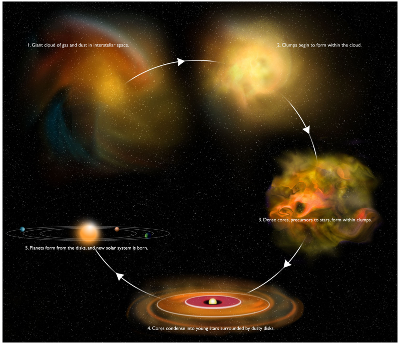
- Hogeschool Fontys -Fontys University of Applied Sciences, Dr. Nico Kuijpers (TLC Eindhoven).

The quest to understand the formation of planets and origin of life!

Circumstellar/Protoplanetary Disks



– Persson, Magnus Vilhelm (2014)



– Bill Saxton, NRAO/AUI/NSF

Quantum chemistry and quantum dynamics simulations

During my PhD work: computationally expensive simulations of molecular interactions.

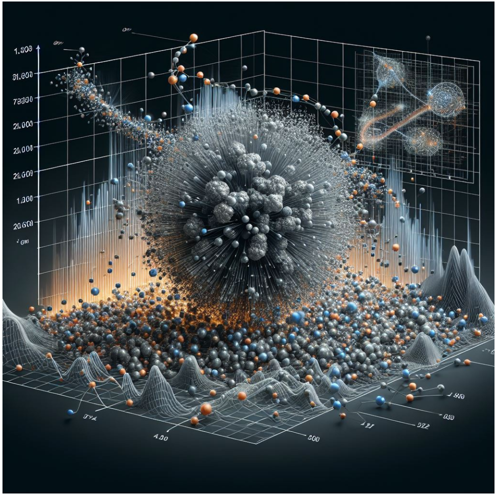


Image credit: Taha Selim, AI-generated image.

Quantum chemistry and quantum dynamics simulations

Would quantum computing help in performing these simulations?

Applications include: material design, drug discovery, $\cdots$.

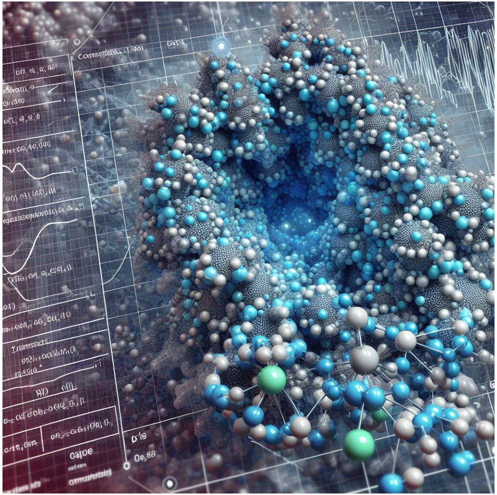


Image credit: Taha Selim, AI-generated image.

It is all about computing!

The need for digital twin, performing simulations and design

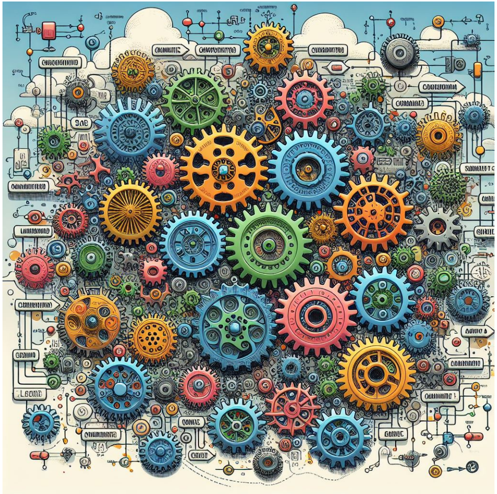


Image credit: Taha Selim, AI-generated image.

Curious Questions!

- What is the fastest computer in the world?
- What is the difference between classical and quantum computing?
- Why do we need Quantum Computing?
- What do you expect to have from quantum computing?
- Can Quantum Computing solve all problems?
- Can Quantum Computing break all encryptions?
- Can current AI be improved by Quantum Computing?
- Which one wins, AI or Quantum Computing?
- What is the future of Quantum Computing?

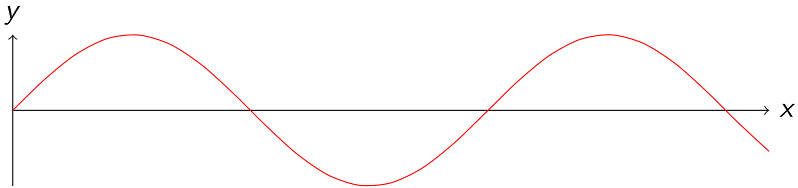
What is Quantum Mechanics?

Quantum mechanics is the language of the microscopic world!

With the language of quantum mechanics, we can do:

- Make designs of quantum sensors.
- Send and encrypt information.
- Design quantum systems to do computations.

What is the functional form of the following wave?



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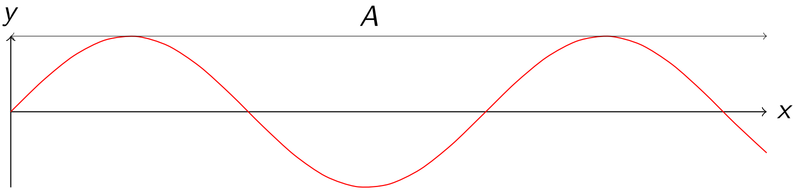
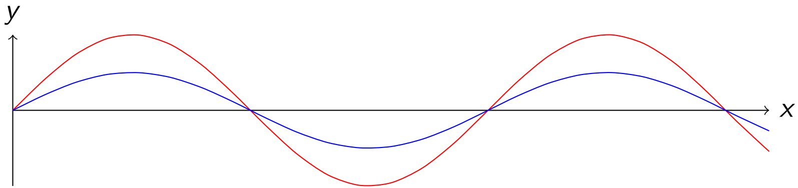


$$y = A\sin(x)$$

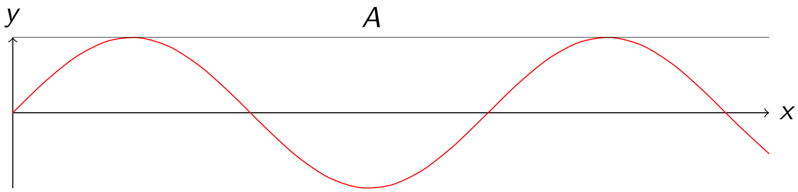
Classical vs quantum superposition

Classical or Quantum superposition?



Classical vs quantum superposition

Classical superposition:
Two physical quantities are added together to make another third physical quantity.



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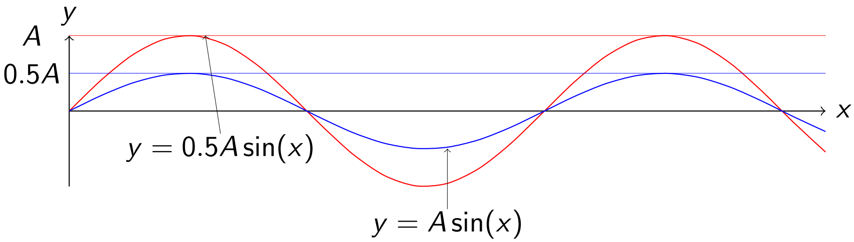
Classical vs quantum superposition

Example of constructive and destructive interference of two waves.



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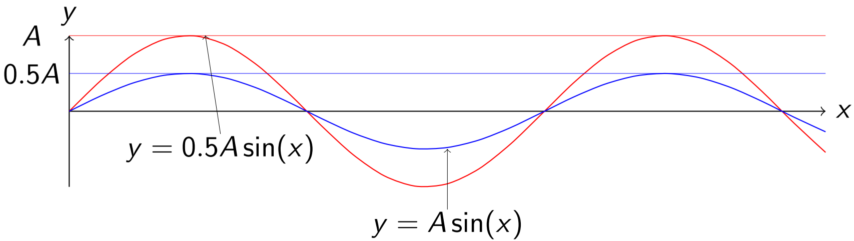
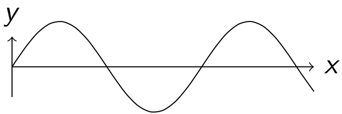
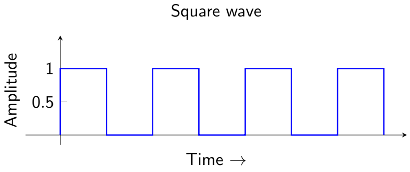


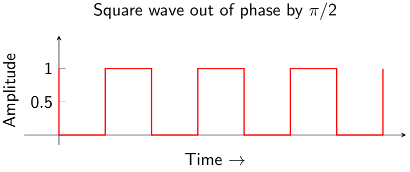
The superposition of two waves: constructive interference,

$$y(x) = A\sin(x) + 0.5A\sin(x)$$



Example of constructive and destructive interference of two square waves.





The result of the superposition of the two waves: destructive interference,

Classical vs quantum superposition

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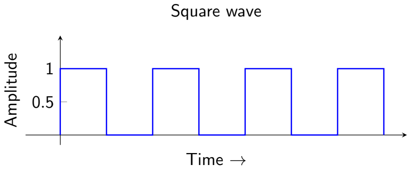

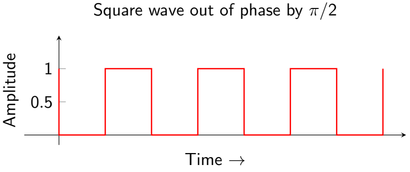

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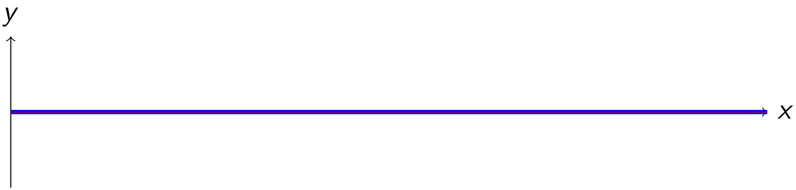
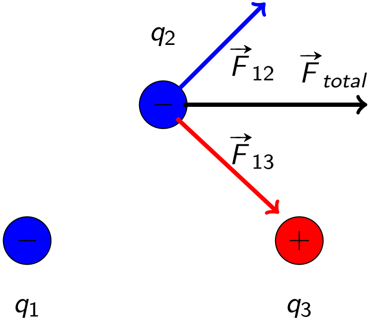
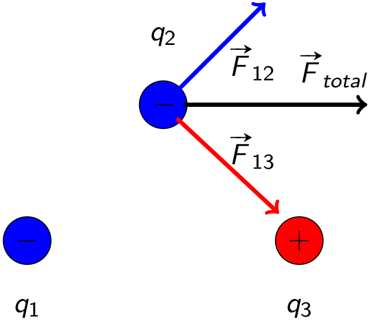


Example of classical superposition in electrostatics.



The resultant force acting on the upper electric charge:

Example of classical superposition in electrostatics.



The resultant force acting on the upper electric charge:

$$\vec{F}_{total} = \vec{F}_{12} + \vec{F}_{13}$$

Classical vs quantum superposition

Quantum superposition:

A quantum system can be in a superposition of two or more states. We can explain it using the following analogy:

A coin has a 50/50 probability of landing as either heads or tails:

Classical vs quantum superposition

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Question: What state is the coin in while it is in the air? Is it heads or tails?

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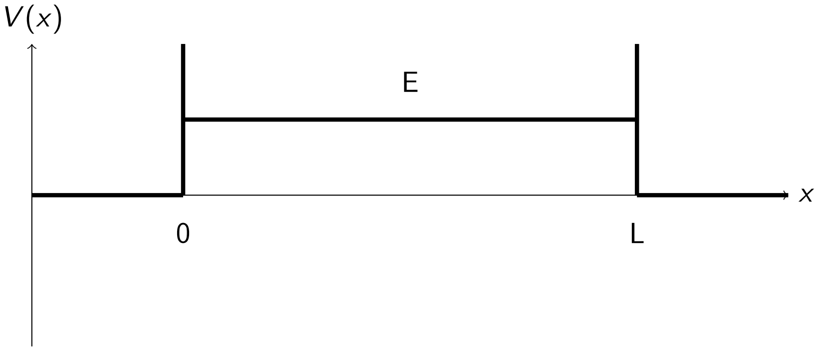
Question: What state is the coin in while it is in the air? Is it heads or tails?
We can say that the coin is in a superposition of both heads and tails. When it lands, it has a definite state, either heads or tails.

After the coin is tossed, it is in a superposition of heads and tails. Only when it fails on the ground, we will know the outcome:

Probability of landing on heads	Probability of landing on tails
$P(H) = 0.5$	$P(T) = 0.5$

Quantum superposition:

Example: Particle (electron) trapped in a box

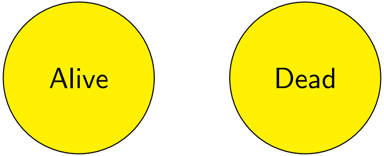


Quantization of energy levels which a trapped particle in 1D well can have.

Classical vs quantum superposition

Quantum superposition:

Schrödinger's cat:



Quantum superposition:

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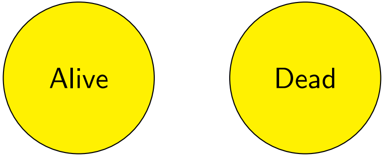

Measurements: destroy the superposition of states.

Entanglement and coherence

Entanglement and coherence are two important concepts in quantum mechanics.

Entanglement is created when two or more particles are generated or interact in a way such that the quantum state of each particle cannot be described independently of the state of the others.
Coherence is a property of a quantum system that allows it to be in a superposition of states.

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Measurements: when a quantum system is measured, it is forced to choose one of the possible states. In other words, the quantum states collapse.

Let's play!

Tik Tok Toe game:

https://tiqtaqtoe.com/

Let's play!

Tik Tok Toe game:

https://tiqtaqtoe.com/

Who wins? How did you win?

Refreshing 1

Discuss among yourselves whether the following quantities are quantized or continuous:

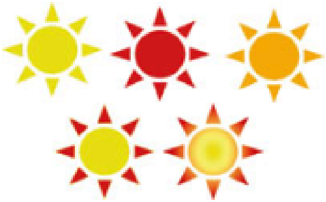
1. electric charge.
2. time.
3. length.
4. energy.
5. cash.
6. paint color.

Question source: Hughes et al. (2021), Springer. Quantum Computing for the Quantum Curious.

Refreshing 2

Question:

An ink is created by mixing together 50% red ink and 50% yellow ink. An artist uses it to stamp a picture of a sun. If the ink behaves like a quantum system in a half-yellow, half-red quantum superposition, what are the different options for what the resulting picture could look like? Some options are shown in the figure.



Question source: Hughes et al. (2021), Springer. Quantum Computing for the Quantum Curious.

Refreshing 3

Question:
If this controversial picture of a dress is always seen as blue/black by Student A and always seen as white/gold by Student B, is the dress in a quantum superposition?
`https://en.wikipedia.org/wiki/The_dress`

Question source: Hughes et al. (2021), Springer. Quantum Computing for the Quantum Curious.

Classical vs quantum superposition

Classical Superposition

- A coin can be in one of two states: heads or tails.
- A bit can be in one of two states: 0 or 1.
- A light switch can be in one of two states: on or off.

Quantum Superposition

- A quantum bit (qubit) can be in a superposition of 0 and 1.
- A quantum coin can be in a superposition of heads and tails.
- A quantum light switch can be in a superposition of on and off.

Wavefunctions

Quantum mechanics:
Wavefunctions are used to describe the state of a quantum system.
Example:
Free particle:

$$\Psi(x, t) = A \sin(kx - \omega t)$$

Wavefunction is a complex-valued function, for example, of position and time.

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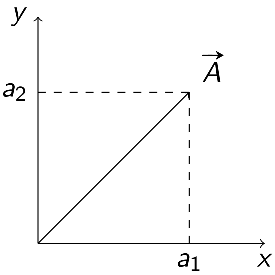
Wavefunction is a complex-valued function, for example, of position and time.

Probability density:
$|\Psi(x, t)|^2$ gives the probability of finding the particle at position x at time t.

Dirac Bra-Ket Notation

A vector $\vec{A}$ in a two-dimensional space can be written as a column matrix:

$$\vec{A} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{1}$$

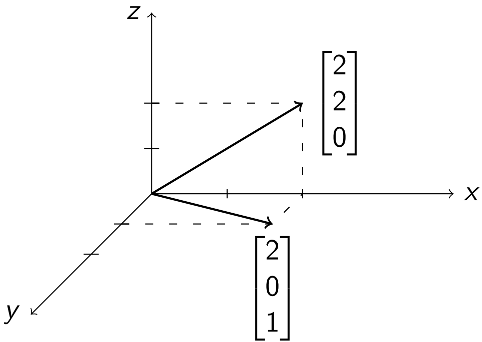


A given vector A can be written as a column matrix:

Dirac Bra-Ket Notation

Similarly, a vector $\vec{B}$ in a three-dimensional space can be written as a column matrix:

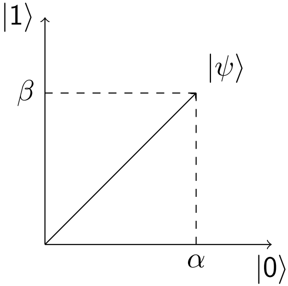
$$\vec{B} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \tag{2}$$



Dirac Bra-Ket Notation

In analogy to vectors, we can write the wavefunction as a ket $|\Psi\rangle$:

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{3}$$



with α and β called the amplitudes of the states and they are generally complex numbers.

Dirac Bra-Ket Notation

Amplitudes give the probability of finding the system in a given state when performing a measurement.

The probability of finding the system in state $|0\rangle$ is $|\alpha|^2$, and the probability of finding the system in state $|1\rangle$ is $|\beta|^2$.

The sum of the probabilities of finding the system in the two states must be equal to 1: $|\alpha|^2 + |\beta|^2 = 1$.
Hence,

$$|\alpha|^2 + |\beta|^2 = 1 \tag{4}$$

The particle exists by itself in a superposition of states.

Dirac Bra-Ket Notation

Question:
The quantum state of a spinning coin can be written as a superposition of heads and tails. Using heads as $|1\rangle$ and tails as $|0\rangle$, the quantum state of the coin is

$$|\text{coin}\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |0\rangle). \tag{5}$$

What is the probability of getting heads?

Dirac Bra-Ket Notation

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What is the probability of getting heads?

The amplitude of the state $|1\rangle$ is $\beta = \frac{1}{\sqrt{2}}$, so the probability of getting heads is $|\beta|^2 = \frac{1}{2}$. So, the probability is is 0.5 , or 50%.

Similarly, the probability of getting tails is also $\frac{1}{2}$, so the sum of the probabilities of getting heads and tails is 1.

Dirac Bra-Ket Notation

Question:
A weighted coin has twice the probability of landing on heads vs. tails. What is the state of the coin in “ket” notation?

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$$
\begin{aligned}
P_{\text{heads}} + P_{\text{tails}} &= 1 \quad (\text{ Normalization Condition }) \\
P_{\text{heads}} &= 2P_{\text{tails}} \quad (\text{ Statement in Example }) \\
\rightarrow P_{\text{tails}} &= \frac{1}{3} = \alpha^2 \\
\rightarrow P_{\text{heads}} &= \frac{2}{3} = \beta^2 \\
\rightarrow \alpha = \sqrt{\frac{1}{3}}, \beta = \sqrt{\frac{2}{3}} &\rightarrow |\text{coin}\rangle = \sqrt{\frac{1}{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle.
\end{aligned}
$$

Dirac Bra-Ket Notation

Question for the creative minds:
How can you use the concept of quantum superposition to describe the composition of a cake?

Dirac Bra-Ket Notation

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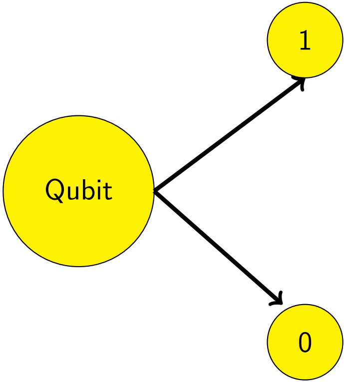
How can you use the concept of quantum superposition to describe the composition of a cake?

$$|\text{cake}\rangle = \alpha|\text{chocolate}\rangle + \beta|\text{vanilla}\rangle + \gamma|\text{strawberry}\rangle + \delta|\text{lemon}\rangle + \epsilon|\text{carrot}\rangle + \cdots \quad (6)$$

Qubit vs. classical bit

Qubit is a quantum bit.
Classical bit: can be in one of two states: 0 or 1.
Qubit: can be in a superposition of 0 and 1.



Matrix representation of qubit

We use matrix algebra to represent qubits.
State of a single qubit:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{7}$$

In a vector form/representation:

$$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \tag{8}$$

The states $|0\rangle$ and $|1\rangle$ are represented by the following column matrices:

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{and} \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{9}$$

The coefficients α and β are complex numbers, and they are generally called the amplitudes of the states.

What is a matrix?

- A matrix is a rectangular array of numbers, symbols, or expressions, arranged in rows and columns.
- The individual items in a matrix are called its elements or entries.
- The horizontal and vertical lines of entries in a matrix are called rows and columns, respectively.
- The size of a matrix is defined by the number of rows and columns that it contains.
- A matrix with m rows and n columns is called an $m \times n$ matrix.
- A matrix can be used to represent a linear map.
- A matrix can be used to represent a property of a mathematical object.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

- $a_{11}, a_{12}, \cdots, a_{1n}$ are the elements of the first row of the matrix A.
- $a_{21}, a_{22}, \cdots, a_{2n}$ are the elements of the second row of the matrix A.
- $a_{m1}, a_{m2}, \cdots, a_{mn}$ are the elements of the mth row of the matrix A.

Matrix operations review

How to perform matrix multiplication?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix} \tag{10}$$

For example, acting with 2x2 matrix on a 2x1 column matrix:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} a\alpha + b\beta \\ c\alpha + d\beta \end{bmatrix} \tag{11}$$

Matrix representation of qubit

Experimentally, we can manipulate qubits using lasers or passing them through optical devices.

Changing the qubit state is equivalent to changing the amplitudes α and β.
This can be done using the action of an unitary matrix U on the qubit.
Let the state of the qubit be $|\psi\rangle$. We can change the state of the qubit to $|\psi'\rangle$ using the action of the unitary matrix U on the qubit:

$$|\psi'\rangle = \boldsymbol{U}|\psi\rangle \tag{12}$$

Unitary means that the matrix U acts on the qubit without changing the norm of the qubit, i.e. $|\alpha|^2+|\beta|^2= 1$.
The matrix U is unitary if its conjugate transpose is equal to its inverse:

$$U^\dagger U^{-1} = U^{-1}U^\dagger = I \tag{13}$$

Example:
What is the conjugate transpose of the following matrix:

$$A = \begin{bmatrix} 1 & i \\ 1 & i \end{bmatrix} \tag{14}$$

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The conjugate transpose of the matrix U is:

$$A^{\dagger} = \begin{bmatrix} 1 & 1 \\ -i & -i \end{bmatrix} \tag{15}$$

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Is the matrix A unitary?

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Is the matrix A unitary?

No, the matrix A is not unitary:

$$AA^\dagger = \begin{pmatrix} 1 & i \\ 1 & i \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -i & -i \end{pmatrix} = 2 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Matrix representation of qubit

Given the state of a qubit in $|0\rangle$. What is the result of applying the unitary operator $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ to the qubit?

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$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle \tag{16}$$

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The result of applying the unitary operator X to the qubit is:

$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle \tag{16}$$

Hence the matrix X flips the state of the qubit from $|0\rangle$ to $|1\rangle$.

Matrix representation of qubit

Let's now perform two successive operations on the qubit in state $|0\rangle$. First, we apply the unitary operator X to the qubit, and then we apply the unitary operator $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ to the qubit.

What is the result of applying the unitary operator Y to the qubit?

Matrix representation of qubit

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$$Y|1\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix} = -i|0\rangle \tag{17}$$

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$$Y|1\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix} = -i|0\rangle \tag{17}$$

Hence the matrix Y flips the state of the qubit from $|1\rangle$ to $-i|0\rangle$.

Technical details

Don't forget to install the following packages in your preamble:

- Qiskit, a Python library for quantum computing.
- Pennylane, a Python library for quantum machine learning.
- Create account of IBM Quantum Experience.
- Create account of Quantum Inspire to try actual quantum computing hardware.

Matrix representation of qubit

Given the following state of two-qubit system:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \tag{18}$$

Is the state of the two-qubit system normalized?

Thank you!