Introduction to Quantum Computing Workshop

Lesson 3: Building Qubits and Gates with Qiskit

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Building Qubits and Gates with Qiskit

Contents

Building Qubits and Gates with Qiskit:

Bra-ket notation

We use bra-ket notation to describe quantum states:

- $|a\rangle$ *ket*: column vector with 1 in entry a and 0 everywhere else.
- $\langle a|$ *bra*: row vector with 1 in entry a and 0 everywhere else.
- $\langle a| |b\rangle$ or $\langle a|b\rangle$ *inner product* of $\langle a|$ and $|b\rangle$: 1 if $a = b$ and 0 if $a \neq b$.
- $|a\rangle \langle b|$ *outer product* of $|a\rangle$ and $\langle b|$: matrix with 1 in entry (a, b) and 0 everywhere else.

Quantum states

A single qubit can be in *superposition* of two basis states: $|0\rangle$ and $|1\rangle$.

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,,$$

where $\alpha, \beta \in \mathbb{C}$ are *probability amplitudes*.

α and β should satisfy the *normalization condition*:

$$|\alpha|^2 + |\beta|^2 = 1.$$

For a quantum circuit of one qubit, state vector $|\psi\rangle$ can be written as

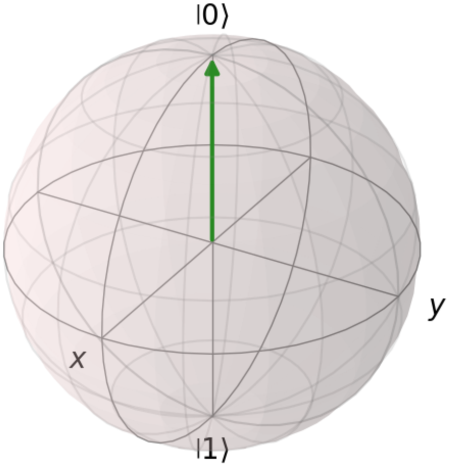
$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \to \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

Bloch sphere

The *Bloch sphere* is used to visualise a quantum state:

- sphere in $\mathbb{R}^3$
- center in origin $(0, 0, 0)$
- radius 1
- state $|0\rangle$ at north pole $(0, 0, 1)$
- state $|1\rangle$ at south pole $(0, 0, -1)$

Bloch sphere: state $|0\rangle$



Bloch sphere

Quantum state $|\psi\rangle$ is represented as a vector from the origin to $(r = 1, \theta, \phi)$, where θ and ϕ are defined by

$$|\psi\rangle = \cos(\frac{\theta}{2})\,|0\rangle + e^{i\phi}\sin(\frac{\theta}{2})\,|1\rangle\,.$$

$0 \leq \theta \leq \pi$ is the angle with the z-axis.

$0 \leq \phi \leq 2\pi$ is the angle with the x-axis.

Assuming $\langle\psi|\psi\rangle = 1$, the *normalization condition*, $r = 1$ and (r, θ, ϕ) is a point on the Bloch sphere.

Unitary operations

A *unitary operation* on a quantum circuit of N qubits is represented by a $2^N \times 2^N$ *unitary* matrix of complex numbers.

A square matrix of complex numbers U is *unitary* if

$$U^\dagger U = \mathbf{I} = UU^\dagger,$$

where $U^\dagger$ is the *conjugate transpose* of U and $\mathbf{I}$ the identity matrix of the same size.

Quantum gates: Pauli operations

Quantum gates are unitary operations that can be performed on one or several qubits of a quantum circuit.

These operations can be applied to a state vector $|\psi\rangle$:

$$|\psi_{\text{new}}\rangle = U\,|\psi\rangle\,.$$

Pauli operations I, X, Y, and Z are unitary operations on a single qubit:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Quantum gates: Hadamard gate

The Hadamard operation H is defined by

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Example: apply Hadamard operation on basis state $|0\rangle$:

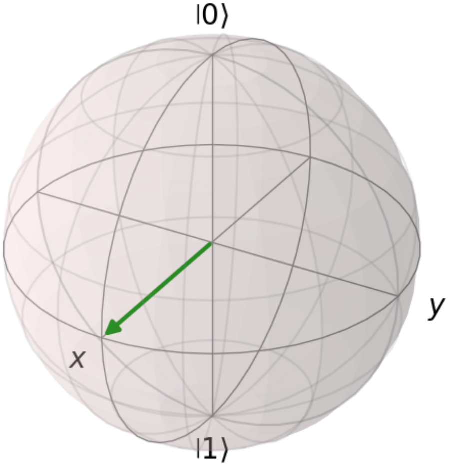
$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

This state is the *plus* state or *Hadamard* state and is denoted as $|+\rangle$.

With respect to basis states $|0\rangle$ and $|1\rangle$, the Hadamard state is defined by

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle .$$

Bloch sphere: state $|+\rangle$



Quantum gates: Phase operations

Phase operations $P(\varphi)$ are described by

$$P(\varphi) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{bmatrix}.$$

The S-gate is the same as $P(\varphi)$ with $\varphi = \frac{\pi}{2}$:

$$S = P(\frac{\pi}{2}) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}.$$

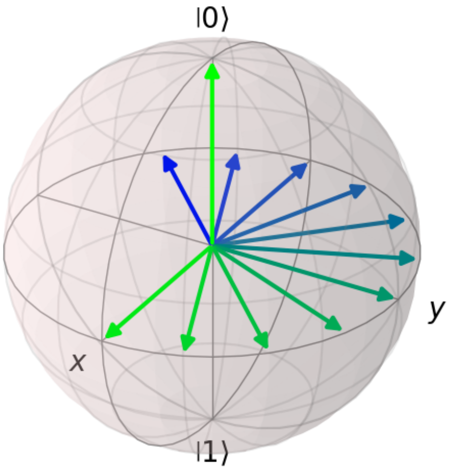
The T-gate is the same as $P(\varphi)$ with $\varphi = \frac{\pi}{4}$:

$$T = P(\frac{\pi}{4}) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{bmatrix}.$$

The Pauli Z operation is the same as $P(\varphi)$ with $\varphi = \pi$:

$$Z = P(\pi) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Bloch sphere: Hadamard gate and 10 times $P(\frac{\pi}{8})$



Quantum gates: Rotations

Rotations $R_x(\varphi)$, $R_y(\varphi)$, and $R_z(\varphi)$ about the x, y, and z axes:

$$R_x(\varphi) = \begin{bmatrix} \cos(\frac{\varphi}{2}) & -i\sin(\frac{\varphi}{2}) \\ -i\sin(\frac{\varphi}{2}) & \cos(\frac{\varphi}{2}) \end{bmatrix}$$

$$R_y(\varphi) = \begin{bmatrix} \cos(\frac{\varphi}{2}) & -\sin(\frac{\varphi}{2}) \\ \sin(\frac{\varphi}{2}) & \cos(\frac{\varphi}{2}) \end{bmatrix}$$

$$R_z(\varphi) = \begin{bmatrix} e^{-i\frac{\varphi}{2}} & 0 \\ 0 & e^{i\frac{\varphi}{2}} \end{bmatrix}$$

Measuring a qubit

Consider quantum state $|\psi\rangle$ of a single qubit in super position:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle.$$

When measuring a qubit, its state collapses either to $|0\rangle$ or $|1\rangle$.

Suppose r is a random value between 0 and 1 drawn from a uniform distribution.

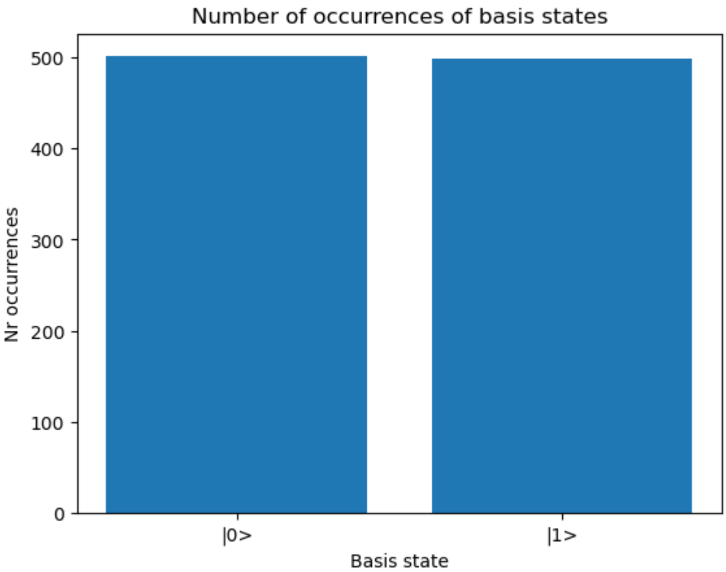
The measured state $|\psi_m\rangle$ is determined as follows:

$$|\psi_m\rangle = \begin{cases} |0\rangle, & \text{if } r < |\alpha|^2 \\ |1\rangle, & \text{otherwise} \end{cases}$$

Note that the normalization condition holds:

$$|\alpha|^2 + |\beta|^2 = 1.$$

Measuring 1000 times: state $|+\rangle$



Qiskit: Quantum Information Software Kit

Qiskit is an open-source software development kit (SDK) for:

- Programming quantum circuits
- Simulating execution of quantum circuits
- Executing quantum circuits on real quantum computers

In this lesson we use Qiskit to simulate the effect of quantum gates applied to a qubit.

Qiskit documentation: `https://docs.quantum.ibm.com/`

To install Qiskit: `pip install qiskit`

Qiskit: Example circuit of one qubit

Import qiskit libraries:

```
from qiskit import QuantumCircuit, ClassicalRegister, QuantumRegister
```

Create a quantum register of 1 qubit:

```
qreg = QuantumRegister(1)
```

Create a classical register of 1 bit:

```
creg = ClassicalRegister(1)
```

Create a quantum circuit with qreg and creg:

```
circuit = QuantumCircuit(qreg, creg)
```

Apply the Hadamard gate to the qubit:

```
circuit.h(qreg[0])
```

Measure the qubit:

```
circuit.measure(qreg[0], creg[0])
```

Qiskit: Draw circuit

To draw the circuit:

```
circuit.draw(output='mpl')
```

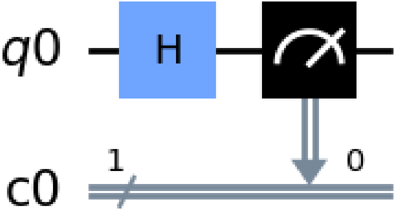


Figure: Circuit with Hadamard gate and measurement

Qiskit: simulate execution

Simulate the quantum circuit:

```
simulator = Aer.get_backend('aer_simulator')
```

Transpile the quantum circuit:

```
job = simulator.run(transpile(circuit, simulator), shots=1000)
```

Get the result:

```
result = job.result()
```

Get the counts:

```
counts = result.get_counts(circuit)
```

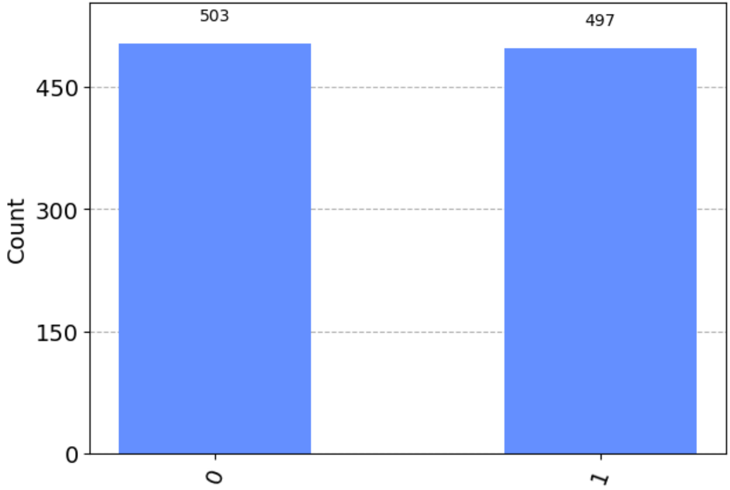
Print the counts

```
print(counts)
```

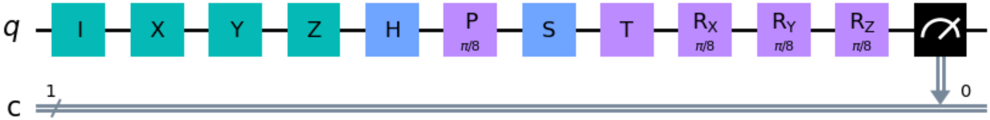
Plot the histogram:

```
plot_histogram(counts)
```

Qiskit: measuring 1000 times: state $|+\rangle$



```
circuit.id(0)              circuit.x(0)         circuit.y(0)          circuit.z(0)
circuit.h(0)               circuit.p(phi,0)     circuit.s(0)          circuit.t(0)
circuit.rx(phi,0)          circuit.ry(phi,0)    circuit.rz(phi,0)
circuit.measure(0,0)
```

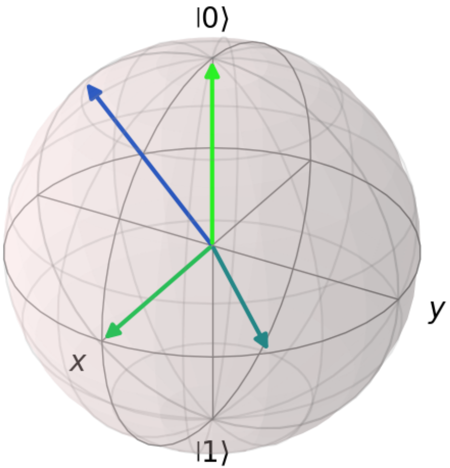


Qiskit: Quantum Phase Estimation (single qubit)

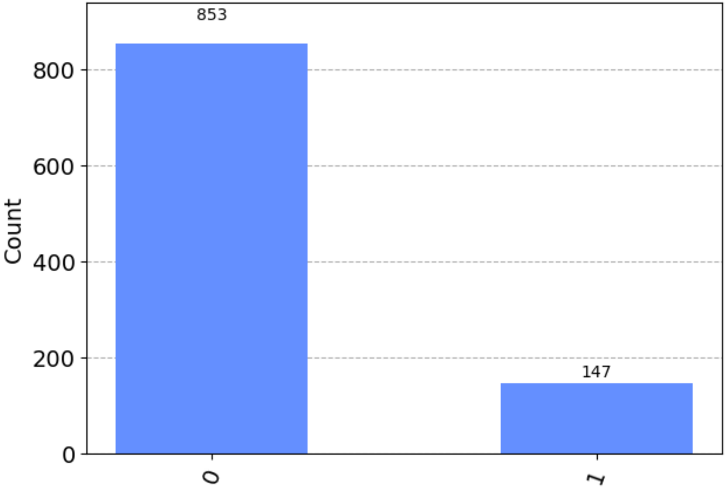
```
phi = np.pi/4
circuit = QuantumCircuit(1, 1)
circuit.h(0)
circuit.p(phi, 0)
circuit.h(0)
```

Can we estimate ϕ by measuring multiple times?

Bloch sphere: Quantum Phase Estimation



Qiskit: measuring 1000 times for phase estimation



Thank you

Thank you for your attention!

Start working with Jupyter notebook for Lesson 3.

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