Amsterdam University of Applied Sciences

Quantum Talent and Learning Center
Intro to Quantum Computing Workshop

Available Time: 30 minutes

Permitted Materials: Simple calculator, graphic and advanced calculators are not permitted

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Name:	Student number:

Generic guidelines:

1. Have fun!
2. If you need questions, please ask your teacher in the room or in the chat.
3. We trust you won't look up the answers online :D

NB. This is an individual exam.

Good luck!

Question 1 Find the complex conjugate transpose of the following matrices:

$$\begin{pmatrix} 1 & 2 & -3 \\ 4i & -5i & 6 \\ 7 & 8 & 9i \end{pmatrix} \tag{1}$$

$$\begin{pmatrix} 3+1i & 2-2i & 1+3i \\ 4-2i & 5+1i & 6-3i \\ 7+1i & 8-2i & 9+3i \end{pmatrix} \tag{2}$$

Answer

First, for the given matrix

$$\begin{pmatrix} 1 & 2 & -3 \\ 4i & -5i & 6 \\ 7 & 8 & 9i \end{pmatrix}$$

the transpose of the matrix is:

$$\begin{pmatrix} 1 & 4i & 7 \\ 2 & -5i & 8 \\ -3 & 6 & 9i \end{pmatrix}$$

and the complex conjugate of the transposed matrix is:

$$\begin{pmatrix} 1 & -4i & 7 \\ 2 & 5i & 8 \\ -3 & 6 & -9i \end{pmatrix}$$

Hence the final answer is

$$\begin{pmatrix} 1 & -4i & 7 \\ 2 & 5i & 8 \\ -3 & 6 & -9i \end{pmatrix}.$$

Second, for the given matrix

$$\begin{pmatrix} 3+1i & 2-2i & 1+3i \\ 4-2i & 5+1i & 6-3i \\ 7+1i & 8-2i & 9+3i \end{pmatrix}$$

the transpose of the matrix is:

$$\begin{pmatrix} 3+1i & 4-2i & 7+1i \\ 2-2i & 5+1i & 8-2i \\ 1+3i & 6-3i & 9+3i \end{pmatrix}$$

and the complex conjugate of the transposed matrix is:

$$\begin{pmatrix} 3-1i & 4+2i & 7-1i \\ 2+2i & 5-1i & 8+2i \\ 1-3i & 6+3i & 9-3i \end{pmatrix}$$

Question 2 Which of the following expressions represent quantum state(s), you can choose more than one if applicable:

1. $|\psi\rangle = |0\rangle$.
2. $|\psi\rangle = |1\rangle$.
3. $|\psi\rangle = |0\rangle + |1\rangle$.
4. $|\psi\rangle = \frac{1}{\sqrt{2}}[|0\rangle - |1\rangle]$.
5. $|\psi\rangle = \frac{1}{\sqrt{2}}[|0\rangle + i\,|1\rangle]$.

Answer

For $|\psi\rangle = |0\rangle$ and $|\psi\rangle = |1\rangle$, they are quantum states. The reason is, quantum states should be normalized.

The expression $|\psi\rangle = |0\rangle + |1\rangle$ does not represent a quantum state since it is not normalized.

The expression $|\psi\rangle = \frac{1}{\sqrt{2}}[|0\rangle - |1\rangle]$ is a quantum state since it is normalized.

Similarly, the expression $|\psi\rangle = \frac{1}{\sqrt{2}}[|0\rangle + i\,|1\rangle]$ is a quantum state since it is normalized.

Question 3 Which of the following matrices are unitary: Note: multiple answers are possible.

1. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
2. $\begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$.
3. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.
4. $\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$.
5. $\begin{pmatrix} 1 & 0 \\ 0 & \frac{i}{\sqrt{2}} \end{pmatrix}$.

Answer

General rule: A matrix is unitary if its conjugate transpose multiplied by the matrix itself gives an identity matrix:

$$U^\dagger U = I. \tag{3}$$

By applying the rule to the given matrices, we can determine which of them are unitary.

The matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is a unitary matrix.

The matrix $\begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$ is a unitary matrix.

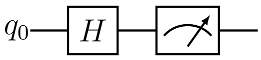
The matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is a unitary matrix.

The matrix $\begin{pmatrix} 1 & 0 \\ 0 & i/\sqrt{2} \end{pmatrix}$ is not a unitary matrix.

Question 4 Let's find the state of the qubit after applying the Hadamard gate as shown in the figure.

The Hadamard gate is defined as:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{4}$$



First, apply the Hadamard gate to the qubit state $|\psi\rangle = |0\rangle$ and determine the new state of the qubit.

Answer

The Hadamard gate is applied to the qubit state $|\psi\rangle = |0\rangle$ as follows:

$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle). \tag{5}$$

Hence, the new state of the qubit is $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$.

Second, apply the Hadamard gate to the qubit state $|\psi\rangle = |1\rangle$ and determine the new state of the qubit.

Answer

The Hadamard gate is applied to the qubit state $|\psi\rangle = |1\rangle$ as follows:

$$H|1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle). \tag{6}$$

Hence, the new state of the qubit is $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$.

Third, what is the probability of measuring the qubit in the state $|0\rangle$ after applying the Hadamard gate to the qubit state $|\psi\rangle = |1\rangle$?

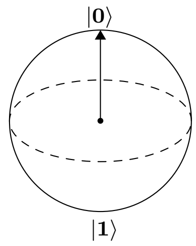
Answer

The probability of measuring the qubit in the state $|0\rangle$ after applying the Hadamard gate to the qubit state $|\psi\rangle = |1\rangle$ is given by:

$$P(|0\rangle) = |\langle 0|H|1\rangle|^2 = \left| \frac{1}{\sqrt{2}} \langle 0|0\rangle - \frac{1}{\sqrt{2}} \langle 0|1\rangle \right|^2 = \left| \frac{1}{\sqrt{2}} \right|^2 = \frac{1}{2}. \tag{7}$$

Hence, the probability of measuring the qubit in the state $|0\rangle$ is $\frac{1}{2}$.

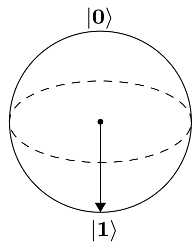
Question 5 Let's find the corresponding quantum state to a given Bloch sphere representation. Write below each Bloch sphere representation the corresponding quantum state in the computational basis. Assume the x-axis is horizontal, the z-axis is vertical, and the y-axis is perpendicular to the paper.



$|\psi\rangle =?$

Answer

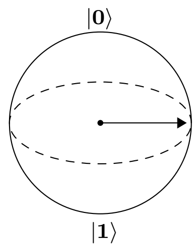
The given Bloch sphere representation corresponds to the quantum state $|\psi\rangle = |0\rangle$.



$|\psi\rangle =?$

Answer

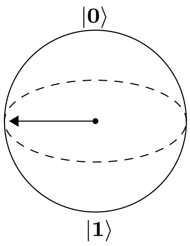
The given Bloch sphere representation corresponds to the quantum state $|\psi\rangle = |1\rangle$.



$|\psi\rangle =?$

Answer

The given Bloch sphere representation corresponds to the quantum state $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$.



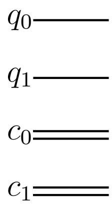
$|\psi\rangle = ?$

Answer

The given Bloch sphere representation corresponds to the quantum state $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$.

Question 6 Let's emulate together a quantum circuit of two qubits and entangle them.

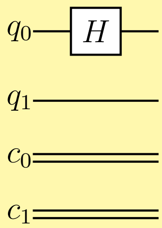
We initialize both qubits in state $|0\rangle$ and two classical bits in the given quantum circuit. What will be the total initial state of the two qubits?



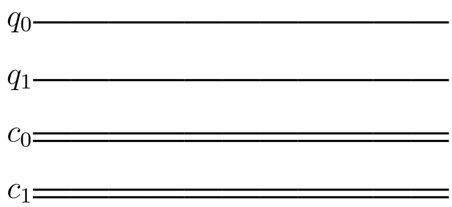
Answer is: $|q_0 q_1\rangle =$

> **Answer**
>
> The total initial state of the two qubits is $|q_0 q_1\rangle = |00\rangle$.

Apply a Hadamard gate to the first qubit. First, draw it in the quantum circuit given below and write the new state of the two qubits.

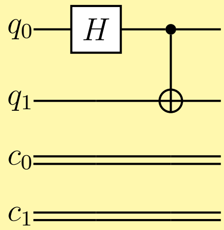
> **Answer**
>
> First, apply a Hadamard gate to the first qubit in the quantum circuit as follows:
>
> 
>
>
> The new state of the two qubits is $|q_0 q_1\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$.

Second, apply a CNOT gate to the two qubits. First, draw it in the quantum circuit given above and write the new state of the two qubits. Note that CNOT gate has the first qubit as the control qubit and the second qubit as the target.
Make your drawing here:



Answer

Second, apply a CNOT gate to the two qubits in the quantum circuit as follows:



The new state of the two qubits is $|q_0 q_1\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

How does the state of the two qubits change after applying the Hadamard and CNOT gates?

Answer

After applying the Hadamard and CNOT gates, the state of the two qubits changes from $|00\rangle$ to $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. The two qubits are now entangled.

Given the final state of the two qubits, how do you see the entanglement?

Answer

The final state of the two qubits is $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. The two qubits are entangled, by knowing the state of one qubit know the state of the other qubit.

——— *End of Examination* ———