Amsterdam University of Applied Sciences

Quantum Talent and Learning Center
Intro to Quantum Computing Workshop

Available Time: 30 minutes

Permitted Materials: Simple calculator, graphic and advanced calculators are not permitted

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Name:	Student number:

Generic guidelines:

1. Have fun!
2. If you need questions, please ask your teacher in the room or in the chat.
3. We trust you won't look up the answers online :D

NB. This is an individual exam.

Good luck!

Question 1 Find the conjugate transpose of the following matrices:

$$\begin{pmatrix} 1 & 2 & -3 \\ 4i & -5i & 6 \\ 7 & 8 & 9i \end{pmatrix} \tag{1}$$

$$\begin{pmatrix} 3+1i & 2-2i & 1+3i \\ 4-2i & 5+1i & 6-3i \\ 7+1i & 8-2i & 9+3i \end{pmatrix} \tag{2}$$

Question 2 Which of the following expressions represent quantum state(s), you can choose more than one if applicable:

1. $|\psi\rangle = |0\rangle$.
2. $|\psi\rangle = |1\rangle$.
3. $|\psi\rangle = |0\rangle + |1\rangle$.
4. $|\psi\rangle = \frac{1}{\sqrt{2}}[|0\rangle - |1\rangle]$.
5. $|\psi\rangle = \frac{1}{\sqrt{2}}[|0\rangle + i\,|1\rangle]$.

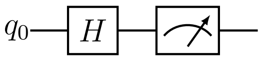
Question 3 Which of the following matrices are unitary: Note: multiple answers are possible.

1. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
2. $\begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$.
3. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.
4. $\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$.
5. $\begin{pmatrix} 1 & 0 \\ 0 & \frac{i}{\sqrt{2}} \end{pmatrix}$.

Question 4 Let's find the state of the qubit after applying the Hadamard gate as shown in the figure.

The Hadamard gate is defined as:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{3}$$

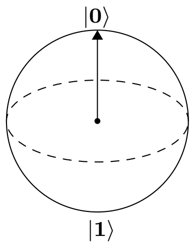


First, apply the Hadamard gate to the qubit state $|\psi\rangle = |0\rangle$ and determine the new state of the qubit.

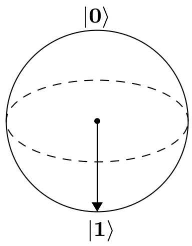
Second, apply the Hadamard gate to the qubit state $|\psi\rangle = |1\rangle$ and determine the new state of the qubit.

Third, what is the probability of measuring the qubit in the state $|0\rangle$ after applying the Hadamard gate to the qubit state $|\psi\rangle = |1\rangle$?

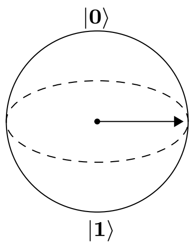
Question 5 Let's find the corresponding quantum state to a given Bloch sphere representation. Write below each Bloch sphere representation the corresponding quantum state in the computational basis. Assume the x-axis is horizontal, the z-axis is vertical, and the y-axis is perpendicular to the paper.



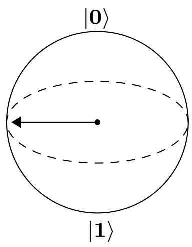
$|\psi\rangle =?$



$|\psi\rangle =?$



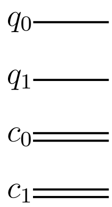
$|\psi\rangle =?$



$|\psi\rangle =?$

Question 6 Let's emulate together a quantum circuit of two qubits and entangle them.

We initialize both qubits in state $|0\rangle$ and two classical bits in the given quantum circuit. What will be the total initial state of the two qubits?

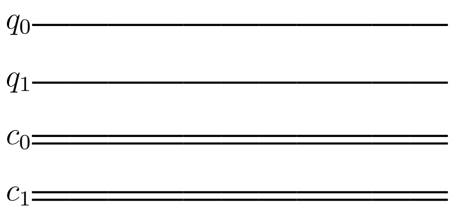


Answer is: $|q_0 q_1\rangle =$

Apply a Hadamard gate to the first qubit. First, draw it in the quantum circuit given below and write the new state of the two qubits.

Second, apply a CNOT gate to the two qubits. First, draw it in the quantum circuit given above and write the new state of the two qubits.

Make your drawing here:



How does the state of the two qubits change after applying the Hadamard and CNOT gates?

Given the final state of the two qubits, how do you see the entanglement?

——— *End of Examination* ———