Introduction to Quantum Computing

Week 5: Constructing quantum circuits and quantum computing programming with Qiskit

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Quantum Talent and Learning Center
Amsterdam University of Applied Sciences

Week 5
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Agenda for today:

- Recap of previous sessions.
- Quick review of the blocksphere.
- Classical vs. quantum computing.
- Quantum computing programming with Qiskit.
- Entanglement.

Every 30 min or so we will have a feed from other QTLC groups.
Feel free to ask questions at any time!
It is an interactive workshop, we all learn from each other!

Workshop facilities

- Join Discord and display your name instead of the nickname or the username.
- Check the invite email for the Discord link.

JupyterHub facilities

- JupyterHub is up and running; please check your accounts.
- If you have any issues, please let us know.
- We should test it between 1:30 until 14:30.

Server's link:

`https://jupyter.snellius.surf.nl`

`https://jupyter.snellius.surf.nl/jhshr001/`

Session recording

- The session is being recorded.

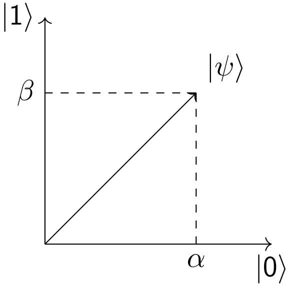
Recap of the previous sessions

- Classical superposition vs. quantum superposition.
- Measurements in quantum mechanics.
- Quantum mechanics relies on linear algebra.
- Matrix operations and manipulations.
- Bra-Ket notation.
- Classical bits vs. qubits.
- Qubit states.
- Bloch Sphere representation.
- Unitary matrices and quantum gates.
- Doing computations with quantum circuits.

Dirac Bra-Ket Notation

In analogy to vectors, we can write the wavefunction as a ket $|\Psi\rangle$:

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



with α and β called the amplitudes of the states and they are generally complex numbers.

Dirac Bra-Ket Notation

Amplitudes give the probability of finding the system in a given state when performing a measurement.

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Hence,

$$|\alpha|^2 + |\beta|^2 = 1$$

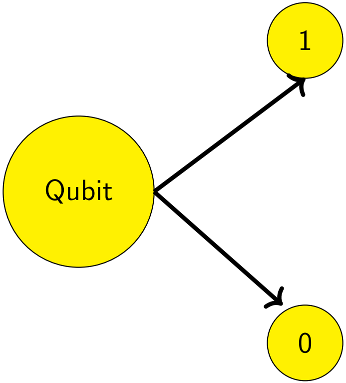
The particle exists by itself in a superposition of states.

Qubit vs. classical bit

Qubit is a quantum bit.

Classical bit: can be in one of two states: 0 or 1.

Qubit: can be in a superposition of 0 and 1.



Matrix representation of qubit

We use matrix algebra to represent qubits.

State of a single qubit:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{1}$$

In a vector form/representation:

$$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \tag{2}$$

The states $|0\rangle$ and $|1\rangle$ are represented by the following column matrices:

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{and} \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{3}$$

The coefficients α and β are complex numbers, and they are generally called the amplitudes of the states.

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Given the state of a qubit in $|0\rangle$. What is the result of applying the unitary operator $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ to the qubit?

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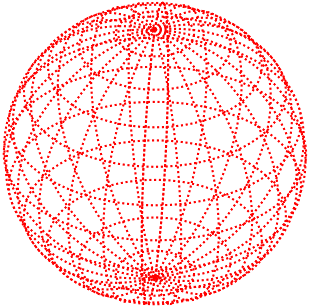
Hence the matrix X flips the state of the qubit from $|0\rangle$ to $|1\rangle$.

Bloch Sphere

Bloch sphere is a geometrical representation of a qubit. It is a representation of the qubit state as a vector on unit sphere.

Before we go into the details of the Bloch sphere, let's first understand the spherical

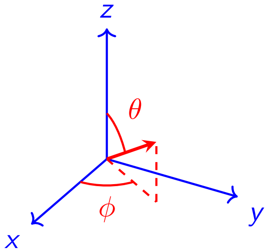
coordinates and the concept of a unit sphere.



Spherical coordinates

Spherical coordinates are a system of three coordinates used to describe the position of a point in space.

Typical coordinate systems are Cartesian, cylindrical, and spherical.
These are orthogonal coordinate systems.



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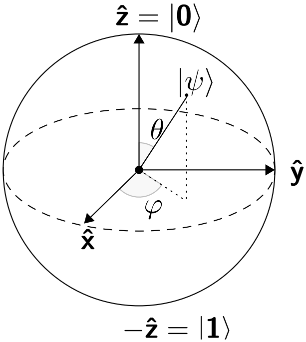
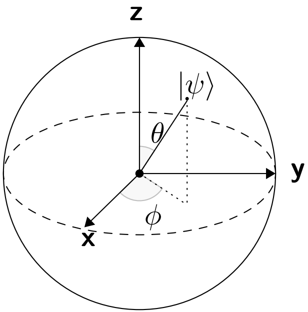


Bloch Sphere



- The spherical coordinates are (r, θ, ϕ).
- The unit sphere is defined by the equation $x^2 + y^2 + z^2 = 1$.
- Spherical coordinates are related to the Cartesian coordinates by the following equations:

$$
\begin{aligned}
x &= r\sin(\theta)\cos(\phi) \\
y &= r\sin(\theta)\sin(\phi) \\
z &= r\cos(\theta)
\end{aligned}
$$

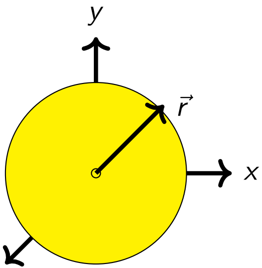
Computer Lab

Now, open your notebook and make a function that converts the Cartesian coordinates to the spherical coordinates using the previously formulas. Make sure you can make another function to plot the transformed point on a sphere.

Note: There is difference in defining the polar angles θ and ϕ between the physics and

mathematics. In physics, the polar angle θ is measured from the positive z-axis, while in mathematics, the polar angle θ is measured from the positive x-axis.

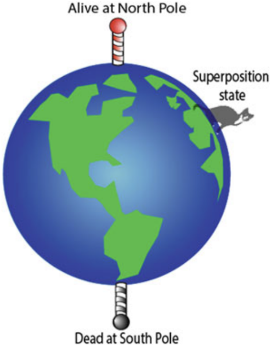
$$x = r\sin(\theta)\cos(\phi)$$
$$y = r\sin(\theta)\sin(\phi)$$
$$z = r\cos(\theta)$$



Schrödinger's cat on a Blochsphere

Schrödinger's cat is determined to be alive.What location on the Earth in the figure could the cat have been before the quantum measurement?

1. Russia.
2. North Pole.
3. Australia.
4. All the above.



Question source: Hughes et al. (2021), Springer. Quantum Computing for the Quantum Curious.

Schrödinger's cat on a Blochsphere

The cat could have been anywhere on Earth except for the South Pole. Notice that in Australia the cat has a smaller probability of being alive since it is further away from the North Pole.

Question source: Hughes et al. (2021), Springer. Quantum Computing for the Quantum Curious.

Blochsphere

The state of a qubit can be represented by a point on the Blochsphere.
This point is defined by the spherical coordinates (θ, ϕ).
It also gives us an idea about the superposition of the qubit.

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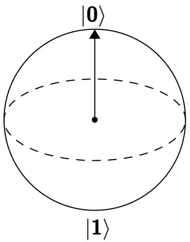

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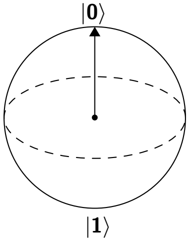

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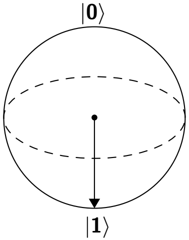

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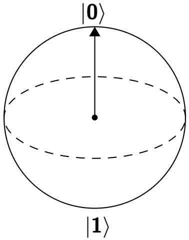

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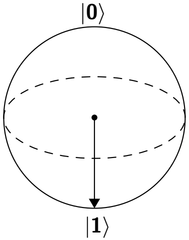

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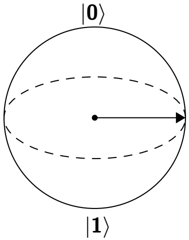

$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle + |1\rangle\right]$

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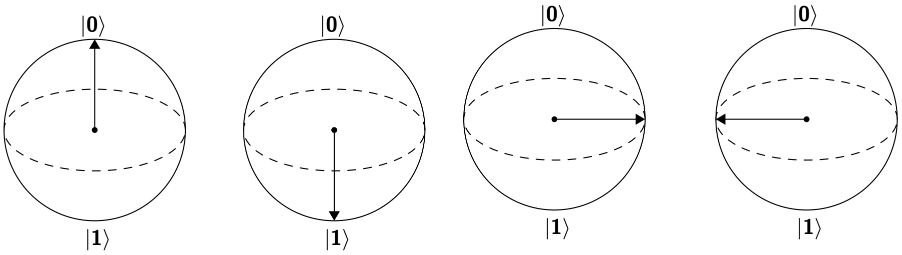


$|\psi\rangle = |0\rangle$ $\quad$ $|\psi\rangle = |1\rangle$ $\quad$ $|\psi\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle + |1\rangle\right]$ $\quad$ $|\psi\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle - |1\rangle\right]$

Question source: Hughes et al. (2021), Springer. Quantum Computing for the Quantum Curious.

Blochsphere

Let's have fun with the Blochsphere!

Go to the following link and play with the Blochsphere:
First, we check the action manually:

`https://attilakun.net/bloch/`

`https://bits-and-electrons.github.io/bloch-sphere-simulator/`

Then, we check the action using the IBM Quantum Experience:

https://quantum.ibm.com/

Doing computations with quantum computing

So far, we have learned how to represent qubits using the Blochsphere.

This time, we are going to learn how to do computations with quantum computing.

Performing quantum computations is different from classical computations. It is more complex and requires a different approach.

Doing computations with quantum computing

In classical computing, we use bits to represent information.
In quantum computing, we use qubits to represent information.

Classical Computing	**Quantum Computing**
Bits	Qubits
0 or 1	0, 1, or superposition of 0 and 1
AND, OR, NOT gates	Quantum gates
classical circuits	quantum circuits

In classical computing, we use AND, OR, and NOT gates to manipulate bits.
In quantum computing, we use quantum gates to manipulate qubits.

Classical vs. quantum gates

Classical gates are used to manipulate bits.
Quantum gates are used to manipulate qubits.

Example:
Classical NOT gate versus quantum NOT (Pauli-X) gate

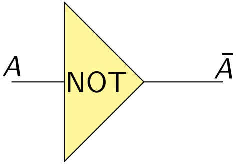
Classical NOT gate	**Quantum NOT gate**
$0 \to 1$	$\lvert 0\rangle \to \lvert 1\rangle$
$1 \to 0$	$\lvert 1\rangle \to \lvert 0\rangle$

Classical vs. quantum circuits

Dealing with quantum computing requires a different approach; it is still primitive and require programming using quantum gates.

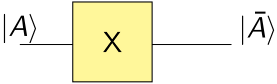
This is different from the digital computing we are used to in the classical (conventional) computing.

Classical NOT Gate



A	$\bar{A}$
0	1
1	0

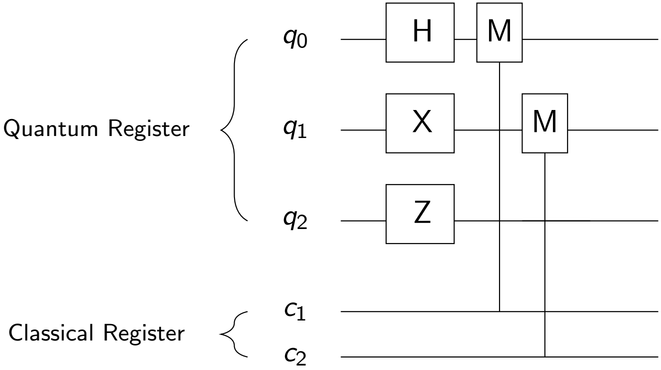
Quantum NOT Gate



$\lvert A\rangle$	$\lvert \bar{A}\rangle$
0	1
1	0

Classical vs. quantum circuits

Classical circuits are made of certain arrangement or configuration classical gates. Similarly, quantum circuits are made of a certain arrangement or configurations of quantum gates.



SURF JupyterHub access

Check your email for credentials you have received on your email.

We are going to learn how to initialize qubits, create quantum circuits, manipulate qubits, and measure them.
First, let's understand the effect of gates on the qubit state using the Blochsphere.

What is a qubit made of?

Classical bits are made of transistors.

It is easy to realize a classical bit in practice. Foe example, if the voltage is above a

certain threshold, the bit is 1, otherwise it is 0.

Qubits are made of quantum systems in a sohpiosticated way.

We cannot simply create a superposition of 0 and 1 by using a transistor in which two currents are there, one flows and the other does not.

Quantum computers are still in their infancy. There are several technologies are being

developed to realize qubits. Still is a long way to go. It is unclear which candidate will provide more benefits than the other or the best qubits.

Answers will be there when quantum computers are more mature and more

commercially available.

What is a qubit made of?

A qubit can be made of different physical systems.

- **Superconducting qubits:**
    - Made of superconducting circuits.
    - They are cooled to very low temperatures.
    - They are the most widely used qubits in quantum computers.
- **Trapped ions:**
    - Made of individual atoms.
    - They are trapped using electromagnetic fields.
    - They are cooled to very low temperatures.

What is a qubit made of?

- **Photonic qubits:**
  - Made of photons.
  - They are used in quantum communication.
- **Topological qubits:**
  - Made of exotic particles called anyons.
  - They are not yet realized in practice.

References

- Hughes, R. J., Nordholt, J. E., Mink, A., & Lanzagorta, M. (2021). Quantum Computing for the Quantum Curious. Springer.
- Nielsen, M. A., & Chuang, I. L. (2010). Quantum Computation and Quantum Information: 10th Anniversary Edition. Cambridge University Press.
- IBM Quantum Experience: `https://quantum-computing.ibm.com/`

Thank you!