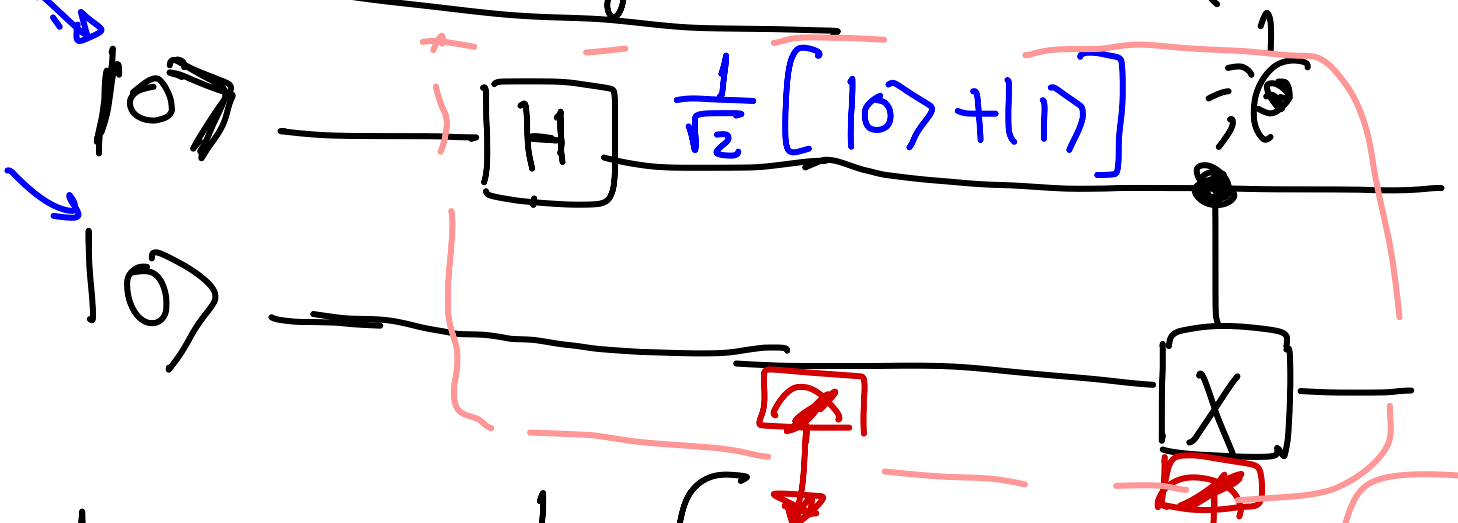
# Quantum Entanglement

Control — $\underline{C}\,\underline{X}$ → X acts on the target

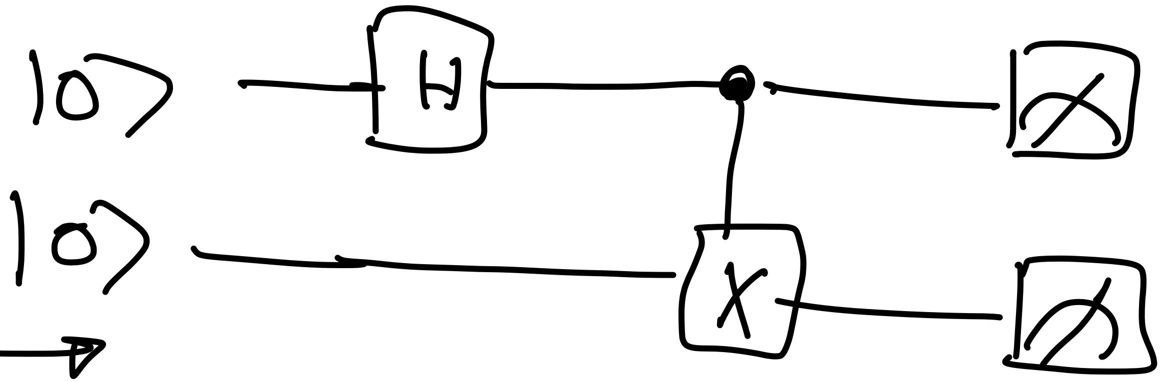



$$|q_0 q_1\rangle = \frac{1}{\sqrt{2}}\left[|00\rangle + |11\rangle\right]$$

$q_1 = |1\rangle$

$q_0 \quad q_1 \qquad q_0 \quad q_1$

2




$$\rightarrow \frac{1}{\sqrt{2}}\left[|00\rangle + |11\rangle\right]$$

Input $|00\rangle$

$$|q_0 q_1\rangle = \frac{1}{\sqrt{3}}\left[|00\rangle + |01\rangle + |11\rangle\right]$$

1 2

$1^{st}$ Situation $\rightarrow$ Second qubit $q_1 = |1\rangle$ $\rightarrow$ $|0\rangle$, $|1\rangle$

$2^{nd}$ Situation first qubit $q_0 = |0\rangle$ $\rightarrow$ $|0\rangle$, $|1\rangle$

first qubit $q_0 = |1\rangle \xrightarrow[\text{for free}]{\text{gives}} q_1 = |1\rangle$

$$|q_0 q_1\rangle = \frac{1}{\sqrt{3}}\left[ |00\rangle + |01\rangle + |11\rangle \right]$$

$q_0$ $q_0$

$|0\rangle$ $|0\rangle$

2nd qubit

$q_0 = |1\rangle$

$q_1 = |1\rangle$

Situation X

$q_1 = |1\rangle$

$q_0 = \begin{cases} |0\rangle \\ |1\rangle \end{cases}$

$$|q_0 q_1\rangle = \frac{1}{\sqrt{3}}\Big[\cancel{|00\rangle} + \cancel{|01\rangle} + \cancel{|11\rangle}\Big]$$

Situation 1

$q_0 \rightarrow |0\rangle$

$q_1 \rightarrow |0\rangle$

$q_1 \rightarrow |1\rangle$

New state

$$\underbrace{N\Big[|0\rangle + |1\rangle\Big]}_{|q_1\rangle}$$

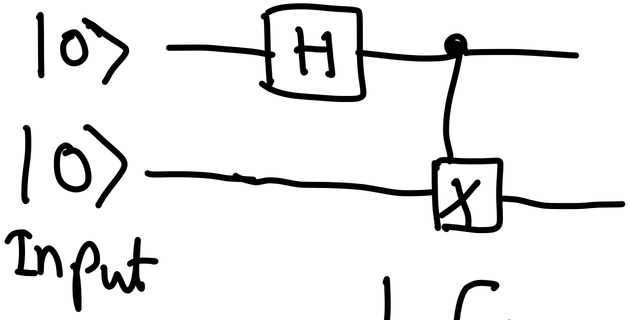
$$|q_0 q_1\rangle = \frac{1}{\sqrt{2}}\left[\,|\underbrace{0}_{q_0}\underbrace{0}_{q_1}\rangle + |\underbrace{1}_{q_0}\underbrace{1}_{q_1}\rangle\,\right] \longrightarrow$$

Situations:

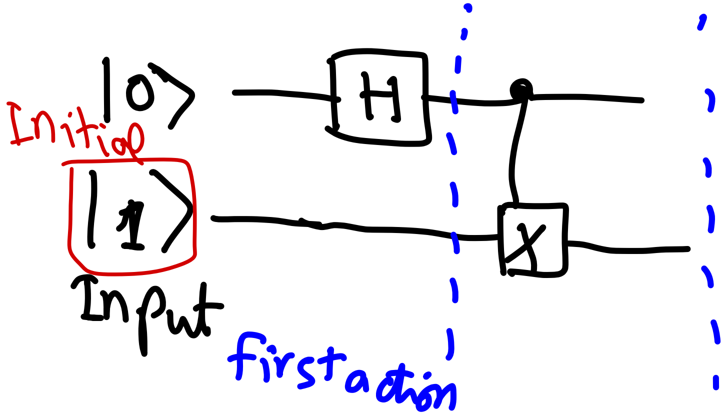
$q_0 \rightarrow |0\rangle \xrightarrow[\text{free}]{\text{for}} q_1 \rightarrow |0\rangle$

$q_1 \rightarrow |0\rangle \xrightarrow[\text{free}]{\text{for}} q_0 \rightarrow |0\rangle$

$q_0 \rightarrow |1\rangle \xrightarrow[\text{free}]{\text{for}} q_1 \rightarrow |1\rangle$




$$\frac{1}{\sqrt{2}}\left[|00\rangle + |11\rangle\right]$$

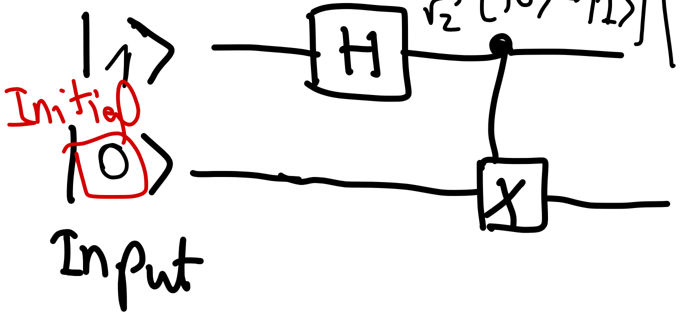



$$q_0 \longrightarrow \frac{1}{\sqrt{2}}\left[|0\rangle + |1\rangle\right]$$

$$\text{CNOT} \longrightarrow \frac{1}{\sqrt{2}}\left[|01\rangle + |10\rangle\right]$$

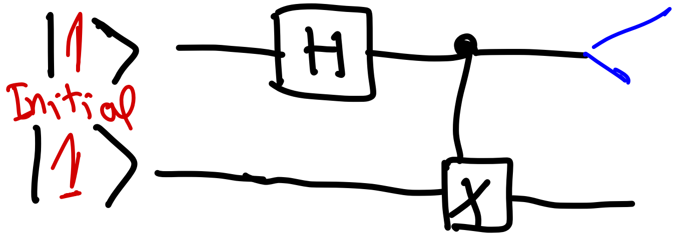
0 1 1 0

$$\frac{1}{\sqrt{2}}\left[|0\rangle - |1\rangle\right] \left[\quad\right] \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



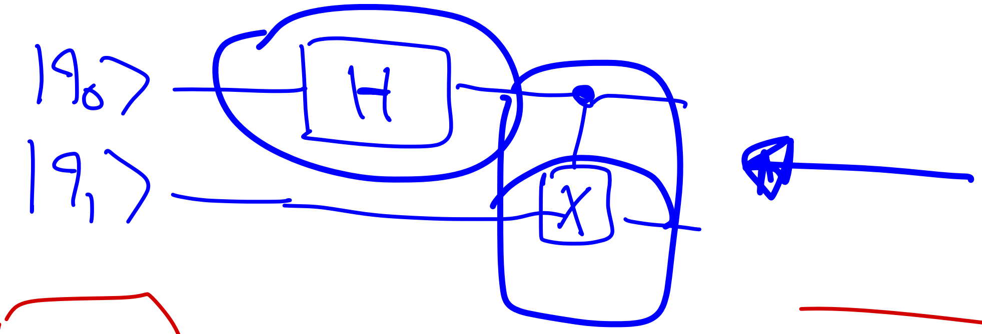

$$\xrightarrow{H} \frac{1}{\sqrt{2}}\left[|0\rangle - |1\rangle\right]$$

$$\xrightarrow{\text{CNOT}} \frac{1}{\sqrt{2}}\left[|00\rangle - |11\rangle\right]$$




$$\xrightarrow{\text{first } H} \frac{1}{\sqrt{2}}\left[|0\rangle - |1\rangle\right]$$

$$\xrightarrow{\text{CNOT}} \frac{1}{\sqrt{2}}\left[|01\rangle - |10\rangle\right]$$




$00 \rightarrow \frac{1}{\sqrt{2}}\left[|00\rangle + |11\rangle\right]$

$01 \rightarrow \frac{1}{\sqrt{2}}\left[|01\rangle + |10\rangle\right]$

$10 \rightarrow \frac{1}{\sqrt{2}}\left[|00\rangle - |11\rangle\right]$

$11 \rightarrow \frac{1}{\sqrt{2}}\left[|01\rangle - |10\rangle\right]$

Comp. basis

Bell Basis

# Quantum Teleportation protocol

Alice

two-qubits

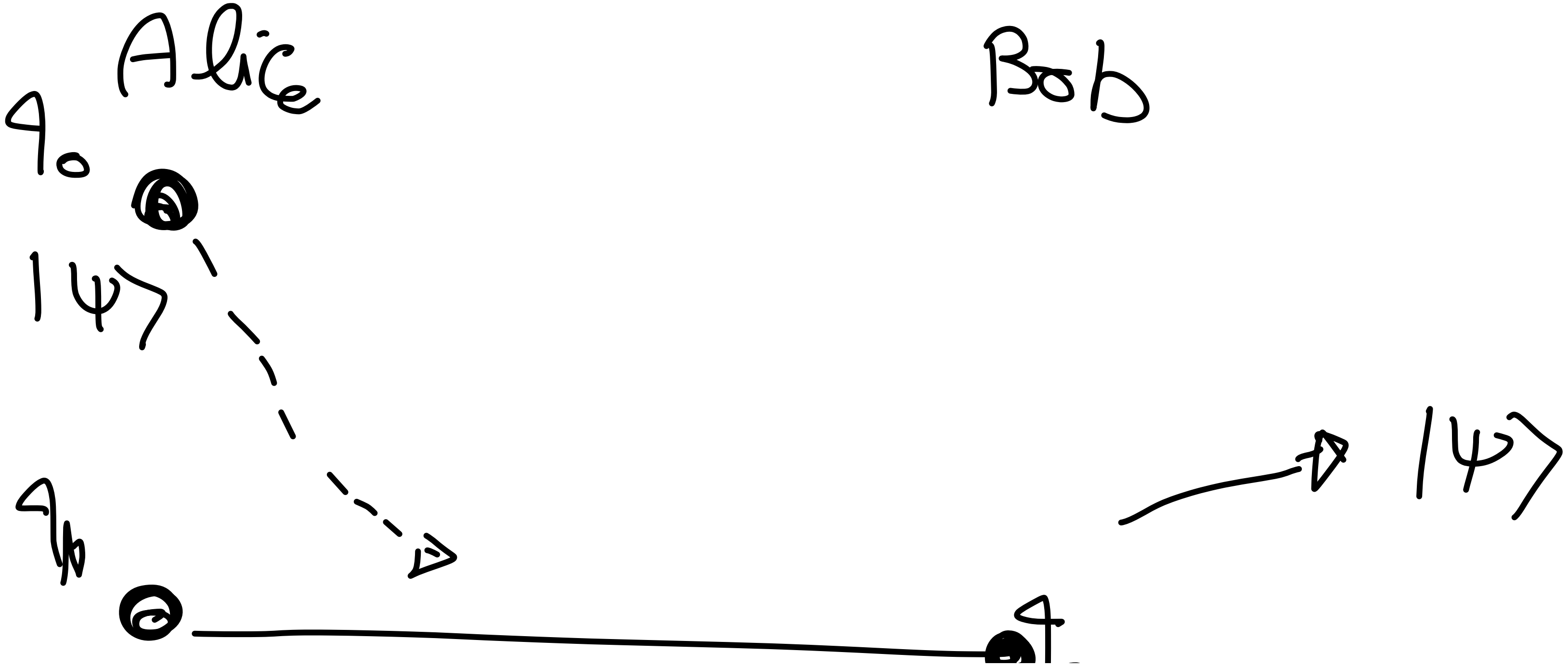
$q_0$ qubit $\longrightarrow$ Unknown State

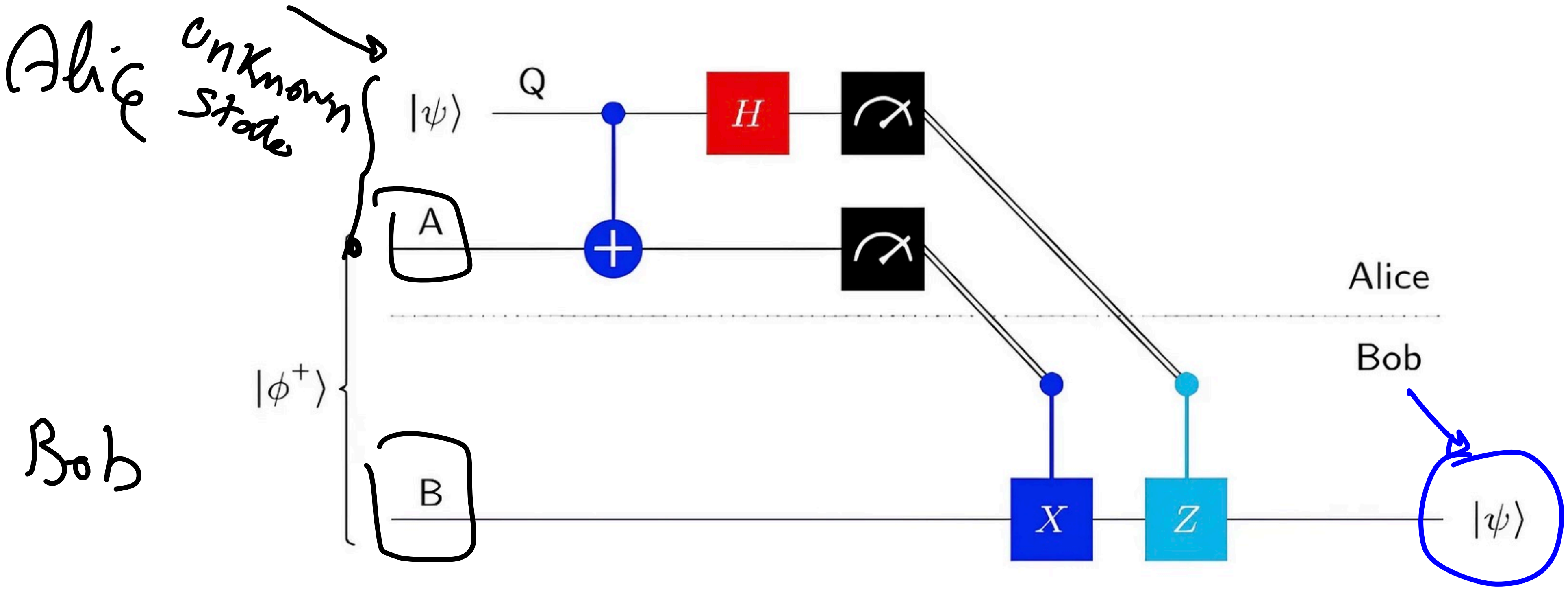
$$|\psi\rangle = a|0\rangle + b|1\rangle$$

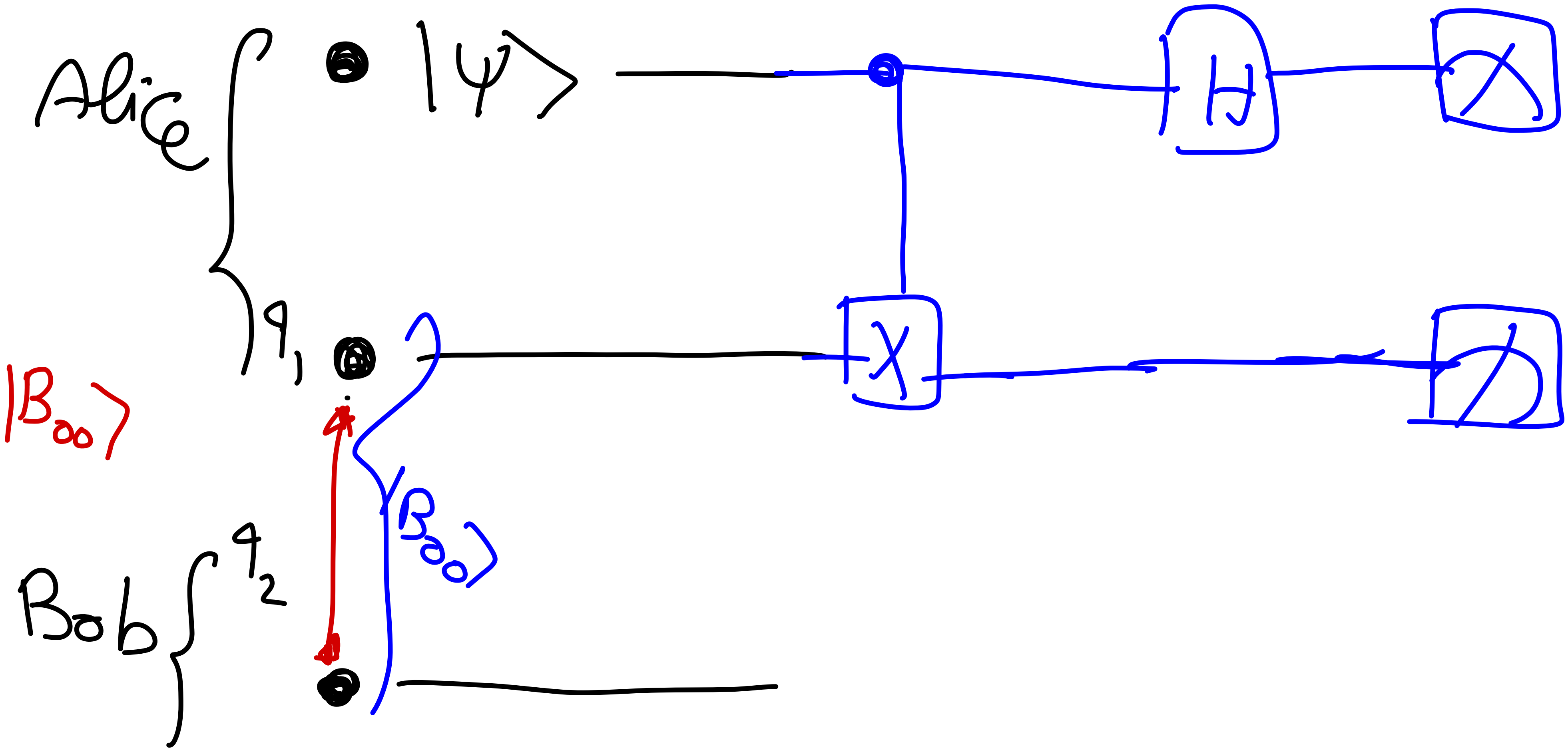
Bob

$q_1$ $\xleftrightarrow{\text{Entangled}}$ $q_2$

Bell states $\frac{1}{\sqrt{2}}\left[|00\rangle + |11\rangle\right]$


Alice
Bob
$q_0$
$|\psi\rangle$
$|\psi\rangle$

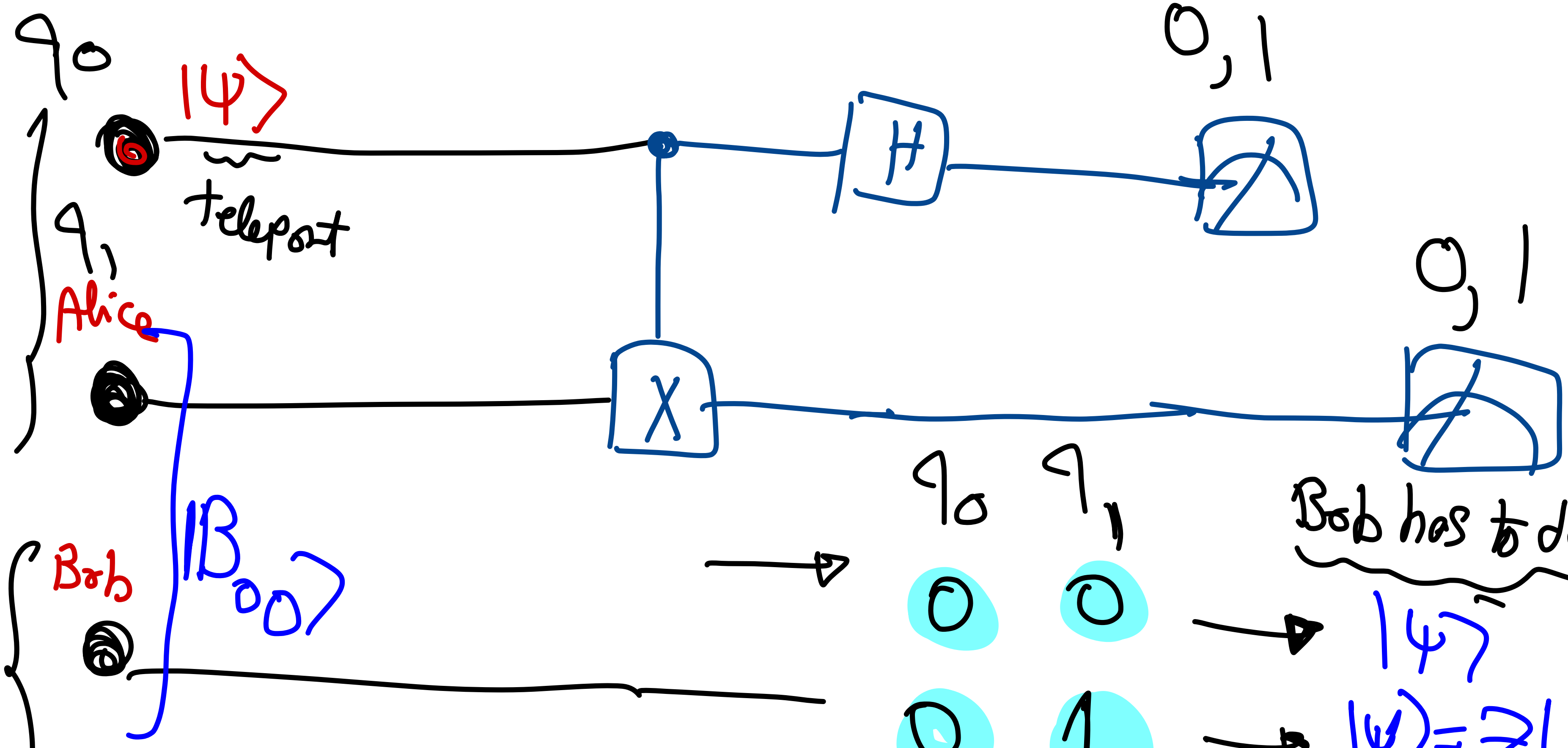

Alice
unknown state
Q
$|\psi\rangle$
H
A
Alice
Bob
$|\phi^+\rangle$
Bob
B
X
Z
$|\psi\rangle$




$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left[ |00\rangle + |11\rangle \right]$$

Alice

Bob




$\frac{1}{\sqrt{2}}\left[|00\rangle + |11\rangle\right]$

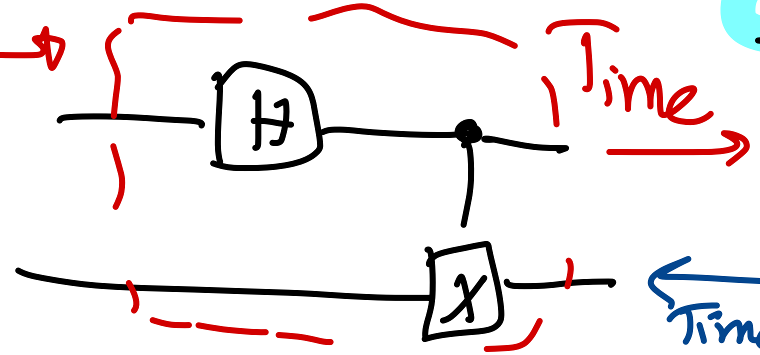
| $q_0$ | $q_1$ | | Bob has to do |
|---|---|---|---|
| 0 | 0 | → | $\|\psi\rangle$ |
| 0 | 1 | → | $\|\psi\rangle = Z\|\psi\rangle$ |
| 1 | 0 | → | $X\|\psi\rangle$ |
| 1 | 1 | → | $XZ\|\psi\rangle$ |

$|\psi\rangle$

$q_0$ $q_1$

$X$ $Z$

Entanglement




← Time Teleportation