Introduction to Quantum Computing

Week 2: Linear Algebra for Quantum Computing

Taha Selim

t.i.m.m.selim2@hva.nl

Quantum Talent and Learning Center
Amsterdam University of Applied Sciences

Week 2
March 22nd, 2024

Agenda for today:

- Recap of the previous session and solving some questions.
- Qubits.
- Coherent and entanglement concepts, pending time.
- Bra-Ket notation and the linear algebra in quantum computing.
- Solving exercises on finding probability of quantum states.
- Matrix representation and matrix operations in quantum computing.

Every 30 min or so we will have a feed from other QTLC groups.
Feel free to ask questions at any time!
It is an interactive workshop, we all learn from each other!

Workshop facilities

- Join Discord and display your name instead of the nickname or the username.
- Check the invite email for the Discord link.

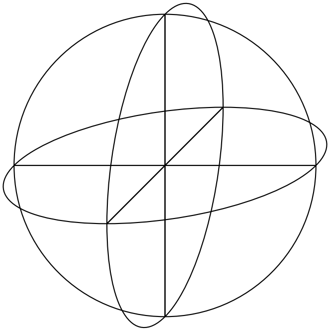
Goede vrijdag

Goede vrijdag (Good Friday): Friday 29 March 2024
No workshop on that day.

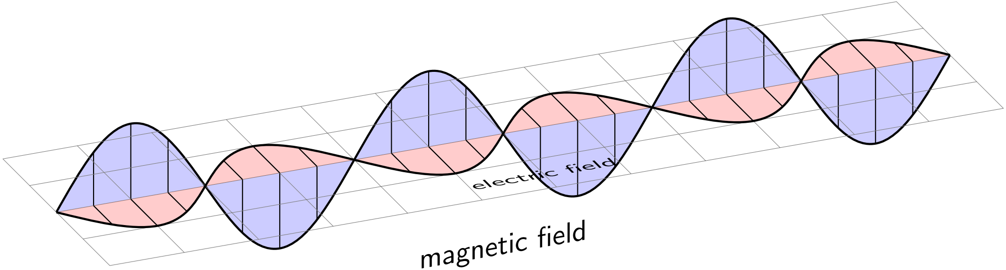
We will resume on Friday 5 April 2024.

We will give you some exercises to work on during the break.

- Classical superposition vs. quantum superposition.
- Measurements in quantum mechanics.



Review, week 1



Qubit vs. classical bit

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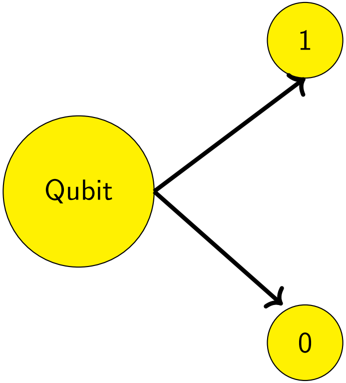

Wavefunctions

Quantum mechanics:
Wavefunctions are used to describe the state of a quantum system.

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Example:Free particle

$$\Psi(x) = Ae^{ikx}$$

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Example:Free particle

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Wavefunction is a complex-valued function, for example, of position and time.

Probability density:
$|\Psi(x,t)|^2$ gives the probability of finding the particle at position x at time t.

Dirac Bra-Ket Notation

We can write the wavefunction as a ket $|\Psi\rangle$:

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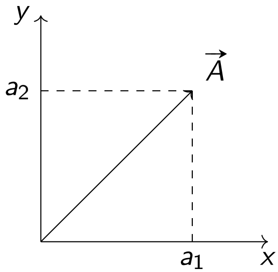
Bra-Ket notation is a standard notation in quantum mechanics.
It is used to describe quantum states and operations.
It is named after Paul Dirac.
Qubit states are represented as kets.

Dirac Bra-Ket Notation

A vector $\vec{A}$ in a two-dimensional space can be written as a column matrix:

$$\vec{A} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$



Using the Dirac notation, we can write the vector $\vec{A}$ as a ket:

$$|A\rangle = a_1|x\rangle + a_2|y\rangle$$

Dirac Bra-Ket Notation

Similarly, a vector $\vec{B}$ in a three-dimensional space can be written as a column matrix:

$$\vec{B} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

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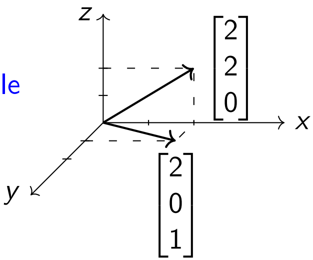

Dirac Bra-Ket Notation

Similarly, a vector $\vec{B}$ in a three-dimensional space can be written as a column matrix:

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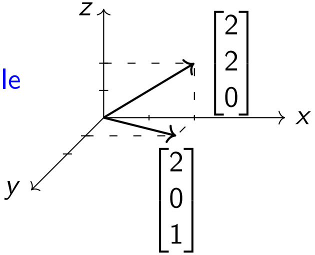


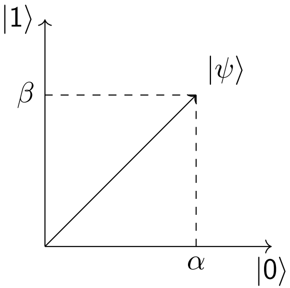
Using the Dirac notation, we can write the vector $\vec{B}$ as a ket:

$$|B\rangle = b_1|x\rangle + b_2|y\rangle + b_3|z\rangle$$

Dirac Bra-Ket Notation

In analogy to vectors, we can write the wavefunction as a ket $|\Psi\rangle$:

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



with α and β called the amplitudes of the states and they are generally complex numbers.

Dirac Bra-Ket Notation

The norm of a 2D vector is:

$$||A|| = \sqrt{a_1^2 + a_2^2}$$

Similarly, the norm of a 3D vector is:

$$||B|| = \sqrt{b_1^2 + b_2^2 + b_3^2}$$

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$$|\alpha|^2 + |\beta|^2 = 1$$

The particle exists by itself in a superposition of states.

Dirac Bra-Ket Notation

Question:
The quantum state of a spinning coin can be written as a superposition of heads and tails. Using heads as $|1\rangle$ and tails as $|0\rangle$, the quantum state of the coin is

$$|\text{coin}\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |0\rangle).$$

What is the probability of getting heads?

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What is the probability of getting heads?

The amplitude of the state $|1\rangle$ is $\beta = \frac{1}{\sqrt{2}}$, so the probability of getting heads is $|\beta|^2 = \frac{1}{2}$. So, the probability is is 0.5 , or 50%.

Similarly, the probability of getting tails is also $\frac{1}{2}$, so the sum of the probabilities of getting heads and tails is 1.

Dirac Bra-Ket Notation

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A weighted coin has twice the probability of landing on heads vs. tails. What is the state of the coin in “ket” notation?

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$$
\begin{aligned}
P_{\text{heads}} + P_{\text{tails}} &= 1 \quad (\text{ Normalization Condition })\\
P_{\text{heads}} &= 2P_{\text{tails}} \quad (\text{ Statement in Example })\\
\rightarrow P_{\text{tails}} &= \frac{1}{3} = \alpha^2\\
\rightarrow P_{\text{heads}} &= \frac{2}{3} = \beta^2\\
\rightarrow \alpha = \sqrt{\frac{1}{3}}, \beta = \sqrt{\frac{2}{3}} &\rightarrow |\text{coin}\rangle = \sqrt{\frac{1}{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle.
\end{aligned}
$$

Dirac Bra-Ket Notation

Question for the creative minds:
How can you use the concept of quantum superposition to describe the composition of a cake?

Dirac Bra-Ket Notation

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How can you use the concept of quantum superposition to describe the composition of a cake?

$$|\text{cake}\rangle = \alpha|\text{chocolate}\rangle + \beta|\text{vanilla}\rangle + \gamma|\text{strawberry}\rangle + \delta|\text{lemon}\rangle + \epsilon|\text{carrot}\rangle + \cdots$$

Qubit vs. classical bit

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Classical bit: can be in one of two states: 0 or 1.
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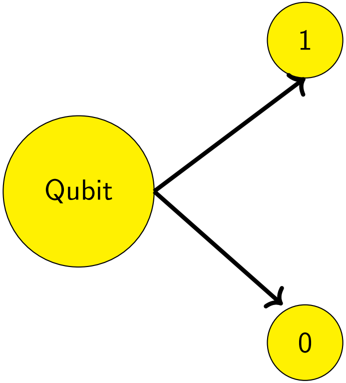

Matrix representation of qubit

We use matrix algebra to represent qubits.
State of a single qubit:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

In a vector form/representation:

$$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

The states $|0\rangle$ and $|1\rangle$ are represented by the following column matrices:

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{and} \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The coefficients α and β are complex numbers, and they are generally called the amplitudes of the states.

What is a matrix?

- A matrix is a rectangular array of numbers, symbols, or expressions, arranged in rows and columns.
- The individual items in a matrix are called its elements or entries.
- The horizontal and vertical lines of entries in a matrix are called rows and columns, respectively.
- The size of a matrix is defined by the number of rows and columns that it contains.
- A matrix with m rows and n columns is called an $m \times n$ matrix.
- A matrix can be used to represent a linear map.
- A matrix can be used to represent a property of a mathematical object.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

- $a_{11}, a_{12}, \cdots, a_{1n}$ are the elements of the first row of the matrix A.
- $a_{21}, a_{22}, \cdots, a_{2n}$ are the elements of the second row of the matrix A.
- $a_{m1}, a_{m2}, \cdots, a_{mn}$ are the elements of the mth row of the matrix A.

Matrix operations review

How to perform matrix multiplication?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

For example, acting with 2x2 matrix on a 2x1 column matrix:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} a\alpha + b\beta \\ c\alpha + d\beta \end{bmatrix}$$

Matrix operations review

How to perform matrix multiplication?

This can be extended to multiplication of 3x3 matrices.

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix} = \begin{bmatrix} aj + bm + cp & ak + bn + cq & al + bo + cr \\ dj + em + fp & dk + en + fq & dl + eo + fr \\ gj + hm + ip & gk + hn + iq & gl + ho + ir \end{bmatrix}$$

Example: multiply the following matrices:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Matrix operations review

Example: multiply the following matrices:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 1 \cdot 5 + 2 \cdot 7 & 1 \cdot 6 + 2 \cdot 8 \\ 3 \cdot 5 + 4 \cdot 7 & 3 \cdot 6 + 4 \cdot 8 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}.$$

Matrix operations review

Example: multiply the following matrices:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix operations review

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$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

$$= \begin{bmatrix} 1 \cdot 1 + 2 \cdot 0 + 3 \cdot 0 & 1 \cdot 0 + 2 \cdot 1 + 3 \cdot 0 & 1 \cdot 0 + 2 \cdot 0 + 3 \cdot 1 \\ 4 \cdot 1 + 5 \cdot 0 + 6 \cdot 0 & 4 \cdot 0 + 5 \cdot 1 + 6 \cdot 0 & 4 \cdot 0 + 5 \cdot 0 + 6 \cdot 1 \\ 7 \cdot 1 + 8 \cdot 0 + 9 \cdot 0 & 7 \cdot 0 + 8 \cdot 1 + 9 \cdot 0 & 7 \cdot 0 + 8 \cdot 0 + 9 \cdot 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$

Matrix operations review

The matrix also can have complex numbers as elements.

Example: multiply the following matrices:

$$\begin{bmatrix} 1 & 2i & 3 \\ 4 & 5 & 6i \\ 7 & 8i & 9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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Solution

$$= \begin{bmatrix} 1\cdot 1 + 2i\cdot 0 + 3\cdot 0 & 1\cdot 0 + 2i\cdot 1 + 3\cdot 0 & 1\cdot 0 + 2i\cdot 0 + 3\cdot 1 \\ 4\cdot 1 + 5\cdot 0 + 6i\cdot 0 & 4\cdot 0 + 5\cdot 1 + 6i\cdot 0 & 4\cdot 0 + 5\cdot 0 + 6i\cdot 1 \\ 7\cdot 1 + 8i\cdot 0 + 9\cdot 0 & 7\cdot 0 + 8i\cdot 1 + 9\cdot 0 & 7\cdot 0 + 8i\cdot 0 + 9\cdot 1 \end{bmatrix} = \begin{bmatrix} 1 & 2i & 3 \\ 4 & 5 & 6i \\ 7 & 8i & 9 \end{bmatrix}.$$

We can also multiply a matrix by a scalar.

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Solution

$$= \begin{bmatrix} 2 \cdot 1 & 2 \cdot 2 \\ 2 \cdot 3 & 2 \cdot 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}.$$

This is equivalent to multiplying each element of the matrix by the scalar.
This is called scaling a matrix.

Matrix operations review

We can also add two matrices.

Example: add the following matrices:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

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Solution

$$= \begin{bmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}.$$

We can also subtract two matrices.

Example: subtract the following matrices:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Solution

$$= \begin{bmatrix} 1-5 & 2-6 \\ 3-7 & 4-8 \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix} = -4 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

Matrix operations review

We can also find the transpose of a matrix.

General rule is to swap the rows and columns of the matrix.
Let A be an $m \times n$ matrix.

The transpose of A, denoted by A^T, is an $n \times m$ matrix.
Example:

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \qquad A^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$$

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$$= \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}.$$

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Matrix operations review

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Solution

$$= \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}.$$

Matrix operations review

Also, we can find the conjugate of a matrix.

The complex conjugate of a matrix is obtained by taking the conjugate of each element of the matrix.
Let A be an $m \times n$ matrix.

Let $T = 1 + 2i$, then the complex conjugate of T is $1 - 2i$.

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Let A be an $m \times n$ matrix.

Let $T = 1 + 2i$, then the complex conjugate of T is $1 - 2i$.

The complex conjugate of A, denoted by A^*, is an $m \times n$ matrix.

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \qquad A^* = \begin{bmatrix} a^* & b^* & c^* \\ d^* & e^* & f^* \end{bmatrix}$$

Example: find the conjugate of the following matrix:

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Matrix operations review

Example: find the conjugate of the following matrix:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^*$$

Solution

$$= \begin{bmatrix} 1^* & 2^* \\ 3^* & 4^* \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

It is a real matrix, so the conjugate of a real matrix is the matrix itself.

Example: find the conjugate of the following matrix:

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Solution

$$= \begin{bmatrix} 1^* & 2i^* & 3^* \\ 4^* & 5^* & 6i^* \\ 7^* & 8i^* & 9^* \end{bmatrix} = \begin{bmatrix} 1 & -2i & 3 \\ 4 & 5 & -6i \\ 7 & -8i & 9 \end{bmatrix}.$$

We can also find the conjugate transpose of a matrix.

The conjugate transpose of a matrix is obtained by taking the conjugate of each element of the matrix and then taking the transpose of the matrix.
Let A be an $m \times n$ matrix.

The conjugate transpose of A, denoted by $A^\dagger$, is an $n \times m$ matrix.

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \qquad A^\dagger = \begin{bmatrix} a^* & d^* \\ b^* & e^* \\ c^* & f^* \end{bmatrix}$$

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Solution

$$= \begin{bmatrix} 1i^{*} & 3^{*} & 5i^{*} \\ 2^{*} & 4i^{*} & 6^{*} \end{bmatrix} = \begin{bmatrix} -i & 3 & i \\ 2 & -4i & 6 \end{bmatrix}.$$

Matrix representation of qubit

Experimentally, we can manipulate qubits using lasers or passing them through optical devices.

Changing the qubit state is equivalent to changing the amplitudes α and β.
This can be done using the action of an unitary matrix U on the qubit.
Let the state of the qubit be $|\psi\rangle$. We can change the state of the qubit to $|\psi'\rangle$ using the action of the unitary matrix U on the qubit:

$$|\psi'\rangle = \boldsymbol{U}|\psi\rangle \tag{1}$$

Unitary means that the matrix U acts on the qubit without changing the norm of the qubit, i.e. $|\alpha|^2+|\beta|^2=1$.
The matrix U is unitary if its conjugate transpose is equal to its inverse:

$$U^\dagger U^{-1} = U^{-1} U^\dagger = I \tag{2}$$

Example:

What is the conjugate transpose of the following matrix:

$$A = \begin{bmatrix} 1 & i \\ 1 & i \end{bmatrix} \tag{3}$$

Matrix representation of qubit

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No, the matrix A is not unitary:
$$AA^\dagger = \begin{bmatrix} 1 & i \\ 1 & i \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -i & -i \end{bmatrix} = 2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle \tag{5}$$

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Hence the matrix X flips the state of the qubit from $|0\rangle$ to $|1\rangle$.

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Let's now perform two successive operations on the qubit in state $|0\rangle$. First, we apply the unitary operator X to the qubit, and then we apply the unitary operator $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ to the qubit.

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Hence the matrix Y flips the state of the qubit from $|1\rangle$ to $-i|0\rangle$.

Thank you

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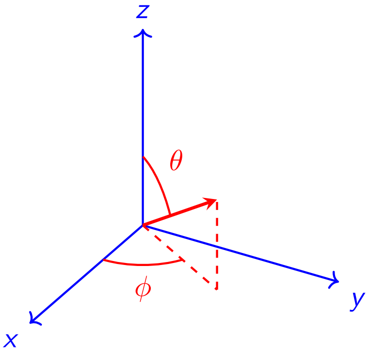

z
θ
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y