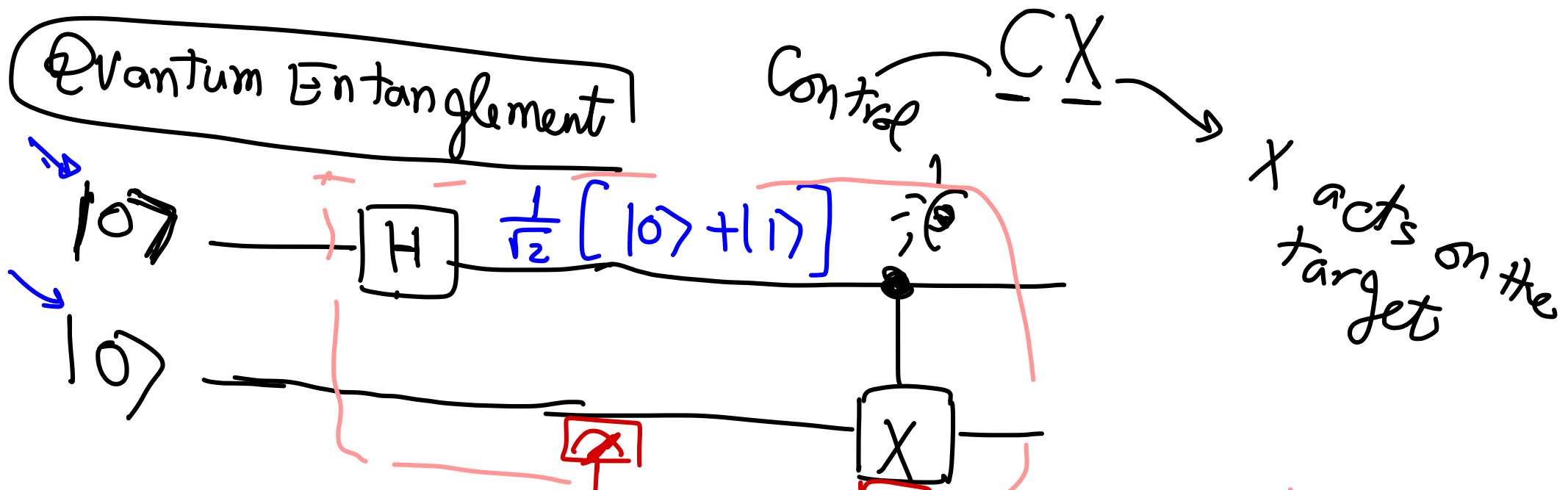


Quantum Entanglement

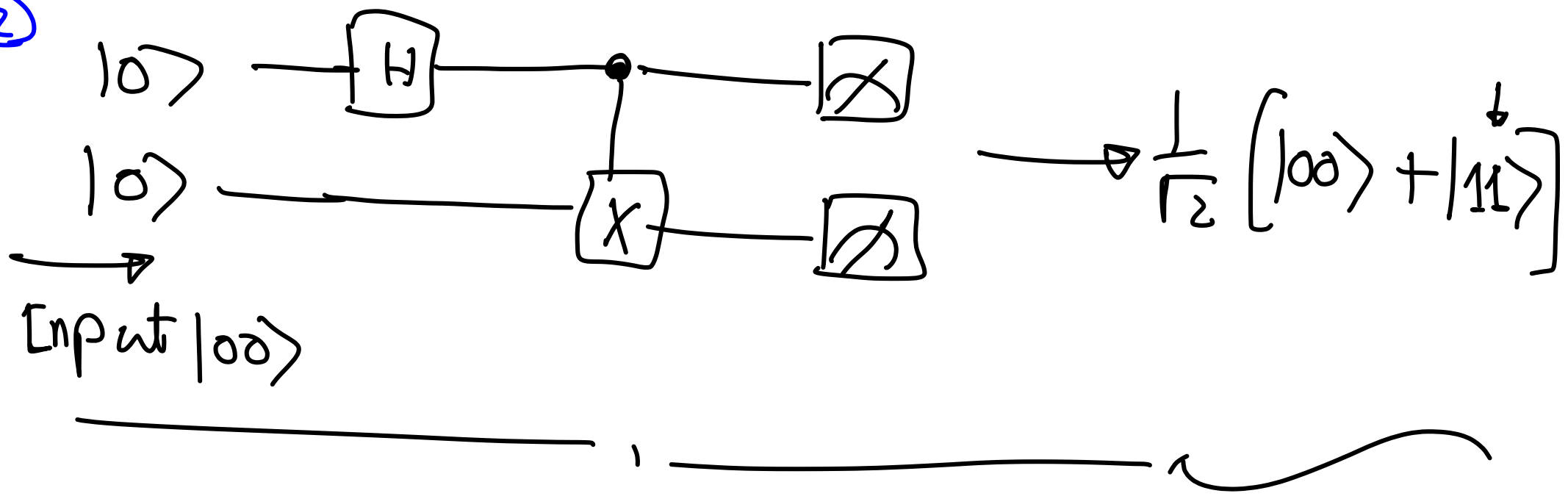


$$|q_0 q_1\rangle = \frac{1}{\sqrt{2}} \left[|00\rangle + |11\rangle \right]$$

q_0 q_1 q_0 q_1

$q_1 = |1\rangle$

②



$$|q_0 q_1\rangle = \frac{1}{\sqrt{3}} \left[|00\rangle + |01\rangle + |11\rangle \right]$$

1st Situation



Second qubit $q_1 = |1\rangle$

2nd Situation

first qubit $q_0 = |0\rangle$

first qubit $q_0 = |1\rangle$ gives $q_1 = |1\rangle$
for free

$$|q_0 q_1\rangle = \frac{1}{\sqrt{3}} \left[|q_0\rangle |00\rangle + |q_0\rangle |01\rangle + |q_0=11\rangle \right]$$

Diagram illustrating the decomposition of the state $|q_0 q_1\rangle$ into a sum of states. The equation shows $|q_0 q_1\rangle = \frac{1}{\sqrt{3}} [|q_0\rangle |00\rangle + |q_0\rangle |01\rangle + |q_0=11\rangle]$. Red arrows point from the $|q_0\rangle$ labels to the $|00\rangle$ and $|01\rangle$ terms. Blue arrows point from the $|q_0=11\rangle$ label to the $|11\rangle$ term. A blue arrow points from the $|00\rangle$ term to the label 2^{nd} q_{vibr} . A red arrow points from the $|01\rangle$ term to the label q_{vibr} . A blue arrow points from the $|11\rangle$ term to the label $q_1 = |1\rangle$.

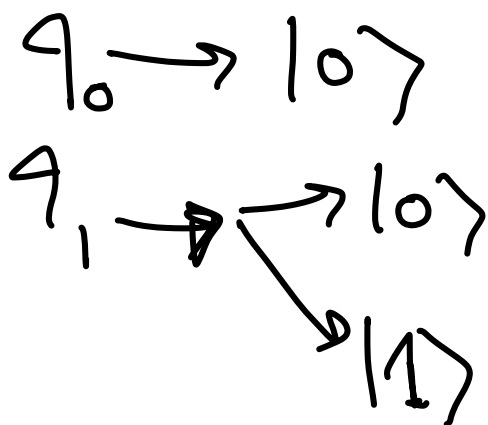
Situation X

$$q_1 = |1\rangle$$

$$q_0 = \begin{cases} |0\rangle \\ |1\rangle \end{cases}$$

$$|q_0 q_1\rangle = \frac{1}{\sqrt{3}} \left[\cancel{|00\rangle} + \cancel{|01\rangle} + \cancel{|11\rangle} \right]$$

Situation 1



New State

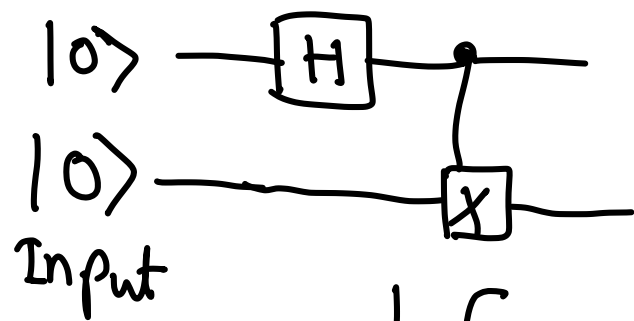
$$N(|0\rangle + |1\rangle)$$

$|q_1\rangle$

$$|q_0 q_1\rangle = \frac{1}{\sqrt{2}} \left[\underbrace{|00\rangle}_{q_0 q_1} + \underbrace{|11\rangle}_{q_0 q_1} \right] \rightarrow$$

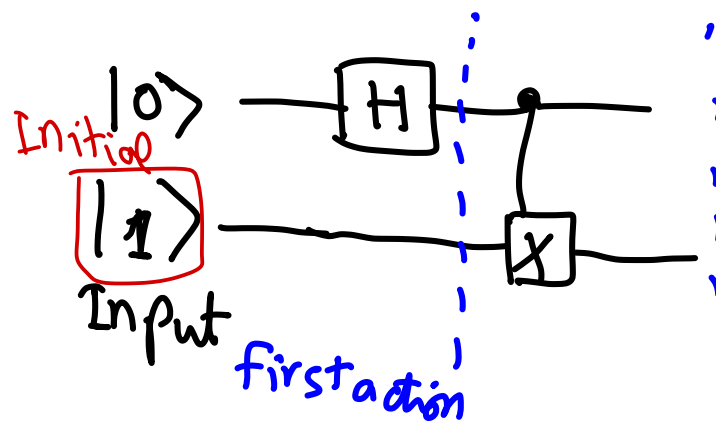
Situations:

$q_0 \rightarrow 0\rangle$	$\xrightarrow{\text{for free}}$	$q_1 \rightarrow 0\rangle$
$q_1 \rightarrow 0\rangle$	$\xrightarrow{\text{for free}}$	$q_0 \rightarrow 0\rangle$
$q_0 \rightarrow 1\rangle$	$\xrightarrow{\text{for free}}$	$q_1 \rightarrow 1\rangle$



$$\frac{1}{\sqrt{2}} [|00\rangle + |11\rangle]$$

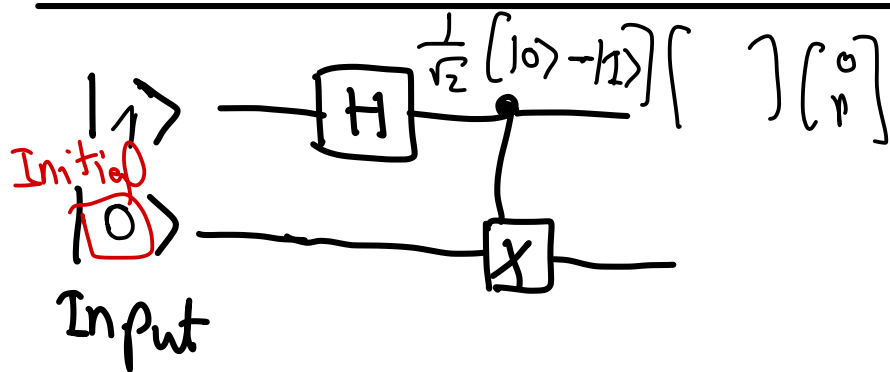
Annotations: Under the first two qubits of $|00\rangle$, there are two blue arrows pointing up. Under the last two qubits of $|11\rangle$, there are two red arrows pointing up.



$$q_0 \rightarrow \frac{1}{\sqrt{2}} [|0\rangle + |1\rangle]$$

$$\text{CNOT} \rightarrow \frac{1}{\sqrt{2}} [|01\rangle + |10\rangle]$$

Annotations: Under $|01\rangle$, there are red arrows pointing up labeled 0 and 1. Under $|10\rangle$, there are red arrows pointing up labeled 1 and 0.

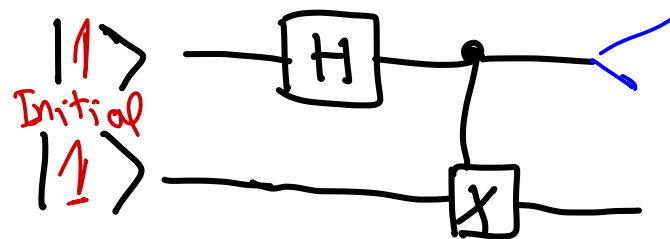


$$H \rightarrow \frac{1}{\sqrt{2}} [|0\rangle - |1\rangle]$$

Annotation: A red arrow points from the $|1\rangle$ term in the expression above to the $|11\rangle$ term in the expression below.

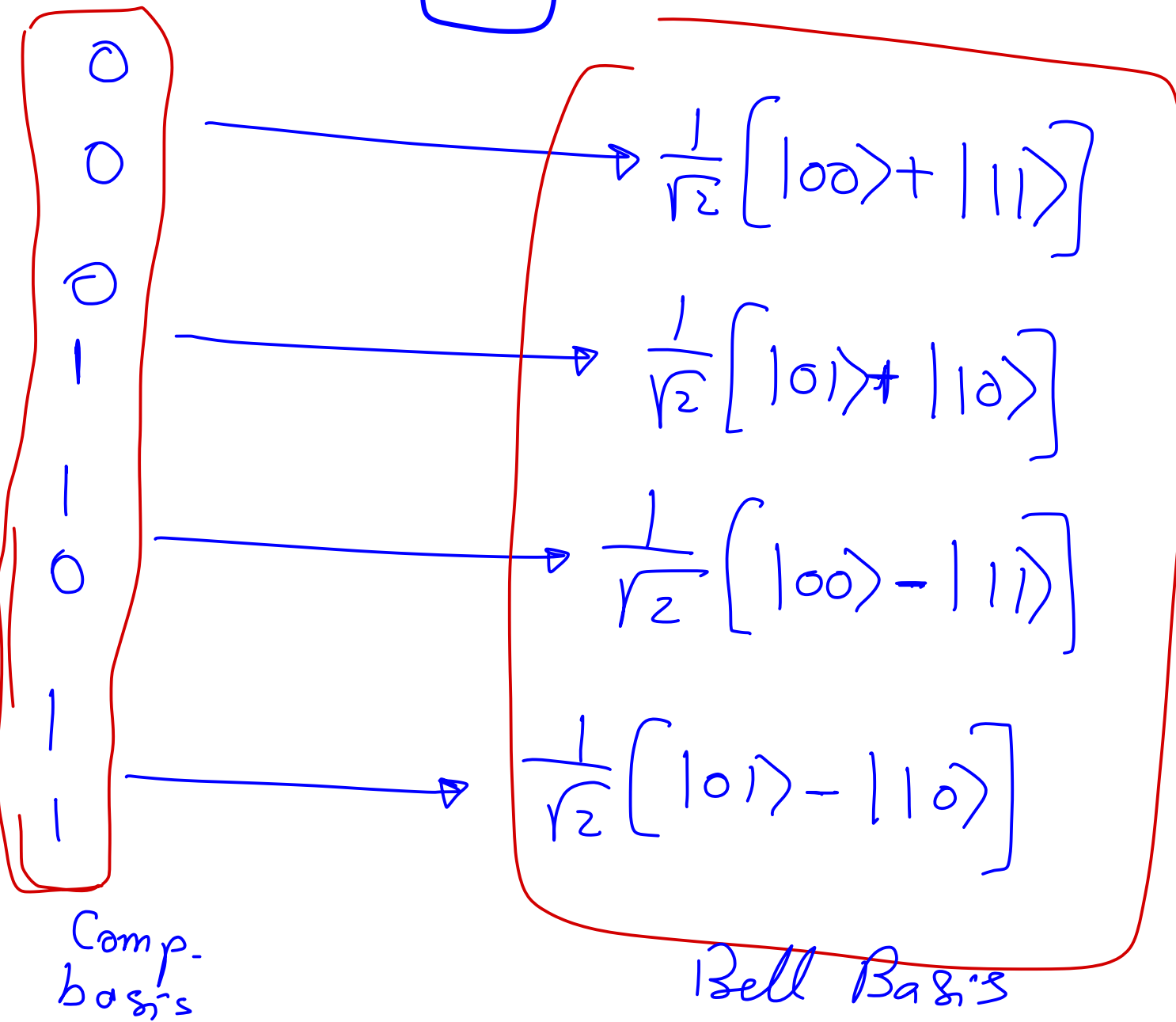
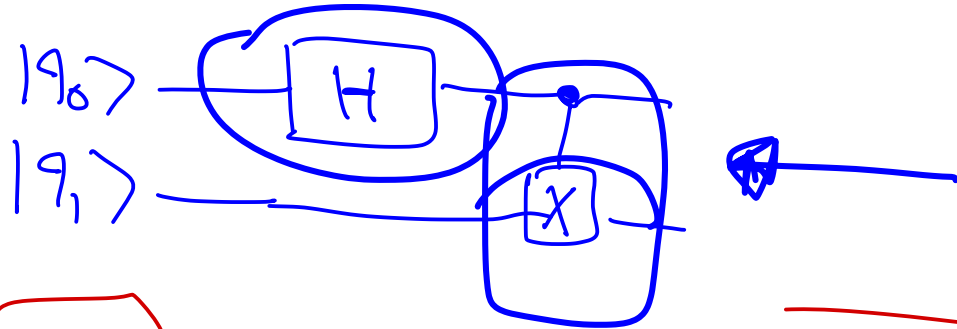
$$\text{CNOT} \rightarrow \frac{1}{\sqrt{2}} [|00\rangle - |11\rangle]$$

Annotations: Under the first two qubits of $|00\rangle$, there are two red arrows pointing up. Under the last two qubits of $|11\rangle$, there are two red arrows pointing up.



$$\text{first H} \rightarrow \frac{1}{\sqrt{2}} [|0\rangle - |1\rangle]$$

$$\text{CNOT} \rightarrow \frac{1}{\sqrt{2}} [|01\rangle - |10\rangle]$$



Quantum Teleportation protocol

Alice

two-qubits

q_0

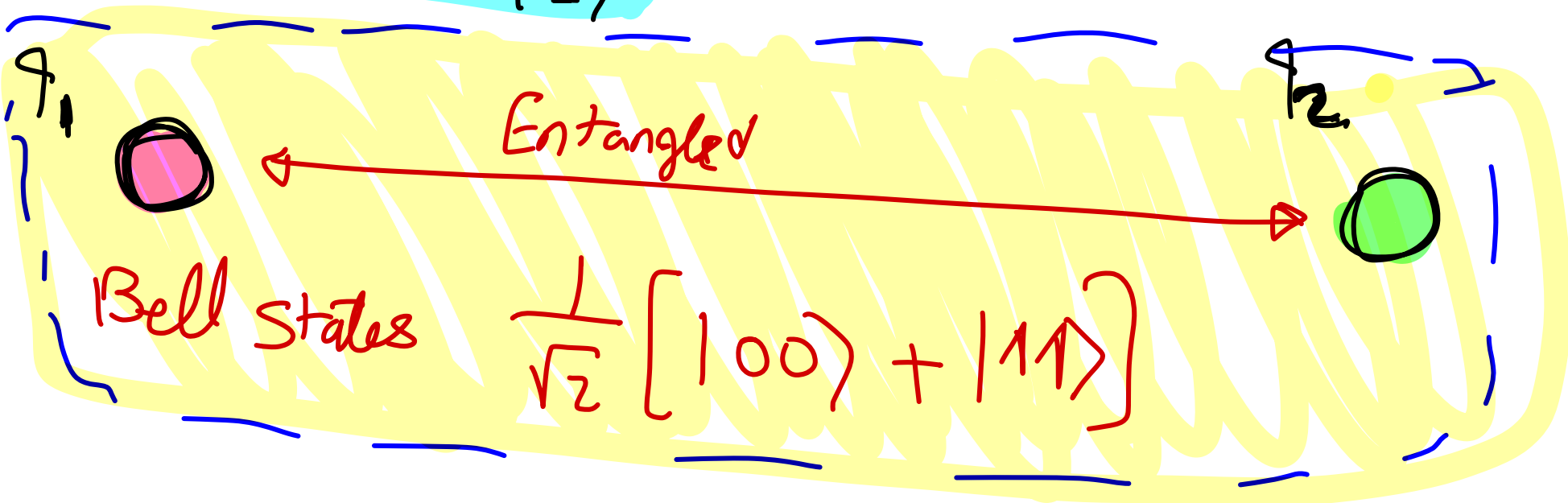


qubit

→ Unknown State

$$|\psi\rangle = a|0\rangle + b|1\rangle$$

Bob



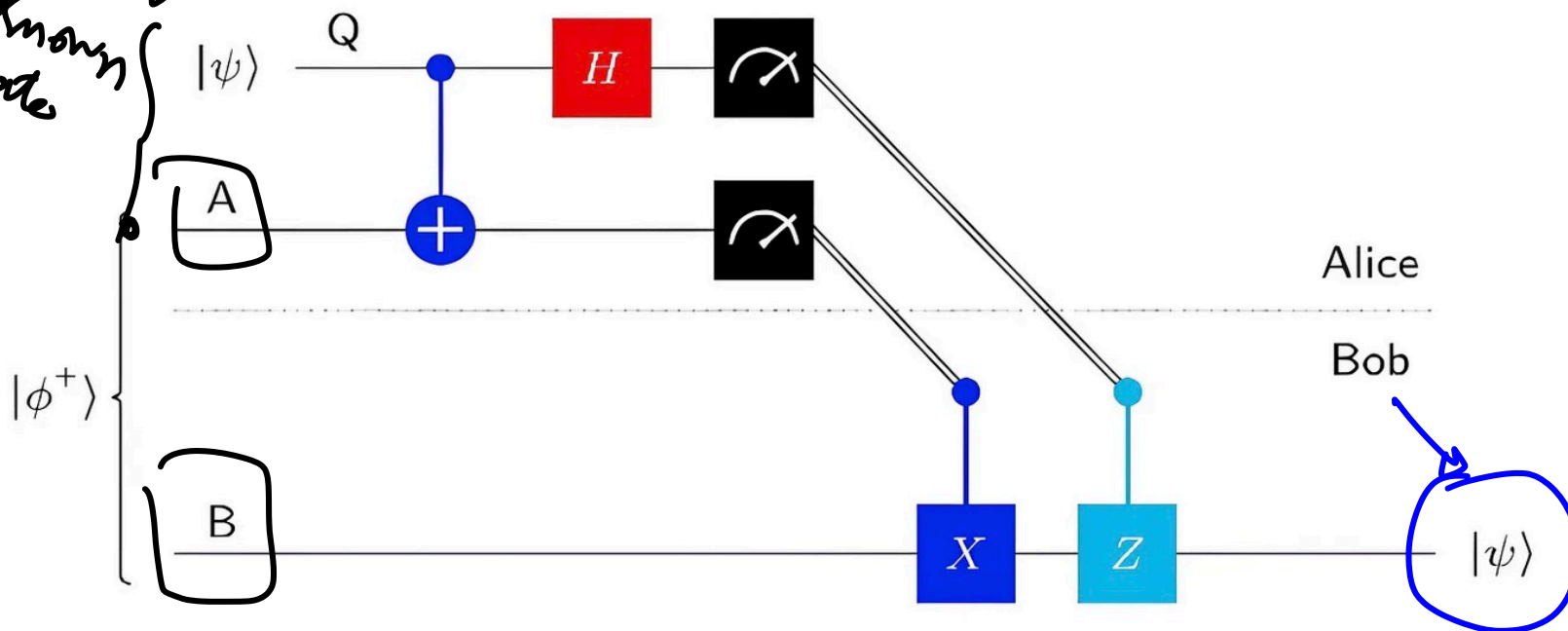


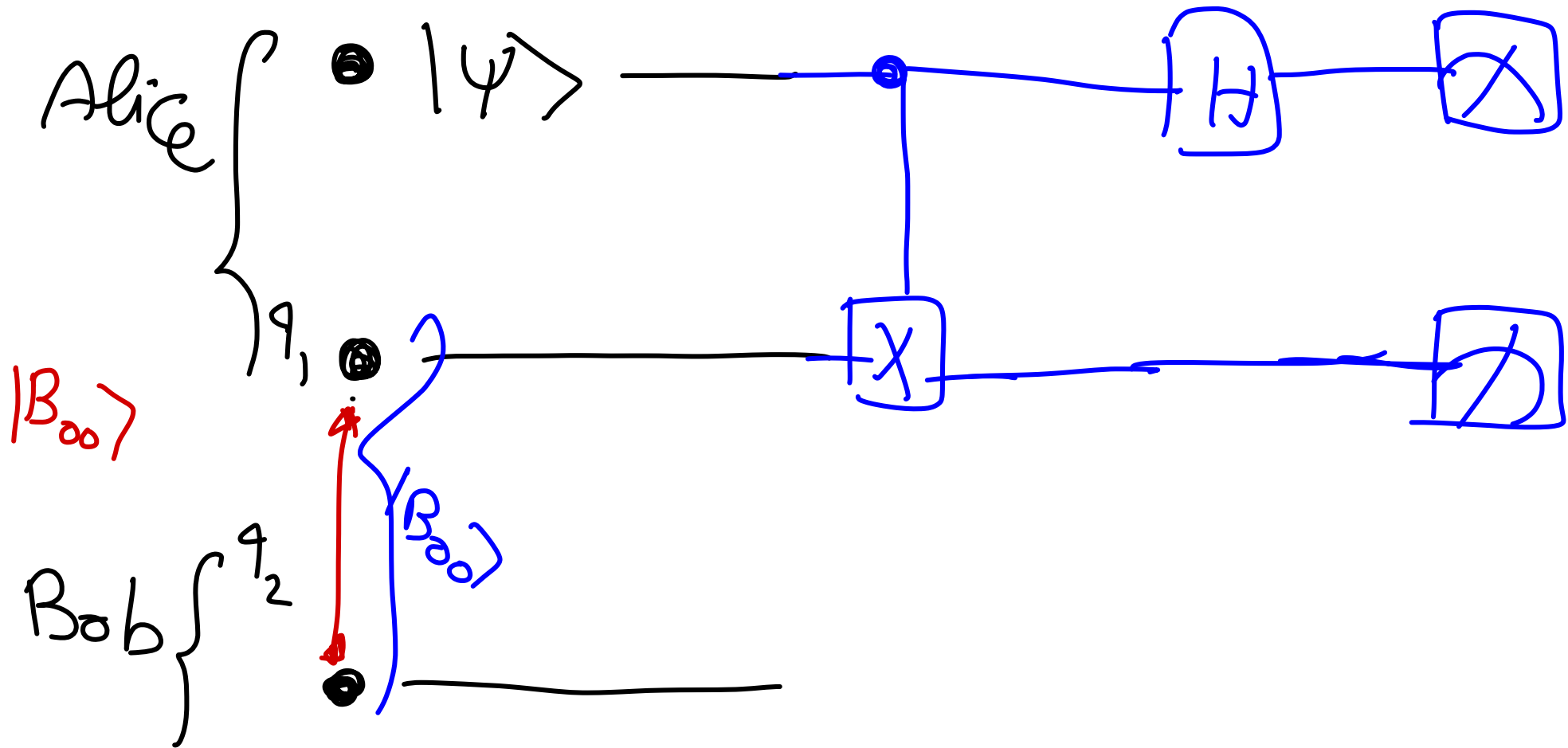
Bob

$|\psi\rangle$

Alice

Unknown State





$$|B_{00}\rangle = \frac{1}{\sqrt{2}} [|00\rangle + |11\rangle]$$

