Amsterdam University of Applied Sciences

Quantum Talent and Learning Center
Intro to Quantum Computing Workshop

Week 8: Quantum Teleportation Protocol

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Agenda for today::

1. A quick recap of previous sessions.
2. Review of the entanglement concept and protocol.
3. Introduction to the quantum teleportation protocol.
4. Quantum Inspire: Bernardo Villalba Frias and Pascal van den Bosch.

Let's review the quantum entanglement concept and protocol.

What is quantum entanglement?

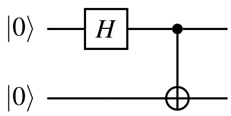
Quantum Entanglement is a quantum mechanical phenomenon in which the quantum states of two or more objects have to be described with reference to each other, even though the individual objects may be spatially separated. This leads to correlations between observable physical properties of the systems.

Quantum entanglement is a key feature of quantum mechanics and is at the heart of quantum information science. It is also one of the pure quantum features.

The entanglement protocol is a way to create entangled states between two or more qubits. We have seen the following state in the previous sessions:

$$|B_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \tag{1}$$

which is the output of the Hadamard gate acting on the first qubit followed by the CNOT gate acting on the second qubit, controlled by the first qubit as shown in the following circuit:



This state is called the Bell state. However, this is not the only Bell state. Depending on the input states, we can have different four Bell states:

$$|B_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \tag{2}$$

$$|B_{01}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \tag{3}$$

$$|B_{10}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \tag{4}$$

$$|B_{11}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \tag{5}$$

Hence, with the Hadamard and CNOT gates, we perform a unitary transformation from the computational basis to the Bell basis. Bell basis are orthonormal basis states.

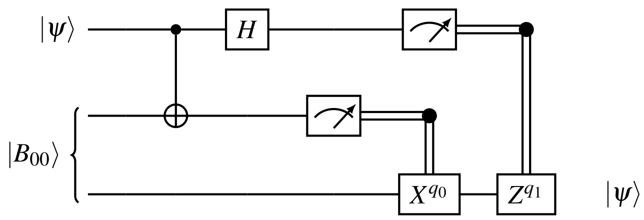
Quantum Teleportation Protocol

The quantum teleportation protocol is a way to transfer the state of a qubit from one location to another without physically moving the qubit. The protocol is based on the entanglement concept.

The protocol consists of the following steps:

1. Alice has two qubits and Bob has one qubit.
2. Alice has a qubit in an unknown state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$. This is the qubit she wants to send to Bob.
3. Alice and Bob share an entangled state between Alice's second qubit and Bob's qubit. This is the Bell state $|B_{00}\rangle$.
4. Alice wants to send the unknown state of the first qubit to Bob.
5. Alice performs a Bell measurement on the qubit she wants to send and her part of the entangled state.
6. Alice sends the measurement results to Bob.
7. Bob applies the necessary operations to his part of the entangled state based on the measurement results to recover the unknown state.

The quantum teleportation protocol is represented by the following circuit:



The protocol is based on the fact that the Bell measurement collapses the state of the qubit to one of the Bell states. The measurement results are two classical bits. These bits are sent to Bob via a classical channel. Bob applies the necessary operations to his part of the entangled state based on the measurement results.

The operations Bob applies to involve the Pauli-X and Pauli-Z gates based on the measurement results by Alice. In general, the Bob applies X^{q_0} and Z^{q_1} to his qubit, where q_0 and q_1 are the measurement results, the classical bits, of the first and second qubits of Alice, respectively.

We can summarize Bob's operations as follows:

- If $q_0 = 0$ and $q_1 = 0$, Bob does nothing.
- If $q_0 = 0$ and $q_1 = 1$, Bob applies Z gate.
- If $q_0 = 1$ and $q_1 = 0$, Bob applies X gate.
- If $q_0 = 1$ and $q_1 = 1$, Bob applies XZ gate.

Why we must perform the Bell measurement on the qubit we want to send and the entangled state?

Quantum teleportation and non-cloning theorem:

There is a no-cloning theorem in quantum mechanics. This theorem states that it is impossible to create an identical copy of an arbitrary unknown quantum state. This is a fundamental principle of quantum mechanics. Hence, we cannot copy an unknown quantum state. The quantum teleportation protocol is a way to transfer the state of a qubit from one location to another without physically moving the qubit. However, it demands the destruction of the original qubit state to transfer the state to another qubit.

*Based on the progress in the lecture, we might have a live demonstration of how the protocol works mathematically.