Introduction to Quantum Computing
Week 3: Linear Algebra and Bloch Sphere manipulation

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Quantum Talent and Learning Center
Amsterdam University of Applied Sciences

Week 3
April 5th, 2024

Agenda for today:

- Recap of previous sessions.
- Review: Bra-Ket notation and the linear algebra in quantum computing.
- Review: Matrix representation and matrix operations in quantum computing.
- Gate actions.
- Bloch Sphere.
- Bloch Sphere manipulation.

Every 30 min or so we will have a feed from other QTLC groups.
Feel free to ask questions at any time!
It is an interactive workshop, we all learn from each other!

Workshop facilities

- Join Discord and display your name instead of the nickname or the username.
- Check the invite email for the Discord link.

JupyterHub facilities

- Check the list of active users from all universities.
- So far, we have the list from Saxion, Fontys, and HvA.
- We are missing the list from HHS.

List needs to be sent to the QTLC team/me by today or so.

Meet my new collegue from HvA/QTLC

Femke Verheijen, Quantum Education Officer

a new member of the QTLC team.

Session recording

- The session is being recorded.

However, the recording will not be shared unless with get the consent of each participant. For that, we will send a form via email for all participant to tell whether

they are ok for distributing the recordings or not.
each TLC can help by distributing a form on their participants

Recap of the previous sessions

- Classical superposition vs. quantum superposition.
- Measurements in quantum mechanics.
- Quantum mechanics relies on linear algebra.
- Matrix operations and manipulations.
- Bra-Ket notation.
- Classical bits vs. qubits.
- Qubit states.

Quantum mechanics:
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Probability density:
$|\Psi(x, t)|^2$ gives the probability of finding the particle at position x at time t.

Dirac Bra-Ket Notation

We can write the wavefunction as a ket $|\Psi\rangle$:

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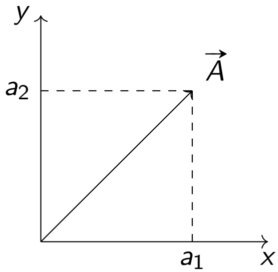
Bra-Ket notation is a standard notation in quantum mechanics.
It is used to describe quantum states and operations.
It is named after Paul Dirac.
Qubit states are represented as kets.

A vector $\vec{A}$ in a two-dimensional space can be written as a column matrix:

$$\vec{A} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$



Using the Dirac notation, we can write the vector $\vec{A}$ as a ket:

$$|A\rangle = a_1|x\rangle + a_2|y\rangle$$

Dirac Bra-Ket Notation

Similarly, a vector $\vec{B}$ in a three-dimensional space can be written as a column matrix:

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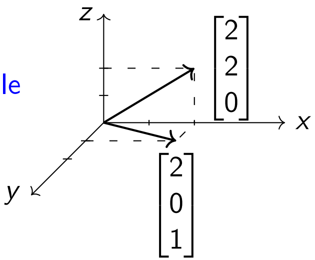

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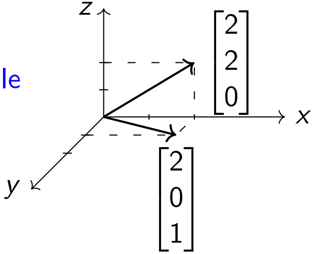


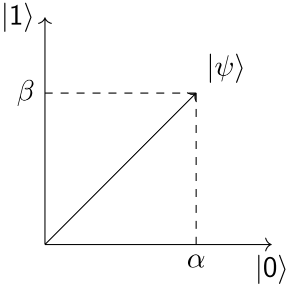
Using the Dirac notation, we can write the vector $\vec{B}$ as a ket:

$$|B\rangle = b_1|x\rangle + b_2|y\rangle + b_3|z\rangle$$

Dirac Bra-Ket Notation

In analogy to vectors, we can write the wavefunction as a ket $|\Psi\rangle$:

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



with α and β called the amplitudes of the states and they are generally complex numbers.

Dirac Bra-Ket Notation

The norm of a 2D vector is:

$$||A|| = \sqrt{a_1^2 + a_2^2}$$

Similarly, the norm of a 3D vector is:

$$||B|| = \sqrt{b_1^2 + b_2^2 + b_3^2}$$

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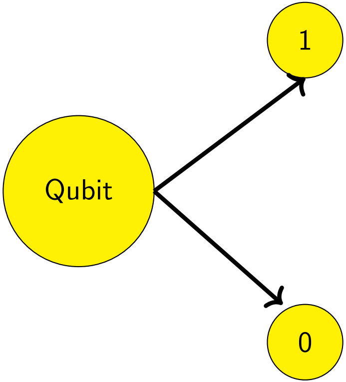
The particle exists by itself in a superposition of states.

Qubit vs. classical bit

Qubit is a quantum bit.

Classical bit: can be in one of two states: 0 or 1.

Qubit: can be in a superposition of 0 and 1.



Matrix representation of qubit

We use matrix algebra to represent qubits.
State of a single qubit:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{1}$$

In a vector form/representation:

$$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \tag{2}$$

The states $|0\rangle$ and $|1\rangle$ are represented by the following column matrices:

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{and} \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{3}$$

The coefficients α and β are complex numbers, and they are generally called the amplitudes of the states.

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Let the state of the qubit be $|\psi\rangle$. We can change the state of the qubit to $|\psi'\rangle$ using the action of the unitary matrix U on the qubit:

$$|\psi'\rangle = \boldsymbol{U}|\psi\rangle \tag{4}$$

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Unitary means that the matrix U acts on the qubit without changing the norm of the qubit, i.e. $|\alpha|^2+|\beta|^2=1$.
The matrix U is unitary if its conjugate transpose is equal to its inverse:

$$U^\dagger U^{-1} = U^{-1} U^\dagger = I \tag{5}$$

Matrix representation of qubit

Example:

What is the conjugate transpose of the following matrix:

$$A = \begin{bmatrix} 1 & i \\ 1 & i \end{bmatrix} \tag{6}$$

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The conjugate transpose of the matrix U is:

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Is the matrix A unitary?

No, the matrix A is not unitary:
$$AA^{\dagger} = \begin{bmatrix} 1 & i \\ 1 & i \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -i & -i \end{bmatrix} = 2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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Hence the matrix X flips the state of the qubit from $|0\rangle$ to $|1\rangle$.

Matrix representation of qubit

Let's now perform two successive operations on the qubit in state $|0\rangle$. First, we apply the unitary operator X to the qubit, and then we apply the unitary operator $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ to the qubit.

What is the result of applying the unitary operator Y to the qubit?

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What is the result of applying the unitary operator Y to the qubit?

The result of applying the unitary operator Y to the qubit is:

$$Y|1\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix} = -i|0\rangle \tag{10}$$

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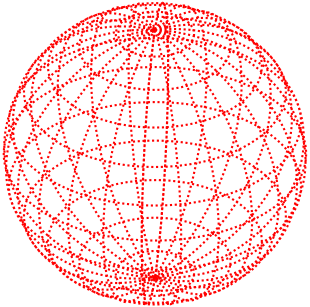
Hence the matrix Y flips the state of the qubit from $|1\rangle$ to $-i|0\rangle$.

Bloch Sphere

Bloch sphere is a geometrical representation of a qubit. It is a representation of the qubit state as a vector on unit sphere.

Before we go into the details of the Bloch sphere, let's first understand the spherical coordinates and the concept of a unit sphere.

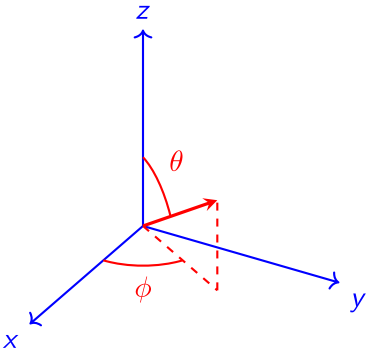


Spherical coordinates

Spherical coordinates are a system of three coordinates used to describe the position of a point in space.

Typical coordinate systems are Cartesian, cylindrical, and spherical.

These are orthogonal coordinate systems.



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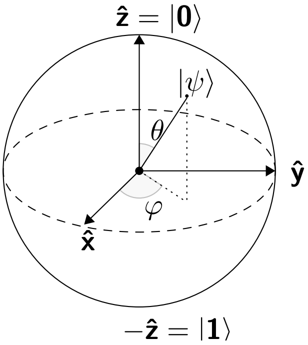
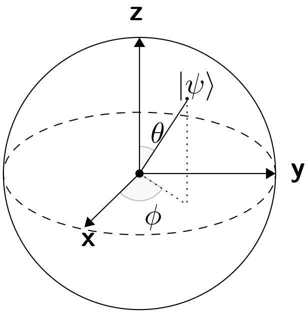


Bloch Sphere



- The spherical coordinates are (r, θ, ϕ).
- The unit sphere is defined by the equation $x^2 + y^2 + z^2 = 1$.
- Spherical coordinates are related to the Cartesian coordinates by the following equations:

$$x = r\sin(\theta)\cos(\phi)$$
$$y = r\sin(\theta)\sin(\phi)$$
$$z = r\cos(\theta)$$

Computer Lab

Now, open your notebook and make a function that converts the Cartesian coordinates to the spherical coordinates using the previously formulas. Make sure you can make another function to plot the transformed point on a sphere.

Note: There is difference in defining the polar angles θ and ϕ between the physics and mathematics. In physics, the polar angle θ is measured from the positive z-axis, while in mathematics, the polar angle θ is measured from the positive x-axis.

$$
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\end{aligned}
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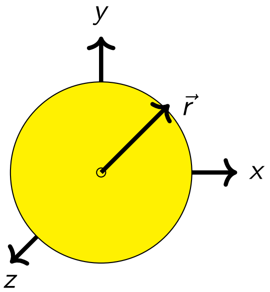
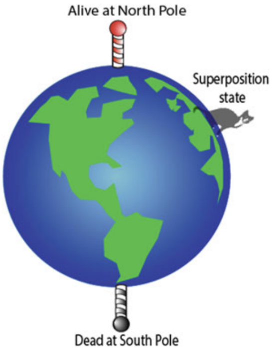


Schrödinger's cat on a Blochsphere

Schrödinger's cat is determined to be alive.What location on the Earth in the figure could the cat have been before the quantum measurement?

1. Russia.
2. North Pole.
3. Australia.
4. All the above.



Question source: Hughes et al. (2021), Springer. Quantum Computing for the Quantum Curious.

Schrödinger's cat on a Blochsphere

The cat could have been anywhere on Earth except for the South Pole. Notice that in Australia the cat has a smaller probability of being alive since it is further away from the North Pole.

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Blochsphere

The state of a qubit can be represented by a point on the Blochsphere.
This point is defined by the spherical coordinates (θ, ϕ).
It also gives us an idea about the superposition of the qubit.

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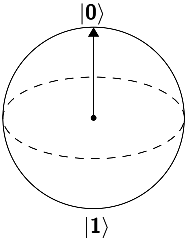

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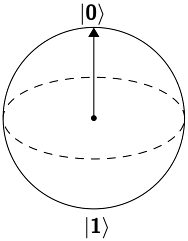

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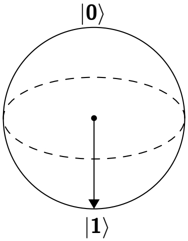

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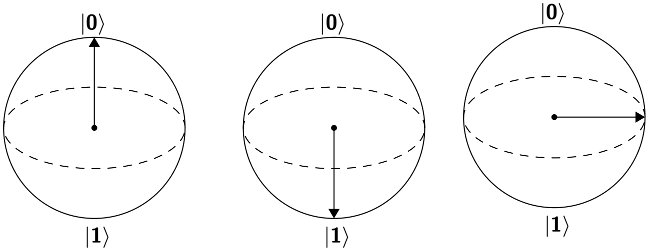

$|\psi\rangle = |0\rangle$ $\quad$ $|\psi\rangle = |1\rangle$ $\quad$ $|\psi\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle + |1\rangle\right]$

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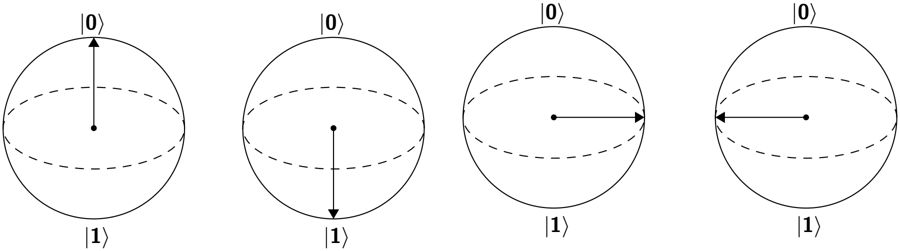

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Question source: Hughes et al. (2021), Springer. Quantum Computing for the Quantum Curious.

Blochsphere

Let's have fun with the Blochsphere!
Go to the following link and play with the Blochsphere:

First, we check the action manually:

`https://attilakun.net/bloch/`

`https://bits-and-electrons.github.io/bloch-sphere-simulator/`

Then, we check the action using the IBM Quantum Experience:

https://quantum.ibm.com/

Next time

Next time, we are going to learn more practical examples of qubits using our JupyterHub.

We are going to learn how to initialize qubits, create quantum circuits, manipulate qubits, and measure them.

We are going to learn how to and how to simulate them.

Also, in case JupyterHub is not open to all of your accounts, we might consider continuing with the IBM Quantum Experience.

Extra questions: Check your understanding!

Check your understanding!

Assume a flipped coin can be measured as either heads (H) or tails (T).

1. If the coin is in a normalized state $\frac{1}{\sqrt{10}}|H\rangle + \frac{3}{\sqrt{10}}|T\rangle$, what is the probability that the coin will be tails?

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1. If the coin is in a normalized state $\frac{1}{\sqrt{10}}|H\rangle + \frac{3}{\sqrt{10}}|T\rangle$, what is the probability that the coin will be tails?
2. During a flip, the coin is in a state $\frac{1}{3}|H\rangle + \frac{2}{3}|T\rangle$. Is this state normalized?

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2. During a flip, the coin is in a state $\frac{1}{3}|H\rangle + \frac{2}{3}|T\rangle$. Is this state normalized?
3. A machine is built to flip coins and put them into a state $\frac{1}{2}|H\rangle + \frac{\sqrt{3}}{2}|T\rangle$ when flipped. If 100 coins are flipped, how many coins should land on tails?

Check your understanding!

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3. A machine is built to flip coins and put them into a state $\frac{1}{2}|H\rangle + \frac{\sqrt{3}}{2}|T\rangle$ when flipped. If 100 coins are flipped, how many coins should land on tails?
4. A coin starts in the state $\frac{1}{\sqrt{10}}|H\rangle + \frac{3}{\sqrt{10}}|T\rangle$. After a measurement is made on the coin, what could be the state of the coin?

What is a qubit made of?

Classical bits are made of transistors.

It is easy to realize a classical bit in practice. Foe example, if the voltage is above a certain threshold, the bit is 1, otherwise it is 0.

Qubits are made of quantum systems in a sohpiosticated way.
We cannot simply create a superposition of 0 and 1 by using a transistor in which two currents are there, one flows and the other does not.

Quantum computers are still in their infancy. There are several technologies are being developed to realize qubits. Still is a long way to go. It is unclear which candidate will provide more benefits than the other or the best qubits.

Answers will be there when quantum computers are more mature and more commercially available.

What is a qubit made of?

A qubit can be made of different physical systems.

- **Superconducting qubits:**
  - Made of superconducting circuits.
  - They are cooled to very low temperatures.
  - They are the most widely used qubits in quantum computers.
- **Trapped ions:**
  - Made of individual atoms.
  - They are trapped using electromagnetic fields.
  - They are cooled to very low temperatures.

- **Photonic qubits:**
  - Made of photons.
  - They are used in quantum communication.
- **Topological qubits:**
  - Made of exotic particles called anyons.
  - They are not yet realized in practice.

Check your understanding!

Your friend gives you many qubits which are in same superposition state. How can you determine what the state is?

Check your understanding!

A qubit is prepared in an unknown state. It is then measured with the outcome $|0>$.

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2. If you tried to measure the same qubit a second time, can you narrow down what the initial state was?

Check your understanding!

A qubit is prepared in an unknown state. It is then measured with the outcome $|0>$.

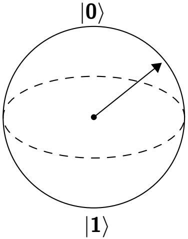
1. Which of the following could be its initial state before the measurement: $|0\rangle$, $\frac{1}{\sqrt{10}}|0\rangle + \frac{3}{\sqrt{10}}|1\rangle$, $\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$ and/or $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$?
2. If you tried to measure the same qubit a second time, can you narrow down what the initial state was?
3. Another qubit is prepared in the same unknown state. It is measured in the $|1>$ state. What can you say about the initial state now?

Check your understanding!

Show by example that applying a non-unitary matrix to a qubit results in probabilities that no longer add up to 100%. (Hint: Start with any initial state, e.g., $|0>$. Measure the probabilities of finding either 0 or 1. Apply a non-unitary matrix to the initial state. Then measure the probabilities of finding either a 0 or 1. Do the probabilities add up to 100%?)

Check your understanding!

If the qubit represented by the figure is measured, what are the possible outcomes? Numerical values for the amplitudes are not needed, only conceptual statements.



References

- Hughes, R. J., Nordholt, J. E., Mink, A., & Lanzagorta, M. (2021). Quantum Computing for the Quantum Curious. Springer.
- Nielsen, M. A., & Chuang, I. L. (2010). Quantum Computation and Quantum Information: 10th Anniversary Edition. Cambridge University Press.
- IBM Quantum Experience: `https://quantum-computing.ibm.com/`

Thank you

Thank you!