Quantum AI Lab

iQafé & Bibliotheca Alexandrina

Available Time: 30 minutes

Permitted Materials: Simple calculator, graphic and advanced calculators are not permitted

Author: Dr. Taha Selim
General Manager, MolKet SAS
Email: tahaselim@molket.io

Question 1 Find the complex conjugate transpose of the following matrices:

$$\begin{pmatrix} 1 & 2 & -3 \\ 4i & -5i & 6 \\ 7 & 8 & 9i \end{pmatrix} \tag{1}$$

$$\begin{pmatrix} 3+1i & 2-2i & 1+3i \\ 4-2i & 5+1i & 6-3i \\ 7+1i & 8-2i & 9+3i \end{pmatrix} \tag{2}$$

Question 2 Which of the following expressions represent the state $|q_0 q_1\rangle$ of the two qubits $|q_0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|q_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$?

1. $|q_0 q_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$.

2. $|q_0 q_1\rangle = \begin{pmatrix} 1 \\ 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$.

3. $|q_0 q_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$.

4. $|q_0 q_1\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \\ 0 \end{pmatrix}$.

Question 3 Which of the following expressions represent quantum state(s), you can choose more than one if applicable:

1. $|\psi\rangle = |0\rangle$.
2. $|\psi\rangle = |1\rangle$.
3. $|\psi\rangle = |0\rangle + |1\rangle$.
4. $|\psi\rangle = \frac{1}{\sqrt{2}}[|0\rangle - |1\rangle]$.
5. $|\psi\rangle = \frac{1}{\sqrt{2}}[|0\rangle + i\,|1\rangle]$.

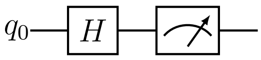
Question 4 Which of the following matrices are unitary: Note: multiple answers are possible.

1. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
2. $\begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$.
3. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.
4. $\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$.
5. $\begin{pmatrix} 1 & 0 \\ 0 & \frac{i}{\sqrt{2}} \end{pmatrix}$.

Question 5 Let's find the state of the qubit after applying the Hadamard gate as shown in the figure.

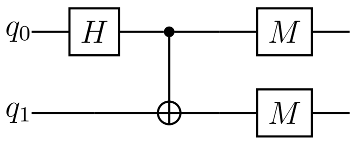
The Hadamard gate is defined as:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{3}$$



First, apply the Hadamard gate to the qubit state $|\psi\rangle = |0\rangle$ and determine the new state of the qubit.
Second, apply the Hadamard gate to the qubit state $|\psi\rangle = |1\rangle$ and determine the new state of the qubit.
Third, what is the probability of measuring the qubit in the state $|0\rangle$ after applying the Hadamard gate to the qubit state $|\psi\rangle = |1\rangle$?

Question 6 Let's explore the matrix representation of the following quantum circuit.



The two qubits are initialized in the state $|00\rangle$ and the Hadamard gate is applied to the first qubit, while the CNOT gate is applied to the two qubits where the first qubit is the control qubit and the second qubit is the target qubit. The Hadamard gate is defined as:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{4}$$

The CNOT gate is defined as:

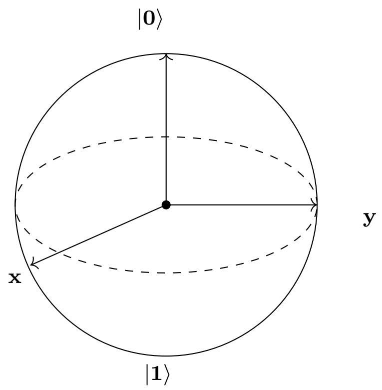
$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{5}$$

The measurement operator is defined as:

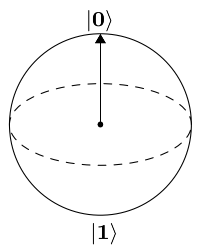
$$M = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{6}$$

First, write the statevector of the two qubits after applying the Hadamard gate to the first qubit and show the mathematical steps.
Second, apply the CNOT gate to the two qubits and show the mathematical steps.

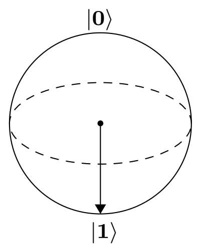
Question 7 Let's find the corresponding quantum state to a given Bloch sphere representation. The bloch sphere representation of a qubit is a geometrical representation of the state of a qubit. The state of a qubit can be represented as a point on the bloch sphere as shown in the figure below.



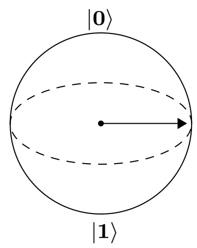
Write below each Bloch sphere representation the corresponding quantum state in the computational basis. Assume the y-axis is horizontal, the z-axis is vertical, and the x-axis is perpendicular to the paper.



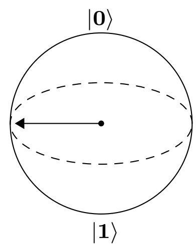
$|\psi\rangle =?$



$|\psi\rangle =?$



$|\psi\rangle =?$



$|\psi\rangle =?$

——— *End of Examination* ———