

# CE 580 COMPUTATIONAL TECHNIQUES FOR FLUID DYNAMICS

## Homework- 6

### Determination of Best Overrelaxation Parameter for PSOR Method

Write a computer program for solution of Poisson type equations using Point Successive Overrelaxation (PSOR) method. Improve your program to determine the **best overrelaxation parameter**,  $\omega_{opt}$ , when the geometry of the computational domain, boundary conditions and the grid distribution are given.

#### A) Determination of $\omega_{opt}$

1. Consider the analytical solution given as:  $U(x,y) = x^2 - y^2$   
on the square domain  $0 \leq x \leq 1$  and  $0 \leq y \leq 1$
2. Generate NxN computational grid using  $\Delta x = \Delta y = 1/(N - 1)$ .
3. Compute the solution  $U_{i,j}$  from the expression given above.
4. Consider the finite difference solution  $S_{i,j}$  and set the boundary values  $S_{1,j}$ ,  $S_{N,j}$ ,  $S_{i,1}$  and  $S_{i,N}$  from  $U_{i,j}$ .
5. Set the initial values  $S_{i,j} = 0$  for all **internal nodes**.
6. Perform 20 iterations of PSOR for a given  $\omega$ .
7. Compute the overall space average error:

$$Error = \frac{\sum_{i=2}^{N-1} \sum_{j=2}^{N-1} |U_{i,j} - S_{i,j}|}{(N-2)(N-2)}$$

8. Print out Error and  $\omega$
9. Repeat steps 5 to 8 for  $1 \leq \omega \leq 2$  with  $\Delta\omega=0.002$
10. Repeat the above procedure for N=21, 31, 41, 51, 61, 81 and 101
11. Make a plot of  $\log(\text{Error})$  vs  $\omega$  for all N and determine the  $\omega_{opt}$  values. Present the results in a table.

#### B) Variation of error with number of iterations

Modify your program to obtain another set of outputs:

12. Set N=51
13. Set the initial data and boundary values as in part A
14. Start PSOR iterations and print out Error and iteration count at each iteration. Perform 1000 iterations.
15. Repeat steps 13 and 14 for  $\omega=1.6, 1.7, 1.8, 1.9, 1.99$
16. Make a plot of  $\log(\text{Error})$  vs iteration count for each  $\omega$  value.
17. Write a discussion on the results you obtained.