

CE 580 COMPUTATIONAL TECHNIQUES FOR FLUID DYNAMICS

Homework- 6

Determination of Best Overrelaxation Parameter for PSOR Method

Write a computer program for solution of Poisson type equations using Point Successive Overrelaxation (PSOR) method. Improve your program to determine the **best overrelaxation parameter**, ω_{opt} , when the geometry of the computational domain, boundary conditions and the grid distribution are given.

A) Determination of ω_{opt}

1. Consider the analytical solution given as: $U(x,y) = x^2 - y^2$
on the square domain $0 \leq x \leq 1$ and $0 \leq y \leq 1$
2. Generate NxN computational grid using $\Delta x = \Delta y = 1/(N-1)$.
3. Compute the solution $U_{i,j}$ from the expression given above.
4. Consider the finite difference solution $S_{i,j}$ and set the boundary values $S_{1,j}$, $S_{N,j}$, $S_{i,1}$ and $S_{i,N}$ from $U_{i,j}$.
5. Set the initial values $S_{i,j} = 0$ for all **internal nodes**.
6. Perform 20 iterations of PSOR for a given ω .
7. Compute the overall space average error:

$$Error = \frac{\sum_{i=2}^{N-1} \sum_{j=2}^{N-1} |U_{i,j} - S_{i,j}|}{(N-2)(N-2)}$$

8. Print out Error and ω
9. Repeat steps 5 to 8 for $1 \leq \omega \leq 2$ with $\Delta\omega=0.002$
10. Repeat the above procedure for $N=21, 31, 41, 51, 61, 81$ and 101
11. Make a plot of $\log(Error)$ vs ω for all N and determine the ω_{opt} values. Present the results in a table.

B) Variation of error with number of iterations

Modify your program to obtain another set of outputs:

12. Set $N=51$
13. Set the initial data and boundary values as in part A
14. Start PSOR iterations and print out Error and iteration count at each iteration. Perform 1000 iterations.
15. Repeat steps 13 and 14 for $\omega=1.6, 1.7, 1.8, 1.9, 1.99$
16. Make a plot of $\log(Error)$ vs iteration count for each ω value.
17. Write a discussion on the results you obtained.