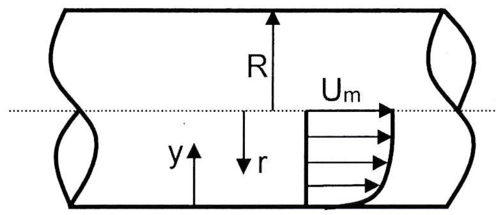
CE 580 COMPUTATIONAL TECHNIQUES FOR FLUID DYNAMICS

Homework 4

Explicit Solution to Turbulent Pipe Flow

Develop a computer program to obtain velocity distribution from an explicit solution to steady, uniform, turbulent flow in a smooth, circular pipe.



1. Use mixing length theory as the turbulence model.
2. Apply the boundary conditions, $u = 0$ at $y = 0$ and $u = U_m$ at $y = R$.
3. Obtain initial data for velocity from the power law: $u(y) = U_m\,(y/R)^{1/7}$
4. Consider the unsteady momentum equation. Obtain a finite difference representation for variable mesh size. Generate the computational grid using constant ratio method.
5. Describe a transient explicit solution to the finite difference equations.
6. Compute the residual error for each iteration step.
7. Compute the diffusion number for all internal nodes (i=2,N-1) for the converged solution.
8. Determine the discharge, average velocity, Reynolds number and the friction factor (f_m) from Swamee-Jain (experimental) formula. Compute the friction factor (f_d) using the computed average velocity and the wall shear stress in the Darcy's friction factor definition. Compare the experimental and the computed friction factors.
9. Run your program for the data given below:

$U_m = 2$ m/s	$R = 0.05$ m	$N = 31$
$\nu = 1.0E\text{-}06$ m^2/s	Grid ratio = 0.82	$\Delta t = 3.50E\text{-}04$ s
$\rho = 1000$ kg/m^3	Number of iterations = 100000	

10. Make a logarithmic plot of the velocity profile (u^+ vs $\log(y^+)$).
11. Make a logarithmic plot of the residual error (log(error) ~ No. of iterations).
12. Make a logarithmic plot of diffusion number (d_f vs $\log(y^+)$).
13. Make a logarithmic plot of turbulent viscosity/molecular viscosity as function of y/R ($\log(\nu_t/\nu)$ vs y/R).
14. Write a discussion on the results you obtained.