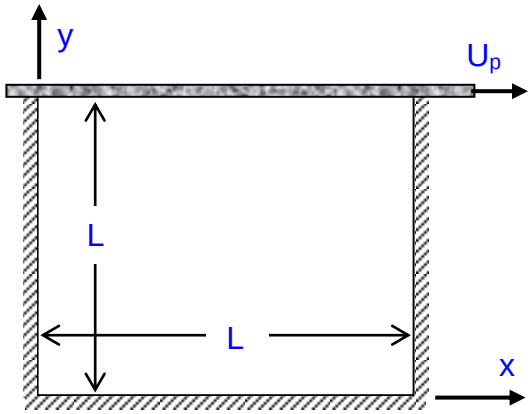
CE 580 COMPUTATIONAL TECHNIQUES FOR FLUID DYNAMICS

Homework - 7

Vorticity-Stream Function Solution to Driven Cavity Flow



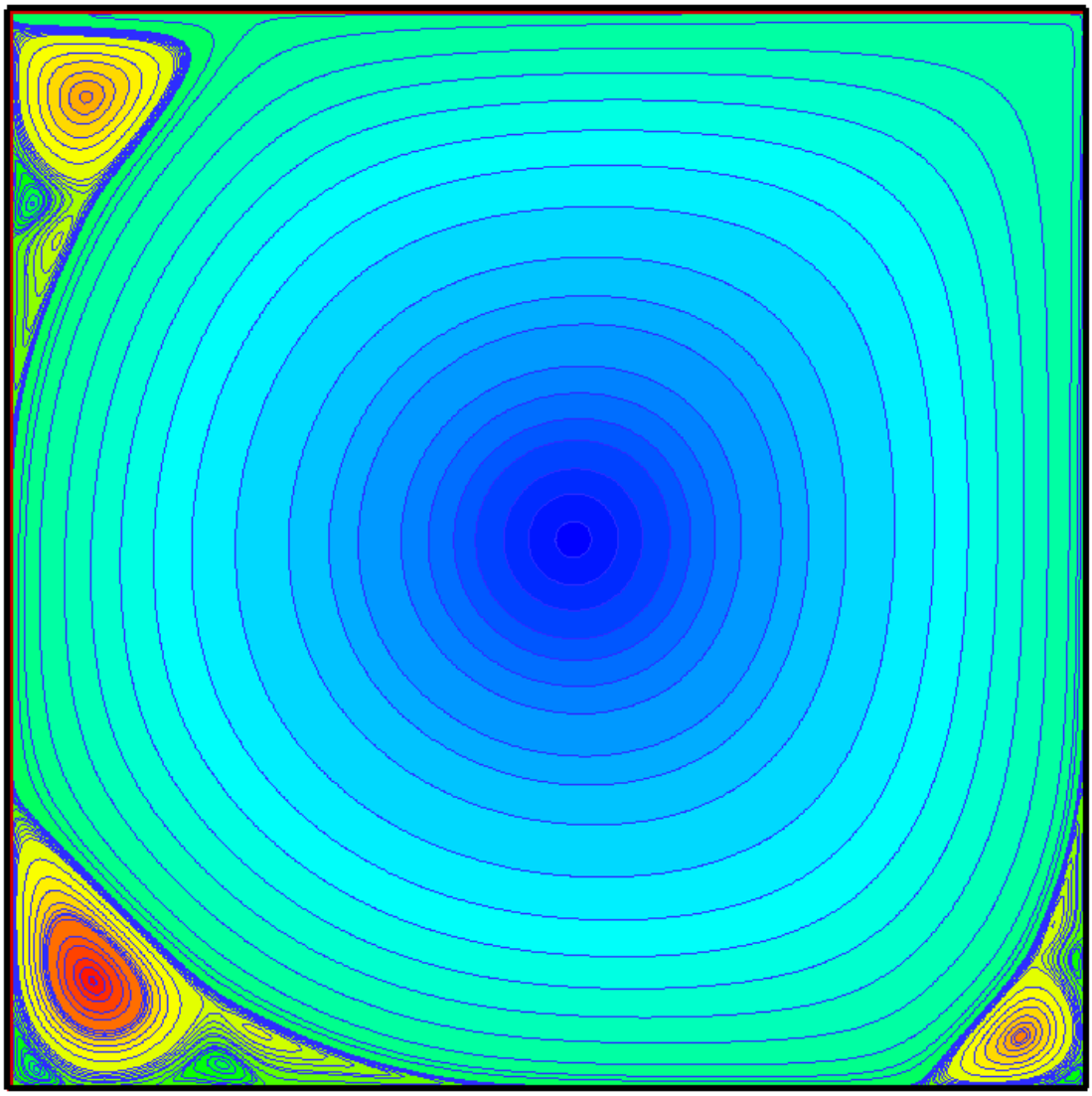
An incompressible fluid is contained in a 2-D square cavity as shown in the figure. The plate on the upper face is moved horizontally at a constant speed U_p. The laminar vortex motion driven by the moving plate is to be computed by solving the Navier-Stokes equations numerically.

1. Write down the vorticity-stream function formulation of the governing equations.
2. Obtain finite-difference equations for vorticity transport equation with first order upwind differences for convective terms. Use constant mesh size.
3. Use ADI method for solution of the vorticity transport equation.
4. Use PSOR method for solution of the Poisson equation for stream function.
5. Define the boundary conditions for vorticity and stream function and indicate their numerical implementation.
6. Define an initial data for vorticity and stream function.
7. Discuss the stability conditions if required.
8. Define an overall error and the condition of convergence.
9. Obtain the solution using 101X101 nodal points for the data given:

$$L = 0.01 \text{ m}, \quad \nu = 1X10^{-6} \text{ m}^2/\text{s}$$
$$U_p = 0.01, 0.02, 0.05, 0.1, 1.0 \text{ m/s}$$

10. Make a contour plot of streamlines for each case.
11. Compute the Reynolds number of the cavity flow for each solution, $R_e=U_pL/\nu$.
12. Determine the maximum stream function value at the core of the main vortex.
13. Determine the drag force (per unit width) on the moving plate.
14. Report your results in a table and write a discussion of results.

Example solution



Lid driven cavity flow $R_e = 10^6$