Middle East Technical University

Department of Aerospace Engineering

Project II - Process Report II

SOR Solution over NACA 0006 Airfoil

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1. Solution Process

In solution process, code of process report 1 has been modified. These modifications include applying necessary boundary conditions to coefficients of unknowns. In addition, incomplete code is modified such that Gauss-Seidel iteration with Successive Over Relaxation method is used.

```
      subroutine  COEFF(i,j)
c..Evaluate the matrix coefficients
      parameter (imx=301,jmx=301)
      common/flow/fsmach,fsmach2,gamma,gmp1
      common/naca/naca,nair,dydx(2,imx)
      common/grid/ig,jg,ile,ite,jal,jau,xg(imx),yg(jmx),dx(imx),dy(jmx)
      common/sol/ circ,mu(imx,jmx),phi(imx,jmx),dphi(imx,jmx)
      common/coef/Cr(jmx),Cs(jmx),Ct(jmx),Cv(jmx),Cw(jmx),Rhs(jmx)

c....apply the Wall BC on the Ctq term
      if ( j .eq. jal .and. i .ge .ile .and. i .le .ite ) then
         Ctq = (1./dy(j))/(dy(j+1)+dy(j))
      elseif (j .eq. jau .and. i .ge .ile .and. i .le .ite ) then
         Ctq = (1./dy(j+1))/(dy(j+1)+dy(j))
      else
         Ctq = (1./dy(j+1)+1./dy(j))/(dy(j+1)+dy(j))
      endif

c.....Eliminate Cr term for i=2
      if (i.eq.2) then
      Cr(j) = 0.
      else
      Cr(j)  = (mu(i-1,j)*An(i-1,j)/dx(i-1))/(dx(i+1)+dx(i))
      endif

      Cs(j)  = ((1-mu(i,j)-mu(i-1,j))*An(i,j)/dx(i) -
     +        mu(i-1,j)*An(i-1,j)/dx(i-1)) / (dx(i+1)+dx(i))

      Ct(j)  = -( (1-mu(i,j))*An(i+1,j)/dx(i+1)
     +            +(1-mu(i,j)-mu(i-1,j))*An(i,j)/dx(i))/(dx(i+1)+dx(i))
     +          - Ctq

c.....Eliminate j-1 derivative on upper surface
      if (j.eq.jau .and. i.ge.ile .and. i.le.ite) then
      Cv(j) = 0.
      else
      Cv(j)  = (1./dy(j))*(1./(dy(j+1)+dy(j)))
      endif

c.....Eliminate j+1 derivative on lower surface
      if (j.eq.jal .and. i.ge.ite .and. i.le.ite) then
      Cw(j) = 0.
      else
      Cw(j)  = (1./dy(j+1))*(1./(dy(j+1)+dy(j)))
      endif

      Rhs(j) = -(          ( (1-mu(i,j))*Pn(i+1,j)
     +            -(1-mu(i,j)-mu(i-1,j))*Pn(i,j)
     +            -             mu(i-1,j)*Pn(i-1,j) ) / (dx(i)+dx(i+1))
     +            +      (  Qn(i,j+1,j) - Qn(i,j,j)) / (dy(j)+dy(j+1)))


      return
      end
```

Table 1 - Subroutine of Calculation of Coefficients

Since detailed derivation of coefficient process is presented in process report 1, in this report only application of derived coefficients to code is presented.

```
      subroutine SOLVER(iter)
      parameter (imx=301,jmx=301)
      common/naca/naca,nair,dydx(2,imx)
      common/grid/ig,jg,ile,ite,jal,jau,xg(imx),yg(jmx),dx(imx),dy(jmx)
      common/sol/ circ,mu(imx,jmx),phi(imx,jmx),dphi(imx,jmx)
      common/coef/Cr(jmx),Cs(jmx),Ct(jmx),Cv(jmx),Cw(jmx),Rhs(jmx)
      common/flow/fsmach,fsmach2,gamma,gmp1
      data omega/1.5/ resl21/0./
      save omega,resl21

      DO i=2,ig-1
      Do j=2,jg-1
c..Evaluate the coefficients; Cr,Cs,Ct,Cv,Cw and Res
         call COEFF(i,j)
c..delta_phi for SOR
         Res   = Rhs(j)  - dphi(i-1,j)*Cs(j) - dphi(i-2,j)*Cr(j) -
     &   dphi(i,j-1)*Cv(j)
         dphi(i,j) = omega*Res/Ct(j)
      ENDDO
       ...
```

Table 2 - Gauss-Seidel Iteration with SOR

In solution process of Gauss-Seidel Iteration, right-hand side of the equation is modified accordingly. Now that solution knows previous steps, these must be included in right-hand side in order to accelerate the solution process.

For successive over relaxation method, numerical value of omega is modified according to flight Mach number. This coefficient is determined by numerical error. As Mach number is increased, solution process becomes very unstable. Therefore, relaxation parameter is reduced and this method is called under relaxation.

Although intended, 0.9 Mach solution is not obtained. However, 0.85 Mach solution is obtained. Therefore, instead of 0.9 Mach solution, 0.85 Mach solution is presented in this report.

Following omega values are used in solution:

- $\omega = 1.5$ for 0.2 Mach
- $\omega = 1.5$ for 0.6 Mach
- $\omega = 0.7$ for 0.85 Mach

All of the solutions performed over NACA 0006 airfoil, which has 51 points at zero degree angle of attack.

For post-processing such as determining coefficient of pressure, disturbance velocities, Mach number, temperature/density/pressure ratio, following code segment is used:

```
      subroutine OUT(nio)
      parameter (imx=301,jmx=301)
      common/grid/ig,jg,ile,ite,jal,jau,xg(imx),yg(jmx),dx(imx),dy(jmx)
      common/sol/ circ,mu(imx,jmx),phi(imx,jmx),dphi(imx,jmx)
      common/flow/fsmach,fsmach2,gamma,gmp1
      real cp(2,imx), u(imx,jmx),v(imx,jmx),mach(imx,jmx),t(imx,jmx),
     + rho(imx,jmx), pressure(imx,jmx), a(imx,jmx)
      character chd*10,cstep*5,filename*32,string*8
      data chd/'0123456789'/
      data Tinf/288.15/
```

```
c..Evaluate u, v, cp, Mach no, Pressure, etc. first

c..calculate the u and v
      do i=1,ig
      do j=1,jg
         if ((i.eq.1).or.(i.eq.ig)) then
             u(i,j)=1.0
         else
             u(i,j)=1.0+(phi(i+1,j)-phi(i-1,j))/(dx(i-1)+dx(i))
         end if

         if ((j.eq.1) .or. (j.eq.jg)) then
           v(i,j)=0
         else
           v(i,j)=(phi(i,j+1)-phi(i,j-1))/(dy(j-1)+dy(j))
         end if
      end do
      end do

c..calculate temperature/density/pressure ratio
c..local speed of sound and local mach number
      do i=1,ig
      do j=1,jg
         t(i,j) = 1 + (gamma-1)/2.0*fsmach2*(1-(u(i,j)**2+v(i,j)**2))
         rho(i,j) = t(i,j)**(1.0/(gamma-1.0))
         pressure(i,j) = t(i,j)**(gamma/(gamma-1.0))
         a(i,j) = 1.0/fsmach2 + (gamma-1.0)/2.0*(1-u(i,j)**2-v(i,j)**2)
         mach(i,j) = sqrt(u(i,j)**2+v(i,j)**2)/a(i,j)
      enddo
      enddo


c..calculate the pressure coefficient on both airfoil surfaces
      do i=ile,ite,1
          cp(1,i) = 2.0/(gamma*fsmach2)*(pressure(i,jal)-1.0)
          cp(2,i) = 2.0/(gamma*fsmach2)*(pressure(i,jau)-1.0)
      end do



c..Construct the filenames and write out the data
      write(string,'(f8.5)') float(nio)/100000
      read(string,'(3x,a5)') cstep
      filename = 'cp-'//cstep//'.dat'
      open(1,file=filename)
      write(1,'(2E13.5)') (xg(i),cp(1,i), i=ite,ile,-1)
      write(1,'(2E13.5)') (xg(i),cp(2,i), i=ile,ite)
      close(1)

      filename='vars-'//cstep//'.tec'
      open(1,file=filename,form='formatted')
c..Output Phi,u,v,pressure,density,Mach in TECPLOT format
      write(1,*) ' variables="x","y","phi","mach",
     + "pressure", "u", "v", "rho", "temp" '
      write(1,*) ' zone i=',ig, ',j=',jg
      do j = 1,jg
      do i = 1,ig
         write(1,'(8E12.4)') xg(i),yg(j),phi(i,j),mach(i,j),
     +     pressure(i,j), u(i,j), v(i,j), rho(i,j), t(i,j)
      enddo
      enddo

      close(1)
      return
      end
```

Table 3 - Post-Processing Subroutine

2. Results

2.1. 0.2 Mach Solution

After code is executed, solution is obtained for 0.2 Mach, which is low subsonic speed. Then, coefficient of pressure, pressure contours and Mach contours are plotted with streamlines.

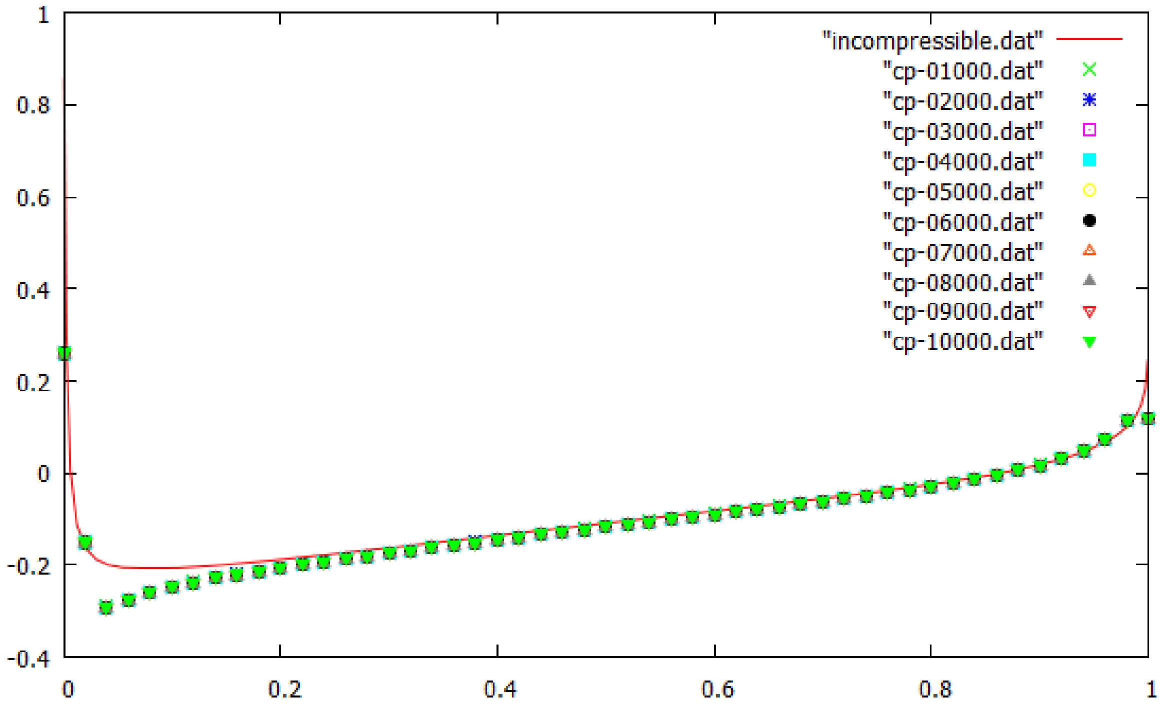

Figure 1 - Coefficient of Pressure

As it can be seen from the figure, for low subsonic Mach number case transonic small disturbance equation gives almost same result as incompressible solution. After a certain number of iterations, solution converges to a certain value which almost overlaps with panel method solution.

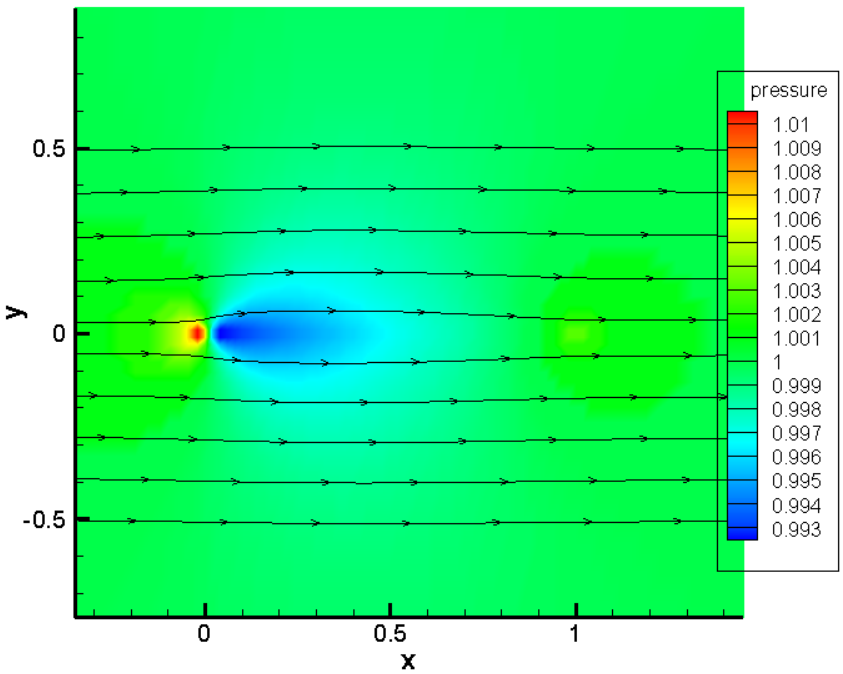

Figure 2 - Pressure Distrubution

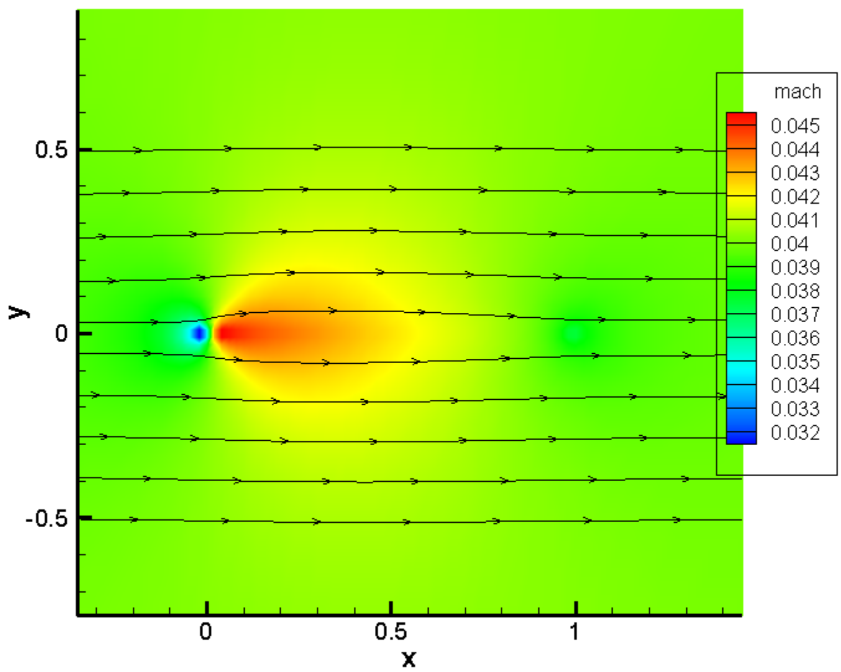

Figure 3 - Mach Number Distribution

2.2. 0.6 Mach Solution

After code is executed, solution is obtained for 0.6 Mach. Then, coefficient of pressure, pressure contours and Mach contours are plotted with streamlines.

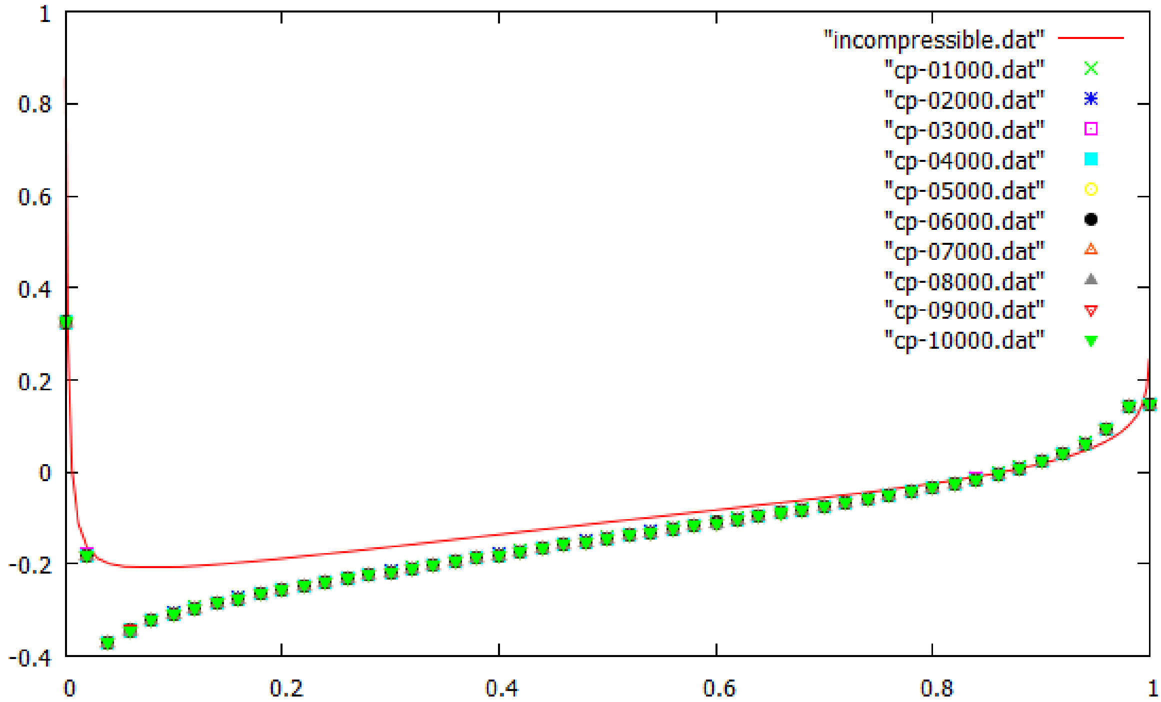

Figure 4 - Coefficient of Pressure

As it can be seen from the figure, as Mach number increases due to compressibility effects coefficient of pressure differs from incompressible solution as expected. After certain number of iterations, solutions converges to a certain number which differs from panel method solution.

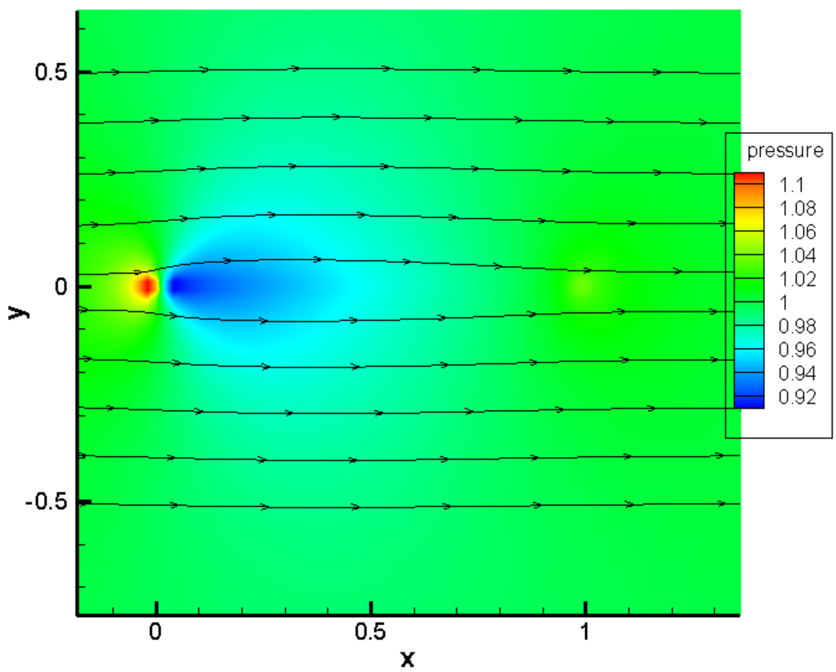

Figure 5 - Pressure Contours

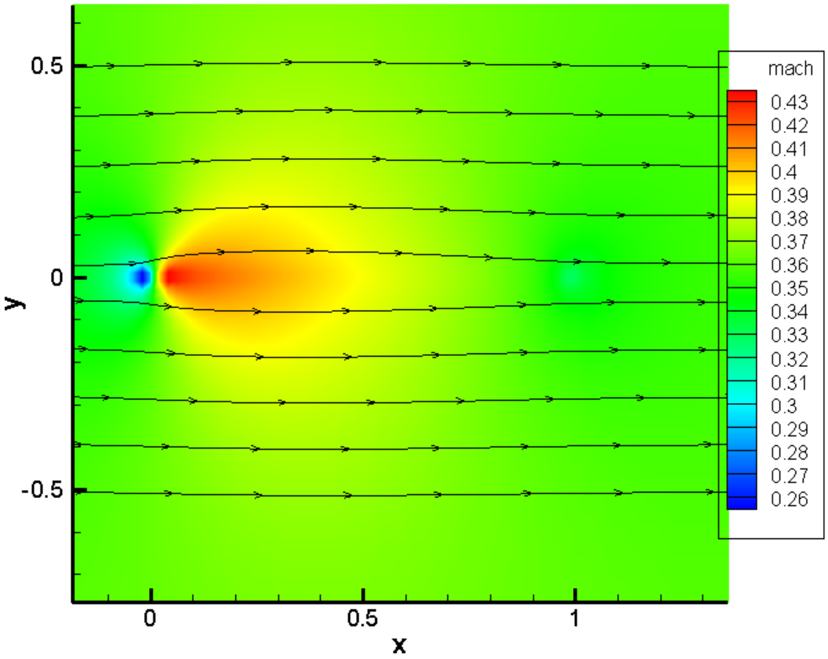

Figure 6 - Mach Contours

2.3. 0.85 Mach Solution

After code is executed, solution is obtained for 0.8 Mach, which is transonic speed. Then, coefficient of pressure, pressure contours and Mach contours are plotted with streamlines.

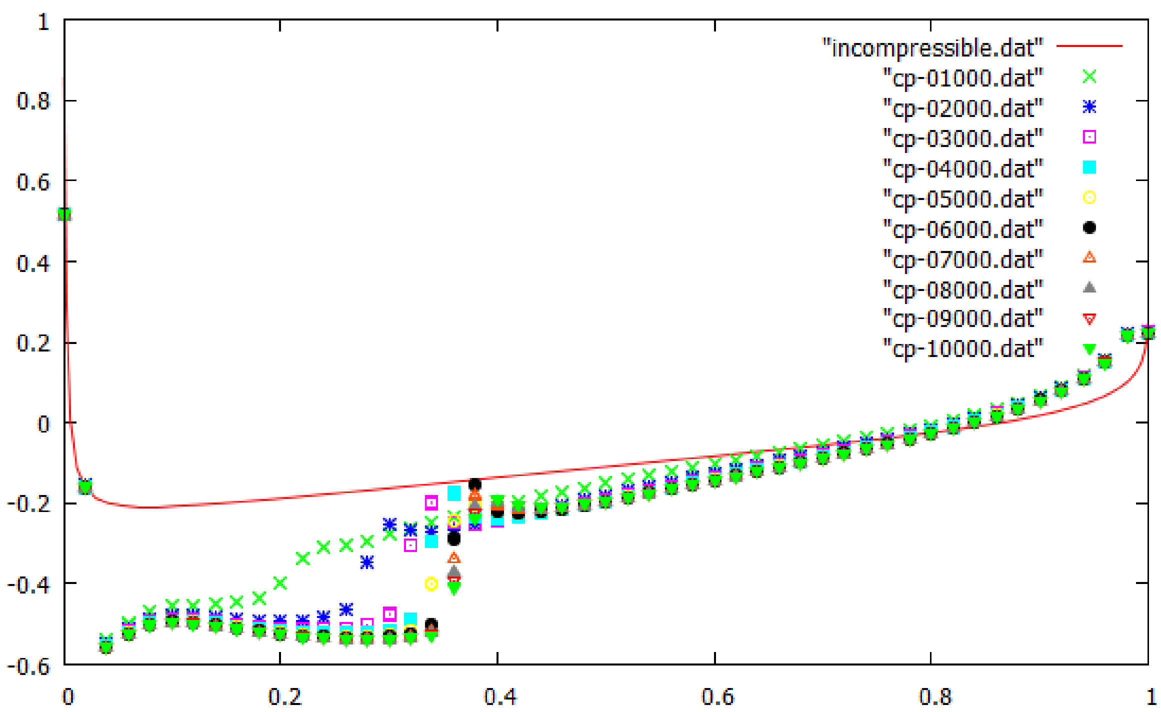

Figure 7 - Coefficient of Pressure

As it can be seen from the figure, due to transonic speeds and increased velocity over the airfoil shock formation occurs. Eventually, this results in sudden increase in coefficient of pressure. After iterations, shock location and shock stregth converges to a certain value. As a result, transonic solution completely differs from incompressible solution, as expected.

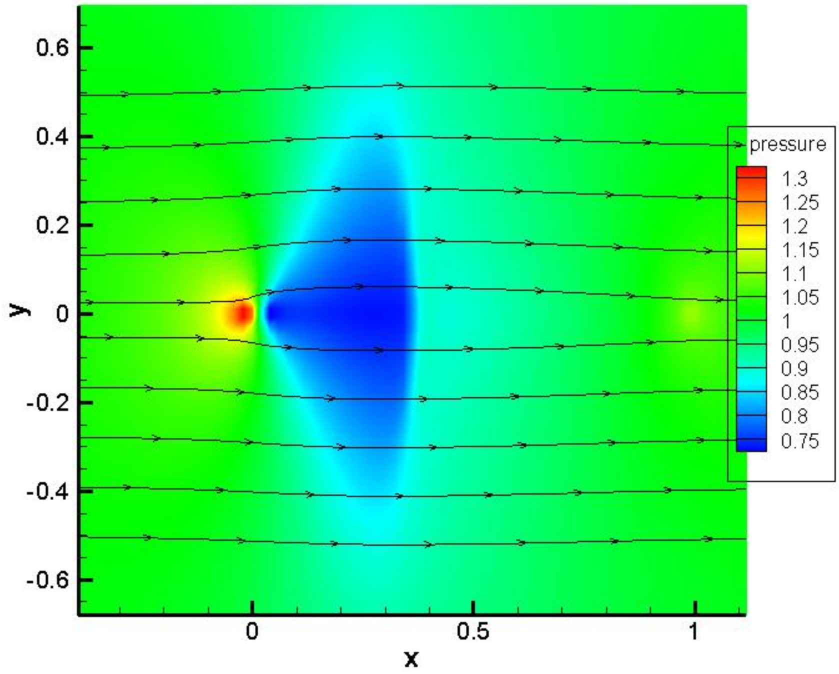

Figure 8 - Pressure Contours

As it can be seen from the figure, shock formations can be seen in pressure contours. Supersonic cone occures over the airfoil. As it can clearly be seen, pressure changes suddenly when flow goes from supersonic to subsonic.

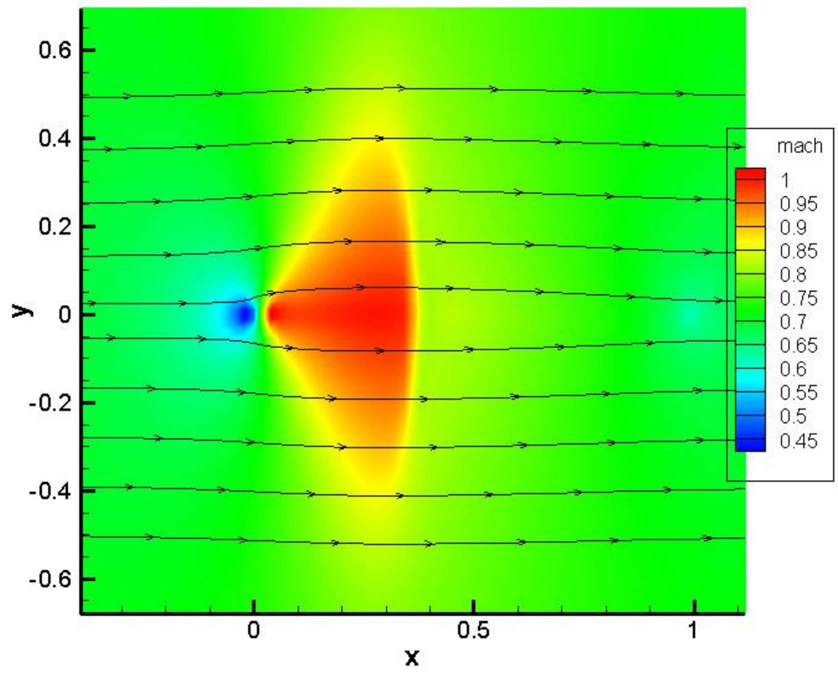

Figure 9 - Mach Contours

As it can clearly be seen, shock formations occur in Mach contours as well. Pressure and Mach contours are in harmony with coefficient of pressure. In coefficient of pressure, shock occured close to 0.4 chord. From above figures, shocks occurs exactly at same location. As it can be seen, there is a supersonic, in this case sonic, cone occurs. After the shock, Mach number become immediately subsonic, which is expected after the shock.

3. Conclusion

It is concluded that, for low subsonic values transonic small disturbance equation solution has similar behavior with incompressible panel method solution.

For high Mach numbers, as expected coefficient of pressure differs from incompressible solution as expected due to compressibility effects.

For transonic Mach numbers, main advantage of transonic small disturbance equation comes to stage. Shock formations observed in coefficient of pressure, pressure contours and Mach contours.

In addition, from compressible aerodynamics, it is known that after formation of shock, temperature and density must increase suddenly. These are also obtained in solution, and following figures completely agrees with compressible aerodynamics:

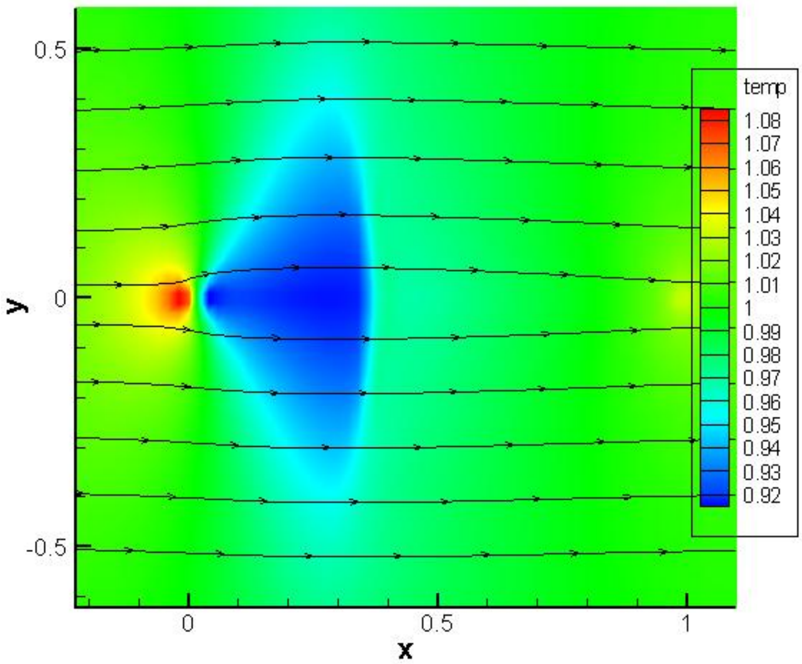

Figure 10 - Temperature Ratio Contours

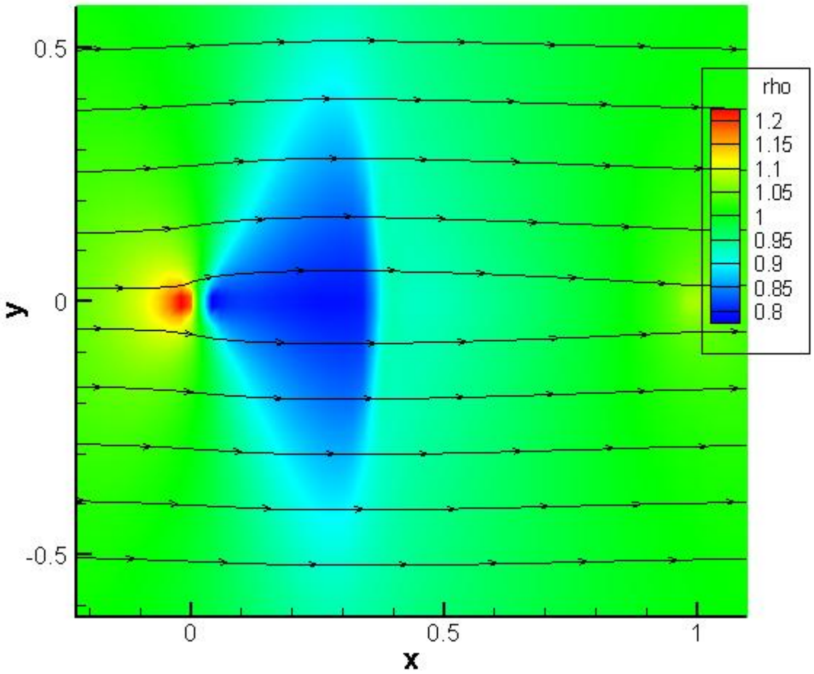

Figure 11 - Density Ratio Contours

All of the ratios are defined as local property to the freestream property.