EXAMINATION COVERSHEET

Winter 2021 Final Examination

UNIVERSITY OF WOLLONGONG IN DUBAI

THIS EXAMINATION CONTENT IS STRICTLY CONFIDENTIAL

Students must comply with requirements stated in the Online Examination Procedures

Student Number:	
First Name:	
Family Name:	
Date of Examination: (DD/MM/YY)	**01-04-21**
Subject Code:	**MATH 141**
Subject Title:	**Foundation of Engineering Mathematics**
Time Permitted to Write Exam:	2 Hours
Time Permitted to Upload Exam Paper:	**10** Minutes
Total Number of Questions:	**10**
Total Number of Pages *(including this page):*	**13**

INSTRUCTIONS TO STUDENTS FOR THE EXAM (*FURTHER TO INSTRUCTIONS POSTED ON MOODLE SITE*)

1) Download the final examination question paper onto your laptop/desktop |
2) Answers should be submitted within the time-frame specified above.
3) *Answers **must** be written (and drawn) in black or blue ink.*
4) Any mistakes must be crossed out. Whitener and ink erasers must **not** be used.
5) All questions are written and you must show your detailed work.
6) All questions are compulsory.
7) Total marks: 100. This Exam is worth 35% of your final marks for MATH 141.

INSTRUCTIONS TO STUDENTS FOR UPLOADING EXAMS

- **A**nswer the questions on paper with clear indication of which solution belongs to which question.
- Ensure to take clear scan or pictures/images of all your working and solutions.
- Convert your solutions of the final examination in a single PDF file for submission.
- Keep your answer sheets for reference in case the images are not readable or corrupted.
- Upload your PDF submission document by the deadline specified above.
- If you are unable to upload the answer paper(s) before the due time, DO NOT PANIC. Email it to the lecturer at assanelo@uowdubai.ac.ae within the time-limit.
- You will be permitted only one single attempt to upload your submission.
- The Lecturer will have the discretion to conduct viva on the submission made, if needed.

(**10pts**)**Problem 1**
Write the following complex numbers in the form $re^{i\theta}$ where $r = |z|$ and $\theta = \arg z$.

1. $z = \dfrac{5 + 11i\sqrt{3}}{7 - 4i\sqrt{3}}$,
2. $z = \dfrac{\sqrt{2}}{1 - i}$

(Show your detailed work)

(**10pts**)**Problem 2**
Solve the quadratic equation

$$z^2 + (3i - 4)z + 1 - 7i = 0. \qquad \text{(Show your detailed work)}$$

(**10pts**)**Problem 3**
Consider the surface whose equation is given by

$$x^2 + y^2 + z^2 - 4x - 12y - 8z = m.$$

For what value (s) of m will the surface be a sphere. (Justify and show your work).

(**10pts**)**Problem 4**

Let $\overrightarrow{a}$ and $\overrightarrow{b}$ be two vectors such that $\|\overrightarrow{a}\| = 3$ and $\left\|\overrightarrow{b}\right\| = 4$. If $\theta = \dfrac{2\pi}{3}$ is the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$, find

1. $\left\|\overrightarrow{a} - \overrightarrow{b}\right\|$
2. $\left(3\overrightarrow{a} - 2\overrightarrow{b}\right) \cdot \left(\overrightarrow{a} + 2\overrightarrow{b}\right)$

(Show your detailed work)

(**10pts**)**Problem 5**

Find the parametric equations of the line passing through the point $A(2,3,5)$ and parallel to the line of intersection of the two planes $3x - y + z = 0$ and $x - y + z = 0$. (Show your detailed work)

(**10pt**s)**Problem 6**
(a) Find the distance from the point $Q(3,1,0)$ to the line with parametric equations

$$\begin{cases} x = 2+3t \\ y = -1+2t \\ z = 5+t \end{cases}$$

(b) Find the distance from the point $Q(3,1,0)$ to the plane $x+2y-5z=1$. (Show your detailed work)

(**10pt**s)**Problem 7**
Use the Gauss elimination method to solve the linear system

$$\begin{cases} x_1 + x_2 + 2x_3 = 5 \\ x_1 - x_2 - x_3 = 1 \\ x_1 + x_3 = 3 \end{cases}$$

(Show your detailed work including the elementary operations)

(10pt$s$)Problem 8
Consider the matrices

$$A = \begin{pmatrix} 4 & 8 \\ 1 & 2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 3 & 9 \\ 1 & 1 \end{pmatrix}$$

(a) Find $A^2 + 2AB + B^2$
(b) Find $(A + B)^2$.
(Show your detailed work)

(**10pt**s)**Problem 9**
Find all the value (s) of x for which the matrix A does not have an inverse.

$$A = \begin{pmatrix} 1 & 3 & x \\ 4 & 5 & -1 \\ 2 & -1 & 5 \end{pmatrix}$$

(10pt$s$)Problem 10

Use elementary operations to find the determinant of the matrix in terms of x and y.

$$A = \begin{pmatrix} 1 & 0 & x & x^2 \\ 0 & 1 & y & y^2 \\ 1 & 0 & 2 & 4 \\ 0 & 1 & 3 & 9 \end{pmatrix}$$

Formulas and Identities

Tangent and Cotangent Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Even/Odd Formulas

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$

$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

Periodic Formulas

If n is an integer.

$$\sin(\theta + 2\pi n) = \sin\theta \qquad \csc(\theta + 2\pi n) = \csc\theta$$

$$\cos(\theta + 2\pi n) = \cos\theta \qquad \sec(\theta + 2\pi n) = \sec\theta$$

$$\tan(\theta + \pi n) = \tan\theta \qquad \cot(\theta + \pi n) = \cot\theta$$

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Half Angle Formulas **(alternate form)**

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \sin^2\theta = \frac{1}{2}\left(1 - \cos(2\theta)\right)$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \cos^2\theta = \frac{1}{2}\left(1 + \cos(2\theta)\right)$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \qquad \tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Product to Sum Formulas

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

$$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$

Sum to Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

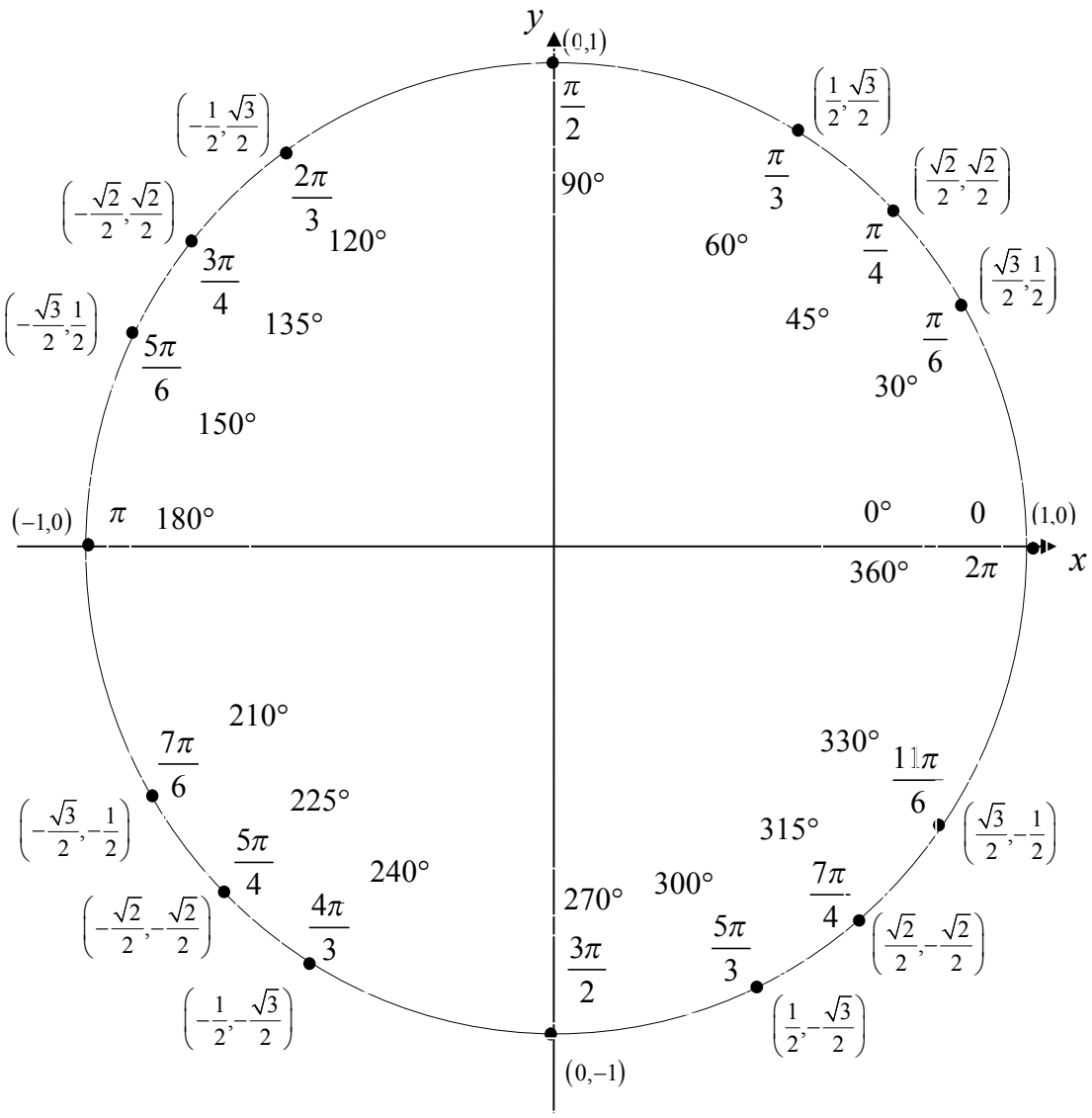
Cofunction Formulas

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

Unit Circle



For any ordered pair on the unit circle (x, y) : $\cos\theta = x$ and $\sin\theta = y$

Example

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2} \qquad \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$