Problem 1

Evaluate the following indefinite integrals

Evaluate $\int_0^2 (x^2 + 2x + 5)\, dx$.

Evaluate $\int_{-1}^1 (x^3 - x + 5)\, dx$.

Evaluate $\int_1^2 \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$.

Evaluate $\int_1^2 \frac{7}{\sqrt{x}}\, dx$.

Evaluate $\int_1^2 \frac{x^4 + 1}{x^3}\, dx$.

Evaluate $\int_1^2 \left(x^2 + 8x + \frac{3}{x^2} \right) dx$.

Evaluate $\int_0^1 (3\sqrt{x} + 1)\, dx$.

Evaluate $\int_1^2 \left(x^2 - \frac{3}{x^4} \right) dx$.

Problem 2
Use u-substitution to evaluate the following integrals

Evaluate $\int_0^1 x\sqrt{9x^2+16}\,db$

Evaluate $\int_0^1 \dfrac{x}{(1+x^2)^2}\,dx$

Evaluate $\int_0^3 x\sqrt{9-x^2}\,dx$.

Evaluate $\int_0^4 \dfrac{x}{\sqrt{9+x^2}}\,dx$.

Evaluate $\int_1^2 \dfrac{x^3}{\sqrt{3x^4+1}}\,dx$.

Evaluate $\int_1^2 x^2\sqrt{x-1}\,dx$.

Evaluate $\int_1^5 x\sqrt{2x-1}\,dx$.

Evaluate $\displaystyle\int \frac{\cos^5 x}{\sqrt{\sin x}}dx, \quad \int \sec^4 x\sqrt[3]{\tan x}dx, \quad \int \frac{\tan^3 x}{\sec x}dx$

Problem 3
Use definite integrals to evaluate

$$\lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n}\sqrt{1-\left(-1+\frac{2i}{n}\right)^2}.$$

If f is an integrable function, the

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{6}{n}f\left(1+\frac{6i}{n}\right)=$$

(a) $\int_1^7 f(x)\,dx$

(b) $\int_1^6 f(x)\,dx$

(c) $\int_1^5 6f(x)\,dx$

(d) $\int_1^7 6f(x)\,dx$

(e) $\int_1^6 6f(x)\,dx$

$$\lim_{n\to\infty}\left[\sum_{i=1}^{n}\left(\frac{1}{n}\right)\frac{1}{\sqrt{1-\left(\frac{i}{2n}\right)^2}}\right]=$$

(a) $\int_0^{\frac{1}{2}} \frac{2}{\sqrt{1-x^2}}\,dx$

(b) $\int_0^{\frac{1}{2}} \frac{1}{1-x^2}\,dx$

(c) $\int_{-\frac{1}{2}}^{0} \frac{1}{\sqrt{1-x^2}}\,dx$

(d) $\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{2}} \frac{2}{\sqrt{1-2x^2}}\,dx$

(e) $\int_{-2}^{2} \frac{2}{\sqrt{x}}\,dx$

Problem 4

Find the modulus and the argument of the complex number

$$w = \frac{-9+3i}{1-2i}.$$

Solve the equation

$$2z^2 - 2iz - 5 = 0$$

Solve the equation

$$2z^2 + (2+3i)\,z + 2i - 1 = 0$$

$$z^2 - (1-i)\,z + 7i - 4 = 0$$

Find x and y given that

(a) $(x+iy)(2+i) = 0$

(b) $x\,(1+i)^2 + y\,(2-i)^2 = 3 + 10i$

Problem 5

Let $P(2,-1,3)$, $Q(3,1,2)$ and $R(2,1,-4)$ be three points in the 3D-space.

(a) Find $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

(b) Find the vector projection of $\overrightarrow{PQ}$ onto $\overrightarrow{PR}$.

(c) Find the area of the triangle with vertices P, Q and R.

(d) Find an equation of the plane containing the points P, Q and R.

Problem 6

Let $\overrightarrow{a}$ be a unit vector and $\overrightarrow{b}$ a vector such that $\left\|\overrightarrow{b}\right\| = 5$. If the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$ is $\theta = \frac{2\pi}{3}$, find the magnitude of the vector $\overrightarrow{a} - \overrightarrow{b}$.

B) The angle between a unit vector $\overrightarrow{a}$ and a vector $\overrightarrow{b}$ is $\frac{\pi}{3}$. If $\left\|\overrightarrow{b}\right\| = 2$, then find tl

The sum of all the values of x for which the two vectors $\overrightarrow{a} = \langle 3,\ 2,\ x\rangle$ and $\overrightarrow{b} = \langle 2x,\ 4,\ x\rangle$ are orthogonal is

$(a)\ -6 \qquad (b)\ 6 \qquad (c)\ 4 \qquad (d)\ -9 \qquad (e)\ -8$

If $\langle a,\ b,\ c\rangle$ is a unit vector orthogonal to both vectors $\langle 1,\ -1,\ 1\rangle$, $\langle 0,\ 4,\ 4\rangle$, then $|a|+|b|+|c|$ is equal to

$(a)\ \frac{2}{\sqrt{6}} \qquad (b)\ \frac{8}{\sqrt{6}} \qquad (c)\ \frac{10}{\sqrt{6}} \qquad (d)\ \frac{4}{\sqrt{6}} \qquad (e)\ \frac{1}{\sqrt{6}}$

Problem 7

) Find a direction vector of the line of intersection of the two planes

$$2x - y + 3z = 1 \quad \text{and} \quad -x + 3y + 3z = 5$$

Find parametric equations for the line through $(2, 0, -3)$ that is parallel to the line of intersection of the planes $x + 2y + 3z + 4 = 0$ and $2x - y - z - 5 = 0$.

Find parametric equations of the line of intersection of the planes

$$x - 3y + 2z = -1 \text{ and } 4x + y + 7z = 9$$

Problem 8
Use the Gauss elimination method to solve the linear system

$$
\begin{aligned}
3x - 2y + 8z &= 9 \\
-2x + 2y + z &= 3 \\
x + 2v - 3z &= 8
\end{aligned}
$$

$$
\begin{cases}
x + y + 2z = 9 \\
2x + 4y - 3z = 1 \\
3x + 6y - 5z = 0
\end{cases} .
$$

Problem 9

Consider the following matrices

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}$$

(a) Use the **Gauss Jordan method** to find C^{-1}. (Do NOT use the formula and show your work)

(b) Compute the following matrices, where possible.

1. $A + 2B^T$, 2. AC

Let

$$A = \begin{bmatrix} 1 & 3 \\ 0 & -1 \\ -1 & 2 \\ 3 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ -2 & 3 \\ 2 & -1 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix}$$

Find, if possible

a) AB **b)** AC

Problem 10

Find the determinant of the matrices

$$A = \begin{bmatrix} 3 & -7 & 8 & 9 & -6 \\ 0 & 2 & -5 & 7 & 3 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 2 & 4 & -1 \\ 0 & 0 & 0 & -2 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & -1 & 1 & 3 \\ 1 & 2 & -1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 1 & 8 & 0 \end{bmatrix}.$$