1. $x^2 - xy + y^2 = 7$ $\qquad y'$ & y''

$$2x - xy' - y + 2yy' = 0$$

$$y'(2y - x) = y - 2x$$

$$y' = \frac{y - 2x}{2y - x}$$

@ $(2, -1)$

$$y' = \frac{-1 - 2(2)}{2(-1) - 2}$$

$$= \frac{-1 - 4}{-2 - 2}$$

$$= \frac{-5}{-4} = \frac{5}{4}$$

$$y'' = \frac{(y' - 2)(2y - x) - (2y' - 1)(y - 2x)}{(2y - x)^2}$$

$$y'' = \frac{\left(\frac{5}{4} - 2\right)(-2 - 2) - \left(\frac{5}{2} - 1\right)(-1 - 4)}{(-2 - 2)^2}$$

$$= \underline{\frac{21}{32}}$$

2. $\frac{dV}{dt} = 4\ m^3/min$

$S = 6x^2 = 24\ m^2$ $\qquad V = x^3$

$\hookrightarrow x^2 = 4\ m^2$

$$\frac{dV}{dt} = 3x^2 \times \frac{dx}{dt}$$

$$\frac{\cancel{4}\, m^{\cancel{3}}}{min} \times \frac{1}{3 \times \cancel{4}\, \cancel{m^2}} = \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{1}{3}\ m/min$$

$$\frac{dS}{dt} = 12\,x\,\frac{dx}{dt}$$

$$= \cancel{12}^{4} \times 2 \times \frac{1}{\cancel{3}}$$

$$= 8\ m^2/min$$

3. $r = \frac{1}{2}h$ $\quad \frac{dV}{dt} = 10\ ft^3/min$ $\quad \frac{dh}{dt} = ?$ $\quad h = 5\,ft$

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{\pi}{3}\left(\frac{h}{2}\right)^2 h$$

$$= \frac{\pi}{12}h^3$$

$$\frac{dV}{dt} = \frac{\pi}{12}\,3h^2 \times \frac{dh}{dt}$$

$$10 = \frac{\pi}{\cancel{12}\,4} \times \cancel{3} \times 25 \times \frac{dh}{dt}$$

$$\frac{\cancel{10}^{2} \times 4}{\pi \times \cancel{25}\,5} = \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{8}{5}\pi\ ft/min$$

4. $f(x) = 4x^3 - 15x^2 - 18x + 10$

$f'(x) = 12x^2 - 30x - 18$

$f'(x) = 0$

$12x^2 - 30x - 18 = 0$

$6(2x^2 - 5x - 3) = 0$

$2x^2 - 5x - 3 = 0$

$x(2x+1) - 3(2x+1) = 0$

$(x-3)(2x+1) = 0$

$x = 3$ or $x = -1/2$

x	$-\infty$	$-1/2$	3	$+\infty$
$2x^2-5x-3$	+	−	+	
$f(x)$	$\uparrow$	$\downarrow$	$\uparrow$	

f increases on $(-\infty, -1/2) \cup (3, +\infty)$

f decreases on $(-1/2, 3)$

5.

$f'(x) = e^{(x-4)^3}(x^2+x+2)^3(x-2)^3(x-1)$

$f'(x) = 0$

$e^u \Rightarrow$ Always positive

$(x^2+x+2)^3 = 0$

$x^2+x+2 = 0 \Rightarrow$ Cannot be factorized

Always positive

$(x-2)^3 = 0$ | $x-1 = 0$

$x-2 = 0$ | $x = 1$

$x = 2$

$\therefore$ f is decreasing on $(1, 2)$

6.

$f(x) = x^4 - 2x^2 + 2$

$f'(x) = 4x^3 - 4x$

$= 4x(x^2 - 1)$

$f'(x) = 0$

$4x(x^2 - 1) = 0$

$4x = 0 \qquad x^2 - 1 = 0$

$x = 0 \qquad x^2 = 1$

$x = \pm 1$

$\therefore$ Critical numbers are $-1, 0, 1$

$f(-1) = 1$

$f(0) = 2$

$f(1) = 1$

$f(2) = 10$

The absolute maximum is 10 at $x = 2$

The absolute minimum is 1 at $x = -1$ and $x = 1$

7.

$f(x) = 3x^5 - 5x^4$

$f'(x) = 15x^4 - 20x^3$

$f'(x) = 0$

$5x^3(3x - 4) = 0$

$5x^3 = 0 \qquad 3x - 4 = 0$

$x^3 = 0 \qquad 3x = 4$

$x = 0 \qquad x = 4/3$

$\therefore$ Critical numbers are $0, 4/3$

x	$-\infty$	0	$4/3$	∞
$5x^3$	$-$	$+$	$+$	
$3x-4$	$-$	$-$	$+$	
$f'(x)$	$+$	$-$	$+$	
$f(x)$	$\uparrow$	$\downarrow$	$\uparrow$	

$f(0) = 0 \Rightarrow$ Local maximum

$f\left(\frac{4}{3}\right) = -\frac{256}{81} \Rightarrow$ Local minimum

8.

$$f'(x) = \cancel{12}^{4} \times \frac{2}{\cancel{3}} x^{-1/3} - 16$$

$$= \frac{8}{\sqrt[3]{x}} - 16$$

@ $x = 0$

$f'(0) \Rightarrow$ undefined

$\therefore$ Critical number is $0, 1/8$

x	$-\infty$	0	$1/8$	$+\infty$
f'	$-$	$+$	$-$	
f	$\downarrow$	$\uparrow$	$\downarrow$	

$f(0) = 0 \Rightarrow$ Local minimum

$f(1/8) = 1 \Rightarrow$ Local maximum

9.

$f'(x) = 3x^2 + 2ax + b$

Rmin @ $x = -1$

$f'(-1) = 0$

$f'(-1) = 3 - 2a + b = 0$

$2a - b = 3 \Rightarrow b = 2a - 3$

Rmax @ $x = -3$

$f'(-3) = 0$

$f'(-3) = 27 - 6a + b = 0$

$27 = 6a - b$

$27 = 6a - 2a + 3$

$24 = 4a$

$a = 6$

$b = 9$

$f'(x) = 3x^2 + 6x^2 + 9x + 1$

$f''(x) = 6x + 12$

$f''(x) = 0$

$6(x+2) = 0$

$x = -2 \Rightarrow$ Point of inflection

x	$-\infty$	-2	∞
f''	$-$		$+$
f	$\cap$		$\cup$

$\therefore f$ is concave down on $(-\infty, -2)$
and concave up on $(-2, \infty)$