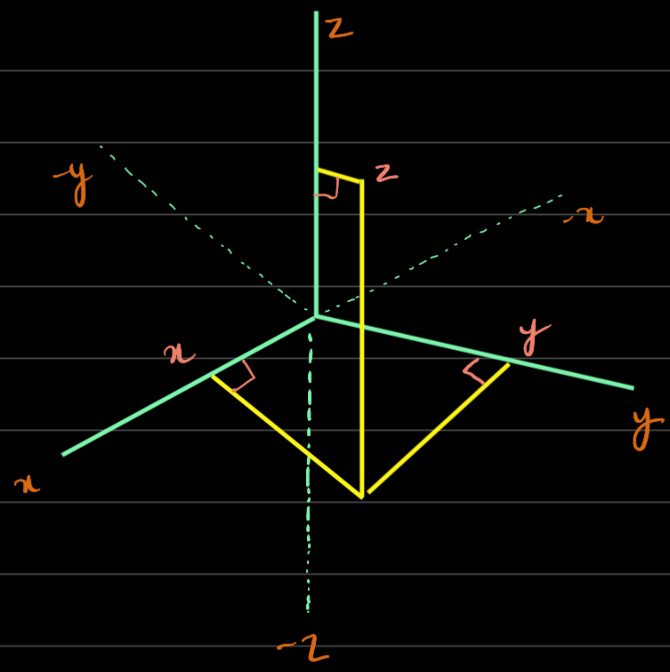
Three Dimensional Geometry

In 3 - Dimensional Space, a point P is represented by the ordered triple (x, y, z)

(x, y) are the coordinates of the projection of P onto the xy - plane



Midpoint & Distance Formula

Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two points in the space. Then the distance from P to Q is given by

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

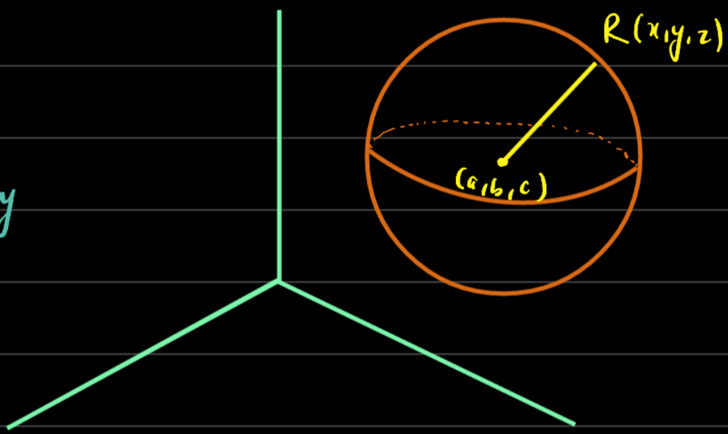
The coordinate of the midpoint of the line segment joining P and Q are

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

Equation of a sphere

The standard form of the equation of the sphere centered at (a,b,c) with radius R is given by

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2$$



Example

Find the equation of the sphere whose diameter has endpoints $P(-1,2,4)$ and $(0,2,1)$

$$\text{Center} = \left(\frac{-1+0}{2}, \frac{2+2}{2}, \frac{4+1}{2}\right)$$

$$= \left(\frac{-1}{2}, 2, \frac{5}{2}\right)$$

$$\text{Radius} = \frac{1}{2}\,\text{Diameter}$$

$$= \frac{1}{2}\sqrt{1+9}$$

$$= \frac{\sqrt{10}}{2}$$

Equation of sphere

$$\left(x+\frac{1}{2}\right)^2 + \left(y-2\right)^2 + \left(z-\frac{5}{2}\right)^2 = \frac{10}{4} = \frac{5}{2}$$

Example

Show that the equation $x^2 + y^2 + z^2 = 4x - 6y + 8z + 1$ represents a sphere and find its center and its radius.

$$x^2 - 4x + y^2 + 6y + z^2 - 8z = 1$$

$$(x-2)^2 - 4 + (y+3)^2 - 9 + (z-4)^2 - 16 = 1$$

$$(x-2)^2 + (y+3)^2 + (z-4)^2 = 30$$

$\therefore$ This is a sphere

Center $= (2, -3, 4)$

Radius $= \sqrt{30}$

Completing the square

$$x^2 + kx = \left(x + \frac{k}{2}\right)^2 - \left(\frac{k}{2}\right)^2$$

Vectors

$$\left.\begin{array}{l}\text{Direction}\\ \text{Distance}\\ \text{Orientation}\end{array}\right\} \text{Direction Vector}$$

The directed line segment from D to T is called a vector. It is denoted by $\vec{DT}$

D is called the initial point.

T is called the terminal point.

The distance between D and T is called the magnitude of the vector $\vec{DT}$

It is denoted by $\|\vec{DT}\|$