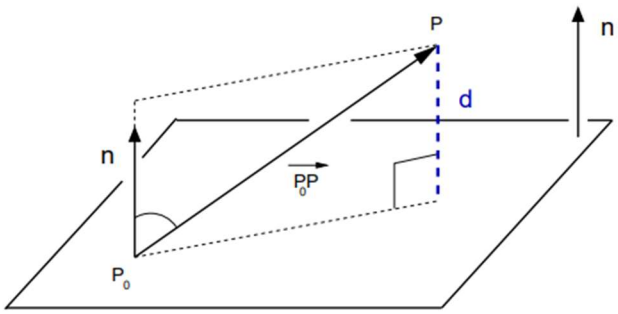
Formula Sheet

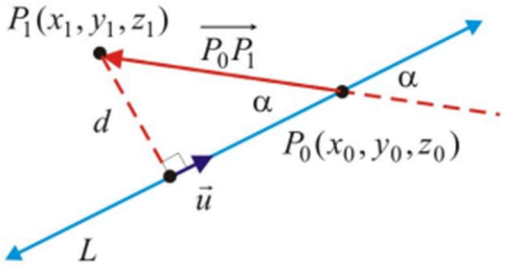
Distance formula from a point to a plane

$$d = \frac{|(\overrightarrow{P_0P}) \cdot \mathbf{n}|}{|\mathbf{n}|}.$$



Distance formula from a point to a line

$$d = \frac{\| \overrightarrow{P_0P_1} \times \vec{u} \|}{\| \vec{u} \|}$$



Vector Formulas

Vector Magnitude

$\|\langle a, b\rangle\| = \sqrt{a^2 + b^2}$

Unit Vector (Normalizing a Vector)

Divide by the magnitude: $\frac{\vec{u}}{\|\vec{u}\|}$

Adding Vectors

$\langle a, b\rangle + \langle c, d\rangle = \langle a + c, b + d\rangle$

$(a\hat{\imath} + b\hat{\jmath}) + (c\hat{\imath} + d\hat{\jmath}) = (a + c)\hat{\imath} + (b + d)\hat{\jmath}$

Scalar Multiplication

$a\langle b, c\rangle = \langle ab, ac\rangle$

$a(b\hat{\imath} + c\hat{\jmath}) = ab\hat{\imath} + ac\hat{\jmath}$

Dot Products

$\langle a, b\rangle \cdot \langle c, d\rangle = ac + bd$

$(a\hat{\imath} + b\hat{\jmath}) \times (c\hat{\imath} + d\hat{\jmath}) = ac + bd$

Alternative formula: $\vec{u} \cdot \vec{v} = \|\vec{u}\|\ \|\vec{v}\| \cos\theta$

Cross Products

$\langle a, b, c\rangle \times \langle d, e, f\rangle = \langle bf - ec,\quad -(af - dc),\quad ae - db\rangle$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a & b & c \\ d & e & f \end{vmatrix} = (bf - ec)\hat{\imath} - (af - dc)\hat{\jmath} + (ae - db)\hat{k}$$

Magnitude of the cross product: $\|\vec{u} \times \vec{v}\| = \|\vec{u}\|\ \|\vec{v}\|\ \sin\theta$

Projections

$proj_u\vec{v} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|^2}\vec{u}$ $\qquad$ $proj_v\vec{u} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\vec{v}$

Vector Angles

Finding the direction of a single vector: $\tan\theta = \frac{y}{x} = \frac{\hat{\jmath}\ term}{\hat{\imath}\ term}$

Finding the angle separating two vectors: $\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\ \|\vec{v}\|}$

Formulas and Identities

Tangent and Cotangent Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Even/Odd Formulas

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$

$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

Periodic Formulas

If n is an integer.

$$\sin(\theta + 2\pi n) = \sin\theta \qquad \csc(\theta + 2\pi n) = \csc\theta$$

$$\cos(\theta + 2\pi n) = \cos\theta \qquad \sec(\theta + 2\pi n) = \sec\theta$$

$$\tan(\theta + \pi n) = \tan\theta \qquad \cot(\theta + \pi n) = \cot\theta$$

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\begin{aligned}\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Half Angle Formulas **(alternate form)**

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \sin^2\theta = \frac{1}{2}\left(1-\cos(2\theta)\right)$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \cos^2\theta = \frac{1}{2}\left(1+\cos(2\theta)\right)$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \qquad \tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Product to Sum Formulas

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

$$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$

Sum to Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

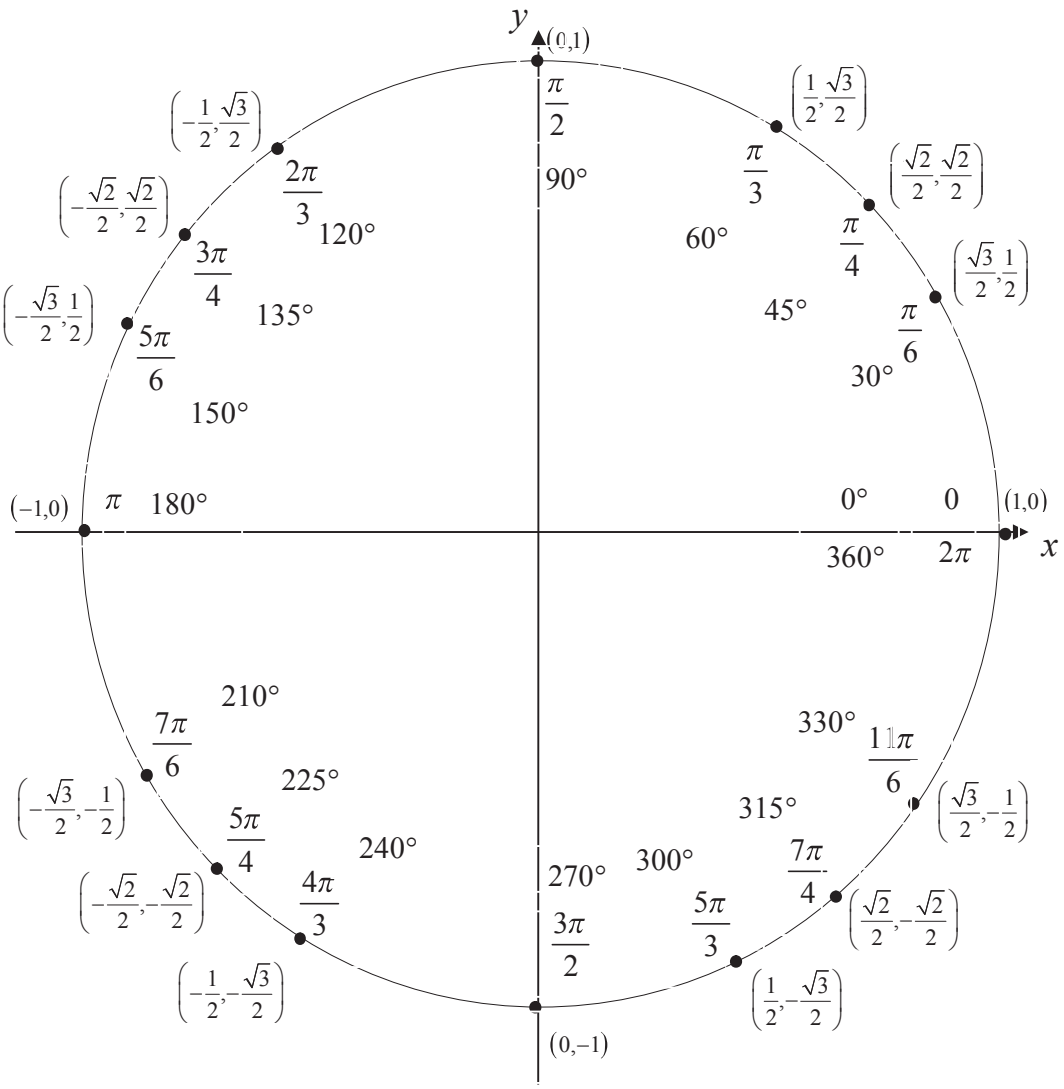
Cofunction Formulas

$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$$

$$\csc\left(\frac{\pi}{2}-\theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2}-\theta\right) = \csc\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$$

Unit Circle



For any ordered pair on the unit circle (x, y) : $\cos\theta = x$ and $\sin\theta = y$