Formulas

1.

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f\left[a + i\left(\frac{b-a}{n}\right)\right]\left(\frac{b-a}{n}\right)$$

2.

$$\frac{d}{dx}\int_{u(x)}^{v(x)} f(t)dt = v'(x)f\left[v(x)\right] - u'(x)f\left[u\left(x\right)\right]$$

3. .

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

4.

Product to Sum Formulas

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos\left(\alpha-\beta\right) - \cos\left(\alpha+\beta\right)\right]$$

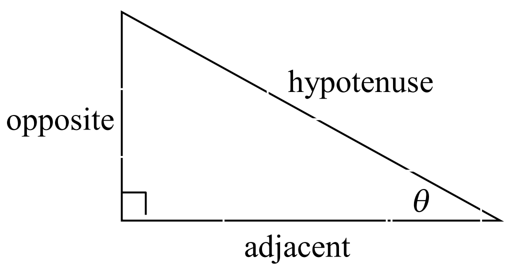
$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos\left(\alpha-\beta\right) + \cos\left(\alpha+\beta\right)\right]$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin\left(\alpha+\beta\right) + \sin\left(\alpha-\beta\right)\right]$$

$$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin\left(\alpha+\beta\right) - \sin\left(\alpha-\beta\right)\right]$$

Right triangle definition

For this definition we assume that $0 < \theta < \frac{\pi}{2}$ or $0° < \theta < 90°$.



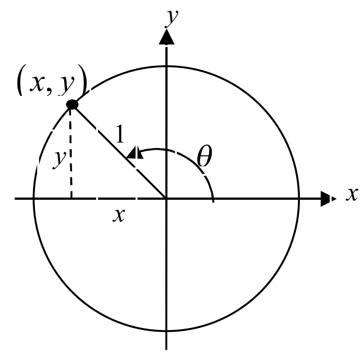
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

Unit circle definition

For this definition θ is any angle.



$$\sin\theta = \frac{y}{1} = y \qquad \csc\theta = \frac{1}{y}$$

$$\cos\theta = \frac{x}{1} = x \qquad \sec\theta = \frac{1}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Facts and Properties

Domain

The domain is all the values of θ that can be plugged into the function.

$\sin\theta$, θ can be any angle

$\cos\theta$, θ can be any angle

$\tan\theta$, $\theta \neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

$\csc\theta$, $\theta \neq n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

$\sec\theta$, $\theta \neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

$\cot\theta$, $\theta \neq n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

Range

The range is all possible values to get out of the function.

$-1 \le \sin\theta \le 1 \qquad \csc\theta \ge 1$ and $\csc\theta \le -1$

$-1 \le \cos\theta \le 1 \qquad \sec\theta \ge 1$ and $\sec\theta \le -1$

$-\infty < \tan\theta < \infty \qquad -\infty < \cot\theta < \infty$

Period

The period of a function is the number, T, such that $f(\theta + T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$$\sin(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cos(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\tan(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

$$\csc(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\sec(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cot(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

Formulas and Identities

Tangent and Cotangent Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Even/Odd Formulas

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$

$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

Periodic Formulas

If n is an integer.

$$\sin(\theta + 2\pi n) = \sin\theta \qquad \csc(\theta + 2\pi n) = \csc\theta$$

$$\cos(\theta + 2\pi n) = \cos\theta \qquad \sec(\theta + 2\pi n) = \sec\theta$$

$$\tan(\theta + \pi n) = \tan\theta \qquad \cot(\theta + \pi n) = \cot\theta$$

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\begin{aligned}\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Half Angle Formulas **(alternate form)**

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \sin^2\theta = \frac{1}{2}\left(1 - \cos(2\theta)\right)$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \cos^2\theta = \frac{1}{2}\left(1 + \cos(2\theta)\right)$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \qquad \tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Product to Sum Formulas

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

$$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$

Sum to Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

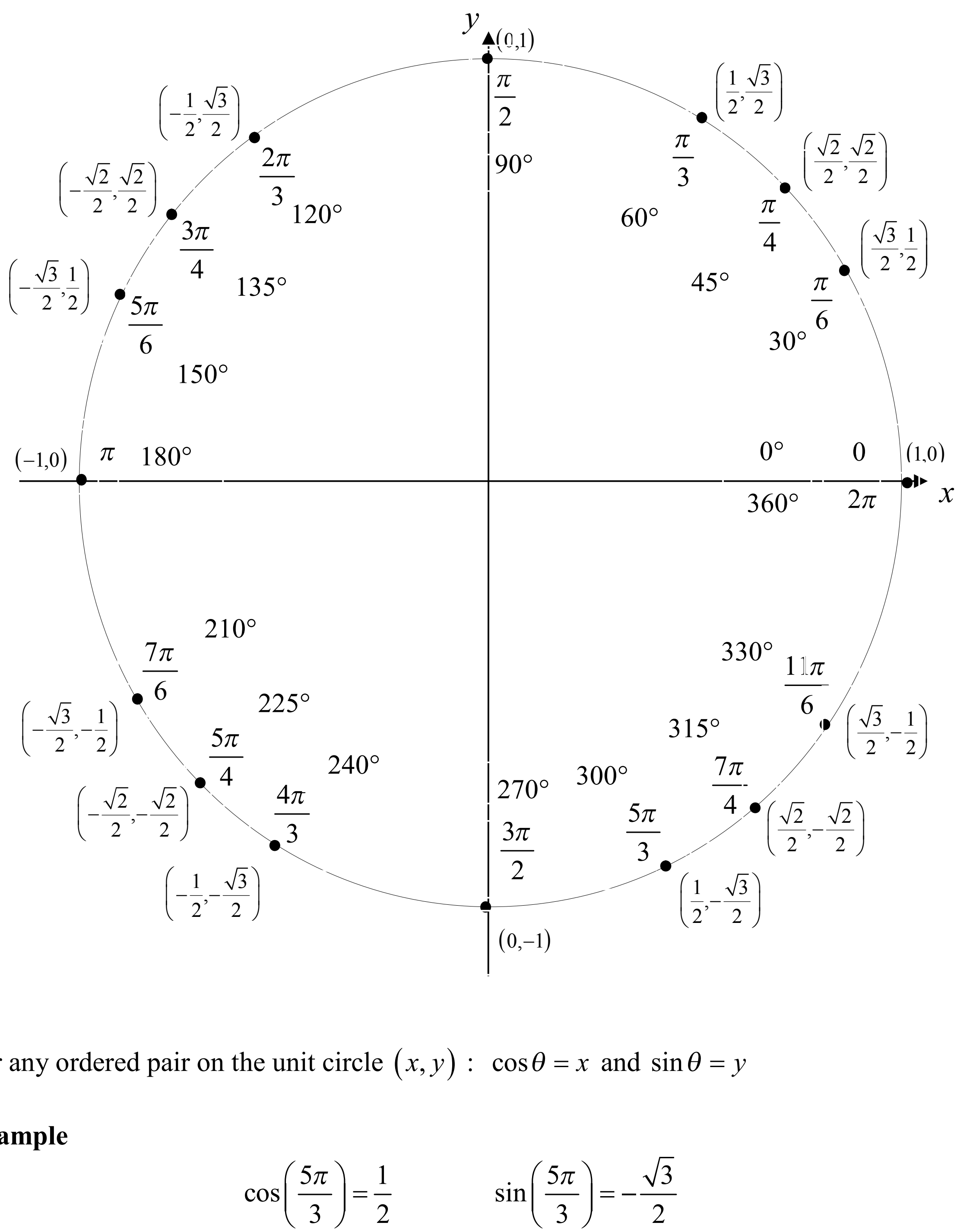
Cofunction Formulas

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

Unit Circle



For any ordered pair on the unit circle (x, y) : $\cos\theta = x$ and $\sin\theta = y$

Example

$$\cos\left(\frac{5\pi}{3}\right)=\frac{1}{2} \qquad \sin\left(\frac{5\pi}{3}\right)=-\frac{\sqrt{3}}{2}$$

Inverse Trig Functions

Definition

$y = \sin^{-1} x$ is equivalent to $x = \sin y$

$y = \cos^{-1} x$ is equivalent to $x = \cos y$

$y = \tan^{-1} x$ is equivalent to $x = \tan y$

Domain and Range

Function	Domain	Range
$y = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

Inverse Properties

$\cos\left(\cos^{-1}(x)\right) = x \qquad \cos^{-1}\left(\cos(\theta)\right) = \theta$

$\sin\left(\sin^{-1}(x)\right) = x \qquad \sin^{-1}\left(\sin(\theta)\right) = \theta$

$\tan\left(\tan^{-1}(x)\right) = x \qquad \tan^{-1}\left(\tan(\theta)\right) = \theta$

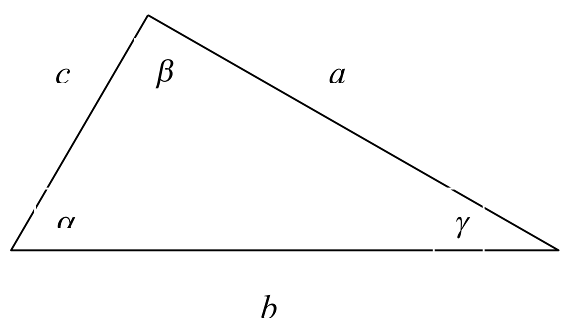
Alternate Notation

$\sin^{-1} x = \arcsin x$

$\cos^{-1} x = \arccos x$

$\tan^{-1} x = \arctan x$

Law of Sines, Cosines and Tangents



Law of Sines

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Mollweide's Formula

$$\frac{a+b}{c} = \frac{\cos\frac{1}{2}(\alpha-\beta)}{\sin\frac{1}{2}\gamma}$$

Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan\frac{1}{2}(\alpha-\beta)}{\tan\frac{1}{2}(\alpha+\beta)}$$

$$\frac{b-c}{b+c} = \frac{\tan\frac{1}{2}(\beta-\gamma)}{\tan\frac{1}{2}(\beta+\gamma)}$$

$$\frac{a-c}{a+c} = \frac{\tan\frac{1}{2}(\alpha-\gamma)}{\tan\frac{1}{2}(\alpha+\gamma)}$$

Calculus Formulas

Power Rules: $\frac{d}{dx}x^n = nx^{n-1}$ and $\int x^n\,dx = \frac{x^{n+1}}{n+1}+c$	***Product Rule:*** $\frac{d}{dx}[f(x)\cdot g(x)] = f(x)\cdot g'(x) + f'(x)\cdot g(x)$
Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{[g(x)]^2}$	***Reciprocal Rule:*** $\frac{d}{dx}\left[\frac{1}{g(x)}\right] = \frac{-g'(x)}{[g(x)]^2}$
Chain Rule: $\frac{d}{dx}(f\circ g)(x) = f'[g(x)]\cdot g'(x)$	***Integration-by-Parts:*** $\int u\,dv = uv - \int v\,du$

Trigonometric Functions		***Inverse Trigonometric Functions***	
Derivative	***Integral***	***Derivative***	***Integral***
$\frac{d}{dx}\sin x = \cos x$	$\int \sin x\,dx = -\cos x + c$	$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-u^2}}\,dx = \sin^{-1}\frac{u}{a} + c$
$\frac{d}{dx}\cos x = -\sin x$	$\int \cos x\,dx = \sin x + c$	$\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$	
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \tan x\,dx = \ln\lvert\sec x\rvert + c$ $\int \sec^2 x\,dx = \tan x + c$	$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$	$\int \frac{1}{a^2+u^2}\,dx = \frac{1}{a}\tan^{-1}\frac{u}{a} + c$
$\frac{d}{dx}\cot x = -\csc^2 x$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + c$ $\int \csc^2 x\,dx = -\cot x + c$	$\frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$	
$\frac{d}{dx}\sec x = \sec x\cdot\tan x$	$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + c$ $\int \sec x\cdot\tan x\,dx = \sec x + c$	$\frac{d}{dx}\sec^{-1}x = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\int \frac{1}{u\sqrt{u^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\frac{u}{a} + c$
$\frac{d}{dx}\csc x = -\csc x\cdot\cot x$	$\int \csc x\,dx = \ln\lvert\csc x - \cot x\rvert + c$ $\int \csc x\cdot\cot x\,dx = -\csc x + c$	$\frac{d}{dx}\csc^{-1}x = \frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$	

Identities:

$$\begin{cases} \sin^2 x + \cos^2 x = 1 & \sin 2x = 2\sin x\cos x & \cos^2 x = \frac{1+\cos 2x}{2} \\ 1+\cot^2 x = \csc^2 x & \cos 2x = \cos^2 x - \sin^2 x & \sin^2 x = \frac{1-\cos 2x}{2} \\ \tan^2 x + 1 = \sec^2 x & \cos(x+y) = \cos x\cos y - \sin x\sin y & \sin(x+y) = \sin x\cos y + \cos x\sin y \end{cases}$$

Exponential Functions		***Logarithmic Functions***	
Derivative	***Integral***	***Derivative***	***Integral***
$\frac{d}{dx}(e^x) = e^x$	$\int e^x\,dx = e^x + c$	$\frac{d}{dx}(\ln\lvert x\rvert) = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + c$
$\frac{d}{dx}(b^x) = (\ln b)b^x$	$\int b^x\,dx = \frac{b^x}{\ln b} + c$	$\frac{d}{dx}(\log_b\lvert x\rvert) = \frac{1}{(\ln b)x}$	

Definition of Log base b: $\log_b N = x \Leftrightarrow b^x = N$	***Change of Base Formula:*** $\log_b x = \frac{\ln x}{\ln b} = \frac{\log x}{\log b}$

Identities:

$$\begin{cases} \ln(e^x) = x & e^{\ln x} = x & \ln e = \log 10 = \log_b b = 1 \\ \log_b(b^x) = x & b^{\log_b x} = x & \ln 1 = \log 1 = \log_b 1 = 0 \end{cases}$$