$$\int \sin^4 x \cos x \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$$\int u^4 \, du$$

$$= \frac{u^5}{5} + C$$

$$= \frac{\sin^5 x}{5} + C$$

$$\int \sin^2 x \cos^3 x \, dx$$

$$\int \sin^2 x \left(1 - \sin^2 x\right) \cos x \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$$\int u^2 \left(1 - u^2\right) du$$

$$= \int u^2 - u^4 \, du$$

$$= \frac{u^3}{3} - \frac{u^5}{5} + C$$

$$= \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$$

$$\int \sin^2 x \cos^2 x \, dx$$

$$\int (\sin x \cos x)^2 \, dx$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos x = \frac{1}{2} \sin 2x$$

$$= \frac{1}{4} \int \sin^2 2x \, dx$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$= \frac{1}{4} \int \left( \frac{1}{2} - \frac{\cos 4x}{2} \right) dx$$

$$\sin^2 2x = \frac{1 - \cos 4x}{2}$$

$$= \frac{1}{8} \int (1 - \cos 4x) \, dx$$

$$= \frac{1}{8} \left[ x - \frac{\sin 4x}{4} \right] + C$$

$$= \frac{x}{8} - \frac{\sin 4x}{32} + C$$

$$\int \tan^3 \sec^2 x \, dx$$

$$u = \tan x$$

$$du = \sec^2 x \, dx$$

$$\int u^3 \, du$$

$$= \frac{u^4}{4} + C$$

$$= \frac{\tan^4 x}{4} + C$$

$$\int \tan^3 \sec^3 x \, dx$$

$$\int \tan^2 x \sec^2 x \cdot \tan x \sec x \, dx$$

$$u = \sec x$$

$$du = \sec x \tan x \, dx$$

$$\int \tan^2 x \, u^2 \, du$$

$$= \int (\sec^2 x - 1) u^2 \, du$$

$$= \int (u^2 - 1) u^2 \, du$$

$$= \int (u^4 - u^2) \, du$$

$$= \frac{u^5}{5} - \frac{u^3}{3} + C$$

$$= \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$$

$$\int \sqrt{\sec x} \tan x \, dx$$

$$u = \sec x$$

$$du = \sec x \tan x \, dx$$

$$\tan x \, dx = \frac{du}{\sec x} = \frac{du}{u}$$

$$\int \frac{\sqrt{u}}{u} \, du$$

$$= \int u^{-1/2} \, du$$

$= \frac{u^{1/2}}{1/2} + C$

$= 2u^{1/2} + C$

$= 2\sqrt{\sec x} + C$

## Complex Numbers

1. $\frac{1+4i}{3+i} \times \frac{3-i}{3-i}$

$= \frac{(1+4i)(3-i)}{3^2 - i^2}$

$= \frac{3 - i + 12i - 4i^2}{9 - (-1)}$

$= \frac{3 + 11i - 4(-1)}{10}$

$= \frac{7 + 11i}{10}$

$= \frac{7}{10} + \frac{11}{10} i$

2. $\sqrt{21 - 20i} = a + ib$

$21 - 20i = (a+ib)^2$

$21 - 20i = a^2 + 2abi - b^2$

$21 - 20i = a^2 - b^2 + 2abi$

$21 = a^2 - b^2$ $\qquad 2ab / = -20 /$

$ab = -10$

$a = 5, \; b = -2$

$\therefore \sqrt{21 - 20i} = 5 - 2i$

3. Multiplicative inverse of $x = \frac{1}{x}$

$$\frac{1}{4+3i}$$

$$= \frac{4-3i}{4^2-(3i)^2}$$

$$= \frac{4-3i}{16+9}$$

$$= \frac{4}{25} - \frac{3}{25}i$$

4. i) $x+3+3i = 5+yi$

$x+3=5$ $\qquad$ $3i = yi$

$x=2$ $\qquad$ $y=3$

ii) $x+2yi = ix+y+1$

$x = y+1$ $\qquad$ $2yi = ix$

$2y = x$

$2y = y+1$

$y = 1$

$x = 2(1) = 2$

$\therefore x=2, \; y=1$

iii) $(x+iy)(1+2i) = -1+8i$

$x+2xi+yi-2y = -1+8i$

$x-2y = -1$ $\qquad$ $i(2x+y) = 8i$

$x+1 = 2y$ $\qquad$ $2x+y = 8$

$2(2y-1)+y = 8$

$5y-2 = 8$

$5y = 10$ $\qquad$ $y = 2$

$x = 2y - 1$

$x = 3$

$\therefore x = 3,\ y = 2$

iv) $(x - iy)(3 - 4i) = 3 - 29i$

$3x - 4ix - 3iy - 4y = 3 - 29i$

$3x - 4y - 4ix - 3iy = 3 - 29i$

$3x - 4y = 3$ ⓘ $\quad -(4x - 3y) = +29$ ⓘ

$4x + 3y = 29$ ⓘ

3ⓘ − 4ⓘ

$12x + 9y = 87$

$12x - 16y = 12$

$25y = 75$

$y = 3$

$x = \frac{3 + 4y}{3}$

$x = \frac{1 + 12}{3} = 1 + 4 = 5$

$\therefore x = 5,\ y = 3$

5. i) $\bar{z} = -2 - 3i$

ii) $z = -2 - 3i + 1$

$= -1 - 3i$

$\bar{z} = -1 + 3i$

iii) $z = -6i + 15$

$\bar{z} = 15 + 6i$

iv)

6. i) $a = -2, b = 0$

$|z| = \sqrt{4} = 2$

ii) $a = 3, b = 2$

$|z| = \sqrt{9+4} = \sqrt{13}$

iii) $a = 0 \quad b = 5$

$|z| = \sqrt{25} = 5$

viii) $\dfrac{(3-5i)(1+i)}{4+2i}$

$= \dfrac{3+3i-5i+5}{4+2i}$

$= \dfrac{8-2i}{4+2i}$

$= \dfrac{(8-2i)(4-2i)}{16+4}$

$= 32-16i-8i-4$

$= \dfrac{28-24i}{20}$

$= \dfrac{28}{20} - \dfrac{24}{20}i$

$= \dfrac{7}{5} - \dfrac{6}{5}i$

$|z| = \sqrt{\dfrac{49}{25} + \dfrac{36}{25}} = \dfrac{\sqrt{85}}{5}$

7. i) $a = 2,\ b = 2\sqrt{3}$

$r = \sqrt{4 + 12} = \sqrt{16} = 4$

$\tan\theta = \frac{b}{a} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

$\theta = \tan^{-1}\sqrt{3}$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{3}\right)\right)$

$= \frac{\pi}{3}$

$z = 4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$= 4\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

ii) $a = 1,\ b = -1$

$r = \sqrt{1 + 1} = \sqrt{2}$

$\tan\theta = \frac{-1}{1}$

$\theta = \tan^{-1}(-1)$

$= \tan^{-1}\left(\tan\left(\frac{5\pi}{4}\right)\right)$

$= \frac{7\pi}{4}$

$z = \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$