Tutorial ③

① $\frac{dy}{dx} = ?$ $\frac{d^2y}{dx^2} = ?$ $P(2,-1)$ $x^2 - xy + y^2 = 7$

$$x^2 - xy + y^2 = 7$$

$$2x - y - xy' + 2yy' = 0$$

$$-xy' + 2yy' = y - 2x$$

$$y'(2y - x) = y - 2x$$

$$y' = \frac{y - 2x}{2y - x} \Rightarrow \boxed{\frac{dy}{dx} = \frac{y - 2x}{2y - x}}$$

$$\left.\frac{dy}{dx}\right|_{x=2,\, y=-1} = \frac{(-1) - 2(2)}{2(-1) - 2}$$

$$= \frac{-1-4}{-4}$$

$$= \frac{5}{4}$$

$$= 1.25$$

$$y'' = \frac{(y' - 2)(2y - x) - (2y' - 1)(y - 2x)}{(2y - x)^2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=2,\, y=-1} = \frac{(1.25 - 2)(2(-1) - 2) - (2(1.25) - 1)(-1-4)}{(2(-1) - 2)^2}$$

$$= \frac{21}{32}$$

2) $V = x^3$ ~ volume of a cube

$S = 6x^2$ ~ surface area of a cube

x ~ edge

V, S and x are functions of time

$\frac{dV}{dt} = 4 \ m^2/min$ $\qquad \frac{dS}{dt} = ?$ $\qquad S = 24 m^2$

$S = 24 = 6x^2 \Rightarrow x^2 = 4$

$x = 2$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

$4 = 3(2)^2 \frac{dx}{dt}$

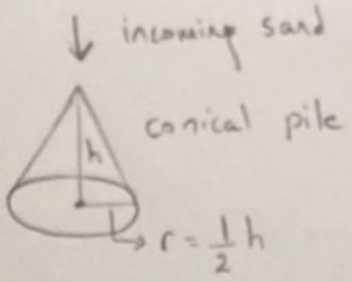
$4 = 12 \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{1}{3} \ m/min$

$\frac{dS}{dt} = 12x \cdot \frac{dx}{dt}$

$= 12(2)\left(\frac{1}{3}\right)$

$= 8 \ m^2/min$

3.



$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = 10 \text{ ft}^3/\text{min} \qquad \frac{dh}{dt} = ? \qquad h = 5 \text{ ft}$$

$$r = \frac{1}{2}h \Rightarrow V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$$

$$V = \frac{1}{12}\pi h^3 \Rightarrow \frac{dV}{dt} = 3\left(\frac{1}{12}\pi\right)h^2\frac{dh}{dt}$$

$$10 = \frac{\pi}{4}(5)^2\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{40}{25\pi}$$

$$= \frac{8}{5\pi} \text{ ft}/\text{min}$$

4. $f(x) = 4x^3 - 15x^2 - 18x + 10$

$f'(x) = 12x^2 - 30x - 18$

$12x^2 - 30x - 18 = 0 \Rightarrow 2x^2 - 5x - 3 = 0$

$2x^2 + 1x - 6x - 3 = 0$

$x(2x+1) - 3(2x+1) = 0$

$(x-3)(2x+1) = 0$

$x = 3$ or $x = -\frac{1}{2}$

x	$-\infty$		$-\frac{1}{2}$		3		∞
f'		$+$	0	$-$	0	$+$	
f		$\nearrow$		$\searrow$		$\nearrow$	

f is increasing on $(-\infty, -\frac{1}{2})$ and $(3, \infty)$

f is decreasing on $(-\frac{1}{2}, 3)$

5) $f'(x) = e^{(x-4)^3}(x^2+x+2)^3(x-2)^3(x-1)$

$f'(x)=0 \Rightarrow e^{(x-4)^3}(x^2+x+2)^3(x-2)^3(x-1)=0 \Rightarrow$

$(x-2)^3(x-1)=0$

$x=2$ or $x=1$

x	$-\infty$		1		2		∞
f'		$+$	0	$-$	0	$+$	
f		$\nearrow$		$\searrow$		$\nearrow$	

f is decreasing on $(1,2)$

6) $f(x) = x^4 - 2x^2 + 2$ $\qquad [-1, 2]$

$f'(x) = 4x^3 - 4x$

$f'(x) = 0 \Rightarrow 4x^3 - 4x = 0$

$4x(x^2-1) = 0 \Rightarrow x = 0$ or $x = \pm 1$

x	$f(x)$
-1	$f(-1) = 1 \rightarrow$ Absolute min. at $x=1$
0	$f(0) = 2$
1	$f(1) = 1 \rightarrow$ Absolute min at $x=1$
2	$f(2) = 10 \rightarrow$ Absolute max at $X=2$

Scanned with CamScanner

7) $f(x) = 3x^5 - 5x^4$

$f'(x) = 15x^4 - 20x^3$

$f'(x) = 0 \Rightarrow 15x^4 - 20x^3 = 0$

$5x^3(3x - 4) = 0$

$x = 0$ or $x = \frac{4}{3}$ C.Ns

x	$-\infty$		0		$\frac{4}{3}$		∞
f'		$+$	0	$-$	0	$+$	
f		$\nearrow$	R.max	$\searrow$	R.min	$\nearrow$	

R.max $= f(0) = 0$

R.min $= f\left(\frac{4}{3}\right) = \frac{-256}{81}$

8) $f(x) = 12x^{\frac{2}{3}} - 16x$

Domain: $(-\infty, \infty)$

$f'(x) = 12\left(\frac{2}{3}\right)x^{-\frac{1}{3}} - 16$

$f'(x) = \frac{8}{x^{1/3}} - 16 \Rightarrow f'(x) = \frac{8 - 16x^{\frac{1}{3}}}{x^{\frac{1}{3}}}$

$f'(x) = 0 \Rightarrow 8 - 16x^{\frac{1}{3}} = 0$

$x^{\frac{1}{3}} = \frac{8}{16} = \frac{1}{2} \Rightarrow x = \frac{1}{8}$ C.N

$f'(x) = \text{undefined} \Rightarrow x^{\frac{1}{3}} = 0 \Rightarrow x = 0$

x	$-\infty$		0		$\frac{1}{8}$		∞
f'		$-$		$+$	0	$-$	
f		$\searrow$		$\nearrow$		$\searrow$	

R.min $= f(0) = 0$

R.max $= f\left(\frac{1}{8}\right) = 1$

9) $f(x) = x^3 + ax^2 + bx + 1$

$f'(x) = 3x^2 + 2ax + b$

R.min at $x = -1 \Rightarrow f'(-1) = 0$

$3(-1)^2 + 2a(-1) + b = 0$

$\boxed{-2a + b = -3}$

R.max at $x = -3 \Rightarrow f'(-3) = 0$

$3(-3)^2 + 2a(-3) + b = 0$

$\boxed{-6a + b = -27}$

$-2a + b = -3$

$\ominus \; -6a + b = -27$

$4a = 24 \rightarrow a = 6$

$a = 6 \Rightarrow b = 9$

$$f'(x) = 3x^2 + 12x + 9$$

$$f''(x) = 6x + 12$$

$$f''(x) = 0 \Rightarrow 6x + 12 = 0$$

$$x = -2$$

x	$-\infty$	-2	∞
f''	$-$	0	$+$
f	$\cap$		$\cup$

f is concave down on $(-\infty, -2)$

f is concave up on $(-2, \infty)$