

TEST BANK
To Accompany

SALAS / HILLE / ETGEN

CALCULUS

One and Several
Variables

Eighth Edition

PREPARED BY
Deborah Betthauser Britt



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TO THE INSTRUCTOR

The Test Bank has been prepared for the eighth edition of Salas, Hille, and Etgen's *Calculus: One and Several Variables*. Each chapter is divided into sections that correspond to the sections in the textbook. In all, the Test Bank consists of 2,223 problems, together with their solutions that appear immediately after the end of each chapter. Numerous charts and illustrations have also been drawn where appropriate to strengthen the presentation. Far more items are offered within each chapter than any user of the Test Bank would ordinarily need. Due to differences in rounding, some of the answers requiring calculations by students may differ slightly from the ones given in the Test Bank.

The entire Test Bank has been incorporated into MICROTEST[®], a computerized test preparation system, by Delta Software. This software has been designed to retrieve the questions from the Test Bank and print your tests and answer keys using a PC running on Windows 3.1 or higher. The questions on your tests printed by the software will be exact duplicates of those shown here. The software includes a host of powerful features and flexibility for selecting questions and producing multiple versions of tests without your having to retype the items or draw the pictures. To obtain MICROTEST[®] software for this textbook, please contact your John Wiley sales representative.

I wish to thank Edwin and Shirley Hackleman of Delta Software for their composition and editorial assistance in preparing the hard copy manuscript and Donald Newell of Delta Software for developing and checking the computerized version of the Test Bank. We have endeavored as a team to produce test items that we trust instructors will find a useful supplement to *Calculus: One and Several Variables*.

DEBORAH BETTHAUSER BRITT

CHAPTER 1

Introduction

1.2 Notions and Formulas from Elementary Mathematics

1. Is the number $\sqrt{13^2 - 12^2}$ rational or irrational?
2. Is the number $5.121122111222 \dots$ rational or irrational?
3. Write $6.27272727 \dots$ in rational form p/q .
4. Find, if any, upper and lower bounds for the set $S = \{x : x^3 > 1\}$.
5. Find, if any, upper and lower bounds for the set $S = \left\{ \frac{2n-1}{n} : n = 1, 2, 3, \dots \right\}$.
6. Rewrite $27 - 8x^3$ in factored form.
7. Rewrite $x^4 - 18x^2 + 81$ in factored form.
8. Evaluate $\frac{4!}{6!}$.
9. Evaluate $\frac{7!}{4!3!}$.
10. What is the ratio of the surface area of a cube of side x to the surface area of a sphere of diameter x ?

1.3 Inequalities

11. Solve $x + 3 < 2x - 8$.
12. Solve $\frac{-2}{3}x \geq \frac{1}{6} - \frac{3}{4}x$.
13. Solve $8 - x^2 > 7x$.
14. Solve $x^2 - 5x + 5 \geq 1$.
15. Solve $3x^2 - 1 \geq \frac{1}{2}(3 + x^2)$.
16. Solve $\frac{2x+3}{4x-1} < 1$.
17. Solve $\frac{1}{x} < \frac{1}{x+1}$.
18. Solve $\frac{1}{2}(3x+2) < 2 - \frac{2}{3}(5-3x)$.

19. Solve $\frac{3}{x+2} < \frac{2}{x-1}$.
20. Solve $\frac{2x-3}{x+2} \geq 1$.
21. Solve $x(2x-1)(3x+2) < 0$.
22. Solve $\frac{x^2}{x^2-9} < 0$.
23. Solve $x^2(x-1)(x+2)^2 > 0$.
24. Solve $\frac{x+2}{x^2(x+3)} > 0$.
25. Solve $|x| > 2$.
26. Solve $|x+1| \geq \frac{1}{3}$.
27. Solve $|x-1| \leq \frac{1}{2}$.
28. Solve $0 < \left|x - \frac{1}{4}\right| < 1$.
29. Solve $|2x-1| < 5$.
30. Solve $|3x-5| \geq \frac{2}{3}$.
31. Find the inequality of the form $|x-c| < \delta$ whose solution is the open interval $(-1, 5)$.
32. Find the inequality of the form $|x-c| < \delta$ whose solution is the open interval $(-2, 3)$.
33. Determine all values of $A > 0$ for which the following statement is true. If $|2x-5| < 1$, then $|4x-10| < A$.
34. Determine all values of $A > 0$ for which the following statement is true. If $|2x-3| < A$ then $|6x-9| < 4$.

1.4 Coordinate Plane; Analytic Geometry

35. Find the distance between the points $P_0(2, -4)$ and $P_1(1, 5)$.
36. Find the midpoint of the line segment from $P_0(a, 2b)$ to $P_1(3a, 5b)$.
37. Find the slope of the line through $P_0(-4, 2)$ and $P_1(3, 5)$.
38. Find the slope of the line through $P_0(-2, -4)$ and $P_1(3, 5)$.
39. Find the slope and the y-intercept for the line $2x + y - 10 = 0$.
40. Find the slope and the y-intercept for the line $8x + 3y = 6$.

41. Write an equation for the line with the slope -3 and y -intercept -4 .
42. Write an equation for the horizontal line 4 units below the x -axis.
43. Write an equation for the vertical line 2 units to the left of the y -axis.
44. Find an equation for the line that passes through the point $P(2, -1)$ and is parallel to the line $3y + 5x - 6 = 0$.
45. Find an equation for the line that passes through the point $P(1, -1)$ and is perpendicular to the line $2x - 3y - 8 = 0$.
46. Find an equation for the line that passes through the point $P(2, 2)$ and is parallel to the line $2x + 3y = 18$.
47. Find an equation for the line that passes through the point $P(3, 5)$ and is perpendicular to the line $6x - 7y + 17 = 0$.
48. Determine the point(s) where the line $y = 2x$ intersects the circle $x^2 + y^2 = 4$.
49. Find the point where the lines l_1 and l_2 intersect. $l_1: x + y - 2 = 0$; $l_2: 3x + y = 0$
50. Find the area of the triangle with vertices $(-1, 1)$, $(4, -2)$, $(3, 6)$.
51. Find the area of the triangle with vertices $(-1, -1)$, $(-4, 3)$, $(3, 3)$.
52. Find an equation for the line tangent to the circle $x^2 + y^2 - 4x - 2y = 0$ at the point $P(4, 2)$.

1.5 Functions

53. If $f(x) = \frac{|x+2|}{x^2+4}$, calculate (a) $f(0)$, (b) $f(1)$, (c) $f(-2)$, (d) $f(-5/2)$.
54. If $f(x) = \frac{x^3}{x^2+2}$, calculate (a) $f(-x)$, (b) $f(1/x)$, (c) $f(a+b)$.
55. $f(x) = |1 - 2x|$ Find the number(s), if any, where f takes on the value 1.
56. $f(x) = 1 - \cos x$ Find the number(s), if any, where f takes on the value 1.
57. Find the exact value(s) of x in the interval $[0, 2\pi)$ which satisfy $\cos 2x = -\frac{\sqrt{3}}{2}$.
58. Find the domain and range for $f(x) = 2 - x - x^2$.
59. Find the domain and range for $f(x) = \sqrt{3x+4}$.
60. Find the domain and range for $h(x) = -\sqrt{4-x^2}$.
61. Find the domain and range for $f(x) = \frac{1}{\sqrt{x}-1}$.
62. Find the domain and range for $f(x) = \frac{1}{(x+3)^2}$.

63. Find the domain and range for $f(x) = \frac{1}{x^2 + 3}$.
64. Find the domain and range for $f(x) = \left| \cos x - \frac{1}{2} \right|$.
65. Sketch the graph of $f(x) = 3 - 4x$.
66. Sketch the graph of $f(x) = -\sqrt{6x - x^2}$.
67. Sketch the graph of $f(x) = x - \frac{4}{x}$.
68. Sketch the graph of $g(x) = 2 + \sin x$.
69. Sketch the graph of $g(x) = \begin{cases} 2 & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2x - 1 & \text{if } x > 1 \end{cases}$ and give its domain and range.
70. Sketch the graph of $f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 2x & \text{if } x \geq 1 \end{cases}$ and give its domain and range.
71. Is an ellipse the graph of a function?
72. Determine whether $f(x) = x^4 - x^2 + 1$ is odd, even, or neither.
73. Determine whether $f(x) = x^5 + x^3 - 3x$ is odd, even, or neither.
74. Determine whether $f(x) = \frac{x^3 - 2x^2 + 5x + 1}{x}$ is odd, even, or neither.
75. Determine whether $f(x) = \cos(x + \pi / 6)$ is odd, even, or neither.
76. Determine whether $f(x) = 3x - 2 \sin x$ is odd, even, or neither.
77. Determine whether $f(x) = \cos x + \sec x$ is odd, even, or neither.
78. A given rectangle is twice as long as it is wide. Express the area of the rectangle as a function of the (a) width, (b) length, (c) diagonal.

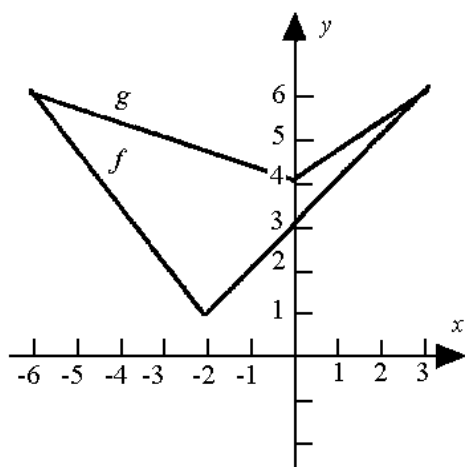
1.6 The Elementary Functions

79. Find all real numbers x for which $R(x) = \frac{3x^2 + 2x + 5}{x - x^2}$ is undefined.
80. Find all real numbers x for which $R(x) = \frac{x^3 - 4x^2 + 3x}{x^2 + x - 2}$ is zero.
81. Find the inclination of the line $x - \sqrt{3}y + 2\sqrt{3} = 0$.

82. Write an equation for the line with inclination 45° and y-intercept -2 .
83. Find the distance between the line $4x + 3y + 4 = 0$ and (a) the origin (b) the point $P(1, 3)$.
84. Find the distance between the line $2x - 5y - 10 = 0$ and (a) the origin (b) the point $P(-2, -1)$.
85. In the triangle with vertices $(0, 0)$, $(2, 6)$, $(7, 0)$, which vertex is farthest from the centroid?

1.7 Combinations of Functions

86. Given that $f(x) = \frac{x^2 - x - 6}{x}$ and $g(x) = x - 3$, find (a) $f + g$ (b) $f - g$ (c) $\frac{f}{g}$
87. Sketch the graphs of the following functions with f and g as shown in the figure.
- (a) $\frac{1}{2}f$ (b) $-g$ (c) $g - f$



88. $f(x) = \sqrt{x+5}$, $g(x) = \sqrt{x}$.
 (a) Form the composition of $f \circ g$. (b) Form the composition of $g \circ f$.
89. $f(x) = x^2 + 1$, $g(x) = \frac{2}{\sqrt{x}}$.
 (a) Form the composition of $f \circ g$. (b) Form the composition of $g \circ f$.
90. $f(x) = \frac{1}{x} + 1$, $g(x) = 3x^2 + 2$.
 (a) Form the composition of $f \circ g$. (b) Form the composition of $g \circ f$.
91. $f(x) = \sqrt{4+x^2}$, $g(x) = \frac{2}{x}$.
 (a) Form the composition of $f \circ g$. (b) Form the composition of $g \circ f$.
92. $f(x) = |x|$, $g(x) = x^3 + 1$.
 (a) Form the composition of $f \circ g$. (b) Form the composition of $g \circ f$.
93. Form the composition of $f \circ g \circ h$ if $f(x) = \frac{1}{4}x$, $g(x) = 2x - 1$, and $h(x) = 3x^2$.

94. Form the composition of $f \circ g \circ h$ if $f(x) = x^2$, $g(x) = 2x + 1$, and $h(x) = 2x^2$.
95. Form the composition of $f \circ g \circ h$ if $f(x) = \frac{2}{x}$, $g(x) = \frac{2}{3x+2}$, and $h(x) = x^2$.
96. Find f such that $f \circ g = F$ given that $g(x) = 2x^2$ and $F(x) = x + 2x^2 + 3$.
97. Find f such that $f \circ g = F$ given that $g(x) = \sqrt{x} + 1$ and $F(x) = x + 2\sqrt{x}$.
98. Find g such that $f \circ g = F$ given that $f(x) = x^2 - 1$ for all real x and $F(x) = 3x - 1$ for $x \geq 0$.
99. Find g such that $f \circ g = F$ given that $f(x) = x^2$ and $F(x) = (2x + 5)^2$.
100. Form $f \circ g$ and $g \circ f$ given that $f(x) = 4x + 1$ and $g(x) = 4x^2$.
101. Form $f \circ g$ and $g \circ f$ given that $f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 0 \\ 2x - 1 & \text{if } x \geq 0 \end{cases}$ and that $g(x) = \begin{cases} 2x & \text{if } x < 1 \\ x^2 & \text{if } x \geq 1 \end{cases}$.
102. Decide whether $f(x) = 4x + 3$ and $g(x) = \frac{1}{4}x - 3$ are inverses of each other.
103. Decide whether $f(x) = (x - 1)^5 + 1$ and $g(x) = (x - 1)^{1/5} + 1$ are inverses of each other.

1.8 **A Note on Mathematical Proof; Mathematical Induction**

104. Show that $3n \leq 3^n$ for all positive integers n .
105. Show that $n(n + 1)(n + 2)(n + 3)$ is divisible by 8 for all positive integers n .
106. Show that $1 + 5 + 9 + \dots + (4n - 3) = 2n^2 - n$ for all positive integers n .

Answers to Chapter 1 Questions

1. rational
2. irrational
3. $69/11$
4. lower bound 1, no upper bound
5. lower bound 1, upper bound 2
6. $(3 - 2x)(9 + 6x + 4x^2)$
7. $(x - 3)^2(x + 3)^2$
8. $1/30$
9. 35
10. $6/\pi$
11. $(11, \infty)$
12. $[2, \infty)$
13. $(-8, 1)$
14. $(-\infty, 1] \cup [4, \infty)$
15. $(-\infty, 1] \cup [1, \infty)$
16. $(-\infty, 1/4] \cup (2, \infty)$
17. $(-1, 0)$
18. $(14/3, \infty)$
19. $(-\infty, -2) \cup (1, 7)$
20. $(-\infty, -2) \cup [5, \infty)$
21. $(-\infty, -2/3] \cup (0, 1/2)$
22. $(-3, 0) \cup (0, 3)$
23. $(1, \infty)$
24. $(-\infty, -3) \cup (-2, 0) \cup (0, \infty)$
25. $(-\infty, -2) \cup (2, \infty)$
26. $(-\infty, -4/3] \cup [-2/3, \infty)$
27. $[1/2, 3/2]$
28. $(-3/4, 1/4) \cup (1/4, 5/4)$
29. $(-1, 4)$
30. $(-\infty, 13/9) \cup (17/9, \infty)$
31. $|x - 2| < 3$
32. $|x - 1/2| < 5/2$
33. $A \geq 2$
34. $0 \leq A \leq 4/3$
35. $\sqrt{82}$
36. $\left(2a, \frac{7}{2}b\right)$
37. $m = 3/7$
38. $m = 9/5$
39. $m = -2$; y-intercept: 10
40. $m = -8/3$; y-intercept: 2
41. $y = -3x - 4$
42. $y = -4$
43. $x = -2$
44. $y = \frac{-5}{3}x + \frac{7}{3}$
45. $y = \frac{-3}{2}x + \frac{1}{2}$
46. $y = \frac{-2}{3}x + \frac{10}{3}$
47. $y = \frac{-7}{6}x + \frac{17}{2}$
48. $\left(\frac{2\sqrt{5}}{5}, \frac{4\sqrt{5}}{5}\right), \left(\frac{-2\sqrt{5}}{5}, \frac{-4\sqrt{5}}{5}\right)$
49. The point of intersection is $P(-1, 3)$.
50. $37/2$
51. 14
52. $y = -2x + 10$

53. (a) $1/2$ (b) $3/5$ (c) 0 (d) $2/41$

54. (a) $\frac{-x^3}{x^2+2}$ (b) $\frac{1}{x+2x^3}$ (c) $\frac{(a+b)^3}{(a+b)^2+2}$

55. $x = 0, 1$

56. $(2n+1)\frac{\pi}{2}, n = \text{integer}$

57. $5\pi/12, 7\pi/12, 17\pi/12, 19\pi/12$

58. domain: $(f) = (-\infty, \infty)$
range: $(f) = (-\infty, \infty)$

59. domain: $(f) = [-4/3, \infty)$
range: $(f) = [0, \infty)$

60. domain: $(f) = [-2, 2]$
range: $(f) = [-2, 0]$

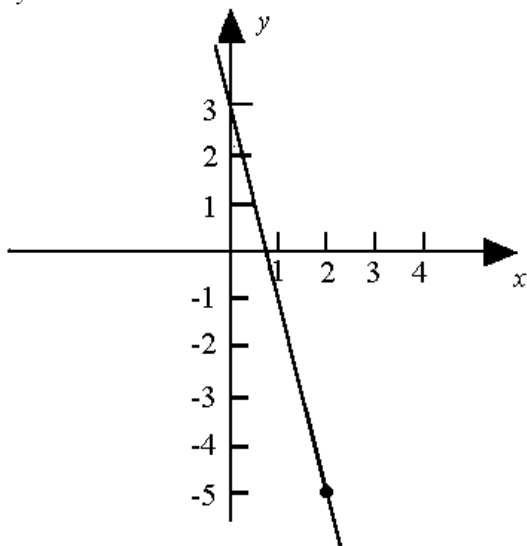
61. domain: $(f) = [0, 1) \cup (1, \infty)$
range: $(f) = [-1, 0) \cup (0, \infty)$

62. domain: $(f) = (-\infty, -3) \cup (-3, \infty)$
range: $(f) = (0, \infty)$

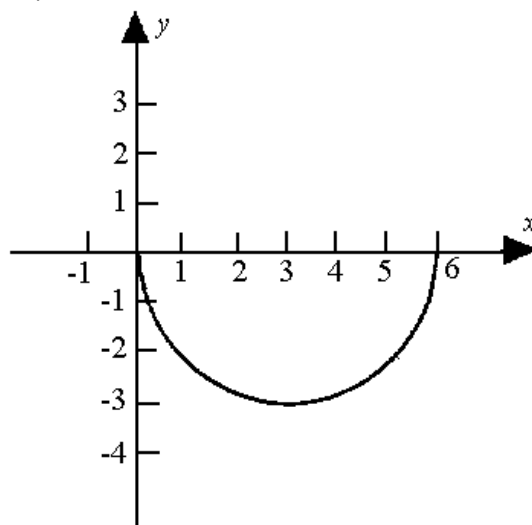
63. domain: $(f) = (-\infty, \infty)$
range: $(f) = (0, 1/3]$

64. domain: $(f) = (-\infty, \infty)$
range: $(f) = [0, 3/2]$

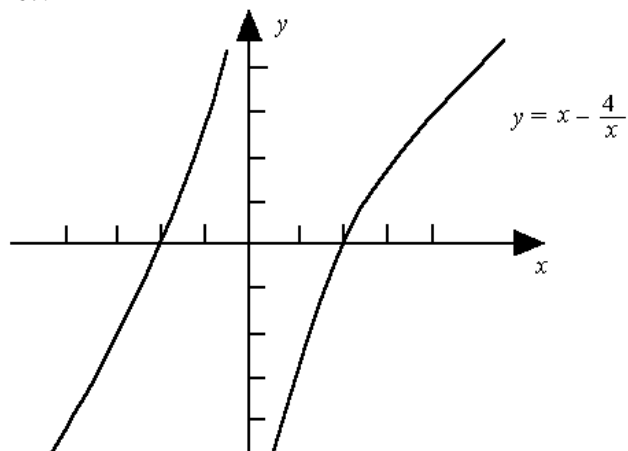
65. $y = 3 - 4x$



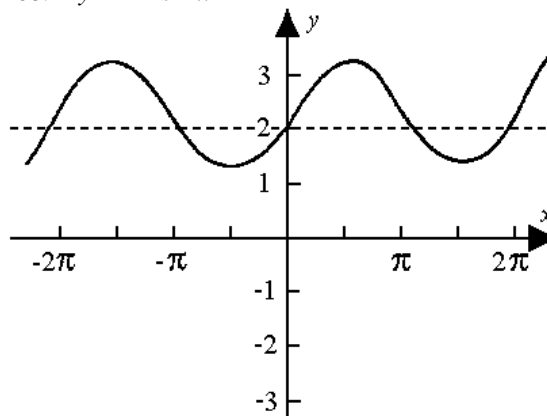
66. $y = -\sqrt{6x - x^2}$



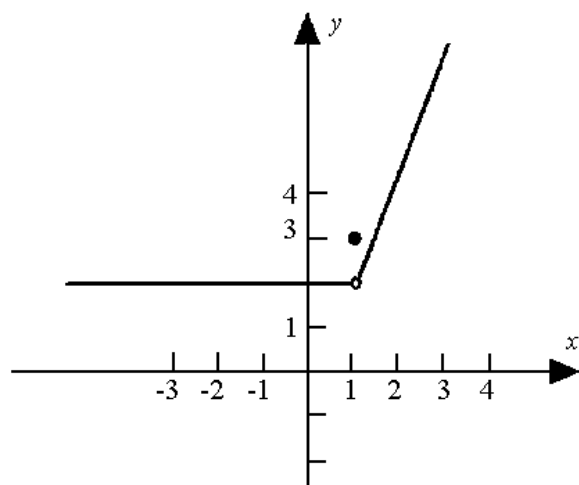
67.



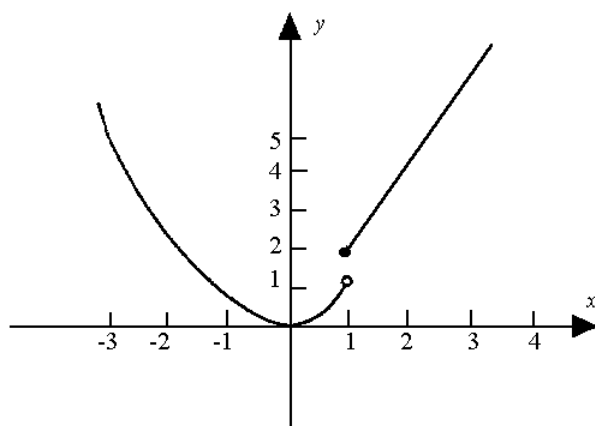
68. $y = 2 + \sin x$



69. domain: $(f) = (-\infty, +\infty)$
range: $(f) = [2, +\infty)$



70. domain: $(f) = (-\infty, +\infty)$
range: $(f) = [0, +\infty)$



71. no

72. even

73. odd

74. neither

75. neither

76. odd

77. even

78. (a) $A = 2w^2$ (b) $A = \frac{l^2}{2}$ (c) $A = \frac{2}{5}d^2$

79. 0, 1

80. 0, 3

81. $\theta = \pi/6$

82. $x - y - 2 = 0$

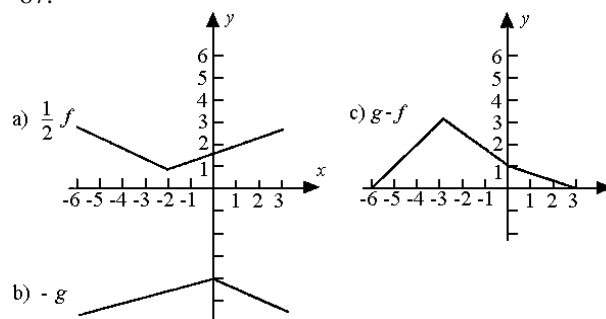
83. (a) $4/5$ (b) $17/5$

84. (a) $\frac{10}{\sqrt{29}}$ (b) $\frac{9}{\sqrt{29}}$

85. $(7, 0)$

86. (a) $\frac{2x^2 - 4x - 6}{x}$ (c) $\frac{x + 26}{x}, x \neq 3$
(b) $\frac{2x - 6}{x}$

87.



88. (a) $\sqrt{\sqrt{x} + 5}$ (b) $\sqrt[4]{x + 5}$

89. (a) $\frac{4}{x} + 1, x > 0$ (b) $\frac{2}{\sqrt{x^2 + 1}}$

90. (a) $\frac{1}{3x^2 + 2} + 1$ (b) $3\left(\frac{1}{x} + 1\right)^2 + 2$

91. (a) $\sqrt{4 + \frac{4}{x^2}}$ (b) $\frac{2}{\sqrt{4 + x^2}}$

92. (a) $|x^3 + 1|$ (b) $|x|^3 + 1$

93. $\frac{1}{4}(6x^2 - 1)$

94. $(4x^2 + 1)^2$

95. $3x^2 + 2$

96. $f(x) = x + 3$

97. $f(x) = x^2 - 1$

98. $g(x) = \sqrt{3x}$

99. $g(x) = 2x + 5$

100. $(f \circ g)(x) = 16x^2 + 1$

$$(g \circ f)(x) = 4(4x + 1)^2$$

$$101. \quad (f \circ g)(x) = \begin{cases} \frac{1}{2x} & \text{if } x < 0 \\ 4x - 1 & \text{if } 0 \leq x < 1 \\ 2x^2 - 1 & \text{if } x \geq 1 \end{cases}$$
$$(g \circ f)(x) = \begin{cases} 2/x & \text{if } x < 0 \\ 2(2x - 1) & \text{if } 0 \leq x < 1 \\ (2x - 1)^2 & \text{if } x \geq 1 \end{cases}$$

102. not inverses

103. inverses

104. True for $n = 1$: $3 \leq 3$. Assume true for n . Then $3(n + 1) = 3n + 3 \leq 3^n + 3 \leq 3^n + 3^n = 2(3^n) < 3(3^n) = 3^{n+1}$, so the inequality is true for $n + 1$. Therefore, by induction, it is true for $n \geq 1$.

105. True for $n = 1$: $1 \cdot 2 \cdot 3 \cdot 4 = 3 \cdot 8$. Assume true for n . Then $(n + 1)(n + 2)(n + 3)(n + 4) = n(n + 1)(n + 2)(n + 3) + 4(n + 1)(n + 2)(n + 3)$. The first term is divisible by 8 by the induction hypothesis, and the second term is divisible by 8 since at least one of $(n + 1)$, $(n + 2)$, $(n + 3)$ is even. Hence the result is true for $n + 1$. Therefore, by induction, it is true for all $n \geq 1$.

106. True for $n = 1$: $1 = 2(1)^2 - 1$. Assume true for n . Then $1 + 5 + 9 + \dots + [4(n + 1) - 3] = 1 + 5 + 9 + \dots + (4n - 3) + [4(n + 1) - 3] = 2n^2 - n + (4n + 1) = 2(n + 1)^2 - (n + 1)$, so the result is true for $n + 1$. Therefore, by induction, it is true for all $n \geq 1$.

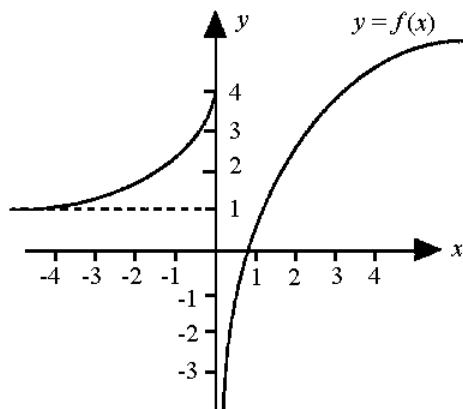
CHAPTER 2

Limits and Continuity

2.1 The Idea of Limit

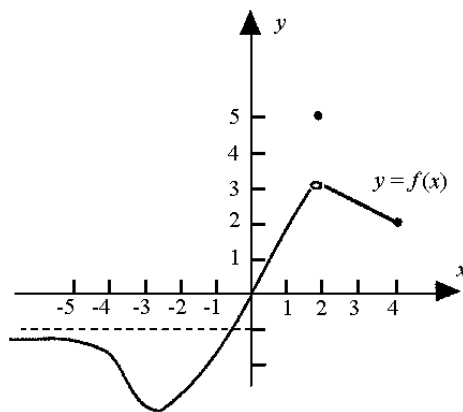
1. For the function f graphed below, $c = 0$. Use the graph of f to find

(a) $\lim_{x \rightarrow c^-} f(x)$ (b) $\lim_{x \rightarrow c^+} f(x)$ (c) $\lim_{x \rightarrow c} f(x)$ (d) $f(c)$



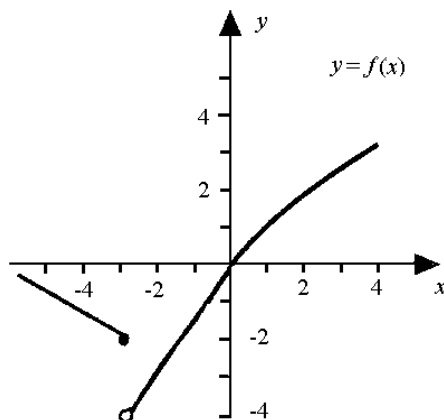
2. For the function f graphed below, $c = 2$. Use the graph of f to find

(a) $\lim_{x \rightarrow c^-} f(x)$ (b) $\lim_{x \rightarrow c^+} f(x)$ (c) $\lim_{x \rightarrow c} f(x)$ (d) $f(c)$



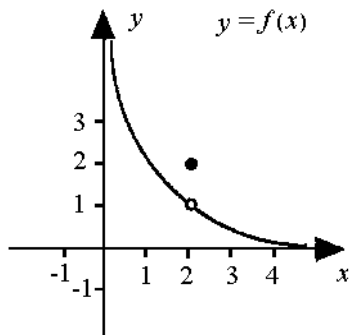
3. For the function f graphed below, $c = -3$. Use the graph of f to find

(a) $\lim_{x \rightarrow c^-} f(x)$ (b) $\lim_{x \rightarrow c^+} f(x)$ (c) $\lim_{x \rightarrow c} f(x)$ (d) $f(c)$



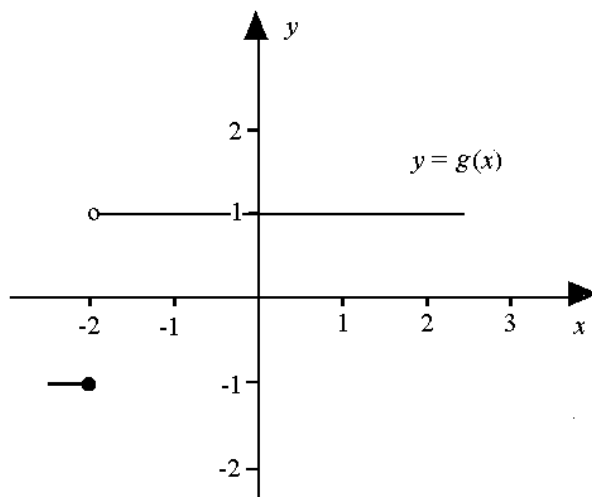
4. For the function f graphed below, $c = 2$. Use the graph of f to find

(a) $\lim_{x \rightarrow c^-} f(x)$ (b) $\lim_{x \rightarrow c^+} f(x)$ (c) $\lim_{x \rightarrow c} f(x)$ (d) $f(c)$



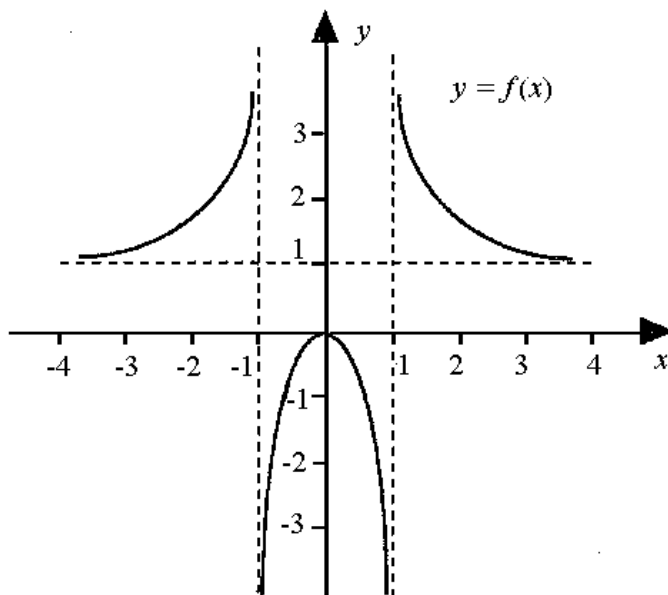
5. For the function g graphed below, $c = -2$. Use the graph of g to find

(a) $\lim_{x \rightarrow c^-} f(x)$ (b) $\lim_{x \rightarrow c^+} f(x)$ (c) $\lim_{x \rightarrow c} f(x)$ (d) $f(c)$



6. For the function f graphed below, $c = -1$. Use the graph of f to find

(a) $\lim_{x \rightarrow c^-} f(x)$ (b) $\lim_{x \rightarrow c^+} f(x)$ (c) $\lim_{x \rightarrow c} f(x)$ (d) $f(c)$



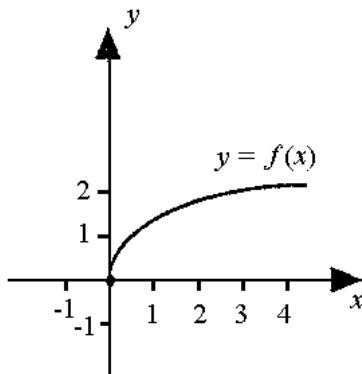
7. For the function f graphed below, $c = 0$. Use the graph of f to find

(a) $\lim_{x \rightarrow c^-} f(x)$

(b) $\lim_{x \rightarrow c^+} f(x)$

(c) $\lim_{x \rightarrow c} f(x)$

(d) $f(c)$



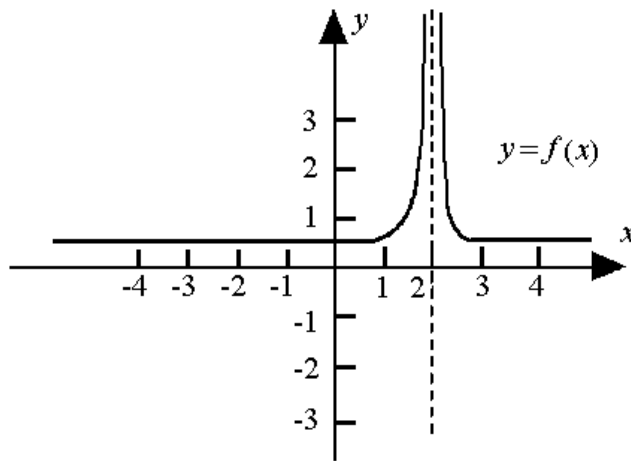
8. For the function f graphed below, $c = 2$. Use the graph of f to find

(a) $\lim_{x \rightarrow c^-} f(x)$

(b) $\lim_{x \rightarrow c^+} f(x)$

(c) $\lim_{x \rightarrow c} f(x)$

(d) $f(c)$



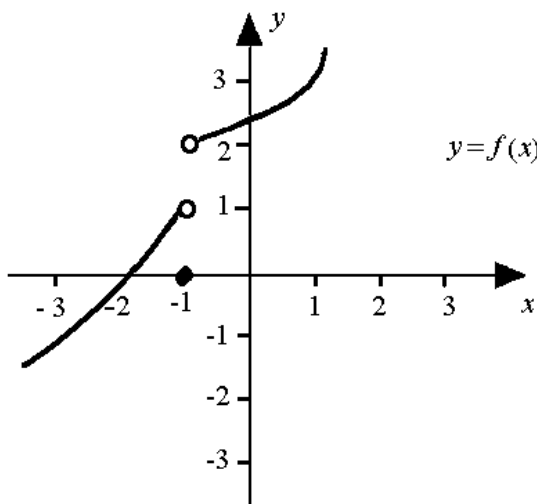
9. For the function f graphed below, $c = -1$. Use the graph of f to find

(a) $\lim_{x \rightarrow c^-} f(x)$

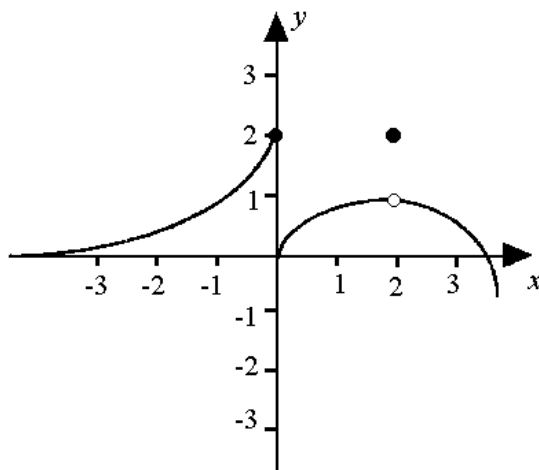
(b) $\lim_{x \rightarrow c^+} f(x)$

(c) $\lim_{x \rightarrow c} f(x)$

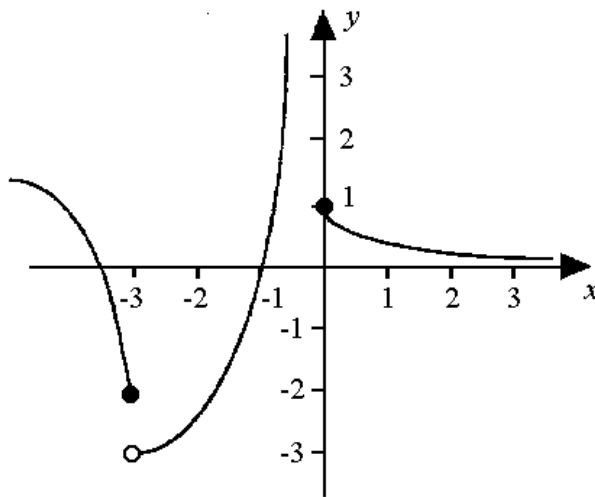
(d) $f(c)$



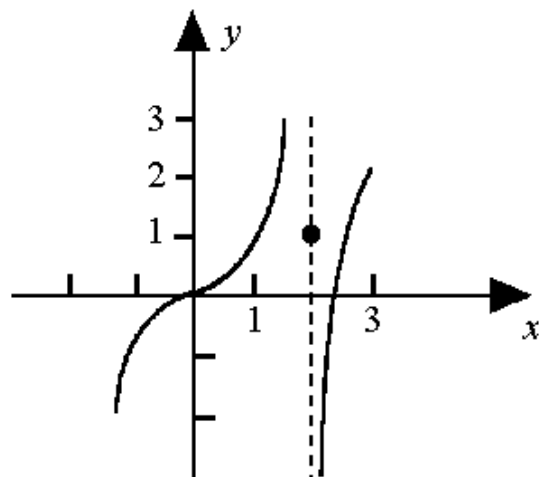
10. Consider the function f graphed below. State the values of c for which $\lim_{x \rightarrow c} f(x)$ does not exist.



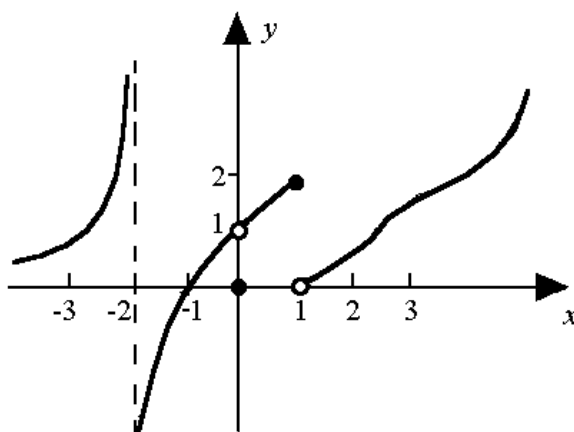
11. Consider the function f graphed below. State the values of c for which $\lim_{x \rightarrow c} f(x)$ does not exist.



12. Consider the function f graphed below. State the values of c for which $\lim_{x \rightarrow c} f(x)$ does not exist.



13. Consider the function f graphed below. State the values of c for which $\lim_{x \rightarrow c} f(x)$ does not exist.



14. Evaluate $\lim_{x \rightarrow 2} (2x - 5)$, if it exists.
15. Evaluate $\lim_{x \rightarrow 0} p^2$, if it exists.
16. Evaluate $\lim_{x \rightarrow -3} x^3$, if it exists.
17. Evaluate $\lim_{x \rightarrow -2} \frac{5}{x - 2}$, if it exists.
18. Evaluate $\lim_{x \rightarrow -2} (x^3 + 6x^2 - 16)$, if it exists.
19. Evaluate $\lim_{x \rightarrow 4} \frac{x^2 + 9}{x^2 - 1}$, if it exists.
20. Evaluate $\lim_{x \rightarrow 2} \frac{3x - 6}{2x - 4}$, if it exists.
21. Evaluate $\lim_{x \rightarrow 0} \frac{7x - 5x^2}{x}$, if it exists.
22. Evaluate $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$, if it exists.
23. Evaluate $\lim_{x \rightarrow -1} \frac{x^2 - 3}{x^2 + 1}$, if it exists.
24. Evaluate $\lim_{x \rightarrow a} \frac{x^2 - a^2}{x - a}$, if it exists.
25. Evaluate $\lim_{x \rightarrow 4} \frac{x^2 - 16}{x^2 + x - 20}$, if it exists.

26. Evaluate $\lim_{x \rightarrow 0} \frac{x^2 + 2x}{x^2 - 2x^2}$, if it exists.
27. Evaluate $\lim_{x \rightarrow 2} f(x)$, if it exists. $f(x) = \begin{cases} 3, & x \neq 2 \\ 2, & x = 2 \end{cases}$
28. Evaluate $\lim_{x \rightarrow -1} f(x)$, if it exists. $f(x) = \begin{cases} 2x - 1, & x < -1 \\ 3x, & x \geq -1 \end{cases}$
29. Evaluate $\lim_{x \rightarrow 1} f(x)$, if it exists. $f(x) = \begin{cases} \frac{1}{x+2}, & x < 1 \\ 1 - 2x, & x > 1 \end{cases}$
30. Evaluate $\lim_{x \rightarrow 1} \frac{\sqrt{x^2 + 4} - \sqrt{5}}{x - 1}$, if it exists.

2.2 Definition of Limit

31. Evaluate $\lim_{x \rightarrow 1} \frac{2x}{3x+1}$, if it exists.
32. Evaluate $\lim_{x \rightarrow 0} \frac{x^3(4x+1)}{5x^2}$, if it exists.
33. Evaluate $\lim_{x \rightarrow 2} \frac{\sqrt{9x}}{\sqrt{x+3}}$, if it exists.
34. Evaluate $\lim_{x \rightarrow 1} \frac{1 - x^2}{x^2 + 5x - 6}$, if it exists.
35. Evaluate $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - 4x + 3}$, if it exists.
36. Evaluate $\lim_{x \rightarrow 1} \frac{x^3 - 3x^2 + 2x}{x - 2}$, if it exists.
37. Evaluate $\lim_{h \rightarrow 1} \frac{|h - 2| - 2}{h}$, if it exists.
38. Evaluate $\lim_{x \rightarrow 2^-} \frac{|x - 2|}{x - 2}$, if it exists.
39. Evaluate $\lim_{x \rightarrow 1^+} \frac{x - 1}{|x - 1|}$, if it exists.
40. Evaluate $\lim_{x \rightarrow 3^-} f(x)$, if it exists, if $f(x) = \begin{cases} \frac{|x-3|}{x-3}, & x < 3 \\ x, & x > 3 \end{cases}$

41. Evaluate the right hand limit at $x = 1$, if it exists, for $f(x) = \begin{cases} 1+x, & x < 1 \\ 6, & x = 1 \\ 1-x, & x > 1 \end{cases}$
42. Evaluate the right hand limit at $x = 0$, if it exists, for $f(x) = \begin{cases} x+1, & x < 0 \\ x^3-1, & x \geq 0 \end{cases}$
43. Evaluate $\lim_{x \rightarrow 1} f(x)$, if it exists, if $f(x) = \begin{cases} 3x^2, & x < 1 \\ 5, & x = 1 \\ 2x^2-1, & x > 1 \end{cases}$
44. Evaluate the largest δ that “works” for a given arbitrary ϵ . $\lim_{x \rightarrow 1} 5x = 5$
45. Evaluate the largest δ that “works” for a given arbitrary ϵ . $\lim_{x \rightarrow \frac{10}{3}} \frac{3}{5}x = 2$
46. Give an ϵ, σ proof for $\lim_{x \rightarrow 2} (3x - 2) = 4$.
47. Give an ϵ, σ proof for $\lim_{x \rightarrow 1} (5x - 2) = 3$.
48. Give an ϵ, σ proof for $\lim_{x \rightarrow 5} |x - 3| = 2$.
49. Give the four equivalent limit statements displayed in (2.2.5), taking $f(x) = \frac{4}{2x+1}, c = 2$.
50. Give an ϵ, σ proof for $\lim_{x \rightarrow 3} x^2 = 9$.
51. Evaluate $\lim_{x \rightarrow 3} f(x)$, if it exists, if $f(x) = \begin{cases} x^2-1, & x < 3 \\ (x-1)^3, & x > 3 \end{cases}$

2.3 Some Limit Theorems

52. Given that $\lim_{x \rightarrow c} f(x) = 0, \lim_{x \rightarrow c} g(x) = 3, \lim_{x \rightarrow c} h(x) = -4$, evaluate the limits that exist.
- (a) $\lim_{x \rightarrow c} [f(x) - g(x)]$ (e) $\lim_{x \rightarrow c} \frac{g(x)}{f(x)}$
- (b) $\lim_{x \rightarrow c} [h(x)]^2$ (f) $\lim_{x \rightarrow c} \frac{1}{g(x) - h(x)}$
- (c) $\lim_{x \rightarrow c} \frac{g(x)}{h(x)}$ (g) $\lim_{x \rightarrow c} [3f(x) - 2g(x) - h(x)]$
- (d) $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$
53. Evaluate $\lim_{x \rightarrow 1} 5$, if it exists.

54. Evaluate $\lim_{x \rightarrow 2} (2 - 3x)^2$, if it exists.
55. Evaluate $\lim_{x \rightarrow 2} (2x^3 - 3x^2 + 2)$, if it exists.
56. Evaluate $\lim_{x \rightarrow -1} 2|2x - 1|$, if it exists.
57. Evaluate $\lim_{x \rightarrow 1} \frac{2x}{3x - 1}$, if it exists.
58. Evaluate $\lim_{x \rightarrow 0} 2x - \frac{5}{x}$, if it exists.
59. Evaluate $\lim_{h \rightarrow 2} \frac{h^3 - 4h}{h^3 - 2h^2}$, if it exists.
60. Evaluate $\lim_{x \rightarrow 0} x \left(2x + \frac{3}{x} \right)$, if it exists.
61. Evaluate $\lim_{x \rightarrow 2} \frac{x - 2}{x^3 - 8}$, if it exists.
62. Evaluate $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$, if it exists.
63. Evaluate $\lim_{x \rightarrow 1} \frac{2x - 3x^2}{(2x - 1)(x^2 - 4)}$, if it exists.
64. Evaluate $\lim_{x \rightarrow -5} \frac{(x^2 + 2x - 15)^2}{x + 5}$, if it exists.
65. Evaluate $\lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{(x + 5)^2}$, if it exists.
66. Evaluate $\lim_{x \rightarrow -5} \frac{x^2 + 2x - 15}{(x + 5)^2}$, if it exists.
67. Evaluate $\lim_{x \rightarrow -3} \frac{(x + 5)^2}{x^2 + 2x - 15}$, if it exists.
68. Evaluate $\lim_{x \rightarrow a} \frac{\frac{1}{x} - \frac{1}{a}}{x - a}$, if it exists.
69. Evaluate $\lim_{h \rightarrow 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h}$, if it exists.

70. Evaluate $\lim_{x \rightarrow -a} \frac{x^3 + a^3}{x + a}$, if it exists.
71. Evaluate $\lim_{x \rightarrow 2} \frac{1 - \frac{4}{x^2}}{1 - \frac{2}{x}}$, if it exists.
72. Evaluate $\lim_{x \rightarrow 0} \frac{x + \frac{2}{x}}{x - \frac{3}{x}}$, if it exists.
73. Evaluate $\lim_{x \rightarrow 3} \left(\frac{2x}{x-3} + \frac{5}{x-3} \right)$, if it exists.
74. Evaluate $\lim_{x \rightarrow 3} \left(\frac{2x}{x-3} + \frac{-6}{x-3} \right)$, if it exists.
75. Evaluate the following limits that exist.
- (a) $\lim_{x \rightarrow 2} \left(\frac{3}{x} - \frac{1}{2} \right)$
- (c) $\lim_{x \rightarrow 2} \left[\left(\frac{3}{x} - \frac{1}{2} \right) (x-2) \right]$
- (b) $\lim_{x \rightarrow 2} \left[\left(\frac{3}{x} - \frac{1}{2} \right) \left(\frac{1}{x-2} \right) \right]$
- (d) $\lim_{x \rightarrow 2} \left[\left(\frac{3}{x} - \frac{1}{2} \right)^2 \left(\frac{1}{x-6} \right) \right]$
76. Given that $f(x) = x^2 - 3x$, evaluate the limits that exist.
- (a) $\lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x - 3}$
- (d) $\lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1}$
- (b) $\lim_{x \rightarrow 5} \frac{f(x) - f(5)}{x - 5}$
- (e) $\lim_{x \rightarrow 2} \frac{f(x) - f(1)}{x - 2}$
- (c) $\lim_{x \rightarrow -3} \frac{f(x) - f(5)}{x + 3}$
- (f) $\lim_{x \rightarrow -2} \frac{f(x) - f(-2)}{x + 2}$

2.4 Continuity

77. Determine whether or not $f(x) = 2x^2 - 3x - 5$ is continuous at $x = 1$. If not, determine whether the discontinuity is a removable discontinuity, a jump discontinuity, or neither.
78. Determine whether or not $f(x) = \sqrt{(x-2)^2 + 2}$ is continuous at $x = 2$. If not, determine whether the discontinuity is a removable discontinuity, a jump discontinuity, or neither.
79. Determine whether or not $f(x) = |5 - 2x^2|$ is continuous at $x = 3$. If not, determine whether the discontinuity is removable discontinuity, a jump discontinuity, or neither.
80. Determine whether or not $f(x) = \frac{9x^2 - 4}{3x - 2}$ is continuous at $x = 2/3$. If not, determine whether the discontinuity is a removable discontinuity, a jump discontinuity, or neither.

81. Determine whether or not $f(x) = \begin{cases} -3, & x < -1 \\ 1, & x = -1 \\ 2, & x > -1 \end{cases}$ is continuous at $x = -1$. If not, determine whether the discontinuity is a removable discontinuity, a jump discontinuity, or neither.

82. Determine whether or not $f(x) = \begin{cases} x^2, & x < 1 \\ 3, & x = 1 \\ 2x+1, & x > 1 \end{cases}$ is continuous at $x = 1$. If not, determine whether the discontinuity is a removable discontinuity, a jump discontinuity, or neither.

83. Determine whether or not $f(x) = \begin{cases} \frac{1}{2}x^3, & x < 2 \\ 1, & x = 2 \\ 2x, & x > 2 \end{cases}$ is continuous at $x = 2$. If not, determine whether the discontinuity is a removable discontinuity, a jump discontinuity, or neither.

84. Determine whether or not $f(x) = \begin{cases} \frac{1}{x-4}, & x \neq 4 \\ 1, & x = 4 \end{cases}$ is continuous at $x = 4$. If not, determine whether the discontinuity is a removable discontinuity, a jump discontinuity, or neither.

85. Determine whether or not $f(x) = \frac{x-1}{x(x+1)}$ is continuous at $x = 0$. If not, determine whether the discontinuity is a removable discontinuity, a jump discontinuity, or neither.

86. Determine whether or not $f(x) = \frac{1}{(x-1)^3}$ is continuous at $x = 1$. If not, determine whether the discontinuity is a removable discontinuity, a jump discontinuity, or neither.

87. Sketch the graph of $f(x) = \frac{x^2-9}{x+3}$ and classify the discontinuities, if any.

88. Sketch the graph of $f(x) = \frac{x+2}{x^2-4}$ and classify the discontinuities, if any.

89. Sketch the graph of $f(x) = |x-3|$ and classify the discontinuities, if any.

90. Sketch the graph of $f(x) = \begin{cases} |x+1|, & x \leq -2 \\ x+1, & -2 < x < 1 \\ 5, & x \geq 1 \end{cases}$ and classify the discontinuities, if any.

91. Sketch the graph of $f(x) = \begin{cases} 2x+1, & x < 1 \\ 1, & x = 1 \\ 2x-1, & x > 1 \end{cases}$ and classify the discontinuities, if any.

92. Sketch the graph of $f(x) = \begin{cases} x^2, & x < 1 \\ 0, & x = 1 \\ 2x, & x > 1 \end{cases}$ and classify the discontinuities, if any.
93. Define $f(x) = \frac{x^3 + 1}{x + 1}$ at $x = -1$ so that it becomes continuous at $x = -1$.
94. Define $f(x) = \frac{x^2 - 9}{x + 3}$ at $x = -3$ so that it becomes continuous at $x = -3$.
95. Define $f(x) = \frac{x^2 + x - 6}{x - 2}$ at $x = 2$ so that it becomes continuous at 2.
96. Let $f(x) = \begin{cases} \frac{x^2 - x - 2}{x + 1}, & x \geq -1 \\ A, & x < -1 \end{cases}$. Find A given that f is continuous at -1 .
97. Prove that if $f(x)$ has a removable discontinuity at c , then $\lim_{x \rightarrow c} (x - c) f(x) = 0$.

2.5 The Pinching Theorem; Trigonometric Limits

98. Evaluate $\lim_{x \rightarrow 0} \frac{\sin 7x}{x}$, if it exists.
99. Evaluate $\lim_{x \rightarrow 0} \frac{x\sqrt{2}}{\sin \frac{x}{2}}$, if it exists.
100. Evaluate $\lim_{x \rightarrow 0} \frac{4x^2}{1 - \cos 3x}$, if it exists.
101. Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos 3x^2}{5x^2}$, if it exists.
102. Evaluate $\lim_{q \rightarrow 0} \frac{\tan q}{q}$, if it exists.
103. Evaluate $\lim_{q \rightarrow 0} \frac{\sin 2q}{\tan q}$, if it exists.
104. Evaluate $\lim_{a \rightarrow 0} \frac{\sin a - \tan a}{\sin^3 a}$, if it exists.
105. Evaluate $\lim_{q \rightarrow 0} q \cot 4q$, if it exists.
106. Evaluate $\lim_{q \rightarrow 0} \frac{\sin \sqrt{2q}}{\sqrt{q}}$, if it exists.

107. Evaluate $\lim_{q \rightarrow 0} \frac{q^2}{\sin 3q^2}$, if it exists.
108. Evaluate $\lim_{q \rightarrow 0} \frac{3}{q \csc q}$, if it exists.
109. Evaluate $\lim_{q \rightarrow 0} \frac{\sin 3q}{\sin 2q}$, if it exists.
110. Evaluate $\lim_{a \rightarrow 0} \frac{a}{\cos a}$, if it exists.
111. Evaluate $\lim_{t \rightarrow 0} \frac{t^2}{1 - \cos^2 t}$, if it exists.
112. Evaluate $\lim_{q \rightarrow 0} \frac{3q}{\cos 2q}$, if it exists.
113. Evaluate $\lim_{q \rightarrow 0} \frac{\sin^2 q}{\tan q}$, if it exists.
114. For $f(x) = \sin x$ and $a = \pi/3$, find $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ and give an equation for the tangent line to the graph of f at $(a, f(a))$.
115. Use the pinching theorem to find $\lim_{x \rightarrow 0} \sqrt{x} \cos \frac{1}{x^2}$.

2.6 Some Basic Properties of Continuous Functions

116. Sketch the graph of a function f that is defined on $[0, 1]$ and meets the following conditions (if possible): f is continuous on $[0, 1]$, minimum value $1/2$, maximum value 1.
117. Sketch the graph of a function f that is defined on $[0, 1]$ and meets the following conditions (if possible): f is continuous on $(0, 1]$, no minimum value, maximum value $1/2$.
118. Sketch the graph of a function f that is defined on $[0, 1]$ and meets the following conditions (if possible): f is continuous on $(0, 1)$, takes on the values $1/2$ and 1 but does not take on the value 0.
119. Sketch the graph of a function f that is defined on $[0, 1]$ and meets the following conditions (if possible): f is continuous on $[0, 1]$, does not take on the value 0, minimum value -1 , maximum value $1/2$.
120. Sketch the graph of a function f that is defined on $[0, 1]$ and meets the following conditions (if possible): f is discontinuous at $x = 3/4$ but takes on both a minimum value and a maximum value.
121. Show the equation $x^3 - \cos^2 x = 0$ has a root in $[0, 2]$.
122. Given that $f(x) = x^4 - x^2 + 5x + 2$, show that there exist at least two real numbers x such that $f(x) = 3$.

Answers to Chapter 2 Questions

1. (a) 4 (c) does not exist
(b) does not exist ($-\infty$) (d) 0
2. (a) 3 (c) 3
(b) 3 (d) 5
3. (a) -2 (c) does not exist
(b) -4 (d) -2
4. (a) 1 (c) 1
(b) 1 (d) 2
5. (a) -1 (c) does not exist
(b) 1 (d) -1
6. (a) does not exist ($+\infty$) (c) does not exist
(b) does not exist ($-\infty$) (d) does not exist
7. (a) does not exist (c) does not exist
(b) 0 (d) 0
8. (a) does not exist ($+\infty$) (c) does not exist ($+\infty$)
(b) does not exist ($+\infty$) (d) does not exist
9. (a) 1 (c) does not exist
(b) 2 (d) 0
10. $c = 0$
11. $c = -3$ and $c = 0$
12. $c = 2$
13. $c = -2$ and $c = 1$
14. -1
15. p^2
16. -27
17. $-5/4$
18. 0
19. $5/3$
20. $3/2$
21. 7
22. 12
23. -1
24. $2a$
25. $8/9$
26. 2
27. 3
28. -3
29. does not exist
30. $\sqrt{5} / 5$
31. $1/2$
32. 0
33. $\frac{3\sqrt{10}}{5}$
34. $-2/7$
35. $-3/2$
36. 2
37. -1
38. -1
39. 1
40. -1
41. 0
42. 1
43. does not exist
44. $\frac{1}{5}e$
45. $\frac{5}{3}e$
46. Since $|(3x - 2) - 4| = |3x - 6| = 3|x - 2|$, we can take $d = \frac{1}{3}e$: if $0 < |x - 2| < \frac{1}{3}e$, then $|(3x - 2) - 4| = 3|x - 2| < e$.

47. Since $|(5x-2)-3| = |5x-5| = 5|x-1|$, we can take $\delta = \frac{1}{5}\epsilon$: if $0 < |x-1| < \frac{1}{5}\epsilon$, then $|(5x-2)-3| = 5|x-1| < \epsilon$.

48. Since $|(x-3)-2| = |x-5|$, we can take $\delta = \epsilon$: if $0 < |x-5| < \epsilon$, then $|(x-3)-2| = |x-5| < \epsilon$.

49. (i) $\lim_{x \rightarrow 2} \frac{4}{2x+1} = \frac{4}{5}$

(ii) $\lim_{h \rightarrow 0} \frac{4}{2(2+h)+1} = \frac{4}{5}$

(iii) $\lim_{x \rightarrow 2} \left(\frac{4}{2x+1} - \frac{4}{5} \right) = 0$

(iv) $\lim_{x \rightarrow 2} \left| \frac{4}{2x+1} - \frac{4}{5} \right| = 0$

50. If $|x-3| < 1$, then $-1 < x-3 < 1$, $2 < x < 4$, $5 < x+3 < 7$, and $|x+3| < 7$.

Take $\delta = \text{minimum of } 1 \text{ and } \epsilon/7$.

If $0 < |x-3| < \delta$, then $2 < x < 4$ and

$|x-3| < \epsilon/7$. Therefore,

$$|x^2-9| = |x+3||x-3| < 7|x-3| < 7(\epsilon/7) = \epsilon.$$

51. 8

52. (a) -3 (e) does not exist
(b) 16 (f) 1/7
(c) -3/4 (g) -2
(d) 0

53. 5

54. 16

55. 6

56. 6

57. 1

58. does not exist

59. 2

60. 3

61. 1/12

62. 12

63. 1/3

64. 0

65. 0

66. does not exist

67. does not exist

68. $\frac{-1}{a^2}$

69. $\frac{-1}{4}$

70. $3a^2$

71. 2

72. $\frac{-2}{3}$

73. does not exist

74. 2

75. (a) 1 (c) 0
(b) does not exist (d) -1/4

76. (a) 3 (d) -1/4
(b) 7 (e) 1
(c) does not exist (f) -7

77. continuous

78. continuous

79. continuous

80. removable discontinuity at $x = 2/3$.

81. jump discontinuity

82. jump discontinuity

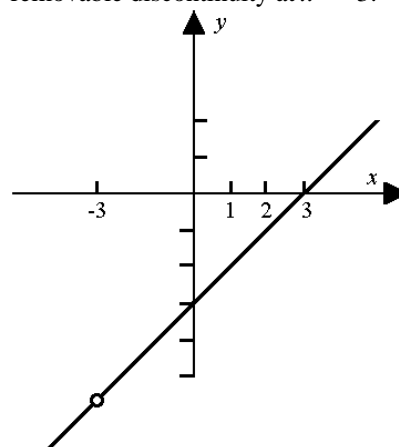
83. removable discontinuity

84. discontinuity of neither type

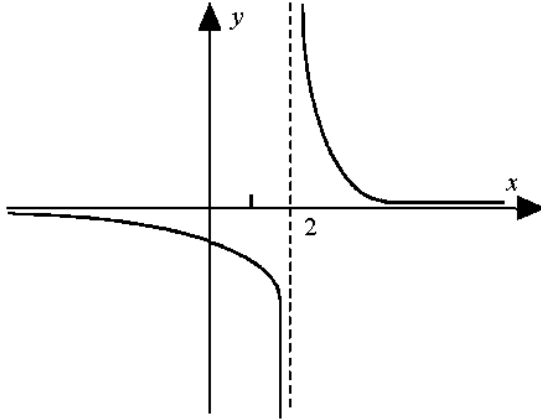
85. discontinuity of neither type

86. discontinuity of neither type

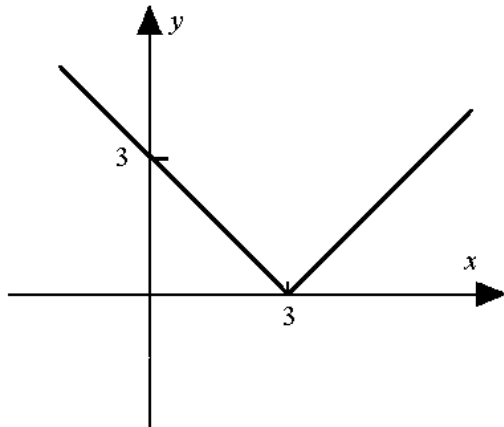
87. removable discontinuity at $x = -3$.



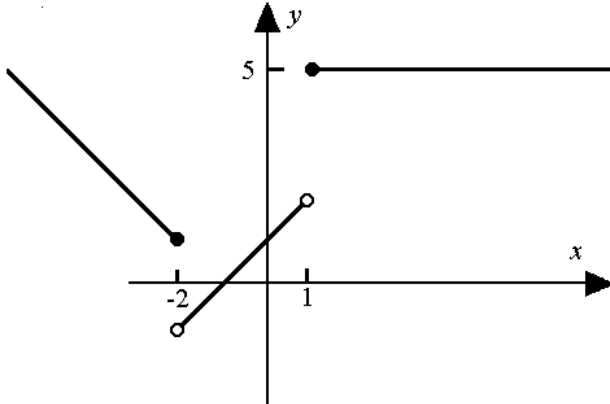
88. nonremovable, nonjump discontinuity at $x = 2$.



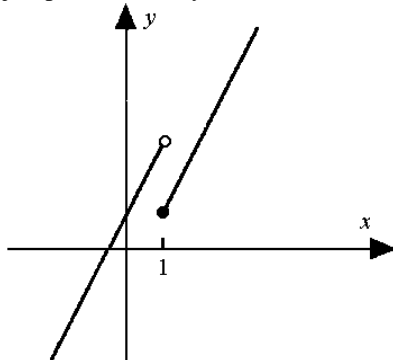
89. no discontinuities



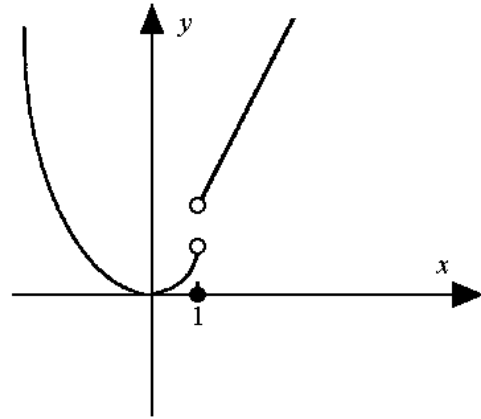
90. jump discontinuity at $x = -2$ and $x = 1$.



91. jump discontinuity at $x = 1$.



92. jump discontinuity at $x = 1$.



93. $f(-1) = 3$

94. $f(-3) = -6$

95. $f(2) = 5$

96. -3

97. Since f has a removable discontinuity at c , for $\lim_{x \rightarrow c} f(x) = L$ some real number L . Then
 $\lim_{x \rightarrow c} (x - c)f(x) = L \cdot \lim_{x \rightarrow c} (x - c) = L \cdot 0 = 0$.

98. 7

99. $2\sqrt{2}$

100. $8/9$

101. 0

102. 1

103. 2

104. $-1/2$

105. $1/4$

106. $\sqrt{2}$

107. $1/3$

108. 3

109. $3/2$

110. 0

111. 1

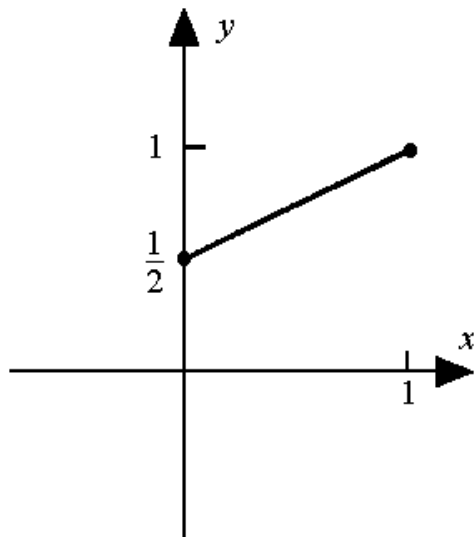
112. 0

113. 0

114. limit: $\frac{1}{2}$; tangent line: $y = \frac{1}{2}\left(x - \frac{p}{3}\right) + \frac{\sqrt{3}}{2}$

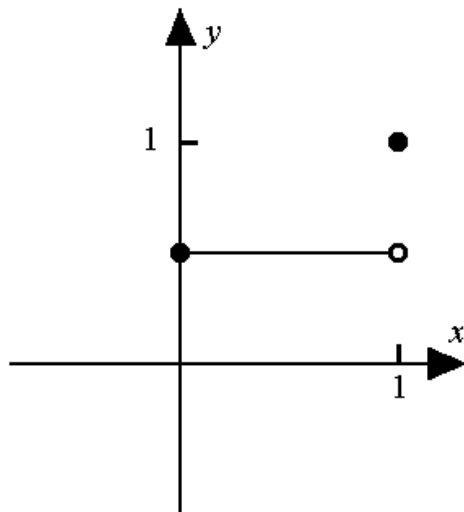
115. 0

116.



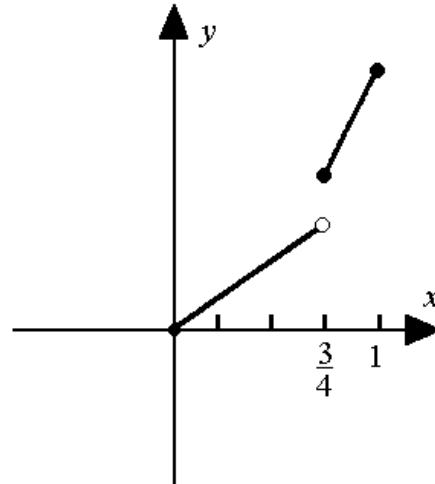
117. impossible

118.



119. impossible

120.


121. If $f(x) = x^3 - \cos^2 x$, then f is continuous, and $f(0) = -1 < 0$, $f(2) = 8 - \cos^2(2) \geq 7 > 0$, so by the intermediate-value theorem $f(c) = 0$ for some c in $[0, 2]$.

122. f is continuous, and $f(0) = 2$, $f(-2) = 4$, $f(1) = 7$, so by the intermediate-value theorem $f(x) = 3$ for some x in $[-2, 0]$ and for some x in $[0, 1]$.

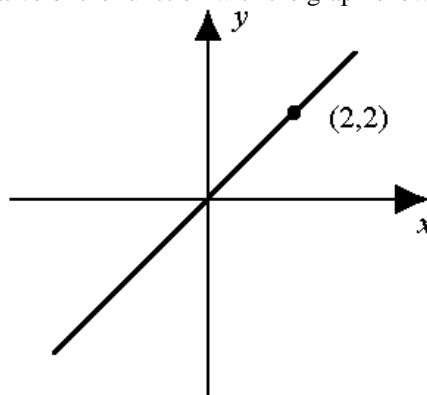
CHAPTER 3

Differentiation

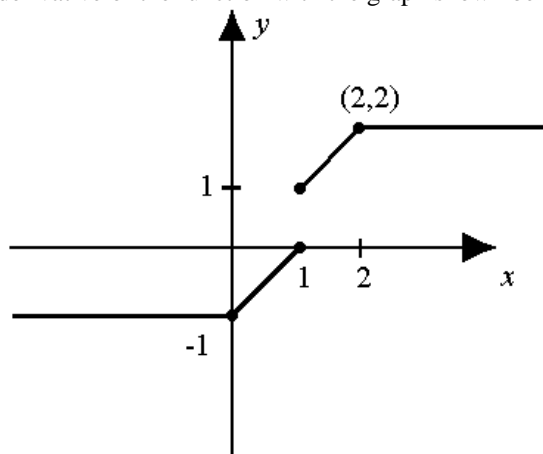
3.1 The Derivative

1. Differentiate $f(x) = 7$ by forming a difference quotient $\frac{f(x+h) - f(x)}{h}$ and taking the limit as h tends to 0.
2. Differentiate $f(x) = 3 - 4x$ by forming a difference quotient $\frac{f(x+h) - f(x)}{h}$ and taking the limit as h tends to 0.
3. Differentiate $f(x) = 2x^2 + x$ by forming a difference quotient $\frac{f(x+h) - f(x)}{h}$ and taking the limit as h tends to 0.
4. Differentiate $f(x) = x^3$ by forming a difference quotient $\frac{f(x+h) - f(x)}{h}$ and taking the limit as h tends to 0.
5. Differentiate $f(x) = \sqrt{x+2}$ by forming a difference quotient $\frac{f(x+h) - f(x)}{h}$ and taking the limit as h tends to 0.
6. Differentiate $f(x) = \frac{1}{x+1}$ by forming a difference quotient $\frac{f(x+h) - f(x)}{h}$ and taking the limit as h tends to 0.
7. Differentiate $f(x) = \frac{1}{2x^2}$ by forming a difference quotient $\frac{f(x+h) - f(x)}{h}$ and taking the limit as h tends to 0.
8. Find $f'(2)$ for $f(x) = (2x+3)^2$ by forming a difference quotient $\frac{f(2+h) - f(2)}{h}$ and taking the limit as $h \rightarrow 0$.
9. Find $f'(2)$ for $f(x) = x^3 - 2x$ by forming a difference quotient $\frac{f(2+h) - f(2)}{h}$ and taking the limit as $h \rightarrow 0$.
10. Find $f'(2)$ for $f(x) = 2x + \sqrt{x+2}$ by forming a difference quotient $\frac{f(2+h) - f(2)}{h}$ and taking the limit as $h \rightarrow 0$.
11. Find $f'(2)$ for $f(x) = \frac{3}{2x+1}$ by forming a difference quotient $\frac{f(2+h) - f(2)}{h}$ and taking the limit as $h \rightarrow 0$.
12. Find equations for the tangent and normal to the graph of $f(x) = 2x^3 + 1$ at the point $(1, f(1))$.
13. Find equations for the tangent and normal to the graph of $f(x) = x^3 - 3x$ at the point $(2, f(2))$.

14. Find equations for the tangent and normal to the graph of $f(x) = \sqrt{2x}$ at the point $(2, f(2))$.
15. Find equations for the tangent and normal to the graph of $f(x) = \sqrt{2x+1}$ at the point $(3/2, f(3/2))$.
16. Draw a graph of $f(x) = |2x-1|$ and indicate where it is not differentiable.
17. Draw a graph of $f(x) = \begin{cases} x^2+1, & x \leq 1 \\ 2, & x > 1 \end{cases}$ and indicate where it is not differentiable.
18. Find $f'(c)$ if it exists. $f(x) = \begin{cases} 2x^2+2, & x \leq 1 \\ x^4+3, & x > 1 \end{cases} \quad c = 1.$
19. Find $f'(c)$ if it exists. $f(x) = \begin{cases} 3x+1, & x \leq -1 \\ 2(x+1)^2, & x > -1 \end{cases} \quad c = -2.$
20. Sketch the graph of the derivative of the function with the graph shown below.



21. Sketch the graph of the derivative of the function with the graph shown below.



22. $\lim_{h \rightarrow 0} \frac{(8+h)^{2/3} - 4}{h}$ represents the derivative of a function f at a point c . Determine f and c .
23. $\lim_{h \rightarrow 0} \frac{\sin h}{h}$ represents the derivative of a function f at a point c . Determine f and c .

3.2 Some Differentiation Formulas

24. Differentiate $F(x) = 1 - 3x$.
25. Differentiate $F(x) = 4x^5 - 8x^2 + 9x$.
26. Differentiate $F(x) = \frac{2}{x^4}$.
27. Differentiate $F(x) = (2x^2 - 1)(3x + 1)$.
28. Differentiate $F(x) = \frac{(2x^2 - 5)}{x^4}$.
29. Differentiate $F(x) = \frac{3x^4 + 5}{x - 1}$.
30. Differentiate $F(x) = \left(1 + \frac{2}{x}\right)\left(1 + \frac{2}{x^2}\right)$.
31. Find $f'(0)$ and $f'(1)$ for $f(x) = x^3(2x + 3)$.
32. Find $f'(0)$ and $f'(1)$ for $f(x) = \frac{4x+1}{2x-1}$.
33. Given that $h(0) = 4$ and $h'(0) = 3$, find $f'(0)$ for $f(x) = 2x^2h(x) - 3x$.
34. Find an equation for the tangent to the graph of $f(x) = 2x^2 - \frac{5}{x}$ at the point $(-1, f(-1))$.
35. Find the points where the tangent to the curve is horizontal for $f(x) = (x + 1)(x^2 - 3x - 8)$.
36. Find the points where the tangent to the curve for $f(x) = -x^3 + 2x$ is parallel to the line $y = 2x + 5$.
37. Find the points where the tangent to the curve for $f(x) = 3x + x^2$ is perpendicular to the line $3x + 2y + 1 = 0$.
38. Find the area of the triangle formed by the x -axis and the lines tangent and normal to the curve $f(x) = 2x + 3x^2$ at the point $(-1, 3)$.

3.3 The d/dx Notation; Derivatives of Higher Order

39. Find $\frac{dy}{dx}$ for $y = \frac{3x^3 + 5y^2 + 2}{x^2}$.
40. Find $\frac{dy}{dx}$ for $y = 5x^3 + \frac{1}{\sqrt{x}}$.
41. Find $\frac{dy}{dx}$ for $y = \frac{x^2 + 3x}{7 - 2x}$.
42. Find $\frac{d}{dx}[-2(x^2 - 5x)(3 + x^7)]$.

43. Find $\frac{d}{dx} \left[\frac{x^2 - 5}{3x^2 - 1} \right]$.
44. Evaluate $\frac{dy}{dx}$ at $x = 2$ for $y = (x^2 + 1)(x^3 - x)$.
45. Find the second derivative for $f(x) = \frac{-8}{x^2} + \frac{1}{5}x^5$.
46. Find the second derivative for $f(x) = (x^2 - 2)(x^3 + 5x)$.
47. Find $\frac{d^3y}{dx^3}$ for $y = x^3 - \frac{1}{x}$.
48. Find $\frac{d^4y}{dx^4}$ for $y = \frac{-1}{x} - 5x^{-2}$.
49. Find $\frac{d}{dx} \left[x^2 \frac{d^2}{dx^2} (3x^2 - x^5) \right]$.
50. Determine the values of x for which (a) $f''(x) = 0$, (b) $f''(x) > 0$, and (c) $f''(x) < 0$ for $f(x) = 2x^4 + 2x^3 - x$.

3.4 The Derivative as a Rate of Change

51. Find the rate of change of the area of a circle with respect to the radius r when $r = 3$.
52. Find the rate of change of the volume of a cube with respect to the length s of a side when $s = 2$.
53. Find the rate of change of the area of a square with respect to the length z of a diagonal when $z = 5$.
54. Find the rate of change of the volume of a ball with respect to the radius r when $r = 4$.
55. Find the rate of change of $y = 6 - x - x^2$ with respect to x at $x = -1$.
56. Find the rate of change of the volume V of a cube with respect to the length w of a diagonal on one of the faces when $w = 2$.
57. The volume of a cylinder is given by the formula $V = \pi r^2 h$ where r is the base radius and h is the height.
(a) Find the rate of change of V with respect to h if r remains constant.
(b) Find the rate of change of V with respect to r if h remains constant.
(c) Find the rate of change of h with respect to r if V remains constant.
58. An object moves along a coordinate line, its position at each time $t \geq 0$ given by $x(t) = 3t^2 - 7t + 4$. Find the position, velocity, acceleration, and speed at time $t_0 = 4$.
59. An object moves along a coordinate line, its position at each time $t \geq 0$ given by $x(t) = t^3 - 6t^2 - 15t$. Determine when, if ever, the object changes direction.
60. An object moves along the x -axis, its position at each time $t \geq 0$ given by $x(t) = t^4 - 12t^3 + 28t^2$. Determine the time interval(s), if any, during which the object moves left.
61. An object moves along the x -axis, its position at each time $t \geq 0$ given by $x(t) = 5t^4 - t^5$. Determine the time interval(s), if any, during which the object is speeding up to the right.

62. An object is dropped and hits the ground 5 seconds later. From what height was it dropped? Neglect air resistance.
63. A stone is thrown upward from ground level. The initial speed is 24 feet per second. (a) In how many seconds will the stone hit the ground? (b) How high will it go? (c) With what minimum speed should the stone be thrown to reach a height of 40 feet?
64. An object is projected vertically upward from ground level with a velocity of 32 feet per second. What is the height attained by the object? (Take g as 32 ft/sec^2).
65. If $C(x) = 700 + 5x + \frac{100}{\sqrt{x}}$ is the cost function for a certain commodity, find the marginal cost at a production level of 400 units, and find the actual cost of producing the 401st unit.
66. If $C(x) = 25,000 + 30x + (0.003)x^2$ is the cost function for a certain commodity and $R(x) = 60x - (0.002)x^2$ is the revenue function, find:
 (a) the profit function
 (b) the marginal profit
 (c) the production level(s) at which the marginal profit is zero.

3.5 The Chain Rule

67. Differentiate $f(x) = (x^3 + 1)^3$: (a) by expanding before differentiation, (b) by using the chain rule. Then reconcile the results.
68. Differentiate $f(x) = (x - x^3)^3$.
69. Differentiate $f(x) = \left[\frac{x+1}{x-1} \right]^2$.
70. Differentiate $f(x) = \left[\frac{1}{x} + \frac{1}{x^2} \right]^4$.
71. Differentiate $f(x) = \left[\frac{x^2+7}{x^2-7} \right]^4$.
72. Differentiate $f(x) = (x+4)^4(3x+2)^3$.
73. Find $\frac{dy}{dx}$ at $x=0$ for $y = \frac{1}{1+u}$ and $u = (3x+1)^3$.
74. Find $\frac{dy}{dt}$ at $t=1$ for $y = u^3 - u^2$, $u = \frac{1-x}{1+x}$ and $x = 2t - 5$.
75. Find $\frac{dy}{dx}$ at $x=1$ for $y = \frac{1+5}{1-5}$, and $s = \sqrt{t+1}$, and $t = \frac{3x^2}{4}$.
76. Given that $f(1) = 2$, $g(1) = 1$, $f'(1) = 3$, $g'(1) = 2$, and $f'(2) = 0$, evaluate $(f \cdot g)'(1)$.
77. Find $\frac{d}{dx}[f(x^3 - 1)]$.
78. Given that $f(x) = (1 + 2x^2)^{-2}$, determine the values of x for which (a) $f'(x) = 0$, (b) $f'(x) > 0$ and (c) $f'(x) < 0$.

79. An object moves along a coordinate line, its position at each time $t \geq 0$ given by $x(t) = (t^2 - 3)^3(t^2 + 1)^2$. Determine when the object changes direction.
80. Differentiate $f(x) = [(x^3 - x^{-3})^2 - x^2]^3$.
81. Find $f''(x)$ for $f(x) = (x^2 + 2x)^{17}$.
82. The edge of a cube is decreasing at the rate of 3 centimeters per second. How is the volume of the cube changing when the edge is 5 centimeters long?
83. The diameter of a sphere is increasing at the rate of 3 centimeters per second. How is the volume of the sphere changing when the diameter is 6 centimeters?

3.6 **Differentiating the Trigonometric Functions**

84. Differentiate $y = x \tan x$.
85. Differentiate $y = \sin x \tan x$.
86. Differentiate $y = \frac{\sin x}{1 - \cos x}$.
87. Differentiate $y = \frac{\sin x}{x^2}$.
88. Differentiate $y = \sec x \tan x$.
89. Find the second derivative for $y = x \sin x$.
90. Find the second derivative for $y = 5 \cos x + 7 \sin x + \frac{x^2}{2}$.
91. Find $\frac{d^3}{dx^3}(\sin x)$.
92. Find an equation for the tangent to the curve $y = \sin x$ at $x = \pi/6$.
93. Determine the numbers x between 0 and 2π on $y = \sin x$, where the tangent to the curve is parallel to the line $y = 0$.
94. An object moves along the y-axis, its position at each time t given by $x(t) = \sin 2t$. Determine those times from $t = 0$ to $t = \pi$ when the object is moving to the right with increasing speed.
95. Find $\frac{dy}{dt}$ for $y = \left[\frac{1}{2}(1 + u) \right]^3$, $u = \sin x$, and $x = 2\pi t$.
96. A rocket is launched 2 miles away from one observer on the ground. How fast is the rocket going when the angle of elevation of the observer's line of sight to the rocket is 50° (from the horizontal) and is increasing at $5^\circ/\text{sec}$?
97. An airplane at a height of 2000 meters is flying horizontally, directly toward an observer on the ground, with a speed of 300 meters per second. How fast is the angle of elevation of the plane changing when this angle is 45° ?

3.7 Implicit Differentiation: Rational Powers

98. Use implicit differentiation to obtain $\frac{dy}{dx}$ in terms of x and y for $x^2 - 4xy + 2y^2 = 5$.
99. Use implicit differentiation to obtain $\frac{dy}{dx}$ in terms of x and y for $x^2y + y^2 = 6$.
100. Use implicit differentiation to obtain $\frac{dy}{dx}$ in terms of x and y for $xy^2 + \sqrt{xy} = 2$.
101. Use implicit differentiation to obtain $\frac{dy}{dx}$ in terms of x and y for $y = \sin(x + y) + \cos x$.
102. Express $\frac{d^2y}{dx^2}$ in terms of x and y for $x^2 + 3y^2 = 10$.
103. Express $\frac{d^2y}{dx^2}$ in terms of x and y for $x^2 + 2xy - y^2 + 8 = 0$.
104. Express $\frac{dy}{dx}$ at the point $P(-1, -1)$ for $3x^2 + xy = y^2 + 3$.
105. Express $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point $P(2, -1)$ for $x^2 - xy + y^2 = 7$.
106. Find the equations for the tangent and normal at the point $P(-1, -1)$ for $2x^2 - 3xy + 3y^2 = 2$.
107. Find $\frac{dy}{dx}$ for $y = (x^4 + x^3)^{3/2}$.
108. Find $\frac{dy}{dx}$ for $y = \sqrt[4]{3x^3 + 2}$.
109. Find $\frac{dy}{dx}$ for $y = (x^2 + 1)^{1/4}(x^2 + 2)^{1/2}$.
110. Compute $\frac{d}{dx} \left(\sqrt[4]{x} + \frac{1}{\sqrt[4]{x}} \right)$.
111. Compute $\frac{d}{dx} \sqrt{\frac{4x+3}{2x-5}}$.
112. Find the second derivative for $y = \sqrt{9 + x^3}$.
113. Find the second derivative for $y = 4\sqrt{4 - x^3}$.
114. Compute $\frac{d}{dx} [f(\sqrt{x} - 1)]$.

115. In economics, the elasticity of demand is given by the formula $e = \frac{P}{Q} \left| \frac{dQ}{dP} \right|$ where P is price and Q quantity.

The demand is said to be $\begin{cases} \text{inelastic where } e < 1 \\ \text{unitary where } e = 1 \\ \text{elastic where } e > 1 \end{cases}$. Describe the elasticity of $Q = (400 - P)^{3/5}$.

3.8 Rates of Change Per Unit Time

116. A shark, looking for dinner, is swimming parallel to a straight beach and is 90 feet offshore. The shark is swimming at a constant speed of 30 feet per second. At time $t = 0$, the shark is directly opposite a lifeguard station. How fast is the shark moving away from the lifeguard station when the distance between them is 150 feet?
117. A boat sails parallel to a straight beach at a constant speed of 12 miles per hour, staying 4 miles offshore. How fast is it approaching a lighthouse on the shoreline at the instant it is exactly 5 miles from the lighthouse?
118. A ladder 13 feet long is leaning against a wall. If the base of the ladder is moving away from the wall at the rate of $\frac{1}{2}$ foot per second, at what rate will the top of the ladder be moving when the base of the ladder is 5 feet from the wall?
119. A spherical balloon is inflated so that its volume is increasing at the rate of 3 cubic feet per minute. How fast is the radius of the balloon increasing at the instant the radius is $\frac{1}{2}$ foot? $V = \frac{4}{3}\pi r^3$
120. Sand is falling into a conical pile so that the radius of the base of the pile is always equal to one-half of its altitude. If the sand is falling at a rate of 10 cubic feet per minute, how fast is the altitude of the pile increasing when the pile is 5 feet deep? $V = \frac{1}{3}\pi r^2 h$
121. A spherical balloon is inflated so that its volume is increasing at the rate of 20 cubic feet per minute. How fast is the surface area of the balloon increasing at the instant the radius is 4 feet? $V = \frac{4}{3}\pi r^3, S = 4\pi r^2$
122. Two ships leave port at noon. One ship sails north at 6 miles per hour, and the other sails east at 8 miles per hour. At what rate are the two ships separating 2 hours later?
123. A conical funnel is 14 inches in diameter and 12 inches deep. A liquid is flowing out at the rate of 40 cubic inches per second. How fast is the depth of the liquid falling when the level is 6 inches deep? $V = \frac{1}{3}\pi r^2 h$
124. A baseball diamond is a square 90 feet on each side. A player is running from home to first base at the rate of 25 feet per second. At what rate is his distance from second base changing when he has run half way to first base?
125. A ship, proceeding southward on a straight course at a rate of 12 miles/hr. is, at noon, 40 miles due north of a second ship, which is sailing west at 15 miles/hr.
(a) How fast are the ships approaching each other 1 hour later?
(b) Are the ships approaching each other or are they receding from each other at 2 o'clock and at what rate?
126. An angler has a fish at the end of his line, which is being reeled in at the rate of 2 feet per second from a bridge 30 feet above water. At what speed is the fish moving through the water towards the bridge when the amount of line out is 50 feet? (Assume the fish is at the surface of the water and that there is no sag in the line.)

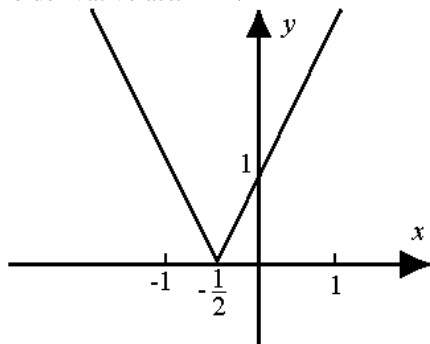
127. A kite is 150 feet high and is moving horizontally away from a boy at the rate of 20 feet per second. How fast is the string being payed out when the kite is 250 feet from him?
128. An ice cube is melting so that its edge length x is decreasing at the rate of 0.1 meters per second. How fast is the volume decreasing when $x = 2$ meters?
129. Consider a rectangle where the sides are changing but the area is always 100 square inches. One side changes at the rate of 3 inches per second. When that side is 20 inches long, how fast is the other side changing?
130. The sides of an equilateral triangle are increasing at the rate of 5 centimeters per hour. At what rate is the area increasing when the side is 10 centimeters?
131. A circular cylinder has a radius r and a height h feet. If the height and radius both increase at the constant rate of 10 feet per minute, at what rate is the lateral surface area increasing? $S = 2\pi rh$
132. The edges of a cube of side x are contracting. At a certain instant, the rate of change of the surface area is equal to 6 times the rate of change of its edge. Find the length of the edge.
133. A particle is moving along the parabola $y = x^2$. If the x -coordinate of its position P is increasing at the rate of 10 m/sec, what is the rate of change of the angle of inclination of the line OP when $x = 3$ m?

3.9 Differentials; Newton-Raphon Approximations

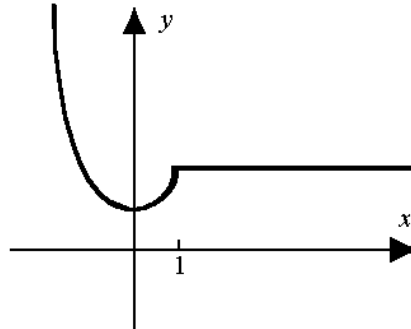
134. Estimate $\sqrt[4]{14}$ by differentials.
135. Estimate $\sqrt[3]{9}$ by differentials.
136. Estimate $\sqrt[5]{30}$ by differentials.
137. Estimate $\sqrt[3]{10}$ by differentials.
138. Use differentials to estimate $\cos 59^\circ$.
139. Use differentials to estimate $\sin 31^\circ$.
140. Use differentials to estimate $\tan 43^\circ$.
141. Estimate $f(3.2)$, given that $f(3) = 2$ and $f'(x) = (x^3 + 5)^{1/5}$.
142. How accurately must we measure the edge of a cube to determine the volume within 1%?
143. Use differentials to estimate the volume of gold needed to cover a sphere of radius 10 cm with a layer of gold 0.05 cm thick.

Answers to Chapter 3 Questions

1. 0
2. -4
3. $4x + 1$
4. $3x^2$
5. $\frac{1}{2\sqrt{x+2}}$
6. $\frac{-1}{(x+1)^2}$
7. $-1/x^3$
8. 28
9. 10
10. 9/4
11. -6/25
12. tangent: $y - 3 = 6(x - 1)$
normal: $y - 3 = \frac{-1}{6}(x - 1)$
13. tangent: $y - 2 = 9(x - 2)$
normal: $y - 2 = \frac{-1}{9}(x - 2)$
14. tangent: $y - 2 = \frac{1}{2}(x - 2)$
normal: $y - 2 = -2(x - 2)$
15. tangent: $y - 2 = \frac{1}{2}\left(x - \frac{3}{2}\right)$
normal: $y - 2 = -2\left(x - \frac{3}{2}\right)$
16. no derivative at $x = -1/2$

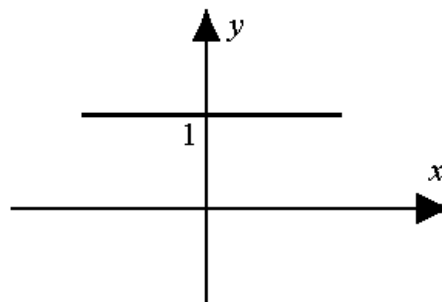


17. no derivative at $x = 1$

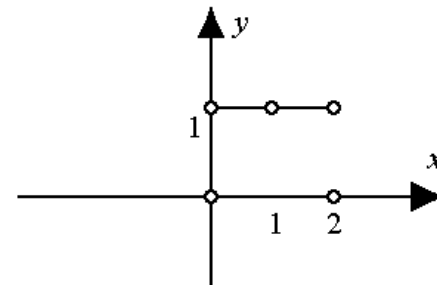


18. $f'(1) = 4$
19. $f'(-2) = 3$

20.



21.



22. $f(x) = x^{2/3}; c = 8$

23. $f(x) = \sin x; c = 0$

24. -3

25. $20x^4 - 16x + 9$

26. $-8/x^5$

27. $18x^2 + 4x - 3$

28. $\frac{-4}{x^3} + \frac{20}{x^5}$

29. $\frac{9x^4 - 12x^3 - 5}{(x-1)^2}$

30. $\frac{-2}{x^2} - \frac{4}{x^3} - \frac{12}{x^4}$
31. $f'(x) = 8x^3 + 9x^2$
 $f'(0) = 0$
 $f'(1) = 17$
32. $f'(x) = \frac{-6}{(2x-1)^2}$
 $f'(0) = -6$
 $f'(1) = -6$
33. $f'(x) = 4xh(x) + 2x^2h'(x) - 3$
 $f'(0) = -3$
34. $y - 7 = (1)(x + 1)$
 $y - 7 = x + 1$
 $y = x + 8$
35. $f'(x) = 0$ at $x = \frac{2 \pm \sqrt{37}}{3}$
36. at $x = 0$
37. at $x = -9/4$
38. $153/8$
39. $3 - 4/x^3$
40. $15x^2 - \frac{1}{2x\sqrt{x}}$
41. $\frac{21 + 14x - 2x^2}{(7 - 2x)^2}$
42. $-18x^8 + 80x^7 - 12x + 30$
43. $\frac{28x}{(3x^2 - 1)^2}$
44. 79
45. $\frac{-48}{x^4} + 4x^3$
46. $20x^3 + 18x$
47. $6 + 6/x^4$
48. $-24x^{-5} - 600x^{-6} = \frac{-24}{x^5} - \frac{600}{x^6}$
49. $12x - 100x^4$
50. (a) $x = 0$ or $x = -1/2$
 (b) $x < -1/2$ or $x > 0$
 (c) $-1/2 < x < 0$
51. $6p$
52. 12
53. 5
54. $64p$
55. 1
56. $3\sqrt{2}$
57. (a) $\frac{dV}{dh} = pr^2$
 (b) $\frac{dV}{dr} = 2prh$
 (c) $\frac{dh}{dr} = \frac{-2V}{pr^3}$
58. $x(4) = 24$; $v(4) = 17$; $a(4) = 6$
 Speed = $|v(4)| = 17$
59. The object changes direction (from left to right) at $t = 5$.
60. $v(t) < 0$ when $2 < t < 7$.
61. $0 < t < 3$
62. 400 ft
63. (a) $1\frac{1}{2}$ sec
 (b) 9 ft
 (c) $16\sqrt{10}$ ft/sec
64. 16 ft
65. \$4.99; \$4.99
66. (a) $P(x) = 30x - (0.005)x^2 - 25,000$
 (b) $P'(x) = 30 - (0.010)x$
 (c) $x = 3000$
67. $f(x) = x^9 + 3x^6 + 3x^3 + 1$
 $f'(x) = 9x^8 + 18x^5 + 9x^2$
 $f(x) = (x^3 + 1)^3$
 $f'(x) = 9x^2(x^3 + 1)^2 = 9x^8 + 18x^5 + 9x^2$
68. $3(x - x^3)^2(1 - 3x^2)$

69. $\frac{-4(x+1)}{(x-1)^3}$

70. $-4\left(\frac{1}{x} + \frac{1}{x^2}\right)^3\left(\frac{1}{x^2} + \frac{2}{x^3}\right)$

71. $\frac{-112x(x^2+7)^3}{(x^2-7)^5}$

72. $(x+4)^3(3x+2)^2(21x+44)$

73. $-9/4$

74. -16

75. $\frac{12}{\sqrt{7}(2-\sqrt{7})^2}$ or $\frac{112+44\sqrt{7}}{21}$

76. 6

77. $3x^2 f'(x^3 - 1)$

78. (a) $x = 0$ (b) $x < 0$ (c) $x > 0$

79. The object changes direction (from left to right) at $t = \frac{\sqrt{15}}{5}$ and at $t = \sqrt{3}$ (from right to left).

80. $3[(x^3 - x^{-3})^2 - x^2]^2 [2(x^3 - x^{-3})(3x^2 + 3x^{-4}) - 2x]$

81. $17(x^2 + 2x)^{15}(66x^2 + 132x + 64)$

82. decreasing at a rate of $225 \text{ cm}^3/\text{sec}$

83. decreasing at a rate of $54p \text{ cm}^3/\text{sec}$

84. $\tan x + x \sec^2 x$

85. $\cos x \tan x + \sin x \sec^2 x = \sin x + \sin x \sec^2 x$
 $= \sin x (1 + \sec^2 x)$

86. $\frac{1}{\cos x - 1}$

87. $\frac{x \cos x - 2 \sin x}{x^3}$

88. $2\sec^3 x - \sec x$

89. $2\cos x - x \sin x$

90. $-5\cos x - 7\sin x + 1$

91. $-\cos x$

92. $y - \frac{1}{2} = \frac{\sqrt{3}}{2}\left(x - \frac{p}{6}\right)$

93. $x = p/2, 3p/2$

94. $3p/4 < t < p$

95. $\frac{3p}{4}(1 + \sin 2pt)^2 \cos 2pt$

96. $\frac{p}{18} \sec^2 50^\circ \cong 1521 \text{ mph}$

97. $3/40 \text{ radian/sec}$

98. $\frac{dy}{dx} = \frac{2y - x}{2y - 2x}$

99. $\frac{dy}{dx} = \frac{-2xy}{x^2 + 2y}$

100. $\frac{dy}{dx} = \frac{-y - 2y^2 \sqrt{xy}}{4x\sqrt{xy} + x}$

101. $\frac{dy}{dx} = \frac{\cos(x+y) - \sin x}{1 - \cos(x+y)}$

102. $\frac{d^2 y}{dx^2} = \frac{-10}{9y^3}$

103. $\frac{16}{(y-x)^3}$

104. 7

105. at $(2, -1)$, $\frac{dy}{dx} = \frac{5}{4}$, $\frac{d^2 y}{dx^2} = \frac{21}{32}$

106. tangent: $y + 1 = \frac{-1}{3}(x + 1)$
normal: $y + 1 = 3(x + 1)$

107. $\frac{3x^2}{2}(4x + 3)(x^4 + x^3)^{1/2}$

108. $\frac{9x^2}{4}(3x^3 + 2)^{-3/4}$

109. $\frac{3x^3 + 4x}{2(x^2 + 1)^{3/4}(x^2 + 2)^{1/2}}$

$$110. \frac{1}{4}x^{-3/4} - \frac{1}{4}x^{-5/4} = \frac{1}{4\sqrt[4]{x^3}} - \frac{1}{4x\sqrt[4]{x}}$$

$$111. \frac{-13}{(2x-5)\sqrt{(4x+3)(2x-5)}}$$

$$112. \frac{3x^4 + 108x}{4(9+x^3)\sqrt{9+x^3}}$$

$$113. \frac{15x^5 - 96x^2}{4(4-x^3)\sqrt{4-x^3}}$$

$$114. \frac{1}{2\sqrt{x}} f'(\sqrt{x} - 1)$$

$$115. \text{ with } 0 < p < 400, \begin{aligned} \epsilon &< 1 \text{ for } P < 250 \\ \epsilon &= 1 \text{ for } P = 250 \\ \epsilon &> 1 \text{ for } P > 250 \end{aligned}$$

$$116. 24 \text{ ft/sec}$$

$$117. \frac{36}{5} \text{ mi/hr}$$

$$118. \frac{-5}{24} \text{ ft/sec}$$

$$119. \frac{3}{p} \text{ ft/min}$$

$$120. \frac{8}{5p} \text{ ft/min}$$

$$121. 10 \text{ ft}^2/\text{min}$$

$$122. 10 \text{ mi/hr}$$

$$123. \frac{-160}{49p} \text{ in/sec}$$

$$124. -5\sqrt{5} \text{ ft/sec}$$

$$125. (a) \frac{111}{\sqrt{1009}} \approx 3.498 \text{ mi/hr}$$

$$(b) \frac{129}{17} \text{ mi/hr (receding from each other)}$$

$$126. 5/2 \text{ ft/sec}$$

$$127. 16 \text{ ft/sec}$$

$$128. -1.2 \text{ m}^3/\text{sec}$$

$$129. \frac{-3}{4} \text{ in/sec}$$

$$130. 25\sqrt{3} \text{ cm}^2/\text{hr}$$

$$131. 20p(r+h) \text{ ft}^2/\text{min}$$

$$132. x = 1/2$$

$$133. 1 \text{ radian/sec (increasing)}$$

$$134. 31/16$$

$$135. 25/12$$

$$136. 79/40$$

$$137. 13/6$$

$$138. \approx \frac{1}{2} + \frac{\sqrt{3}p}{360} \approx 0.515$$

$$139. \approx \frac{1}{2} + \frac{\sqrt{3}}{2} \left(\frac{p}{180} \right) \approx 0.515$$

$$140. \approx 1 - \frac{p}{45} \approx 0.930$$

$$141. 2.4$$

$$142. \text{ within } \frac{1}{3} \%$$

$$143. \text{ approximately } 20p \text{ cm}^3$$

CHAPTER 4

The Mean-Value Theorem and Applications

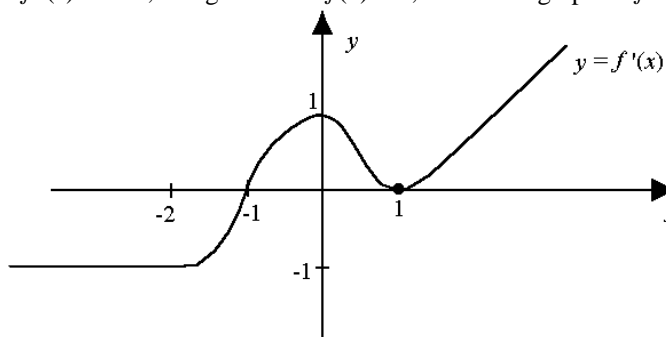
4.1 The Mean-Value Theorem

1. Determine whether the function $f(x) = x^3 - 3x + 2$ satisfies the conditions of the mean-value theorem on the interval $[-2, 3]$. If so, find the admissible values of c .
2. Determine whether the function $f(x) = x^2 + 2x - 1$ satisfies the conditions of the mean-value theorem on the interval $[0, 1]$. If so, find the admissible values of c .
3. Determine whether the function $f(x) = 1/x^2$ satisfies the conditions of the mean-value theorem on the interval $[-1, 1]$. If so, find the admissible values of c .
4. Determine whether the function $f(x) = x^2 + 4$ satisfies the conditions of the mean-value theorem on the interval $[0, 2]$. If so, find the admissible values of c .
5. Determine whether the function $f(x) = x^3 - 3x + 1$ satisfies the conditions of the mean-value theorem on the interval $[-2, 2]$. If so, find the admissible values of c .
6. Determine whether the function $f(x) = x^3 - 2x + 4$ satisfies the conditions of the mean-value theorem on the interval $[1, 2]$. If so, find the admissible values of c .
7. Determine whether the function $f(x) = x^3 - 3x^2 - 3x + 1$ satisfies the conditions of the mean-value theorem on the interval $[0, 2]$. If so, find the admissible values of c .
8. Determine whether the function $f(x) = \sqrt{x}$ satisfies the conditions of the mean-value theorem on the interval $[0, 4]$. If so, find the admissible values of c .
9. Determine whether the function $f(x) = \sqrt[3]{x}$ satisfies the conditions of the mean-value theorem on the interval $[-1, 1]$. If so, find the admissible values of c .
10. Determine whether the function $f(x) = x^3 - x$ satisfies the conditions of the mean-value theorem on the interval $[-1, 1]$. If so, find the admissible values of c .
11. Determine whether the function $f(x) = x^3 - 4x$ satisfies the conditions of Rolle's theorem on the interval $[-2, 2]$. If so, find the admissible values of c .
12. Determine whether the function $f(x) = \sqrt[3]{x}$ satisfies the conditions of the mean-value theorem on the interval $[0, 1]$. If so, find the admissible values of c .

4.2 Increasing and Decreasing Functions

13. Find the intervals on which $f(x) = x^4 - 24x^2$ increases and the intervals on which f decreases.
14. Find the intervals on which $f(x) = x^4 - 4x^3$ increases and the intervals on which f decreases.
15. Find the intervals on which $f(x) = x^4 - 6x^2 + 2$ increases and the intervals on which f decreases.
16. Find the intervals on which $f(x) = 5x^4 - x^5$ increases and the intervals on which f decreases.

17. Find the intervals on which $f(x) = 4x^3 - 15x^2 - 18x + 10$ increases and the intervals on which f decreases.
18. Find the intervals on which $f(x) = x(x - 6)^2$ increases and the intervals on which f decreases.
19. Find the intervals on which $f(x) = x^2 + \frac{2}{x}$ increases and the intervals on which f decreases.
20. Find the intervals on which $f(x) = x(x - 4)^4 + 4$ increases and the intervals on which f decreases.
21. Find the intervals on which $f(x) = \sin 2x$, $0 \leq x \leq \pi$, increases and the intervals on which f decreases.
22. Find f given that $f'(x) = 3x^2 - 10x + 3$ for all real x and $f(0) = 1$.
23. Find f given that $f'(x) = 12x^3 - 12x^2$ for all real x and $f(0) = 1$.
24. Find f given that $f'(x) = 3(x - 2)^2$ for all real x and $f(0) = 1$.
25. Find the intervals on which f increases and the intervals on which f decreases given that
- $$f(x) = \begin{cases} x + 2, & x < 0 \\ 7 - 2x, & 0 \leq x < 3 \\ (x - 1)^2, & 3 \leq x \end{cases}$$
26. Given the graph of $f'(x)$ below, and given that $f(0) = 0$, sketch the graph of f .



27. Sketch the graph of a differentiable function f that satisfies $f(2) = 1$, $f(-1) = 0$, and $f'(x) > 0$, for all x , if possible.
28. Sketch the graph of a differentiable function f that satisfies $f(0) = 0$, $f'(x) < 0$ for $x < 0$, and $f'(x) > 0$ for $x > 0$, if possible.

4.3 Local Extreme Values

29. Find the critical numbers and the local extreme values of $f(x) = 3x^5 - 5x^4$.
30. Find the critical numbers and the local extreme values of $f(x) = 12x^{2/3} - 16x$.
31. Find the critical numbers and the local extreme values of $f(x) = x^{2/3}(5 - x)$.
32. Find the critical numbers and the local extreme values of $f(x) = \frac{1}{3}x^{4/3} - \frac{4}{3}x^{1/3}$.
33. Find the critical numbers and the local extreme values of $f(x) = \frac{x^4}{4} - 2x^2 + 1$.

34. Find the critical numbers and the local extreme values of $f(x) = (x + 1)(x - 1)^3$.
35. Find the critical numbers and the local extreme values of $f(x) = 2x + 2x^{2/3}$.
36. Find the critical numbers and the local extreme values of $f(x) = \frac{1}{x} - \frac{1}{3x^3}$.
37. Find the critical numbers and the local extreme values of $f(x) = x^{4/3} - 4x^{-1/3}$.
38. Find the critical numbers and the local extreme values of $f(x) = 6x^2 - 9x + 5$.
39. Find the critical numbers and the local extreme values of $f(x) = x^4 - 6x^2 + 17$.
40. Find the critical numbers and the local extreme values of $f(x) = x - \frac{1}{x}$.
41. Find the critical numbers and the local extreme values of $f(x) = (x + 1)^{2/3}$.
42. Find the critical numbers and the local extreme values of $f(x) = x - \sin 2x$, $0 < x < \pi$.
43. Show that $f(x) = x^3 - 4x^2 + 2x - 5$ has exactly one critical number in $(0, 1)$.

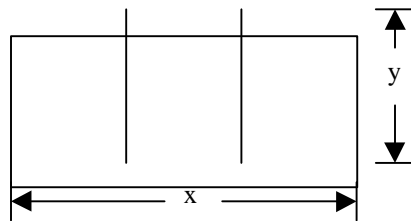
4.4 Endpoint and Absolute Extreme Values

44. Find the critical numbers and classify the extreme values for $f(x) = \frac{x}{2} + 2$, $x \in [0, 100]$.
45. Find the critical numbers and classify the extreme values for $f(x) = 2x^3 - 3x^2 - 12x + 8$, $x \in [-2, 2]$.
46. Find the critical numbers and classify the extreme values for $f(x) = \frac{x^3}{3} - x^2 - 3x + 1$, $x \in [-1, 2]$.
47. Find the critical numbers and classify the extreme values for $f(x) = x^3 - 6x^2 + 5$, $x \in [-1, 5]$.
48. Find the critical numbers and classify the extreme values for $f(x) = 2x^3 - 3x^2 - 12x + 5$, $x \in [0, 4]$.
49. Find the critical numbers and classify the extreme values for $f(x) = 4x^3 - 6x^2 - 9x$, $x \in [-1, 2]$.
50. Find the critical numbers and classify the extreme values for $f(x) = x^3 - 3x + 6$, $x \in [0, 3/2]$.
51. Find the critical numbers and classify the extreme values for $f(x) = 1 - x^{2/3}$, $x \in [-1, 1]$.
52. Find the critical numbers and classify the extreme values for $f(x) = x^3 - 12x + 8$, $x \in [-4, 3]$.
53. Find the critical numbers and classify the extreme values for $f(x) = x^{4/3} - 3x^{1/3}$, $x \in [-1, 8]$.
54. Find the critical numbers and classify the extreme values for $f(x) = \frac{\sqrt{x}}{x^2 + 3}$, $x \in [0, \infty]$.
55. Find the critical numbers and classify the extreme values for $f(x) = \frac{x}{x^2 + 1}$, $x \in [0, 2]$.

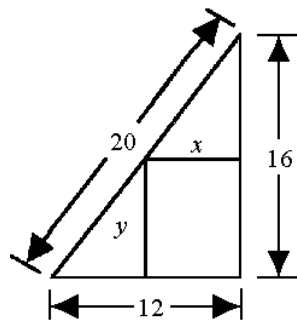
56. Find the critical numbers and classify the extreme values for $f(x) = \frac{1}{x - x^2}$, $x \in [0, 1]$.
57. Find the critical numbers and classify the extreme values for $f(x) = \begin{cases} x^2, & x < 0 \\ x^3, & x \geq 0 \end{cases}$.
58. Find the critical numbers and classify the extreme values for $f(x) = \begin{cases} -x - 1, & x < -1 \\ 1 - x^2, & -1 \leq x \leq 1, \\ x - 1, & x > 1 \end{cases}$, $x \in [-2, 2]$.
59. Find the critical numbers and classify the extreme values for $f(x) = \begin{cases} -1 - x^2, & x < 0 \\ x^3 - 1, & x \geq 1 \end{cases}$, $x \in [-2, 1]$.

4.5 **Some Max-Min Problems**

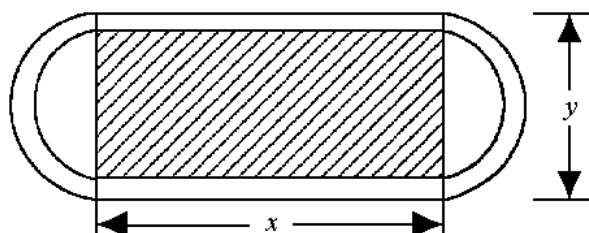
60. Find the dimensions of the rectangle of greatest area that can be inscribed in a circle of radius a .
61. Find the dimensions of the rectangle of greatest area that can be inscribed in a semicircle of radius 1.
62. An open field is to be surrounded with a fence that also divides the enclosure into three equal areas as shown in the figure below. The fence is 4000 feet long. For what value of x will the total area be a maximum?



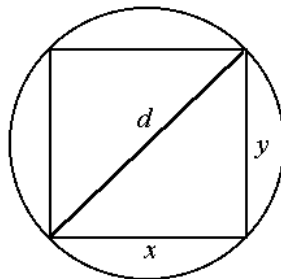
63. Find the dimension of the rectangle of maximum area that may be embedded in a right triangle with sides of length 12, 16, and 20 feet as shown in the figure below.



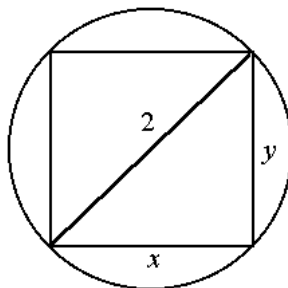
64. The infield of a 440-yard track consists of a rectangle and two semicircles as shown below. To what dimensions should the track be built in order to maximize the area of the rectangle?



65. Find the dimensions of the largest circular cylinder that can be inscribed in a hemisphere of radius 1.
66. A long strip of copper 8 inches wide is to be made into a rain gutter by turning up the sides to form a trough with a rectangular cross section. Find the dimensions of the cross section if the carrying capacity of the trough is to be a maximum.
67. An isosceles triangle is drawn with its vertex at the origin and its base parallel to the x -axis. The vertices of the base are on the curve $5y = 25 - x^2$. Find the area of the largest such triangle.
68. The strength of a beam with a rectangular cross section varies directly as x and as the square of y . What are the dimensions of the strongest beam that can be sawed out of a round log whose diameter is d ? See the figure below.

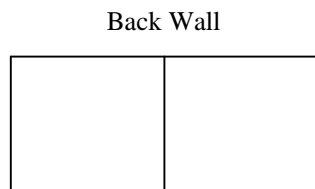


69. Find the area of the largest possible isosceles triangle with 2 sides equal to 6.
70. A lighthouse is 8 miles off a straight coast and a town is located 18 miles down the seacoast. Supplies are to be moved from the town to the lighthouse on a regular basis and at a minimum time. If the supplies can be moved at the rate of 7 miles/hour on water and 25 miles/hour over land, how far from the town should a dock be constructed for shipment of supplies?
71. Find the circular cylinder of largest lateral area that can be inscribed in a sphere of radius 4 feet. [Surface area of a cylinder, $S = 2\pi rh$, where r = radius, h = height].
72. If three sides of a trapezoid are 10 inches long, how long should the fourth side be if the area is to be a maximum? [Area of a trapezoid = $(a + b)h/2$ where a and b are the lengths of the parallel sides and h = height].
73. The stiffness of a beam of rectangular cross section is proportional to the product xy^3 . Find the stiffest beam that can be cut from a round log two feet in diameter. See the figure below.



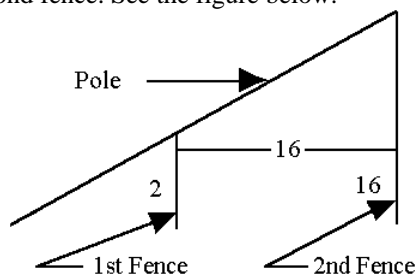
74. Find the dimensions of the maximum rectangular area that can be laid out within a triangle of base 12 and altitude 4 if one side of the rectangle lies on the base of the triangle.
75. Find the dimensions of the rectangle of greatest area with its base on the x -axis and its other two vertices above the x -axis and on $4y = 16 - x^2$.
76. Find the dimensions of the trapezoid of greatest area with its longer base on the x -axis and its other two vertices above the x -axis on $4y = 16 - x^2$. [Area of a trapezoid = $(a + b)h/2$ where a and b are the lengths of the parallel sides and h = height].

77. A poster is to contain 50 in^2 of printed matter with margins of 4 in. each at top and bottom and 2 in. at each side. Find the overall dimensions if the total area of the poster is to be a minimum.
78. A rancher is going to build a 3-sided cattle enclosure with a divider down the middle as shown below.



- The cost per foot of the three side walls will be \$6/foot, while the back wall, being taller, will be \$10/foot. If the rancher wishes to enclose an area of 180 ft^2 , what dimensions of the enclosure will minimize his cost?
79. A can containing 16 in^3 of tuna and water is to be made in the form of a circular cylinder. What dimensions of the can will require the least amount of material? ($V = \pi r^2 h$, $S = 2\pi r h$, $A = \pi r^2$)
80. Find the maximum sum of two numbers given that the first plus the square of the second is equal to 30.
81. An open-top shipping crate with a square bottom and rectangular sides is to hold 32 in^3 and requires a minimum amount of cardboard. Find the most economical dimensions.
82. Find the minimum distance from the point $(3, 0)$ to $y = \sqrt{x}$.
83. The product of two positive numbers is 48. Find the numbers, if the sum of one number and the cube of the other is to be minimized.
84. Find the values for x and y such that their product is a minimum, if $y = 2x - 10$.
85. A container with a square base, vertical sides, and an open top is to be made from 192 ft^2 of material. Find the dimensions of the container with greatest volume.
86. The cost of fuel used in propelling a dirigible varies as the square of its speed and costs \$200/hour when the speed is 100 miles/hour. Other expenses amount to \$300/hour. Find the most economical speed for a voyage of 1000 miles.
87. A rectangular garden is to be laid out with one side adjoining a neighbor's lot and is to contain 675 ft^2 . If the neighbor agrees to pay for half of the dividing fence, what should the dimensions of the garden be to ensure a minimum cost of enclosure?
88. A rectangle is to have an area of 32 in^2 . What should be its dimensions if the distance from one corner to the midpoint of the nonadjacent side is to be a minimum?
89. A slice of pizza, in the form of a sector of a circle, is to have a perimeter of 24 inches. What should be the radius of the pan to make the slice of pizza the largest? (Hint: the area of a sector of circle is $A = r^2 q / 2$ where q is the central angle in radians and the arc length along a circle is $C = r q$ with q in radians).
90. Find the minimum value for the slope of the tangent to the curve of $f(x) = x^5 + x^3 - 2x$.
91. A line is drawn through the point $P(3, 4)$ so that it intersects the y -axis at $A(0, y)$ and the x -axis at $B(x, 0)$. Find the triangle formed if x and y are positive.
92. An open cylindrical trashcan is to hold 6 ft^3 of material. What should be its dimension if the cost of material used is to be a minimum? [Surface Area, $S = 2\pi r h$ where r = radius and h = height].

93. Two fences, 16 feet apart, are to be constructed so that the first fence is 2 feet high and the second fence is higher than the first. What is the length of the shortest pole that has one end on the ground, passes over the first fence and reaches the second fence. See the figure below.



94. A line is drawn through the point $(3, 4)$ so that it intersects the y -axis at $A(0, y)$ and the x -axis at $B(x, 0)$. Find the equation of the line through AB if the triangle is to have a minimum area and both x and y are positive.

4.6 Concavity and Points of Inflection

95. Describe the concavity of the graph of $f(x) = x^4 - 24x^2$ and find the points of inflection, if any.
96. Describe the concavity of the graph of $f(x) = x^4 - 4x^3$ and find the points of inflection, if any.
97. Describe the concavity of the graph of $f(x) = x^4 - 6x^2 + 2$ and find the points of inflection, if any.
98. Describe the concavity of the graph of $f(x) = 5x^4 - x^5$ and find the points of inflection, if any.
99. Describe the concavity of the graph of $f(x) = 4x^3 - 15x^2 - 18x + 10$ and find the points of inflection, if any.
100. Describe the concavity of the graph of $f(x) = x(x - 6)^2$ and find the points of inflection, if any.
101. Describe the concavity of the graph of $f(x) = x^3 - 5x^2 + 3x + 1$ and find the points of inflection, if any.
102. Describe the concavity of the graph of $f(x) = 3x^4 - 4x^3 + 1$ and find the points of inflection, if any.
103. Describe the concavity of the graph of $f(x) = x^2 + 2/x$ and find the points of inflection, if any.
104. Describe the concavity of the graph of $f(x) = (x - 2)^3 + 1$ and find the points of inflection, if any.
105. Describe the concavity of the graph of $f(x) = (x - 4)^4 + 4$ and find the points of inflection, if any.
106. Describe the concavity of the graph of $f(x) = \sin 2x$, $x \in (0, \pi)$ and find the points of inflection, if any.

4.7 Vertical and Horizontal Asymptotes; Vertical Tangents and Cusps

107. Find the vertical and horizontal asymptotes for $f(x) = \left(\frac{x-3}{x-1}\right)^2$.
108. Find the vertical and horizontal asymptotes for $f(x) = \frac{x^2}{x^2 + 1}$.
109. Find the vertical and horizontal asymptotes for $f(x) = \frac{x^2 - x}{(x + 1)^2}$.

110. Find the vertical and horizontal asymptotes for $f(x) = \frac{3x^2}{x^2 - 4}$.
111. Find the vertical and horizontal asymptotes for $f(x) = \frac{8}{4 - x^2}$.
112. Find the vertical and horizontal asymptotes for $f(x) = \frac{x^2}{x^2 - 9}$.
113. Find the vertical and horizontal asymptotes for $f(x) = \frac{x - 1}{x - 2}$.
114. Determine whether the graph of $f(x) = 1 + (x - 2)^{1/3}$ has a vertical tangent or a vertical cusp at $c = 2$.
115. Determine whether the graph of $f(x) = (x + 1)^{1/3}(x - 4)$ has a vertical tangent or a vertical cusp at $c = -1$.
116. Determine whether the graph of $f(x) = (x + 1)^{2/3}$ has a vertical tangent or a vertical cusp at $c = -1$.
117. Determine whether the graph of $f(x) = (x - 2)^{2/3} - 1$ has a vertical tangent or a vertical cusp at $c = 2$.

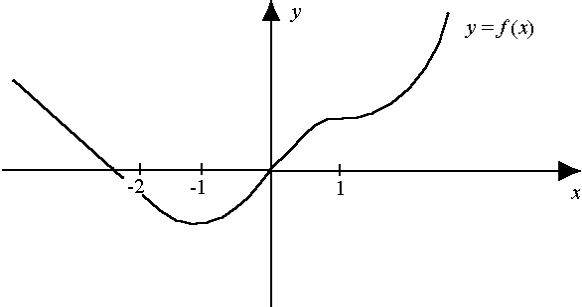
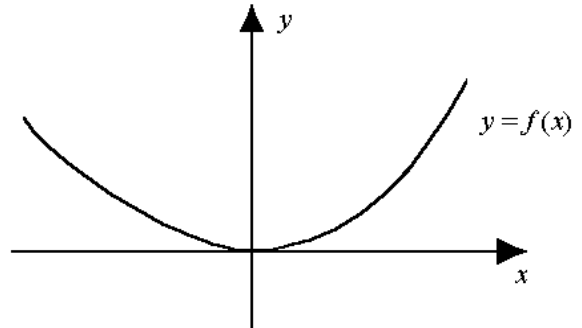
4.8 **Curve Sketching**

When graphing the following functions, you need not indicate the extrema or inflection points, but show all asymptotes (vertical, horizontal, or oblique).

118. Sketch the graph of $f(x) = 5 - 2x - x^2$.
119. Sketch the graph of $f(x) = x^3 - 9x^2 + 24x - 7$.
120. Sketch the graph of $f(x) = x^3 + 6x^2$.
121. Sketch the graph of $f(x) = x^3 - 5x^2 + 8x - 4$.
122. Sketch the graph of $f(x) = x^3 - 12x^2 + 6$.
123. Sketch the graph of $f(x) = x^3 - 6x^2 + 9x + 6$.
124. Sketch the graph of $f(x) = 3x^4 - 4x^3 + 1$.
125. Sketch the graph of $f(x) = x^2(9 - x^2)$.
126. Sketch the graph of $f(x) = x^4 - 2x^2 + 7$.
127. Sketch the graph of $f(x) = x^3 + \frac{3}{2}x^2 - 6x + 12$.
128. Sketch the graph of $f(x) = x^{1/3}(x + 4)$.
129. Sketch the graph of $f(x) = x^{2/3}(x + 5)$.
130. Sketch the graph of $f(x) = x(x - 3)^{2/3}$.
131. Sketch the graph of $f(x) = \sqrt{1 - x}$.

-
132. Sketch the graph of $f(x) = \sqrt{1 - x^2}$.
133. Sketch the graph of $f(x) = \sqrt{4 - x^2}$.
134. Sketch the graph of $f(x) = \sqrt{\frac{x}{4 - x}}$.
135. Sketch the graph of $f(x) = \frac{x - 3}{x + 2}$.
136. Sketch the graph of $f(x) = \frac{x^3 - 1}{3x^2 - 3x - 6}$.

Answers to Chapter 4 Questions

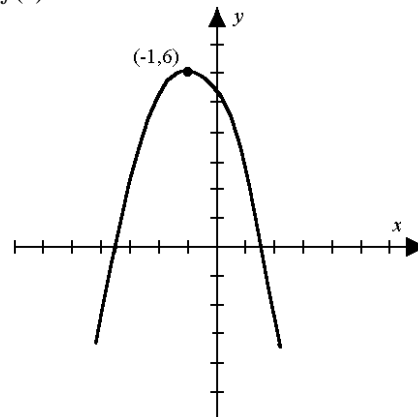
1. $c = \pm \sqrt{\frac{7}{3}} = \pm \frac{\sqrt{21}}{3}$
2. $c = 1/2$
3. Since f is not differentiable at $x = 0$, which is in $(-1, 1)$, the function $f(x)$ does not satisfy the conditions of the mean-value theorem.
4. $c = 1$
5. $c = \pm \frac{2\sqrt{3}}{3}$
6. $c = \sqrt{\frac{7}{3}} = \frac{\sqrt{21}}{3}$
7. $c = \frac{3 \pm \sqrt{3}}{3}$
8. $c = 1$
9. Since f is not differentiable at $x = 0$, which is in $(-1, 1)$, the function $f(x)$ does not satisfy the conditions of the mean-value theorem.
10. $c = \pm \frac{\sqrt{3}}{3}$
11. $c = \pm \frac{2\sqrt{3}}{3}$
12. $c = \frac{\sqrt{3}}{9}$
13. f increases on $[-2\sqrt{3}, 0]$ and $[2\sqrt{3}, \infty)$
 f decreases on $[-\infty, -2\sqrt{3}]$ and $[0, 2\sqrt{3}]$
14. f increases on $[3, \infty)$
 f decreases on $(-\infty, 0)$ and $[0, 3]$
15. f increases on $[-\sqrt{3}, 0]$ and $[\sqrt{3}, \infty)$
 f decreases on $[-\infty, -\sqrt{3}]$ and $[0, \sqrt{3}]$
16. f increases on $[0, 4]$
 f decreases on $(-\infty, 0)$ and $[4, \infty)$
17. f increases on $(-\infty, -1/2]$ and $[3, \infty)$
 f decreases on $[-1/2, 3]$
18. f increases on $(-\infty, 2]$ and $[6, \infty)$
 f decreases on $[2, 6]$
19. f increases on $[1, \infty)$
 f decreases on $(-\infty, 0]$ and $(0, 1]$
20. f increases on $[4, \infty)$
 f decreases on $(-\infty, 4]$
21. f increases on $[0, p/4]$ and $[3p/4, p]$
 f decreases on $[p/4, 3p/4]$
22. $f(x) = x^3 - 5x^2 + 3x + 1$
23. $f(x) = 3x^4 - 4x^3 + 1$
24. $f(x) = (x - 2)^3 + 9$
25. f increases on $(-\infty, 0)$ and $[3, \infty)$
 f decreases on $[0, 3]$
26. 
27. impossible
28. 
29. critical numbers $x = 0, 4/3$;
 local maximum $f(0) = 0$;
 local minimum $f(4/3) = -256/81$

30. critical numbers $x = 0, 1/8$;
local maximum $f(1/8) = 1$;
local minimum $f(0) = 0$
31. critical numbers $x = 0, 2$;
local maximum $f(2) = 3(2)^{2/3}$;
local minimum $f(0) = 0$
32. critical numbers $x = 0, 1$;
local minimum $f(1) = -1$;
no local extreme at $x = 0$
33. critical numbers $x = -2, 0, 2$;
local minimum $f(-2) = -3$;
local maximum $f(0) = 1$;
local minimum $f(2) = -3$
34. critical numbers $x = -1/2, 1$;
local minimum $f(-1/2) = -27/16$;
no local extreme at $x = 1$
35. critical numbers $x = -8/27, 1$;
local maximum $f(-8/27) = 8/27$;
local minimum $f(0) = 0$
36. critical numbers $x = -1, 1$;
local minimum $f(-1) = -2/3$;
local maximum $f(1) = 2/3$
37. critical number $x = -1$;
local minimum $f(-1) = 5$
38. critical number $x = 3/4$;
local minimum $f(3/4) = 13/8$
39. critical numbers $x = -\sqrt{3}, 0, \sqrt{3}$;
local minimum $f(-\sqrt{3}) = 8$;
local maximum $f(0) = 17$;
local minimum $f(\sqrt{3}) = 8$
40. no critical numbers; no local extreme values
41. critical number $x = -1$;
local maximum $f(-1) = 0$
42. critical numbers $x = p/6, 5p/6$;
local minimum $f(p/6) = p/6 - \sqrt{3}/2$;
local maximum $f(5p/6) = 5p/6 + \sqrt{3}/2$
43. $f'(x) = 3x^2 - 8x + 2$, $f'(0) = 2$, $f'(1) = -3$, so f' has at least one zero in $(0, 1)$. $f''(x) = 6x - 8 < 0$ on $(0, 1)$ so f' is decreasing on $(0, 1)$, and therefore, it has exactly one zero in $(0, 1)$. Hence f has exactly one critical number in $(0, 1)$.
44. no critical numbers; $f(0) = 2$ endpoint minimum and absolute minimum; $f(100) = 52$ endpoint maximum and absolute maximum
45. critical numbers $x = -1, 2$; $f(-2) = 4$ endpoint minimum; $f(-1) = 15$ local and absolute maximum; $f(2) = -12$ endpoint and absolute minimum
46. critical numbers $x = -1, 3$ but $x = 3$ is outside the interval; $f(-1) = 8/3$ endpoint and absolute maximum; $f(2) = -19/3$ endpoint and absolute minimum
47. critical numbers $x = 0, 4$; $f(-1) = -2$ endpoint minimum; $f(0) = 5$ local and absolute maximum; $f(4) = -27$ endpoint and absolute minimum; $f(5) = -20$ endpoint maximum
48. critical numbers $x = -1, 2$ but $x = -1$ is outside the interval; $f(0) = 5$ endpoint maximum; $f(2) = -15$ local and absolute minimum; $f(4) = 37$ local and absolute maximum
49. critical numbers $x = -1/2, 3/2$; $f(-1) = -1$ endpoint minimum; $f(-1/2) = 5/2$ local and absolute maximum; $f(3/2) = -27/2$ local and absolute minimum; $f(2) = -10$ endpoint maximum
50. critical numbers $x = -1, 1$ but $x = -1$ is outside the interval; $f(0) = 6$ endpoint maximum; $f(1) = 4$ local and absolute minimum; $f(3/2) = 39/8$ endpoint minimum
51. critical number $x = 0$; $f(-1) = 0$ endpoint minimum; $f(0) = 1$ absolute maximum; $f(1) = 0$ endpoint minimum
52. critical numbers $x = -2, 2$; $f(-4) = -8$ endpoint and absolute minimum; $f(-2) = 24$ local and absolute maximum; $f(2) = -8$ local and absolute minimum; $f(3) = -1$ endpoint maximum
53. critical numbers $x = 3/4, 0$; $f(-1) = 4$ endpoint maximum; $f(3/4) = 9/4(3/4)^{1/3} \approx -2.04$ local and absolute minimum; $f(8) = 10$ endpoint and absolute maximum
54. critical numbers $x = -1, 0, 1$ but $x = -1, 0$ are outside the interval; $f(1) = 1/4$ local and absolute maximum; no minimum
55. critical numbers $x = -1, 1$ but $x = -1$ is outside the interval; $f(0) =$ endpoint and absolute minimum; $f(1) = 1/2$ local and absolute maximum; $f(2) = 2/5$ endpoint minimum

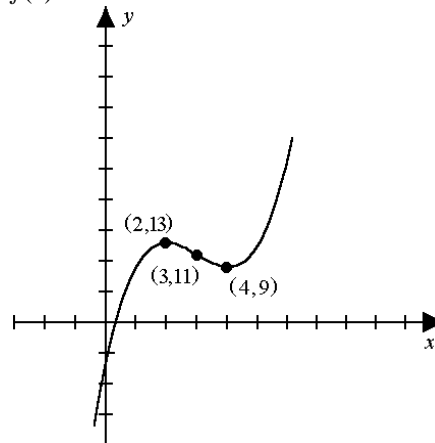
56. critical number $x = 1/2$; $f(1/2) = 4$ local and absolute minimum; no maximum
57. critical number $x = 0$; $f(x) = 0$ local and absolute minimum; no maximum
58. critical numbers $x = -1, 0, 1$; $f(-2) = 1$ endpoint maximum; $f(-1) = 0$ local and absolute minimum; $f(0) = 1$ local and absolute maximum; $f(1) = 0$ local and absolute minimum; $f(2) = 1$ endpoint maximum
59. critical number $x = 0$; $f(-2) = -5$ endpoint minimum; $f(1) = 0$ endpoint maximum
60. $a\sqrt{2}$ by $a\sqrt{2}$
61. $\sqrt{2}$ by $\sqrt{2}/2$
62. 1000 ft
63. $x = 6, y = 8$
64. $x = 110, y = 220/\pi$
65. $\sqrt{3}/3$ by $\sqrt{6}/3$
66. 2 by 4
67. $50\sqrt{3}/9$
68. $x = \frac{\sqrt{3}}{3}d, y = \frac{\sqrt{6}}{3}d$
69. 18
70. $15\frac{2}{3}$ mi
71. largest lateral area $= 32\pi$
 $h = 4\sqrt{2}, r = 2\sqrt{2}$
72. 20
73. $x = 1, y = \sqrt{3}$
74. 6 by 2
75. $x = \frac{4\sqrt{3}}{3}, y = \frac{8}{3}; \frac{8\sqrt{3}}{3}$ by $\frac{8}{3}$
76. $x = 4/3, y = 32/9$; Vertices of the trapezoid are at $(-4, 0), (-4/3, 32/9), (4/3, 32/9), (4, 0)$; Lengths of the bases are $8\frac{8}{3}$. Lengths of the sides are $40/9$ and $40/9$.
77. 5 in. by 10 in.
78. 18 ft by 10 ft
79. $r = \frac{2}{\sqrt[3]{p}}$ in., $h = \frac{4}{\sqrt[3]{p}}$ in.
80. Numbers are $1/2$ and $119/4$.
81. 4 in. by 4 in. by 2 in.
82. Minimum distance is $\frac{26}{\sqrt{2}}$, when
 $x = \frac{5}{2}$ and $y = \sqrt{\frac{5}{2}}$
83. Numbers are 24 and 2.
84. $x = 5/2$ and $y = -5$
85. 8 ft by 8 ft by 4 ft
86. $50\sqrt{6}$ miles/hr
87. 30 ft by $45/2$ ft
88. 4 in. by 8 in.
89. $r = 6$ in
90. Minimum slope of the tangent is -2 when $x = 0$.
91. $x = 6$ and $y = 8$
92. $r = \sqrt[3]{\frac{6}{p}}$ ft and $h = \sqrt[3]{\frac{6}{p}}$ ft
93. $10\sqrt{5}$ ft
94. $3y - 4x - 48 = 0$
95. concave up on $(-\infty, -2)$; concave down on $(-2, 2)$; concave up on $(2, \infty)$; points of inflection $(-2, -80)$ and $(2, 80)$
96. concave up on $(-\infty, 0)$; concave down on $(0, 2)$; concave up on $(2, \infty)$; points of inflection $(0, 0)$ and $(2, -16)$
97. concave up on $(-\infty, -1)$; concave down on $(-1, 1)$; concave up on $(1, \infty)$; points of inflection $(-1, -3)$ and $(1, -3)$

98. concave up on $(-\infty, 0)$; concave up on $(0, 3)$; concave down on $(3, \infty)$; point of inflection $(3, 162)$
99. concave down on $(-\infty, 5/4)$; concave up on $(5/4, \infty)$; point of inflection $(5/4, -28 \frac{1}{4})$
100. concave down on $(-\infty, 4)$; concave up on $(4, \infty)$; point of inflection $(4, 16)$
101. concave down on $(-\infty, 5/3)$; concave up on $(5/3, \infty)$; point of inflection $(5/3, -88/27)$
102. concave up on $(-\infty, 0)$; concave down on $(0, 2/3)$; concave up on $(2/3, \infty)$; points of inflection $(0, 1)$ and $(2/3, 11/27)$
103. concave up on $(-\infty, -\sqrt[3]{2})$; concave down on $(-\sqrt[3]{2}, 0)$; concave up on $(0, \infty)$; point of inflection $(-\sqrt[3]{2}, 0)$
104. concave down on $(-\infty, 2)$; concave up on $(2, \infty)$; point of inflection $(2, \infty)$
105. concave up on $(-\infty, 4)$; concave up on $(4, \infty)$; no points of inflection
106. concave down on $(0, \pi/2)$; concave up on $(\pi/2, \pi)$; point of inflection $(\pi/2, 0)$
107. vertical: $x = 1$; horizontal: $y = 1$
108. vertical: none; horizontal: $y = 1$
109. vertical: $x = -1$; horizontal: $y = 1$
110. vertical: $x = \pm 2$; horizontal: $y = 3$
111. vertical: $x = \pm 2$; horizontal: $y = 0$
112. vertical: $x = \pm 3$; horizontal: $y = 1$
113. vertical: $x = 2$; horizontal: $y = 1$
114. vertical tangent at $(2, 1)$
115. vertical tangent at $(-1, 0)$
116. cusp at $(-1, 0)$
117. cusp at $(2, -1)$

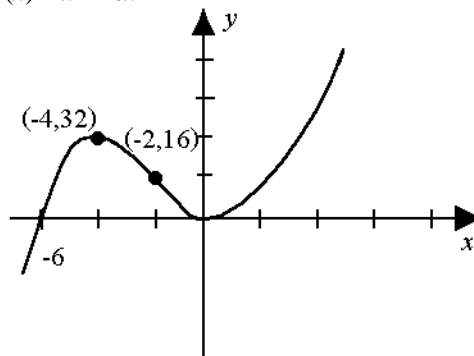
118. $f(x) = 5 - 2x - x^2$



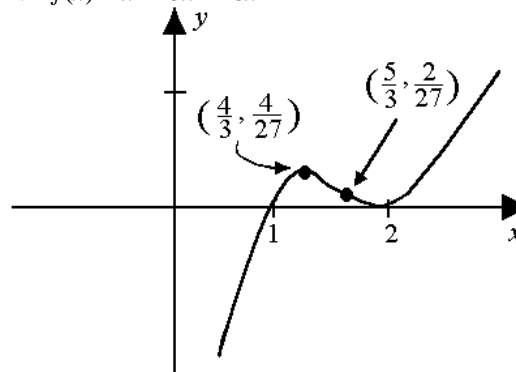
119. $f(x) = x^3 - 9x^2 + 24x - 7$



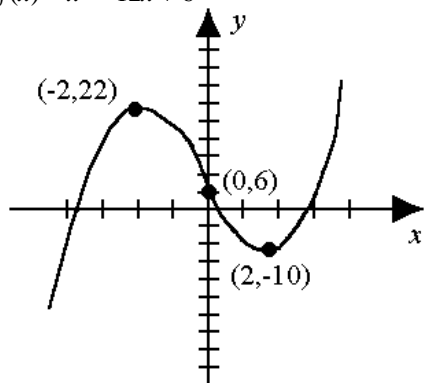
120. $f(x) = x^3 + 6x^2$



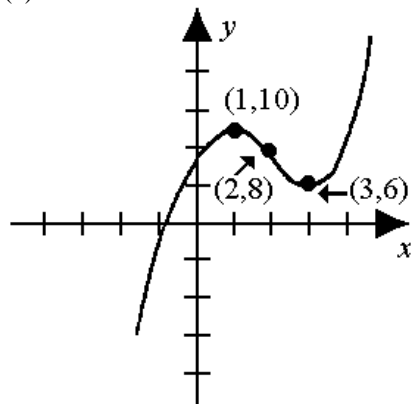
121. $f(x) = x^3 - 5x^2 + 8x - 4$



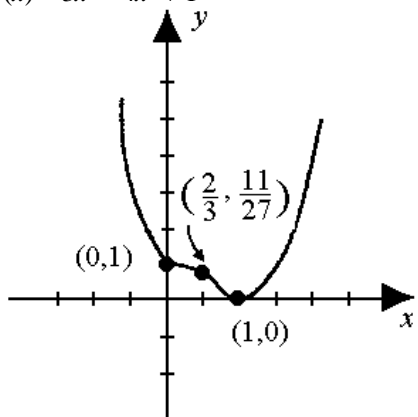
122. $f(x) = x^3 - 12x + 6$



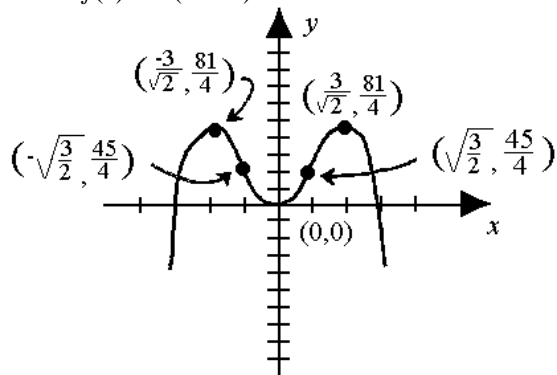
123. $f(x) = x^3 - 6x^2 + 9x + 6$



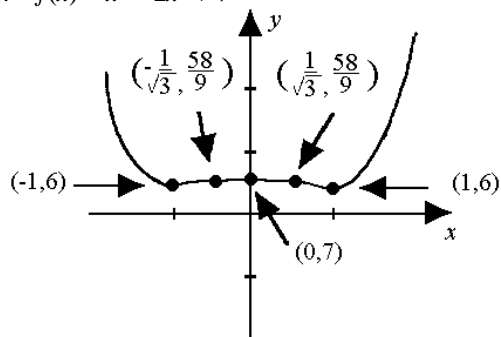
124. $f(x) = 3x^4 - 4x^3 + 1$



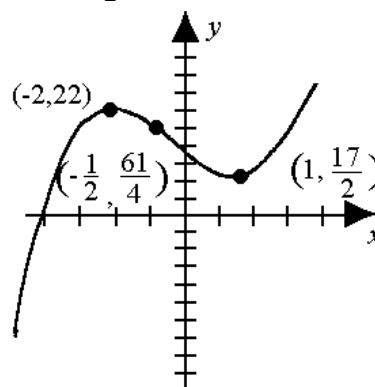
125. $f(x) = x^2(9 - x^2)$



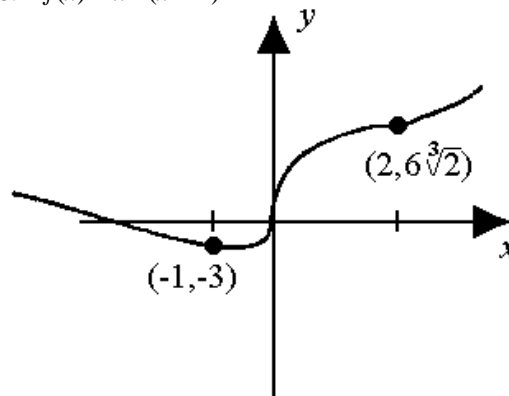
126. $f(x) = x^4 - 2x^2 + 7$



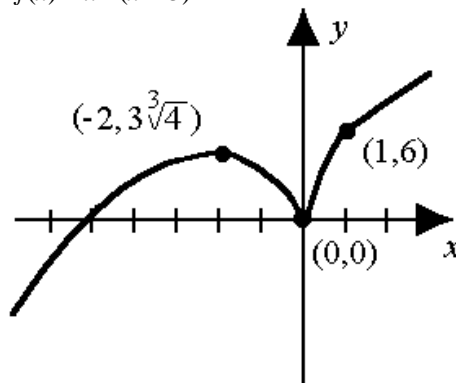
127. $f(x) = x^3 + \frac{3}{2}x^2 - 6x + 12$



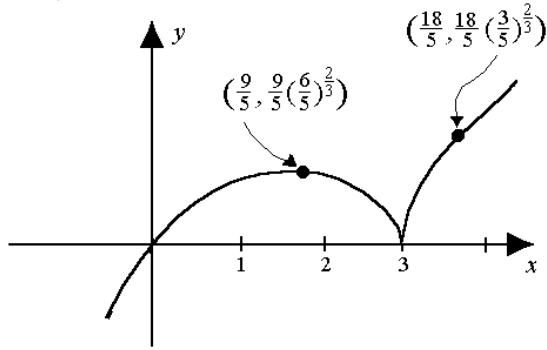
128. $f(x) = x^{1/3}(x + 4)$



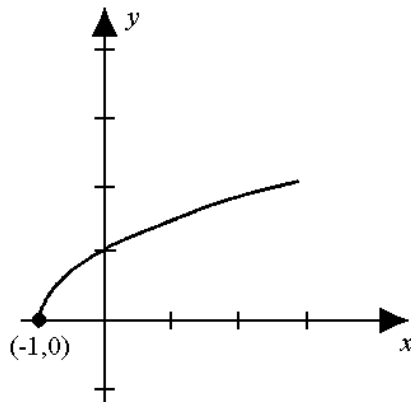
129. $f(x) = x^{2/3}(x + 5)$



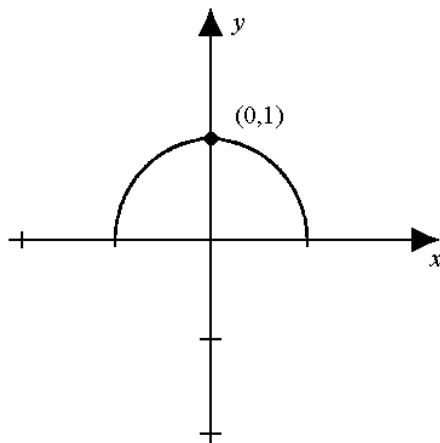
130. $f(x) = x(x-3)^{2/3}$



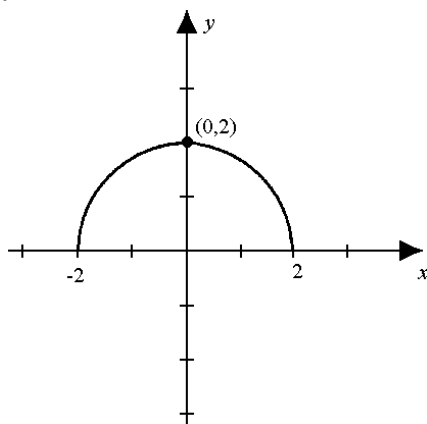
131. $f(x) = \sqrt{1+x}$



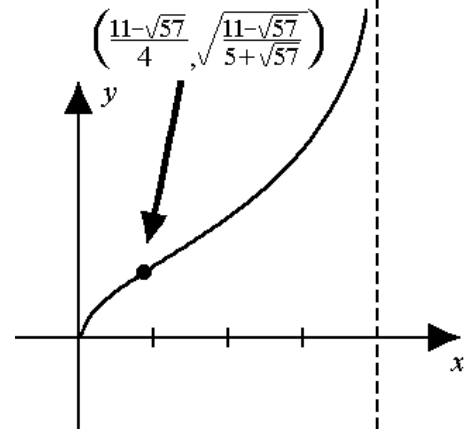
132. $f(x) = \sqrt{1-x^2}$



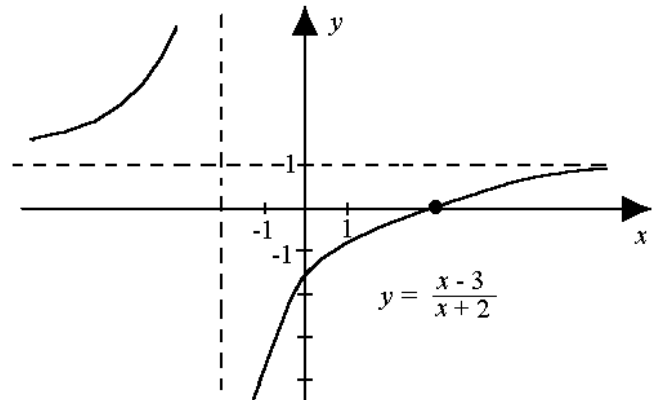
133. $f(x) = \sqrt{4-x^2}$



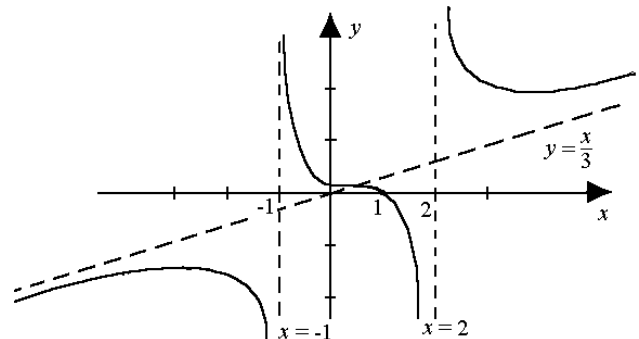
134. $f(x) = \sqrt{\frac{x}{4-x}}$



135.



136.



CHAPTER 5

Integration

5.1 The Definite Integral of a Continuous Function

1. Find $L_f(P)$ and $U_f(P)$ for $f(x) = 3x$, $x \in [-1, 0]$; $P = \left\{-1, \frac{-3}{4}, \frac{-1}{2}, \frac{-1}{4}, 0\right\}$.
2. Find $L_f(P)$ and $U_f(P)$ for $f(x) = 1 + x$, $x \in [0, 1]$; $P = \left\{0, \frac{1}{4}, \frac{1}{2}, 1\right\}$.
3. Find $L_f(P)$ and $U_f(P)$ for $f(x) = 1 + x^2$, $x \in [0, 2]$; $P = \left\{0, \frac{1}{2}, 1, \frac{3}{2}, 2\right\}$.
4. Find $L_f(P)$ and $U_f(P)$ for $f(x) = \sqrt{x-1}$, $x \in [1, 2]$; $P = \left\{1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2\right\}$.
5. Find $L_f(P)$ and $U_f(P)$ for $f(x) = |x|$, $x \in [0, 1]$; $P = \left\{0, \frac{1}{4}, \frac{3}{4}, 1\right\}$.
6. Find $L_f(P)$ and $U_f(P)$ for $f(x) = x^2$, $x \in [0, 1]$; $P = \left\{0, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, 1\right\}$.

5.2 The Function $F(x) = \int_a^x f(t) dt$

7. Given that $\int_0^1 f(x) dx = 5$, $\int_0^3 f(x) dx = 3$, $\int_3^7 f(x) dx = 1$ find each of the following:
(a) $\int_0^7 f(x) dx$ (b) $\int_1^3 f(x) dx$ (c) $\int_1^7 f(x) dx$ (d) $\int_3^0 f(x) dx$
8. Given that $\int_1^5 f(x) dx = 7$, $\int_3^5 f(x) dx = 3$, $\int_1^7 f(x) dx = 12$ find each of the following:
(a) $\int_5^7 f(x) dx$ (b) $\int_1^3 f(x) dx$ (c) $\int_3^7 f(x) dx$ (d) $\int_5^3 f(x) dx$
9. For $x > -1$, set $F(x) = \int_0^x \sqrt{2t+2} dt$
(a) Find $F(0)$ (b) Find $F'(x)$ (c) Find $F'(1)$
10. For $x > 0$, set $F(x) = \int_1^x \frac{dt}{t+1}$
(a) Find $F(1)$ (b) Find $F'(x)$ (c) Find $F'(1)$
11. For the function $F(x) = \int_0^x \frac{dt}{t^2+4}$
(a) Find $F'(-1)$ (b) Find $F'(0)$ (c) Find $F'(1)$ (d) Find $F''(x)$
12. For the function $F(x) = \int_0^x t\sqrt{t^2+9} dt$
(a) Find $F'(-1)$ (b) Find $F'(0)$ (c) Find $F'(1)$ (d) Find $F''(x)$

13. For the function $F(x) = \int_0^x t\sqrt{t^2 + 4} \, dt$
(a) Find $F'(-1)$ (b) Find $F'(0)$ (c) Find $F'(1)$ (d) Find $F''(x)$
14. For the function $F(x) = \int_0^x (t + 2)^2 \, dt$
(a) Find $F'(-1)$ (b) Find $F'(0)$ (c) Find $F'(1)$ (d) Find $F''(x)$

5.3 **The Fundamental Theorem of Integral Calculus**

15. Evaluate $\int_0^1 (3x - 2) \, dx$.
16. Evaluate $\int_{-2}^{-1} 3x^5 \, dx$.
17. Evaluate $\int_0^2 (x^2 + 2x + 5) \, dx$.
18. Evaluate $\int_{-1}^1 (x^3 - x + 5) \, dx$.
19. Evaluate $\int_1^2 \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) \, dx$.
20. Evaluate $\int_1^2 \frac{7}{\sqrt{x}} \, dx$.
21. Evaluate $\int_1^2 \frac{x^4 + 1}{x^3} \, dx$.
22. Evaluate $\int_1^2 \left(x^2 + 8x + \frac{3}{x^2} \right) \, dx$.
23. Evaluate $\int_0^1 (3\sqrt{x} + 1) \, dx$.
24. Evaluate $\int_1^2 \left(x^2 - \frac{3}{x^4} \right) \, dx$.
25. Evaluate $\int_1^2 \frac{7}{t^4} \, dt$.
26. Evaluate $\int_0^1 (x + 1)^{11} \, dx$.
27. Evaluate $\int_0^1 x^3 |2x - 1| \, dx$.
28. Evaluate $\int_0^1 (x^2 + 1)^2 \, dx$.
29. Evaluate $\int_{p/3}^{3p/2} \cos x \, dx$.

-
30. Evaluate $\int_0^{p/4} \sin x \, dx$.
31. Evaluate $\int_0^{p/4} \sec^2 x \, dx$.
32. Evaluate $\int_0^{p/6} \sec x \tan x \, dx$.
33. Evaluate $\int_1^3 f(x) \, dx$, where $f(x) = \begin{cases} (x+1)^2, & 1 \leq x \leq 2 \\ 3-x^2, & 2 < x \leq 3 \end{cases}$.
34. Evaluate $\int_0^p f(x) \, dx$, where $f(x) = \begin{cases} x, & 0 \leq x \leq p/3 \\ \sin x, & p/3 < x \leq p \end{cases}$.
35. Find the area between the graph of $f(x) = x^2$ and the x -axis for $x \in [0, 2]$.
36. Find the area between the graph of $f(x) = \frac{1}{x^2}$ and the x -axis for $x \in [1, 2]$.
37. Find the area between the graph of $f(x) = x^3 + 2$ and the x -axis for $x \in [1, 4]$.
38. Find the area between the graph of $f(x) = x^2 - x$ and the x -axis for $x \in [3, 8]$.
39. Find the area between the graph of $f(x) = x^2 - x - 6$ and the x -axis for $x \in [0, 2]$.
40. Find the area between the graph of $f(x) = \frac{1}{x^2}$ and the x -axis for $x \in [1, 4]$.
41. Find the area between the graph of $f(x) = (2x + 1)^{-2}$ and the x -axis for $x \in [0, 2]$.
42. Find the area between the graph of $f(x) = \sin x$ and the x -axis for $x \in [p/6, 2p/3]$.
43. Find the area between the graph of $f(x) = \sqrt{x + 3}$ and the x -axis for $x \in [1, 6]$.
44. Find the area between the graph of $f(x) = 2(x + 5)^{-1/2}$ and the x -axis for $x \in [-1, 4]$.
45. Sketch the region bounded by the curves $y = \frac{1}{2}x^2$ and $y = x + 4$, and find its area.
46. Sketch the region bounded by the curves $y = x^2$ and $2x - y + 3 = 0$, and find its area.
47. Sketch the region bounded by the curves $x^2 = 8y$ and $x = 2y - 8$, and find its area.
48. Sketch the region bounded by the curves $y = x^2 - 4x + 4$ and $y = x$, and find its area.
49. Sketch the region bounded by the curves $y = x + 5$ and $y = x^2 - 1$ and find its area.
50. Sketch the region bounded by the curves $y = x^3$, $x = -1$, $x = 2$, and $y = 0$ and find its area.
51. Sketch the region bounded by the curves $y = 2 - x^2$ and $y = -x$ and find its area.
52. Sketch the region bounded by the curves $y = x^3 + 1$, $x = -1$, $x = 2$, and the x -axis, and find its area.

5.4 Indefinite Integrals

53. Calculate $\int \frac{dx}{x^5}$.
54. Calculate $\int (3x + 2)^2 dx$.
55. Calculate $\int (2x^2 + 5) dx$.
56. Calculate $\int \frac{2dx}{\sqrt{4x + 5}}$.
57. Calculate $\int \frac{x^5 + 2}{x^7} dx$.
58. Calculate $\int \frac{3x^3 + 2x}{x^2} dx$.
59. Calculate $\int \frac{(1 + x)^2}{x^{1/2}} dx$.
60. Calculate $\int (x^3 + 2)^2 dx$.
61. Calculate $\int \frac{x^2 - 4}{\sqrt[3]{x^2}} dx$.
62. Calculate $\int (x + 1)\sqrt{x} dx$.
63. Calculate $\int (\sqrt{x} + 2)^2 dx$.
64. Find f given that $f'(x) = 3x + 1$ and $f(2) = 3$.
65. Find f given that $f'(x) = 2x^2 + 3x + 1$ and $f(0) = 2$.
66. Find f given that $f'(x) = \sin x$ and $f(\pi/2) = 2$.
67. Find f given that $f''(x) = 4x - 1$, $f'(1) = 3$, and $f(0) = 1$.
68. Find f given that $f''(x) = x^2 + 2x$, $f'(0) = 3$, and $f(2) = 3$.
69. Find f given that $f''(x) = \cos x$, $f'(\pi) = 2$, and $f(\pi) = 1$.
70. An object moves along a coordinate line with velocity $v(t) = 2t^2 - 6t - 8$ units per second. Its initial position (position at time $t = 0$) is 3 units to the left of the origin.
(a) Find the position of the object 2 seconds later.
(b) Find the total distance traveled by the object during those 2 seconds.
71. An object moves along a coordinate line with acceleration $a(t) = \frac{1}{2}(t + 1)^3$ units per second per second.
(a) Find the velocity function given that the initial velocity is 4 units per second.
(b) Find the position function given that the initial velocity is 4 units per second and the initial position is the origin.

72. An object moves along a coordinate line with acceleration $a(t) = (2t + 1)^{-1/2}$ units per second per second.
(a) Find the velocity function given that the initial velocity is 2 units per second.
(b) Find the position function given that the initial velocity is 2 units per second and the initial position is the origin.
73. An object moves along a coordinate line with velocity $v(t) = 3 - 2t^2$ units per second. Its initial position is 3 units to the left of the origin.
(a) Find the position of the object 5 seconds later.
(b) Find the total distance traveled by the object during those 5 seconds.
74. A ball is rolled across a level floor with an initial velocity of 28 feet per second. How far will the ball roll if the speed diminishes by 4 feet/sec² due to friction?
75. A particle, initially moving at 16 cm/sec, is slowing down at the rate of 0.8 m/sec². How far will the particle travel before coming to rest.
76. A jet plane moves with constant acceleration a from rest to a velocity of 300 ft/sec in a distance of 450 ft. Find a .
77. A particle which starts at the origin moves along the x -axis from time $t = 0$ to time $t = 3$ with velocity $v(t) = t^2 - t - 2$. Determine the final position of the particle and the total distance traveled.
78. A particle which starts at the origin moves along the x -axis from time $t = 0$ to time $t = 6$ with velocity $v(t) = 4 - t$. Determine the final position of the particle and the total distance traveled.
79. A particle which starts at the origin moves along the x -axis from time $t = 0$ to time $t = 5$ with velocity $v(t) = 8 - 2t$. Determine the final position of the particle and the total distance traveled.
80. A particle which starts at the origin moves along the x -axis from time $t = 0$ to time $t = 3$ with velocity $v(t) = t^2 - 3t + 2$. Determine the final position of the particle and the total distance traveled.
81. A particle which starts at the origin moves along the x -axis from time $t = 0$ to time $t = 4$ with velocity $v(t) = t^2 - 4t + 3$. Determine the final position of the particle and the total distance traveled.
82. A particle which starts at the origin moves along the x -axis from time $t = 0$ to time $t = 2$ with velocity $v(t) = t^2 + t - 2$. Determine the final position of the particle and the total distance traveled.
83. A particle which starts at the origin moves along the x -axis from time $t = 1$ to time $t = 3$ with velocity $v(t) = t - 8/t^2$. Determine the final position of the particle and the total distance traveled.
84. A rapid transit trolley moves with a constant acceleration and covers the distance between two points 300 feet apart in 8 seconds. Its velocity as it passes the second point is 50 ft/sec.
(a) Find its acceleration.
(b) Find the velocity of the trolley as it passes the first point.

5.5 The u-Substitution; Change of Variables

85. Calculate $\int 3x\sqrt{1 - 2x^2} \, dx$.
86. Calculate $\int t^2(2 - 3t^3)^3 \, dt$.
87. Calculate $\int \frac{4x}{\sqrt[3]{8 - x^2}} \, dx$.
88. Calculate $\int x^3\sqrt{5x^4 - 18} \, dx$.

89. Calculate $\int x\sqrt{x-5} \, dx$.
90. Calculate $\int \frac{dx}{(x+1)^2}$.
91. Calculate $\int (x^2+1)(x^3+3x)^{10} \, dx$.
92. Calculate $\int x^3\sqrt{x-2} \, dx$.
93. Calculate $\int \frac{x^2}{\sqrt{x+1}} \, dx$.
94. Calculate $\int \frac{x-2}{(x^2-4x+4)^2} \, dx$.
95. Evaluate $\int_0^1 \frac{dx}{\sqrt{x+1}}$.
96. Evaluate $\int_1^2 (x^2+1)\sqrt{2x^3+6x} \, dx$.
97. Evaluate $\int_0^3 \sqrt{x^4+2x^2+1} \, dx$.
98. Evaluate $\int_0^1 x\sqrt{9x^2+16} \, dx$.
99. Evaluate $\int_0^1 \frac{x}{(1+x^2)^2} \, dx$.
100. Evaluate $\int_0^3 x\sqrt{9-x^2} \, dx$.
101. Evaluate $\int_0^4 \frac{x}{\sqrt{9+x^2}} \, dx$.
102. Evaluate $\int_1^2 \frac{x^3}{\sqrt{3x^4+1}} \, dx$.
103. Evaluate $\int_1^2 x^2\sqrt{x-1} \, dx$.
104. Evaluate $\int_1^5 x\sqrt{2x-1} \, dx$.
105. Calculate $\int \sin(5x+3) \, dx$.
106. Calculate $\int \cos^2(2x+1) \, dx$.
107. Calculate $\int \cos^2 2x \sin 2x \, dx$.

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108. Calculate $\int x^{1/2} \sin x^{3/2} dx$.
109. Calculate $\int (2 + \cos 3x)^{3/2} \sin 3x dx$.
110. Calculate $\int \frac{\cos 2x}{(1 + \sin 2x)^2} dx$.
111. Calculate $\int \frac{dx}{\cos^2 x}$.
112. Calculate $\int (x^{-2} \sec^2 x + 3 \sin 2x) dx$.
113. Calculate $\int \frac{dx}{\sin^2 3x}$.
114. Calculate $\int (2 + \sin 3t)^{1/2} \cos 3t dt$.
115. Calculate $\int \csc 2t \cot 2t dt$.
116. Calculate $\int \tan^3 5x \sec^2 5x dx$.
117. Calculate $\int \frac{\sin x dx}{\cos^3 x}$.
118. Calculate $\int x \sec^2 x^2 dx$.
119. Calculate $\int x^3 \sin(x^4 + 2) dx$.
120. Evaluate $\int_{\pi/6}^{\pi/3} (\cos t - \csc t \cot t) dt$.
121. Evaluate $\int_0^{\pi/4} \sec^2 t dt$.
122. Evaluate $\int_{\pi/3}^{2\pi/3} (t - \csc t \cot t) dt$.
123. Evaluate $\int_{4\pi}^{2\pi} \frac{\sin(1/t)}{t^2} dt$.
124. Evaluate $\int_0^{\pi/4} \cos^2 3t \sin 3t dt$.
125. Evaluate $\int_{\sqrt{\pi/3}}^{\sqrt{\pi/2}} \frac{t}{\sin^2(t^2/2)} dt$.
126. Evaluate $\int_0^{\pi/8} (2x + \sec 2x \tan 2x) dx$.
127. Find the area bounded by $y = \cos \pi x$, $y = \sin \pi x$, $x = 1/4$, and $x = 1/2$.

128. Find the area bounded by $y = \sin x$, $y = 2x/p$, and $x = 0$.
129. Find the area bounded by $y = \frac{1}{2} \cos^2 px$, $y = -\sin^2 px$, $x = 0$, and $x = \frac{1}{2}$.

5.6 Additional Properties of the Definite Integral

130. Calculate $\frac{d}{dx} \left[\int_1^x (t^3 + 1) dt \right]$.
131. Calculate $\frac{d}{dx} \left[\int_0^x (t + 1)^{1/2} dt \right]$.
132. Calculate $\frac{d}{dx} \left[\int_2^{3x^2} (t^2 - 4)^{2/3} dt \right]$.
133. Calculate $\frac{d}{dx} \left(\int_x^{x^2} \frac{1}{\sqrt{1 - 3t^2}} dt \right)$.
134. Calculate $\frac{d}{dx} \left(\int_{1-x}^{1+x} \frac{dt}{\sqrt{2t + 5}} \right)$.
135. Calculate $\frac{d}{dx} \left(\int_{1/x}^{1/\sqrt{x}} \sin t^2 dt \right)$.
136. Find $H'(2)$ given that $H(x) = \int_{\sqrt{x}}^{x^2+x} \frac{3}{2 + \sqrt{2t}} dt$.
137. Find $H'(2)$ given that $H(x) = \frac{1}{x} \int_2^x [2t + H'(t)] dt$.
138. Suppose f is continuous and $\int_a^b |f(x)| dx = 0$.
- (a) Does it necessarily follow that $\int_a^b f(x) dx = 0$?
- (b) What can you conclude about $f(x)$ on $[a, b]$?
139. Let Ω be the region below the graph of $f(x) = 2x + 3$, $x \in [0, 2]$. Draw a figure showing the Riemann sum $S^*(P)$ as an estimate for this area. Take $P = \left\{ 0, \frac{1}{4}, \frac{3}{4}, 1, \frac{3}{2}, 2 \right\}$ and let the x_i^* be the midpoints of the subintervals. Evaluate the Riemann sum.
140. Set $f(x) = 3x + 1$, $x \in [0, 1]$. Take $P = \left\{ 0, \frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, 1 \right\}$ and set $x_1^* = \frac{1}{16}$, $x_2^* = \frac{3}{16}$, $x_3^* = \frac{5}{16}$, $x_4^* = \frac{7}{16}$, $x_5^* = \frac{9}{16}$. Calculate the following:
- (a) $\Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4, \Delta x_5$
- (b) $\|P\|$
- (c) m_1, m_2, m_3, m_4, m_5
- (d) $f(x_1^*), f(x_2^*), f(x_3^*), f(x_4^*), f(x_5^*)$
- (e) M_1, M_2, M_3, M_4, M_5
- (f) $L_f(P)$

- (g) $S^*(P)$
- (h) $U_f(P)$
- (i) $\int_0^1 f(x) \, dx$

5.7 Mean-Value Theorems for the Integrals; Average Values

- 141. Determine the average value of $f(x) = \sqrt{4x+1}$ on the interval $[0, 2]$ and find a point c in this interval at which the function takes on this average value.
- 142. Determine the average value of $f(x) = x^3 \sqrt{3x^4 + 1}$ on the interval $[-1, 2]$.
- 143. Determine the average value of $f(x) = x^3 + 1$ on the interval $[0, 2]$ and find a point c in this interval at which the function takes on this average value.
- 144. Determine the average value of $f(x) = x \cos x^2$ on the interval $[0, \pi/2]$.
- 145. Determine the average value of $f(x) = \cos x$ on the interval $[0, \pi/2]$ and find a point c in this interval at which the function takes on this average value.
- 146. Find the average distance of the parabolic arc $y = 2(x+1)^2$, $x \in [0, \sqrt{2}]$ from (a) the x -axis; and (b) the y -axis.
- 147. A rod lies on the x -axis from $x = 1$ to $x = L > 1$. If the density at any point x on the rod is $3/x^3$, find the center of mass of the rod.

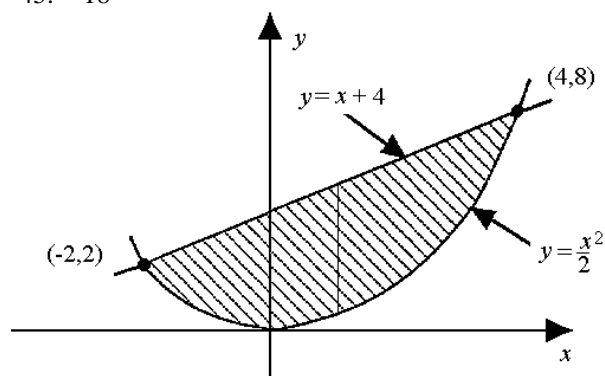
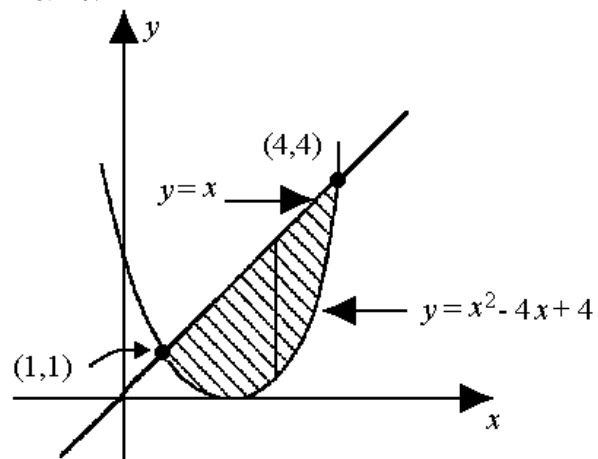
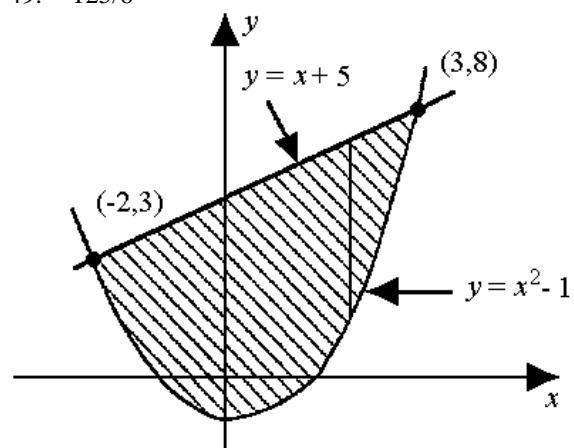
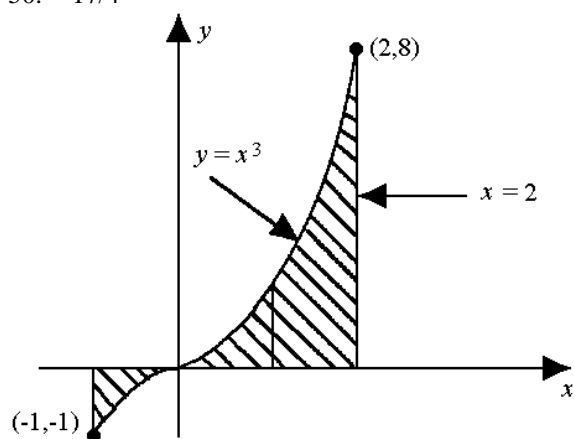
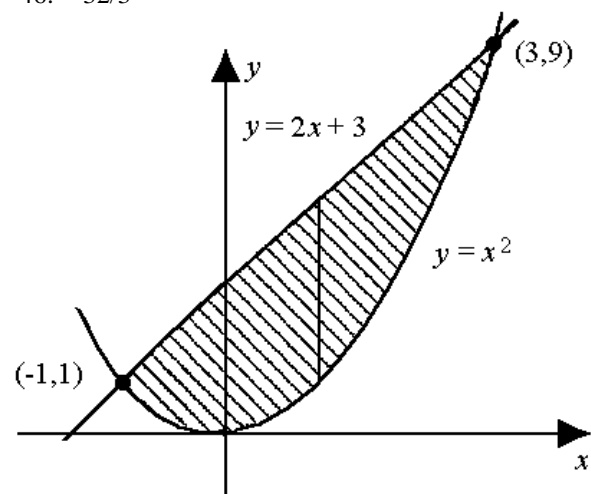
Answers to Chapter 5 Questions

17. $50/3$

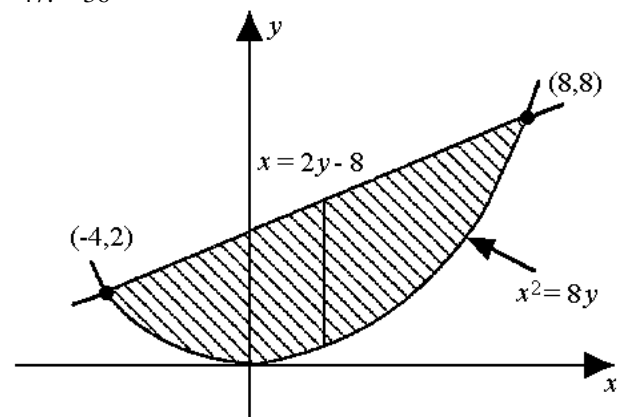
43. $38/3$

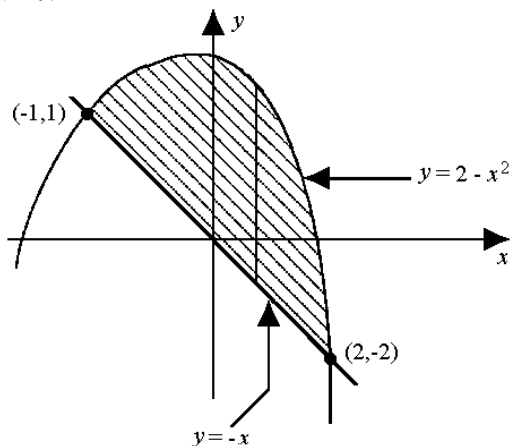
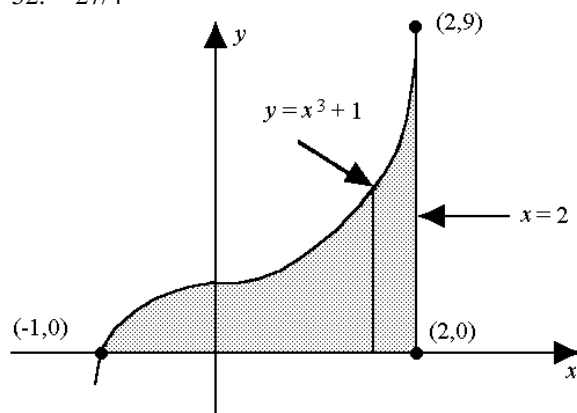
44. 4

45. 18


 48. $9/2$

 49. $125/6$

 50. $17/4$

 46. $32/3$


47. 36



51. $9/2$ 52. $27/4$ 53. $-\frac{1}{4x^4} + C$ 54. $\frac{1}{9}(3x+2)^3 + C$ 55. $\frac{2}{3}x^3 + 5x + C$ 56. $\sqrt{4x+5} + C$ 57. $-\frac{1}{x} - \frac{1}{3x^6} C$ 58. $\frac{3}{2}x^2 + 2\ln x + C$ 59. $2\sqrt{x} + \frac{4}{3}x^{3/2} + \frac{2}{5}x^{5/2} + C$ 60. $\frac{x^7}{7} + x^4 + 4x + C$ 61. $\frac{3}{7}x^{7/3} - 12x^{1/3} + C$ 62. $\frac{2}{5}x^{5/3} + \frac{2}{3}x^{3/2} + C$ 63. $\frac{x^2}{2} + \frac{8}{3}x^{3/2} + 4x + C$ 64. $\frac{3}{2}x^2 + x - 5$ 65. $\frac{2}{3}x^3 + \frac{3}{2}x^2 + x + 2$ 66. $-\cos x + 2$ 67. $\frac{2}{3}x^3 + \frac{x^2}{2} + 2x + 1$ 68. $\frac{x^4}{12} + \frac{x^3}{3} + 3x - 7$ 69. $-\cos x + 2x - 2\pi$ 70. (a) $-77/3$ (b) $68/3$ 71. (a) $\frac{1}{8}(t+1)^4 + \frac{31}{8}$
(b) $\frac{1}{40}(t+1)^5 + \frac{31}{8}t - \frac{1}{40}$ 72. (a) $\sqrt{2t+1} + 1$
(b) $\frac{1}{3}(2t+1)^{3/2} + t - \frac{1}{3}$ 73. (a) $-71 \frac{1}{3}$ units to the left of the origin
(b) $\frac{205 + 3\sqrt{6}}{3} \approx 70.78$ units

74. 98 ft

75. 160 cm

76. 100 ft/sec²77. $-3/2$; $31/6$

78. 6; 10

79. 15; 17

80. $3/2$; $11/6$ 81. $4/3$; 482. $2/3$; 3

83. $-4/3; 11/3$
84. (a) $25/8 \text{ ft/sec}^2$ (b) 25 ft/sec
85. $-\frac{1}{2}(1-2x^2)^{3/2} + C$
86. $-\frac{1}{36}(2-3t^3)^4 + C$
87. $-3(8-x^2)^{2/3} + C$
88. $\frac{1}{30}(5x^4-18)^{3/2} + C$
89. $\frac{2}{5}(x-5)^{5/2} + \frac{10}{3}(x-5)^{3/2} + C$
90. $-\frac{1}{x+1} + C$
91. $\frac{1}{33}(x^3+3x)^{11} + C$
92. $\frac{2}{9}(x-2)^{9/2} + \frac{12}{7}(x-2)^{7/2} + \frac{24}{5}(x-2)^{5/2} + \frac{16}{3}(x-2)^{3/2} + C$
93. $\frac{2}{5}(x+1)^{5/2} + \frac{4}{3}(x+1)^{3/2} + 2(x+1)^{1/2} + C$
94. $\frac{-1}{2(x^2-4x+4)} + C$
95. $2(\sqrt{2}-1)$
96. $\frac{1}{9}(28^{3/2}-8^{3/2}) \approx 13.95$
97. 12
98. $61/27$
99. $1/4$
100. 9
101. 2
102. $5/6$
103. $184/105$
104. $428/15$
105. $-\frac{1}{5}\cos(5x+3) + C$
106. $\frac{x}{2} + \frac{1}{8}\sin 2(2x+1) + C$
107. $-\frac{1}{6}\cos^3 2x + C$
108. $-\frac{2}{3}\cos x^{3/2} + C$
109. $-\frac{2}{15}(2+\cos 3x)^{5/2} + C$
110. $-\frac{1}{2(1+\sin 2x)} + C$
111. $\tan x + C$
112. $-\frac{1}{x} + \tan x - \frac{3}{2}\cos 2x + C$
113. $-\frac{1}{3}\cot 3x + C$
114. $\frac{2}{9}(2+\sin 3t)^{3/2} + C$
115. $-\frac{1}{2}\csc 2t + C$
116. $\frac{1}{20}\tan^4 5x + C$
117. $\frac{1}{2}\sec^2 x + C$
118. $\frac{1}{2}\tan x^2 + C$
119. $-\frac{1}{4}\cos(x^4+2) + C$
120. $\frac{7\sqrt{3}-15}{6} \approx -0.479$
121. 1
122. $1 + \frac{5p^2}{72} - \frac{2}{\sqrt{3}} \approx -0.5307$

123. $\frac{-\sqrt{2}}{2}$

124. $7/72$

125. $\sqrt{3} - 1$

126. $\frac{p^2}{64} + \frac{\sqrt{2}}{2} - \frac{1}{2} \approx -0.3613$

127. $\frac{1}{p}(\sqrt{2} - 1) \approx 0.1318$

128. $1 - \pi/4$

129. $3/8$

130. $x^3 + 1$

131. $(x + 1)^{1/2}$

132. $6x(9x^4 - 4)^{2/3}$

133. $\frac{2x}{\sqrt{1-3x^4}} - \frac{1}{\sqrt{1-3x^2}}$

134. $\frac{1}{\sqrt{2x+7}} + \frac{1}{\sqrt{-2x+7}}$

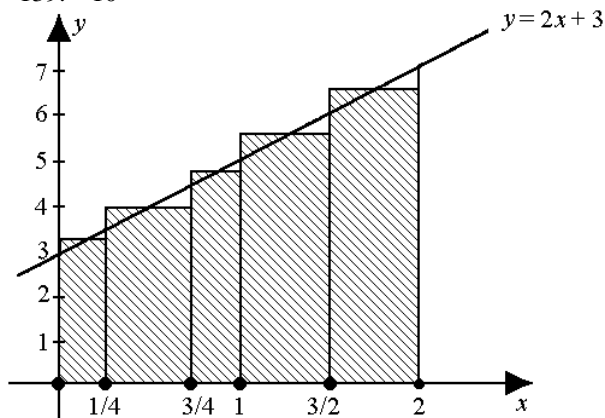
135. $\frac{\sin(1/x^2)}{x^2} - \frac{\sin(1/x)}{2x^{3/2}}$

136. $\frac{15}{2+2\sqrt{3}} - \frac{3}{4\sqrt{2}+4(2^{1/4})}$

137. 4

138. (a) yes
(b) $f(x) = 0$ for all $x \in [a, b]$

139. 10



140. (a) $\Delta x_1 = 1/8, \Delta x_2 = 1/8, \Delta x_3 = 1/8, \Delta x_4 = 1/8, \Delta x_5 = 1/2$

(b) $\|P\| = 1/2$

(c) $m_1 = 1, m_2 = 11/8, m_3 = 7/4, m_4 = 17/8, m_5 = 5/2$

(d) $f(x_1^*) = 19/16, f(x_2^*) = 25/16, f(x_3^*) = 17/8, f(x_4^*) = 23/8, f(x_5^*) = 13/4$

(e) $M_1 = 11/8, M_2 = 7/4, M_3 = 17/8, M_4 = 5/2, M_5 = 4$

(f) $L_f(P) = 65/32$

(g) $S^*(P) = 83/32$

(h) $U_f(P) = 95/32$

(i) $\int_0^1 f(x)dx = 5/2 = 80/32$

141. $13/6; c = 133/144$

142. $335/54$

143. $3; c = \sqrt[3]{2} \approx 1.26$

144. $\frac{1}{\sqrt{2p}}$

145. $2/\pi; c \approx 0.8807 \text{ rad}$

146. (a) $\frac{10+6\sqrt{2}}{3} \approx 6.1618$ (b) $2/3$

147. $2L(L+1)$

CHAPTER 6

Some Applications of the Interval

6.1 More on Area

1. Sketch the region bounded by $y = x^2 - 4x + 5$ and $y = 2x - 3$. Represent the area of the region by one or more integrals (a) in terms of x ; (b) in terms of y .
2. Sketch the region bounded by $x = y^2 - 4y + 2$ and $x = y - 2$. Represent the area of the region by one or more integrals (a) in terms of x ; (b) in terms of y .
3. Sketch the region bounded by $y = 2x - x^2$ and $y = -3$. Represent the area of the region by one or more integrals (a) in terms of x ; (b) in terms of y .
4. Sketch the region bounded by $y = x + 4/x^2$, the x -axis, $x = 2$, and $x = 4$.
(a) Represent the area of the region by one or more integrals. (b) Find the area.
5. Sketch the region bounded by $y = 4x - x^2$ and $y = 3$. Represent the area of the region by one or more integrals (a) in terms of x ; (b) in terms of y . (c) Find the area.
6. Sketch the region bounded by $x = y^2 - 4y$ and $x = y$ and find its area.
7. Sketch the region bounded by $y = 3 - x^2$, $y = -x + 1$, $x = 0$ and $x = 2$, and find its area.
8. Sketch the region bounded by $x = 3y - y^2$ and $x + y = 3$, and find its area.

6.2 Volume by Parallel Cross Sections; Discs and Washers

9. Sketch the region Ω bounded by $x + y = 4$, $y = 0$, and $x = 0$, and find the volume of the solid generated by revolving the region about the x -axis.
10. Sketch the region Ω bounded by $y^2 = 4x$, $y = 2$, and $x = 4$, and find the volume of the solid generated by revolving the region about the x -axis.
11. Sketch the region Ω bounded by $y = 4 - x^2$ and $y = x + 2$, and find the volume of the solid generated by revolving the region about the x -axis.
12. Sketch the region Ω bounded by $y = x^2$, $y = 4$, and $x = 0$, and find the volume of the solid generated by revolving the region about the x -axis.
13. Sketch the region Ω bounded by $y^2 = x^3$, $x = 1$, and $y = 0$, and find the volume of the solid generated by revolving the region about the x -axis.
14. Sketch the region Ω bounded by $y^2 = x^2$, $x = 0$, and $y = 4$, and find the volume of the solid generated by revolving the region about the y -axis.
15. Sketch the region Ω bounded by $y = \sqrt{x}$, $y = 0$, and $x = 9$, and find the volume of the solid generated by revolving the region about the y -axis.
16. Sketch the region Ω bounded by $y^2 = 4x$, $x = 4$, and $y = 0$, and find the volume of the solid generated by revolving the region about the y -axis.

17. Sketch the region Ω bounded by $y^2 = x^3$, $x = 1$, and $y = 0$, and find the volume of the solid generated by revolving the region about the y -axis.
18. Sketch the region Ω bounded by $y = x^3$, $x = 2$, and $y = 0$, and find the volume of the solid generated by revolving the region about the line $x = 2$.
19. Sketch the region Ω bounded by $y = x^2$, $y = 0$, and $x = 2$, and find the volume of the solid generated by revolving the region about the line $x = 2$.
20. Sketch the region Ω bounded by $y = x^3$, $x = 1$, and $y = -1$, and find the volume of the solid generated by revolving the region about the line $y = -1$.
21. Sketch the region Ω bounded by $y = x^3/2$, $x = 2$, and $y = 0$, and find the volume of the solid generated by revolving the region about the line $y = 4$.
22. The base of a solid is a circle of radius 2. All sections that are perpendicular to the diameter are squares. Find the volume of the solid.
23. The steeple of a church is constructed in the form of a pyramid 45 feet high. The cross sections are all squares, and the base is a square of side 15 feet. Find the volume of the steeple.

6.3 Volume by the Shell Method

24. Sketch the region Ω bounded by $y = \sqrt{x}$, $y = 0$, and $x = 9$, and use the shell method to find the volume of the solid generated by revolving Ω about the y -axis.
25. Sketch the region Ω bounded by $y^2 = 4x$, $x = 4$, and $y = 0$, and use the shell method to find the volume of the solid generated by revolving Ω about the y -axis.
26. Sketch the region Ω bounded by $y^2 = 4x$, $y = 2$, and $x = 4$, and use the shell method to find the volume of the solid generated by revolving Ω about the y -axis.
27. Sketch the region Ω bounded by $y = 2x + 3$, $x = 1$, and $x = 4$, and $y = 0$, and use the shell method to find the volume of the solid generated by revolving Ω about the y -axis.
28. Sketch the region Ω bounded by $y = \sqrt{x + 1}$, $x = 0$, $y = 0$, and $x = 3$, and use the shell method to find the volume of the solid generated by revolving Ω about the y -axis.
29. Sketch the region Ω bounded by $y = x^2$, $y = 4$, and $x = 0$, and use the shell method to find the volume of the solid generated by revolving Ω about the x -axis.
30. Sketch the region Ω bounded by $y = x^2$ and $x = y^2$, and use the shell method to find the volume of the solid generated by revolving Ω about the x -axis.
31. Sketch the region Ω bounded by $y = x^3$ and $y = x$, and use the shell method to find the volume of the solid generated by revolving Ω about the x -axis.
32. Sketch the region Ω bounded by the first quadrant of the circle $x^2 + y^2 = r^2$ and use the shell method to find the volume of the solid generated by revolving Ω about the x -axis.
33. Sketch the region Ω bounded by $x = 2y - y^2$ and $x = 0$, and use the shell method to find the volume of the solid generated by revolving Ω about the x -axis.
34. Sketch the region Ω bounded by $y = 2x$, $x = 0$, and $y = 2$, use the shell method to find the volume of the solid generated by revolving Ω about $x = 1$.

35. Sketch the region Ω bounded by $y^2 = 4x$, $y = x$, and use the shell method to find the volume of the solid generated by revolving Ω about $x = 4$.
36. Sketch the region Ω bounded by $y = x^2$, $y = 0$, and $x = 2$, and use the shell method to find the volume of the solid generated by revolving Ω about $x = 2$.
37. Sketch the region Ω bounded by $y = x^3$ and $x = 2$, and use the shell method to find the volume of the solid generated by revolving Ω about $x = 2$.

6.4 The Centroid of a Region; Pappus's Theorem on Volumes

38. Sketch the region bounded by $y = x^2$, $y = \sqrt{x}$. Determine the centroid of the region and the volume generated by revolving the region about each of the coordinate axes.
39. Sketch the region bounded by $y^2 = x^3$, $x = 1$, and $y = 0$. Determine the centroid of the region and the volume generated by revolving the region about each of the coordinate axes.
40. Sketch the region bounded by $y^2 = 8x$, $y = 0$, and $x = 2$. Determine the centroid of the region and the volume generated by revolving the region about each of the coordinate axes.
41. Sketch the region bounded by $y = 2x$, $y = 4$, $y = 0$, and $x = 2$. Determine the centroid of the region and the volume generated by revolving the region about each of the coordinate axes.
42. Sketch the region bounded by $x^2 = 2y$ and $2x - y = 0$. Determine the centroid of the region and the volume generated by revolving the region about each of the coordinate axes.
43. Find the centroid of the bounded region determined by $y = 2x^2$ and $x - 2y + 3 = 0$.
44. Find the centroid of the bounded region determined by $y + 1 = 0$ and $x^2 + y = 0$.
45. Find the centroid of the bounded region determined by $y + x^2 + 2x$ and $y = 2x + 1$.
46. Locate the centroid of a solid cone of base radius 2 cm and height 4 cm.
47. Locate the centroid of a solid generated by revolving the region below the graph of $f(x) = 2 - x^2$, $x \in [0, 1]$.
 - (a) about the x -axis.
 - (b) about the y -axis.

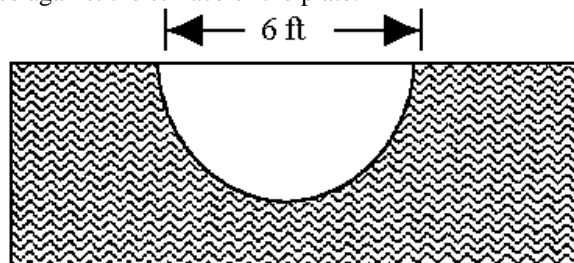
6.5 The Notion of Work

48. Find the work done by the force $F(x) = (x + 1) / \sqrt{x}$ pounds in moving an object from $x = 1$ foot to $x = 4$ feet along the x -axis.
49. Find the work done by the force $F(x) = x\sqrt{x + 3}$ Newtons in moving an object from $x = 1$ meter to $x = 6$ meters along the x -axis.
50. A spring exerts a force of 2 pounds when stretched 6 inches. How much work is required in stretching the spring from a length of 1 foot to a length of 2 feet?
51. A spring whose natural length is 10 feet exerts a force of 400 pounds when stretched 0.4 feet. How much work is required to stretch the spring from its natural length to 12 feet?
52. A spring exerts a force of 1 ton when stretched 10 feet beyond its natural length. How much work is required to stretch the spring 8 feet beyond its natural length?

53. A spring whose natural length is 18 inches exerts a force of 10 pounds when stretched 16 inches. How much work is required to stretch the spring from 4 inches beyond its natural length?
54. A dredger scoops a shovel full of mud weighing 2000 pounds from the bottom of a river at a constant rate. Water leaks uniformly at such a rate that half the weight of the contents is lost when the scoop has been lifted 25 feet. How much work is done by the dredger in lifting the mud this distance?
55. A 60-foot length of steel chain weighing 10 pounds per foot is hanging from the top of a building. How much work is required to pull half of it to the top?
56. A 50-foot chain weighing 10 pounds per foot supports a steel beam weighing 1000 pounds. How much work is done in winding 40 feet of the chain onto a drum?
57. A bucket weighing 1000 pounds is to be lifted from the bottom of a shaft 20 feet deep. The weight of the cable used to hoist it is 10 pounds per foot. How much work is done lifting the bucket to the top of the shaft?
58. A cylindrical tank 8 feet in diameter and 10 feet high is filled with water weighing 62.4 lbs/ft^3 . How much work is required to pump the water over the top of the tank?
59. A cylindrical tank is to be filled with gasoline weighing 50 lbs/ft^3 . If the tank is 20 feet high and 10 feet in diameter, how much work is done by the pump in filling the tank through a hole in the bottom of the tank?
60. A cylindrical tank 5 feet in diameter and 10 feet high is filled with oil whose density is 48 lbs/ft^3 . How much work is required to pump the water over the top of the tank?
61. A conical tank (vertex down) has a diameter of 9 feet and is 12 feet deep. If the tank is filled with water of density 62.4 lbs/ft^3 , how much work is required to pump the water over the top?
62. A conical tank (vertex down) has a diameter of 8 feet and is 10 feet deep. If the tank is filled to a depth of 6 feet with water of density 62.4 lbs/ft^3 , how much work is required to pump the water over the top?

6.6 Fluid Pressure and Fluid Forces

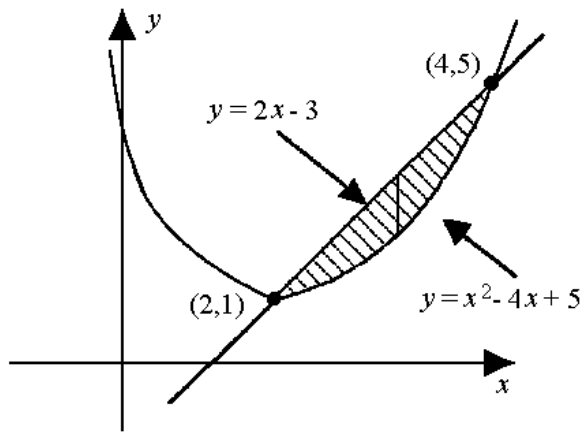
63. A flat rectangular plate, 6 feet long and 3 feet wide, is submerged vertically in water (density 62.4 lbs/ft^3) with the 3-foot edge parallel to and 2 feet below the surface. Find the force against the surface of the plate.
64. A flat rectangular plate, 6 feet long and 3 feet wide, is submerged vertically in water (density 62.4 lbs/ft^3) with the 6-foot edge parallel to and 2 feet below the surface. Find the force against the surface of the plate.
65. A flat triangular plate whose dimensions are 5, 5, and 6 feet is submerged vertically in water (density 62.4 lbs/ft^3) so that its longer side is at the surface and parallel to it. Find the force against the surface of the plate.
66. A flat triangular plate whose dimensions are 5, 5, and 6 feet is submerged vertically in water (density 62.4 lbs/ft^3) so that its longer side is at the bottom and parallel to the surface, and its vertex is 2 feet below the surface. Find the force against the surface of the plate.
67. A flat plate, shaped in the form of a semicircle 6 feet in diameter, is submerged in water (density 62.4 lbs/ft^3) as shown. Find the force against the surface of the plate.



Answers to Chapter 6 Questions

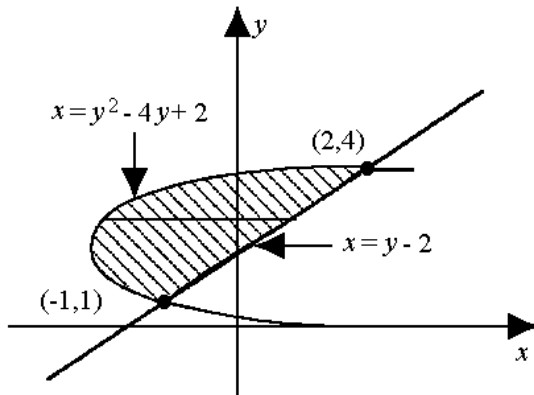
1. (a) $\int_2^4 [(2x-3) - (x^2-4x+5)]dx$

(b) $\frac{1}{2} \int_1^5 (1+2\sqrt{y-1}-y)dy$



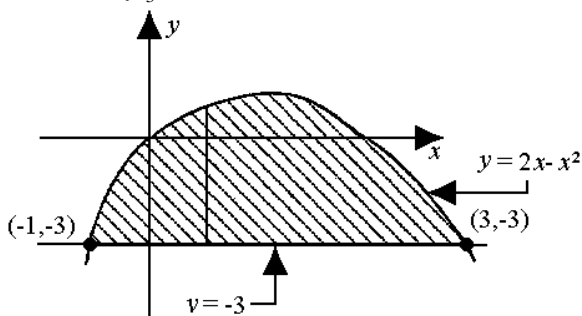
2. (a) $2 \int_{-2}^{-1} \sqrt{x+2} dx + \int_{-1}^2 (\sqrt{x+2} - x)dx$

(b) $\int_1^4 [(y-2) - (y^2-4y+2)]dy$



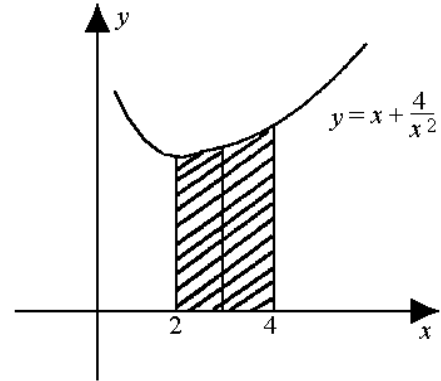
3. (a) $\int_{-1}^3 [(2x-x^2) - (-3)]dx$

(b) $2 \int_{-3}^1 \sqrt{1-y} dy$



4. (a) $\int_2^4 \left(x + \frac{4}{x^2}\right)dx$

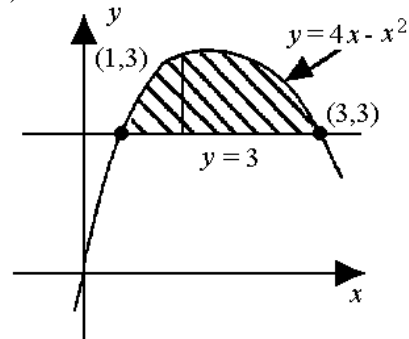
(b) 7



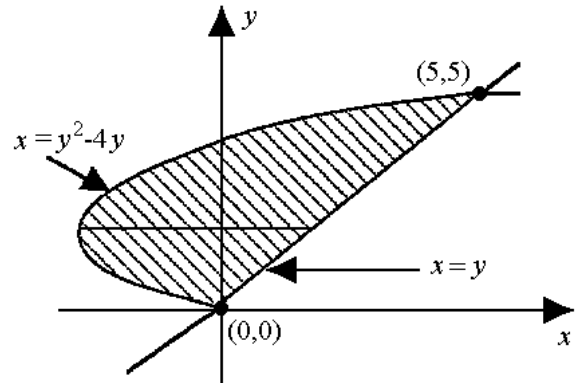
5. (a) $\int_1^3 (4x - x^2 - 3)dx$

(b) $2 \int_3^4 \sqrt{4-y} dy$

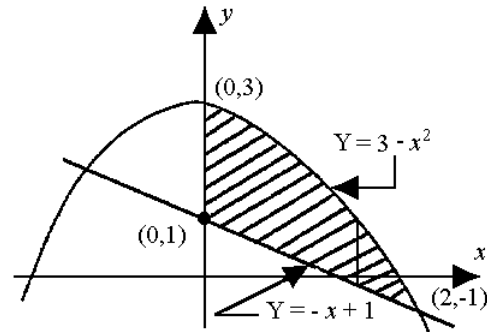
(c) $4/3$



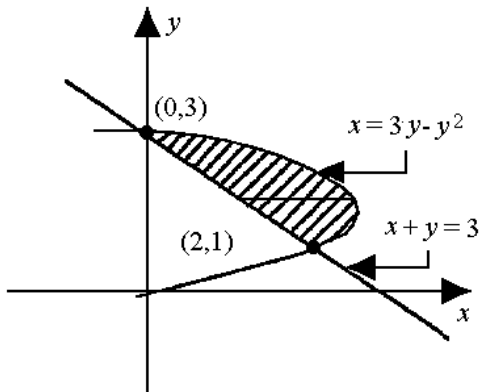
6. $125/6$



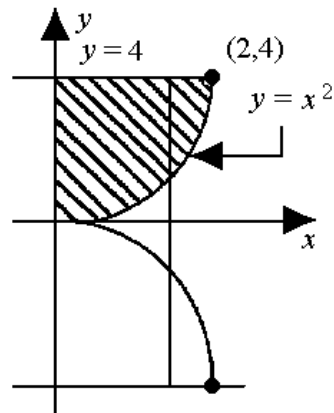
7. $10/3$



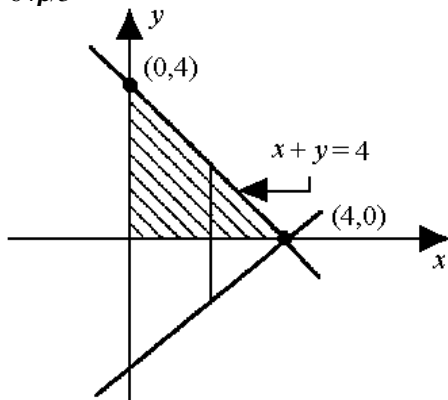
8. $4/3$



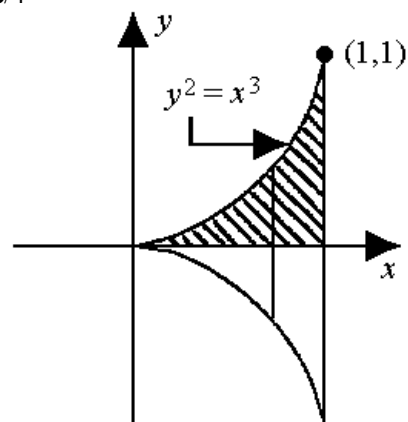
12. $128p/5$



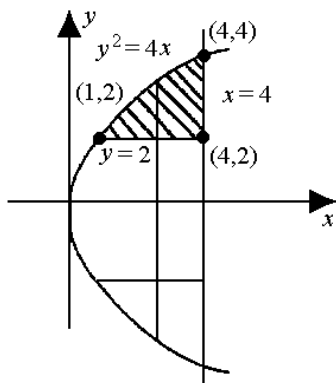
9. $64p/3$



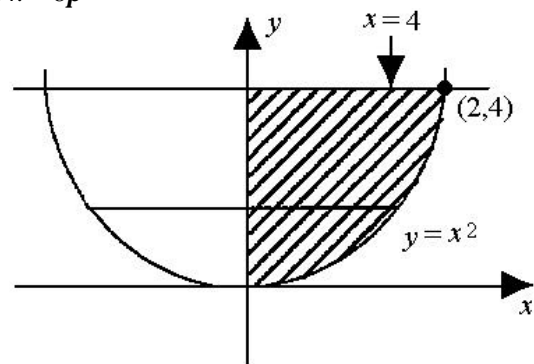
13. $p/4$



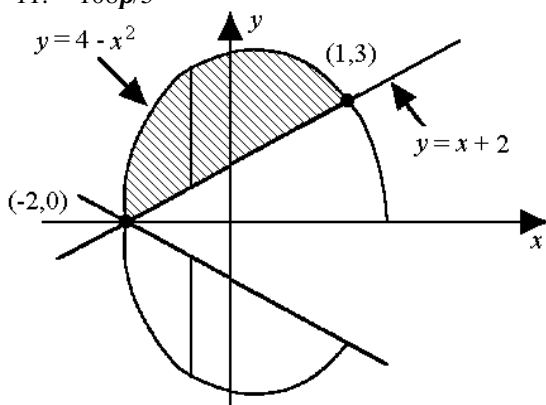
10. $18p$



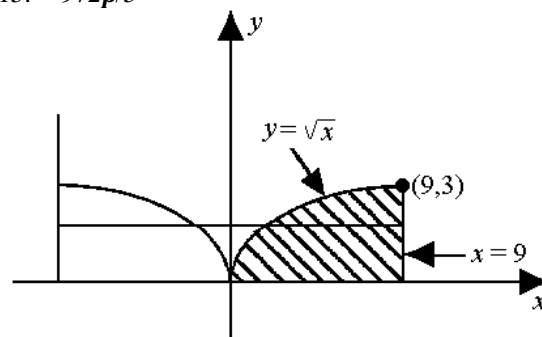
14. $8p$

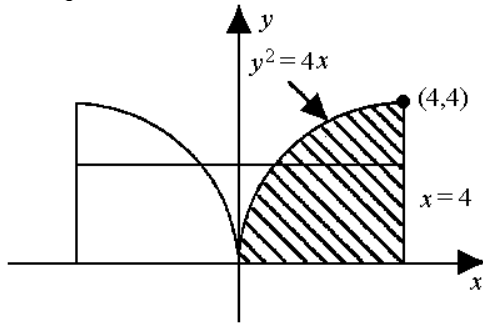
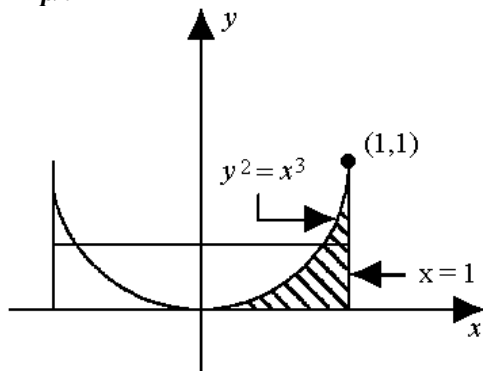
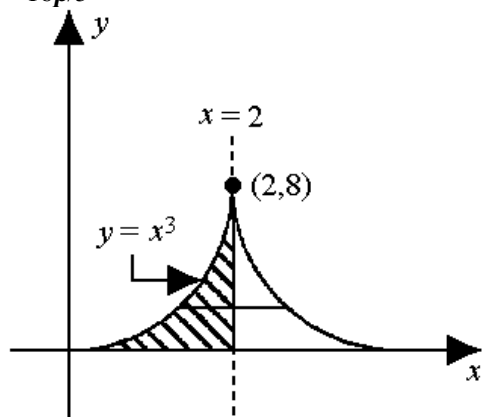
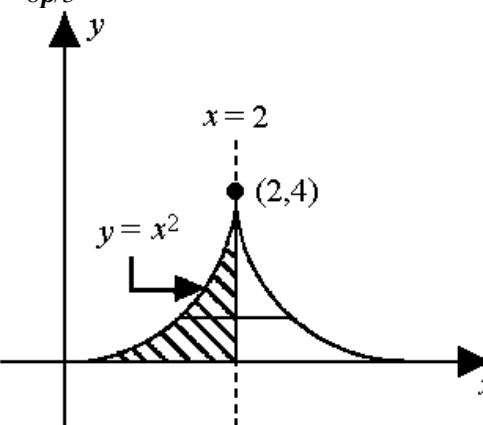
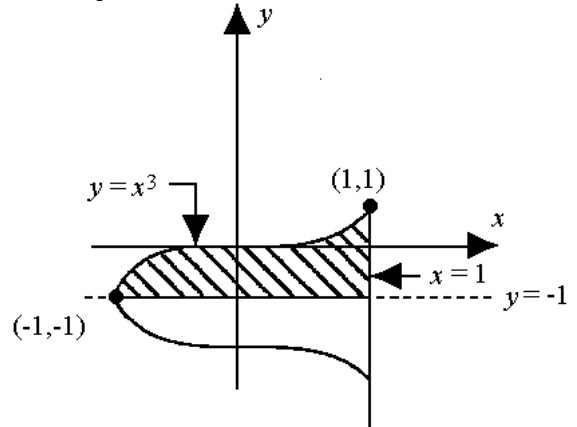
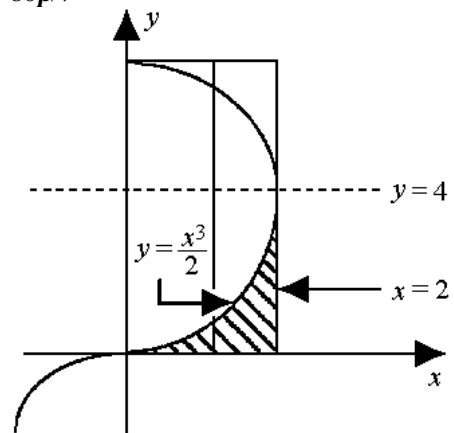
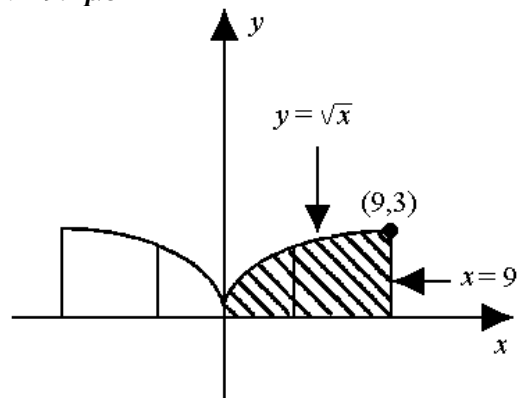


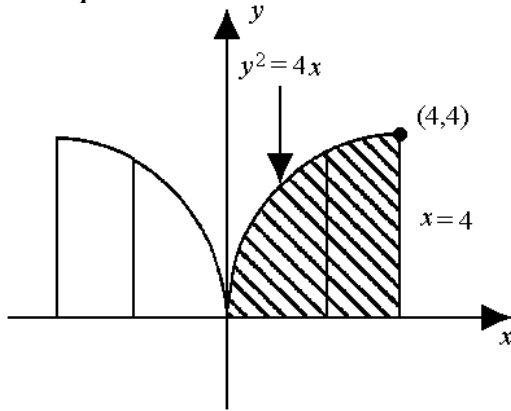
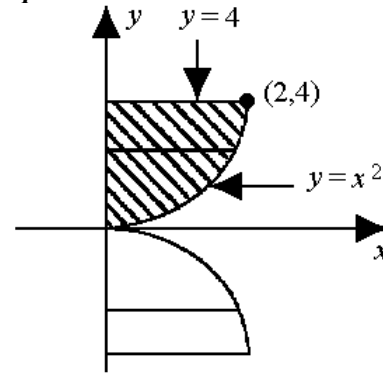
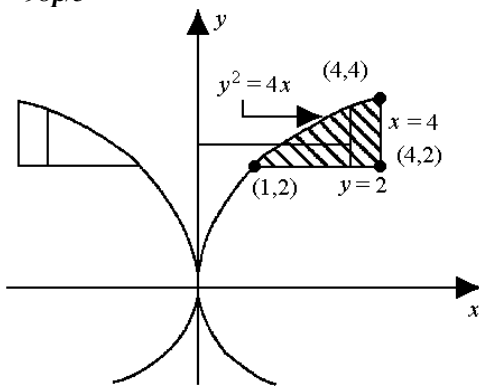
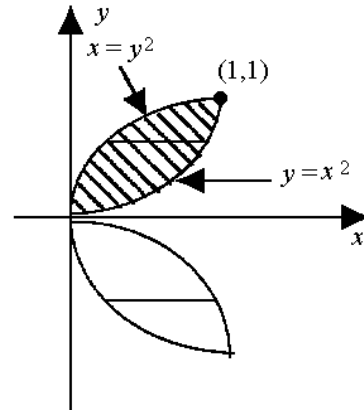
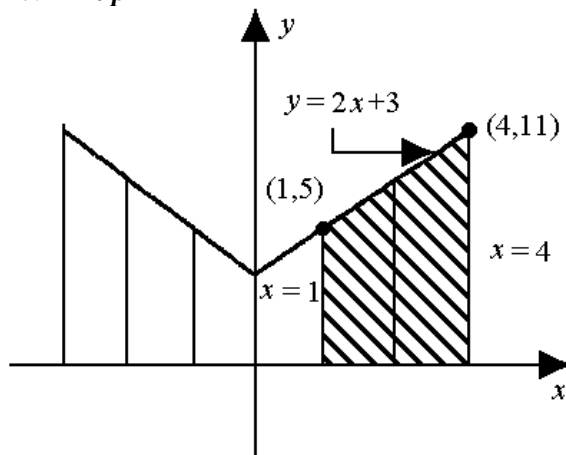
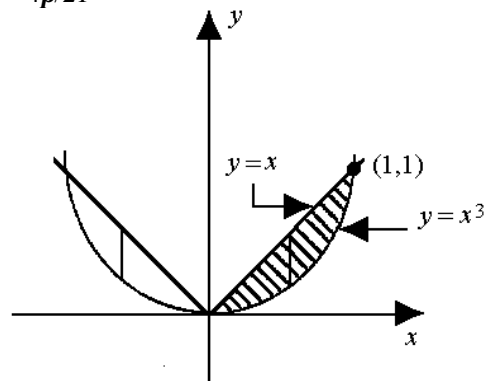
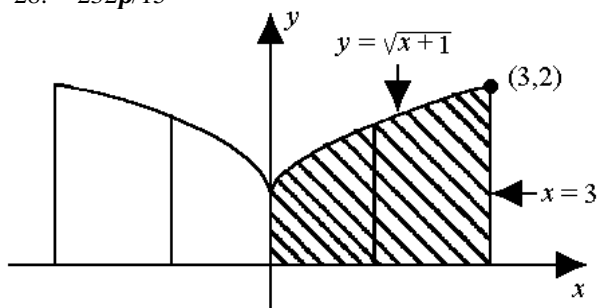
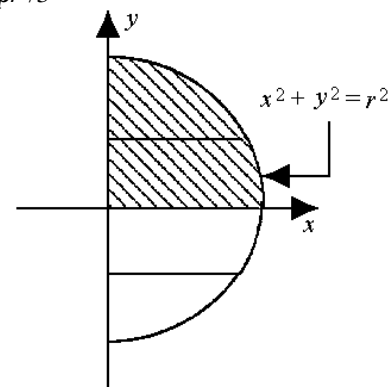
11. $108p/5$

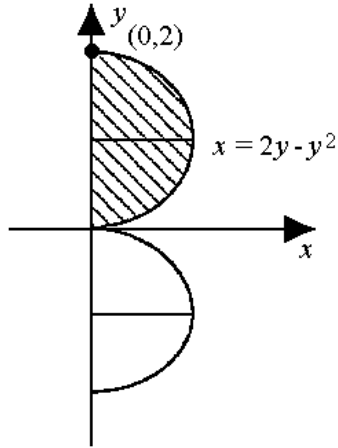
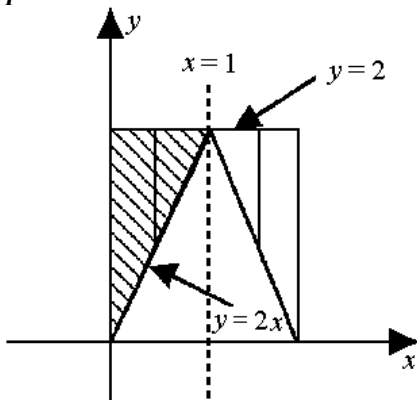
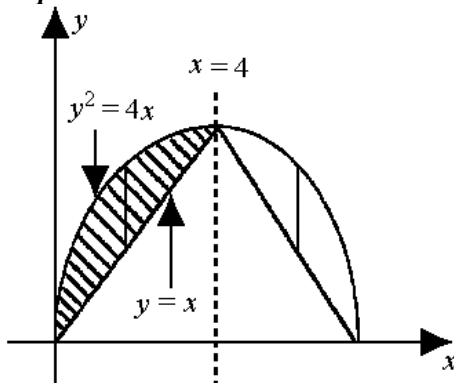
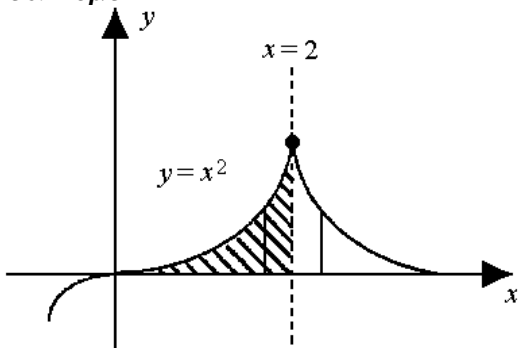
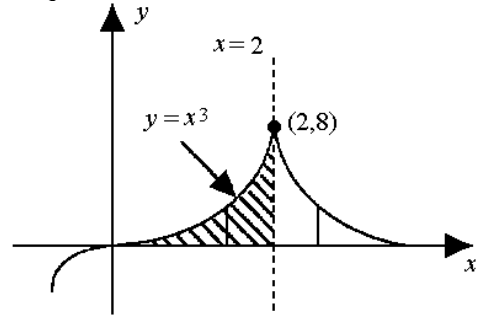
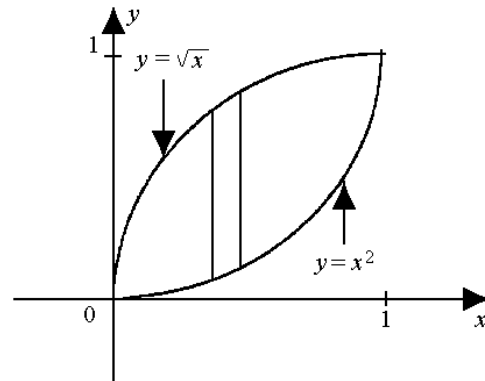
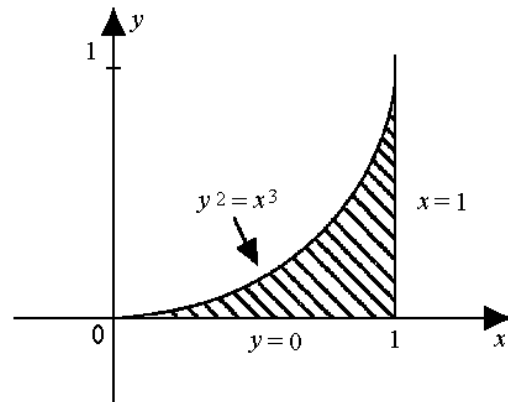
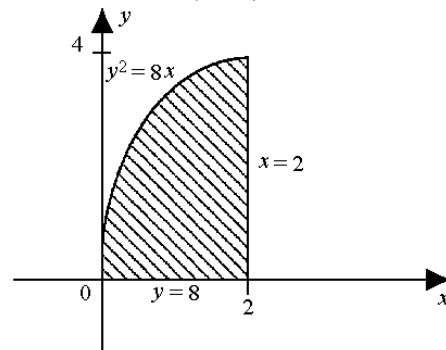


15. $972p/5$

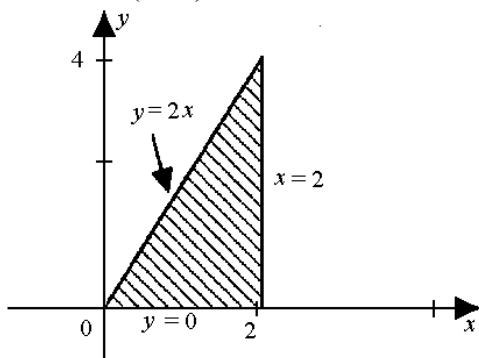


16. $256\pi/5$ 17. $4\pi/7$ 18. $16\pi/5$ 19. $8\pi/3$ 20. $16\pi/7$ 21. $80\pi/7$ 22. $128/3$ 23. 3375 ft^3 24. $972\pi/5$ 

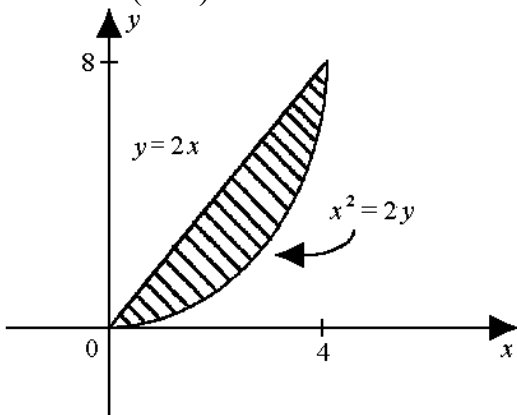
25. $256p/5$ 29. $128p/5$ 26. $98p/5$ 30. $3p/10$ 27. $129p$ 31. $4p/21$ 28. $232p/15$ 32. $2pr^3/3$ 

33. $8p/3$ 34. $4p/3$ 35. $64p/5$ 36. $8p/3$ 37. $16p/5$ 38. $(\bar{x}, \bar{y}) = \left(\frac{9}{20}, \frac{9}{20}\right); V_x = V_y = \frac{3p}{10}$ 39. $(\bar{x}, \bar{y}) = \left(\frac{5}{7}, \frac{5}{16}\right); V_x = \frac{p}{4}, V_y = \frac{4p}{7}$ 40. $(\bar{x}, \bar{y}) = \left(\frac{6}{5}, \frac{3}{2}\right); V_x = 16p, V_y = \frac{64p}{5}$ 

41. $(\bar{x}, \bar{y}) = \left(\frac{4}{3}, \frac{4}{3}\right); V_x = V_y = \frac{32p}{3}$



42. $(\bar{x}, \bar{y}) = \left(2, \frac{16}{5}\right); V_x = \frac{512p}{15}, V_y = \frac{64p}{3}$



43. $(\bar{x}, \bar{y}) = \left(\frac{1}{8}, \frac{19}{20}\right)$

44. $(\bar{x}, \bar{y}) = \left(0, -\frac{3}{5}\right)$

45. $(\bar{x}, \bar{y}) = \left(0, \frac{3}{5}\right)$

46. $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, 1)$

47. (a) $(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{9}{20}, 0, 0\right)$

(b) $(\bar{x}, \bar{y}, \bar{z}) = \left(0, \frac{43}{50}, 0\right)$

48. $20/3$ ft-lbs

49. $232/5$ joules

50. 6 ft-lbs

51. 2000 ft-lbs

52. 6400 ft-lbs

53. $5/12$ ft-lbs

54. 37,500 ft-lbs

55. 13,500 ft-lbs

56. 52,000 ft-lbs

57. 22,000 ft-lbs

58. $49,920p$ ft-lbs

59. $250,000p$ ft-lbs

60. $15,000p$ ft-lbs

61. $15,163.2p$ ft-lbs

62. $3953.7p$ ft-lbs

63. 2995.2 lbs

64. 3931.2 lbs

65. 998.4 lbs

66. 3494.4 lbs

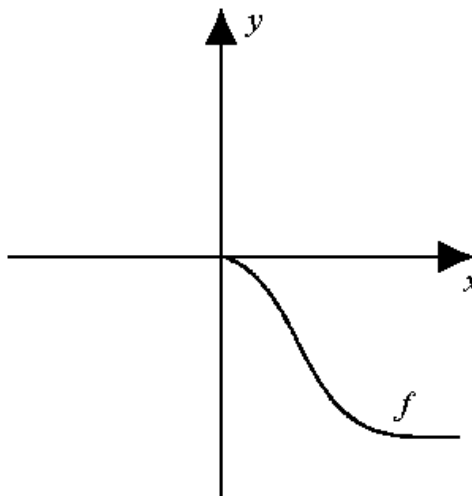
67. 1123.2 lbs

CHAPTER 7

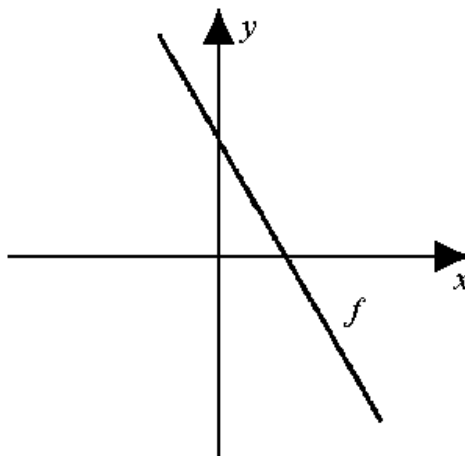
The Transcendental Functions

7.1 One-to-One Functions; Inverses

1. Determine whether or not $f(x) = 4x + 3$ is one-to-one, and, if so, find its inverse.
2. Determine whether or not $f(x) = 5x - 7$ is one-to-one, and, if so, find its inverse.
3. Determine whether or not $f(x) = 2x^2 - 1$ is one-to-one, and, if so, find its inverse.
4. Determine whether or not $f(x) = 2x^3 + 1$ is one-to-one, and, if so, find its inverse.
5. Determine whether or not $f(x) = (1 - 3x)^3$ is one-to-one, and, if so, find its inverse.
6. Determine whether or not $f(x) = (x + 1)^4$ is one-to-one, and, if so, find its inverse.
7. Determine whether or not $f(x) = (x - 1)^5 + 1$ is one-to-one, and, if so, find its inverse.
8. Determine whether or not $f(x) = (4x + 5)^3$ is one-to-one, and, if so, find its inverse.
9. Determine whether or not $f(x) = 2 + (x + 1)^{5/3}$ is one-to-one, and, if so, find its inverse.
10. Determine whether or not $f(x) = \frac{1}{x - 2}$ is one-to-one, and, if so, find its inverse.
11. Determine whether or not $f(x) = \frac{2x - 3}{3x + 1}$ is one-to-one, and, if so, find its inverse.
12. Determine whether or not $f(x) = \frac{1}{x - 1} + 2$ is one-to-one, and, if so, find its inverse.
13. Determine whether or not $f(x) = \frac{2}{x^3 - 1}$ is one-to-one, and, if so, find its inverse.
14. Sketch the graph of f^{-1} given the graph of f shown below.



15. Sketch the graph of f^{-1} given the graph of f shown below.



16. Given that f is one-to-one and $f(2) = 1, f'(2) = 3, f(3) = 2, f'(3) = 4$, deduce, if possible, $(f^{-1})'(2)$.
17. Show that $f(x) = x^3 + 2x - 5$ has an inverse and find $(f^{-1})'(7)$.
18. Show that $f(x) = \sin 2x - 4x$ has an inverse and find $(f^{-1})'(0)$.
19. Find a formula for $(f^{-1})'(x)$ given that f is one-to-one and satisfies $f'(x) = \frac{1}{f(x)}$.
20. Show that $f(x) = \int_{p/2}^x (1 + \sin^2 t) dt$ has an inverse and find $(f^{-1})'(0)$.

7.2 The Logarithm Function; Part I

21. Estimate $\ln 25$ on the basis of the following table.

n	$\ln n$	n	$\ln n$
1	0.00	6	1.79
2	0.69	7	1.95
3	1.10	8	2.08
4	1.39	9	2.20
5	1.61	10	2.30

22. Estimate $\ln 2.2$ on the basis of the following table.

n	$\ln n$	n	$\ln n$
1	0.00	6	1.79
2	0.69	7	1.95
3	1.10	8	2.08
4	1.39	9	2.20
5	1.61	10	2.30

23. Estimate $\ln 5^4$ on the basis of the following table.

n	$\ln n$	n	$\ln n$
1	0.00	6	1.79
2	0.69	7	1.95
3	1.10	8	2.08
4	1.39	9	2.20
5	1.61	10	2.30

24. Estimate $\ln \sqrt{425}$ on the basis of the following table.

n	$\ln n$	n	$\ln n$
1	0.00	6	1.79
2	0.69	7	1.95
3	1.10	8	2.08
4	1.39	9	2.20
5	1.61	10	2.30

25. Estimate $\ln 3.5 = \int_1^{3.5} \frac{dt}{t}$ using the approximation $\frac{1}{2} [L_f(P) + U_f(P)]$ with $P = \left\{ 1 = \frac{4}{4}, \frac{5}{4}, \frac{6}{4}, \frac{7}{4}, \frac{8}{4}, \frac{9}{4}, \frac{10}{4} = \frac{5}{2} \right\}$
26. Taking $\ln 4 \approx 1.39$, use differentials to estimate (a) $\ln 4.25$; (b) $\ln 3.75$.
27. Solve $\ln x + \ln (x + 2) = 0$ for x .
28. Solve $\ln (x + 1) - \ln (x - 2) = 1$ for x .
29. Solve $2 \ln x = \ln (x + 2)$ for x .
30. Solve $(5 - \ln x)(2 \ln x) = 0$ for x .

7.3 The Logarithm Function; Part II

31. Determine the domain and find the derivative of $f(x) = \ln(x^3 - 1)$.
32. Determine the domain and find the derivative of $f(x) = \ln(1 - x^2)$.
33. Determine the domain and find the derivative of $f(x) = \ln 2x\sqrt{2 + x}$.
34. Determine the domain and find the derivative of $f(x) = x \ln(2x + x^2)$.
35. Determine the domain and find the derivative of $f(x) = \ln 3x\sqrt{3 - x^2}$.
36. Determine the domain and find the derivative of $f(x) = \ln(\sec x)$.
37. Calculate $\int \frac{dx}{5 - 3x}$.
38. Calculate $\int \frac{(4x - 2)dx}{x^2 - x}$.
39. Calculate $\int \frac{x^2}{3x^3 - 5} dx$.
40. Calculate $\int \cot 3x dx$.
41. Calculate $\int \frac{\sin x + \cos x}{\sin x} dx$.

42. Calculate $\int_0^1 \frac{x}{6x^2 + 1} dx$.
43. Calculate $\int_{1/3}^{1/2} \frac{x}{1 - 3x^2} dx$.
44. Calculate $\int_0^{p/2} \frac{\sin x}{2 - \cos x} dx$.
45. Calculate $g'(x)$ by logarithmic differentiation if $g(x) = \frac{x\sqrt{x^2 + 1}}{(x + 1)^{2/3}}$.
46. Calculate $g'(x)$ by logarithmic differentiation if $g(x) = \sqrt[3]{\frac{3(x^2 + 5)\cos^4 2x}{(x^3 - 8)^2}}$.
47. Calculate $g'(x)$ by logarithmic differentiation if $g(x) = \sqrt[5]{\frac{\sin x}{(1 + x^5)^3}}$.
48. The region bounded by the graph of $f(x) = \frac{1}{5 - x^2}$ and the x -axis for $0 \leq x \leq 2$ is revolved around the y -axis. Find the volume of the solid that is generated.
49. Calculate $\int \tan 5x \, dx$.
50. Calculate $\int \sec \frac{2px}{3} \, dx$.
51. Calculate $\int \csc\left(2p - \frac{x}{2}\right) dx$.
52. Calculate $\int \cot(2px - 3) \, dx$.
53. Calculate $\int e^{2x} \sin e^{2x} \, dx$.
54. Calculate $\int \frac{\sec^2 2x \, dx}{2 + \tan 2x}$.
55. Calculate $\int \frac{3 \sec x \tan x \, dx}{(2 + \sec x)^3}$.
56. Evaluate $\int_0^{\ln p^{1/3}} e^{2x} \csc e^{2x} \, dx$.
57. Evaluate $\int_{7p/12}^{5p/6} \tan(2x - p) \, dx$.
58. Evaluate $\int_{p/4}^{p/2} \frac{3 \csc x \cot x \, dx}{2 + \csc x}$.

7.4 The Exponential Function

59. Differentiate $y = e^{-3x/2}$.

60. Differentiate $y = e^{2x^2-x}$.

61. Differentiate $y = e^{\sqrt[3]{x}+1}$.

62. Differentiate $y = e^{\sqrt[3]{x}} \ln \sqrt{x^3}$.

63. Differentiate $y = (e^{x^3+x} + 1)^2$.

64. Differentiate $y = \frac{e^{3x} - e^{2x}}{e^{3x} + e^{2x}}$.

65. Differentiate $y = e^{x \sin x}$.

66. Differentiate $y = e^{-2x} \sin 3x$.

67. Differentiate $y = e^{\sin 2x}$.

68. Differentiate $y = e^{1/x}$.

69. Differentiate $y = \frac{e^{2x}}{2x}$.

70. Calculate $\int \frac{e^x}{\sqrt{1-e^x}} dx$.

71. Calculate $\int \frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}} dx$.

72. Calculate $\int e^{5x/2} dx$.

73. Calculate $\int \frac{e^{\sqrt[3]{x}}}{\sqrt[3]{x^2}} dx$.

74. Calculate $\int \left(\frac{1}{2} e^{2x} - e^{-2x} \right)^2 dx$.

75. Calculate $\int \frac{e^{3x}}{3e^{3x} - 2} dx$.

76. Calculate $\int \frac{2xe^{3x^2}}{e^{3x^2} + 2} dx$.

77. Evaluate $\int_0^1 e^{x/2} dx$.
78. Evaluate $\int_0^{\ln p/2} e^{-3x} dx$.
79. Evaluate $\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$.
80. Evaluate $\int_0^1 \frac{e^{2x}}{2 + e^{2x}} dx$.
81. Sketch the region bounded by $y = e^{3x}$, $y = e^x$, and $x = 2$, and find its area.
82. Sketch the graph of $f(x) = xe^{-2x}$.

7.5 **Arbitrary Powers; Other Bases**

83. Find $\log_{10} 0.001$
84. Find $\log_9 243$
85. Find the derivative of $f(x) = 3^{\cos \sqrt{x}}$.
86. Find the derivative of $f(x) = 5^{2^x}$.
87. Calculate $\int 4^{2x} dx$.
88. Calculate $\int 3^{-x/2} dx$.
89. Calculate $\int 3x5^{-x^2} dx$.
90. Calculate $\int \frac{\log_3 2x}{x} dx$.
91. Calculate $\int \frac{\log_4 \sqrt{2x+2}}{x} dx$.
92. Find $f'(e)$ if $f(x) = e^x + x^e$.
93. Find $f'(e)$ if $f(x) = \ln(\ln(\ln x^2))$.
94. Find $\frac{d}{dx}[(\sin x)^x]$ by logarithmic differentiation.
95. Find $\frac{d}{dx}(x^4 4^x)$ by logarithmic differentiation.
96. Find $\frac{d}{dx}[(\tan x)^x]$ by logarithmic differentiation.

97. Evaluate $\int_1^2 3^{-x} dx$.

98. Evaluate $\int_0^1 2^{3x} dx$.

99. Evaluate $\int_0^5 \frac{4a^{\sqrt{2x-1}}}{\sqrt{2x-1}} dx$.

100. Evaluate $9^{(\ln 5) / (\ln 3)}$.

7.6 Exponential Growth and Decay

101. Find the amount of interest earned by \$700 compounded continuously for 10 years (a) at 8% (b) at 9% (c) at 10%.
102. The population of a certain city increases at a rate proportional to the number of its inhabitants at any time. If the population of the city was originally 10,000 and it doubled in 15 years, in how many years will it triple?
103. A certain radioactive substance has a half-life of 1300 years. Assume an amount y_0 was initially present. (a) Find a formula for the amount of substance present at any time. (b) In how many years will only 1/10 of the original amount remain?
104. A tank initially contains 100 gal. of pure water. At time $t = 0$, a solution containing 4 lb. of dissolved salt per gal. flows into the tank at 3 gal./min. The well-stirred mixture is pumped out of the tank at the same rate. (a) How much salt is present at the end of 30 min.? (b) How much salt is present after a very long time?
105. A tank initially contains 150 gal. of brine in which there is dissolved 30 lb. of salt. At $t = 0$, a brine solution containing 3 lb. of dissolved salt per gallon flows into the tank at 4 gal./min. The well-stirred mixture flows out of the tank at the same rate. How much salt is in the tank at the end of 10 min.?
106. An object of unknown temperature is put in a room held at 30°F . After 10 minutes, the temperature of the object is 0°F ; 10 minutes later it is 15°F . What was the object's initial temperature?
107. Show that $\frac{dy}{dx} - e^{-y} \sec^2 x = 0$ is a separable equation and then find its solution.

7.7 The Inverse Trigonometric Functions

108. Determine the exact value for $\tan^{-1}(\sqrt{3} / 3)$.
109. Determine the exact value for $\cot[\sin^{-1}(-1 / 4)]$.
110. Determine the exact value for $\tan[\sin^{-1}(-1 / 4)]$.
111. Determine the exact value for $\tan[\sec^{-1}(3 / 2)]$.
112. Determine the exact value for $\sin^{-1}[\cot(\pi / 4)]$.
113. Determine the exact value for $\sec[\sin^{-1}(3 / 4)]$.

114. Determine the exact value for $\cos\left[\sin^{-1}(-3/4)\right]$.
115. Determine the exact value for $\sin 2\left[\tan^{-1}(1/3)\right]$.
116. Determine the exact value for $\sin\left[\tan^{-1}(-2/3)\right]$.
117. Determine the exact value for $\cos\left[\sin^{-1}(-3/4)\right]$.
118. Taking $0 < x < 1$, calculate $\sin(\cos^{-1} x)$.
119. Taking $0 < x < 1/2$, calculate $\tan(\cos^{-1} 2x)$.
120. Differentiate $f(x) = x \tan^{-1} 3x$.
121. Differentiate $f(x) = x \sin^{-1} 2x$.
122. Differentiate $f(x) = \sec(\tan^{-1} x)$.
123. Differentiate $y = \cos^{-1}(\cos x)$.
124. Differentiate $f(x) = e^{3x} \sin^{-1} 2x$.
125. Differentiate $f(x) = \tan^{-1}\left(\frac{x-1}{x+1}\right)$.
126. Differentiate $f(x) = \sin^{-1} x + \sqrt{1-x^2}$.
127. Differentiate $y = \sin^{-1} \frac{1}{\sqrt{1+x^2}}$.
128. Differentiate $f(x) = \ln(x^2 + 4) - x \tan^{-1} \frac{x}{2}$.
129. Evaluate $\int_0^1 \frac{dx}{16+9x^2}$.
130. Evaluate $\int_0^{1/4} \frac{dx}{\sqrt{1-4x^2}}$.
131. Evaluate $\int_{-1/2}^0 \frac{dx}{\sqrt{1+4x^2}}$.
132. Evaluate $\int_1^3 \frac{dx}{x^2 - 6x + 13}$.
133. Calculate $\int \frac{x}{1+x^4} dx$.
134. Calculate $\int \frac{dx}{1+9x^2}$.

135. Calculate $\int \frac{dx}{x\sqrt{1 - (\ln x)^2}}$.

7.8 The Hyperbolic Sine and Cosine Functions

136. Differentiate $y = \sinh x^3$.

137. Differentiate $y = \cosh x^2$.

138. Differentiate $y = e^{2\ln \sinh 3x}$.

139. Differentiate $y = \cosh^2 3x$.

140. Differentiate $y = x \sinh \sqrt{x}$.

141. Differentiate $y = (\sinh 2x)^{3x}$.

142. Differentiate $y = \cosh (e^{-2x})$.

143. Differentiate $y = 2^{\cosh 3x}$.

144. Differentiate $y = \sinh 2x \cosh^2 2x$.

145. Calculate $\int \frac{\sinh x}{1 + \cosh x} dx$.

146. Calculate $\int \sqrt{\sinh 2x} \cosh 2x dx$.

147. Calculate $\int \sinh^5 px \cosh px dx$.

7.9 The Other Hyperbolic Functions

148. Differentiate $y = \operatorname{sech} x \tanh x$.

149. Differentiate $y = e^{3x} \tanh 2x$.

150. Differentiate $y = \coth(\sqrt{2x^2 + 1})$.

151. Differentiate $y = \operatorname{csch}(\tan e^{3x})$.

152. Differentiate $y = \tanh^2 x$.

153. Differentiate $y = \tanh^3(3x + 2)$.

154. Differentiate $y = x \operatorname{sech} x$.

155. Differentiate $y = \sqrt{1 + x^2} + \tanh x$.

156. Differentiate $y = (\tanh 3x)^2$.

157. Differentiate $y = \tanh(\sin 2x)$.

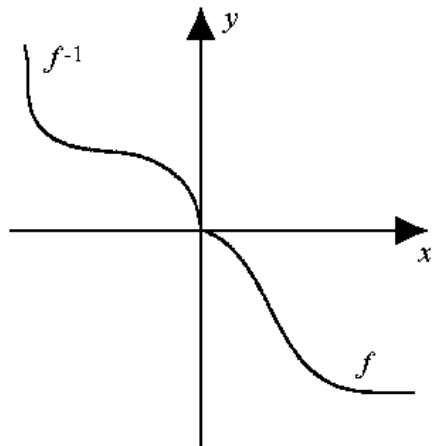
158. Calculate $\int \tanh 5x \, dx$.

159. Calculate $\int x \tanh x^2 \operatorname{sech}^2 x^2 \, dx$.

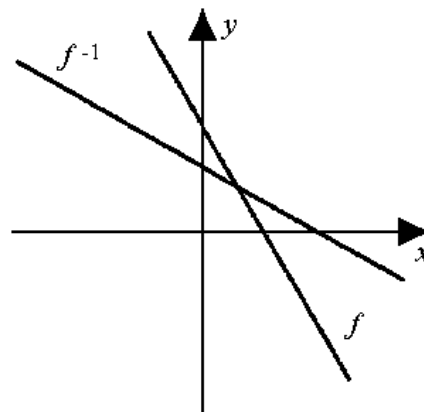
Answers to Chapter 7 Questions

1. one-to-one; $f^{-1}(x) = \frac{x-3}{4}$
2. one-to-one; $f^{-1}(x) = \frac{x+7}{5}$
3. not one-to-one
4. one-to-one; $f^{-1}(x) = \sqrt[3]{\frac{x-1}{2}}$
5. one-to-one; $f^{-1}(x) = \frac{1-\sqrt[3]{x}}{3}$
6. not one-to-one
7. one-to-one; $f^{-1}(x) = (x-1)^{1/5} + 1$
8. one-to-one; $f^{-1}(x) = \frac{\sqrt[3]{x}-5}{4}$
9. one-to-one; $f^{-1}(x) = (x-2)^{3/5} - 1$
10. one-to-one; $f^{-1}(x) = \frac{1+2x}{x}$
11. one-to-one; $f^{-1}(x) = \frac{x+3}{2-3x}$
12. one-to-one; $f^{-1}(x) = \frac{x-1}{x-2}$
13. one-to-one; $f^{-1}(x) = \sqrt[3]{\frac{2+x}{x}}$

14.



15.



16. $1/4$
17. $f'(x) = 3x^2 + 2 > 0$, so f is increasing and therefore, one-to-one. $(f^{-1})'(7) = 1/14$.
18. $f'(x) = 2\cos 2x - 4 < 0$, so f is decreasing and therefore, one-to-one. $(f^{-1})'(0) = -1/2$.
19. $(f^{-1})'(x) = x$
20. $f'(x) = 1 + \sin^2 x > 0$, so f is increasing and therefore, one-to-one. $(f^{-1})'(0) = 1/2$.
21. 3.22
22. 0.79
23. 6.44
24. 3.03
25. 1.3
26. (a) 1.45 (b) 1.33
27. $x = -1 + \sqrt{2}$ ($-1 - \sqrt{2}$ is not a solution because $h(-1 - \sqrt{2})$ is undefined.)
28. $x = \frac{2e+1}{e-1}$
29. $x = 2$ (-1 is not a solution because $\ln(-1)$ is undefined.)
30. $x = 1, e^5$
31. $\text{dom}(f) = (1, \infty)$; $f'(x) = \frac{3x^2}{x^3 - 1}$

32. $\text{dom}(f) = (-1, 1); f'(x) = \frac{2x}{x^2 - 1}$
33. $\text{dom}(f) = (0, \infty);$
 $f'(x) = \frac{1}{x} + \frac{1}{2(2+x)} = \frac{4+3x}{2x(2+x)}$
34. $\text{dom}(f) = (-\infty, -2) \cup (0, \infty);$
 $f'(x) = \ln(2x + x^2) + \frac{2+2x}{2x+x^2}$
35. $\text{dom}(f) = (0, \sqrt{3}); f'(x) = \frac{3-2x^2}{x(3-x^2)}$
36. $\text{dom}(f) = (2pk - p/2, 2pk + p/2),$
 $k = 0, \pm 1, \pm 2, \dots; f'(x) = \tan x$
37. $-\frac{1}{3} \ln |5-3x| + C$
38. $2 \ln |x^2 - x| + C$
39. $\frac{1}{9} \ln |3x^3 - 5| + C$
40. $\frac{1}{3} \ln |\sin 3x| + C$
41. $x + \ln |\sin x| + C$
42. $\frac{1}{12} \ln 7$
43. $\frac{1}{6} \ln \frac{8}{3}$
44. $\ln 2$
45. $\frac{x\sqrt{x^2+1}}{(x+1)^{2/3}} \left(\frac{1}{x} + \frac{x}{x^2+1} - \frac{2}{3(x+1)} \right)$
46. $\frac{1}{3} \sqrt[3]{\frac{3(x^2+5)\cos^4 2x}{(x^3-8)^2}} \left(\frac{2x}{x^2+5} - 8 \tan 2x - \frac{6x^2}{x^3-8} \right)$
47. $\sqrt[5]{\frac{\sin x}{(1+x^5)^3}} \left(\frac{1}{5} \cot x - \frac{3x^4}{(1+x)^5} \right)$
48. $V = p \ln 5$
49. $\frac{1}{5} \ln |\sec 5x| + C$
50. $\frac{3}{2p} \ln \left| \sec \frac{2px}{3} + \tan \frac{2px}{3} \right| + C$
51. $-2 \ln \left| \csc \left(2p - \frac{x}{2} \right) - \cot \left(2p - \frac{x}{2} \right) \right| + C$
52. $\frac{-1}{2p} \ln |\sin(2px-3)| + C$
53. $\frac{-1}{2} \cos e^{2x} + C$
54. $\frac{1}{2} \ln |2 + \tan 2x| + C$
55. $\frac{-3}{2} \frac{1}{(2 + \sec x)^2} + C$
56. $\frac{1}{2} \ln \left| \frac{\csc \frac{p^2}{9} - \cot \frac{p^2}{9}}{\csc 1 - \cot 1} \right|$
57. $\frac{1}{4} \ln 3$
58. $3 \ln \frac{2+\sqrt{2}}{3}$
59. $-\frac{3}{2} e^{-3x/2}$
60. $(4x-1)e^{2x^2-x}$
61. $\frac{1}{3} x^{-2/3} e^{\sqrt[3]{x}+1}$
62. $e^{\sqrt[3]{x}} \left(x^{-2/3} \ln x^2 + \frac{3}{2x} \right)$
63. $2(e^{x^3+x} + 1)(3x^2 + 1)e^{x^3+x}$
64. $\frac{2e^{5x}}{(e^{3x} + e^{2x})^2}$
65. $e^{x \sin x} (x \cos x + \sin x)$
66. $e^{-2x} (3 \cos 3x - 2 \sin 3x)$
67. $2e^{\sin 2x} \cos 2x$

68. $\frac{-e^{1/x}}{x^2}$

69. $\frac{1}{2}e^{2x} \frac{2x-1}{x^2}$

70. $-2\sqrt{1-e^x} + C$

71. $\frac{1}{2}\ln(e^{2x} + e^{-2x}) + C$

72. $\frac{2}{5}e^{5x/2} + C$

73. $3e^{\sqrt[3]{x}} + C$

74. $\frac{1}{16}e^{4x} - x - \frac{1}{4}e^{-4x} + C$

75. $\frac{1}{9}\ln|3e^{3x} - 2| + C$

76. $\frac{1}{3}\ln(e^{3x^2} + 2) + C$

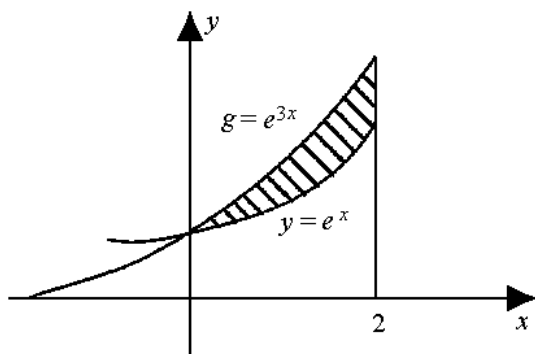
77. $2(e^{1/2} - 1)$

78. $\frac{1}{3}\left[1 - \left(\frac{2}{p}\right)^3\right]$

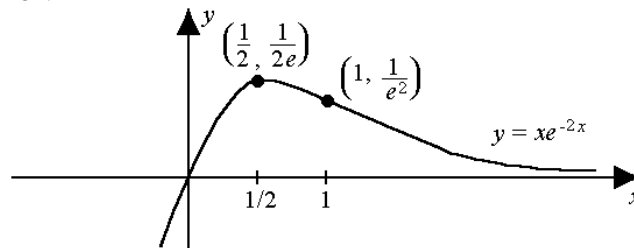
79. $2(e - 1)$

80. $\frac{1}{2}\ln \frac{2+e^2}{3}$

81. $\frac{1}{3}e^6 - e^2 + \frac{2}{3}$



82.

83. -3 84. $5/2$

85. $\frac{-3^{\cos \sqrt{x}} \sin \sqrt{x} (\ln 3)}{2\sqrt{x}}$

86. $5^{2^x} 2^x (\ln 5)(\ln 2)$

87. $\frac{1}{2 \ln 4} 4^{2x} + C$

88. $\frac{-2}{\ln 3} 3^{-x/2} + C$

89. $\frac{-3}{2 \ln 5} 5^{-x^2} + C$

90. $\frac{1}{2 \ln 3} (\ln 2x)^2 + C$

91. $\frac{1}{4 \ln 4} [\ln(2x)]^2 + 2 \ln x + C$

92. $e^e \left(1 + \frac{1}{e}\right)$

93. $\frac{1}{e \ln 2}$

94. $(\sin x)^x [\ln \sin x + x \cot x]$

95. $x^4 4^x \left(\frac{4}{x} + \ln 4\right)$

96. $(\tan x)^x [\ln \tan x + x(\cot x + \tan x)]$

97. $\frac{2}{9 \ln 3}$

98. $\frac{7}{3 \ln 2}$

99. $\frac{4}{\ln a} (a^3 - a)$

100. 25

101. (a) \$857.88 (b) \$1021.72 (c) \$1202.80

102. 23.8 yrs

103. (a) $y(t) = y_0 e^{-0.0005332t}$
(b) approximately 4319 yrs

104. (a) 237.4 lbs (b) 400 lbs

105. approximately 128.3 lbs

106. -30° F

107. The equation can be rewritten as $e^y dy = \sec^2 x \, dx$;
 $y = \ln(\tan x + C)$

108. $p/6$

109. $-\sqrt{15}$

110. $\frac{-1}{\sqrt{15}} = \frac{-\sqrt{15}}{15}$

111. $\frac{\sqrt{5}}{2}$

112. $p/2$

113. $\frac{4}{\sqrt{7}} = \frac{4\sqrt{7}}{7}$

114. $\frac{\sqrt{7}}{4}$

115. $3/5$

116. $\frac{-2}{\sqrt{13}} = \frac{-2\sqrt{13}}{13}$

117. $\frac{\sqrt{7}}{4}$

118. $\sqrt{1-x^2}$

119. $\frac{\sqrt{1-4x^2}}{2x}$

120. $\frac{3x}{1+9x^2} + \tan^{-1} 3x$

121. $\frac{2x}{\sqrt{1-4x^2}} + \sin^{-1} 2x$

122. $\frac{x}{\sqrt{x^2+1}}$

123. 1

124. $\frac{2e^{3x}}{\sqrt{1-4x^2}} + 3e^{3x} \sin^{-1} 2x$

125. $\frac{1}{x^2+1}$

126. $\sqrt{\frac{1-x}{1+x}}$

127. $-\frac{1}{1+x^2}$

128. $-\tan^{-1} \frac{x}{2}$

129. $\frac{1}{12} \tan^{-1} \frac{3}{4}$

130. $p/12$

131. $p/8$

132. $p/8$

133. $\frac{1}{2} \tan^{-1} x^2 + C$

134. $\frac{1}{3} \tan^{-1} 3x + C$

135. $\sin^{-1}(\ln x) + C$

136. $3x^2 \cosh x^2$

137. $2x \sinh x^2$

138. $3 \sinh 6x$

139. $6 \cosh 3x \sinh 3x$

140. $\frac{\sqrt{x}}{2} \cosh \sqrt{x} + \sinh \sqrt{x}$

141. $(\sinh 2x)^{3x} (6x \coth 2x + 3 \ln \sinh 2x)$

142. $-2e^{-2x} \sinh(e^{-2x})$

-
143. $2^{\cosh 3x} (3 \ln 2 \sinh 3x)$
144. $4 \sinh^2 2x \cosh 2x + 2 \cosh^3 2x$
145. $\ln (1 + \cosh x) + C$
146. $\frac{1}{3} (\sinh 2x)^{3/2} + C$
147. $\frac{1}{6p} \sinh^6 px + C$
148. $\operatorname{sech}^2 x - \operatorname{sech} x \tanh^2 x$
149. $2e^{3x} \operatorname{sech}^2 2x + 3e^{3x} \tanh 2x$
150. $\frac{-1}{\sqrt{2x^2 + 1}} \operatorname{csch}^2(\sqrt{2x^2 + 1})$
151. $-3e^{3x} \operatorname{csch}(\tan e^{3x}) \coth(\tan e^{3x}) \sec^2 e^{3x}$
152. $2 \tanh x \operatorname{sech}^2 x$
153. $9 \tanh^2 (3x + 2) \operatorname{sech}^2 (3x + 2)$
154. $\operatorname{sech} x - x \operatorname{sech} x \tanh x = \operatorname{sech} x(1 - x \tanh x)$
155. $\frac{x}{\sqrt{1 + x^2}} + \operatorname{sech}^2 x$
156. $-6 \tanh 3x \operatorname{sech}^2 3x$
157. $2 \operatorname{sech}^2 (\sin 2x) \cos 2x$
158. $\frac{1}{5} \ln(\cosh 5x) + C$
159. $\frac{1}{4} \tanh^2 x^2 + C$

CHAPTER 8

Techniques of Integration

8.2 Integration by Parts

1. Calculate $\int x e^{-2x} dx$.
2. Calculate $\int x \sin 2x dx$.
3. Calculate $\int \ln(1+x^2) dx$.
4. Calculate $\int x\sqrt{1+x} dx$.
5. Calculate $\int e^{-x} \cos 2x dx$.
6. Calculate $\int \ln x^2 dx$.
7. Calculate $\int \frac{x^3}{\sqrt{x^2+1}} dx$.
8. Calculate $\int x^2 2x dx$.
9. Calculate $\int \sqrt{x} \ln x^2 dx$.
10. Calculate $\int e^{5x} \sin 2x dx$.
11. Evaluate $\int_1^2 x^3 \ln x dx$.
12. Evaluate $\int_0^{p/2} x^2 \sin 2x dx$.
13. Evaluate $\int_1^2 x \sec^{-1} x dx$.
14. Find the centroid of the region under the graph of $f(x) = e^{2x}$, $x \in [0, 1]$.
15. Find the volume generated by revolving the region under the graph of $f(x) = e^{2x}$, $x \in [0, 1]$ about the y-axis.

8.3 Powers and Products of Trigonometric Functions

16. Calculate $\int \cos^3(2x) \sin^2(2x) dx$.
17. Calculate $\int \sin^3 x \cos^5 x dx$.

18. Calculate $\int \sin^4 \frac{q}{2} \cos^3 \frac{q}{2} dq$.
19. Calculate $\int \sin^3 3q dq$.
20. Evaluate $\int_{p/4}^{p/3} \frac{dx}{\cos^2 x}$.
21. Calculate $\int \cos^4 2x dx$.
22. Calculate $\int x \sin^2 x^2 \cos^2 x^2 dx$.
23. Evaluate $\int_0^{p/2} \frac{\cot^2 q}{\csc^2 q} dq$.
24. Calculate $\int \cos^2(3x) \sin^2(3x) dx$.
25. Calculate $\int \sin^2 \frac{x}{2} \cos \frac{x}{2} dx$.
26. Calculate $\int \sin^2 \frac{t}{2} \cos^5 \frac{t}{2} dt$.
27. Calculate $\int \frac{\sin x}{\cos^5 x} dx$.
28. Calculate $\int \sin^4 2x dx$.
29. Evaluate $\int_0^{p/2} \sin^2 2q \cos^2 2q dq$.
30. Evaluate $\int_{p/3}^{2p/3} \sin^4 q \cos^3 q dq$.
31. Evaluate $\int_{p/6}^{p/3} (\tan^2 x - \sec^2 x)^4 dx$.
32. Calculate $\int \csc^3(4x) \cot^3(4x) dx$.
33. Calculate $\int \tan^3(2x) \sec^6(2x) dx$.
34. Calculate $\int \sec^3 \frac{x}{2} \tan \frac{x}{2} dx$.
35. Calculate $\int \tan^5 x dx$.
36. Calculate $\int \tan^3 3q dq$.
37. Calculate $\int \sec^6 \frac{x}{3} \tan^2 \frac{x}{3} dx$.

38. Calculate $\int \frac{1}{\sec 2x \tan 2x} dx$.
39. Calculate $\int \tan^3 \frac{x}{2} \sec^4 \frac{x}{2} dx$.
40. Evaluate $\int_0^{p/4} (\tan x + \sec x)^2 dx$.
41. Calculate $\int \cot^4 2q dq$.
42. Calculate $\int \tan^5 t \sec^4 t dt$.
43. Calculate $\int \cot^2 2x dx$.

8.4 Trigonometric Substitutions

44. Calculate $\int \frac{x^3}{\sqrt{25 - 4x^2}} dx$.
45. Calculate $\int \frac{1}{(x^2 + 4)^{3/2}} dx$.
46. Calculate $\int \frac{1}{(4x^2 - 9)^{3/2}} dx$.
47. Calculate $\int \frac{1}{x^2 \sqrt{x^2 + 25}} dx$.
48. Calculate $\int \frac{1}{\sqrt{2 + 4x^2}} dx$.
49. Calculate $\int \frac{\sqrt{x^2 - 4}}{x} dx$.
50. Evaluate $\int_0^4 \sqrt{16 - x^2} dx$.
51. Calculate $\int x^3 \sqrt{1 + x^2} dx$.
52. Calculate $\int \frac{x^2}{\sqrt{x^2 - 3}} dx$.
53. Calculate $\int \frac{x^2}{\sqrt{3 - x^2}} dx$.
54. Calculate $\int \frac{1}{x^2 \sqrt{4 - x^2}} dx$.

55. Evaluate $\int_2^4 \frac{1}{x^2 \sqrt{x^2 + 4}} dx$.

56. Calculate $\int \frac{x^3}{\sqrt{4 - x^2}} dx$.

57. Calculate $\int \frac{1}{x^2 \sqrt{9 - x^2}} dx$.

58. Calculate $\int \frac{1}{x \sqrt{x^2 - 4}} dx$.

59. Calculate $\int x^3 \sqrt{x^2 - 1} dx$.

60. Evaluate $\int_2^3 \frac{dx}{\sqrt{x^2 - 1}}$.

61. Calculate $\int \frac{1}{x \sqrt{4x^2 + 9}} dx$.

8.5 **Partial Fractions**

62. Calculate $\int \frac{x^2 - 6}{x(x - 1)^2} dx$.

63. Calculate $\int \frac{x + 3}{(x - 1)(x^2 - 4x + 4)} dx$.

64. Calculate $\int \frac{x + 2}{x - x^3} dx$.

65. Calculate $\int \frac{x^2}{x^2 - 2x + 1} dx$.

66. Calculate $\int \frac{4x^2 - 3x}{(x - 2)(x^2 + 1)} dx$.

67. Calculate $\int \frac{2x - 3}{x^3 - 3x^2 + 2x} dx$.

68. Calculate $\int \frac{2x + 1}{x^3 + x^2 + 2x + 2} dx$.

69. Calculate $\int \frac{x}{(x + 1)^2} dx$.

70. Calculate $\int \frac{x+1}{x^2+2x-3} dx$.

71. Calculate $\int \frac{x+4}{x^3+3x^2-10x} dx$.

72. Calculate $\int \frac{x+1}{x^2(x-1)} dx$.

73. Calculate $\int \frac{1}{(x+1)(x^2+1)} dx$.

74. Calculate $\int \frac{\cos q}{\sin^2 q + 4 \sin q - 5} dq$.

75. Calculate $\int \frac{4x}{x^3-x^2-x+1} dx$.

76. Calculate $\int \frac{x+4}{x^3+x} dx$.

77. Calculate $\int \frac{x^2+3x-1}{x^3-1} dx$.

8.6 Some Rationalizing Substitutions

78. Calculate $\int \frac{dx}{2+\sqrt{3x}}$.

79. Calculate $\int \frac{\sqrt{x} dx}{3(1+\sqrt{x})}$.

80. Calculate $\int \sqrt{3+e^{2x}} dx$.

81. Calculate $\int \frac{dx}{x^2(\sqrt{x}-1)}$.

82. Calculate $\int (x+2)\sqrt{x-3} dx$.

83. Calculate $\int \frac{\sqrt{2x}}{\sqrt{2x}+1} dx$.

84. Calculate $\int x^3 \sqrt{x+1} dx$.

85. Calculate $\int \frac{dx}{\sqrt{2-e^{2x}}}$.

86. Evaluate $\int_0^2 \frac{x}{(4x+3)^{3/2}} dx$.

87. Evaluate $\int_7^{10} \frac{x}{\sqrt{3x-5}} dx$.
88. Calculate $\int \frac{dx}{1 - \sin x}$.
89. Calculate $\int \frac{3}{2 \cos x + 1} dx$.
90. Calculate $\int \frac{dx}{\tan x - \sin x}$.
91. Calculate $\int \frac{\cot x}{1 + \sin x} dx$.
92. Evaluate $\int_0^{p/2} \frac{\cos x}{2 - \cos x} dx$.

8.7 Numerical Integration

93. Estimate $\int_0^2 \sqrt{4 + x^3} dx$ using
(a) the left-endpoint estimate, $n = 4$.
(b) the right-endpoint estimate, $n = 4$.
(c) the midpoint estimate, $n = 4$.
(d) the trapezoidal rule, $n = 4$.
94. Estimate $\int_0^4 \sqrt{4 + x^4} dx$ using
(a) the left-endpoint estimate, $n = 4$.
(b) the right-endpoint estimate, $n = 4$.
95. Estimate $\int_0^1 \frac{1}{1+x^2} dx = \frac{p}{4}$ using
(a) the trapezoidal rule, $n = 8$.
(b) Simpson's rule, $n = 8$.
96. Estimate $\int_0^2 \frac{1}{1+x^3} dx$ using
(a) the trapezoidal rule, $n = 4$.
(b) Simpson's rule, $n = 4$.
97. Estimate $\int_0^3 (1+x^2)^{3/2} dx$ using
(a) the left-endpoint estimate, $n = 6$.
(b) the right-endpoint estimate, $n = 6$.
(c) the trapezoidal rule, $n = 6$.
(d) Simpson's rule, $n = 6$.
98. Determine the values of n for which a theoretical error less than 0.001 can be guaranteed if the integral is estimated using (a) the trapezoidal rule; (b) Simpson's rule.

8.8 Differential Equations; First-Order Linear Equations

99. Find the general solution of $y' - 3y = 6$.
100. Find the general solution of $y' - 2xy = x$.
101. Find the general solution of $y' + \frac{4}{x}y = x^4$.
102. Find the general solution of $y' + \frac{2}{10+2x}y = 4$.
103. Find the general solution of $y' - y = -e^x$.
104. Find the general solution of $x \ln x y' + y = \ln x$.
105. Find the particular solution of $y' + 10y = 20$ determined by the side condition $y(0) = 2$.
106. Find the particular solution of $y' - y = -e^x$ determined by the side condition $y(0) = 3$.
107. Find the particular solution of $xy' - 2y = x^3 \cos 4x$ determined by the side condition $y(\pi) = 1$.
108. A 100-gallon mixing tank is full of brine containing 0.8 pounds of salt per gallon. Find the amount of salt present t minutes later if pure water is poured into the tank at the rate of 4 gallons per minute and the mixture is drawn off at the same rate.
109. Determine the velocity of time t and the terminal velocity of a 2-kg object dropped with a velocity 3 m/s, if the force due to air resistance is $-50v$ Newtons.
110. Use a suitable transformation to solve the Bernoulli equation $y' + xy = xy^2$.
111. Use a suitable transformation to solve the Bernoulli equation $xy' + y = x^3y^6$.

8.9 Separable Equations

112. Find the general solution of $y' = y^2x^3$.
113. Find the general solution of $y' = \frac{x^2 + 7}{y^9 - 3y^4}$.
114. Find the general solution of $y' = y^2 + 1$.
115. Find the general solution of $x(y^2 + 1)y' + y^3 - 2y = 0$.
116. Find the particular solution of $e^x dx - y dy = 0$ determined by the side condition $y(0) = 1$.
117. Find the particular solution of $y' = y(x - 2)$ determined by the side condition $y(2) = 5$.
118. Verify that the equation $y' = \frac{y+x}{x}$ is homogeneous, then solve it.
119. Verify that the equation $y' = \frac{2y^4 + x^4}{xy^3}$ is homogeneous, then solve it.

120. Verify that the equation $y' + \frac{y}{x + \sqrt{xy}}$ is homogeneous, then solve it.
121. Verify that the equation $[2x \sinh (y/x) + 3y \cosh (y/x)]dx - 3x \cosh (y/x)dy = 0$ is homogeneous, then solve it.
122. Find the orthogonal trajectories for the family of curves $x^2 + y^2 = C$.
123. Find the orthogonal trajectories for the family of curves $x^2 + y^2 = Cx$.

Answers to Chapter 8 Questions

1. $\frac{-x}{2}e^{-2x} - \frac{1}{4}e^{-2x} + C$
2. $\frac{-x}{2}\cos 2x + \frac{1}{4}\sin 2x + C$
3. $x \ln(1+x^2) - 2x + 2 \tan^{-1} x + C$
4. $\frac{2x}{3}(1+x)^{3/2} - \frac{4}{15}(1+x)^{5/2} + C$
5. $\frac{e^{-x}}{5}(2 \sin 2x - \cos 2x) + C$
6. $x^2 \ln x - 2x \ln x + 2x + C$
7. $x^2(x^2+1)^{1/2} - \frac{2}{3}(x^2+1)^{3/2} + C$
8. $\frac{2x}{\ln 2} \left[x^2 - \frac{2x}{\ln 2} + \frac{2}{(\ln 2)^2} \right] + C$
9. $x^{3/2} \left[\frac{2}{3} \ln x^2 - \frac{8}{9} \right] + C$
10. $\frac{1}{29}e^{5x}(5 \sin 2x - 2 \cos 2x) + C$
11. $4 \ln 2 - 15/16$
12. $p^2/8 - 1/2$
13. $\frac{2p}{3} - \frac{\sqrt{3}}{2}$
14. $(\bar{x}, \bar{y}) = \left[\frac{e^2 + 1}{2(e^2 - 1)}, \frac{e^2 + 1}{4} \right]$
15. $\frac{p}{2}(e^2 + 1)$
16. $\frac{1}{6}\sin^3 2x + \frac{1}{10}\sin^5 2x + C$
17. $-\frac{1}{6}\cos^6 x + \frac{1}{8}\sin^8 x + C$
18. $\frac{2}{5}\sin^5 \frac{q}{2} + \frac{2}{7}\sin^7 \frac{q}{2} + C$
19. $\frac{-1}{3}\cos 3q + \frac{1}{9}\cos^3 3q + C$
20. $\sqrt{3} - 1$
21. $\frac{3x}{8} + \frac{1}{8}\sin 4x + \frac{1}{64}\sin 8x + C$
22. $\frac{x^2}{16} - \frac{1}{64}\sin 4x^2 + C$
23. $p/4$
24. $\frac{x}{8} - \frac{1}{96}\sin 12x + C$
25. $\frac{2}{3}\sin^3 \frac{x}{2} + C$
26. $\frac{2}{3}\sin^2 \frac{t}{2} - \frac{4}{5}\sin^5 \frac{t}{2} + \frac{2}{7}\sin^7 \frac{t}{2} + C$
27. $\frac{1}{4}\sec^4 x + C$
28. $\frac{3x}{8} - \frac{1}{8}\sin 4x + \frac{1}{64}\sin 8x + C$
29. $p/16$
30. 0
31. $p/6$
32. $\frac{1}{20}\csc^5 4x + \frac{1}{12}\csc^3 4x + C$
33. $\frac{1}{16}\tan^8 2x + \frac{1}{6}\tan^6 2x + \frac{1}{8}\tan^4 2x + C$
34. $\frac{2}{3}\sec^3 \frac{x}{2} + C$
35. $\frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x - \ln|\cos x| + C$
36. $\frac{1}{6}\tan^2 3q + \frac{1}{3}\ln|\cos 3q| + C$
37. $\frac{3}{7}\tan^7 \frac{x}{3} + \frac{6}{5}\tan^5 \frac{x}{3} + \tan^3 \frac{x}{3} + C$

$$38. \frac{1}{2} \ln |\csc 2x - \cot 2x| + \frac{1}{2} \cos 2x + C$$

$$39. \frac{1}{3} \tan^6 \frac{x}{2} + \frac{1}{2} \tan^4 \frac{x}{2} + C$$

$$40. 2\sqrt{2} - p / 4$$

$$41. \frac{-1}{6} \cos^3 2q + \frac{1}{2} \cos 2q + q + C$$

$$42. \frac{1}{8} \tan^8 t + \frac{1}{6} \tan^6 t + C$$

$$43. -\frac{1}{2} \cot 2x - x + C$$

$$44. \frac{-25}{16} (25 - 4x^2)^{1/2} - \frac{1}{48} (25 - 4x^2)^{3/2} + C$$

$$45. \frac{x}{4\sqrt{x^2 + 4}} + C$$

$$46. \frac{-x}{9\sqrt{4x^2 - 9}} + C$$

$$47. \frac{\sqrt{25 + x^2}}{25x} + C$$

$$48. \frac{1}{2} \ln |\sqrt{1 + 2x^2} + \sqrt{2}x| + C$$

$$49. \sqrt{x^2 - 4} - 2 \sec^{-1} \frac{x}{2} + C$$

$$50. 4p$$

$$51. \frac{1}{5} (1 + x^2)^{5/2} - \frac{1}{3} (1 + x^2)^{3/2} + C$$

$$52. \frac{x}{2} \sqrt{x^2 - 3} + \frac{3}{2} \ln |x + \sqrt{x^2 - 3}| + C$$

$$53. \frac{3}{2} \sin^{-1} \frac{x}{\sqrt{3}} - \frac{x}{2} \sqrt{3 - x^2} + C$$

$$54. \frac{-\sqrt{4 - x^2}}{4x} + C$$

$$55. \frac{2\sqrt{2} - \sqrt{5}}{8}$$

$$56. -x^2 \sqrt{4 - x^2} - \frac{2}{3} (4 - x^2)^{3/2} + C$$

$$57. \frac{-\sqrt{9 - x^2}}{9x} + C$$

$$58. \frac{1}{2} \sec^{-1} \frac{x}{2} + C$$

$$59. \frac{x^2}{3} (x^2 - 1)^{3/2} - \frac{2}{15} (x^2 - 1)^{5/2} + C$$

$$60. \ln \left(\frac{3 + \sqrt{8}}{2 + \sqrt{3}} \right)$$

$$61. \frac{1}{3} \ln \left| \frac{\sqrt{4x^2 + 9}}{2x} - \frac{3}{2x} \right| + C$$

$$62. -6 \ln |x| + 7 \ln |x - 1| + \frac{5}{x - 1} + C$$

$$63. 4 \ln \left| \frac{x - 1}{x - 2} \right| - \frac{5}{(x - 2)} + C$$

$$64. 2 \ln |x| - \frac{3}{2} \ln |1 - x| - \frac{1}{2} \ln |1 + x| + C$$

$$65. x + 2 \ln |x - 1| - \frac{1}{x - 1} + C$$

$$66. 2 \ln |x - 2| + \ln |x^2 + 1| + \tan^{-1} x + C$$

$$67. \frac{-3}{2} \ln |x| + \ln |x - 1| + \frac{1}{2} \ln |x - 2| + C$$

$$68. -\frac{1}{3} \ln |x + 1| + \frac{1}{6} \ln (x^2 + 2) + \frac{5}{3\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} + C$$

$$69. \ln |x + 1| + \frac{1}{x + 1} + C$$

$$70. \frac{1}{2} \ln |x^2 + 2x - 3| + C$$

$$71. -\frac{2}{5} \ln |x| + \frac{3}{7} \ln |x - 2| - \frac{1}{35} \ln |x + 5| + C$$

$$72. 2 \ln \frac{x - 1}{x} + \frac{1}{x} + C$$

$$73. \quad \frac{1}{2} \ln|x+1| - \frac{1}{4} \ln(x^2+1) + \frac{1}{2} \tan^{-1} x + C$$

$$74. \quad \frac{1}{6} \ln \left| \frac{\sin q - 1}{\sin q + 5} \right| + C$$

$$75. \quad \ln|x-1| - \frac{2}{x-1} - \ln|x+1| + C$$

$$76. \quad 4 \ln|x| - 2 \ln(x^2+1) + \tan^{-1} x + C$$

$$77. \quad \ln|x-1| + \frac{4}{\sqrt{3}} \tan^{-1} \frac{2x+1}{\sqrt{3}} + C$$

$$78. \quad \frac{2\sqrt{x}}{\sqrt{3}} - \frac{4}{3} \ln|2 + \sqrt{3}x| + C$$

$$79. \quad \frac{x}{3} - \frac{2}{3} \sqrt{x} + \frac{2}{3} \ln|\sqrt{x}+1| + C$$

$$80. \quad \sqrt{3+e^{2x}} + \frac{\sqrt{3}}{2} \ln \left| \frac{\sqrt{3+e^{2x}} - \sqrt{3}}{\sqrt{3+e^{2x}} + \sqrt{3}} \right| + C$$

$$81. \quad \frac{1}{x} + 2 \left(\frac{1}{\sqrt{x}} + \ln \left| \frac{\sqrt{x}-1}{\sqrt{x}} \right| \right) + C$$

$$82. \quad \frac{2}{5} (x-3)^{5/2} + \frac{10}{3} (x-3)^{3/2} + C$$

$$83. \quad x - \sqrt{2x} + \ln|\sqrt{2x}+1| + C$$

$$84. \quad \frac{2}{9} (x+1)^{9/2} - \frac{6}{7} (x+1)^{7/2} + \frac{6}{5} (x+1)^{5/2} - \frac{2}{3} (x+1)^{3/2} + C$$

$$85. \quad \frac{1}{2\sqrt{2}} \ln \left| \frac{\sqrt{2-e^{2x}} - \sqrt{2}}{\sqrt{2-e^{2x}} + \sqrt{2}} \right| + C$$

$$86. \quad \frac{1}{4} \left(\frac{7}{\sqrt{11}} - \sqrt{3} \right)$$

$$87. \quad 152/27$$

$$88. \quad \frac{2}{1 - \tan(x/2)} + C$$

$$89. \quad \sqrt{3} \ln \left| \frac{\tan(x/2) + \sqrt{3}}{\tan(x/2) - \sqrt{3}} \right| + C$$

$$90. \quad -\frac{1}{4} \left(\tan \frac{x}{2} \right)^{-2} - \frac{1}{2} \ln \left| \tan \frac{x}{2} \right| + C$$

$$91. \quad \ln \left| \frac{\tan(x/2)}{[1 + \tan(x/2)]^2} \right| + C$$

$$92. \quad p \left(\frac{4}{3\sqrt{3}} - \frac{1}{2} \right)$$

$$93. \quad \begin{array}{ll} \text{(a) } 4.4914 & \text{(c) } 4.8030 \\ \text{(b) } 5.2234 & \text{(d) } 4.8574 \end{array}$$

$$94. \quad \begin{array}{ll} \text{(a) } 15.5927 & \text{(b) } 30.6239 \end{array}$$

$$95. \quad \begin{array}{ll} \text{(a) } 0.7847 & \text{(b) } 0.7854 \end{array}$$

$$96. \quad \begin{array}{ll} \text{(a) } 1.0865 & \text{(b) } 1.0968 \end{array}$$

$$97. \quad \begin{array}{ll} \text{(a) } 20.8933 & \text{(c) } 28.0490 \\ \text{(b) } 36.2047 & \text{(d) } 27.9586 \end{array}$$

$$98. \quad \begin{array}{ll} \text{(a) } n \geq 19 & \text{(b) } n \geq 2 \end{array}$$

$$99. \quad y = Ce^{-3x}$$

$$100. \quad y = Ce^{x^2} - \frac{1}{2}$$

$$101. \quad y = \frac{C}{x^4} + \frac{1}{9} x^5$$

$$102. \quad y = \frac{40x + 4x^2 + C}{10 + 2x}$$

$$103. \quad y = (C-x)e^x$$

$$104. \quad y = \frac{\ln^2 x + C}{2 \ln x}$$

$$105. \quad y = 2 \text{ (identically)}$$

$$106. \quad y = (3-x)e^x$$

$$107. \quad y = \frac{1}{4} x^2 \sin 4x + \left(\frac{x}{p} \right)^2$$

$$108. \quad 80e^{-0.04t} \text{ pounds}$$

$$109. \quad v = 0.392 + 2.608e^{-25t};$$

terminal velocity 0.392 m/s

$$110. \quad y = \frac{1}{1 + Ce^{x^2/2}}$$

$$111. \quad y = \left(\frac{5}{2} x^3 + Cx^5 \right)^{-1/5}$$

112. $y = \frac{-4}{x^4 + C}$

113. $\frac{1}{10}y^{10} - \frac{3}{5}y^5 - \frac{1}{3}x^3 + 7x = C$

114. $y = \tan(x + C)$

115. $(y^2 - 2)^3 x^4 = Cy^2$

116. $y = \sqrt{2e^x - 1}, x > \ln \frac{1}{2}$

117. $y = 5e^{(x-2)^2/2}$

118. $y = x \ln |Cx|$

119. $x^8 = C(y^4 + x^4)$

120. $-2\sqrt{x/y} + \ln|y| = C$

121. $x^2 = C \sinh^3(y/x)$

122. $y = Kx$

123. $x^2 + y^2 = Ky$

CHAPTER 9

Conic Sections; Polar Coordinates; Parametric Equations

9.1 Translations; The Parabola

1. Sketch and give an equation for the parabola with vertex $(0, 0)$ and directrix $x = 5/2$.
2. Sketch and give an equation for the parabola with vertex $(1, 2)$ and focus $(1, 4)$.
3. Sketch and give an equation for the parabola with focus $(6, -2)$ and directrix $y = 2$.
4. Sketch and give an equation for the parabola with vertex $(-1, 2)$ and focus $(2, 2)$.
5. Find the vertex, focus, axis, and directrix for the parabola $y^2 + 6y + 6x = 0$.
6. Find the vertex, focus, axis, and directrix for the parabola $x^2 - 4x - 2y - 8 = 0$.
7. Find the vertex, focus, axis, and directrix for the parabola $2x^2 - 10x + 5y = 0$.
8. Find an equation for the parabola with directrix $x = -2$ and vertex $(1, 3)$. Where is the focus?
9. Find an equation for the parabola with directrix $y = 3$ and vertex $(-2, 2)$. Where is the focus?
10. Find an equation for the parabola with directrix $x = 5$ and focus $(-1, 0)$. Where is the vertex?
11. Find an equation for the parabola that has vertex $(2, 1)$, passes through $(5, -2)$, and has axis of symmetry parallel to the x -axis.
12. Find the length of the latus rectum for the parabola $(y - 2) = 3(x + 1)^2$.

9.2 The Ellipse and Hyperbola

13. For the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, find
 - (a) the center
 - (b) the foci
 - (c) the length of the major axis
 - (d) the length of the minor axisThen sketch the figure.
14. For the ellipse $5x^2 + 3y^2 = 15$, find
 - (a) the center
 - (b) the foci
 - (c) the length of the major axis
 - (d) the length of the minor axisThen sketch the figure.
15. For the ellipse $36(x - 1)^2 + 4y^2 = 144$, find
 - (a) the center
 - (b) the foci
 - (c) the length of the major axis
 - (d) the length of the minor axisThen sketch the figure.

16. For the ellipse $9x^2 + 16y^2 - 36x + 96y + 36 = 0$, find
(a) the center
(b) the foci
(c) the length of the major axis
(d) the length of the minor axis
Then sketch the figure.
17. For the ellipse $9x^2 + 5y^2 + 36x - 30y + 36 = 0$, find
(a) the center
(b) the foci
(c) the length of the major axis
(d) the length of the minor axis
Then sketch the figure.
18. Find an equation for the ellipse with foci at $(0, 3)$, $(0, -3)$ and major axis 10.
19. Find an equation for the ellipse with foci at $(1, 0)$, vertices at $(-1, 0)$, $(3, 0)$ and foci at $(1 - \sqrt{3}, 0)$ and $(1 + \sqrt{3}, 0)$.
20. Find an equation for the ellipse with focus $(2, 2)$, center at $(2, 1)$, and major axis 10.
21. Find an equation for the ellipse with foci at $(1, -1)$, $(7, -1)$ and minor axis 6.
22. Find the equation of the parabola that has vertex at the origin and passes through the ends of the minor axis of the ellipse $y^2 - 10y + 25x^2 = 0$.
23. Determine the eccentricity of the ellipse $\frac{x^2}{25} + \frac{(y-1)^2}{9} = 1$.
24. Write an equation for the ellipse with major axis from $(-2, 0)$ to $(2, 0)$, eccentricity $\frac{1}{2}$.
25. Find an equation for the hyperbola with foci at $(3, 0)$, $(-3, 0)$ and transverse axis 4.
26. Find an equation for the hyperbola with asymptotes $y = \pm \frac{4}{3}x$ and foci at $(10, 0)$, $(-10, 0)$.
27. Find an equation for the hyperbola with center at $(2, 2)$, a vertex at $(2, 10)$, and a focus at $(2, 11)$.
28. Find an equation for the hyperbola with vertices at $(7, -1)$, $(-5, -1)$ and a focus at $(9, -1)$.
29. For the hyperbola $x^2 - \frac{y^2}{4} = 1$, find the center, the vertices, the foci, the asymptotes, and the length of the transverse axis. Then sketch the figure.
30. For the hyperbola $\frac{(x-1)^2}{4} - \frac{y^2}{16} = 1$, find the center, the vertices, the foci, the asymptotes, and the length of the transverse axis. Then sketch the figure.
31. For the hyperbola $9x^2 - 16y^2 = 144$, find the center, the vertices, the foci, the asymptotes, and the length of the transverse axis. Then sketch the figure.
32. For the hyperbola $16x^2 - 9y^2 - 160x - 72y + 112 = 0$, find the center, the vertices, the foci, the asymptotes, and the length of the transverse axis. Then sketch the figure.

33. For the hyperbola $\frac{(x-3)^2}{9} - \frac{(y+4)^2}{16} = 1$, find the center, the vertices, the foci, the asymptotes, and the length of the transverse axis. Then sketch the figure.
34. For the hyperbola $4y^2 - 9x^2 - 36x - 8y - 68 = 0$, find the center, the vertices, the foci, the asymptotes, and the length of the transverse axis. Then sketch the figure.
35. Determine the eccentricity of the hyperbola $\frac{x^2}{49} - y^2 = 1$.
36. Determine the eccentricity of the hyperbola $\frac{y^2}{16} - \frac{x^2}{12} = 1$.

9.3 Polar Coordinates

37. Find the rectangular coordinates of the point with polar coordinates $[4, 2p/3]$.
38. Find the rectangular coordinates of the point with polar coordinates $[3, -p/4]$.
39. Find the rectangular coordinates of the point with polar coordinates $[-2, -p/3]$.
40. Find all possible polar coordinates for the point with rectangular coordinates $(-4, 4\sqrt{3})$.
41. Find all possible polar coordinates for the point with rectangular coordinates $(2, -2)$.
42. Find the point $[r, q]$ symmetric to the point $[2, p/6]$ about
 (a) the x -axis
 (b) the y -axis
 (c) the origin
 Express your answer with $r > 0$ and $q \in [0, 2p]$.
43. Find the point $[r, q]$ symmetric to the point $[2/3, 5p/4]$ about
 (a) the x -axis
 (b) the y -axis
 (c) the origin
 Express your answer with $r > 0$ and $q \in [0, 2p]$.
44. Test the curve $r = 3 + 2 \cos q$ for symmetry about the coordinate axes and the origin.
45. Test the curve $r \sin 2q = 1$ for symmetry about the coordinate axes and the origin.
46. Write the equation $x + y^2 = x - y$ in polar coordinates.
47. Write the equation $x^2 + y^2 - 6y = 0$ in polar coordinates.
48. Write the equation $x^2 + y^2 + 8y = 0$ in polar coordinates.
49. Write the equation $x^4 + x^2y^2 = y^2$ in polar coordinates.
50. Write the equation $y^3 + x^2y = x$ in polar coordinates.
51. Write the equation $y^6 + x^2y^4 = x^4$ in polar coordinates.
52. Write the equation $(x^2 + y^2)^2 = 4xy$ in polar coordinates.

53. Write the equation $x(x^2 + y^2) = 2(3x^2 - y^2)$ in polar coordinates.
54. Write the equation $(x^2 + y^2)^{3/2} = x^2 - y^2 - 2xy$ in polar coordinates.
55. Identify the curve given by $r \sin \theta = 2$ and write the equation in rectangular coordinates.
56. Identify the curve given by $r = 4 \cos \theta$ and write the equation in rectangular coordinates.
57. Identify the curve given by $\theta^2 = \frac{4}{9}r^2$ and write the equation in rectangular coordinates.
58. Identify the curve given by $r = 4 \sin \theta - 6 \cos \theta$ and write the equation in rectangular coordinates.
59. Identify the curve given by $r = 3 \cos \theta - \sin \theta$ and write the equation in rectangular coordinates.
60. Identify the curve given by $r = \frac{1}{\cos \theta - 1}$ and write the equation in rectangular coordinates.
61. Identify the curve given by $r = \frac{10}{2 + \cos \theta}$ and write the equation in rectangular coordinates.
62. Identify the curve given by $r = \frac{1}{1 - \cos \theta}$ and write the equation in rectangular coordinates.
63. The parabola $r = \frac{1}{1 + \cos \theta}$ has focus at the pole and directrix $x = 2$. Without resorting to xy -coordinates,
(a) locate the vertex of the parabola
(b) find the width of the latus rectum
(c) sketch the parabola.
64. The ellipse $r = \frac{10}{8 + 5 \cos \theta}$ has right focus at the pole, major axis horizontal. Without resorting to xy -coordinates,
(a) find the eccentricity of the ellipse
(b) locate the ends of the major axis
(c) locate the center of the ellipse
(d) locate the second focus
(e) determine the length of the minor axis
(f) determine the width of the ellipse at the foci
(g) sketch the ellipse
65. The hyperbola $r = \frac{9}{2 + 6 \cos \theta}$ has left focus at the pole, transverse axis horizontal. Without resorting to xy -coordinates,
(a) find the eccentricity of the hyperbola
(b) locate the ends of the transverse axes
(c) locate the center of the hyperbola
(d) locate the second focus
(e) determine the width of the hyperbola at the foci, and sketch the hyperbola.
66. Find the points at which the curves $r = 2 \cos \theta$ and $r = -1$ intersect. Express your answers in rectangular coordinates.
67. Find the points at which the curves $r = 1 + \cos \theta$ and $r = 2 \cos \theta$ intersect. Express your answers in rectangular coordinates.

68. Find the points at which the curves $r = \sin 3q$ and $r = 2 \sin q$ intersect. Express your answers in rectangular coordinates.
69. Find the points at which the curves $r = \frac{1}{2} + \cos q$ and $q = \pi/4$ intersect. Express your answers in rectangular coordinates.
70. Find the points at which the curves $r = \frac{1}{1 + \cos q}$ and $r \sin q = 2$ intersect. Express your answers in rectangular coordinates.

9.4 Graphing in Polar Coordinates

71. Sketch and identify the polar curve $r^2 = 9 \sin 2q$.
72. Sketch and identify the polar curve $r = 1 + \cos q$.
73. Sketch and identify the polar curve $r = 2 \cos q$.
74. Sketch and identify the polar curve $r = \sin 3q$.
75. Sketch and identify the polar curve $r = 4 + 4 \cos q$.
76. Sketch and identify the polar curve $r = \sqrt{2}$.
77. Sketch and identify the polar curve $r^2 = 4 \cos 2q$.
78. Sketch and identify the polar curve $r = 2 - 4 \sin q$.
79. Sketch and identify the polar curve $r = \cos 3q$.
80. Sketch and identify the polar curve $r = 2 \sin 2q$.
81. Sketch and identify the polar curve $r = 2 + 4 \sin q$.
82. Sketch and identify the polar curve $r = 4 + 2 \sin q$.
83. Sketch and identify the polar curve $r = 3 \sin q$.
84. Sketch and identify the polar curve $r = 1 - 2 \cos q$.
85. Sketch and identify the polar curve $r = 2 + 4 \cos q$.
86. Sketch and identify the polar curve $r = 3 + 2 \cos q$.
87. Sketch and identify the polar curve $r = 4(1 - \cos q)$.
88. Sketch and identify the polar curve $r = 4(1 - \sin q)$.

9.5 Area in Polar Coordinates

89. Find the area of the region enclosed by $r = 4 \sin 3q$.
90. Find the area of the region enclosed by $r = 2 + \sin q$.

91. Find the area of the region that is inside $r = 5 \sin q$ but outside $r = 2 + \sin q$.
92. Find the area of the region that is outside $r = 1 + \sin q$ but inside $r = 3 + \sin q$.
93. Find the area of the region that is common to $r = 3 \cos q$ and $r = 1 + \cos q$.
94. Find the area of the region that is common to $r = 1 + \sin q$ and $r = 1$.
95. Find the area of the region that is inside $r = 1$ but outside $r = 1 - \cos q$.
96. Find the area of the region enclosed by $r = 2 + \cos q$.
97. Find the area of the region that is inside $r = 2 \cos q$ but outside $r = \sin q$.
98. Find the area of the region enclosed by $r = 1 - \sin q$.
99. Find the area of the region that is common to $r = 3a \cos q$ and $r = a(1 + \cos q)$.
100. Find the area of the region that is inside $r = 2$ but outside $r = 1 + \cos q$.
101. Find the area of the region enclosed by $r = 2 \cos 3q$.
102. Find the area of the region that is inside $r = 3(1 + \sin q)$ but outside $r = 3 \sin q$.
103. Find the area of the region that is common to $r = a \cos 3q$ and $r = a/2$. Take $a > 0$.
104. Find the area of the region enclosed by $r^2 = \cos 2q$.
105. Find the area of the region enclosed by the inner loop of $r = 1 - 2 \sin q$.
106. Find the centroid of the region enclosed by $r = 2 \cos q$.

9.6 Curves Given Parametrically

107. Express the curve by an equation in x and y : $x(t) = 2t \sin t$, $y(t) = 3 - \cos t$.
108. Express the curve by an equation in x and y : $x(t) = e^t - 1$, $y(t) = 3 + e^{2t}$.
109. Express the curve by an equation in x and y : $x(t) = 2 \cos t$, $y(t) = 3 \sin t$.
110. Express the curve by an equation in x and y : $x(t) = 3t \cosh t$, $y(t) = 2t \sinh t$.
111. Express the curve by an equation in x and y : $x(t) = 3 + \cos t$, $y(t) = 3 - 2 \sin t$.
112. Express the curve by an equation in x and y : $x(t) = 2t^2 + t - 3$, $y(t) = t - 1$.
113. Express the curve by an equation in x and y : $x(t) = \cos 2t$, $y(t) = \sin t$.
114. Express the curve by an equation in x and y : $x(t) = -1 + 3 \cos t$, $y(t) = \sin t$.
115. Find the parametrization $x = x(t)$, $y = y(t)$, $t \in [0, 1]$, for the line segment from $(2, 5)$ to $(5, 8)$.
116. Find the parametrization $x = x(t)$, $y = y(t)$, $t \in [0, 1]$, for the curve $y = x^2$ from $(1, 1)$ to $(3, 9)$.

9.7 Tangents to Curves Given Parametrically

117. Find an equation in x and y for the line tangent to the curve $x(t) = 3t$, $y(t) = t^2 - 1$ at $t = 1$.
118. Find an equation in x and y for the line tangent to the curve $x(t) = 2t^2$, $y(t) = (1 - t)^2$ at $t = 1$.
119. Find an equation in x and y for the line tangent to the curve $x(t) = 2e^t$, $y(t) = \frac{1}{2}e^{-t}$ at $t = 0$.
120. Find an equation in x and y for the line tangent to the curve $x(t) = 2/t$, $y(t) = 2t^2 + 3$ at $t = 1$.
121. Find an equation in x and y for the line tangent to the polar curve $r = 3 + 2 \sin \theta$ at $\theta = \pi/2$.
122. Find an equation in x and y for the line tangent to the polar curve $r = 3 \sin 3\theta$ at $\theta = \pi/6$.
123. Find an equation in x and y for the line tangent to the polar curve $r = \frac{3}{2 + \cos \theta}$ at $\theta = \pi/2$.
124. Find the points (x, y) at which the curve $x(t) = 2t - t^3$, $y(t) = t - 1$ has (a) a horizontal tangent; (b) a vertical tangent.
125. Find the points (x, y) at which the curve $x(t) = 4 + 3 \sin t$, $y(t) = 3 + 4 \cos t$ has (a) a horizontal tangent; (b) a vertical tangent.
126. Find the tangent(s) to the curve $x(t) = t^2 - 2t$, $y(t) = 1 - t$ at the point $(-1, 0)$.
127. Calculate $\frac{d^2y}{dx^2}$ at the point $t = 1$ without eliminating the parameter if $x(t) = e^t - 1$ and $y(t) = 3 + e^{2t}$.
128. Calculate $\frac{d^2y}{dx^2}$ at the point $t = 3\pi/4$ without eliminating the parameter if $x(t) = 5 - 2 \cos t$ and $y(t) = 3 + \sin t$.

9.8 Arc Length and Speed

129. Find the arc length of the curve $f(x) = 2x^{3/2}$, $x \in [0, 8/9]$ and compare it to the straight-line distance between the endpoints.
130. Find the arc length of the curve $f(x) = \frac{x^3}{6} + \frac{1}{2x}$, $x \in [1, 3]$ and compare it to the straight-line distance between the endpoints.
131. Find the arc length of the curve $f(x) = \frac{2}{3}(x + 1)^{3/2}$, $x \in [1, 2]$ and compare it to the straight-line distance between the endpoints.
132. Find the arc length of the curve $f(x) = x^{2/3}$, $x \in [0, 8]$ and compare it to the straight-line distance between the endpoints.
133. Find the arc length of the curve $f(x) = \frac{2}{3}x^{3/2} - \frac{1}{2}x^{1/2}$, $x \in [1, 4]$ and compare it to the straight-line distance between the endpoints.

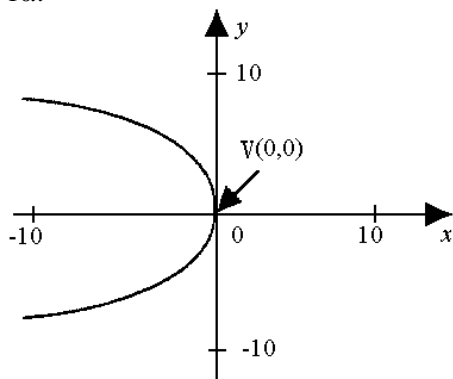
134. Find the arc length of the curve $f(y) = \frac{3}{5}y^{5/3} - \frac{3}{4}x^{1/3}$, $y \in [1, 8]$ and compare it to the straight-line distance between the endpoints.
135. Find the arc length of the curve $f(x) = 2x^{3/2}$, $x \in [0, 3]$ and compare it to the straight-line distance between the endpoints.
136. Find the arc length of the curve $f(x) = \frac{1}{3}(x^2 + 2)^{3/2}$, $x \in [0, 3]$ and compare it to the straight-line distance between the endpoints.
137. The equations $x(t) = 2 + \sin t$, $y(t) = 3 - \cos t$ give the position of a particle at time t from $t = 0$ to $t = p/2$. Find the initial speed of the particle, the terminal speed, and the distance traveled.
138. The equations $x(t) = e^t \sin t$, $y(t) = e^t \cos t$ give the position of a particle at time t from $t = 0$ to $t = p$. Find the initial speed of the particle, the terminal speed, and the distance traveled.
139. The equations $x(t) = t^2 + 2$, $y(t) = t^3 - 3$ give the position of a particle at time t from $t = 0$ to $t = 1$. Find the initial speed of the particle, the terminal speed, and the distance traveled.
140. The equations $x(t) = 3(t - 1)^2$, $y(t) = 8t^{3/2}$ give the position of a particle at time t from $t = 0$ to $t = 1$. Find the initial speed of the particle, the terminal speed, and the distance traveled.
141. Find the length of the polar curve $r = 2e^{3\theta}$ from $\theta = 0$ to $\theta = p$.
142. Find the length of the polar curve $r = 2 \cos 2\theta$ from $\theta = 0$ to $\theta = 2p$.

9.9 The Area of a Surface of Revolution; The Centroid of a Curve; Pappus's Theorem on Surface Area

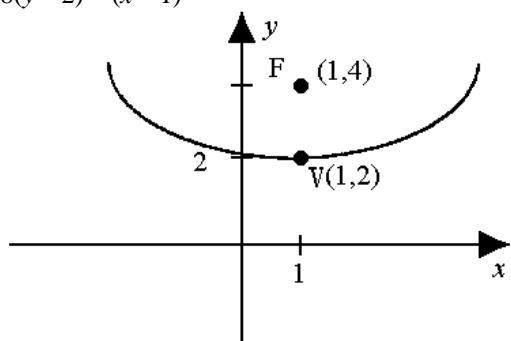
142. Find the length of the curve, locate its centroid, and determine the area of the surface generated by revolving $f(x) = 3x$, $x \in [0, 1]$ about the x -axis.
144. Find the length of the curve, locate its centroid, and determine the area of the surface generated by revolving $y = \frac{1}{2}x$, $x \in [0, 2]$ about the x -axis.
145. Find the area of the surface generated by revolving $f(x) = 2\sqrt{x+1}$, $x \in [-1, 1]$ about the x -axis.
146. Find the area of the surface generated by revolving $y = \sin x$, $x \in [0, p/2]$ about the x -axis.

Answers to Chapter 9 Questions

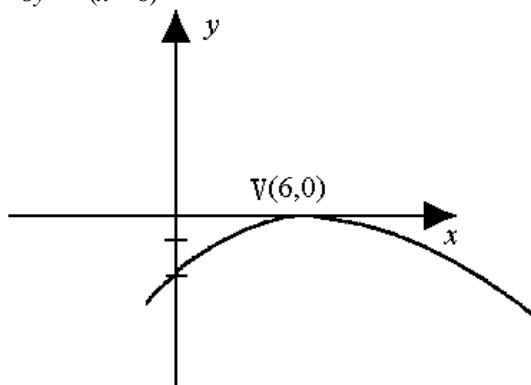
1. $y^2 = -10x$



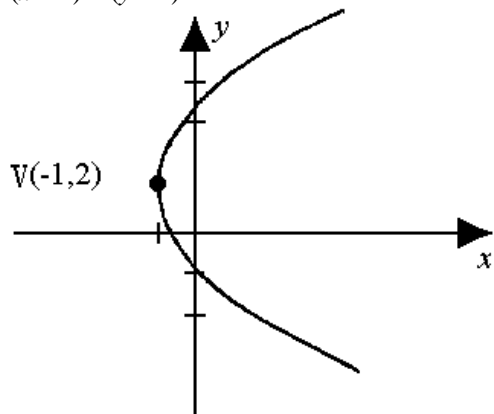
2. $8(y-2) = (x-1)^2$



3. $8y = -(x-6)^2$



4. $12(x+1) = (y-2)^2$



5. $V: (3/2, -3)$ $F: (-3/2, -3)$
axis: $y = -3$ directrix: $x = 9/2$

6. $V: (2, -6)$ $F: (2, -11/2)$
axis: $x = 2$ directrix: $y = -13/2$

7. $V: (5/2, 5/2)$ $F: (5/2, 15/8)$
axis: $x = 5/2$ directrix: $y = 25/8$

8. $(y-3)^2 = 12(x-1)$; $F: (4, 3)$

9. $(x+2)^2 = -4(y-2)$; $F: (-2, 1)$

10. $y^2 = -12(x-2)$; $V: (0, 2)$

11. $(y-1)^2 = 3(x-2)$

12. 3

13. (a) $(0, 0)$ (c) 8
(b) $(\sqrt{7}, 0), (-\sqrt{7}, 0)$ (d) 6

14. (a) $(0, 0)$ (c) $2\sqrt{5}$
(b) $(0, \sqrt{2}), (0, -\sqrt{2})$ (d) $2\sqrt{3}$

15. (a) $(1, 0)$ (c) 12
(b) $(1, 4\sqrt{2}), (1, -4\sqrt{2})$ (d) 4

16. (a) $(2, -3)$ (c) 8
(b) $(2 + \sqrt{7}, -3), (2 - \sqrt{7}, -3)$ (d) 6

17. (a) $(-2, 3)$ (c) 6
(b) $(-2, 1), (-2, 5)$ (d) $2\sqrt{5}$

18. $\frac{x^2}{25} + \frac{y^2}{34} = 1$

19. $\frac{(x-1)^2}{4} + y^2 = 1$

20. $\frac{(x-2)^2}{25} + (y-1)^2 = 1$

21. $\frac{(x-4)^2}{18} + \frac{(y+1)^2}{9} = 1$

22. $x^2 = \frac{1}{5}y$

23. $4/5$

24. $\frac{x^2}{4} + \frac{y^2}{3} = 1$

25. $\frac{x^2}{4} - \frac{y^2}{5} = 1$

26. $\frac{x^2}{36} - \frac{y^2}{64} = 1$

27. $\frac{(y-2)^2}{64} - \frac{(x-2)^2}{17} = 1$

28. $\frac{(x-1)^2}{36} - \frac{(y+1)^2}{28} = 1$

29. center: (0, 0)
 vertices: (-1, 0), (1, 0)
 foci: $(-\sqrt{5}, 0), (\sqrt{5}, 0)$
 asymptotes: $y = \pm 2x$
 length of transverse axis: 2

30. center: (1, 0)
 vertices: (-1, 0), (3, 0)
 foci: $(1-2\sqrt{5}, 0), (1+2\sqrt{5}, 0)$
 asymptotes: $y = \pm 2(x-1)$
 length of transverse axis: 4

31. center: (0, 0)
 vertices: (-4, 0), (4, 0)
 foci: (-5, 0), (5, 0)
 asymptotes: $y = \pm \frac{3}{4}x$
 length of transverse axis: 8

32. center: (5, -4)
 vertices: (2, -4), (8, -4)
 foci: (0, -4), (10, -4)
 asymptotes: $y = -4 \pm \frac{4}{3}(x-5)$
 length of transverse axis: 6

33. center: (3, -4)
 vertices: (0, -4), (6, -4)
 foci: (-2, -4), (8, -4)
 asymptotes: $y = -4 \pm \frac{4}{3}(x-3)$
 length of transverse axis: 6

34. center: (-2, 1)
 vertices: (-2, -2), (-2, 4)
 foci: $(-2, 1-\sqrt{13}), (-2, 1+\sqrt{13})$
 asymptotes: $y = 1 \pm \frac{3}{2}(x+2)$
 length of transverse axis: 6

35. $\frac{5\sqrt{2}}{7}$

36. $\frac{\sqrt{7}}{2}$

37. $(-2, 2\sqrt{3})$

38. $(\sqrt{2}, -\sqrt{2})$

39. $(-1, \sqrt{3})$

40. $\left[8, \frac{2p}{3} + 2np\right], \left[-8, \frac{5p}{3} + 2np\right]$

41. $\left[2\sqrt{2}, \frac{7p}{4} + 2np\right], \left[-2\sqrt{2}, \frac{3p}{4} + 2np\right]$

42. (a) $\left[2, \frac{11p}{6}\right]$ (b) $\left[2, \frac{5p}{6}\right]$ (c) $\left[2, \frac{7p}{6}\right]$

43. (a) $\left[\frac{2}{3}, \frac{3p}{4}\right]$ (b) $\left[\frac{2}{3}, \frac{7p}{4}\right]$ (c) $\left[\frac{2}{3}, \frac{p}{4}\right]$

44. x -axis only

45. symmetric with respect to x -axis, y -axis, origin

46. $r = \frac{\cos q - \sin q}{1 + 2 \sin q \cos q}$

47. $r = 6 \cos q$

48. $r = -8 \sin q$

49. $r = \tan q$

50. $r^2 = \cot q$

51. $r = \cot^2 q$

52. $r^2 = 2 \sin 2q$

53. $r = 2(3 \cos q - \sin q \tan q)$

54. $r = \cos q - \sin 2q$

55. $y = 2$; line

56. circle; center (2, 0)
 $x^2 + y^2 = 4x$ or $(x-2)^2 + y^2 = 4$

57. half lines; $q = \pm \frac{2p}{3}$

$$y = -3\sqrt{x} \quad (x \leq 0)$$

$$y = 3\sqrt{x} \quad (x \leq 0)$$

58. $(x+3)^2 + (y-2)^2 = 13$; circle

59. $(x-3/2)^2 + (y+1/2)^2 = 5/2$; circle

60. $y^2 = -2(x-1/2)$; parabola

61. $\frac{(x+10/3)^2}{400/9} + \frac{y^2}{400/12} = 1$; ellipse

62. $y^2 = 2(x+1/2)$; parabola

63. (a) (1, 0) (b) 4

64. (a) 5/8 (d) $(-100/39, 0)$

(b) $(-10/3, 0), (10/13, 0)$ (e) $20/\sqrt{39}$

(c) $(-50/39, 0)$ (f) 5/2

65. (a) 3 (d) $(27/8, 0)$

(b) $(9/8, 0), (9/4, 0)$ (e) 9

(c) $(27/16, 0)$

66. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

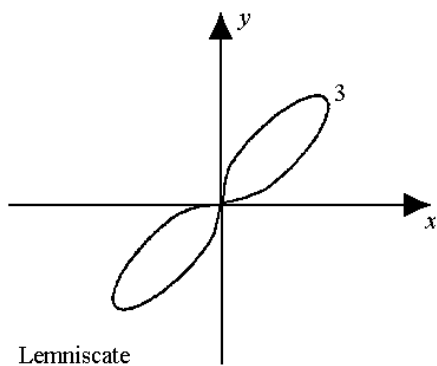
67. (2, 0), (0, 0)

68. $\left(\frac{-\sqrt{2}}{2}, \frac{1}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), (0, 0)$

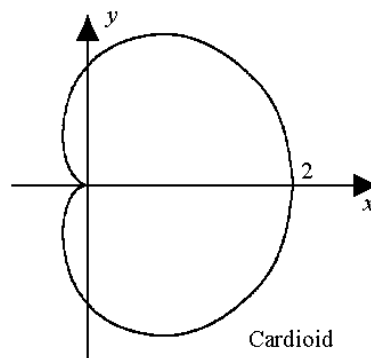
69. $(0, 0), \left(\frac{2+\sqrt{2}}{4}, \frac{2+\sqrt{2}}{4}\right), \left(\frac{2-\sqrt{2}}{4}, \frac{2-\sqrt{2}}{4}\right)$

70. $(-3/2, 2)$

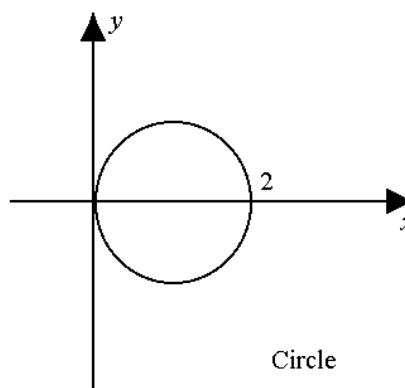
71.



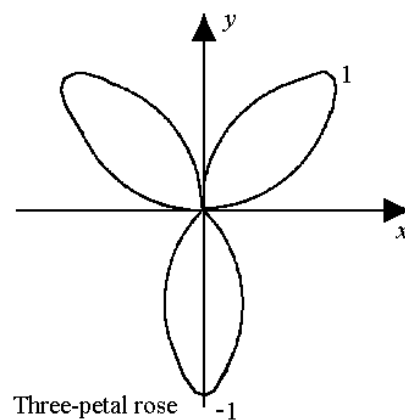
72.



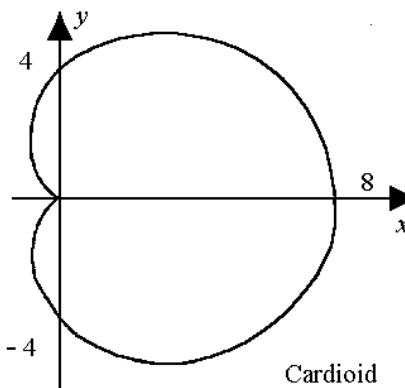
73.



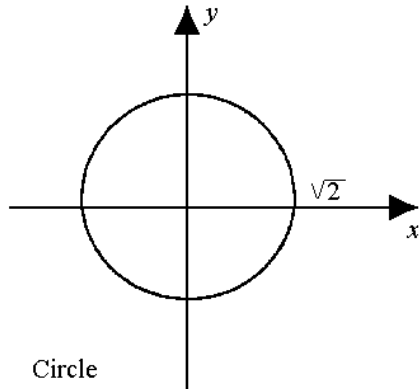
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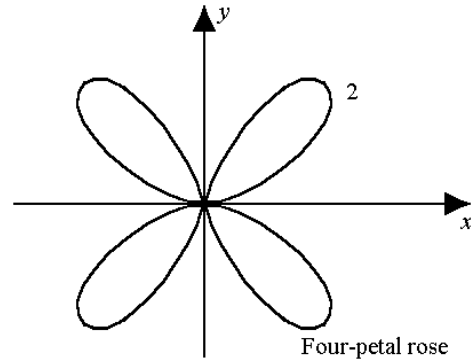
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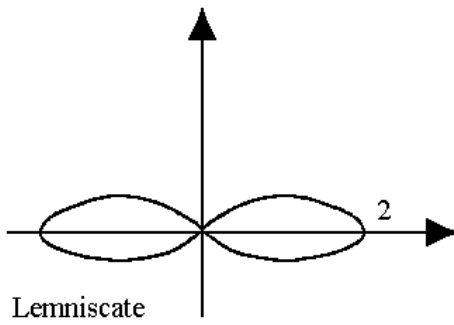
76.



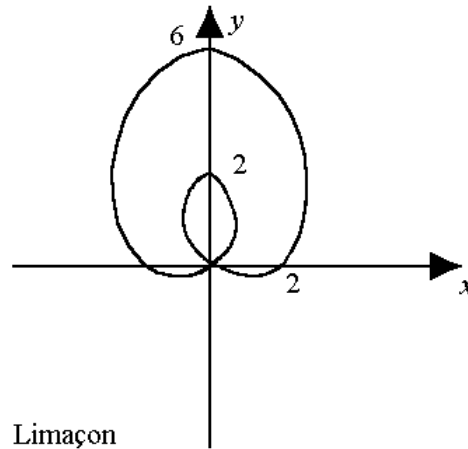
80.



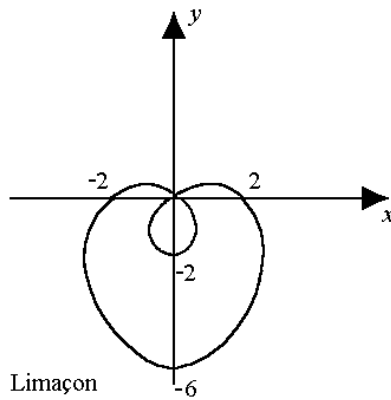
77.



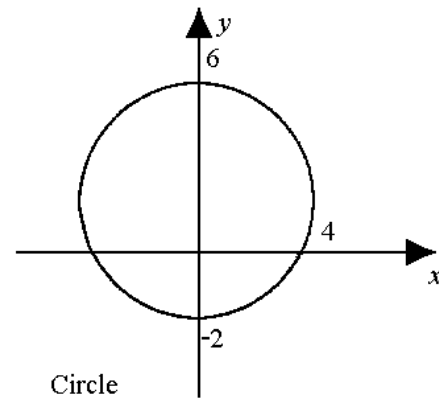
81.



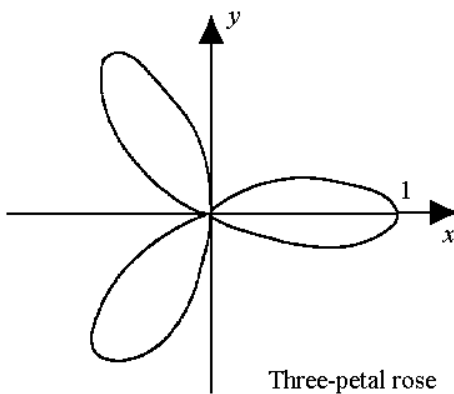
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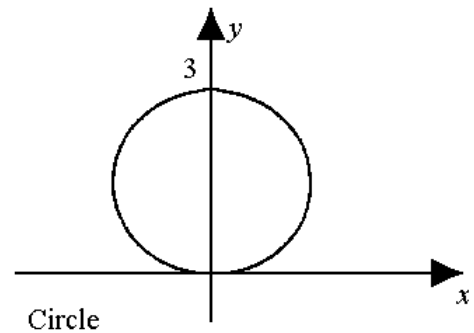
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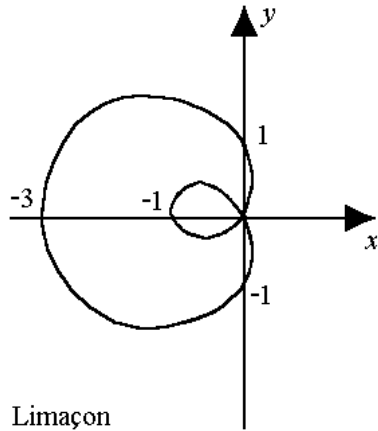
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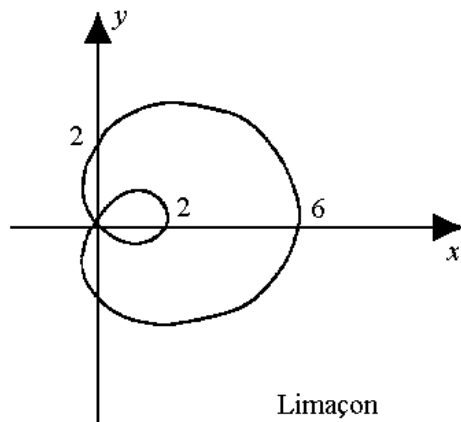
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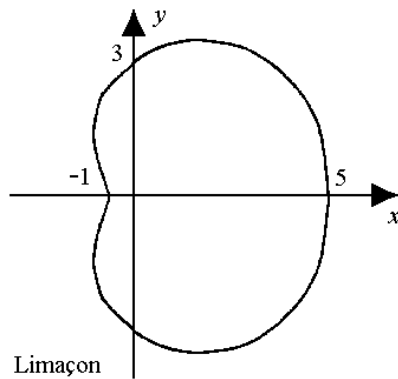
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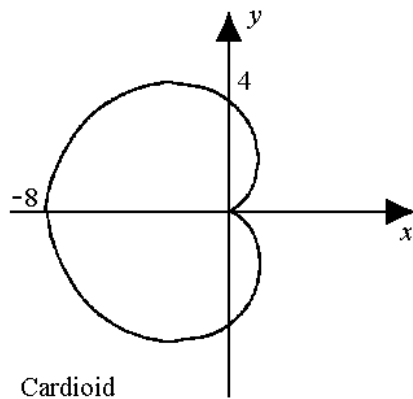
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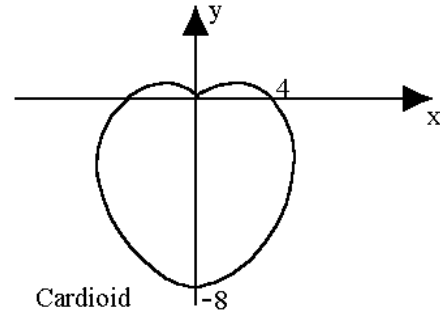
86.



87.



88.

89. 4π 90. $9\pi/2$ 91. $8\pi/3 + \sqrt{3}$ 92. 8π 93. $5\pi/4$ 94. $5\pi/4 - 2$ 95. $2 - \pi/4$ 96. $9\pi/2$ 97. $\frac{p}{2} + \frac{3}{4} \tan^{-1} 2 + \frac{1}{2}$ 98. $3\pi/2$ 99. $5\pi a^2/4$ 100. $5\pi/2$ 101. π 102. $45\pi/4$ 103. $a^2 \left(\frac{p}{6} - \frac{\sqrt{3}}{8} \right)$

104. 1

105. $p - \frac{3\sqrt{3}}{2}$

106. (1, 0)

107. $(x-2)^2 + (y-3)^2 = 1$; circle108. $y = x^2 + 2x + 4 = (x+1)^2 + 3$; parabola

109. $x^2/4 + y^2/9 = 1$; ellipse
110. $(x-3)^2 - (y-2)^2 = 1$; hyperbola
111. $(x-3)^2 + \frac{(y-3)^2}{4} = 1$; ellipse
112. $x = 2y^2 + 5y$; parabola
113. $x = 1 - 2y^2$; parabola
114. $\frac{(x+1)^2}{9} + \frac{y^2}{1} = 1$; ellipse
115. $x = 2 + 3t$; $y = 5 + 3t$
116. $x = 2 + 3t$; $y = 1 + 4t + 4t^2$
117. $2x - 3y = 6$
118. $y = 0$
119. $x + 4y = 4$
120. $2x + y = 9$
121. $y = 5$
122. $\sqrt{3}x + y = 6$
123. $x + 2y = 3$
124. (a) None
(b) $\left(\frac{4\sqrt{2}}{3\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}} - 1 \right); \left(\frac{-4\sqrt{2}}{3\sqrt{3}}, \frac{-\sqrt{2}}{\sqrt{3}} - 1 \right)$
125. (a) (4, 7), (4, -11)
(b) (7, 3), (1, 3)
126. vertical tangent
127. 2
128. $\frac{-\sqrt{2}}{2}$
129. 52/27
130. 14/3
131. $\frac{2}{3}(8 - 3\sqrt{3})$
132. $\frac{8}{27}(10\sqrt{10} - 1)$
133. 31/6
134. 387/20
135. $\frac{2}{27}(56\sqrt{7} - 1)$
136. 12
137. initial speed = 1; terminal speed = 1
distance = $\pi/2$
138. initial speed = $\sqrt{2}$; terminal speed = $e^p \sqrt{2}$
distance = $\sqrt{2}(e^p - 1)$
139. initial speed = 0; terminal speed = $\sqrt{13}$
distance = $\frac{1}{27}(13\sqrt{13} - 8)$
140. initial speed = 0; terminal speed = 12
distance = 9
141. $\frac{2\sqrt{10}}{3}(e^3 p - 1)$
142. 4π
143. length: $\sqrt{10}$; centroid: $\left(\frac{1}{2}, \frac{3}{2} \right)$
area: $3p\sqrt{10}$
144. length: $\sqrt{5}$; centroid: $\left(1, \frac{1}{2} \right)$
area: $p\sqrt{5}$
145. $8p\left(\sqrt{3} - \frac{1}{3} \right)$
146. $p[\sqrt{2} + \ln(1 + \sqrt{2})]$

CHAPTER 10

Sequences; Intermediate Forms; Improper Integrals

10.1 The Least Upper Bound Axiom

1. Find the least upper bound (if it exists) and the greatest lower bound (if it exists) for the interval $(0, 4)$.
2. Find the least upper bound (if it exists) and the greatest lower bound (if it exists) for the set $\{x: x^2 < 5\}$.
3. Find the least upper bound (if it exists) and the greatest lower bound (if it exists) for the set $\{x: x^2 > 9\}$.
4. Find the least upper bound (if it exists) and the greatest lower bound (if it exists) for the set $\{x: |x - 2| > 1\}$.
5. Find the least upper bound (if it exists) and the greatest lower bound (if it exists) for the set $\{x: |x - 3| < 1\}$.
6. Find the least upper bound (if it exists) and the greatest lower bound (if it exists) for the set $\{x: x^2 + 3x + 2 \geq 0\}$.

10.2 Sequences of Real Numbers

7. Give an explicit formula for the n th term of the sequence $\{1, 4/5, 6/8, 8/11, 10/14, 12/17, \dots\}$.
8. Give an explicit formula for the n th term of the sequence $\{1/2, -1/4, 1/8, -1/16, \dots\}$.
9. Give an explicit formula for the n th term of the sequence $\{1, 4, 1/9, 16, 1/25, 36, \dots\}$.
10. Determine the boundedness and monotonicity of the sequence $\frac{2n}{n+1}$.
11. Determine the boundedness and monotonicity of the sequence $\frac{2n}{2n-1}$.
12. Determine the boundedness and monotonicity of the sequence $\frac{2n-5}{3n+2}$.
13. Determine the boundedness and monotonicity of the sequence $1 - \frac{2}{n}$.
14. Determine the boundedness and monotonicity of the sequence $\frac{3^n}{2^n + 3}$.
15. Determine the boundedness and monotonicity of the sequence $\frac{1}{3n} - \frac{1}{3n+1}$.
16. Determine the boundedness and monotonicity of the sequence $\frac{4^n}{(n+2)^2}$.
17. Determine the boundedness and monotonicity of the sequence $\frac{3^n}{e^n}$.

18. Determine the boundedness and monotonicity of the sequence $\frac{3^n}{(n+1)!}$.
19. Determine the boundedness and monotonicity of the sequence $\frac{n+2}{e^n}$.
20. Determine the boundedness and monotonicity of the sequence $\left(\frac{9}{10}\right)^n$.
21. Determine the boundedness and monotonicity of the sequence $\sin \frac{n\pi}{3}$.
22. Write the first six terms of the sequence given by $a_1 = 1$; $a_{n+1} = \frac{n+2}{n+1}a_n$, and then give the general term.
23. Write the first six terms of the sequence given by $a_1 = 1$; $a_{n+1} = 2a_n - 1$, and then give the general term.
24. Write the first six terms of the sequence given by $a_1 = 1$; $a_{n+1} = 2a_n - n(n+1)$, and then give the general term.
25. Write the first six terms of the sequence given by $a_1 = 1$; $a_{n+1} = 1 - 2a_n$, and then give the general term.
26. Write the first six terms of the sequence given by $a_1 = 1$; $a_2 = 4$; $a_{n+1} = 2a_n - a_{n-1}$, and then give the general term.
27. Write the first six terms of the sequence given by $a_1 = 2$; $a_2 = 2$; $a_{n+1} = 2a_n - a_{n-1}$, and then give the general term.
28. Write the first six terms of the sequence given by $a_1 = 3$; $a_2 = 5$; $a_{n+1} = 3a_n - 2a_{n-1} - 2$, and then give the general term.

10.3 Limit of a Sequence

29. State whether or not the sequence $\frac{1}{n} \sin \frac{\pi}{n}$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
30. State whether or not the sequence $\frac{\ln n}{n}$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
31. State whether or not the sequence $(-1)^{n+1} \frac{n}{n+2}$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
32. State whether or not the sequence $(1+n)^{1/n}$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
33. State whether or not the sequence $\frac{n}{n+2}$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
34. State whether or not the sequence $1 + (-1)^n$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
35. State whether or not the sequence $\frac{n^3 + 6n^2 + 11n + 6}{2n^3 + 3n^2 + 1}$ converges as $n \rightarrow \infty$, and, if it does, find the limit.

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36. State whether or not the sequence $\frac{\sqrt{n}}{\ln n}$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
37. State whether or not the sequence $\frac{1-n^2}{2+3n^2}$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
38. State whether or not the sequence $n \sin \frac{1}{n}$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
39. State whether or not the sequence $\frac{2n}{\sqrt{n^2-1}}$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
40. State whether or not the sequence $\frac{n}{2n+1}$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
41. State whether or not the sequence $\frac{\sin n}{n}$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
42. State whether or not the sequence $\frac{n+6}{2n+3}$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
43. State whether or not the sequence $\frac{(n+1)^2}{4^n}$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
44. State whether or not the sequence $\frac{3^n}{n!}$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
45. State whether or not the sequence $\frac{2n-5}{3n+2}$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
46. State whether or not the sequence $\frac{e^n}{\sqrt{n}}$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
47. State whether or not the sequence $\frac{1-n^2}{2+3n^2}$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
48. State whether or not the sequence $\ln\left(\frac{2n}{n+1}\right)$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
49. State whether or not the sequence $\ln\left(\frac{2n}{4n+1}\right)$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
50. State whether or not the sequence $\ln\left(\frac{n^2}{n+1}\right)$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
51. State whether or not the sequence defined recursively by $a_1 = 1$, $a_{n+1} = -\frac{1}{3}a_n$ converges as $n \rightarrow \infty$, and, if it does, find the limit.
52. State whether or not the sequence defined recursively by $a_1 = 1$, $a_{n+1} = (-1)^n a_n$ converges as $n \rightarrow \infty$, and, if it does, find the limit.

53. State whether or not the sequence defined recursively by $a_1 = 1$, $a_{n+1} = 1 - \frac{1}{2}a_n$ converges as $n \rightarrow \infty$, and, if it does, find the limit.

10.4 The Indeterminate Form (0 / 0)

54. Find $\lim_{x \rightarrow e} \left[\frac{\ln(\ln x)}{\ln x - 1} \right]$.
55. Find $\lim_{x \rightarrow \infty} \frac{x^{-3/2}}{\sin \frac{1}{x}}$.
56. Find $\lim_{x \rightarrow 0} \frac{x}{1 + \sin x}$.
57. Find $\lim_{x \rightarrow 0} \frac{\sin x - x}{\tan x - x}$.
58. Find $\lim_{x \rightarrow 0} \frac{\tan x}{x}$.
59. Find $\lim_{x \rightarrow 0} \frac{x - \sin x}{2 + 2x + x^2 - 2e^x}$.
60. Find $\lim_{x \rightarrow 0} \frac{5^x - 3^x}{x}$.
61. Find $\lim_{x \rightarrow 0} \left[\frac{x - \ln(1+x)}{x^2} \right]$.
62. Find $\lim_{x \rightarrow \pi/4} \frac{1 - \tan x}{\cos 2x}$.
63. Find $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x^2 - \sin x}$.
64. Find $\lim_{x \rightarrow 0} \frac{xe^{3x}}{1 - e^{3x}}$.
65. Find $\lim_{x \rightarrow 4} \frac{x^2 - 16}{x^2 + x - 20}$.
66. Find $\lim_{x \rightarrow 1} \frac{\ln x}{x - 1}$.
67. Find $\lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x^2 + 3x - 4}$.
68. Find $\lim_{x \rightarrow 0} \frac{\sinh x}{\cosh x - 1}$.

69. Find $\lim_{x \rightarrow 0} \frac{xe^x}{1 - e^x}$.

70. Find $\lim_{x \rightarrow 0} \frac{x - \tan x}{1 - \cos x}$.

71. Find $\lim_{x \rightarrow a} \frac{\frac{1}{x} - \frac{1}{a}}{x - a}$.

10.5 The Indeterminate Form (∞/∞); Other Indeterminate Forms

72. Find $\lim_{x \rightarrow \infty} \frac{4x^3 - 2x + 1}{4x^3 + 2}$.

73. Find $\lim_{x \rightarrow \infty} \frac{e^x}{x^3}$.

74. Find $\lim_{x \rightarrow \frac{\pi}{2}^-} (\sec x - \tan x)$.

75. Find $\lim_{x \rightarrow 0} (\csc x - 1/x)$.

76. Find $\lim_{x \rightarrow 1^-} (x - 1) \tan \frac{\pi x}{2}$.

77. Find $\lim_{x \rightarrow 0^+} \sin x \ln x$.

78. Find $\lim_{x \rightarrow 0^+} x \ln x$.

79. Find $\lim_{x \rightarrow \infty} (x + e^x)^{2/x}$.

80. Find $\lim_{x \rightarrow \frac{\pi}{2}^+} (\tan x)^{\cos x}$.

81. Find $\lim_{x \rightarrow 0} \left(\frac{1}{2} - \frac{1}{x} \right)$.

82. Find $\lim_{x \rightarrow 0} (\cos 3x)^{1/x}$.

83. Find $\lim_{x \rightarrow 0^+} x \ln \sin x$.

84. Find $\lim_{x \rightarrow 0} (e^x + 3x)^{1/x}$.

85. Find $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x^2} \right)^{x^2}$.

86. Find $\lim_{x \rightarrow 0} (\sin x)^x$.
87. Find $\lim_{x \rightarrow 0} (\sin 2x + 1)^{1/x}$.
88. Find $\lim_{x \rightarrow \infty} (2e^x + x^2)^{3/x}$.
89. Find $\lim_{x \rightarrow 0} (\cosh x)^{4/x}$.

10.7 Improper Integrals

90. Evaluate $\int_0^1 \frac{x}{\sqrt{1-x^2}} dx$.
91. Evaluate $\int_0^{p/4} \frac{\sec^2 x}{\sqrt{\tan x}} dx$.
92. Evaluate $\int_1^\infty \frac{dx}{x^3}$.
93. Evaluate $\int_0^1 \frac{1}{\sqrt{x}} dx$.
94. Evaluate $\int_1^4 \frac{1}{\sqrt[3]{x-3}} dx$.
95. Evaluate $\int_0^3 \frac{x}{(x^2-1)^{2/3}} dx$.
96. Evaluate $\int_3^4 \frac{1}{(x-4)^3} dx$.
97. Evaluate $\int_0^8 \frac{1}{x^{1/3}} dx$.
98. Evaluate $\int_0^\infty x e^{-x^2} dx$.
99. Evaluate $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$.
100. Evaluate $\int_{-1}^2 \frac{1}{x^2} dx$.
101. Evaluate $\int_{-2}^0 \frac{1}{x+2} dx$.
102. Evaluate $\int_0^4 \frac{1}{(x-1)^3} dx$.

103. Evaluate $\int_1^{\infty} \frac{1}{\sqrt{x}} dx$.

104. Evaluate $\int_1^2 \frac{1}{x \ln x} dx$.

105. Evaluate $\int_0^{\infty} \frac{1}{x^{1/3}} dx$.

106. Evaluate $\int_2^{\infty} \frac{1}{(x-1)^3} dx$.

107. Evaluate $\int_{-\infty}^1 e^{(x-e^x)} dx$.

Answers to Chapter 10 Questions

1. lub: 4 ; glb: 0
2. lub: $\sqrt{5}$; glb: $-\sqrt{5}$
3. lub: none; glb: none
4. lub: none; glb: none
5. lub: 4 ; glb: 2
6. lub: none; glb: none
7. $\frac{2n}{3n-1}$
8. $\frac{(-1)^{n+1}}{2^n}$
9. $n^{2(-1)^n}$
10. Bounded: below by 1, above by 2;
monotone increasing
11. Bounded: below by 1, above by 2;
monotone decreasing
12. Bounded: below by $-3/5$, above by $2/3$;
monotone increasing
13. Bounded: below by -1 , above by 1;
monotone increasing
14. Bounded: below by $3/5$, no upper bound;
monotone increasing
15. Bounded: below by 0, above by $1/12$;
monotone decreasing
16. Bounded: below by $4/9$, no upper bound;
monotone increasing
17. Bounded: below by $3/e$, no upper bound;
monotone increasing
18. Bounded: below by 0, above by $3/2$;
monotone decreasing
19. Bounded: below by 0, above by $3/e$;
monotone decreasing
20. Bounded: below by 0, above by $9/10$;
monotone decreasing
21. Bounded: below by $-\frac{\sqrt{3}}{2}$, above by $\frac{\sqrt{3}}{2}$;
not monotone
22. 1, $3/2$, 2, $5/2$, 3, $7/2$; $a_n = \frac{n+1}{2}$
23. 1, 1, 1, 1, 1, 1; $a_n = 1$
24. 1, 0, -6 , -24 , -68 , -166 ;
 $a_n = -\frac{7}{2}(2)^n + n^2 + 3n + 4$
25. 1, -1 , 3, -5 , 11, -21 ; $a_n = \frac{1}{3}[1 - (-2)^n]$
26. 1, 4, 7, 10, 13, 16; $a_n = 3n - 2$
27. 2, 2, 2, 2, 2, 2; $a_n = 2$
28. 3, 5, 7, 9, 11, 13; $a_n = 2n + 1$
29. converges; 0
30. converges; 0
31. diverges
32. converges; 1
33. converges; 1
34. diverges
35. converges; $1/2$
36. diverges
37. converges; $-1/3$
38. converges; 1
39. converges; 2
40. converges; $1/2$
41. converges; 0
42. converges; $1/2$
43. converges; 0
44. converges; 0
45. converges; $2/3$

- | | |
|------------------------|----------------------------------|
| 46. diverges | 75. 0 |
| 47. converges; $-1/3$ | 76. $-2/p$ |
| 48. converges; $\ln 2$ | 77. 0 |
| 49. converges; 0 | 78. 0 |
| 50. diverges | 79. e^2 |
| 51. converges; 0 | 80. 1 |
| 52. diverges | 81. $+\infty$ |
| 53. converges; $2/3$ | 82. 1 |
| 54. 1 | 83. 0 |
| 55. 0 | 84. e^4 |
| 56. 0 | 85. e |
| 57. $-1/2$ | 86. 1 |
| 58. 1 | 87. e^2 |
| 59. $-1/2$ | 88. e^3 |
| 60. $\ln 5/3$ | 89. 1 |
| 61. $1/2$ | 90. 1 |
| 62. 1 | 91. 2 |
| 63. -2 | 92. $1/2$ |
| 64. $-1/3$ | 93. 2 |
| 65. $8/9$ | 94. $\frac{3}{2}(1-\sqrt[3]{4})$ |
| 66. 1 | 95. $9/2$ |
| 67. $4/5$ | 96. divergent |
| 68. $+\infty$ | 97. 6 |
| 69. -1 | 98. $1/2$ |
| 70. 0 | 99. $p/2$ |
| 71. $\frac{-1}{a^2}$ | 100. divergent |
| 72. 1 | 101. divergent |
| 73. $+\infty$ | 102. divergent |
| 74. 0 | 103. divergent |

104. divergent

105. divergent

106. $\frac{1}{2}$

107. $1 - \frac{1}{e^e}$

CHAPTER 11

Infinite Series

11.1 Infinite Series

1. Evaluate $\sum_{k=0}^3 (4k + 2)$.
2. Evaluate $\sum_{k=1}^4 (5k - 2)$.
3. Evaluate $\sum_{k=0}^3 (3^k + 1)$.
4. Evaluate $\sum_{k=0}^3 (-1)^k 3^k$.
5. Evaluate $\sum_{k=1}^4 (-1)^{k+1} 3^{k+1}$.
6. Evaluate $\sum_{k=3}^5 \left(\frac{1}{3}\right)^{2k}$.
7. Evaluate $\sum_{k=0}^4 \frac{2}{e^k}$.
8. Evaluate $\sum_{k=0}^3 \frac{3^k}{2^{k+1}}$.
9. Express $2^0 + 2^1 + \dots + 2^{10}$ in sigma notation.
10. Express $2x^2 - 2^2 x^3 + \dots - 2^6 x^7$ in sigma notation.
11. Express $\frac{3^3}{e^2} + \frac{3^2}{e^3} + \dots + \frac{1}{3^2 e^7}$ in sigma notation.
12. Express $\frac{1}{3(4)} + \frac{1}{4(5)} + \dots + \frac{1}{9(10)}$ in sigma notation.
13. Determine whether $\sum_{k=1}^{\infty} \frac{1}{2k(k+1)}$ converges or diverges. If it converges, find the sum.
14. Determine whether $\sum_{k=2}^{\infty} \frac{(-1)^k}{5^k}$ converges or diverges. If it converges, find the sum.
15. Determine whether $\sum_{k=0}^{\infty} \frac{3^{k+2}}{4^{k+1}}$ converges or diverges. If it converges, find the sum.

16. Determine whether $\sum_{k=0}^{\infty} \frac{3}{10^k}$ converges or diverges. If it converges, find the sum.
17. Determine whether $\sum_{k=1}^{\infty} \frac{3}{e^k}$ converges or diverges. If it converges, find the sum.
18. Determine whether $\sum_{k=1}^{\infty} \frac{1}{(k+3)(k+4)}$ converges or diverges. If it converges, find the sum.
19. Determine whether $\sum_{k=1}^{\infty} \frac{2^k}{5}$ converges or diverges. If it converges, find the sum.
20. Determine whether the series given by $\sum_{k=0}^{\infty} u_k = 1 - \frac{2}{5} + \frac{4}{25} - \frac{8}{125} + \dots$ converges or diverges. If it converges, find the sum.
21. Determine whether $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{4^k}$ converges or diverges. If it converges, find the sum.
22. Determine whether $\sum_{k=3}^{\infty} \left(\frac{1}{2}\right)^k$ converges or diverges. If it converges, find the sum.
23. Determine whether $\sum_{k=1}^{\infty} \frac{2}{(2k-1)(2k+1)}$ converges or diverges. If it converges, find the sum.
24. Determine whether $\sum_{k=1}^{\infty} \left(-\frac{3}{7}\right)^{k+1}$ converges or diverges. If it converges, find the sum.
25. Determine whether $\sum_{k=1}^{\infty} 4^{k-1}$ converges or diverges. If it converges, find the sum.
26. Determine whether $\sum_{k=1}^{\infty} \left(-\frac{2}{3}\right)^{k+1}$ converges or diverges. If it converges, find the sum.
27. Determine whether $\sum_{k=1}^{\infty} \frac{1}{(k+1)(k+2)}$ converges or diverges. If it converges, find the sum.
28. Write the decimal fraction 0.21212121 ... as an infinite series and express the sum as the quotient of two integers.
29. Write the decimal fraction 0.251251251251 ... as an infinite series and express the sum as the quotient of two integers.
30. Write the decimal fraction 0.315315315315 ... as an infinite series and express the sum as the quotient of two integers.
31. Find a series expansion for $\frac{x}{1-x^3}$, $|x| < 1$.

32. Find a series expansion for $\frac{2}{3+x}$, $|x| < 3$.

11.2 The Integral Test; Comparison Tests

33. Determine whether $\sum_{k=1}^{\infty} \frac{3}{3k+2}$ converges or diverges. Justify your answer.
34. Determine whether $\sum_{k=1}^{\infty} \frac{1}{k\sqrt{k}}$ converges or diverges. Justify your answer.
35. Determine whether $\sum_{k=1}^{\infty} \frac{1}{3k+4}$ converges or diverges. Justify your answer.
36. Determine whether $\sum_{k=1}^{\infty} \frac{k^2}{2k+1}$ converges or diverges. Justify your answer.
37. Determine whether $\sum_{k=1}^{\infty} \frac{k}{\sqrt{2k^2+1}}$ converges or diverges. Justify your answer.
38. Determine whether $\sum_{k=1}^{\infty} \frac{3}{e^k}$ converges or diverges. Justify your answer.
39. Determine whether $\sum_{k=1}^{\infty} \frac{1}{4k+1}$ converges or diverges. Justify your answer.
40. Determine whether $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^3}$ converges or diverges. Justify your answer.
41. Determine whether $\sum_{k=1}^{\infty} \frac{1}{(2k+3)^3}$ converges or diverges. Justify your answer.
42. Determine whether $\sum_{k=1}^{\infty} \frac{k+1}{k(k+2)}$ converges or diverges. Justify your answer.
43. Find the sum of $\sum_{k=0}^{\infty} \left(\frac{5}{10^k} - \frac{6}{100^k} \right)$.
44. Determine whether $\sum_{k=1}^{\infty} \frac{1}{\sqrt{(k+1)^3}}$ converges or diverges. Justify your answer.
45. Determine whether $\sum_{k=1}^{\infty} \frac{2k}{1+k^4}$ converges or diverges. Justify your answer.
46. Determine whether $\sum_{k=1}^{\infty} \frac{1}{k\sqrt{k^2-1}}$ converges or diverges. Justify your answer.

47. Determine whether $\sum_{k=1}^{\infty} \frac{k}{e^k}$ converges or diverges. Justify your answer.
48. Determine whether $\sum_{k=1}^{\infty} \frac{1}{\cosh^2 k}$ converges or diverges. Justify your answer.
49. Determine whether $\sum_{k=1}^{\infty} \frac{\ln k}{k}$ converges or diverges. Justify your answer.
50. Which of the following statements about series is true?
(a) If $\lim_{k \rightarrow \infty} u_k = 0$, then $\sum u_k$ converges.
(b) If $\lim_{k \rightarrow \infty} u_k \neq 0$, then $\sum u_k$ diverges.
(c) If $\sum u_k$ diverges, then $\lim_{k \rightarrow \infty} u_k \neq 0$.
(d) $\sum u_k$ converges if and only if $\lim_{k \rightarrow \infty} u_k = 0$.
(e) None of the preceding.
51. Determine whether $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^2 + 1}$ converges or diverges. Justify your answer.
52. Determine whether $\sum_{k=1}^{\infty} \frac{k^2}{(2k^2 + 1)^2}$ converges or diverges. Justify your answer.
53. Determine whether $\sum_{k=1}^{\infty} \frac{1}{(k+3)(k+4)}$ converges or diverges. Justify your answer.
54. Determine whether $\sum_{k=1}^{\infty} \frac{1}{(k+3)(k-4)}$ converges or diverges. Justify your answer.
55. Determine whether $\sum_{k=1}^{\infty} \frac{1}{3k+2}$ converges or diverges. Justify your answer.
56. Determine whether $\sum_{k=1}^{\infty} \frac{1}{3^k + 2}$ converges or diverges. Justify your answer.
57. Determine whether $\sum_{k=1}^{\infty} \frac{\ln k}{k}$ converges or diverges. Justify your answer.
58. Determine whether $\sum_{k=1}^{\infty} \frac{k^2}{(k+2)(k+4)}$ converges or diverges. Justify your answer.
59. Determine whether $\sum_{k=1}^{\infty} \frac{k+1}{k^3 + 1}$ converges or diverges. Justify your answer.
60. Determine whether $\sum_{k=1}^{\infty} \frac{1}{1 + \sqrt{k}}$ converges or diverges. Justify your answer.
61. Determine whether $\sum_{k=1}^{\infty} \frac{7k+2}{2k^5 + 7}$ converges or diverges. Justify your answer.

62. Determine whether $\sum_{k=1}^{\infty} \frac{1}{3^k - 2}$ converges or diverges. Justify your answer.
63. Determine whether $\sum_{k=1}^{\infty} \frac{4k - 3}{k^3 - 5k - 7}$ converges or diverges. Justify your answer.
64. Determine whether $\sum_{k=1}^{\infty} \frac{1}{k^2 - 1}$ converges or diverges. Justify your answer.
65. Determine whether $\sum_{k=2}^{\infty} \frac{1}{k - \ln k}$ converges or diverges. Justify your answer.
66. Determine whether $\sum_{k=1}^{\infty} \frac{1}{3k^{3/2} + 1}$ converges or diverges. Justify your answer.
67. Determine whether $\sum_{k=1}^{\infty} \frac{k^2 + 3}{k(k+1)(k+2)}$ converges or diverges. Justify your answer.
68. Which of the following statements about $\sum_{k=2}^{\infty} \frac{1}{k \ln k}$ is true?
- (a) converges because $\lim_{k \rightarrow \infty} \frac{1}{k \ln k} = 0$.
 - (b) converges because $\frac{1}{k \ln k} < \frac{1}{k}$.
 - (c) converges by ratio test.
 - (d) diverges by ratio test.
 - (e) diverges by integral test.

11.3 The Root Test; The Ratio Test

69. Determine whether $\sum_{k=1}^{\infty} \frac{k^2}{e^k}$ converges or diverges. Justify your answer by citing a relevant test.
70. Determine whether $\sum_{k=1}^{\infty} \frac{k}{2^k}$ converges or diverges. Justify your answer by citing a relevant test.
71. Determine whether $\sum_{k=1}^{\infty} \frac{k!}{10^{4k}}$ converges or diverges. Justify your answer by citing a relevant test.
72. Determine whether $\sum_{k=1}^{\infty} \frac{1}{2 + 3^{-k}}$ converges or diverges. Justify your answer by citing a relevant test.
73. Determine whether $\sum_{k=1}^{\infty} \left(\frac{k}{2k + 100} \right)^k$ converges or diverges. Justify your answer by citing a relevant test.
74. Determine whether $\sum_{k=1}^{\infty} \left(\frac{3k}{2k + 1} \right)^k$ converges or diverges. Justify your answer by citing a relevant test.

75. Determine whether $\sum_{k=5}^{\infty} \left(\frac{3k+2}{2k-9} \right)^k$ converges or diverges. Justify your answer by citing a relevant test.
76. Determine whether $\sum_{k=1}^{\infty} \frac{k!}{2^k}$ converges or diverges. Justify your answer by citing a relevant test.
77. Determine whether $\sum_{k=1}^{\infty} \frac{k^2}{2^k}$ converges or diverges. Justify your answer by citing a relevant test.
78. Determine whether $\sum_{k=1}^{\infty} \frac{1}{2k+9}$ converges or diverges. Justify your answer by citing a relevant test.
79. Determine whether $\sum_{k=1}^{\infty} \frac{e^k}{k!}$ converges or diverges. Justify your answer by citing a relevant test.
80. Determine whether $\sum_{k=1}^{\infty} \frac{10^k}{k!}$ converges or diverges. Justify your answer by citing a relevant test.
81. Determine whether $\sum_{k=1}^{\infty} \frac{k^3}{3^k}$ converges or diverges. Justify your answer by citing a relevant test.
82. Determine whether $\sum_{k=1}^{\infty} \frac{k!}{k^2}$ converges or diverges. Justify your answer by citing a relevant test.
83. Determine whether $\sum_{k=1}^{\infty} \left(\frac{\ln k}{k} \right)^k$ converges or diverges. Justify your answer by citing a relevant test.
84. Determine whether $\sum_{k=1}^{\infty} \frac{k^k}{k!}$ converges or diverges. Justify your answer by citing a relevant test.
85. Determine whether $\sum_{k=1}^{\infty} \frac{3^{2k}}{(2k)!}$ converges or diverges. Justify your answer by citing a relevant test.

11.4 Absolute and Conditional Convergence; Alternating Series

86. Determine whether $\sum_{k=1}^{\infty} \frac{(-1)^k}{k + \sqrt{k}}$ converges absolutely, converges conditionally, or diverges. Justify your answer by citing a relevant test.
87. Determine whether $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k + 2}{k(k+1)}$ converges absolutely, converges conditionally, or diverges. Justify your answer by citing a relevant test.
88. Determine whether $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} 2^k}{3^k + 1}$ converges absolutely, converges conditionally, or diverges. Justify your answer by citing a relevant test.

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89. Determine whether $\sum_{k=2}^{\infty} \frac{(-1)^k \ln k}{k}$ converges absolutely, converges conditionally, or diverges. Justify your answer by citing a relevant test.
90. Determine whether $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} 2^k}{k^2}$ converges absolutely, converges conditionally, or diverges. Justify your answer by citing a relevant test.
91. Determine whether $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}}$ converges absolutely, converges conditionally, or diverges. Justify your answer by citing a relevant test.
92. Determine whether $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{3^k}$ converges absolutely, converges conditionally, or diverges. Justify your answer by citing a relevant test.
93. Determine whether $\sum_{k=1}^{\infty} \frac{(-1)^k k^2}{(2k^2 + 1)^2}$ converges absolutely, converges conditionally, or diverges. Justify your answer by citing a relevant test.
94. Determine whether $\sum_{k=1}^{\infty} \frac{(-1)^k k^3}{3^k}$ converges absolutely, converges conditionally, or diverges. Justify your answer by citing a relevant test.
95. Determine whether $\sum_{k=1}^{\infty} \frac{(-1)^k}{k\sqrt{k}}$ converges absolutely, converges conditionally, or diverges. Justify your answer by citing a relevant test.
96. Determine whether $\sum_{k=1}^{\infty} \frac{(-1)^k k}{k+2}$ converges absolutely, converges conditionally, or diverges. Justify your answer by citing a relevant test.
97. Determine whether $\sum_{k=1}^{\infty} \frac{(-1)^k k 2}{e^k}$ converges absolutely, converges conditionally, or diverges. Justify your answer by citing a relevant test.
98. Determine whether $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} (2k-1)}{k^3 + 1}$ converges absolutely, converges conditionally, or diverges. Justify your answer by citing a relevant test.
99. Determine whether $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{3k+4}$ converges absolutely, converges conditionally, or diverges. Justify your answer by citing a relevant test.
100. Determine whether $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^2}{2k+1}$ converges absolutely, converges conditionally, or diverges. Justify your answer by citing a relevant test.

101. Determine whether $\sum_{k=1}^{\infty} \frac{(-1)^k k!}{(2k+3)!}$ converges absolutely, converges conditionally, or diverges. Justify your answer by citing a relevant test.
102. Determine whether $\sum_{k=1}^{\infty} \frac{(-1)^k 2\sqrt{k}}{k^2 + 1}$ converges absolutely, converges conditionally, or diverges. Justify your answer by citing a relevant test.
103. Determine whether $\sum_{k=1}^{\infty} \frac{(-1)^k k^2}{e^k}$ converges absolutely, converges conditionally, or diverges. Justify your answer by citing a relevant test.
104. Estimate the error if the partial sum S_{20} is used to approximate the sum of the series $\sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{3k+1}}$.
105. Find the smallest integer N such that S_N will approximate the sum of the series $\sum_{k=0}^{\infty} \frac{(-1)^k}{k^2 + 2}$ within 0.01.

11.5 Taylor Polynomials in x ; Taylor Series in x

106. Find the Taylor polynomial $P_4(x)$ for $\ln(1+x)$.
107. Find the Taylor polynomial $P_4(x)$ for $\cos x \sin x$.
108. Find the Taylor polynomial $P_5(x)$ for $\ln(1+x)^{-2}$.
109. Find the Taylor polynomial $P_5(x)$ for $\sin^{-1} x$.
110. Find the Taylor polynomial $P_5(x)$ for $\cosh x$.
111. Determine $P_0(x), P_1(x), P_2(x), P_3(x)$ for $2x^3 + x^2 - 2x + 5$.
112. Determine $P_0(x), P_1(x), P_2(x), P_3(x)$ for $(x+2)^3$.
113. Use Taylor polynomials to estimate $\sin 1.3$ within 0.01.
114. Use Taylor polynomials to estimate $\ln 2.4$ within 0.01.
115. Use Taylor polynomials to estimate $e^{0.2}$ within 0.01.
116. Find the Lagrange form of the remainder R_3 for the function $f(x) = \sqrt{2-x}$.
117. Find the Lagrange form of the remainder R_{n+1} for the function $f(x) = e^{-3x}$.

11.6 Taylor Polynomials and Taylor Series in $x - a$

118. Expand $g(x) = 2x^4 - x^3 + 3x^2 - x + 1$ in powers of $x - 2$ and specify the values of x for which the expansion is valid.
119. Expand $g(x) = 1/x$ in powers of $x - 3$ and specify the values of x for which the expansion is valid.
120. Expand $g(x) = \ln(x - 1)$ in powers of $x - 2$ and specify the values of x for which the expansion is valid.

121. Expand $g(x) = e^{2x}$ in powers of $x - 3$ and specify the values of x for which the expansion is valid.
122. Expand $g(x) = \cos x$ in powers of $x - \frac{\pi}{3}$ and specify the values of x for which the expansion is valid.
123. Expand $g(x) = e^x \sin \pi x$ in powers of $x - 1$ and specify the values of x for which the expansion is valid.
124. Expand $g(x) = \tan x$ in powers of $x - \frac{\pi}{3}$.
125. Expand $g(x) = \ln x$ in powers of $x - 2$.
126. Expand $g(x) = e^{x^2}$ in powers of $x - 1$.

11.7 Power Series

127. Find the interval of convergence for $\sum_{k=1}^{\infty} \frac{k}{2^k} (x-1)^k$.
128. Find the interval of convergence for $\sum_{k=1}^{\infty} \frac{(-1)^k 2^k}{k^2} (x-2)^k$.
129. Find the interval of convergence for $\sum_{k=1}^{\infty} \frac{(-1)^k (x-2)^k}{k+1}$.
130. Find the interval of convergence for $\sum_{k=1}^{\infty} (-1)^k \frac{k^{x^k}}{k}$.
131. Find the interval of convergence for $\sum_{k=1}^{\infty} \frac{2^k x^k}{\sqrt{k}}$.
132. Find the interval of convergence for $\sum_{k=1}^{\infty} \frac{2^k x^k}{3^k}$.
133. Find the interval of convergence for $\sum_{k=1}^{\infty} \frac{(x-1)^k}{2^k + 1}$.
134. Find the interval of convergence for $\sum_{k=1}^{\infty} \frac{(x-3)^k}{k+1}$.
135. Find the interval of convergence for $\sum_{k=1}^{\infty} \frac{(x-3)^k}{(k+1)!}$.
136. Find the interval of convergence for $\sum_{k=1}^{\infty} \frac{k!}{(2k)!} x^k$.
137. Find the interval of convergence for $\sum_{k=1}^{\infty} k 3^k (x-2)^k$.
138. Find the interval of convergence for $\sum_{k=1}^{\infty} \frac{(-1)^k x^k}{e^k}$.

139. Find the interval of convergence for $\sum_{k=0}^{\infty} \left(\frac{x-1}{3}\right)^k$.
140. Find the interval of convergence for $\sum_{k=1}^{\infty} \frac{x^{k+1}}{2^k(k+1)}$.
141. Find the interval of convergence for $\sum_{k=1}^{\infty} \frac{2^k(x-3)^k}{k^2}$. For which values of x is the convergence absolute?
142. Find the interval of convergence for $\sum_{k=1}^{\infty} \frac{(-1)^k(x-1)^k}{3^k \sqrt[3]{k}}$.
143. Find the interval of convergence for $\sum_{k=1}^{\infty} \frac{k^2 + k}{x^k}$.
144. Find the interval of convergence for $\sum_{k=1}^{\infty} \frac{(x+1)^k}{3^k k^2}$.
145. Find the interval of convergence for $\sum_{k=0}^{\infty} \frac{k!x^k}{2^k}$.

11.8 Differentiation and Integration of Power Series

146. Expand $\frac{1}{(1-x)^4}$ in powers of x , basing your calculation on the geometric series
- $$\frac{1}{(1-x)} = 1 + x + x^2 + \dots + x^n + \dots$$
147. Expand $\ln(1-2x^2)$ in powers of x , basing your calculation on the geometric series
- $$\frac{1}{(1-x)} = 1 + x + x^2 + \dots + x^n + \dots$$
148. Find $f^7(0)$ for $f(x) = x \sin x^2$.
149. Expand $e^x \sin x$ in powers of x .
150. Expand $e^{-x} \cos x$ in powers of x .
151. Expand $\frac{e^x}{1-x}$ in powers of x .
152. Expand $\frac{\cos x}{\sqrt{1+x}}$ in powers of x .
153. Expand $\coth x$ in powers of x .
154. Expand $\sec^2 x$ in powers of x .
155. Estimate $\int_0^1 \cos x^2 dx$ within 0.001.

156. Estimate $\int_0^1 \frac{\sin x}{x} dx$ within 0.001.
157. Estimate $\int_0^{1/2} \cos x^3 dx$ within 0.001.
158. Use a series to show $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$.
159. Use a series to show $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$ by first obtaining a series for $\tan x$.
160. Use a series to show $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin x} = 1$.

11.9 The Binomial Series

161. Expand $\sqrt{1+x^3}$ in powers of x up to x^6 .
162. Expand $\sqrt{1-x^3}$ in powers of x up to x^6 .
163. Expand $\frac{1}{\sqrt{1-x}}$ in powers of x up to x^4 .
164. Expand $\frac{1}{\sqrt[3]{1-x}}$ in powers of x up to x^4 .
165. Expand $\sqrt[5]{1-x}$ in powers of x up to x^4 .
166. Expand $\frac{1}{\sqrt[5]{1+x}}$ in powers of x up to x^4 .
167. Estimate $\sqrt{102}$ by using the first three terms of a binomial expansion, rounding off your answer to four decimal places.
168. Estimate $\sqrt[5]{28}$ by using the first three terms of a binomial expansion, rounding off your answer to four decimal places.

Answers to Chapter 11 Questions

1. 32
2. 42
3. 44
4. -20
5. -180
6. $91/3^{10}$
7. $\frac{2}{e^4}(e^4 + e^3 + e^2 + e + 1) \approx 3.1426$
8. $65/16$
9. $\sum_{k=0}^{10} 2^k$
10. $\sum_{k=1}^6 (-1)^{k+1} 2^k x^{k+1}$
11. $\sum_{k=0}^5 3^{3-k} e^{-(2+k)}$
12. $\sum_{k=3}^9 \frac{1}{k(k+1)}$
13. $1/2$
14. $1/30$
15. 9
16. $10/3$
17. $\frac{3}{e-1}$
18. $1/4$
19. diverges
20. $5/7$
21. $1/5$
22. $1/4$
23. 1
24. $9/70$
25. diverges
26. $4/15$
27. $1/2$
28. $\frac{21}{10^2} + \frac{21}{10^4} + \frac{21}{10^6} + \cdots = \frac{7}{33}$
29. $\frac{251}{10^3} + \frac{251}{10^6} + \frac{251}{10^9} + \cdots = \frac{251}{999}$
30. $\frac{315}{10^3} + \frac{315}{10^6} + \frac{315}{10^9} + \cdots = \frac{35}{111}$
31. $\sum_{k=0}^{\infty} x^{3k+1}$
32. $2 \sum_{k=0}^{\infty} \frac{(-1)^k x^k}{3^{k+1}}$
33. diverges
34. converges
35. diverges
36. diverges
37. diverges
38. converges
39. diverges
40. converges
41. converges
42. diverges
43. $\frac{-50}{99}$
44. converges
45. converges
46. converges
47. converges
48. converges

49. diverges
50. b
51. converges
52. converges
53. converges
54. converges
55. diverges
56. converges
57. diverges
58. diverges
59. converges
60. diverges
61. converges
62. converges
63. converges
64. converges
65. diverges
66. converges
67. diverges
68. e
69. converges by ratio test
70. converges by ratio test
71. diverges by ratio test
72. diverges since $\lim_{k \rightarrow \infty} u_k \neq 0$
73. converges by root test
74. diverges by root test
75. diverges by root test
76. diverges by ratio test
77. diverges by ratio test
78. diverges by integral test
79. converges by ratio test
80. converges by ratio test
81. converges by ratio test
82. diverges by ratio test
83. converges by root test
84. diverges by ratio test
85. converges by ratio test
86. converges conditionally by limit comparison test with $\sum_{k=1}^{\infty} \frac{1}{k}$ and by alternating series test
87. converges conditionally by limit comparison test with $\sum_{k=1}^{\infty} \frac{1}{k}$ and by alternating series test
88. converges absolutely by comparison with geometric series $\sum_{k=1}^{\infty} \left(\frac{2}{3}\right)^k$
89. converges conditionally: converges by alternating series test but $\sum_{k=1}^{\infty} \frac{\ln k}{k}$ diverges by integral test
90. diverges by ratio test
91. converges conditionally: converges by alternating series test but $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$ divergent p series
92. converges absolutely by geometric series with $|r| = \frac{1}{3} < 1$
93. converges absolutely by comparison with p series $\frac{1}{4} \sum_{k=1}^{\infty} \frac{1}{k^2}$
94. converges absolutely by ratio test
95. converges absolutely: convergent p series

96. divergent since $\lim_{k \rightarrow \infty} \frac{k}{k+2} = 1 \neq 0$

97. converges absolutely by geometric series with $|r| = \frac{1}{e} < 1$

98. converges absolutely by limit comparison test with convergent p series, $\sum_{k=1}^{\infty} \frac{1}{k^2}$

99. converges conditionally: converges by alternating series test but diverges by integral test

100. diverges by integral test

101. converges absolutely by ratio test

102. converges absolutely by limit comparison test with converging p series $2 \sum_{k=1}^{\infty} \frac{1}{k^{3/2}}$

103. converges absolutely by ratio test

104. $1/8$

105. $N = 9$

106. $x - \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4}$

107. $x - \frac{2}{3}x^3$

108. $1 - 2x + 3x^2 - 4x^3 + 5x^4 - 6x^5$

109. $x - \frac{x^3}{6} + \frac{3}{40}x^5$

110. $x + \frac{x^2}{2!} + \frac{x^4}{4!}$

111. $P_0(x) = 5; P_1(x) = 5 - 2x; P_2(x) = 5 - 2x + x^2; P_3(x) = 5 - 2x + x^2 + 2x^3$

112. $P_0(x) = 8; P_1(x) = 8 + 12x; P_2(x) = 8 + 12x + 6x^2; P_3(x) = 8 + 12x + 6x^2 + x^3$

113. 0.96

114. 0.88

115. 1.22

116. $R_3(x) = \frac{-x^3}{16(2-c)^{5/2}}$

117. $R_{n+1}(x) = \frac{(-3)^{n+1} e^{-3c}}{(n+1)!} x^{n+1}$

118. $35 + 63(x-2) + 45(x-2)^2 + 15(x-2)^3 + 2(x-2)^4$ valid for $-\infty < x < \infty$

119. $\sum_{k=0}^{\infty} (-1)^k \frac{1}{3^{k+1}} (x-3)^k$; valid for $0 < x < 6$

120. $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k} (x-2)^k$; valid for $1 < x < 3$

121. $e^6 \sum_{k=0}^{\infty} \frac{2^k}{k!} (x-3)^k$; valid for $-\infty < x < \infty$

122. $\frac{1}{2} \sum_{k=0}^{\infty} (-1)^k \frac{\left(x - \frac{\pi}{3}\right)^{2k}}{2k!} - \frac{\sqrt{3}}{2} \sum_{k=0}^{\infty} (-1)^k \frac{\left(x - \frac{\pi}{3}\right)^{2k+1}}{(2k+1)!}$

valid for $-\infty < x < \infty$

123. $-\pi e \left[(x-1) + \frac{2}{2!}(x-1)^2 - \frac{(\pi^2-3)(x-1)^3}{3!} - 4 \frac{(\pi^2-1)(x-1)^4}{4!} \dots \right]$

valid for $-\infty < x < \infty$

124. $\sqrt{3} + 4 \left(x - \frac{\pi}{3}\right) + 4\sqrt{3} \left(x - \frac{\pi}{3}\right)^2 + \frac{40}{3} \left(x - \frac{\pi}{3}\right)^3 + \frac{44}{3} \sqrt{3} \left(x - \frac{\pi}{3}\right)^4 + \dots$

125. $\ln 2 + \sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{2^k k!} (x-2)^k$

126. $e \left[1 + 2(x-1) + 3(x-1)^2 + \frac{10}{3}(x-1)^3 + \frac{19}{6}(x-1)^4 + \dots \right]$

127. $(-1, 3)$

128. $[3/2, 5/2)$

129. $(1, 3]$

130. $(-1, 1]$

131. $[-1/2, 1/2]$

132. $(-3/2, 3/2)$

133. $[0, 2)$

134. $[2, 4)$

135. $(-\infty, +\infty)$

136. $(-\infty, +\infty)$

137. $(5/3, 7/3)$

138. $(-e, e)$

139. $(-2, 4)$

140. $[-2, 2)$

141. $[5/2, 7/2]$

142. $(-2, 4]$

143. $(-\infty, -1) \cup (1, +\infty)$

144. $[-4, 2]$

145. $x = 0$

146. $\frac{1}{6} \sum_{k=1}^{\infty} k(k+1)(k+2)x^{k-1}$

147. $-\sum_{k=1}^{\infty} \frac{(2x^2)^k}{k}$

148. -840

149. $x + x^2 + \frac{x^3}{3} - \frac{x^5}{30} + \dots$

150. $1 - x + \frac{x^3}{3} - \frac{x^4}{6} + \dots$

151. $1 + 2x + \frac{5}{2}x^2 + \frac{8}{3}x^3 + \frac{65}{24}x^4 + \dots$

152. $1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 + \frac{49}{384}x^4 + \dots$

153. $\frac{1}{x} + \frac{1}{3}x - \frac{1}{45}x^3 + \frac{2}{945}x^5 + \dots$

154. $1 + x^2 + \frac{2}{3}x^4 + \frac{17}{45}x^6 + \frac{62}{315}x^8 + \dots$

155. 0.905

156. 0.946

157. 0.499

158. $\lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \left(1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \dots \right) = 1$

159. $\lim_{x \rightarrow 0} \frac{\tan x}{x} = \lim_{x \rightarrow 0} \left(1 - \frac{x^2}{3} + \frac{2x^4}{15} - \frac{17x^6}{315} + \dots \right) = 1$

160.

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin x} = \lim_{x \rightarrow 0} \frac{x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots}{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots} =$$

$$\lim_{x \rightarrow 0} \frac{1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + \dots}{1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \dots} = 1$$

161. $1 + \frac{x^3}{2} - \frac{x^6}{8}$

162. $1 - \frac{x^3}{2} - \frac{x^6}{8}$

163. $1 + \frac{x}{2} + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \frac{35}{128}x^4$

164. $1 + \frac{1}{3}x + \frac{2}{9}x^2 + \frac{14}{81}x^3 + \frac{35}{243}x^4$

165. $1 + \frac{1}{5}x - \frac{2}{25}x^2 + \frac{6}{125}x^3 - \frac{21}{625}x^4$

166. $1 - \frac{1}{5}x + \frac{3}{25}x^2 - \frac{11}{125}x^3 + \frac{44}{625}x^4$

167. 10.0995

168. 1.9475

CHAPTER 12

Vectors

12.1 Cartesian Space Coordinates

1. Plot points $A(2, 7, 8)$ and $B(3, 9, 7)$ on a right-handed coordinate system. Then calculate the length of the line segment \overline{AB} and find the midpoint.
2. Plot points $A(-3, -2, 4)$ and $B(9, 7, 2)$ on a right-handed coordinate system. Then calculate the length of the line segment \overline{AB} and find the midpoint.
3. Plot points $A(-1, 1, 1)$ and $B(-1, 4, 4)$ on a right-handed coordinate system. Then calculate the length of the line segment \overline{AB} and find the midpoint.
4. Find an equation for the plane through $(2, -1, -2)$ that is parallel to the xy -plane.
5. Find an equation for the plane through $(-3, 2, -1)$ that is perpendicular to the z -axis.
6. Find an equation for the plane through $(-2, -4, 3)$ that is parallel to the yz -plane.
7. Find an equation for the sphere centered at $(2, 1, 3)$ with radius 4.
8. Find an equation for the sphere that is centered at $(-4, 0, 6)$ and passes through $(2, 2, 3)$.
9. Find an equation for the sphere that is centered at $(5, 1, -4)$ and passes through $(3, -5, -1)$.
10. Find an equation for the sphere that has the line segment joining $(4, 3, 0)$ and $(2, 4, -4)$ as a diameter.
11. Find an equation for the sphere that is centered at $(-2, 1, 4)$ and is tangent to the plane $x = 2$.
12. The points $P(a, b, c)$ and $Q(3, 2, -1)$ are symmetric about the xy -plane. Find a, b, c .
13. The points $P(a, b, c)$ and $Q(-3, 2, -1)$ are symmetric about the yz -plane. Find a, b, c .
14. The points $P(a, b, c)$ and $Q(-3, -2, 1)$ are symmetric about the xz -plane. Find a, b, c .
15. The points $P(a, b, c)$ and $Q(1, 2, -4)$ are symmetric about the z -axis. Find a, b, c .
16. The points $P(a, b, c)$ and $Q(2, -1, 3)$ are symmetric about the plane $x = 2$. Find a, b, c .
17. The points $P(a, b, c)$ and $Q(-2, 1, -3)$ are symmetric about the plane $y = -3$. Find a, b, c .
18. The points $P(a, b, c)$ and $Q(4, 2, 2)$ are symmetric about the point $(0, 2, 1)$. Find a, b, c .

12.3 Vectors

19. Simplify $(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) - 2(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$.
20. Simplify $2(\mathbf{i} - 3\mathbf{k}) - 3(2\mathbf{i} + \mathbf{j} - \mathbf{k})$.
21. Calculate the norm of the vector $4\mathbf{i} - 3\mathbf{j}$.
22. Calculate the norm of the vector $3\mathbf{i} - \mathbf{j} + \mathbf{k}$.

23. Calculate the norm of $2(2\mathbf{i} - \mathbf{j} + \mathbf{k}) - (-2\mathbf{i} - \mathbf{j})$.
24. Let $\mathbf{a} = (-2, 3, 5)$, $\mathbf{b} = (3, 5, -2)$, $\mathbf{c} = (2, 1, 2)$, $\mathbf{d} = (-3, 0, -1)$. Express $\mathbf{a} - 2\mathbf{b} + 2\mathbf{c} + 3\mathbf{d}$ as a linear combination of \mathbf{i} , \mathbf{j} , \mathbf{k} .
25. Given that $\mathbf{a} = (1, 2, 5)$ and $\mathbf{b} = (-1, 0, 3)$, calculate
- (a) $\|\mathbf{a}\|$
 - (b) $\|\mathbf{b}\|$
 - (c) $\|2\mathbf{a} - 3\mathbf{b}\|$
 - (d) $\|3\mathbf{a} + \mathbf{b}\|$
26. Find α given that $3\mathbf{i} + 2\mathbf{j}$ and $-2\mathbf{i} + \alpha\mathbf{j}$ have the same length.
27. Find the unit vector in the direction of $2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$.
28. Given that $\mathbf{a} = 3\mathbf{i} - 5\mathbf{k}$ and $\mathbf{b} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, find the unit vector in the direction of $\mathbf{a} - 2\mathbf{b}$.
29. Given that $\mathbf{a} = 2\mathbf{i} + 9\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 7\mathbf{j} + 8\mathbf{k}$, find the unit vector in the direction of $2\mathbf{a} + \mathbf{b}$.
30. Find the vector of norm 2 in the direction of $3\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$.
31. Find the vector of norm 2 parallel to $5\mathbf{i} - 12\mathbf{j} + \mathbf{k}$.

12.4 The Dot Product

32. Simplify $(2\mathbf{a} \cdot 2\mathbf{b}) + \mathbf{a} \cdot (\mathbf{a} + 2\mathbf{b})$.
33. Simplify $(\mathbf{a} - 2\mathbf{b}) \cdot \mathbf{c} + \mathbf{b} \cdot (\mathbf{a} - \mathbf{c}) - 2\mathbf{a} \cdot (\mathbf{b} - 3\mathbf{c})$.
34. Taking $\mathbf{a} = \mathbf{i} + 2\mathbf{j}$, $\mathbf{b} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$, $\mathbf{c} = -2\mathbf{j} + \mathbf{k}$, calculate:
- (a) the three dot products $\mathbf{a} \cdot \mathbf{b}$, $\mathbf{a} \cdot \mathbf{c}$, $\mathbf{b} \cdot \mathbf{c}$
 - (b) the cosines of the angles between these vectors.
 - (c) the component of \mathbf{a} (i) in the \mathbf{b} direction, (ii) in the \mathbf{c} direction
 - (d) the projection of \mathbf{a} (i) in the \mathbf{b} direction, (ii) in the \mathbf{c} direction
35. Taking $\mathbf{a} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\mathbf{c} = -2\mathbf{j} - \mathbf{k}$, calculate:
- (e) the three dot products $\mathbf{a} \cdot \mathbf{b}$, $\mathbf{a} \cdot \mathbf{c}$, $\mathbf{b} \cdot \mathbf{c}$
 - (f) the cosines of the angles between these vectors.
 - (g) the component of \mathbf{a} (i) in the \mathbf{b} direction, (ii) in the \mathbf{c} direction
 - (h) the projection of \mathbf{a} (i) in the \mathbf{b} direction, (ii) in the \mathbf{c} direction
36. Find the angle between the vectors $2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$.
37. Find the angle between the vectors $2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $3\mathbf{i} + \mathbf{j} + 8\mathbf{k}$.
38. Find the direction angles of the vector $\sqrt{2}\mathbf{i} - \mathbf{j} + \mathbf{k}$.
39. Find the direction angles of the vector $\sqrt{3}\mathbf{i} - 2\mathbf{k}$.
40. Find the unit vectors \mathbf{u} that are perpendicular to both $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$.
41. Find the cosine of the angle between $\mathbf{u} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = -\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$.
42. A 100 Newton force is applied along a rope making a 30° angle with the horizontal to pull a box a distance of 5 meters along the ground. What is the work done?

43. Find the work done by the force $\mathbf{F} = 3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ in moving an object from the point $P(2, 0, 2)$ to the point $Q(1, 4, 5)$.

12.5 The Cross Product

44. Calculate $(\mathbf{i} - \mathbf{k}) \times (\mathbf{j} + \mathbf{k})$.
45. Calculate $(\mathbf{j} \times \mathbf{k}) \cdot \mathbf{j}$.
46. Calculate $(\mathbf{k} \times \mathbf{j}) \times \mathbf{i}$.
47. Calculate $(\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}) \times (2\mathbf{i} + \mathbf{j})$.
48. Calculate $[(\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \times (\mathbf{i} + \mathbf{j} + \mathbf{k})] \times (\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$.
49. Calculate $(3\mathbf{i} - 4\mathbf{j} - 4\mathbf{k}) \times [(2\mathbf{i} - 6\mathbf{j}) \times (\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})]$.
50. Calculate $(\mathbf{i} - \mathbf{j}) \cdot [(3\mathbf{i} - 4\mathbf{j}) \times (\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})]$.
51. Calculate $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}) \cdot [(-\mathbf{i} + \mathbf{j} + 2\mathbf{k}) \times (\mathbf{i} - \mathbf{j} + \mathbf{k})]$.
52. Calculate $(3\mathbf{i} + 2\mathbf{k}) \times [(2\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \times (\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})]$.
53. Use a cross product to find the area of triangle PQR , $P(1, 2, 3)$, $Q(-1, 0, 1)$, $R(2, -2, -1)$.
54. Use a cross product to find the area of triangle PQR , $P(1, 1, 1)$, $Q(2, -1, 3)$, $R(2, 3, -4)$.
55. Find the volume of the parallelepiped with edges determined by $3\mathbf{i} - 4\mathbf{j} - \mathbf{k}$, $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{i} + \mathbf{j}$.
56. Find the volume of the parallelepiped with vertices $A(0, 0, 0)$, $B(1, -1, 1)$, $C(2, 1, -2)$ and $D(-1, 2, -1)$.
57. Find the volume of the parallelepiped with edges determined by $\mathbf{i} + 2\mathbf{k}$, $4\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$, $3\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$.
58. Find the volume of the parallelepiped with edges determined by $2\mathbf{i} + \mathbf{k}$, $3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$, $-\mathbf{i} + 2\mathbf{k}$.
59. Find the area of the triangle with vertices $P(1, -2, 3)$, $Q(2, 4, 1)$, $R(2, 0, 1)$.
60. Find the area of the triangle with vertices $P(1, 2, 1)$, $Q(2, 4, 3)$, $R(5, -1, 4)$.

12.6 Lines

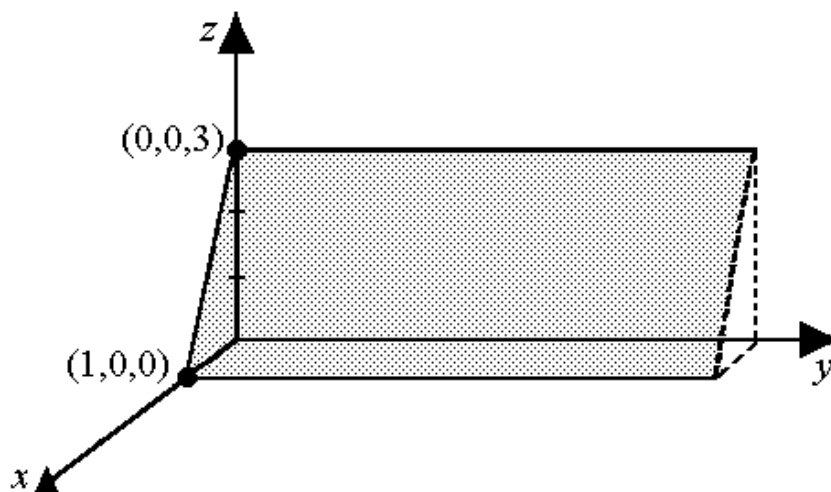
61. Which of the points $P(-1, 3, -1)$, $Q(3, 2, -1)$, $R(3, 0, -2)$ lie on the line $l: \mathbf{r}(t) = (2\mathbf{i} + \mathbf{j}) + t(3\mathbf{i} - 2\mathbf{j} + \mathbf{k})$?
62. Determine whether the lines are parallel.
 $l_1: \mathbf{r}_1(t) = (\mathbf{i} - 2\mathbf{k}) + t(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k})$
 $l_2: \mathbf{r}_2(u) = (3\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) + u(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$
63. Find a vector parametrization for the line that passes through $P(2, 3, 3)$ and is parallel to the line $\mathbf{r}(t) = (2\mathbf{i} - \mathbf{j}) + t\mathbf{k}$.
64. Find a vector parametrization for the line that passes through the origin and $P(3, 1, 8)$.
65. Find a vector parametrization for the line that passes through $P(4, 0, 5)$ and $Q(2, 3, 1)$.
66. Find a vector parametrization for the line that passes through $P(3, 3, 1)$ and $Q(4, 0, 2)$.

67. Find a set of scalar parametric equations for the line that passes through $P(1, 4, 6)$ and $Q(2, -1, 3)$.
68. Find a set of scalar parametric equations for the line that passes through $P(-3, -1, 0)$ and $Q(-1, 2, 1)$.
69. Find a set of scalar parametric equations for the line that passes through $P(4, -2, -1)$ and is perpendicular to the xy -plane.
70. Find a set of scalar parametric equations for the line that passes through $P(-1, 2, -3)$ and is perpendicular to the xz -plane.
71. Give a vector parametrization for the line that passes through $P(1, -2, 3)$ and is parallel to the line $3(x - 2) = 2(y + 2) = 5z$.
72. Find the point where l_1 and l_2 intersect and give the angle of intersection:
 $l_1: x_1(t) = 3 - t, y_1(t) = 5 + 3t, z_1(t) = -1 - 4t$
 $l_2: x_2(u) = 8 + 2u, y_2(u) = -6 - 4u, z_2(u) = 5 + u$.
73. Where does the line that passes through $(1, 4, 2)$ and is parallel to $3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ intersect the xy -plane?
74. Where does the line that passes through $(3, 5, -1)$ and is parallel to $\mathbf{i} - \mathbf{j} + \mathbf{k}$ intersect the xz -plane?
75. Find scalar parametric equations for all lines that are perpendicular to the line $x(t) = 5 + 2t, y(t) = -5t, z(t) = -t$ and intersect the line at the point $P(-3, 2, 2)$.
76. Find the distance from $P(4, -3, 1)$ to the line through the origin parallel to $4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$.
77. Find the distance from $P(3, -4, 1)$ to the line $\mathbf{r}(t) = 2\mathbf{i} - \mathbf{j} + t(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$.
78. Find the standard vector parametrization for the line through $P(-1, 2, 4)$ parallel to $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$.
79. Find the cosine of the angle between the lines $x_1(t) = 2 + t, y_1(t) = 3 + t, z_1(t) = -1 + 2t$ and $x_2(u) = 2 + 2u, y_2(u) = 3 - u, z_2(u) = -1 + 3u$.
80. Find the cosine of the angle between the line $x(t) = 2t, y(t) = 3t, z(t) = t$ and the y -axis.

12.7 Planes

81. Which of the points $P(-2, 3, -1), Q(2, 3, 4), R(3, 4, 1)$ lie on the plane $2(x - 2) + 3(y - 2) - 2(z + 3) = 0$?
82. Which of the points $P(4, 1, 0), Q(2, 1, -3), R(4, 1, -2), S(0, 2, -1)$ lie on the plane $\mathbf{N} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$ if $\mathbf{N} = 2\mathbf{i} - 4\mathbf{j} + \mathbf{k}$ and $\mathbf{r}_0 = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$?
83. Write an equation for the plane that passes through the point $P(2, 1, 3)$ and is perpendicular to $3\mathbf{i} + \mathbf{j} - 5\mathbf{k}$.
84. Write an equation for the plane that passes through the point $P(5, -2, -1)$ and is perpendicular to the plane $3x - y + 6z + 8 = 0$.
85. Find the unit normals for the plane $3x + 3y - 5z - 6 = 0$.
86. Write the equation of the plane $5x - 3y - 2z - 1 = 0$ in intercept form.
87. Where does the plane $4x + 3y - 2z + 4 = 0$ intersect the coordinate axes?
88. Find the angle between the planes $3(x - 1) - 2(y - 5) + 2(z + 1) = 0$ and $2x + 5(y - 1) + (z + 4) = 0$.
89. Find the angle between the planes $x - 2y + 3z = 5$ and $2x + y - z = 7$.

90. Determine whether or not the vectors are coplanar: $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$, $\mathbf{i} - 2\mathbf{j}$, $4\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.
91. Find an equation in x, y, z for the plane that passes through the points $P_1(1, 1, 1)$, $P_2(2, 4, 3)$, $P_3(-1, -2, -1)$.
92. Find a set of scalar parametric equations for the line formed by the two intersecting planes:
 $P_1: 3x - 2y + z = 0$, $P_2: 8x + 2y + z - 11 = 0$.
93. Let l be the line determined by P_1, P_2 , and let p be the plane determined by Q_1, Q_2, Q_3 . Where, if anywhere, does l intersect p ?
 $P_1(2, 5, -2)$, $P_2(1, -2, 2)$; $Q_1(2, 1, -4)$, $Q_2(1, 2, 3)$, $Q_3(-1, 2, 1)$.
94. Find an equation in x, y, z for the plane that passes through $(1, 2, -3)$ and is perpendicular to the line $x(t) = 1 + 2t$, $y(t) = 2 + t$, $z(t) = -3 - 5t$.
95. Find an equation in x, y, z for the plane that passes through $(2, 1, 5)$ and the line $x(t) = -1 + 3t$, $y(t) = -2$, $z(t) = 2 + 4t$.
96. Find a vector equation for the line through $(1, 1, 1)$ that is parallel to the line of intersection of the planes $3x - 4y + 2z - 2 = 0$ and $4x - 3y - z - 5 = 0$.
97. Find parametric equations for the line through $(2, 0, -3)$ that is parallel to the line of intersection of the planes $x + 2y + 3z + 4 = 0$ and $2x - y - z - 5 = 0$.
98. Find an equation for the plane that passes through $(3, 0, 1)$ and is perpendicular to the line $x(t) = 2t$, $y(t) = 1 - t$, $z(t) = 4 - 3t$.
99. Find an equation for the plane that contains the point $(-2, 1, 1)$ and the line $\mathbf{r}(t) = 2\mathbf{i} + \mathbf{j} + \mathbf{k} + t(-\mathbf{i} + 4\mathbf{j} + 4\mathbf{k})$.
100. Find an equation for the plane that contains $P_1(1, 1, 1)$ and $P_2(-1, 2, 1)$ and is parallel to the line of intersection of the planes $2x + y - z - 4 = 0$ and $3x - y + z - 2 = 0$.
101. Find an equation for the plane that contains $P_1(3, 1, 2)$ and $P_2(-1, 2, -1)$ and is parallel to the line of intersection of the planes $2x - y - z - 2 = 0$ and $3x + 2y - 2z - 4 = 0$.
102. Sketch the graph of $20x + 12y + 15z - 60 = 0$.
103. Find the equation of the plane pictured below.



Answers to Chapter 12 Questions

1. $\sqrt{6}; \left(\frac{5}{2}, 8, \frac{15}{2}\right)$
2. $\sqrt{229}; \left(3, \frac{5}{2}, 3\right)$
3. $3\sqrt{2}; \left(-1, \frac{5}{2}, \frac{5}{2}\right)$
4. $z = -2$
5. $z = -1$
6. $x = -2$
7. $(x-2)^2 + (y-1)^2 + (z-3)^2 = 16$
8. $(x+4)^2 + y^2 + (z-6)^2 = 49$
9. $(x-5)^2 + (y-1)^2 + (z+4)^2 = 49$
10. $(x-3)^2 + (y-7/2)^2 + (z+2)^2 = 21/4$
11. $(x+2)^2 + (y-1)^2 + (z-4)^2 = 16$
12. $(3, 2, 1)$
13. $(3, 2, -1)$
14. $(-3, 2, 1)$
15. $(-1, -2, -4)$
16. $(2, -1, 3)$
17. $(-2, -7, -3)$
18. $(-4, 2, 1)$
19. $\mathbf{i} + 3\mathbf{j}$
20. $-4\mathbf{i} - 3\mathbf{j} - 3\mathbf{k}$
21. 5
22. $\sqrt{11}$
23. $\sqrt{73}$
24. $-13\mathbf{i} - 5\mathbf{j} + 10\mathbf{k}$
25. (a) $\sqrt{30}$ (b) $\sqrt{10}$ (c) $\sqrt{42}$ (d) $2\sqrt{91}$
26. ± 3
27. $\frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$
28. $\frac{5}{\sqrt{162}}\mathbf{i} - \frac{4}{\sqrt{162}}\mathbf{j} - \frac{11}{\sqrt{162}}\mathbf{k}$
29. $\frac{1}{\sqrt{30}}\mathbf{i} + \frac{5}{\sqrt{30}}\mathbf{j} + \frac{2}{\sqrt{30}}\mathbf{k}$
30. $\frac{6}{\sqrt{29}}\mathbf{i} + \frac{8}{\sqrt{29}}\mathbf{j} + \frac{4}{\sqrt{29}}\mathbf{k}$
31. $\frac{10}{\sqrt{170}}\mathbf{i} - \frac{24}{\sqrt{170}}\mathbf{j} + \frac{2}{\sqrt{170}}\mathbf{k}$
32. $\mathbf{a} \cdot \mathbf{a} + 6\mathbf{a} \cdot \mathbf{b}$
33. $-\mathbf{a} \cdot \mathbf{b} + 7\mathbf{a} \cdot \mathbf{c} - 3\mathbf{b} \cdot \mathbf{c}$
34. (a) 0, -4, 5
(b) $\cos(\mathbf{a}, \mathbf{b}) = 0$; $\cos(\mathbf{a}, \mathbf{c}) = -4/5$
 $\cos(\mathbf{b}, \mathbf{c}) = \frac{\sqrt{70}}{14}$
(c) (i) 0; (ii) $\frac{-4}{\sqrt{5}}$
(d) (i) 0; (ii) $\frac{8}{5}\mathbf{j} + \frac{4}{5}\mathbf{k}$
35. (a) 11, -8, -5
(b) $\cos(\mathbf{a}, \mathbf{b}) = \frac{11}{\sqrt{21}\sqrt{14}}$; $\cos(\mathbf{a}, \mathbf{c}) = \frac{-8}{\sqrt{21}\sqrt{5}}$
 $\cos(\mathbf{b}, \mathbf{c}) = \frac{-5}{\sqrt{14}\sqrt{5}}$
(c) (i) $\frac{11}{\sqrt{14}}$; (ii) $\frac{-8}{\sqrt{5}}$
(d) (i) $\left(\frac{33}{14}, \frac{-22}{14}, \frac{1}{14}\right)$; (ii) $\left(0, \frac{-16}{5}, \frac{8}{5}\right)$
36. $\approx 47.12^\circ$ or 0.8225 radians
37. $\approx 58.12^\circ$ or 1.014 radians
38. $\pi/4, 2\pi/3, \pi/3$
39. $\pi/6, 0, \pi/3$

40. $\frac{6}{\sqrt{53}}\mathbf{i} - \frac{1}{\sqrt{53}}\mathbf{j} - \frac{4}{\sqrt{53}}\mathbf{k}$ or $\frac{-6}{\sqrt{53}}\mathbf{i} + \frac{1}{\sqrt{53}}\mathbf{j} + \frac{4}{\sqrt{53}}\mathbf{k}$
41. $\frac{-8}{3\sqrt{21}}$
42. $250\sqrt{3}$ Joules
43. $W = 23$
44. $\mathbf{i} - \mathbf{j} + \mathbf{k}$
45. 0
46. 0
47. $2\mathbf{i} - 4\mathbf{j} + 9\mathbf{k}$
48. $-2\mathbf{i} - 7\mathbf{j} + 8\mathbf{k}$
49. $-12\mathbf{i} + 6\mathbf{j} - 54\mathbf{k}$
50. -2
51. 15
52. $14\mathbf{i} + 26\mathbf{j} - 21\mathbf{k}$
53. $5\sqrt{2}$
54. $\frac{1}{2}\sqrt{101}$
55. 17
56. 4
57. 54
58. 10
59. $2\sqrt{5}$
60. $\frac{\sqrt{290}}{2}$
61. $P(-1, 3, -1)$
62. No
63. $\mathbf{r}(t) = 2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k} + t\mathbf{k}$
64. $\mathbf{r}(t) = t(3\mathbf{i} + \mathbf{j} + 8\mathbf{k})$
65. $\mathbf{r}(t) = 4\mathbf{i} + 5\mathbf{k} + t(-2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$
66. $\mathbf{r}(t) = 3\mathbf{i} + 3\mathbf{j} + \mathbf{k} + t(\mathbf{i} - 3\mathbf{j} + \mathbf{k})$
67. $x(t) = 1 + t; y(t) = 4 - 5t; z(t) = 6 - 3t$
68. $x(t) = -3 + 2t; y(t) = -1 + 3t; z(t) = t$
69. $x(t) = 4; y(t) = -2; z(t) = -1 + t$
70. $x(t) = -1; y(t) = 2 + t; z(t) = -3$
71. $\mathbf{r}(t) = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + t(10\mathbf{i} + 15\mathbf{j} + 6\mathbf{k})$
72. $(4, 2, 3); \mathbf{q} = \cos^{-1}\left(\frac{-3}{\sqrt{273}}\right)$
73. $(4, 6, 0)$
74. $(8, 0, 4)$
75. $x(t) = -3 + t\mathbf{a}$
 $y(t) = 2 + t\mathbf{b}$
 $z(t) = 2 + t(2\mathbf{a} - 5\mathbf{b}); \mathbf{a}, \mathbf{b} \in \Re$
76. 0
77. $\sqrt{2}$
78. $\mathbf{r}(t) = -\mathbf{i} + 2\mathbf{j} + 4\mathbf{k} + t(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$
79. $\frac{7}{2\sqrt{21}}$
80. $\frac{3}{\sqrt{14}}$
81. R
82. S
83. $3x + y - 5z + 8 = 0$
84. $3x + y + 6z - 7 = 0$
85. $\left(\frac{3}{\sqrt{43}}, \frac{3}{\sqrt{43}}, \frac{-5}{\sqrt{43}}\right)\left(\frac{-3}{\sqrt{43}}, \frac{-3}{\sqrt{43}}, \frac{5}{\sqrt{43}}\right)$
86. $\frac{x}{1/5} + \frac{y}{-1/3} + \frac{z}{-1/2} = 1$
87. $x = -1, y = -4/3, z = 2$

88. $\cos^{-1} \frac{2}{\sqrt{510}} \approx 84.92^\circ$

89. $\cos^{-1} \frac{3}{2\sqrt{21}} \approx 70.89^\circ$

90. No

91. $2y - 3z + 1 = 0$

92. $x = 1 - 4t, y = 3/2 + 5t, z = 22t$

93. $\left(\frac{92}{61}, \frac{95}{61}, \frac{-2}{61} \right)$

94. $2x + y - 5z - 19 = 0$

95. $10x - 3y - 9z + 28 = 0$

96. $\mathbf{i} + \mathbf{j} + \mathbf{k} + t(10\mathbf{i} + 11\mathbf{j} + 7\mathbf{k})$

97. $x = 2 + t, y = 7t, z = -3 - 5t$

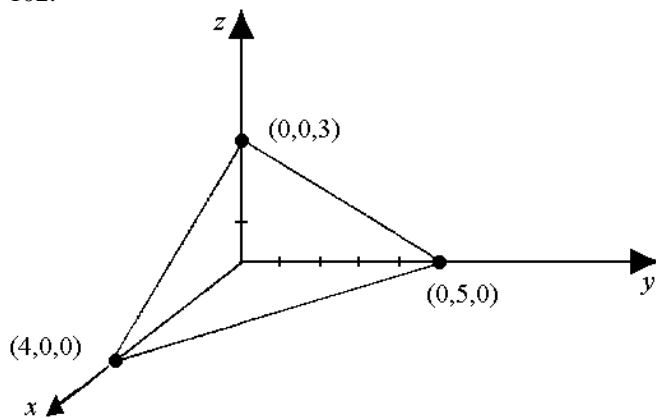
98. $2x - y - 3z - 3 = 0$

99. $y - z = 0$

100. $x + 2y - 2z - 1 = 0$

101. $5x + 8y - 4z - 15 = 0$

102.



103. $3x + z = 3$

CHAPTER 13

Vector Calculus

13.1 Vector Functions

1. Differentiate $\mathbf{f}(t) = (3 - 4t) \mathbf{i} + 5t \mathbf{j} + (2 - 5t) \mathbf{k}$.
2. Differentiate $\mathbf{f}(t) = (1 - t)^{-1/2} \mathbf{i} + \sqrt{1 + 2t^2} \mathbf{j} - t^{3/2} \mathbf{k}$.
3. Differentiate $\mathbf{f}(t) = e^{-t} \mathbf{i} + \ln(2t^2 - t) \mathbf{j} + \sin^{-1} t \mathbf{k}$.
4. Differentiate $\mathbf{f}(t) = \sin^2 t^2 \mathbf{i} + \cos 2t \mathbf{j} + t^2 e^{-2t} \mathbf{k}$.
5. Calculate $\int_1^3 \mathbf{f}(t) dt$ for $\mathbf{f}(t) = (1 + 2t^2) \mathbf{i} - 5t \mathbf{k}$.
6. Calculate $\int_{p/4}^{p/2} \mathbf{r}(t) dt$ for $\mathbf{r}(t) = t \sin 2t^2 \mathbf{i} + \sin 3t \mathbf{j} + e^{2t} \mathbf{k}$.
7. Calculate $\int_0^1 \mathbf{g}(t) dt$ for $\mathbf{g}(t) = te^{-2t^2} \mathbf{i} + t^2 e^{4t^3} \mathbf{j} - \cos \frac{2t}{3} \mathbf{k}$.
8. Find $\lim_{t \rightarrow 1} \mathbf{f}(t)$ if it exists. $\mathbf{f}(t) = \ln t \mathbf{i} - \sqrt[3]{t} \mathbf{j} + e^{4t} \mathbf{k}$.
9. Find $\lim_{t \rightarrow 0} \mathbf{r}(t)$ if it exists. $\mathbf{r}(t) = t \cos t \mathbf{i} - e^{-t} \mathbf{j} + \frac{3t^2 - 2t + 1}{e^t} \mathbf{k}$.
10. Find a vector-valued function \mathbf{f} that traces out the curve $16x^2 + 4y^2 = 64$ in (a) a counterclockwise direction and (b) a clockwise direction.
11. Find a vector-valued function \mathbf{f} that traces out the curve $y = 2(x - 1)^2$ in a direction from (a) left to right and (b) right to left.
12. Find a vector-valued function \mathbf{f} that traces out the directed line segment from $(2, -1, 3)$ to $(1, 4, -2)$.
13. Find $\mathbf{f}(t)$ given that $\mathbf{f}'(t) = (t^2 + 1) \mathbf{i} + 2t^2(1 + t^3)^{-1} \mathbf{j} + t^2 e^{t^3/2} \mathbf{k}$ and $\mathbf{f}(0) = 2 \mathbf{i} - 3 \mathbf{j} + \frac{1}{2} \mathbf{k}$.
14. Find $\mathbf{f}(t)$ given that $\mathbf{f}'(t) = 2t^2 \mathbf{i} - 3(t + 1) \mathbf{j} + e^t(t + 1) \mathbf{k}$ and $\mathbf{f}(0) = \mathbf{i} - 2 \mathbf{j} + 2 \mathbf{k}$.
15. Sketch the curve represented by $\mathbf{r}(t) = 3t \mathbf{i} + t^3 \mathbf{j}$ and indicate the orientation.
16. Sketch the curve represented by $\mathbf{r}(t) = 5 \sin t \mathbf{i} + 3 \cos t \mathbf{j}$ and indicate the orientation.

13.2 Differentiation Formulas

17. Find $\mathbf{f}'(t)$ and $\mathbf{f}''(t)$ for $\mathbf{f}(t) = e^t \mathbf{i} + e^{2t} \mathbf{j} + \mathbf{k}$.
18. Find $\mathbf{f}'(t)$ and $\mathbf{f}''(t)$ for $\mathbf{f}(t) = \sin t \mathbf{i} + \sinh 2t \mathbf{j} + \operatorname{sech} 2t \mathbf{k}$.
19. Find $\mathbf{f}'(t)$ and $\mathbf{f}''(t)$ for $\mathbf{f}(t) = \sqrt{t^2 + 2t} \mathbf{i} + \ln \sqrt{t^2 + 2t} \mathbf{j} + t \mathbf{k}$.

20. Find $\mathbf{f}'(t)$ and $\mathbf{f}''(t)$ for $\mathbf{f}(t) = t \mathbf{i} + \ln \cos 2t \mathbf{j} + \ln \sin 2t \mathbf{k}$.
21. Find $\mathbf{f}'(t)$ and $\mathbf{f}''(t)$ for $\mathbf{f}(t) = (\cos t + t \sin t) \mathbf{i} + (\sin t - t \cos t) \mathbf{j} + t^2 \mathbf{k}$.
22. Find $\mathbf{f}'(t)$ and $\mathbf{f}''(t)$ for $\mathbf{f}(t) = (t^2 \mathbf{i} + \cos t \mathbf{j}) \times (e^t \mathbf{j} + \sin t \mathbf{k})$.
23. Find $\mathbf{f}'(t)$ and $\mathbf{f}''(t)$ for $\mathbf{f}(t) = [(\sqrt{t} \mathbf{i} - t^{3/2} \mathbf{j}) \cdot (\sin t \mathbf{j} - t \mathbf{k})] \mathbf{j}$.
24. Find the derivative $\frac{d^2}{dt^2} [e^t \sin^2 t \mathbf{i} + 2t^2 \mathbf{j}]$.
25. Given $\mathbf{g}(t) = 2t \mathbf{i} + \frac{2}{3} t^3 \mathbf{j} - (1 + t^2) \mathbf{k}$ and $\mathbf{u}(t) = \frac{1}{4} t^2$, $\mathbf{f}(t) = t \mathbf{i} + \frac{1}{t} \mathbf{j} + e^{3t} \mathbf{k}$, find
- $(\mathbf{f} + \mathbf{g})'(t)$
 - $(2\mathbf{f})'(2t)$
 - $(\mathbf{u}\mathbf{f})'(t)$
 - $(\mathbf{f} \cdot \mathbf{g})'(t)$
 - $(\mathbf{f} \times \mathbf{g})'(t)$
 - $(\mathbf{g} \times \mathbf{f})'(t)$
 - $(\mathbf{f}^0 \mathbf{u})'(t)$
26. Find $\mathbf{r}(t)$ given that $\mathbf{r}'(t) = t \mathbf{i} - \mathbf{j} + e^{2t} \mathbf{k}$ and $\mathbf{r}(0) = \mathbf{i} - \frac{1}{4} \mathbf{j} + 2\mathbf{k}$.
27. Find $\mathbf{r}(t)$ given that $\mathbf{r}'(t) = \sin 2t \mathbf{i} + \cos 2t \mathbf{j} - t \mathbf{k}$ and $\mathbf{r}(0) = \mathbf{i} + \mathbf{j} + \mathbf{k}$.
28. Find $\mathbf{r}(t)$ given that $\mathbf{r}'(t) = \frac{1}{1+t^2} \mathbf{i} + 2 \tan 2t \mathbf{j} + e^{-t} \mathbf{k}$ and $\mathbf{r}(0) = \mathbf{i} + \mathbf{j} + \mathbf{k}$.
29. Calculate $\mathbf{r}(t) \cdot \mathbf{r}'(t)$ and $\mathbf{r}(t) \times \mathbf{r}'(t)$ given that $\mathbf{r}(t) = (\sin t + t \cos t) \mathbf{i} + t \mathbf{j}$.
30. Find $\mathbf{f}'(\mathbf{p}/3)$ for $\mathbf{f}(t) = t \mathbf{i} + \ln \sin 2t \mathbf{j} + \cos^2 2t \mathbf{k}$.
31. Find $\mathbf{f}'(t)$ for $\mathbf{f}(t) = \sin^{-1} 2t \mathbf{i} + \tan^{-1} 2t \mathbf{j}$.

13.3 Curves

32. Find the tangent to the vector $\mathbf{r}'(t)$ and the tangent line for $\mathbf{r}(t) = 3t \cos t \mathbf{i} + 3t \sin t \mathbf{j} + 4t \mathbf{k}$ at $t = \pi$.
33. Find the tangent to the vector $\mathbf{r}'(t)$ and the tangent line for $\mathbf{r}(t) = \sin^{-1} \frac{t}{2} \mathbf{i} + \tan^{-1} 3t \mathbf{j} - 3t \mathbf{k}$ at $t = 1$.
34. Find the tangent to the vector $\mathbf{r}'(t)$ and the tangent line for $\mathbf{r}(t) = 6 \sin 2t \mathbf{i} + 6 \cos 2t \mathbf{j} + \frac{2t^2}{p} \mathbf{k}$ at $t = \pi/4$.
35. Find the tangent to the vector $\mathbf{r}'(t)$ and the tangent line for $\mathbf{r}(t) = t^2 \mathbf{i} + 4t^{-2} \mathbf{j} + (1 - t^3) \mathbf{k}$ at $t = 1$.
36. Find the tangent to the vector $\mathbf{r}'(t)$ and the tangent line for $\mathbf{r}(t) = t^2 \mathbf{i} + 2t \mathbf{j} + e^{2t} \mathbf{k}$ at $t = 1$.
37. Find the points on the curve $\mathbf{r}(t) = t \mathbf{j}$ at which $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ are perpendicular.
38. Find the point at which the curves
 $\mathbf{r}_1(t) = (1 - t) \mathbf{i} + (1 + t) \mathbf{j} + (1 - t) \mathbf{k}$ and
 $\mathbf{r}_2(u) = 2u^3 \mathbf{i} + (1 - u^2) \mathbf{j} + (1 + u^2) \mathbf{k}$ intersect and find the angle of intersection.

39. Find a vector parametrization for the curve $x^2 = y - 1$, $x \geq 1$.
40. Find a vector parametrization for the curve $r = 2 \cos 2q$, $q \in [0, p]$ (polar coordinates).
41. Find an equation in x and y for the curve $\mathbf{r}(t) = t^2 \mathbf{i} + 2t \mathbf{j}$. Draw the curve. Does the curve have a tangent vector at the origin? If so, what is the unit tangent vector?
42. At $t = p/4$ find the unit tangent vector for the curve $\mathbf{r}(t) = \sin 2t \mathbf{i} + \cos 3t \mathbf{j} + \tan t \mathbf{k}$.
43. At $t = p/6$ find the unit tangent vector for the curve $\mathbf{r}(t) = \cos 2t \mathbf{i} + \sin 2t \mathbf{j} - 3t \mathbf{k}$.
44. Find the tangent vector $\mathbf{r}'(t)$ and the tangent line for $\mathbf{r}(t) = \sec 2t \mathbf{i} + \cos 2t \mathbf{j} + 2t \mathbf{k}$ at $t = 0$.
45. Find the unit tangent vector, the principal normal vector, and an equation in x, y, z for the osculating plane of the curve $\mathbf{r}(t) = t^2 \mathbf{i} + 2t \mathbf{j} + t \mathbf{k}$ at $t = 1$.
46. Find the unit tangent to the curve $\mathbf{r}(t) = \ln t \mathbf{i} + t \mathbf{j} + \sin pt \mathbf{k}$ at $t = 2$.
47. Find the unit tangent vector, the principal normal vector, and an equation in x, y, z for the osculating plane of the curve $\mathbf{r}(t) = e^t \sin 2t \mathbf{i} + 2e^t \cos 2t \mathbf{j} + 2e^t \mathbf{k}$ at $t = 0$.

13.4 Arc Length

48. Find the length of the curve $\mathbf{r}(t) = 3 \cos t \mathbf{i} + 3 \sin t \mathbf{j} + t \mathbf{k}$ from $t = 0$ to $t = 2p$.
49. Find the length of the curve $\mathbf{r}(t) = 5t \mathbf{i} + 4 \sin 3t \mathbf{j} + 4 \cos 3t \mathbf{k}$ from $t = 0$ to $t = 2p$.
50. Find the length of the curve $\mathbf{r}(t) = \frac{t^3}{3} \mathbf{i} + \frac{t^2}{\sqrt{2}} \mathbf{j} + t \mathbf{k}$ from $t = 0$ to $t = 3$.
51. Find the length of the curve $\mathbf{r}(t) = 6 \sin 2t \mathbf{i} + 6 \cos 2t \mathbf{j} + 5t \mathbf{k}$ from $t = 0$ to $t = p$.
52. Find the length of the curve $\mathbf{r}(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + t^{3/2} \mathbf{k}$ from $t = 0$ to $t = 20/3$.
53. Find the length of the curve $\mathbf{r}(t) = (2 + \cos 3t) \mathbf{i} + (3 - \sin t) \mathbf{j} + 4t \mathbf{k}$ from $t = 0$ to $t = 2p/3$.
54. Find the length of the curve $\mathbf{r}(t) = \sin^3 2t \mathbf{i} + \cos^3 2t \mathbf{j}$ from $t = 0$ to $t = p/4$.
55. Find the length of the curve $\mathbf{r}(t) = 2e^{-t} \mathbf{i} + (4 - 2t) \mathbf{j} + e^t \mathbf{k}$ from $t = 0$ to $t = 2$.
56. Find the curvature of $y = e^{2x}$.
57. Find the curvature of $y = \ln \sin 2x$.
58. Find the curvature of $y = 2x^2 - x + 1$.
59. Find the curvature of $y = 4 \sin 3x$.
60. Find the radius of curvature of $y = x^2/4$ at $x = 1$.
61. Find the radius of curvature of $y^2 = 4x$ at $(1, 2)$.
62. Find the radius of curvature of $xy = 6$ at $x = 2$.
63. Find the radius of curvature of $y = e^x$ at $x = 0$.

64. Find the curvature of $\mathbf{r}(t) = 2 \cos t \mathbf{i} + \cos 2t \mathbf{j}$ at $t = p/4$.
65. Find the curvature of $\mathbf{r}(t) = t^3 \mathbf{i} + 2t^2 \mathbf{j}$ at $t = 1$.
66. Find the curvature of $\mathbf{r}(t) = (t^2 + 1) \mathbf{i} + (t - 2) \mathbf{j}$ at $t = 2$.
67. Find the curvature of $\mathbf{r}(t) = 6 \cos 2t \mathbf{i} + \sin 2t \mathbf{j} + 3t \mathbf{k}$ at $t = p$.
68. Find the curvature of $\mathbf{r}(t) = \sin t \mathbf{i} + \cos t \mathbf{j} + \ln \cos t \mathbf{k}$ at $t = 0$.
69. Find the curvature of $\mathbf{r}(t) = e^t \mathbf{i} + e^t \cos t \mathbf{j} + e^t \sin t \mathbf{k}$ at $t = 0$.
70. Interpret $\mathbf{r}(t)$ as the position of a moving object at time t . Find the curvature of $\mathbf{r}(t) = t^2 \mathbf{i} + \frac{1}{t} \mathbf{j}$ at $t = 1/2$ and determine the tangential and normal components of acceleration.
71. Interpret $\mathbf{r}(t)$ as the position of a moving object at time t . Find the curvature of $\mathbf{r}(t) = 2e^t \mathbf{i} + 2e^{-t} \mathbf{j}$ at $t = 0$ and determine the tangential and normal components of acceleration.
72. Find the curvature of $\mathbf{r}(t) = \ln t \mathbf{i} + t \mathbf{j}$ at $t = 2$.
73. Find the radius of curvature of $\mathbf{r}(t) = e^t \mathbf{i} + \sqrt{2} t \mathbf{j} + e^{-t} \mathbf{k}$ at $t = 0$.
74. Find the radius of curvature of $\mathbf{r}(t) = 4 \sin t \mathbf{i} + (2t - \sin 2t) \mathbf{j} + \cos 2t \mathbf{k}$ at $t = p/2$.
75. Find the radius of curvature of $\mathbf{r}(t) = 2 \cos t \mathbf{i} + 3 \sin t \mathbf{j}$ at $t = p/2$.
76. Find the curvature of $x^2 + y^2 = 10x$ at $(2, -4)$.
77. Find the curvature of $y = 3 \cosh x/3$ at $x = 0$.

13.5 Curvilinear Motion; Vector Calculus in Mechanics

78. A particle moves so that $\mathbf{r}(t) = t^2 \mathbf{i} - 2t \mathbf{j}$. Find the velocity, speed, acceleration, and the magnitude of the acceleration at the time $t = 2$.
79. An object moves so that $\mathbf{r}(t) = 4 \cos t \mathbf{i} + \sin t \mathbf{j}$. Sketch the curve and then compute and sketch the velocity and acceleration vectors at $t = p/2$.
80. An object moves so that $\mathbf{r}(t) = t^2 \mathbf{i} + t^3 \mathbf{j}$. Sketch the curve and then compute and sketch the velocity and acceleration vectors at $t = 1$.
81. An object moves so that $\mathbf{r}(t) = 6 \cos 2t \mathbf{i} + 6 \sin 2t \mathbf{j} + 5t \mathbf{k}$. Find the velocity, speed, acceleration, and the magnitude of the acceleration at the time $t = p$.
82. An object moves so that $\mathbf{r}(t) = 2t \mathbf{i} + 4 \sin 3t \mathbf{j} + 4 \cos 3t \mathbf{k}$. Find the velocity, speed, acceleration, and the magnitude of the acceleration at the time $t = p/2$.
83. An object moves so that $\mathbf{r}(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + t^{3/2} \mathbf{k}$. Find the velocity, speed, acceleration, and the magnitude of the acceleration at the time $t = p/2$.
84. An object moves so that $\mathbf{r}(t) = e^t \mathbf{i} + e^t \cos t \mathbf{j} + e^t \sin t \mathbf{k}$, $t \geq 0$. Find
(a) the initial velocity
(b) the initial position
(c) the initial speed
(d) the acceleration throughout the motion
(e) the acceleration at $t = 0$.

85. Find the force required to propel a particle of mass m so that $\mathbf{r}(t) = t^2 \mathbf{i} + t^3 \mathbf{j}$.
86. At each point $P(x(t), y(t), z(t))$ of its motion, an object of mass m is subject to a force $\mathbf{F}(t) = m\mathbf{p}^2[2 \cos \mathbf{p} t \mathbf{i} + 3 \sin \mathbf{p} t \mathbf{j}]$. Given that $\mathbf{v}(0) = 2 \mathbf{i} - 3\mathbf{p}/2 \mathbf{j} + \frac{1}{2}\mathbf{k}$ and $\mathbf{r}(0) = 3 \mathbf{i} + 2 \mathbf{j}$, find the following at time $t = 1$:
- velocity
 - speed
 - acceleration
 - momentum
 - angular momentum
 - torque.
87. At each point $P(x(t), y(t), z(t))$ of its motion, an object of mass m is subject to a force $\mathbf{F}(t) = 2m\mathbf{p}^2[\frac{3}{2} \cos \mathbf{p} t \mathbf{i} - 2 \sin \mathbf{p} t \mathbf{j}]$. Given that $\mathbf{v}(0) = 3 \mathbf{i} + 2\mathbf{p} \mathbf{j} - \mathbf{k}$ and $\mathbf{r}(0) = 3 \mathbf{j} - 2 \mathbf{k}$, find the following at time $t = 1$:
- velocity
 - speed
 - acceleration
 - momentum
 - angular momentum
 - torque.
88. Show that the position vector and the velocity vector of the particle whose position is given by $\mathbf{r}(t) = \sin t \cos t \mathbf{i} + \cos^2 t \mathbf{j} + \sin t \mathbf{k}$ are at right angles.
89. Solve the initial values problem: $\mathbf{F}(t) = m \mathbf{r}''(t) = m(e^t \mathbf{i} + e^{2t} \mathbf{j})$
 $\mathbf{r}_0 = \mathbf{r}(0) = 2 \mathbf{i}$
 $\mathbf{v}_0 = \mathbf{v}(0) = 3 \mathbf{j} + \mathbf{k}$
90. Solve the initial values problem: $\mathbf{F}(t) = m \mathbf{r}''(t) = m(3 \cos 2t \mathbf{i} + 3 \sin 2t \mathbf{j})$
 $\mathbf{r}_0 = \mathbf{r}(0) = \mathbf{i} - 2 \mathbf{j}$
 $\mathbf{v}_0 = \mathbf{v}(0) = 2 \mathbf{i} - \frac{2}{3} \mathbf{k}$

Answers to Chapter 13 Questions

1. $-4\mathbf{i} + 5\mathbf{j} - 5\mathbf{k}$

2. $\frac{1}{2}(1-t)^{-3/2}\mathbf{i} + \frac{2t}{\sqrt{1+2t^2}}\mathbf{j} - \frac{3}{2}t^{1/2}\mathbf{k}$

3. $-e^{-t}\mathbf{i} + \frac{4t-1}{2t^2-t}\mathbf{j} - \frac{1}{\sqrt{1-t^2}}\mathbf{k}$

4. $4t \sin t^2 \cos t^2 \mathbf{i} - 2 \sin 2t \mathbf{j} + 2te^{-2t}(1-t)\mathbf{k}$

5. $\frac{58}{3}\mathbf{i} - 20\mathbf{k}$

6. $-\frac{1}{4}\left(\cos \frac{p^2}{2} - \cos \frac{p^2}{8}\right)\mathbf{i} + \frac{\sqrt{2}}{6}\mathbf{j} + \frac{1}{2}(e^{pt} - e^{(p/2)t})\mathbf{k}$

7. $\frac{1}{4}(1-e^{-2})\mathbf{i} + \frac{1}{12}(e^4-1)\mathbf{j} - \frac{3}{2}\left(\sin \frac{2}{3}\right)\mathbf{k}$

8. $-\mathbf{j} + e^4\mathbf{k}$

9. $-\mathbf{j} + \mathbf{k}$

10. (a) $\mathbf{f}(t) = 2 \cos t \mathbf{i} + 4 \sin t \mathbf{j}$
(b) $\mathbf{g}(t) = 2 \cos t \mathbf{i} - 4 \sin t \mathbf{j}$

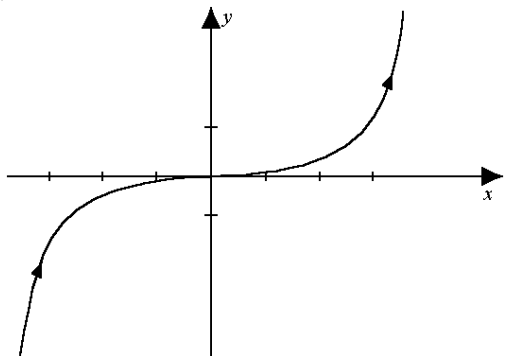
11. (a) $\mathbf{f}(t) = (t+1)\mathbf{i} + 2t^2\mathbf{j}$
(b) $\mathbf{g}(t) = (1-t)\mathbf{i} + 2t^2\mathbf{j}$

12. $\mathbf{f}(t) = (2-t)\mathbf{i} + (-1+5t)\mathbf{j} + (3-5t)\mathbf{k}, t \in [0, 1]$

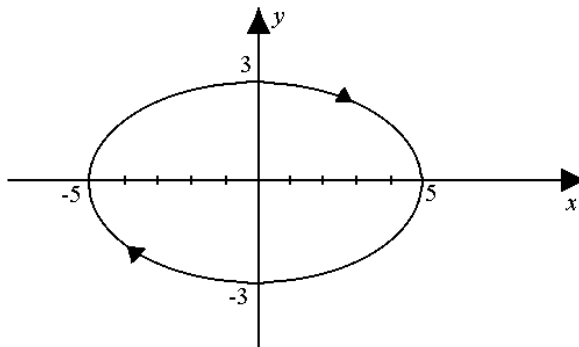
13. $\mathbf{f}(t) = \left(\frac{t^3}{3} + t + 2\right)\mathbf{i} + \left(\frac{2}{3}\ln|1+t^3| - 3\right)\mathbf{j}$
 $+ \left(\frac{2}{3}e^{t^3/2} - \frac{1}{6}\right)\mathbf{k}$

14. $\mathbf{f}(t) = \left(\frac{2}{3}t^2 + 1\right)\mathbf{i} - \left[3\left(\frac{t^2}{2} + t\right) + 2\right]\mathbf{j} + (te^t + 2)\mathbf{k}$

15.



16.



17. $\mathbf{f}'(t) = e^t \mathbf{i} + 2e^{2t} \mathbf{j}$
 $\mathbf{f}''(t) = e^t \mathbf{i} + 4e^{2t} \mathbf{j}$

18. $\mathbf{f}'(t) = \cos t \mathbf{i} + 2 \cosh 2t \mathbf{j} - 2 \operatorname{sech} 2t \tanh 2t \mathbf{k}$
 $\mathbf{f}''(t) = -\sin t \mathbf{i} + 4 \sinh 2t \mathbf{j} + (4 \operatorname{sech} 2t \tanh^2 2t - 4 \operatorname{sech}^3 2t) \mathbf{k}$

19. $\mathbf{f}'(t) = \frac{t+1}{\sqrt{t^2+2t}}\mathbf{i} - \frac{t+1}{\sqrt{t^2+2t}}\mathbf{j} + \mathbf{k}$
 $\mathbf{f}''(t) = \frac{-1}{(t^2+2t)^{3/2}}\mathbf{i} - \frac{t^2+t+1}{(t^2+2t)^2}\mathbf{j}$

20. $\mathbf{f}'(t) = \mathbf{i} - 2 \tan 2t \mathbf{j} - 2 \cot 2t \mathbf{k}$
 $\mathbf{f}''(t) = -4 \sec^2 2t \mathbf{j} - 4 \csc^2 2t \mathbf{k}$

21. $\mathbf{f}'(t) = t \cos t \mathbf{i} + t \sin t \mathbf{j} + 2t \mathbf{k}$
 $\mathbf{f}''(t) = (\cos t - t \sin t) \mathbf{i} + (\sin t + t \cos t) \mathbf{j} + 2 \mathbf{k}$

22. $\mathbf{f}'(t) = \cos 2t \mathbf{i} - (2t \sin t + t^2 \cos t) \mathbf{j} + e'(t^2 + 2t) \mathbf{k}$
 $\mathbf{f}''(t) = -2 \sin 2t \mathbf{i} - (2 \sin t + 4t \cos t + t^2 \sin t) \mathbf{j}$
 $+ e'(t^2 + 4t + 2) \mathbf{k}$

23. $\mathbf{f}'(t) = t^{-3/2} \left(\frac{3 \sin t}{2t} - \cos t \right) \mathbf{j}$
 $\mathbf{f}''(t) = t^{-3/2} \left(\frac{-15 \sin t}{4t^2} + \frac{3 \cos t}{t} + \sin t \right) \mathbf{j}$

24. $e'(\sin^2 t + 2 \sin 2t + 2 \cos 2t) \mathbf{i} + 4 \mathbf{j}$

25. (a) $3\mathbf{i} + \left(\frac{-1}{t^2} + 2t^2\right)\mathbf{j} + (3e^{3t} - 2t)\mathbf{k}$
(b) $2\mathbf{i} - \frac{2}{t^2}\mathbf{j} + 6e^{3t}\mathbf{k}$
(c) $\frac{3}{4}t^2\mathbf{i} + \frac{1}{4}\mathbf{j} + \left(\frac{3}{4}t^2e^{3t} + \frac{t}{2}e^{3t}\right)\mathbf{k}$

(d) $4t - \frac{4}{3}t - e^{3t}(3 + 2t + 3t^2)$

- (e) $\left(1 + \frac{1}{t^2} - 2t^2 e^{3t} - 2t^3 e^{3t}\right) \mathbf{i} + (2e^{3t} + 6te^{3t} + 1 + 3t^2) \mathbf{j} + \frac{8}{3}t^3 \mathbf{k}$
- (f) $\left(-1 - \frac{1}{t^2} + 2t^2 e^{3t} + 2t^3 e^{3t}\right) \mathbf{i} + (-2e^{3t} - 6te^{3t} - 1 - 3t^2) \mathbf{j} - \frac{8}{3}t^3 \mathbf{k}$
- (g) $\frac{t}{2} \mathbf{i} + \frac{8}{t^3} \mathbf{j} + \frac{3t}{2} e^{3t^{2/4}} \mathbf{k}$
26. $\mathbf{r}(t) = \left(\frac{t^2}{2} + 1\right) \mathbf{i} + \left(t + \frac{1}{4}\right) \mathbf{j} + \left(\frac{1}{2}e^{2t} + \frac{3}{2}\right) \mathbf{k}$
27. $\mathbf{r}(t) = \left(\frac{3}{2} - \frac{1}{2}\cos 2t\right) \mathbf{i} + \left(1 + \frac{1}{2}\sin 2t\right) \mathbf{j} + \left(1 - \frac{t^2}{2}\right) \mathbf{k}$
28. $\mathbf{r}(t) = (1 + \tan^{-1} t) \mathbf{i} + (1 - \ln \cos 2t) \mathbf{j} + (2 - e^{-t}) \mathbf{k}$
29. $\mathbf{r}(t) \cdot \mathbf{r}'(t) = 3t \cos^2 t + \left(1 - \frac{t^2}{2}\right) \sin 2t$
 $\mathbf{r}(t) \times \mathbf{r}'(t) = [(t^2 + 1)\sin t - t \cos t] \mathbf{k}$
30. $\mathbf{i} + \frac{2}{\sqrt{3}} \mathbf{j} + \sqrt{3} \mathbf{k}$
31. $\frac{2}{\sqrt{1-4t^2}} \mathbf{i} + \frac{2}{1+4t^2} \mathbf{j}$
32. $\mathbf{r}'(t) = 3(\cos^2 t - t \sin t) \mathbf{i} + 3(\sin t + t \cos t) \mathbf{j} + 4 \mathbf{k}$
 tangent line: $-(3p + 3t) \mathbf{i} - 3pt \mathbf{j} + (4p + 4t) \mathbf{k}$
33. $\mathbf{r}'(t) = \frac{1}{\sqrt{4-t^2}} \mathbf{i} + \frac{3}{1+9t^2} \mathbf{j} - 3 \mathbf{k}$
 tangent line: $\left(\frac{p}{6} - \frac{4}{\sqrt{15}}t\right) \mathbf{i} + \left(\tan^{-1} 3 + \frac{3}{10}t\right) \mathbf{j} - 3(1+t) \mathbf{k}$
34. $\mathbf{r}'(t) = 12\cos 2t \mathbf{i} - 12\sin 2t \mathbf{j} + \frac{4t}{p} \mathbf{k}$
 tangent line: $6 \mathbf{i} - 12t \mathbf{j} + \left(\frac{p}{8} + t\right) \mathbf{k}$
35. $\mathbf{r}'(t) = 2t \mathbf{i} - \frac{8}{t^3} \mathbf{j} - 3t^2 \mathbf{k}$
 tangent line: $(1 + 2t) \mathbf{i} + (4 - 8t) \mathbf{j} - 3t \mathbf{k}$
36. $\mathbf{r}(t) = 2t \mathbf{i} + 2 \mathbf{j} + 2e^{2t} \mathbf{k}$
 tangent line: $(1+t) \mathbf{i} + (2+t) \mathbf{j} + e^2(1+t) \mathbf{k}$
37. $(x, y) = (0, 0), (1/4, -1/2)$
38. $(2, 0, 2); \mathbf{q} = \cos^{-1} \frac{-5}{\sqrt{33}} \approx 29.5^\circ$
39. $\mathbf{r}(t) = t \mathbf{i} + (t^2 + 1) \mathbf{j}$
40. $\mathbf{r}(t) = t \mathbf{u}_q + (2 + \cos 2t) \mathbf{u}_r$
41. $x = y^2/4$, tangent vector at origin is \mathbf{j}
42. $\mathbf{T}(p/4) = \frac{-3}{\sqrt{17}} \mathbf{j} + \frac{2\sqrt{2}}{\sqrt{17}} \mathbf{k}$
43. $\mathbf{T}(p/6) = \frac{-\sqrt{3}}{4} \mathbf{i} + \frac{1}{2} \mathbf{j} + \frac{3}{4} \mathbf{k}$
44. $\mathbf{r}'(t) = (2 \sec 2t \tan 2t) \mathbf{i} - 2\sin 2t \mathbf{j} + 2 \mathbf{k}$
 tangent line: $\mathbf{i} + \mathbf{j} + 2t \mathbf{k}$
45. $\mathbf{T}(1) = \frac{2}{3} \mathbf{i} + \frac{2}{3} \mathbf{j} + \frac{1}{3} \mathbf{k}$
 $\mathbf{N}(1) = \frac{5}{3\sqrt{5}} \mathbf{i} + \frac{4}{3\sqrt{5}} \mathbf{j} + \frac{2}{3\sqrt{5}} \mathbf{k}$
 osculating plane: $5x - 4y - 2z + 5 = 0$
46. $\mathbf{T}(2) = \frac{1}{\sqrt{5+4p^2}} \mathbf{i} + \frac{2}{\sqrt{5+4p^2}} \mathbf{j} + \frac{2p}{\sqrt{5+4p^2}} \mathbf{k}$
47. $\mathbf{T}(0) = \frac{1}{\sqrt{3}} \mathbf{i} + \frac{1}{\sqrt{3}} \mathbf{j} + \frac{1}{\sqrt{3}} \mathbf{k}$
 $\mathbf{N}(0) = \frac{2}{\sqrt{14}} \mathbf{i} + \frac{3}{\sqrt{14}} \mathbf{j} + \frac{1}{\sqrt{14}} \mathbf{k}$
 osculating plane: $2x - 3y + z + 4 = 0$
48. $2p\sqrt{10}$
49. $26p$
50. 12
51. $13p$
52. $56/3$
53. $10p/3$
54. $3/2$

55. $e^2 + 1 - e^{-2}$

56. $\frac{4e^{2x}}{(1+4e^{4x})^{3/2}}$

57. $\frac{4\csc^2 2x}{(1+4\cot^2 2x)^{3/2}}$

58. $\frac{4}{(2+8x+16x^2)^{3/2}}$

59. 0

60. $\frac{5\sqrt{5}}{4}$

61. $4\sqrt{2}$

62. $\frac{13\sqrt{13}}{12}$

63. $2\sqrt{2}$

64. $\frac{1}{3\sqrt{3}}$

65. $\frac{12}{125}$

66. $\frac{12}{17\sqrt{17}}$

67. $\frac{24}{169}$

68. $\sqrt{2}$

69. $\frac{\sqrt{2}}{3}$

70. $\frac{24}{17\sqrt{17}}, a_T = \frac{-62}{\sqrt{17}}, a_N = \frac{24}{\sqrt{17}}$

71. $\frac{1}{\sqrt{8}}, a_T = 0, a_N = \sqrt{8}$

72. $\frac{2}{5\sqrt{5}}$

73. $2\sqrt{2}$

74. $2\sqrt{2}$

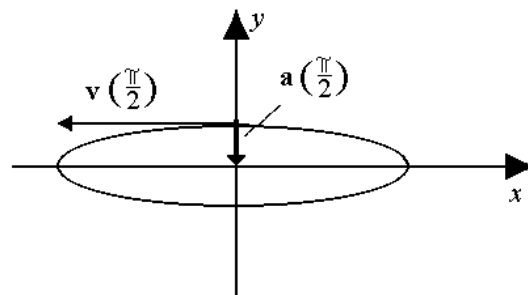
75. $4/3$

76. $1/5$

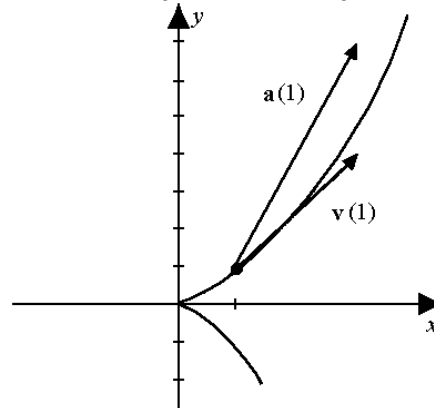
77. $1/3$

78. $\mathbf{v}(2) = 4\mathbf{i} + 2\mathbf{j}; \|\mathbf{v}(2)\| = 2\sqrt{5}$
 $\mathbf{a}(2) = 2\mathbf{i}; \|\mathbf{a}(2)\| = 2$

79. $\mathbf{v}(p/2) = -4\mathbf{i}; \mathbf{a}(p/2) = -\mathbf{j}$



80. $\mathbf{v}(1) = 2\mathbf{i} + 3\mathbf{j}; \mathbf{a}(1) = 2\mathbf{i} + 6\mathbf{j}$



81. $\mathbf{v}(p) = 12\mathbf{j} + 5\mathbf{k}; \|\mathbf{v}(p)\| = 13$
 $\mathbf{a}(p) = -24\mathbf{i}; \|\mathbf{a}(p)\| = 24$

82. $\mathbf{v}(p/2) = 2\mathbf{i} + 12\mathbf{k}; \|\mathbf{v}(p/2)\| = 2\sqrt{37}$
 $\mathbf{a}(p/2) = 36\mathbf{j}; \|\mathbf{a}(p/2)\| = 36$

83. $\mathbf{v}(p/2) = -\mathbf{i} + \frac{3}{2}\sqrt{\frac{p}{2}}\mathbf{k}; \|\mathbf{v}(p/2)\| = \sqrt{1 + \frac{9}{8}p}$
 $\mathbf{a}(p/2) = -\mathbf{j} + \frac{3}{4}\sqrt{\frac{2}{p}}\mathbf{k}; \|\mathbf{a}(p/2)\| = \sqrt{1 + \frac{9}{8p}}$

84. (a) $\mathbf{i} + \mathbf{j} + \mathbf{k}$
(b) $(1, 1, 0)$
(c) $\sqrt{3}$
(d) $\mathbf{a}(t) = e^t\mathbf{i} - 2e^t \sin t\mathbf{j} + 2e^t \cos t\mathbf{k}$
(e) $\mathbf{a}(0) = \mathbf{i} + 2\mathbf{k}$

85. $\mathbf{F}(t) = 2m \mathbf{i} + 6mt \mathbf{j}$

86. (a) $2 \mathbf{i} + 9p/2 \mathbf{j} + 1/2 \mathbf{k}$

(b) $\frac{1}{2} \sqrt{17 + 81p^2}$

(c) $-2p^2 \mathbf{i}$

(d) $2m \mathbf{i} + \frac{9}{2}pm \mathbf{j} + \frac{m}{2} \mathbf{k}$

(e) $\left(1 - \frac{3p}{2}\right)m \mathbf{i} + \frac{1}{2}m \mathbf{j} + \left(\frac{75p}{2} - 4\right)m \mathbf{k}$

(f) $-mp^2 \mathbf{j}$

87. (a) $3 \mathbf{i} - 6p \mathbf{j} - \mathbf{k}$

(b) $\sqrt{10 + 36p^2}$

(c) $-3p^2 \mathbf{i}$

(d) $3m \mathbf{i} - 6pm \mathbf{j} + m \mathbf{k}$

(e) $-(3 + 16p)m \mathbf{i} - (9 + 48p)m \mathbf{k}$

(f) $9p^2m \mathbf{j} + (9p^2 - 6p^3)m \mathbf{k}$

88. $\mathbf{r}(t) = \frac{1}{2} \sin 2t \mathbf{i} + \frac{1}{2} (1 + \cos 2t) \mathbf{j} + \cos t \mathbf{k}$

$\mathbf{v}(t) = \cos 2t \mathbf{i} - \sin 2t \mathbf{j} + \sin t \mathbf{k}$

$\mathbf{r}(t) \cdot \mathbf{v}(t) = 0$

89. $\mathbf{v}(t) = (e^t - 1) \mathbf{i} + \frac{1}{2} (e^{2t} + 5) \mathbf{j} + \mathbf{k}$

$\mathbf{r}(t) = (e^t - t + 1) \mathbf{i} + \frac{1}{4} (e^{2t} + 10t - 1) \mathbf{j} + t \mathbf{k}$

90. $\mathbf{v}(t) = \left(\frac{3}{2} \sin 2t + 2\right) \mathbf{i} + \left(\frac{3}{2} \cos 2t - 1\right) \mathbf{j} - \frac{2}{3} \mathbf{k}$

$\mathbf{r}(t) = \left(-\frac{3}{4} \cos 2t + 2t + \frac{7}{4}\right) \mathbf{i} - \frac{3}{4} \left(\sin 2t - 2t + \frac{8}{3}\right) \mathbf{j}$
 $- \frac{2}{3} t \mathbf{k}$

CHAPTER 14

Functions of Several Variables

14.1 Elementary Examples

1. Find the domain and range of $f(x, y, z) = 2 \tan^{-1} \frac{y}{x} + \ln(x^2 + z^2)$; find $f(1, 1, 1)$ and $f(1, -1, 1)$.
2. Find the domain and range of $f(x, y, z) = ze^x \cos y$; find $f(\ln 2, 0, -1)$.
3. Find the domain and range of $f(x, y, z) = yze^{\ln(x^2 + y^2)}$; find $f(1, -1, 2)$ and $f(0, 1, 4)$.
4. Find the domain and range of $f(x, y) = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{x}{y}\right)$; find $f(1, 1)$.
5. Find the domain and range of $f(x, y) = 4 - \frac{2}{3}x - \frac{1}{2}y$; find $f(2, 3)$.
6. Find the domain and range of $f(x, y) = 1 - y^2$; find $f(2, 0)$.
7. Find the domain and range of $f(x, y, z) = \sqrt{4 - x^2 - y^2 - z^2}$; find $f(1, 1, \frac{1}{2})$.
8. Find the domain and range of $f(x, y) = \sqrt{\frac{x+y}{x-y}}$; find $f(3, 1)$.
9. Find the domain and range of $f(x, y) = 2xy - \frac{y}{x}$; if $x(t) = 2t$ and $y(t) = t^2$.
10. Determine a function f of x and y giving the volume of a cone of base diameter x and height y .
11. Determine a function f of x , y , and z giving the surface area of a box of length x , width y , and volume z .

14.2 A Brief Catalogue of Quadratic Surfaces; Projections

12. Identify the surface $x^2 + 4y^2 + 9z^2 + 2x + 16y - 18z - 10 = 0$.
13. Identify the surface $5x^2 + 4y^2 + 20z^2 - 20x + 32y + 40z + 56 = 0$.
14. Identify the surface $9x^2 + 4y^2 - 54x - 16y - 36z + 277 = 0$.
15. Identify the surface $3x^2 - 2y^2 + 3z^2 + 30x - 8y - 24z + 131 = 0$.
16. Identify the surface $6x^2 + 4y^2 - 3z^2 + 36x - 16y - 6z + 55 = 0$.
17. Identify the surface $6x^2 + 4y^2 - 2z^2 - 6x - 4y + z = 0$.
18. Identify the surface $3x^2 - 2y^2 - z^2 - 6x + 8y - 2z + 6 = 0$.
19. Identify the surface $x^2 + y^2 - z^2 - 2x + 4y - 2z = 0$.

20. Sketch the cylinder $9x^2 + 4z^2 - 36 = 0$.
21. Identify and sketch the surface $z = 4x^2 + y^2$.
22. Identify and sketch the surface $z^2 = x^2 + y^2$.
23. Identify and sketch the surface $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} = 1$.
24. Identify the surface and find the traces: $2x^2 + y^2 - 4z = 0$.
25. Identify the surface and find the traces: $x^2 - y^2 + z^2 + 2y = 1$.
26. Identify the surface and find the traces: $x^2 + y^2 + z - 5 = 0$.
27. Identify the surface and find the traces: $x^2 + 4y^2 + z^2 - 8y = 0$.
28. Write an equation for the surface obtained by revolving the parabola $y - 4z^2 = 0$ about the y -axis.
29. The planes $2x + y + z = 4$ and $x + y - z = 1$ intersect in a space curve C . Determine the projection of C onto the xy -plane.
30. The sphere $x^2 + (y - 1)^2 + z^2 = 6$ and the hyperboloid $x^2 - y^2 + z^2 = 1$ intersect in a space curve C . Determine the projection of C onto the xy -plane.
31. The sphere $x^2 + (y - 2)^2 + z^2 = 2$ and the cone $y^2 + z^2 = 5x^2$ intersect in a space curve C . Determine the projection of C onto the xy -plane.
32. The cone $x^2 + y^2 = 2z^2$ and the plane $y + 4z = 5$ intersect in a space curve C . Determine the projection of C onto the xy -plane.

14.3 Graphs; Level Curves and Level Surfaces

33. Sketch the graph of $f(x, y) = \sqrt{16 - x^2 - y^2}$.
34. Sketch the graph of $f(x, y) = \sqrt{16 - x^2 - 2y^2}$.
35. Identify the level curves and sketch some of them. $f(x, y) = x^2 + y^2$
36. Identify the level curves and sketch some of them. $f(x, y) = 4x^2 + y^2$
37. Identify the level curves and sketch some of them. $f(x, y) = \sqrt{\frac{x+y}{x-y}}$
38. Identify the level curves and sketch some of them. $f(x, y) = xy$
39. Identify the c -level surface for $f(x, y, z) = 2x - 3y + z$ and $c = 1$.
40. Identify the c -level surface for $f(x, y, z) = x^2 - y^2 + z^2$ and $c = 1$.
41. Identify the c -level surface for $f(x, y, z) = \frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{2}$ and $c = 1$.

14.4 Partial Derivatives

42. Calculate the partial derivatives of $z = \frac{x}{y} \sin(xy^2)$.
43. Calculate the partial derivatives of $f(x, y) = x^y$.
44. Calculate the partial derivatives of $z = x^3 + xy - y \cos xy$.
45. Calculate the partial derivatives of $z = y^2 e^{-x} + y$.
46. Calculate the partial derivatives of $f(x, y) = \sqrt{16 - 4x^2 - 9y^2}$.
47. Calculate the partial derivatives of $f(x, y) = \sin(x^2 y)$.
48. Calculate the partial derivatives of $f(x, y) = (1 + x^2 + y)^{5/3}$.
49. Find $f_x(1, 1)$ and $f_y(1, 1)$ given that $f(x, y) = y - e^{xy^2} + \sqrt{x^2 + 1}$.
50. Find $f_x(4, 2)$ and $f_y(4, 2)$ given that $f(x, y) = \ln(xy - 1) + e^y \sqrt{x}$.
51. Find $f_x(4, 2)$ and $f_y(4, 2)$ given that $f(x, y) = y \ln(x + y^2) + y^2 \sqrt{x}$.
52. Find $f_x(x, y)$ and $f_y(x, y)$ by forming the appropriate difference quotient and taking the limit as h tends to zero (Definition 14.4.1). $f(x, y) = xy^2$.
53. Find $f_x(x, y, z)$, $f_y(x, y, z)$, and $f_z(x, y, z)$ by forming the appropriate difference quotient and taking the limit as h tends to zero (Definition 14.4.2). $f(x, y, z) = x^2 yz$.
54. Let C be the curve of intersection of the surface $z = 3xy^2$ with the plane $y = 2$. Find an equation for the tangent line to C at the point $(1, 2, 12)$.
55. Let C be the curve of intersection of the surface $z = \sqrt{x^2 - y^2}$ with the plane $x = 3$. Find an equation for the tangent line to C at the point $(3, 1, 2\sqrt{2})$.
56. Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ given that $x^2 z^2 - 2xyz + y^2 z^3 = 3$.
57. Use implicit differentiation to find $\frac{\partial z}{\partial x}$ at $(1, -2, 1)$ given that $x^3 z - 3xy^2 - (yz)^3 = -3$.
58. Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ given that $x^2 + y^2 + z^2 - 2xyz = 5$.
59. Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ given that $x^{1/3} - y^{1/3} + z^{1/3} = 16$.
60. Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ given that $(x + y)^2 = (y - z)^3$.

61. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ given that $x^3z^2 - 2xyz^2 + z^3y^2 = 2z$.
62. Evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $(3, 3, 2)$ given that $x^3 + y^3 + z^3 - 3xyz = 8$.
63. Evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $(1, 0, \pi/6)$ given that $x^2 \cos^2 z - y^2 \sin z = \sin^2 2z$.
64. Verify that $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = z$ given that $z = x \sin\left(\frac{x}{y}\right) + ye^{y/x}$.
65. Verify that $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 3z$ given that $z = x^3 + 2x^2y - 3xy^2 + y^3$.
66. Verify that $4 \frac{\partial z}{\partial x} - 3 \frac{\partial z}{\partial y} = 0$ given that $z = (3x + 4y)^4$.
67. The area of a triangle is given by the formula $A = \frac{1}{2}bc \sin q$. At time t_0 we have $b_0 = 5$ cm, $c_0 = 10$ cm, and $q_0 = \pi/6$ radians.
- (a) Find the area of the triangle at time t_0 .
 - (b) Find the rate of change of the area with respect to b at time t_0 if c and q remain constant.
 - (c) Find the rate of change of the area with respect to q at time t_0 if b and c remain constant.
 - (d) Using the rate found in (c), calculate (by differentials) the approximate change in area if the angle is increased by one degree.
 - (e) Find the rate of change of c with respect to b at time t_0 if the area and the angle are to remain constant.

14.5 Open Sets and Closed Sets

68. Specify the interior and the boundary of the set $\{(x, y) : 1 \leq x \leq 3, 2 \leq y \leq 4\}$. State whether the set is open, closed, or neither. Then sketch the set.
69. Specify the interior and the boundary of the set $\{(x, y) : 4 < x^2 + y^2 < 9\}$. State whether the set is open, closed, or neither. Then sketch the set.
70. Specify the interior and the boundary of the set $\{(x, y, z) : x^2 + y^2 \leq 2, 0 \leq z \leq 2\}$. State whether the set is open, closed, or neither. Then sketch the set.
71. Specify the interior and the boundary of the set $\{(x, y, z) : x^2 + (y - 1)^2 + z^2 < 1\}$. State whether the set is open, closed, or neither. Then sketch the set.
72. Specify the interior and the boundary of the set $\{y : 2 < y < 4\}$. State whether the set is open, closed, or neither.
73. Specify the interior and the boundary of the set $\{x : x \leq 2\}$. State whether the set is open, closed, or neither.
74. Specify the interior and the boundary of the set $\{x : x > -1\}$. State whether the set is open, closed, or neither.

14.6 Limits and Continuity; Equality of Mixed Partial

75. Find the second partials of $f(x, y) = \sqrt{16 - 9x^2 - 4y^2}$.
76. Find the second partials of $f(x, y) = \cos(xy^2)$.
77. Find the second partials of $f(x, y) = (1 + x + y^2)^{4/3}$.
78. Find the second partials of $f(x, y, z) = 2 \tan^{-1} \frac{y}{x} + \ln(x^2 + z^2)$.
79. Find the second partials of $f(x, y, z) = xz^2 \cosh(\ln y)$.
80. Find the second partials of $f(x, y) = \frac{1}{x^2 + y^2 - 4}$.
81. Find the second partials of $f(x, y, z) = \ln(2x + 3y + 2z)$.
82. Find the second partials of $f(x, y) = (x^2 + xy)^{5/2}$.
83. Find the second partials of $f(x, y, z) = \sqrt{x^2 + y^2 + 2z}$.
84. Evaluate $\lim_{(x,y) \rightarrow (1,2)} (x^2 + 3y)$.
85. Evaluate $\lim_{(x,y) \rightarrow (1,1)} \frac{4 + x - y}{3 + x - 3y}$.
86. Evaluate $\lim_{(x,y) \rightarrow (1, \pi/2)} x^3 \sin \frac{y}{x}$.
87. Evaluate $\lim_{(x,y) \rightarrow (0,0)} \frac{2x + y}{x^3 + y^3}$.
88. Evaluate $\lim_{(x,y) \rightarrow (0,0)} \frac{\tan(x^2 + y^2)}{x^2 + y^2}$.
89. Evaluate $\lim_{(x,y) \rightarrow (1,1)} \frac{\sin^{-1}(xy - 2)}{\tan^{-1}(3xy - 4)}$.
90. Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{x^2 + y^2}$ does not exist.
91. Show that the function $f(x, y) = e^{-2y} \cos 2x$ is harmonic.
92. For what value of $c > 0$ is $f(x, t) = \sin(2x + 3t)$ a solution of the wave equation $\frac{\partial^2 f}{\partial t^2} - c^2 \frac{\partial^2 f}{\partial x^2} = 0$?

Answers to Chapter 14 Questions

1. $\text{dom}(f) = \{(x, y, z) : x \neq 0\}$
 $\text{ran}(f) = (-\infty, \infty)$
 $f(1, 1, 1) = p/2 + \ln 2$
 $f(1, -1, 1) = -p/2 + \ln 2$
2. $\text{dom}(f) = \{(x, y, z) : x, y, z \in \mathbb{R}\}$
 $\text{ran}(f) = (-\infty, \infty)$
 $f(\ln 2, 0, -1) = -2$
3. $\text{dom}(f) = \{(x, y, z) : x^2 + y^2 \neq 0\}$
 $\text{ran}(f) = (-\infty, \infty)$
 $f(1, -1, 2) = -4$
 $f(0, 1, 4) = 4$
4. $\text{dom}(f) = \{(x, y) : |x/y| \leq 1\}$
 $\text{ran}(f) = [-3p/4, 3p/4]$
 $f(1, 1) = 3p/4$
5. $\text{dom}(f) = \{(x, y) : x, y \in \mathbb{R}\}$
 $\text{ran}(f) = (-\infty, \infty)$
 $f(2, 3) = 7/6$
6. $\text{dom}(f) = \{(x, y) : x, y \in \mathbb{R}\}$
 $\text{ran}(f) = (-\infty, 1]$
 $f(2, 0) = 1$
7. $\text{dom}(f) = \{(x, y, z) : x^2 + y^2 + z^2 \leq 4\}$
 $\text{ran}(f) = [0, 2]$
 $f(1, 1, 1/2) = \frac{\sqrt{7}}{2}$
8. $\text{dom}(f) = \{(x, y) : -x \leq y \leq x, x > 0;$
 $\text{or } x \leq y \leq -x, x < 0\}$
 $\text{ran}(f) = [0, \infty)$
 $f(3, 1) = \sqrt{2}$
9. $\text{dom}(f) = \{(x, y) : x \neq 0\} = \{t : t \neq 0\}$
 $\text{ran}(f) = (-\infty, \infty)$
10. $f(x, y) = \frac{1}{12}px^2y$
11. $f(x, y, z) = 2xy + 2\frac{z}{y} + 2\frac{z}{x}$
12. ellipsoid
13. ellipsoid
14. elliptic paraboloid
15. hyperboloid of two sheets

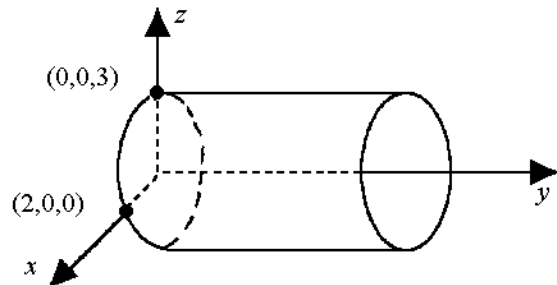
16. hyperboloid of one sheet

17. hyperboloid of one sheet

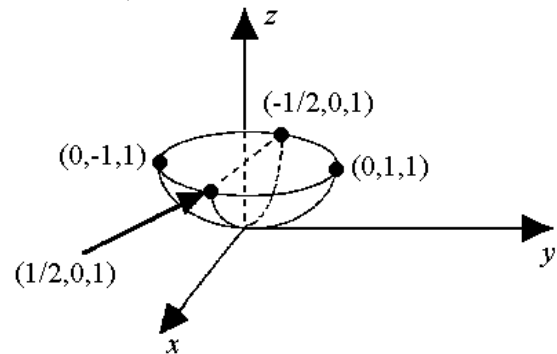
18. hyperboloid of one sheet

19. hyperboloid of one sheet

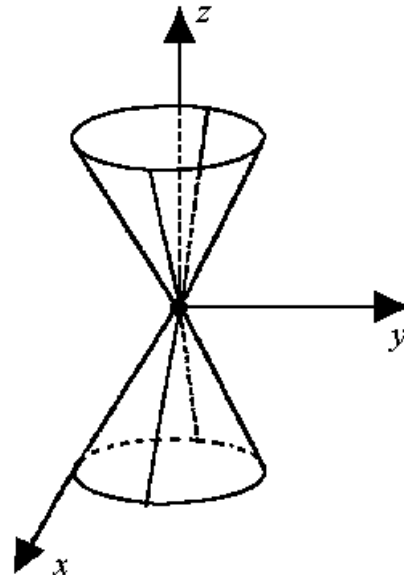
20. $\frac{x^2}{4} + \frac{z^2}{9} = 1$



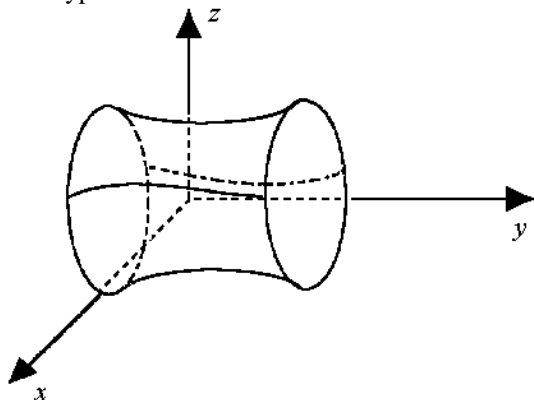
21. $z = \frac{x^2}{1/4} + y^2$; elliptic paraboloid



22. circular cone



23. hyperboloid of one sheet



24. elliptic paraboloid

traces: origin, $z = \frac{y^2}{4}$, $z = \frac{x^2}{2}$

25. circular cone

traces: $z = \pm(y-1)$, $x^2 + z^2 = 1$, $x = \pm(y-1)$

26. circular paraboloid

traces: $z = 5 - y^2$, $z = 5 - x^2$, $x^2 + y^2 = 5$

27. ellipsoid

traces: $(y-1)^2 + z^2/4 = 1$, origin, $x^2/4 + (y-1)^2 = 1$

28. $4x^2 + 4z^2 = y$

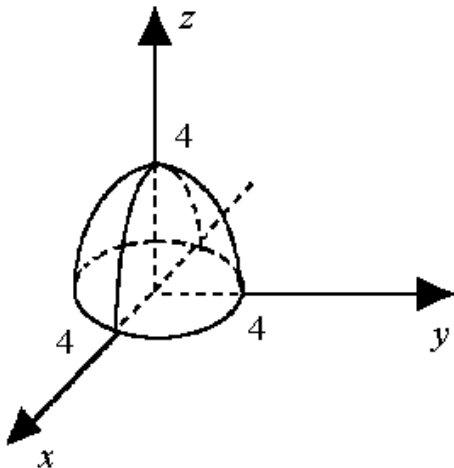
29. $3x + 2y = 5$

30. $x = 0$

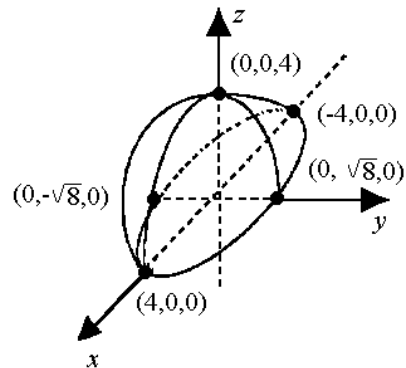
31. $x^2 = \frac{2}{3}\left(y - \frac{1}{2}\right)$

32. $\frac{x^2}{25/7} = \frac{(y+5/7)^2}{200/49} = 1$; an ellipse

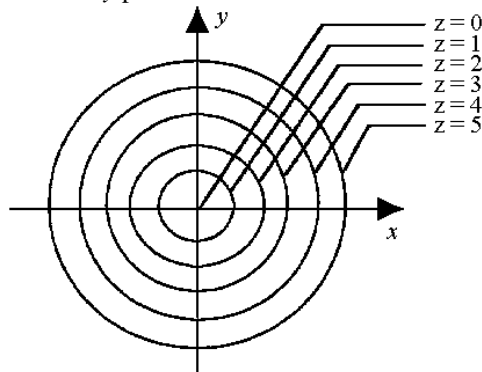
33.



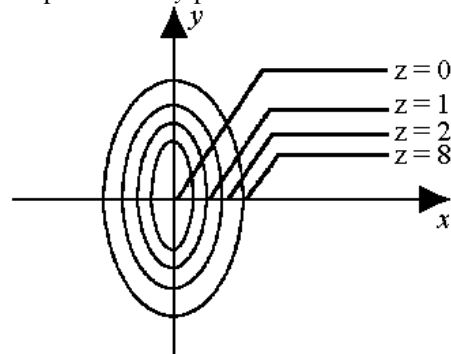
34.



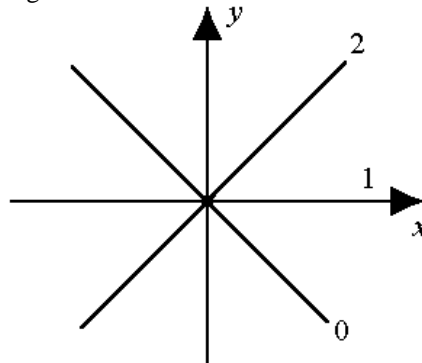
35. The level curves for $z \geq 0$ are concentric circles in the xy plane.



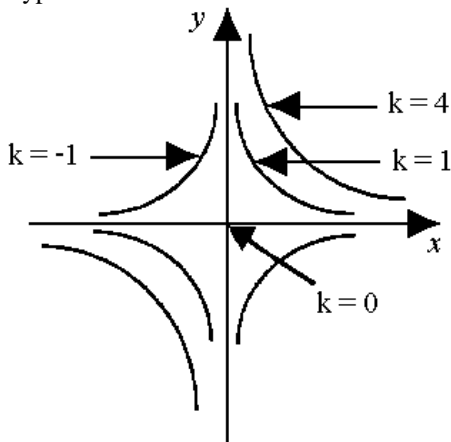
36. The level curves for $z \geq 0$ are concentric ellipses in the xy plane.



37. The level curves are straight lines through the origin.



38. hyperbolas



39. plane

40. hyperboloid of one sheet

41. ellipsoid

$$42. \quad \frac{\partial z}{\partial x} = z_x = xy \cos(xy^2) + \frac{1}{y} \sin(xy^2)$$

$$\frac{\partial z}{\partial y} = z_y = 2x^2 \cos(xy^2) + \frac{x}{y^2} \sin(xy^2)$$

$$43. \quad f_x = yx^{y-1}; \quad f_y = x^y \ln x$$

$$44. \quad z_x = 3x^2 + y + y^2 \sin xy$$

$$z_y = x + xy \sin xy - \cos xy$$

$$45. \quad z_x = -y^2 e^{-x}$$

$$z_y = 2ye^{-x} + 1$$

$$46. \quad f_x = \frac{-4x}{\sqrt{16-4x^2-9y^2}}$$

$$f_y = \frac{-9x}{\sqrt{16-4x^2-9y^2}}$$

$$47. \quad f_x = 2xy \cos(x^2y)$$

$$f_y = x^2 \cos(x^2y)$$

$$48. \quad f_x = \frac{10x}{3}(1+x^2+y^2)^{2/3}$$

$$f_y = \frac{5}{3}(1+x^2+y^2)^{2/3}$$

$$49. \quad f_x(1,1) = -e + \frac{\sqrt{2}}{2}$$

$$f_y(1,1) = 1 - 2e$$

$$50. \quad f_x(4,2) = \frac{2}{7} + \frac{e^2}{4}; \quad f_y(4,2) = \frac{4}{7} + 2e^2$$

$$51. \quad f_x(4,2) = 5/4; \quad f_y(4,2) = \ln 8 + 9$$

$$52. \quad f_x(x,y) = y^2; \quad f_y(x,y) = 2xy$$

$$53. \quad f_x(x,y,z) = 2xyz; \quad f_y(x,y,z) = x^2z$$

$$f_z(x,y,z) = x^2y$$

$$54. \quad \mathbf{r}(t) = t \mathbf{i} + 2 \mathbf{j} + 12t \mathbf{k}$$

$$55. \quad \mathbf{r}(t) = 3 \mathbf{i} + t \mathbf{j} + \frac{\sqrt{2}}{4}(9-t) \mathbf{k}$$

$$56. \quad \frac{\partial z}{\partial x} = \frac{2yz - 2xz^2}{2x^2z - 2xy + 3y^2z^2}$$

$$\frac{\partial z}{\partial y} = \frac{2xz - 2yz^3}{2x^2z - 2xy + 3y^2z^2}$$

$$57. \quad \frac{\partial z}{\partial x} = \frac{9}{25}$$

$$58. \quad \frac{\partial z}{\partial x} = \frac{x - yz}{xy - z}, \quad \frac{\partial z}{\partial y} = \frac{xz - y}{z - xy}$$

$$59. \quad \frac{\partial z}{\partial x} = -\left(\frac{z}{x}\right)^{2/3}, \quad \frac{\partial z}{\partial y} = -\left(\frac{z}{y}\right)^{2/3}$$

$$60. \quad \frac{\partial z}{\partial x} = \frac{-2(x+y)}{3(y-z)^2}, \quad \frac{\partial z}{\partial y} = \frac{1-2(x+y)}{3(y-z)^2}$$

$$61. \quad \frac{\partial z}{\partial x} = \frac{3x^2z^2 - 2yz^2}{2 - 2x^3z + 4xyz - 3z^2y^2}$$

$$\frac{\partial z}{\partial y} = \frac{3z^3y - 2yz^2}{2 - 2x^3z + 4xyz - 3z^2y^2}$$

$$62. \quad \frac{\partial z}{\partial x} = \frac{3}{5}, \quad \frac{\partial z}{\partial y} = \frac{3}{5}$$

$$63. \quad \frac{\partial z}{\partial x} = \frac{1}{\sqrt{3}}, \quad \frac{\partial z}{\partial y} = 0$$

$$64. \quad x \left(\sin \frac{x}{y} + \frac{x}{y} \cos \frac{x}{y} - \frac{x^2}{y^2} e^{y/x} \right) +$$

$$y \left(\frac{-x^2}{y^2} \cos \frac{x}{y} + e^{y/x} + \frac{y}{x} e^{y/x} \right) = x \sin \frac{x}{y} + ye^{y/x} = z$$

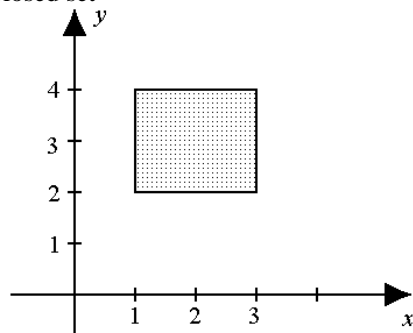
$$65. \quad x(3x^2 + 4xy + 3y^2) + y(2x^2 + 6xy + 3y^2) = z$$

66. $4[12(3x + 4y)^3] - 3[16(3x + 4y)^3] = 0$

67. (a) $25/2 \text{ cm}^2$ (d) $\frac{5p\sqrt{3}}{72} \text{ cm}^2$
 (b) $5/2 \text{ cm}^2/\text{cm}$ (e) $-c/b$
 (c) $\frac{25\sqrt{3}}{2} \text{ cm}^2/\text{rad}$

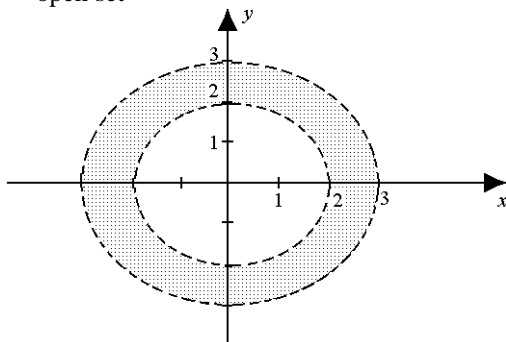
68. Interior: $\{(x, y): 1 < x < 3, 2 < y < 4\}$
 Boundary: $\{(x, y): 1 \leq x \leq 3, y = 2 \text{ or } y = 4\}$
 $\cup \{(x, y): 2 < y < 4, x = 1 \text{ or } x = 3\}$

closed set



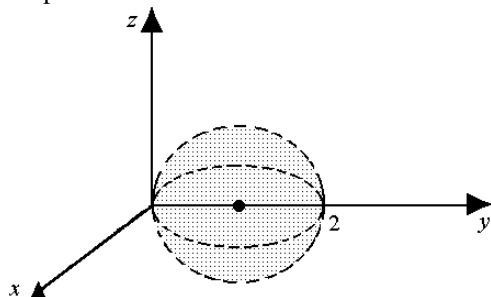
69. Interior: $\{(x, y): 4 < x^2 + y^2 < 9\}$
 Boundary: $\{(x, y): x^2 + y^2 = 4\}$
 $\cup \{(x, y): x^2 + y^2 = 9\}$

open set



70. Interior: $\{(x, y, z): x^2 + y^2 < 2, 0 < z < 2\}$
 Boundary: $\{(x, y, z): x^2 + y^2 = 2, 0 < z < 2\}$
 $\cup \{(x, y, z): x^2 + y^2 \leq 2, z = 0 \text{ or } z = 2\}$
 closed set

71. Interior: $\{(x, y): x^2 + (y - 1)^2 + z^2 < 1\}$
 Boundary: $\{(x, y): x^2 + (y - 1)^2 + z^2 = 1\}$
 open set



72. Interior: $\{(y): 2 < y < 4\}$
 Boundary: $y = 2, y = 4$
 open set

73. Interior: $\{(x): x < 2\}$
 Boundary: $x = 2$
 closed set

74. Interior: $\{(x): x > -1\}$
 Boundary: $x = -1$
 open set

75. $f_{xx} = \frac{36y^2 - 144}{(16 - 9x^2 - 4y^2)^{3/2}}$
 $f_{yy} = \frac{36x^2 - 64}{(16 - 9x^2 - 4y^2)^{3/2}}$
 $f_{xy} = \frac{-36xy}{(16 - 9x^2 - 4y^2)^{3/2}}$

76. $f_{xx} = -y^4 \cos(xy^2)$
 $f_{yy} = -4x^2y \cos(xy^2) - 2x \sin(xy^2)$
 $f_{xy} = f_{yx} = -2xy^3 \cos(xy^2) - 2y \sin(xy^2)$

77. $f_{xx} = \frac{4}{9}(1 + x + y^2)^{-2/3}$
 $f_{yy} = \frac{24 + 24x + 40y^2}{9(1 + x + y^2)^{2/3}}$
 $f_{xy} = f_{yx} = \frac{8y}{9(1 + x + y^2)^{2/3}}$

78. $f_{xx} = \frac{4xy}{(x^2 + y^2)^2} + \frac{2z^2 - 2x^2}{(x^2 + y^2)^2}$
 $f_{yy} = \frac{-4xy}{(x^2 + y^2)^2}, f_{zz} = \frac{2x^2 - 2z^2}{(x^2 + z^2)^2}$
 $f_{xy} = \frac{-2x^2 + 2y^2}{(x^2 + y^2)^2}, f_{xz} = \frac{-4xz}{(x^2 + z^2)^2}$
 $f_{yz} = 0$

79. $f_{xx} = 0, f_{xy} = \frac{z^2}{y} \sinh(\ln y)$
 $f_{yy} = \frac{xz^2}{y^2} [\cosh(\ln y) - \sinh(\ln y)]$
 $f_{xz} = 2z \cosh(\ln y), f_{zz} = 2x \cosh(\ln y)$
 $f_{yz} = \frac{2xz}{y} [\sinh(\ln y)]$

$$80. \quad f_{xx} = \frac{6x^2 - 2y^2 + 8}{(x^2 + y^2 - 4)^3}, f_{yy} = \frac{6y^2 - 2x^2 + 8}{(x^2 + y^2 - 4)^3}$$

$$f_{xy} = \frac{8xy}{(x^2 + y^2 - 4)^3}$$

$$81. \quad f_{xx} = \frac{-4}{(2x + 3y + 2z)^2}, f_{yy} = \frac{-9}{(2x + 3y + 2z)^2}$$

$$f_{zz} = \frac{-4}{(2x + 3y + 2z)^2}, f_{xy} = \frac{-6}{(2x + 3y + 2z)^2}$$

$$f_{xz} = \frac{-4}{(2x + 3y + 2z)^2}, f_{yz} = \frac{-6}{(2x + 3y + 2z)^2}$$

$$82. \quad f_{xx} = \frac{15}{4}(x^2 + xy)^{1/2}(2x + y)^2 + 5(x^2 + xy)^{3/2}$$

$$f_{xy} = \frac{15x}{4}(x^2 + xy)^{1/2}(2x + y) + \frac{5}{2}(x^2 + xy)^{3/2}$$

$$f_{yy} = \frac{15x^2}{4}(x^2 + xy)^{1/2}$$

$$83. \quad f_{xx} = \frac{y^2 + 2z}{(x^2 + y^2 + 2z)^{3/2}}, f_{yy} = \frac{x^2 + 2z}{(x^2 + y^2 + 2z)^{3/2}}$$

$$f_{xy} = \frac{-xy}{(x^2 + y^2 + 2z)^{3/2}}, f_{xz} = \frac{-x}{(x^2 + y^2 + 2z)^{3/2}}$$

$$f_{yz} = \frac{-y}{(x^2 + y^2 + 2z)^{3/2}},$$

$$84. \quad 7$$

$$85. \quad 4$$

$$86. \quad 1$$

$$87. \quad \text{does not exist}$$

$$88. \quad 1$$

$$89. \quad 2$$

$$90. \quad \text{along } y = 0 \quad \lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{x^2} = 1$$

$$\text{along } y = x \quad \lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{2x^2} = \frac{1}{2}$$

limit does not exist in either case

$$91. \quad \frac{\partial^2 f}{\partial x^2} = -4e^{-2y} \cos 2x = -\frac{\partial^2 f}{\partial y^2}$$

$$\text{so } \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

$$92. \quad c = 3/2$$

CHAPTER 15

Gradients; Extreme Values; Differentials

15.1 Differentiability and Gradient

1. Find the gradient of $f(x, y) = x^2 e^{xy}$.
2. Find the gradient of $f(x, y) = 2x^3 + 5xy - 2y^2$.
3. Find the gradient of $f(x, y) = xy \sin xy^2$.
4. Find the gradient of $f(x, y, z) = 2x^2 yz$.
5. Find the gradient of $f(x, y, z) = \frac{1}{\sqrt{x+y+z}}$.
6. Find the gradient of $f(x, y, z) = x^3 y + yz^2 - xy^2 z$.
7. Find the gradient of $f(x, y, z) = e^y \ln xz$.
8. Find the gradient of $f(x, y) = (x^2 - y) \cos (x + y)$.
9. Find the gradient of $f(x, y, z) = xe^{x+y+z}$.
10. Find the gradient of $f(x, y) = \sqrt{x^2 + y^2}$.
11. Find the gradient vector at $(1, 2)$ of $f(x, y) = 3x^2 - 2xy + 4y^2$.
12. Find the gradient vector at $(1, 2)$ of $f(x, y) = \frac{2x+y}{x^2+y^2}$.
13. Find the gradient vector at $(1/2, 1)$ of $f(x, y) = (x+y) \sin px$.
14. Find the gradient vector at $(1, -1, 2)$ of $f(x, y, z) = e^x \sin (y + z^2)$.
15. Find the gradient vector at $(1, 1, 2)$ of $f(x, y, z) = \ln (x + y + z^2)$.
16. Calculate (a) $\nabla (\ln r^3)$; (b) $\nabla (\sin 2r)$; (c) $\nabla (e^{2r^2+r})$
where $r = \sqrt{x^2 + y^2 + z^2}$

15.2 Gradients and Directional Derivatives

17. Find the directional derivative of $f(x, y) = e^x \sin y$ at $(0, \pi/3)$ in the direction of $5\mathbf{i} - 2\mathbf{j}$.
18. Find the directional derivative of $f(x, y) = \ln \sqrt[3]{x^2 + y^2}$ at $(3, 4)$ in the direction of $4\mathbf{i} + 3\mathbf{j}$.
19. Find the directional derivative of $f(x, y) = \frac{x^2}{16} + \frac{y^2}{9}$ at $(4, 3)$ in the direction of $\mathbf{i} + \mathbf{j}$.

20. Find the directional derivative of $f(x, y) = e^x \cos y$ at $(2, \mathbf{p})$ in the direction of $2\mathbf{i} + 3\mathbf{j}$.
21. Find the directional derivative of $f(x, y) = 3xy^2 - 4x^3y$ at $(1, 2)$ in the direction of $3\mathbf{i} + 4\mathbf{j}$.
22. Find the directional derivative of $f(x, y) = e^x \sin \mathbf{p}y$ at $(0, 1/3)$ toward the point $(3, 7/3)$.
23. Find the directional derivative of $f(x, y) = x \tan^{-1} y/x$ at $(1, 1)$ in the direction of $\mathbf{a} = 2\mathbf{i} - \mathbf{j}$.
24. Find the rate of change of $f(x, y) = \frac{2x}{x-y}$ at $(1, 0)$ in the direction of a vector at an angle of 60° from the positive x -axis.
25. Find the rate of change of $f(x, y) = \frac{x+y}{2x-y}$ at $(1, 1)$ in the direction of a vector at an angle of 150° from the positive x -axis.
26. Find the rate of change of $f(x, y) = 2xy - \frac{y}{x}$ at $(1, 2)$ in the direction of a vector at an angle of 120° from the positive x -axis.
27. Find the directional derivative of $f(x, y, z) = x \sin(\mathbf{p}yz) + yz \tan(\mathbf{p}x)$ at $(1, 2, 3)$ in the direction of $2\mathbf{i} + 6\mathbf{j} - 9\mathbf{k}$.
28. Find the directional derivative of $f(x, y, z) = x^2 - 2y^2 + z^2$ at $(3, 3, 1)$ in the direction of $2\mathbf{i} + \mathbf{j} - \mathbf{k}$.
29. Find the directional derivative of $f(x, y, z) = x^2y^3 + \sqrt{xz}$ at $(1, -2, 3)$ in the direction of $5\mathbf{j} + \mathbf{k}$.
30. Find the directional derivative of $f(x, y, z) = x^2y + xy^2 + z^2$ at $(1, 1, 1)$ toward the point $(3, 1, 2)$.
31. The temperature, T , at a point (x, y) on a semi-circular plate is given by $T(x, y) = 3x^2y - y^3 + 273$ degrees Celsius.
 - (a) Find the temperature at $(1, 2)$.
 - (b) Find the rate of change of temperature at $(1, 2)$ in the direction of $\mathbf{i} - 2\mathbf{j}$.
 - (c) Find a unit vector in the direction in which the temperature increases most rapidly at $(1, 2)$ and find this maximum rate of increase in temperature at $(1, 2)$.
32. The temperature, T , at a point (x, y) in the xy -plane is given by $T(x, y) = xy - x$. Find a unit vector in the direction in which the temperature increases most rapidly at $(1, 1)$ and find this maximum rate of increase in temperature at $(1, 1)$.
33. Find the unit vector in the direction in which $f(x, y, z) = 4e^{xy} \cos z$ decreases most rapidly at $(0, 1, \mathbf{p}/4)$ and find the rate of decrease of f in that direction.
34. Find the unit vector in the direction in which $f(x, y, z) = \ln(1 + x^2 + y^2 - z^2)$ increases most rapidly at $(1, -1, 1)$ and find the rate of increase of f in that direction.

15.3 The Mean Value Theorem; Chain Rules

35. Find the rate of change of $f(x, y) = xy^2$ with respect to t along the curve $\mathbf{r}(t) = e^t \mathbf{i} + t^2 \mathbf{j}$.
36. Find the rate of change of $f(x, y, z) = x^2 + y^2 + z^2$ with respect to t along the curve $\mathbf{r}(t) = t \mathbf{i} + t^2 \mathbf{j} - t^3 \mathbf{k}$.
37. Find the rate of change of $f(x, y, z) = xy + yz - xz$ with respect to t along the curve $\mathbf{r}(t) = 2 \cos \mathbf{w} t \mathbf{i} + 3 \sin \mathbf{w} t \mathbf{j} - 2 \mathbf{w} t \mathbf{k}$.

38. Find the rate of change of $f(x, y, z) = x^2 \cos(y + z)$ with respect to t along the curve $\mathbf{r}(t) = t \mathbf{i} - \sin t \mathbf{j} - t^2 \mathbf{k}$.
39. Use the chain rule to find $\frac{\partial u}{\partial t}$ if $u = \sqrt{x^2 + y^2}$; $x = e^t$, $y = \sin t$.
40. Use the chain rule to find $\frac{\partial u}{\partial t}$ if $u = y^2 e^x$; $x = \cos t$, $y = t^3$.
41. Use the chain rule to find $\frac{\partial u}{\partial t}$ if $u = xy$ and $x = e^x \cos x$.
42. Use the chain rule to find $\frac{\partial u}{\partial t}$ if $u = xy$ and $x = y \sin y$.
43. Use the chain rule to find $\frac{\partial u}{\partial t}$ at $t = 1$ if $u = x^3 y^2$; $x = t^2 + 1$, $y = t^3 + 2$.
44. Use the chain rule to find $\frac{\partial w}{\partial t}$ at $w = \tan^{-1}(xyz)$ and $x = t^2$, $y = t^3$, $z = t^4$.
45. Use the chain rule to find $\frac{\partial w}{\partial t}$ at $w = \sin xy + \ln xz + z$ and $x = e^t$, $y = t^2$, $z = 1$.
46. At $t = 0$, the position of a particle on a rectangular membrane is given by $P(x, y) = \sin \frac{px}{3} \sin \frac{py}{5}$. Find the rate at which P changes if the particle moves from $(3/4, 15/4)$ in a direction of a vector at an angle of 30° from the positive x -axis.
47. Use the chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ if $z = x \sin y$, $x = se^t$, $y = se^{-t}$.
48. Use the chain rule to find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ if $z = x^2 \tan y$, $x = u^2 + v^3$, $y = \ln(u^2 + v^2)$.
49. Use the chain rule to find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial q}$ if $z = \frac{xy}{x^2 + y^2}$, $x = r \cos q$, $y = r \sin q$.
50. Use the chain rule to find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ if $z = x \cos y + y \sin x$, $x = uv^2$, $y = u + v$.
51. Use the chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ if $z = x^2 + y^3$, $x = s + t$, $y = s - t$.
52. Use the chain rule to find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ if $z = x^3 + xy + y^2$, $x = 2u + v$, $y = u - 2v$.
53. Verify that $z = f(x^3 - y^2)$ satisfies the equation $2y \frac{\partial z}{\partial x} + 3x^2 \frac{\partial z}{\partial y} = 0$.
54. Verify that $z = f(y/x)$ satisfies the equation $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 0$.
55. Use the chain rule to find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ if $w = \ln(x^2 + y^2 + 2z)$, $x = r + s$, $y = r - s$, $z = 2rs$.

56. Use the chain rule to find $\frac{\partial w}{\partial r}$, $\frac{\partial w}{\partial q}$, and $\frac{\partial w}{\partial z}$ if $w = xy + yz$, $x = r \cos q$, $y = r \sin q$, $z = z$.
57. Use the chain rule to find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ if $w = \ln(x^2 + y^2 + z^2)$, $x = e^r \cos s$, $y = e^r \sin s$, $z = e^s$.
58. Use the chain rule to find $\frac{\partial w}{\partial t}$ if $w = x^2 + y^2 + z^2$, $x = e^t \cos t$, $y = e^t \sin t$, $z = e^t$.
59. Use the chain rule to find $\frac{\partial w}{\partial t}$, $\frac{\partial w}{\partial u}$, and $\frac{\partial w}{\partial v}$ if $w = 2x + y - z$, $x = t^2 + u^2$, $y = u^2 + v^2$, $z = v^2 + t^2$.
60. Use the chain rule to find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ if $w = 2x - 3y + z$, $x = u \sin v$, $y = v \sin u$, $z = \sin u \sin v$.
61. Use the chain rule to find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ if $w = \sqrt{x^2 + y^2 + z^2}$, $x = r \cos s$, $y = r \sin s$, $z = r \tan s$.
62. Use the chain rule to find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ if $w = x^2 + y^2 + z^2$, $x = r \cos s$, $y = r \sin s$, $z = rs$.
63. Evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $(\frac{1}{2}, -1, 2)$ if $e^{xz} + \ln(yz + 3) = y + 1 + e$ and z is a differentiable function of x and y .
64. Use the chain rule to find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ if $z = x^2 y^3 + x \sin y$, $x = u^2$, $y = uv$.
65. Show that $u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$ satisfies $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.
66. Let $z = f(x, y)$ with $x = r \cos q$ and $y = r \sin q$. Show that $\left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial z}{\partial q}\right)^2 = \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$.

15.4 The Gradient as a Normal; Tangent Lines and Tangent Planes

67. Write an equation for the plane tangent to the surface $4x^2 + 9y^2 + z = 17$ at the point $(-1, 1, 4)$.
68. Write an equation for the plane tangent to the surface $z = e^x \sin py$ at the point $(2, 1, 0)$.
69. Write an equation for the plane tangent to the surface $z = e^{-x} y^2 + y$ at the point $(0, 2, 6)$.
70. Write an equation for the plane tangent to the surface $z = x^2 + y^2$ at the point $(2, -1, 5)$.
71. Write an equation for the plane tangent to the surface $z = xe^{\sin y}$ at the point $(2, p, 2)$.
72. Write an equation for the plane tangent to the surface $z = 3x^2 + 2y^2$ at the point $(2, -1, 4)$.
73. Find a normal vector at the point indicated. Write an equation for the normal line and an equation for the plane tangent to the surface $z = \frac{y^2}{3} - x$ at the point $(0, 0, 0)$.

74. Find a normal vector at the point indicated. Write an equation for the normal line and an equation for the plane tangent to the surface $zx^2 - xy^2 - yz - 18 = 0$ at the point $(0, -2, 3)$.
75. Find a normal vector at the point indicated. Write an equation for the normal line and an equation for the plane tangent to the surface $xyz + x + y + z - 3 = 0$ at the point $(1, -2, 2)$.
76. Find a normal vector at the point indicated. Write an equation for the normal line and an equation for the plane tangent to the surface $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{36} = 1$ at the point $(2, 3, 6)$.
77. The surfaces $2x^2 - 2y^2 + z^3 = 4$ and $3x^2 - 2y^2 + 3z = 1$ intersect in a curve that passes through the point $(1, 2, 2)$. What are the equations of the respective tangent planes to the two surfaces at this point?
78. Find a point on the surface $z = 16 - 4x^2 - y^2$ at which the tangent plane is perpendicular to the line $x = 3 + 4t$, $y = 2t$, $z = 2 - t$.
79. Find a point on the surface $z = 9 - x^2 - y^2$ at which the tangent plane is parallel to the plane $2x + 3y + 2z = 6$.

15.5 Maximum and Minimum Values

80. Find the stationary points and determine the local extreme values for the function $f(x, y) = 2x^2 - 3y + y^2$.
81. Find the stationary points and determine the local extreme values for the function $f(x, y) = x^2 - 2x - y^3$.
82. Find the stationary points and determine the local extreme values for the function $f(x, y) = x^2 + xy + y^2 - 2x - 2y + 6$.
83. Find the stationary points and determine the local extreme values for the function $f(x, y) = x^2 - xy + y^2 - 2x - 2$.
84. Find the stationary points and determine the local extreme values for the function $f(x, y) = x^3 + 3x - 2y$.
85. Find the stationary points and determine the local extreme values for the function $f(x, y) = 3xy - 5x^2 - y^2 + 5x - 2y$.
86. Find the stationary points and determine the local extreme values for the function $f(x, y) = x^2 - xy + y^2 - 5x + 2$.
87. Find the stationary points and determine the local extreme values for the function $f(x, y) = 2x^2 + y^2 + 4y$.
88. Find the stationary points and determine the local extreme values for the function $f(x, y) = x^2 - xy + y^2 + 5x + 1$.
89. Find the stationary points and determine the local extreme values for the function $f(x, y) = y \sin 2x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$.
90. Find the absolute extreme values taken on by $f(x, y) = \sqrt{x^2 + y^2}$ on the set $D = \{(x, y): 2 \leq x \leq 4, 1 \leq y \leq 4\}$.
91. Find the absolute extreme values taken on by $f(x, y) = 2x^2 - 3y^2$ on the set $D = \{(x, y): -2 \leq x \leq 2\}$.
92. Find the absolute extreme values taken on by $f(x, y) = (x - 2)^2 + (y - 1)^2$ on the set $D = \{(x, y): x^2 + y^2 \leq 1\}$.

93. Find the absolute extreme values taken on by $f(x, y) = (x - 2)^2 + y^2$ on the set $D = \{(x, y): 0 \leq x \leq 1, x^2 \leq y \leq 4x\}$.
94. Find the absolute extreme values taken on by $f(x, y) = (x - y)^2$ on the set $D = \{(x, y): 0 \leq x \leq 2, 0 \leq y \leq 4 - x\}$.
95. Find the distance from the point $(1, 1, 1)$ to the sphere $x^2 + y^2 + z^2 = 4$.
96. Find the distance from the point $(\frac{1}{2}, 1, \frac{1}{2})$ to the sphere $(x - 1)^2 + y^2 + z^2 = 2$.
97. Find the stationary points and the local extreme values for $f(x, y) = 5xy - 7x^2 - y^2 + 3x - 6y + 2$.
98. Find the stationary points and the local extreme values for $f(x, y) = x^3 + y^2 - 12x - 6y + 7$.
99. Find the stationary points and the local extreme values for $f(x, y) = x^2 + 3xy + 3y^2 - 6x + 3y - 6$.
100. Find the stationary points and the local extreme values for $f(x, y) = x^2 - xy + y^2 + 2x + 2y - 4$.
101. Find the stationary points and the local extreme values for $f(x, y) = x^2 - 2y^2 - 6x + 8y + 3$.
102. Find the stationary points and the local extreme values for $f(x, y) = x^2 + 3xy + y^2 - 10x - 10y$.
103. Find the stationary points and the local extreme values for $f(x, y) = 2x^2 + y^2 - 4x - 6y$.
104. Find the stationary points and the local extreme values for $f(x, y) = x^3 - 9xy + y^3$.
105. Find the stationary points and the local extreme values for $f(x, y) = x^2 + \frac{1}{3}y^3 - 2xy - 3y$.
106. A rectangular box, open at the top, is to contain 256 cubic inches. Find the dimensions of the box for which the surface area is a minimum.
107. Find the point on the plane $2x - 3y + z = 19$ that is closest to $(1, 1, 0)$.
108. Find the shortest distance to $2x + y - z = 5$ from $(1, 1, 1)$.
109. An open rectangular box containing 18 cubic inches is to be constructed so that the base material costs 3 cents per square inch, the front face costs 2 cents per square inch, and the sides and back each cost 1 cent per square inch. Find the dimensions of the box for which the cost of construction will be a minimum.
110. Find the points on $z^2 = x^2 + y^2$ that are closest to $(2, 2, 0)$.
111. Find the point on $x + 2y + z = 1$ that is closest to the origin.
112. Find the maximum product of x , y , and z where x , y , and z are positive numbers such that $4x + 3y + z = 108$.
113. Find the maximum sum of $9x + 5y + 3z$ if x , y , and z are positive numbers such that $xyz = 25$.
114. Find the maximum product of x^2yz if x , y , and z are positive numbers such that $3x + 2y + z = 24$.

15.6 Maxima and Minima with Side Conditions

115. Maximize xy on the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

116. Minimize xy on the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.
117. Maximize x^2y on the circle $x^2 + y^2 = 4$.
118. Minimize x^2y on the circle $x^2 + y^2 = 4$.
119. Maximize xyz^2 on the sphere $x^2 + y^2 + z^2 = 4$.
120. Use the Lagrange multiplier method to find the point on the surface $z = xy + 1$ that is closest to the origin.
121. Use the Lagrange multiplier method to find the point on the plane $x + 2y + z = 1$ that is closest to the point $(1, 1, 0)$.
122. Use the Lagrange multiplier method to find three positive numbers whose sum is 12 for which x^2yz is a maximum.
123. Use the Lagrange multiplier method to find the maximum for $x^2 + y^2 + z^2$ if $x + 2y + 2z = 12$.
124. An open rectangular box is to contain 256 cubic inches. Use the Lagrange multiplier method to find the dimensions of the box that uses the least amount of material.
125. An open rectangular box containing 18 cubic inches is constructed of material costing 3 cents per square inch for the base, 2 cents per square inch for the front face, and 1 cent per square inch for the sides and back. Use the Lagrange multiplier method to find the dimensions of the box for which the cost of construction is a minimum.
126. Use the Lagrange multiplier method to find the maximum possible volume for a rectangular box inscribed in the ellipsoid $2x^2 + 3y^2 + 4z^2 = 12$.
127. Use the Lagrange multiplier method to find the maximum possible volume for a rectangular box inscribed in the ellipsoid $2x^2 + 3y^2 + 6z^2 = 18$.
128. Use the Lagrange multiplier method to find the point on the plane $2x - 3y + z = 19$ that is closest to $(1, 1, 0)$.
129. Use the Lagrange multiplier method to find the shortest distance from $(1, 1, 1)$ to $2x + y - z = 5$.
130. Use the Lagrange multiplier method to find the points on $z^2 = x^2 + y^2$ that are closest to $(2, 2, 0)$.
131. Use the Lagrange multiplier method to find three positive numbers whose sum is 36 and whose product is as large as possible.
132. Use the Lagrange multiplier method to find three positive numbers whose product is 64 and whose sum is as small as possible.
133. Use the Lagrange multiplier method to find three positive numbers x , y , and z whose product is as large as possible given that $2x + 2y + z = 84$.
134. The base of a rectangular box costs three times as much per square foot as do the sides and top. Use the Lagrange multiplier method to find the dimensions of the box with least cost if the box is to contain 54 cubic feet.

15.7 Differentials

135. Find the differential df given that $f(x, y, z) = e^{2z} \sqrt[3]{x^2 + y^2}$.
136. Find the differential df given that $f(x, y, z) = x^2 + 3xy - 2y^2 + 3xz + z^2$.

137. Find the differential df given that $f(x, y, z) = z^4 - 3yz^2 + y \sin z$.
138. Find the differential df given that $f(x, y, z) = 3x^2 + y^2 + z^2 - 3xy + 4xz - 15$.
139. Find the differential df given that $f(x, y, z) = x \sin^{-1} y + x^2 y$.
140. Compute Δu and du for $u = 2x^2 + 3xy - y^2$ at $x = 2$, $y = -2$, $\Delta x = -0.2$, $\Delta y = 0.1$.
141. The radius and height of a right-circular cylinder are measured with errors of at most 0.1 inches. If the height and radius are measured to be 10 inches and 2 inches, respectively, use differentials to approximate the maximum possible error in the calculated value of the volume.
142. The power consumed in an electrical resistor is given by $P = E^2 / R$ watts. Suppose $E = 200$ volts and $R = 8$ ohms, approximate the change in power if E is decreased by 5 volts and R is decreased by 0.20 ohm.
143. Let $f(x, y) = \sqrt{x + 2y}$. Use a total differential to approximate the change in $f(x, y)$ as (x, y) varies from $(3, 5)$ to $(2.98, 5.1)$.
144. The legs of a right triangle are measured to be 6 and 8 inches with a maximum error of 0.10 inches in each measurement. Use differentials to estimate the maximum possible error in the calculated value of the hypotenuse and the area of the triangle.
145. Find the differential df given that $f(x, y) = \ln \sqrt[3]{1 + xy}$.

15.8 Restructuring a Function from its Gradient

146. Determine whether the vector function $x^2 y^3 \mathbf{i} + x^3 y^2 \mathbf{j}$ is the gradient $\nabla f(x, y)$ of a function everywhere defined. If so, find such a function.
147. Determine whether the vector function $e^{2x} \mathbf{i} - e^{-2y} \mathbf{j}$ is the gradient $\nabla f(x, y)$ of a function everywhere defined. If so, find such a function.
148. Determine whether the vector function $\frac{x}{y^2} \mathbf{i} - \left(\frac{x^2}{y^3} + y^2 \right) \mathbf{j}$ is the gradient $\nabla f(x, y)$ of a function everywhere defined. If so, find such a function.
149. Determine whether the vector function $\sin(x + y - 2z) \mathbf{i} - \sin(x + y - 2z) \mathbf{j} + 2 \cos(x + y - 2z) \mathbf{k}$ is the gradient $\nabla f(x, y, z)$ of a function everywhere defined. If so, find such a function.
150. Determine whether the vector function $(3x^2 y e^{z^2} - z) \mathbf{i} - (x^3 e^{z^2} + 1) \mathbf{j} + (2x^3 y z e^{z^2} - x) \mathbf{k}$ is the gradient $\nabla f(x, y, z)$ of a function everywhere defined. If so, find such a function.
151. Determine whether the vector function $\left(\frac{2x}{x^2 + y^2 + 1} + \frac{yz}{1 + x^2 y^2 z^2} \right) \mathbf{i} + \left(\frac{2y}{x^2 + y^2 + 1} + \frac{xz}{1 + x^2 y^2 z^2} \right) \mathbf{j} + \frac{xy}{1 + x^2 y^2 z^2} \mathbf{k}$ is the gradient $\nabla f(x, y, z)$ of a function everywhere defined. If so, find such a function.

15.9 Exact Differential Equations

152. Verify that the equation $2xy + (1 + x^2)y' = 0$ is exact, then solve it.
153. Verify that the equation $ye^{xy} + xe^{xy}y' = 0$ is exact, then solve it.

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154. Verify that the equation $(\cos y + y \cos x) + (\sin x - x \sin y) y' = 0$ is exact, then solve it.
155. Verify that the equation $e^{x^3} (3x^3 y - x^2) + e^{x^3} y' = 0$ is exact, then solve it.
156. Verify that the equation $(3 + 2x \ln xy + x) + \frac{x^2}{y} y' = 0$ is exact, then solve it.
157. Solve $(y + x^4) - xy' = 0$.
158. Solve $(x^2 + y^2 + x) + xy y' = 0$.
159. Solve $(2xy^4 e^y + 2xy^3 + y) + (x^2 y^4 e^y - x^2 y^2 - 3x) y' = 0$.
160. Solve $(y + \ln x) - xy' = 0$.
161. Find the integral curve of $(x + \sin y) + x(\cos y - 2y) y' = 0$ that passes through $(2, p)$.
162. Find the integral curve of $(y^2 - y) + xy' = 0$ that passes through $(-1, 2)$.
163. Find the integral curve of $(2y - 3x) + xy' = 0$ that passes through $(-5, 3)$.

Answers to Chapter 15 Questions

1. $(2x + x^2y)e^{xy} \mathbf{i} + x^3e^{xy} \mathbf{j}$
2. $(6x^2 + 5y) \mathbf{i} + (5x - 4y) \mathbf{j}$
3. $[y \sin(xy^2) + xy^3 \cos(xy^2)] \mathbf{i}$
 $+ [x \sin(xy^2) + 2x^2y^2 \cos(xy^2)] \mathbf{j}$
4. $4xyz \mathbf{i} + 2x^2z \mathbf{j} + 2x^2y \mathbf{k}$
5. $\frac{-1}{2(x+y+z)^{3/2}} (\mathbf{i} + \mathbf{j} + \mathbf{k})$
6. $(3x^2y - y^2z) \mathbf{i} + (x^3 + z^2 - 2xyz) \mathbf{j} + (2yz - xy^2) \mathbf{k}$
7. $\frac{e^y}{x} \mathbf{i} + e^y \ln xz \mathbf{j} + \frac{e^y}{z} \mathbf{k}$
8. $[2x \cos(x+y) - (x^2 - y) \sin(x+y)] \mathbf{i}$
 $- [\cos(x+y) - (x^2 - y) \sin(x+y)] \mathbf{j}$
9. $e^{x+y+z} (\mathbf{i} + \mathbf{j} + \mathbf{k})$
10. $\frac{x}{\sqrt{x^2 + y^2}} \mathbf{i} + \frac{y}{\sqrt{x^2 + y^2}} \mathbf{j}$
11. $2 \mathbf{i} + 14 \mathbf{j}$
12. $\frac{2}{25} \mathbf{i} - \frac{11}{25} \mathbf{j}$
13. $\mathbf{i} + \mathbf{j}$
14. $e \sin 3\mathbf{i} + e \cos 3\mathbf{j} + 4e \cos 3\mathbf{k}$
15. $\frac{1}{6} \mathbf{i} + \frac{1}{6} \mathbf{j} + \frac{2}{3} \mathbf{k}$
16. (a) $\frac{3}{r^2} (x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$
 (b) $\frac{2}{r} \cos 2r (x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$
 (c) $e^{2r^{2+r}} \left(4 + \frac{1}{r} \right) (x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$
17. $\frac{5\sqrt{3}-2}{2\sqrt{29}}$
18. $\frac{16}{125}$
19. $\frac{7}{6\sqrt{2}}$
20. $\frac{-2e^2}{\sqrt{13}}$
21. $-4/5$
22. $\frac{3\sqrt{3}+2p}{2\sqrt{13}}$
23. $\frac{p}{2\sqrt{5}} - \frac{3}{2\sqrt{5}}$
24. $\sqrt{3}$
25. $\frac{3+3\sqrt{3}}{2}$
26. $\frac{\sqrt{3}-6}{2}$
27. $\frac{12p}{11}$
28. $\frac{-\sqrt{6}}{3}$
29. $\frac{60}{\sqrt{26}} + \frac{1}{2\sqrt{78}}$
30. $\frac{8\sqrt{5}}{5}$
31. (a) 271°C
 (b) $6\sqrt{5}$
 (c) $\mathbf{u} = \frac{4}{5} \mathbf{i} - \frac{3}{5} \mathbf{j}$; 5
32. $\mathbf{u} = \mathbf{j}$; 1
33. $\mathbf{u} = \frac{-\sqrt{2}}{2} \mathbf{i} - \frac{\sqrt{2}}{2} \mathbf{k}$; 4
34. $\mathbf{u} = \frac{1}{\sqrt{3}} (\mathbf{i} - \mathbf{j} - \mathbf{k})$; $\sqrt{3}$
35. $e'(t^4 + 4t^3)$
36. $2t + 4t^3 + 6t^5$

$$37. \quad 6\mathbf{w} \cos 2\mathbf{w}t - 4\mathbf{w}^2 t - 6\mathbf{w}^2 t \cos \mathbf{w}t \\ - 6\mathbf{w} \sin \mathbf{w}t - 4\mathbf{w} \cos \mathbf{w}t$$

$$38. \quad 2t \cos(t^2 - \sin t) + (t^2 \cos t - 2t^3) \sin(t^2 - \sin t)$$

$$39. \quad \frac{e^{2t} + \cos t + \sin t}{\sqrt{e^{2t} + \sin^2 t}}$$

$$40. \quad 6t^5 e^{\cos t} - t^6 e^{\cos t} \sin t$$

$$41. \quad e^x \cos x + xe^x \cos x - xe^x \sin x$$

$$42. \quad y^2 \cos y + 2y \sin y$$

$$43. \quad 360$$

$$44. \quad \frac{1}{1+t^2}$$

$$45. \quad t^2 e^t \cos t^2 e^t + 2te^t \cos t^2 e^t + 3t^2$$

$$46. \quad \frac{(5\sqrt{3}-3)\mathbf{p}}{60}$$

$$47. \quad \frac{\partial z}{\partial s} = e^t \sin(se^{-t}) + s \cos(se^{-t}) \\ \frac{\partial z}{\partial t} = se^t \sin(se^{-t}) + s^2 \cos(se^t)$$

$$48. \quad \frac{\partial z}{\partial u} = \frac{4u(u^2 + v^3) \tan \ln(u^2 + v^2)}{u^2 + v^2} \\ + \frac{2u(u^2 + v^3) \sec^2 \ln(u^2 + v^2)}{u^2 + v^2} \\ \frac{\partial z}{\partial v} = \frac{6v^2(u^2 + v^3) \tan \ln(u^2 + v^2)}{u^2 + v^2} \\ + \frac{2v(u^2 + v^3) \sec^2 \ln(u^2 + v^2)}{u^2 + v^2}$$

$$49. \quad \frac{\partial z}{\partial r} = 0; \frac{\partial z}{\partial q} = \cos 2q$$

$$50. \quad \frac{\partial z}{\partial v} = 2uv \cos(u+v) + 2uv(u+v) \cos uv^2 \\ - uv^2 \sin(u+v) + \sin uv^2$$

$$\frac{\partial z}{\partial u} = v^2 \cos(u+v) + v^2(u+v) \cos uv^2 \\ - uv^2 \sin(u+v) + \sin uv^2$$

$$51. \quad \frac{\partial z}{\partial s} = 2(s+t) + 3(s-t)^2; \frac{\partial z}{\partial t} = 2(s+t) - 3(s-t)^2$$

$$52. \quad \frac{\partial z}{\partial u} = 6(2u+v)^2 + 6u - 7v$$

$$\frac{\partial z}{\partial v} = 3(2u+v)^2 - 7u + 4v$$

$$53. \quad 2y \cdot 3x^2 f'(x^3 - y^2) + 3x^2(-2y) f'(x^3 - y^2) = 0$$

$$54. \quad x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \frac{-y}{x} f\left(\frac{y}{x}\right) + \frac{y}{x} f\left(\frac{y}{x}\right) = 0$$

$$55. \quad \frac{\partial \mathbf{w}}{\partial r} = \frac{2}{r+s}; \frac{\partial \mathbf{w}}{\partial s} = \frac{2}{r+s}$$

$$56. \quad \frac{\partial \mathbf{w}}{\partial r} = r \sin 2q + z \sin q \\ \frac{\partial \mathbf{w}}{\partial q} = r^2 \cos 2q + rz \cos q \\ \frac{\partial \mathbf{w}}{\partial z} = r \sin q$$

$$57. \quad \frac{\partial \mathbf{w}}{\partial r} = \frac{2e^{2r}}{e^{2r} + e^{2s}}, \frac{\partial \mathbf{w}}{\partial s} = \frac{2e^{2s}}{e^{2r} + e^{2s}}$$

$$58. \quad 4e^{2t}$$

$$59. \quad \frac{\partial \mathbf{w}}{\partial t} = 2t, \frac{\partial \mathbf{w}}{\partial u} = 6u, \frac{\partial \mathbf{w}}{\partial v} = 0$$

$$60. \quad \frac{\partial \mathbf{w}}{\partial r} = 2u \cos v - 3 \sin u \sin u \cos v \\ \frac{\partial \mathbf{w}}{\partial u} = 2 \sin v - 3v \cos u + \cos u \sin v$$

$$61. \quad \frac{\partial \mathbf{w}}{\partial r} = \sec s; \frac{\partial \mathbf{w}}{\partial s} = \tan s \sec s$$

$$62. \quad \frac{\partial \mathbf{w}}{\partial r} = 2r + 2rs^2; \frac{\partial \mathbf{w}}{\partial s} = 2r^2 s$$

$$63. \quad \frac{\partial z}{\partial x} = \frac{4e}{2-e}; \frac{\partial z}{\partial y} = \frac{2e}{2-e}$$

$$64. \quad \frac{\partial z}{\partial u} = 7u^6 v^3 + 2u \sin uv + u^2 v \cos uv \\ \frac{\partial z}{\partial v} = 3u^7 v^2 + u^3 \cos uv$$

$$65. \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \\ = \frac{2x^2 - 2x^2 + 2y^2 - 2y^2 + 2z^2 - 2z^2}{(x^2 + y^2 + z^2)^{3/2}} = 0$$

66. $f_x^2(\cos^2 \theta + \sin^2 \theta) + f_y^2(\cos^2 \theta + \sin^2 \theta) = \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$
67. $8x - 18y - z + 30 = 0$
68. $pe^2y + z - pe^2 = 0$
69. $4x - 5y + z + 4 = 0$
70. $4x - 2y - z - 5 = 0$
71. $x - 2y - z + 2p = 0$
72. $12x - 4y - z - 14 = 0$
73. normal vector: $\mathbf{i} + \mathbf{k}$
normal line: $x = t, z = t$
tangent plane: $x + z = 0$
74. normal vector: $4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$
normal line: $x = 4t, y = -2 + 3t, z = 3 - 2t$
tangent plane: $4x + 3y - 2z + 12 = 0$
75. normal vector: $3\mathbf{i} - 3\mathbf{j} + \mathbf{k}$
normal line: $x = 1 + 3t, y = -2 - 3t, z = 2 + t$
tangent plane: $3x + 3y - z - 11 = 0$
76. normal vector: $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$
normal line: $x = 2 + 3t, y = 3 + 2t, z = 6 - t$
tangent plane: $3x + 2y - z - 6 = 0$
77. $4x - 3y + 6z - 10 = 0$
 $6x - 8y + 3z + 4 = 0$
78. $(-1/2, -1, 14)$
79. $(1/2, 3/4, 131/16)$
80. stationary point: $(0, 3/2, -9/4)$ local minimum
81. stationary point: $(1, 0, -1)$ local minimum
82. stationary point: $(2/3, 2/3, 14/3)$ local minimum
83. stationary point: $(4/3, 2/3, -10/3)$ local minimum
84. No stationary points
85. stationary point: $(4/11, -5/11, 15/11)$
local maximum
86. stationary point: $(10/3, -5/3, -44/3)$
local minimum
87. stationary point: $(1, -2, -4)$ local minimum
88. stationary point: $(-10/3, -5/3, -22/3)$ local minimum
89. stationary points: $(-p/2, 0), (-p/4, 0), (0, 0), (p/4, 0), (p/2, 0)$
no local extrema
90. absolute minimum at $(0, 0, 0)$
absolute maximum at $(4, 4, 4\sqrt{2})$
91. absolute maximum at $(-2, 0, 8)$ and $(2, 0, 8)$
no absolute minimum
92. absolute maximum at $(\frac{-2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}, 6 + 2\sqrt{5})$
absolute minimum at $(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 6 - 2\sqrt{5})$
93. absolute maximum at $(1, 4, 17)$
absolute minimum at $(0.83512, 0.6974, 1.8433)$
94. absolute maximum at $(0, 4, 16)$
absolute minimum at $(x, x, 0), 0 \leq x \leq 2$
95. $x = y = z = \frac{2\sqrt{3}}{3}$, distance: $2 - \sqrt{3}$
96. $\frac{2\sqrt{2} - \sqrt{6}}{2}$
97. $f(-8, -23) = 59$, relative maximum
98. $f(2, -3) = -32$, relative minimum;
 $(-2, -3)$ is a saddle point
99. $f(15, -8) = -63$, relative minimum
100. $f(-2, -2) = -8$, relative minimum
101. $(3, 2)$ is a saddle point
102. $(2, 2)$ is a saddle point
103. $f(1, 3) = -11$, relative minimum
104. $f(3, 3) = -27$, relative minimum;
 $(0, 0)$ is a saddle point
105. $f(3, 3) = -9$, relative minimum;
 $(-1, -1)$ is a saddle point
106. 8 in. by 8 in. by 4 in.
107. $(27/7, -23/7, 10/7)$

108. $\frac{\sqrt{6}}{2}$
109. 2 in. by 3 in. by 3 in.
110. $(1, 1, \sqrt{2})$ and $(1, 1, -\sqrt{2})$
111. $(1/6, 1/3, 1/6)$
112. $x = 9, y = 12, z = 36$; product = 3888
113. $x = 5/3, y = 3, z = 5$; sum = 45
114. $x = 4, y = 3, z = 6$; product = 288
115. $(x, y) = \left(-\sqrt{2}, \frac{-3}{\sqrt{2}}\right)$ or $\left(-\sqrt{2}, \frac{-3}{\sqrt{2}}\right)$; $xy = 3$
116. $(x, y) = \left(\sqrt{2}, \frac{-3}{\sqrt{2}}\right)$ or $\left(-\sqrt{2}, \frac{-3}{\sqrt{2}}\right)$; $xy = -3$
117. $(x, y) = \left(\frac{-2\sqrt{2}}{\sqrt{3}}, \frac{-2}{\sqrt{3}}\right)$ or $\left(\frac{2\sqrt{2}}{\sqrt{3}}, \frac{-2}{\sqrt{3}}\right)$;
 $x^2y = \frac{16\sqrt{3}}{9}$
118. $(x, y) = \left(\frac{-2\sqrt{2}}{\sqrt{3}}, \frac{-2}{\sqrt{3}}\right)$ or $\left(\frac{2\sqrt{2}}{\sqrt{3}}, \frac{-2}{\sqrt{3}}\right)$;
 $x^2y = \frac{-16\sqrt{3}}{9}$
119. $(x, y, z) = (-1, -1, \pm\sqrt{2}), (1, 1, \pm\sqrt{2})$; $xyz^2 = 2$
120. $(0, 0, 1)$
121. $(2/3, 1/3, -1/3)$
122. $x = 6, y = 3, z = 3$; product = 324
123. $x = 4/3, y = 8/3, z = 8/3$; sum = 16
124. 8 in. by 8 in. by 4 in.
125. 2 in. by 3 in. by 3 in.
126. $x = \sqrt{2}, y = \frac{2}{\sqrt{3}}, z = 1$; volume = $\frac{16\sqrt{6}}{3}$
127. $x = \sqrt{2}, y = \sqrt{3}, z = 1$; volume = $8\sqrt{6}$
128. $[27/7, -23/7, 10/7]$
129. $\frac{\sqrt{6}}{2}$
130. $(1, 1, \sqrt{2})$ and $(1, 1, -\sqrt{2})$
131. $x = y = z = 12$; product = 1728
132. $x = y = z = 4$; sum = 12
133. $x = 14, y = 14, z = 28$; product = 5488
134. 3 ft. by 3 ft. by 6 ft.
135. $\frac{2xe^{2z}}{3(x^2 + y^2)^{2/3}}dx + \frac{2ye^{2z}}{3(x^2 + y^2)^{2/3}}dy$
 $+ 2e^{2z}(x^2 + y^2)^{1/3}dz$
136. $(2x + 3y + 3z)dx + (3x - 4y)dy + (3x + 2z)dz$
137. $0dx + (\sin z - 3z^2)dy + (4z^3 - 6yz + y \cos z)dz$
138. $(6x - 3y + 4z)dx + (2y - 3x)dy + (2z + 4x)dz$
139. $(\sin^{-1} y + 2xy)dx + \left(\frac{x}{\sqrt{1-y^2}} + x^2\right)dy$
140. $\Delta u = 0.61$; $du = 0.60$
141. $4.4p$
142. decreased by 125 watts
143. ≈ 0.025
144. error in hypotenuse = 0.14; error in area = 0.7
145. $\frac{y}{3(1+xy)}dx + \frac{y}{3(1+xy)}dy$
146. $f(x, y) = \frac{x^3}{3}y^3 + C$
147. $f(x, y) = \frac{1}{2}e^{2x} + \frac{1}{2}e^{2y} + C$
148. It is the gradient of $f(x, y) = \frac{x^2}{2y^2} - \frac{y^3}{3} + C$,
but f is only defined for $y \neq 0$.
149. not a gradient
150. $f(x, y, z) = x^3ye^{z^2} - xz + y + C$

151. $f(x, y, z) = \ln(x^2 + y^2 + 1) + \tan^{-1}(xyz) + C$

152. $y = \frac{C}{x^2 + 1}$

153. $e^{xy} = C$

154. $x \cos y + y \sin x = C$

155. $y = Ce^{-x^3} + \frac{1}{3}$

156. $3x + x^2 \ln xy = C$

157. $y = \frac{1}{3}x^4 - Cx$

158. $3x^4 + 4x^3 + 6x^2y^2 = C$

159. $x^2e^y + \frac{x^2}{y} + \frac{x}{y^3} = C$

160. $y = Cx - \ln x - 1$

161. $\frac{1}{2}x^2 + x \sin y - y^2 = 2 - p^2$

162. $y = \frac{x}{x + 1/2}$

163. $y = \frac{x^3 + 200}{x^2}$

CHAPTER 16

Double and Triple Integrals

16.1 Multiple-Sigma Notation

- Let $P_1 = \{x_0, x_1, \dots, x_m\}$ be a partition of $[a_1, a_2]$
 Let $P_2 = \{y_0, y_1, \dots, y_n\}$ be a partition of $[b_1, b_2]$
 Let $P_3 = \{z_0, z_1, \dots, z_q\}$ be a partition of $[c_1, c_2]$
 Take $\Delta x_i = x_i - x_{i-1}$, $\Delta y_j = y_j - y_{j-1}$, $\Delta z_k = z_k - z_{k-1}$

Evaluate the sum, $\sum_{i=1}^m \sum_{k=1}^q \Delta x_i \Delta z_k$

- Let $P_1 = \{x_0, x_1, \dots, x_m\}$ be a partition of $[a_1, a_2]$
 Let $P_2 = \{y_0, y_1, \dots, y_n\}$ be a partition of $[b_1, b_2]$
 Let $P_3 = \{z_0, z_1, \dots, z_q\}$ be a partition of $[c_1, c_2]$
 Take $\Delta x_i = x_i - x_{i-1}$, $\Delta y_j = y_j - y_{j-1}$, $\Delta z_k = z_k - z_{k-1}$

Evaluate the sum, $\sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^q \Delta x_i \Delta y_j \Delta z_k$

- Let $P_1 = \{x_0, x_1, \dots, x_m\}$ be a partition of $[a_1, a_2]$
 Let $P_2 = \{y_0, y_1, \dots, y_n\}$ be a partition of $[b_1, b_2]$
 Let $P_3 = \{z_0, z_1, \dots, z_q\}$ be a partition of $[c_1, c_2]$
 Take $\Delta x_i = x_i - x_{i-1}$, $\Delta y_j = y_j - y_{j-1}$, $\Delta z_k = z_k - z_{k-1}$

Evaluate the sum, $\sum_{j=1}^n \sum_{k=1}^q (z_k + z_{k-1}) \Delta y_j \Delta z_k$

- Let $P_1 = \{x_0, x_1, \dots, x_m\}$ be a partition of $[a_1, a_2]$
 Let $P_2 = \{y_0, y_1, \dots, y_n\}$ be a partition of $[b_1, b_2]$
 Let $P_3 = \{z_0, z_1, \dots, z_q\}$ be a partition of $[c_1, c_2]$
 Take $\Delta x_i = x_i - x_{i-1}$, $\Delta y_j = y_j - y_{j-1}$, $\Delta z_k = z_k - z_{k-1}$

Evaluate the sum, $\sum_{i=1}^m \sum_{j=1}^n (4\Delta x_i - 3\Delta y_j)$

- Let $P_1 = \{x_0, x_1, \dots, x_m\}$ be a partition of $[a_1, a_2]$
 Let $P_2 = \{y_0, y_1, \dots, y_n\}$ be a partition of $[b_1, b_2]$
 Let $P_3 = \{z_0, z_1, \dots, z_q\}$ be a partition of $[c_1, c_2]$
 Take $\Delta x_i = x_i - x_{i-1}$, $\Delta y_j = y_j - y_{j-1}$, $\Delta z_k = z_k - z_{k-1}$

Evaluate the sum, $\sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^q (y_j + y_{j-1}) \Delta x_i \Delta y_j \Delta z_k$

16.2 The Double Integral

- Take $f(x, y) = 3x - 2y$ on R : $0 \leq x \leq 1$, $0 \leq y \leq 2$, and P as the partition $P = P_1 \times P_2$. Find $L_f(P)$ and $U_f(P)$ if $P_1 = \{0, 1/4, 1/2, 3/4, 1\}$, $P_2 = \{0, 1/2, 1, 3/2, 2\}$.
- Take $f(x, y) = 3x - 2y$ on R : $0 \leq x \leq 1$, $0 \leq y \leq 2$, and P as the partition $P = P_1 \times P_2$. $P_1 = \{x_0, x_1, \dots, x_m\}$ is an arbitrary partition of $[0, 1]$, and $P_2 = \{y_0, y_1, \dots, y_n\}$ is an arbitrary partition of $[0, 2]$.

(a) Find $L_f(P)$ and $U_f(P)$

(b) Evaluate the double integral $\iint_R (3x - 2y) dx dy$ using part (a).

8. Take $f(x, y) = 2x(y - 1)$ on $R: 0 \leq x \leq 2, 0 \leq y \leq 1$, and P as the partition $P = P_1 \times P_2$. Find $L_f(P)$ and $U_f(P)$ if $P_1 = \{0, 1, 3/2, 2\}$ $P_2 = \{0, 1/2, 1\}$.
9. Take $f(x, y) = 2x(y - 1)$ on $R: 0 \leq x \leq 2, 0 \leq y \leq 1$, and P as the partition $P = P_1 \times P_2$. $P_1 = \{x_0, x_1, \dots, x_m\}$ is an arbitrary partition of $[0, 2]$, and $P_2 = \{y_0, y_1, \dots, y_n\}$ is an arbitrary partition of $[0, 1]$.
 (a) Find $L_f(P)$ and $U_f(P)$
 (b) Evaluate the double integral $\iint_R 2x(y-1)dx dy$ using part (a).
10. Take $f(x, y) = 2x^2 - 3y^2$ on $R: 0 \leq x \leq 1, 0 \leq y \leq 1$, and P as the partition $P = P_1 \times P_2$. Find $L_f(P)$ and $U_f(P)$ if $P_1 = \{0, 1/2, 3/4, 1\}$ $P_2 = \{0, 1/2, 1\}$.
11. Take $f(x, y) = 2x^2 - 3y^2$ on $R: 0 \leq x \leq 1, 0 \leq y \leq 1$, and P as the partition $P = P_1 \times P_2$. $P_1 = \{x_0, x_1, \dots, x_m\}$ is an arbitrary partition of $[0, 2]$, and $P_2 = \{y_0, y_1, \dots, y_n\}$ is an arbitrary partition of $[0, 1]$.
 (a) Find $L_f(P)$ and $U_f(P)$
 (b) Evaluate the double integral $\iint_R (2x^2 - 3y^2)dx dy$ using part (a).

16.3 The Evaluation of a Double Integral by Repeated Integrals

12. Evaluate $\iint_{\Omega} (x^2 - y^2) dx dy$ taking $\Omega: 0 \leq x \leq 1, 0 \leq y \leq 1$.
13. Evaluate $\iint_{\Omega} y \cos xy dx dy$ taking $\Omega: 0 \leq x \leq 1, 0 \leq y \leq p$.
14. Evaluate $\iint_{\Omega} x^2 y^2 dx dy$ taking $\Omega: -1 \leq x \leq 1, 0 \leq y \leq p/2$.
15. Evaluate $\iint_{\Omega} (3 - y)x^2 dx dy$ taking $\Omega: 2 \leq x \leq 4, 0 \leq y \leq 3$.
16. Evaluate $\iint_{\Omega} (x - 1) dy dx$ taking $\Omega: 0 \leq x \leq 1, 0 \leq y \leq 2$.
17. Evaluate $\iint_{\Omega} e^{x+y} dy dx$ taking $\Omega: 0 \leq x \leq 1, 0 \leq y \leq 1$.
18. Evaluate $\iint_{\Omega} x\sqrt{x^2 + y} dx dy$ taking $\Omega: 0 \leq x \leq 1, 0 \leq y \leq 3$.
19. Evaluate $\iint_{\Omega} (x^2 - y) dx dy$ taking $\Omega: 0 \leq x \leq 3, 1 \leq y \leq 4$.
20. Evaluate $\iint_{\Omega} \frac{1}{\sqrt{1-x^2}} dy dx$ taking $\Omega: 0 \leq x \leq 1/2, 0 \leq y \leq 2$.
21. Evaluate $\iint_{\Omega} \frac{1}{1+y^2} dy dx$ taking $\Omega: 0 \leq x \leq 4, 0 \leq y \leq 1$.
22. Evaluate $\iint_{\Omega} \frac{1}{\sqrt{4-x^2}} dx dy$ taking $\Omega: 0 \leq x \leq 1, 0 \leq y \leq 2$.

23. Calculate the average value of $f(x, y) = x \cos y$ over the region Ω : $0 \leq x \leq 1, 0 \leq y \leq \pi/4$.
24. Calculate the average value of $f(x, y) = e^x$ over the region Ω : $0 \leq x \leq \ln y, 0 \leq y \leq e$.
25. Evaluate $\iint_{\Omega} (x^2 + 2) dx dy$ taking Ω : the bounded region between $y^2 = 2x$ and $y^2 = 8 - 2x$.
26. Evaluate $\iint_{\Omega} (2xy - x^2) dx dy$ taking Ω : the bounded region between $y = x^3$ and $y = x^2$.
27. Evaluate $\int_0^1 \int_y^1 e^{x^2} dx dy$ by first sketching the region of integration Ω then changing the order of integration.
28. Evaluate $\int_0^1 \int_{x^2}^x (x^2 + y^2) dx dy$ by first sketching the region of integration Ω then changing the order of integration.
29. Evaluate $\int_0^1 \int_0^{2\sqrt{1-y^2}} x dx dy$ by first sketching the region of integration Ω then changing the order of integration.
30. Evaluate $\int_1^2 \int_0^{\sqrt{x}} y \ln x^2 dy dx$.
31. Evaluate $\int_0^1 \int_{2y}^2 \cos(x^2) dx dy$ by first expressing it as an equivalent double integral with order of integration reversed.
32. Evaluate $\int_0^1 \int_0^x y \sqrt{x^2 + y^2} dy dx$.
33. Sketch the region of integration Ω and express $\int_0^{\pi/4} \int_{\sin x}^{\cos x} f(x, y) dy dx$ as an equivalent double integral with order of integration reversed.
34. Sketch the region of integration Ω and express $\int_0^1 \int_{1-y}^{2-y} f(x, y) dx dy$ as an equivalent double integral with order of integration reversed.
35. Use a double integral to find the area bounded by $y = x^2$ and $y = \sqrt{x}$.
36. Use a double integral to find the area bounded by $x = y - y^2$ and $x + y = 0$.
37. Find the volume of the solid bounded by $y = x^2 - x, y = x, z = 0$, and $z = x + 1$.
38. Find the volume of the solid in the first octant bounded by $y = x^2/4, z = 0, y = 4, x = 0$, and $x - y + 2z = 2$.
39. Find the volume of the solid bounded by $x = 0, z = 0, z = 4 - x^2, y = 2x$, and $y = 4$.
40. Find the volume of the solid bounded by $y = x^2 - x + 1, y = x + 1, z = 0$, and $z = x + 1$.
41. Find the volume of the solid in the first octant bounded by $z = x^2 + y^2, z = 0$, and $x + y = 1$.
42. Find the volume of the solid in the first octant bounded by $z = 4 - y^2, z = 0, x = 0$, and $y = x$.
43. Find the volume of the solid in the first octant bounded by $x^2 + y^2 = 4, y = z$, and $z = 0$.

44. Find the volume bounded by $x^2 + y^2 = 1$ and $y^2 + z^2 = 1$.

16.4 Double Integrals in Polar Coordinates

45. Calculate $\int_{-2}^2 \int_0^{\sqrt{4-x^2}} e^{-(x^2+y^2)} dy dx$ by changing to polar coordinates.
46. Calculate $\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \frac{1}{\sqrt{x^2+y^2}} dx dy$ by changing to polar coordinates.
47. Calculate $\int_{-3}^3 \int_0^{\sqrt{9-x^2}} y dy dx$ by changing to polar coordinates.
48. Calculate $\int_0^2 \int_0^{\sqrt{4-x^2}} (x^2 + y^2) dy dx$ by changing to polar coordinates.
49. Integrate $f(x, y) = 2(x + y)$ over Ω , the region bounded by $x^2 + y^2 = 9$ and $x \geq 0$.
50. Find the volume in the first octant bounded by $x = 0$, $y = 0$, and $z = 0$, the plane $z + y = 3$, and the cylinder $x^2 + y^2 = 4$.
51. Use a double integral in polar coordinates to find the volume in the first octant of the solid bounded by $x^2 + y^2 = 4$, $y = z$, and $z = 0$.
52. Use a double integral in polar coordinates to find the volume of the solid bounded by $x^2 + y^2 = 5 - z$, and $z = 1$.
53. Use a double integral in polar coordinates to find the volume of the solid between the sphere $x^2 + y^2 + z^2 = 9$ and the cylinder $x^2 + y^2 = 1$.
54. Use a double integral in polar coordinates to find the volume of the solid bounded by the paraboloid $z = 4 - x^2 - y^2$ and $z = 0$.
55. Use a double integral in polar coordinates to find the volume of the solid in the first octant bounded by the ellipsoid $9x^2 + 9y^2 + 4z^2 = 36$ and the planes $x = \sqrt{3}y$, $x = 0$, and $z = 0$.
56. Use a double integral in polar coordinates to find the volume bounded by the sphere $x^2 + y^2 + z^2 = 16$ and the cylinder $(x - 2)^2 + y^2 = 4$.
57. Use a double integral in polar coordinates to find the volume bounded by $z = 0$, $x + 2y - z = -4$, and the cylinder $x^2 + y^2 = 1$.
58. Use a double integral in polar coordinates to find the volume that is inside the sphere $x^2 + y^2 + z^2 = 9$, outside the cylinder $x^2 + y^2 = 4$, and above $z = 0$.
59. Use a double integral in polar coordinates to find the area bounded by the limaçon $r = 4 + \sin \theta$.
60. Use a double integral in polar coordinates to find the area that is inside $r = 1 + \cos \theta$ and outside $r = 1$.
61. Use a double integral in polar coordinates to find the area that is inside $r = 3 \sin 3\theta$.

16.5 Some Applications of Double Integration

62. Find the center of mass of a plate of mass M bounded by $x = 0$, $x = 4$, $y = 0$, and $y = 3$ if its density is given by $\lambda(x, y) = k(x + y^2)$.
63. Find the center of mass of a plate of mass M bounded by $y^2 = 4x$, $x = 4$, and $y = 0$ if its density is given by $\lambda(x, y) = ky$.
64. Find the center of mass of a homogeneous plate of mass M bounded by $x = 0$, $x = 4$, $y = 0$, and $y = 3$ if its density is given by $\lambda(x, y) = kx^2y$.
65. Find the center of mass of a plate of mass M bounded by $y = \sin x$, $y = 0$, and $0 \leq x \leq p$ if its density is proportional to the distance from the x -axis.
66. Find the center of mass of a plate of mass M bounded by $r = a \cos q$, $0 \leq q \leq p/2$ if its density is proportional to the distance from the origin.
67. Find the mass of a homogeneous plate of mass M in the first quadrant that is inside $r = 8 \cos q$ and outside $r = 4$ if the density of the region is given by $\lambda(r, q) = \sin q$.
68. Find the mass of a homogeneous plate of mass M cut from the circle $x^2 + y^2 = 36$ by the line $x = 3$ if its density is given by $I(x, y) = \frac{x^2}{x^2 + y^2}$.
69. Find the centroid of the region bounded by $x = 4y - y^2$ and the y -axis.
70. Find the centroid of the region bounded by $y = 4 - x$, $x = 0$, and $y = 0$.
71. Find the centroid of the region bounded by $y = x^2$ and the line $y = 4$.
72. Find the centroid of the region bounded by $y = x^3$, $x = 2$, and the $y = 0$.
73. Find the centroid of the region bounded by $y = x^2 - 2x$ and $y = 0$.
74. Find the centroid of the region bounded by $x^2 = 8y$, $y = 0$, and $x = 4$.
75. Find the centroid of the region bounded by $y = \sqrt{4 - x^2}$ and $y = 0$.
76. Find the centroid of the region enclosed by the cardioid $r = 2 - 2 \cos q$.
77. Find the moments of inertia I_x , I_y , I_z of the plate of mass density $\lambda(x, y) = 6x + 6y + 6$ occupying the region Ω : $0 \leq x \leq 1$, $0 \leq y \leq 2x$.
78. Find the moments of inertia I_x , I_y , I_z of the plate of mass density $\lambda(x, y) = y + 1$ occupying the region Ω bounded by $y = x$, $y = -x$, $y = 1$.

16.6 Triple Integrals

79. Evaluate $\int_0^1 \int_0^z \int_0^{\sqrt{yz}} x \, dx \, dy \, dz$.
80. Evaluate $\iiint_T x \, dx \, dy \, dz$ where T is the solid in the first octant bounded by $x + y + z = 3$ and the coordinate planes.

16.7 Reduction to Repeated Integrals

81. Evaluate $\iiint_T yz \, dx \, dy \, dz$ where T is the solid in the first octant bounded by $y = 0$, $y = \sqrt{1 - x^2}$, and $z = x$.
82. Evaluate $\iiint_T y \, dx \, dy \, dz$ where T is the solid in the first octant bounded by $y = 1$, $y = x$, $z = x + 1$, and the coordinate planes.
83. Use a triple integral to find the volume of the solid bounded by $z = 0$, $y = 4 - x^2$, $y = 3x$, and $z = x + 4$.
84. Use a triple integral to find the volume of the solid whose base is the region in the xy -plane bounded by $y = x^2 - x + 1$ and $y = x + 1$, and whose height is given by $z = x + 1$.
85. Use a triple integral to find the volume of the solid bounded by $z = x^2 + y^2$, $y = x^2$, $z = 0$, and $y = x$.
86. Use a triple integral to find the volume of the solid bounded by $y = x^2$, $x = y^2$, $z = 0$, and $z = 3$.
87. Use a triple integral to find the volume of the solid bounded by $z = \frac{4}{y^2 + 1}$, $z = 0$, $y = x$, $y = 3$, and $x = 0$.
88. Use a triple integral to find the volume of the solid bounded by $z = 0$, $y = x^2 - x$, $y = x$, and $z = x + 1$.
89. Use a triple integral to find the volume of the solid bounded by $x^2 = 4y$, $y + z = 1$, and $z = 0$.
90. Use a triple integral to find the volume of the solid bounded by $y^2 = 4x$, $z = 0$, $z = x$, and $x = 4$.
91. Find the centroid of the tetrahedron bounded by $2x + 2y + z = 6$ and the coordinate planes.
92. Use a triple integral to find the volume of the solid in the first octant bounded by $z = y$, $y^2 = x$, and $x = 1$.
93. Use a triple integral to find the volume of the solid in the first octant bounded by the cylinder $x = 4 - y^2$, and the planes $z = y$, $x = 0$, and $z = 0$.
94. Use a triple integral to find the volume of the solid in the first octant bounded by $z = x^2 + y^2$, $y = x$, and $x = 1$.
95. Use a triple integral to find the volume of the solid in the first octant bounded by the cylinder $x = 4 - y^2$, and the planes $y = x$, $z = 0$, and $x = 0$.
96. Use a triple integral to find the volume of the tetrahedron bounded by the plane $3x + 6y + 4z = 12$ and the coordinate planes.
97. Find the centroid of the solid bounded below by the paraboloid $z = x^2 + y^2$ and above by the plane $z = 4$.
98. Find the centroid of the solid bounded by $z = 4y^2$, $z = 4$, $x = -1$, and $x = 1$.
99. Find the center of mass of the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ if the mass density is proportional to the distance from the yz -plane.
100. Find the moments of inertia about its three edges of a homogeneous box of mass M with edges of lengths a , b , and c .
101. Find the moments of inertia I_x , I_y , I_z of the homogeneous tetrahedron bounded by the coordinate planes and the plane $x + y + z = 1$.

102. Find the moment of inertia about the y -axis of the homogeneous solid bounded by $z = 1 - x^2$, $z = 0$, $y = -1$, and $y = 1$.
103. Find the moment of inertia about the z -axis of the cube $0 \leq x \leq 1$, $0 \leq y \leq 1$, $0 \leq z \leq 1$ if the mass density is $\lambda(x, y, z) = kz$.

16.8 Triple Integrals in Cylindrical Coordinates

104. Find the cylindrical coordinates (r, θ, z) of the point with rectangular coordinates $(2, 1, 2)$.
105. Find the rectangular coordinates of the point with cylindrical coordinates $(2, \theta/4, 2)$.
106. Evaluate $\int_0^{\pi/4} \int_1^{\cos \theta} \int_1^r \frac{1}{r^2 z^2} dz dr d\theta$.
107. Evaluate $\int_0^{\pi/2} \int_1^2 \int_0^5 e^z r dz dr d\theta$.
108. Use cylindrical coordinates to find the volume of the solid in the first octant bounded by the coordinate planes, the cylinder $x^2 + y^2 = 4$, and the plane $z + y = 3$.
109. Use cylindrical coordinates to find the volume and centroid of the cylinder bounded by $x^2 + y^2 = 4$, $z = 0$, and $z = 4$.
110. Use cylindrical coordinates to find the volume inside $x^2 + y^2 = 4x$, above $z = 0$, and below $x^2 + y^2 = 4z$.
111. Use cylindrical coordinates to find the volume of the solid cut from the sphere $x^2 + y^2 + z^2 = 4$, bounded below $z = 0$, and on the sides by the cylinder $x^2 + y^2 = 1$.
112. Use cylindrical coordinates to evaluate $\iiint_T \sqrt{x^2 + y^2} dx dy dz$ where T is the solid bounded by $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$.
113. Use cylindrical coordinates to find the volume and centroid of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.
114. Use cylindrical coordinates to find the volume and centroid of the solid bounded by $z = \sqrt{x^2 + y^2}$, and the plane $z = 1$.

16.9 Triple Integrals in Spherical Coordinates

115. Find the spherical coordinates (ρ, θ, ϕ) of the point with rectangular coordinates $(2, -1, 1)$.
116. Find the rectangular coordinates of the point with spherical coordinates $(2, 2\pi/3, \pi/4)$.
117. Find the spherical coordinates of the point with cylindrical coordinates $(1, \pi/6, 2)$.
118. Find the cylindrical coordinates of the point with spherical coordinates $(3, \pi/3, \pi/6)$.
119. Evaluate $\int_0^{\pi/2} \int_0^{\pi} \int_{2 \sin \phi}^2 \rho \sin^2 \phi \cos \phi d\rho d\phi d\theta$.
120. Evaluate $\int_0^{\pi/2} \int_0^{\pi} \int_1^2 \rho^4 \cos^2 \phi \sin \phi d\rho d\phi d\theta$.

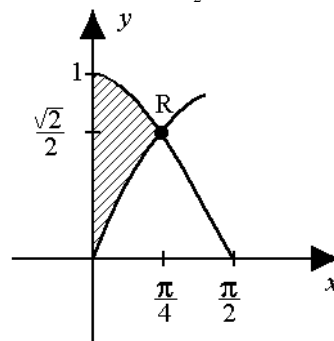
121. Use spherical coordinates to find the mass of the ball bounded by $x^2 + y^2 + z^2 \leq 9$ if its density is given by $\lambda(x, y, z) = \frac{z^2}{x^2 + y^2 + z^2}$.
122. Use spherical coordinates to find the mass of the ball bounded by $x^2 + y^2 + z^2 = 2z$ if its density is given by $\lambda(x, y, z) = \sqrt{x^2 + y^2 + z^2}$.
123. Use spherical coordinates to find the mass and center of mass of the ball bounded by $x^2 + y^2 + z^2 \leq 4$ if its density is given by $\lambda(x, y, z) = x^2 + y^2$.
124. Use spherical coordinates to find the mass of a ball of radius 4 if its density is proportional to the distance from its center. Take k as the constant of proportionality.

16.10 Jacobians; Changing Variables in Multiple Integration

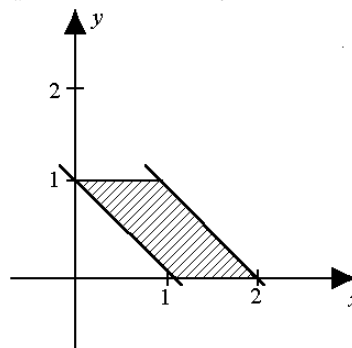
125. Find the Jacobian of the transformation $x = 2u + 3v, y = -u + 4v$.
126. Find the Jacobian of the transformation $x = u^2v, y = u^2 + v^2$.
127. Find the Jacobian of the transformation $x = u \ln v, y = \ln u + v$.
128. Find the Jacobian of the transformation $x = u^2 - v^2, y = uv$.
129. Take Ω as the parallelogram bounded by $x + y = 0, x + y = 1, x - y = 0, x - y = 2$.
Evaluate $\iint_{\Omega} (x + y) dx dy$.
130. Take Ω as the parallelogram bounded by $x - y = 0, x - y = \mathbf{p}, x + 2y = 0, x + 2y = \frac{1}{2}\mathbf{p}$.
Evaluate $\iint_{\Omega} (x^2 - y^2) dx dy$.
131. Take Ω as the parallelogram bounded by $x - y = 0, x - y = \mathbf{p}, x + 2y = 0, x + 2y = \frac{1}{2}\mathbf{p}$.
Evaluate $\iint_{\Omega} 2x^2 y dx dy$.
132. Take Ω as the parallelogram bounded by $x + y = 0, x + y = 1, x - y = 0, x - y = 2$.
Evaluate $\iint_{\Omega} \sin 2x dx dy$.

Answers to Chapter 16 Questions

1. $(a_2 - a_1)(c_2 - c_1)$
2. $(a_2 - a_1)(b_2 - b_1)(c_2 - c_1)$
3. $(b_2 - b_1)(c_2^2 - c_1^2)$
4. $4n(a_2 - a_1) - 3m(b_2 - b_1)$
5. $(a_2 - a_1)(b_2^2 - b_1^2)(c_2 - c_1)$
6. $L_f(P) = -11/4$; $U_f(P) = 3/4$
7. (a) $L_f(P) = \frac{3}{2}(a_2 - a_1)^2(b_2 - b_1) - (a_2 - a_1)(b_2 - b_1)^2 - \frac{3}{2}(a_2 - a_1)(b_2 - b_1)\Delta x - (a_2 - a_1)(b_2 - b_1)\Delta y$
(b) -1
8. $L_f(P) = -33/8$; $U_f(P) = -5/8$
9. (a) $L_f(P) = 2 \sum_{i=1}^m x_i \Delta x_i \sum_{j=1}^n (y_{j-1} - 1) \Delta y_j$
 $U_f(P) = 2 \sum_{i=1}^m x_{i-1} \Delta x_i \sum_{j=1}^n (y_j - 1) \Delta y_j$
(b) -2
10. $L_f(P) = -47/32$; $U_f(P) = 21/32$
11. (a) $L_f(P) = 2 \sum_{i=1}^m x_{i-1}^2 \Delta x_i \sum_{j=1}^n \Delta y_j - 3 \sum_{i=1}^m \Delta x_i \sum_{j=1}^n y_j^2 \Delta y_j$
 $U_f(P) = 2 \sum_{i=1}^m x_i^2 \Delta x_i \sum_{j=1}^n \Delta y_j - 3 \sum_{i=1}^m \Delta x_i \sum_{j=1}^n y_{j-1}^2 \Delta y_j$
(b) $-1/3$
12. $2/3$
13. 2
14. $p^3/36$
15. 84
16. -1
17. $e^2 - 2e + 1$
18. $\frac{62}{15} - \frac{6\sqrt{3}}{5}$
19. $9/2$
20. $p/3$
21. p
22. $p/3$
23. $\sqrt{2}p$
24. $e^2/2 - e + 1/2$
25. $7744/105$
26. $1/120$
27. $\frac{1}{2}(e-1)$
28. $3/35$
29. $4/3$
30. $2 \ln 2 - 3/4$
31. $\frac{1}{4} \sin 4$
32. $\frac{2\sqrt{2}-1}{12}$
33. $\int_0^{\frac{\sqrt{2}}{2}} \int_0^{\sin^{-1} y} f(x, y) dx dy + \int_{\frac{\sqrt{2}}{2}}^1 \int_0^{\cos^{-1} y} f(x, y) dx dy$



$$34. \int_0^1 \int_{1-x}^1 f(x, y) dy dx + \int_1^2 \int_0^{2-x} f(x, y) dy dx$$



- | | |
|-------------------------------|---|
| 35. $1/3$ | 61. $9p/4$ |
| 36. $4/3$ | 62. $(34/15, 39/20)$ |
| 37. $8/3$ | 63. $(8/3, 32/75)$ |
| 38. $232/15$ | 64. $(3, 2)$ |
| 39. $40/3$ | 65. $(p/2, 16/9p)$ |
| 40. $8/3$ | 66. $(3a5, 9a/40)$ |
| 41. $1/6$ | 67. $16/3$ |
| 42. 4 | 68. $3p + \frac{9\sqrt{3}}{2}$ |
| 43. $8/3$ | 69. $(8/5, 2)$ |
| 44. $16/3$ | 70. $(4/3, 4/3)$ |
| 45. $\frac{p}{2}(1 - e^{-4})$ | 71. $(0, 12/5)$ |
| 46. $3p$ | 72. $(8/5, 16/7)$ |
| 47. 18 | 73. $(1, -2/5)$ |
| 48. $2p$ | 74. $(3, 3/5)$ |
| 49. 36 | 75. $(0, 8/3p)$ |
| 50. $3p - \frac{8}{3}$ | 76. $(\bar{x}, \bar{y}) = \left(-\frac{5}{3}, 0\right)$ |
| 51. $8/3$ | 77. $I_x = 12, I_y = 39/5, I_z = 99/5$ |
| 52. $8p$ | 78. $I_x = 9/10, I_y = 3/10, I_z = 6/5$ |
| 53. $\frac{64p}{3}\sqrt{2}$ | 79. $1/16$ |
| 54. $8p$ | 80. $27/8$ |
| 55. $4p/3$ | 81. $1/30$ |
| 56. $\frac{128}{9}(3p - 4)$ | 82. $11/24$ |
| 57. $4p$ | 83. $625/12$ |
| 58. $\frac{10\sqrt{5}}{3}p$ | 84. $8/3$ |
| 59. $33p/2$ | 85. $3/35$ |
| 60. $2 + p/4$ | 86. 1 |
| | 87. $2 \ln 10$ |
| | 88. $8/3$ |

89. $16/15$
90. $256/5$
91. $(3/4, 3/4, 3/2)$
92. $1/4$
93. 4
94. $1/3$
95. 4
96. 4
97. $(0, 0, 8/3)$
98. $(0, 0, 12/5)$
99. $(2/5, 1/5, 1/5)$
100. $I_a = \frac{M}{3}(b^2 + c^2), I_b = \frac{M}{3}(a^2 + c^2), I_c = \frac{M}{3}(a^2 + b^2)$
101. $I_x = I_y = I_z = 1/30$
102. $I_y = 8/7$
103. $k/3$
104. $\left(\sqrt{5}, \tan^{-1} \frac{1}{2}, 2\right)$
105. $(\sqrt{2}, \sqrt{2}, 2)$
106. $\frac{1}{2} + \frac{p}{8} - \ln(\sqrt{2} + 1)$
107. $3p(e^5 - 1)$
108. $3p - 8/3$
109. $16p; (0, 0, 2)$
110. $6p$
111. $\frac{2p}{3}(8 - 3\sqrt{3})$
112. $16p$
113. $8p; (0, 0, 8/3)$
114. $p/3; (0, 0, 3/4)$
115. $\left(\sqrt{6}, \tan^{-1}\left(-\frac{1}{2}\right), \cos^{-1}\left(\frac{1}{\sqrt{6}}\right)\right)$
116. $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{6}}{2}, \sqrt{2}\right)$
117. $\left(\sqrt{5}, \frac{p}{6}, \cos^{-1} \frac{2}{\sqrt{5}}\right)$
118. $\left(\frac{3}{2}, \frac{p}{3}, \frac{3\sqrt{3}}{2}\right)$
119. $1/10$
120. $124p/15$
121. $12p$
122. $8p/5, (0, 0, 8/7)$
123. $256p/15$
124. $256pk$
125. 11
126. $4uv^2 - 2u^3$
127. $\ln v - 1/v$
128. $2u^2 + 2v^2$
129. $1/2$
130. $7p^4/216$
131. $\frac{-1}{96}p^5$
132. $\frac{-1}{2}(\sin 3 - \sin 2 - \sin 1)$

CHAPTER 17

Line Integrals and Surface Integrals

17.1 Line Integrals

1. Integrate $\mathbf{h}(x, y) = x^2 \mathbf{i} + y \mathbf{j}$ over
(a) $\mathbf{r}(u) = u^2 \mathbf{i} - 2u \mathbf{j}$, $u \in [0, 1]$
(b) the line segment from $(2, 3)$ to $(1, 2)$.
2. Integrate $\mathbf{h}(x, y) = 2xy \mathbf{i} + 3y^2 \mathbf{j}$ over
(a) $\mathbf{r}(u) = e^u \mathbf{i} + e^{-u} \mathbf{j}$, $u \in [0, 2]$
(b) the line segment from $(1, 2)$ to $(2, 3)$.
3. Integrate $\mathbf{h}(x, y) = (2xy - y) \mathbf{i} + 3xy \mathbf{j}$ over
(a) $\mathbf{r}(u) = (1 - u) \mathbf{i} + 2u \mathbf{j}$, $u \in [0, 1]$
(b) the line segment from $(1, 1)$ to $(2, 2)$.
4. Integrate $\mathbf{h}(x, y) = x^{-2} y^{-2} \mathbf{i} - x^{-1} y^{-1} \mathbf{j}$ over
(a) $\mathbf{r}(u) = \sqrt{2u-1} \mathbf{i} + \sqrt{2+u} \mathbf{j}$, $u \in [1, 3]$
(b) the line segment from $(2, 3)$ to $(4, 5)$.
5. Integrate $\mathbf{h}(x, y) = 2y \mathbf{i} - 3x \mathbf{j}$ over the triangle with vertices $(-2, 0)$, $(2, 0)$, $(0, 2)$ traversed counterclockwise.
6. Integrate $\mathbf{h}(x, y) = e^{x-2y} \mathbf{i} - e^{2x+y} \mathbf{j}$ over the line segment from $(-1, 1)$ to $(1, 2)$.
7. Integrate $\mathbf{h}(x, y) = (x^2 + y) \mathbf{i} + (2y^2 - xy) \mathbf{j}$ over the closed curve that begins at $(-2, 0)$, goes along the x -axis to $(2, 0)$, and returns to $(-2, 0)$ by the upper part of the circle.
8. Integrate $\mathbf{h}(x, y) = 2xy^2 \mathbf{i} + (xy^2 - 2x^3) \mathbf{j}$ over the square with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$, $(0, 1)$ traversed counterclockwise.
9. Integrate $\mathbf{h}(x, y, z) = xz^2 \mathbf{i} + y^2z \mathbf{j} + xy \mathbf{k}$ over
(a) $\mathbf{r}(u) = 2u \mathbf{i} - u^2 \mathbf{j} + u^3 \mathbf{k}$, $u \in [0, 1]$
(b) the line segment from $(0, 0, 0)$ to $(1, 1, 1)$.
10. Integrate $\mathbf{h}(x, y, z) = xe^x \mathbf{i} + e^z \mathbf{j} + e^{-y} \mathbf{k}$ over
(a) $\mathbf{r}(u) = 2u \mathbf{i} - 3u \mathbf{j} + 2u \mathbf{k}$, $u \in [0, 1]$
(b) the line segment from $(0, 0, 0)$ to $(1, 1, 1)$.
11. Calculate the work done by the force $F(x, y, z) = xy \mathbf{i} - y^2 \mathbf{j} - xyz \mathbf{k}$ applied to an object that moves in a straight line from $(0, 2, -1)$ to $(2, 1, 1)$.
12. Calculate the work done by the force $F(x, y, z) = x^2 \mathbf{i} + xy \mathbf{j} + z \mathbf{k}$ applied to an object that moves in a straight line from $(-1, 1, 2)$ to $(2, -1, -1)$.
13. An object of mass m moves from time $t = 0$ to $t = 1$ so that its position at time t is given by the vector function $\mathbf{r}(t) = 2t \mathbf{i} - t^2 \mathbf{j}$. Find the total force acting on the object at time t and calculate the work done by that force during the time interval $[0, 1]$.

17.2 The Fundamental Theorem for Line Integrals

14. Calculate the line integral of $\mathbf{h}(x, y) = 2xy \mathbf{i} + (x^2 + y) \mathbf{j}$ over the curve $\mathbf{r}(u) = 2 \cos u \mathbf{i} + \sin u \mathbf{j}$, $u \in [0, 2\pi]$.
15. Calculate the line integral of $\mathbf{h}(x, y) = (y^2 + 2xy) \mathbf{i} + (x^2 + 2xy) \mathbf{j}$ over the curve $\mathbf{r}(u) = \sin u \mathbf{i} + (2 - 2 \cos u) \mathbf{j}$, $u \in [0, \pi/2]$.
16. Calculate the line integral of $\mathbf{h}(x, y) = (\sin y + y \cos x) \mathbf{i} + (\sin x + x \cos y) \mathbf{j}$ over the straight-line segment from $(\pi/2, \pi/2)$ to (π, π) .
17. Calculate the line integral of $\mathbf{h}(x, y, z) = 8xz \mathbf{i} - 2yz \mathbf{j} + (4x^2 - y^2) \mathbf{k}$ over the curve $\mathbf{r}(u) = (1 + 3u) \mathbf{i} + 2u^{5/2} \mathbf{j} + (2 + u) \mathbf{k}$, $u \in [0, 1]$.
18. Calculate the line integral of $\mathbf{h}(x, y, z) = (y + z) \mathbf{i} + (x + z) \mathbf{j} + (x + y) \mathbf{k}$ over the curve $\mathbf{r}(u) = u^4 \mathbf{i} + 2 \sin \frac{\pi}{2} u \mathbf{j} + 3u^2 \mathbf{k}$, $u \in [0, 1]$.
19. Calculate the work done by the force $F(x, y, z) = z \mathbf{i} + y \mathbf{j} + x \mathbf{k}$ applied to a particle that moves along the curve $\mathbf{r}(u) = \mathbf{i} + \sin u \mathbf{j} + \cos u \mathbf{k}$ for $0 \leq u \leq \pi/3$.
20. Calculate the work done by the force $F(x, y, z) = 3x^2 \mathbf{i} + \frac{z^2}{y} \mathbf{j} + 2z \ln y \mathbf{k}$ applied to a particle that moves from the point $(0, 1, 1)$ to the point $(2, 2, 1)$.

17.4 Line Integrals with Respect to Arc Length

21. Evaluate $\int_C 2xy \, dx + (e^x + x^2) \, dy$, where C is the line segment from $(0, 0)$ to $(1, 1)$.
22. Evaluate $\int_C y^2 \, dx - x^2 \, dy$, where C is the line segment from $(0, 1)$ to $(1, 0)$.
23. Evaluate $\int_C xy \, dx - y^2 \, dy$, where C is the line segment from $(0, 0)$ to $(2, 1)$.
24. Evaluate $\int_C (x^2 - y^2) \, dx - 2xy \, dy$, where C is the parabola $y = 2x^2$ from $(0, 0)$ to $(1, 2)$.
25. Evaluate $\int_C (3x^2 + y) \, dx + 4xy \, dy$, where C is the broken line path from $(0, 0)$ to $(2, 0)$ to $(0, 4)$ to $(0, 0)$.
26. Evaluate $\int_C (e^x - 3y) \, dx + (e^y + 6x) \, dy$, where C is the broken line path from $(0, 0)$ to $(1, 0)$ to $(0, 2)$ to $(0, 0)$.
27. Evaluate $\int_C -yz \, dx - xz \, dy + (1 + xy) \, dz$, where C is the circular helix $\mathbf{r}(t) = 2 \cos t \mathbf{i} + 2 \sin t \mathbf{j} + 3t \mathbf{k}$ from $(2, 0, 0)$ to $(2, 0, 6\pi)$.
28. Evaluate $\int_C x^2 y \, dx + 4 \, dy$, where C is the curve $\mathbf{r}(t) = e^t \mathbf{i} + e^{-t} \mathbf{j}$ for $0 \leq t \leq 1$.
29. Evaluate $\int_C z \, dx + x \, dy + y \, dz$, where C is the helix $\mathbf{r}(t) = \sin t \mathbf{i} + 3 \sin t \mathbf{j} + \sin^2 t \mathbf{k}$ for $0 \leq t \leq \pi/2$.
30. Evaluate $\int_C -y \, dx - x \, dy + z \, dz$, where C is the circle $x = \cos t$, $y = \sin t$ for $0 \leq t \leq 2\pi$.

31. Evaluate $\int_C 4xy \, dx - 8y \, dy + 3 \, dz$, where C is the curve given by $y = 2x, z = 3$ from $(0, 0, 3)$ to $(3, 6, 3)$.
32. Evaluate $\int_C x \sin y \, dx + \cos y \, dy + (x + y) \, dz$, where C is the straight line $x = y = z$ from $(0, 0, 0)$ to $(1, 1, 1)$.
33. Find the length and centroid of a wire shaped like the helix $x = 3 \cos u, y = 3 \sin u, z = 4u, u \in [0, 2\pi]$.
34. Find the mass, center of mass, and moments of inertia I_x, I_y, I_z of a wire shaped like the first-quadrant portion of the circle $x^2 + y^2 = a^2$ with mass density $\lambda(x, y) = kxy$.
35. Find the moment of inertia about the z -axis of a thin homogeneous rod of mass M that lies along the interval $0 \leq x \leq L$ of the x -axis.
36. Find the center of mass and moments of inertia I_x, I_y, I_z of a wire shaped like the curve $\mathbf{r}(u) = (u^2 - 1)\mathbf{j} + 2u\mathbf{k}, u \in [0, 1]$ if the mass density is $\lambda(x, y, z) = \sqrt{y + 2}$.

17.5 Green's Theorem

37. Use Green's Theorem to evaluate $\oint_C (3x^2 + y) \, dx + 4xy \, dy$, where C is the triangular region with vertices $(0, 0), (2, 0)$, and $(0, 4)$. Assume that the curve is traversed in a counterclockwise manner.
38. Use Green's Theorem to evaluate $\oint_C (2xy - y^2) \, dx + (x^2 - y^2) \, dy$, where C is the boundary of the region enclosed by $y = x$ and $y = x^2$. Assume that the curve C is traversed in a counterclockwise manner.
39. Use Green's Theorem to evaluate $\oint_C (3x^2 + y) \, dx + 4y^2 \, dy$, where C is the boundary of the region enclosed by $x = y^2$ and $y = x/2$ traversed in a counterclockwise manner.
40. Use Green's Theorem to evaluate $\oint_C (y - \sin x) \, dx + \cos x \, dy$, where C is the boundary of the region with vertices $(0, 0), (\pi/2, 0)$, and $(\pi/2, 1)$ traversed in a counterclockwise manner.
41. Use Green's Theorem to evaluate $\oint_C (e^x - 3y) \, dx + (e^y + 6x) \, dy$, where C is the boundary of the triangular region with vertices $(0, 0), (1, 0)$, and $(0, 2)$ traversed in a counterclockwise manner.
42. Use Green's Theorem to evaluate $\oint_C (2xy - y^2) \, dx + x^2 \, dy$, where C is the boundary of the region enclosed by $y = x + 1$ and $y = x^2 + 1$ traversed in a counterclockwise manner.
43. Use Green's Theorem to evaluate $\oint_C (x^3 - 3y) \, dx + (x + \sin y) \, dy$, where C is the boundary of the triangular region with vertices $(0, 0), (1, 0)$, and $(0, 2)$ traversed in a counterclockwise manner.
44. Use Green's Theorem to evaluate $\oint_C (x^2 - \cosh y) \, dx + (y + \sin x) \, dy$, where C is the boundary of the region enclosed by $0 \leq x \leq \pi$ and $0 \leq y \leq 1$ traversed in a counterclockwise manner.
45. Use Green's Theorem to evaluate $\oint_C (-xy^2 \, dx + x^2 y \, dy)$, where C is the boundary of the region in the first quadrant enclosed by $y = 1 - x^2$ traversed in a counterclockwise manner.

46. Use Green's Theorem to evaluate $\oint_C [y^3 dx + (x^3 + 3xy^2) dy]$, where C is the boundary of the region enclosed by $y = x^2$ and $y = x$ traversed in a counterclockwise manner.
47. Use Green's Theorem to evaluate $\oint_C (-x^2 y dx + xy^2 dy)$, where C is the circle $x^2 + y^2 = 16$ traversed in a counterclockwise manner.
48. Use Green's Theorem to evaluate $\oint_C [2xy dx + (e^x + x^2) dy]$, where C is the boundary of the triangular region with vertices $(0, 0)$, $(1, 0)$, and $(1, 1)$ traversed in a counterclockwise manner.
49. Use a line integral to find the area of the region in the first quadrant enclosed by $y = x$ and $y = x^3$.
50. Use a line integral to find the area of the region enclosed by $y = 1 - x^4$ and $y = 0$.
51. Use Green's Theorem to evaluate $\oint_C [2 \tan^{-1} \frac{y}{x} dx + \ln(x^2 + y^2) dy]$, where C is the boundary of the circle $(x - 2)^2 + y^2 = 1$ traversed in a counterclockwise manner.
52. Use a line integral to find the area of the region enclosed by $x^2 + 4y^2 = 4$.
53. Use a line integral to find the area of the region enclosed by $y = x$ and $y = x^2$.
54. Use a line integral to find the area of the region enclosed by $y = \sin x$, $y = \cos x$, and $x = 0$.

17.6 Parameterized Surfaces; Surface Area

55. Find the surface area cut from the plane $z = 4x + 3$ by the cylinder $x^2 + y^2 = 25$.
56. Find the surface area of that portion of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 1$.
57. Find the surface area cut from the plane $2x - y - z = 0$ by the cylinder $x^2 + y^2 = 4$.
58. Find the surface area of that portion of the plane $3x + 4y + 6z = 12$ that lies in the first octant.
59. Find the surface area of that portion of the paraboloid $z = 25 - x^2 - y^2$ for which $z \geq 0$.
60. Find the surface area of that portion of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 2x$ and above the xy -plane.
61. Find the surface area of that portion of the paraboloid $z = 25 - x^2 - y^2$ that lies inside the cylinder $x^2 + y^2 = 9$ and above the xy -plane.
62. Find the surface area of the surface $z = \frac{1}{a}(y^2 - x^2)$ cut by the cylinder $x^2 + y^2 = a^2$ that lies above the xy -plane.
63. Find the surface area of that portion of the cylinder $y^2 + z^2 = 4$ in the first octant cut out by the planes $x = 0$ and $y = x$.
64. Find the surface area of that portion of the cylinder $x^2 + z^2 = 25$ that lies inside the cylinder $x^2 + y^2 = 25$.
65. Find the surface area of the surface $z = 2x + y^2$ that lies above the triangular region with vertices at $(0, 0, 0)$, $(0, 1, 0)$, and $(1, 1, 1)$.

66. Find the surface area of that portion of the plane $z = x + y$ in the first octant that lies inside the cylinder $4x^2 + 9y^2 = 36$.
67. Find the surface area of that portion of the cylinder $y^2 + z^2 = 4$ that lies above the region in the xy -plane enclosed by the lines $x + y = 1$, $x = 0$, and $y = 0$.
68. Find the surface area of that portion of the cylinder $z = y^2$ that lies above the triangular region with vertices at $(0, 0, 0)$, $(0, 1, 0)$, and $(1, 1, 0)$.
69. Find the surface area of that portion of the cylinder $x^2 = 1 - z$ that lies above the triangular region with vertices at $(0, 0, 0)$, $(1, 0, 0)$, and $(1, 1, 0)$.
70. Find the surface area of that portion of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 2y$.
71. Find the surface area of that portion of the sphere $x^2 + y^2 + z^2 = 4$ that lies in the first octant between the planes $y = 0$, and $y = x$.
72. Find the surface area of that portion of the sphere $x^2 + y^2 + z^2 = 4$ that lies in the first octant between the planes $y = 0$, and $y = \sqrt{3}x$.

17.7 Surface Integrals

73. Evaluate the surface integral $\iint_S (x^2 + y^2) d\mathbf{S}$ where S is the portion of the cone $z = \sqrt{3(x^2 + y^2)}$ for $0 \leq z \leq 3$.
74. Evaluate the surface integral $\iint_S 8x d\mathbf{S}$ where S is the surface enclosed by $z = x^2$, $0 \leq x \leq 2$, and $-1 \leq y \leq 2$.
75. Evaluate the surface integral $\iint_S 3x^3 \sin y d\mathbf{S}$ where S is the surface enclosed by $z = x^3$, $0 \leq x \leq 2$, and $0 \leq y \leq \pi$.
76. Evaluate the surface integral $\iint_S (\cos x + \sin y) d\mathbf{S}$ where S is that portion of the plane $x + y + z = 1$ that lies in the first octant.
77. Evaluate the surface integral $\iint_S \tan^{-1} \frac{y}{x} d\mathbf{S}$ where S is that portion of the paraboloid $z = x^2 + y^2$ enclosed by $1 \leq z \leq 9$.
78. Evaluate the surface integral $\iint_S x d\mathbf{S}$ where S is that portion of the plane $x + 2y + 3z = 6$ that lies in the first octant.
79. Evaluate the surface integral $\iint_S (x^2 + y^2) d\mathbf{S}$ where S is that portion of the plane $z = 4x + 20$ intercepted by the cylinder $x^2 + y^2 = 9$.
80. Evaluate the surface integral $\iint_S y d\mathbf{S}$ where S is that portion of the plane $z = x + y$ inside the elliptic cylinder $4x^2 + 9y^2 = 36$ that lies in the first octant.

81. Evaluate the surface integral $\iint_S y \, d\mathbf{s}$ where S is that portion of the cylinder $y^2 + z^2 = 4$ that lies above the region in the xy -plane enclosed by the lines $x + y = 1$, $x = 0$, and $y = 0$.
82. Evaluate the surface integral $\iint_S y^4 \, d\mathbf{s}$ where S is that portion of the surface $z = y^4$ that lies above the triangle in the xy -plane with vertices $(0, 0)$, $(0, 1)$, and $(1, 1)$.
83. Evaluate the surface integral $\iint_S x^2 \, d\mathbf{s}$ where S is that portion of the surface $z = x^3$ that lies above the triangle in the xy -plane with vertices $(0, 0)$, $(1, 0)$, and $(1, 1)$.
84. Evaluate the surface integral $\iint_S x^2 \, d\mathbf{s}$ where S is that portion of the plane $x + y + z = 1$ that lies inside the cylinder $x^2 + y^2 = 1$.
85. Evaluate the surface integral $\iint_S y^2 \, d\mathbf{s}$ where S is that portion of the plane $x + y + z = 1$ that lies in the first octant.
86. Evaluate the surface integral $\iint_S y^2 \, d\mathbf{s}$ where S is that portion of the cylinder $y^2 + z^2 = 1$ that lies above the xy -plane between $x = 0$ and $x = 5$.
87. Evaluate the surface integral $\iint_S (x^2 + y^2) \, d\mathbf{s}$ where S is that portion of the cylinder $x^2 + z^2 = 1$ that lies above the xy -plane enclosed by $0 \leq y \leq 5$.
88. Evaluate the surface integral $\iint_S (y^2 + z^2) \, d\mathbf{s}$ where S is the portion of the cone $x = \sqrt{3(x^2 + z^2)}$ for $0 \leq x \leq 3$.
89. Evaluate the surface integral $\iint_S 8x \, d\mathbf{s}$ where S is the surface enclosed by $y = x^2$, $0 \leq x \leq 2$, and $-1 \leq z \leq 2$.
90. Evaluate the surface integral $\iint_S (\sin y + \cos z) \, d\mathbf{s}$ where S is that portion of the plane $x + y + z = 1$ that lies in the first octant.
91. Evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) \, d\mathbf{s}$ where $\mathbf{v} = y\mathbf{i} - x\mathbf{j} + 8\mathbf{k}$ and S is that portion of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 4$. Take \mathbf{n} as the downward unit normal.
92. Evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) \, d\mathbf{s}$ where $\mathbf{v} = y\mathbf{i} - x\mathbf{j} + 9\mathbf{k}$ and S is that portion of the paraboloid $z = 4 - x^2 - y^2$ that lies above $z = 0$. Take \mathbf{n} as the upward unit normal.
93. Evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) \, d\mathbf{s}$ where $\mathbf{v} = x\mathbf{i} - y\mathbf{j} + z\mathbf{k}$ and S is that portion of the plane $2x + 3y + 4z = 12$ that lies in the first octant. Take \mathbf{n} as the upward unit normal.

94. Evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v} = yz \mathbf{i} - xz \mathbf{j} + xy \mathbf{k}$ and S is that portion of the hemisphere $z = \sqrt{4 - x^2 - y^2}$ that lies above the xy -plane. Take \mathbf{n} as the upward unit normal.
95. Evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v} = y \mathbf{i} - x \mathbf{j} - 4z^2 \mathbf{k}$ and S is that portion of the cone $z = \sqrt{x^2 + y^2}$ that lies above the square in the xy -plane with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$ and $(0, 1)$. Take \mathbf{n} as the downward unit normal.
96. Evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v} = y \mathbf{i} - x \mathbf{j} - \mathbf{k}$ and S is that portion of the hemisphere $z = -\sqrt{4 - x^2 - y^2}$ that lies above the plane $z = 0$. Take \mathbf{n} as the downward unit normal.
97. Evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v} = z \mathbf{i} + x \mathbf{j} + y \mathbf{k}$ and S is that portion of the cylinder $x^2 + y^2 = 4$ in the first octant between $z = 0$ and $z = 4$. Take \mathbf{n} as the outward unit normal.
98. Evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$ and S is that portion of the cone $z = \sqrt{x^2 + y^2}$ that lies in the first octant between $z = 1$ and $z = 2$. Take \mathbf{n} as the downward unit normal.
99. Evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v} = -xy^2 \mathbf{i} + z \mathbf{j} + xz \mathbf{k}$ and S is that portion of the surface $z = xy$ bounded by $0 \leq x \leq 3$ and $0 \leq y \leq 2$. Take \mathbf{n} as the upward unit normal.
100. Evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v} = y \mathbf{i} + 2x \mathbf{j} + xy \mathbf{k}$ and S is that portion of the cylinder $x^2 + y^2 = 9$ in the first octant between $z = 1$ and $z = 4$.
101. Evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v} = y \mathbf{i} + z \mathbf{j} + y \mathbf{k}$ and S is that portion of the cone $x = \sqrt{y^2 + z^2}$ that lies in the first octant between $x = 1$ and $x = 3$. Take \mathbf{n} as the unit normal that points away from the yz -plane.
102. Calculate the flux of $\mathbf{v} = x \mathbf{i} + y \mathbf{j} - 2z \mathbf{k}$ across the portion of the sphere $x^2 + y^2 + z^2 = 9$ that lies above the xy -plane, with upward unit normal.
103. Evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v} = x \mathbf{i} + 4 \mathbf{j} + 2x^2 \mathbf{k}$ and S is that portion of the paraboloid $z = x^2 + y^2$ that lies above the xy -plane enclosed by the parabolas $y = 1 - x^2$ and $y = x^2 - 1$. Take \mathbf{n} as the downward unit normal.
104. Evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v} = 2 \mathbf{i} - z \mathbf{j} + y \mathbf{k}$ and S is that portion of the paraboloid $x = y^2 + z^2$ between $x = 0$ and $x = 4$. Take \mathbf{n} as the unit normal that points away from the yz -plane.
105. Calculate the flux of $\mathbf{v} = 9 \mathbf{i} - z \mathbf{j} + y \mathbf{k}$ across the portion of the paraboloid $x = 4 - y^2 - z^2$ for which $x \geq 0$, in the direction pointing away from the xy -plane.
106. Evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v} = -x \mathbf{i} - 2x \mathbf{j} + (z - 1) \mathbf{k}$ and S is the surface enclosed by that portion of the paraboloid $z = 4 - y^2$ that lies in the first octant and is bounded by the coordinate planes and the plane $y = x$. Take \mathbf{n} as the upward unit normal.

17.8 The Vector Differential Operator ∇

107. Given that $\mathbf{v}(x, y) = x^2 \mathbf{i} + 2y \mathbf{j}$, find $\nabla \cdot \mathbf{v}$ and $\nabla \times \mathbf{v}$.
108. Given that $\mathbf{v}(x, y) = 3y \mathbf{i} - 2x^2 \mathbf{j}$, find $\nabla \cdot \mathbf{v}$ and $\nabla \times \mathbf{v}$.
109. Given that $\mathbf{v}(x, y, z) = yz \mathbf{i} - xz \mathbf{j} + 3xy \mathbf{k}$, find $\nabla \cdot \mathbf{v}$ and $\nabla \times \mathbf{v}$.
110. Given that $\mathbf{v}(x, y, z) = -2xy^2 \mathbf{i} + z \mathbf{j} + xz \mathbf{k}$, find $\nabla \cdot \mathbf{v}$ and $\nabla \times \mathbf{v}$.
111. Given that $\mathbf{v}(x, y, z) = -x \mathbf{i} + \mathbf{j} - 2x^2 \mathbf{k}$, find $\nabla \cdot \mathbf{v}$ and $\nabla \times \mathbf{v}$.
112. Given that $\mathbf{v}(x, y, z) = -x \mathbf{i} - 2x \mathbf{j} + (z - 1) \mathbf{k}$, find $\nabla \cdot \mathbf{v}$ and $\nabla \times \mathbf{v}$.
113. Given that $\mathbf{v}(x, y, z) = (2x + \cos z) \mathbf{i} + (y - e^x) \mathbf{j} - (2z - \ln y) \mathbf{k}$, find $\nabla \cdot \mathbf{v}$ and $\nabla \times \mathbf{v}$.
114. Given that $\mathbf{v}(x, y, z) = e^x \mathbf{i} - ye^x \mathbf{j} + 3yz \mathbf{k}$, find $\nabla \cdot \mathbf{v}$ and $\nabla \times \mathbf{v}$.
115. Given that $\mathbf{v}(x, y, z) = (x^3 + 3xy^2) \mathbf{i} + z^3 \mathbf{k}$, find $\nabla \cdot \mathbf{v}$ and $\nabla \times \mathbf{v}$.
116. Given that $\mathbf{v}(x, y, z) = -y^3 \mathbf{i} + x^3 \mathbf{j} - (x + z) \mathbf{k}$, find $\nabla \cdot \mathbf{v}$ and $\nabla \times \mathbf{v}$.
117. Given that $f(x, y, z) = x^3 + y^3 + z^3$, calculate the Laplacian $\nabla^2 f$.
118. Given that $f(x, y, z) = 2x^2 y^3 z$, calculate the Laplacian $\nabla^2 f$.
119. Given that $f(x, y, z) = 2(x^2 + y^2)$, calculate the Laplacian $\nabla^2 f$.
120. Given that $f(r) = \sin r$, calculate the Laplacian $\nabla^2 f$.

17.9 The Divergence Theorem

121. Find the divergence of $\mathbf{v}(x, y, z) = x^2 y \mathbf{i} + xy^2 \mathbf{j} + xyz \mathbf{k}$.
122. Find the divergence of $\mathbf{v}(x, y, z) = \cosh x \mathbf{i} + \sinh y \mathbf{j} + \ln(xy) \mathbf{k}$.
123. Find the divergence of $\mathbf{v}(x, y, z) = e^x \cos y \mathbf{i} + e^x \sin y \mathbf{j} + z \mathbf{k}$.
124. Use the divergence theorem to evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v}(x, y, z) = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$, \mathbf{n} is the outer unit normal to S , and S is the surface of the paraboloid $z = x^2 + y^2$ that is inside the cylinder $x^2 + y^2 = 1$.
125. Use the divergence theorem to evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v}(x, y, z) = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$, \mathbf{n} is the outer unit normal to S , and S is the surface of the cube $-1 \leq x \leq 1, -1 \leq y \leq 1, -1 \leq z \leq 1$.
126. Use the divergence theorem to evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v}(x, y, z) = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$, \mathbf{n} is the outer unit normal to S , and S is the surface formed by the intersection of two paraboloids $z = x^2 + y^2$ and $z = 4 - (x^2 + y^2)$.
127. Use the divergence theorem to evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v}(x, y, z) = (2x + z) \mathbf{i} + y \mathbf{j} - (2z + \sin x) \mathbf{k}$, \mathbf{n} is the outer unit normal to S , and S is the surface of the cylinder $x^2 + y^2 = 4$ enclosed between the planes $z = 0$ and $z = 4$.

128. Use the divergence theorem to evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v}(x, y, z) = \frac{x^3}{3} \mathbf{i} + \frac{y^3}{3} \mathbf{j} - \frac{z^3}{3} \mathbf{k}$, \mathbf{n} is the outer unit normal to S , and S is the surface of the cylinder $x^2 + y^2 = 1$ enclosed between the planes $z = 0$ and $z = 1$.
129. Use the divergence theorem to evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v}(x, y, z) = x \mathbf{i} + y \mathbf{j} - z \mathbf{k}$, \mathbf{n} is the outer unit normal to S , and S is the surface of the solid bounded by $x + y + z = 1$, $x = 0$, $y = 0$, and $z = 0$.
130. Use the divergence theorem to evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v}(x, y, z) = (x^3 + 3xy^2) \mathbf{i} + z^3 \mathbf{k}$, \mathbf{n} is the outer unit normal to S , and S is the surface of the sphere of radius a centered at the origin.
131. Use the divergence theorem to evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v}(x, y, z) = e^x \mathbf{i} + ye^x \mathbf{j} + 4x^2z \mathbf{k}$, \mathbf{n} is the outer unit normal to S , and S is the surface of the solid enclosed by $x^2 + y^2 = 4$ and the planes $z = 0$ and $z = 9$.
132. Use the divergence theorem to calculate the total flux of $\mathbf{v}(x, y, z) = e^x \mathbf{i} - ye^x \mathbf{j} + 3z \mathbf{k}$, out of the sphere $x^2 + y^2 + z^2 = 9$.
133. Use the divergence theorem to calculate the total flux of $\mathbf{v}(x, y, z) = x^2 \mathbf{i} - y^2 \mathbf{j} + z^2 \mathbf{k}$, out of the cube $0 \leq x \leq 1$, $0 \leq y \leq 1$, $0 \leq z \leq 1$.
134. Use the divergence theorem to evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v}(x, y, z) = x^3 \mathbf{i} + x^2y \mathbf{j} + x^2z \mathbf{k}$, \mathbf{n} is the outer unit normal to S , and S is the surface of the solid enclosed by the cylinder $x^2 + y^2 = 2$ and the planes $z = 0$ and $z = 2$.
135. Use the divergence theorem to evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v}(x, y, z) = x(x^2 + y^2 + z^2) \mathbf{i} + y(x^2 + y^2 + z^2) \mathbf{j} + z(x^2 + y^2 + z^2) \mathbf{k}$, \mathbf{n} is the outer unit normal to S , and S is the sphere $x^2 + y^2 + z^2 = 16$.
136. Use the divergence theorem to evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v}(x, y, z) = x^3 \mathbf{i} + x^2y \mathbf{j} + x^2z \mathbf{k}$, \mathbf{n} is the outer unit normal to S , and S is the surface of the solid enclosed by the hemisphere $z = \sqrt{4 - x^2 - y^2}$ and the xy -plane.
137. Use the divergence theorem to evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v}(x, y, z) = x^2 \mathbf{i} + y^2 \mathbf{j} + z^2 \mathbf{k}$, \mathbf{n} is the outer unit normal to S , and S is the surface of the solid enclosed by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $z = 5$.
138. Use the divergence theorem to evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v}(x, y, z) = yz \mathbf{i} + xy \mathbf{j} + xz \mathbf{k}$, \mathbf{n} is the outer unit normal to S , and S is the solid enclosed by the cylinder $x^2 + z^2 = 1$ and the planes $y = -1$ and $y = 1$.
139. Use the divergence theorem to evaluate $\iint_S (\mathbf{v} \cdot \mathbf{n}) dS$ where $\mathbf{v}(x, y, z) = y^2x \mathbf{i} + yz^2 \mathbf{j} + x^2y^2 \mathbf{k}$, \mathbf{n} is the outer unit normal to S , and S is the sphere $x^2 + y^2 + z^2 = 4$.

17.10 Stokes's Theorem

140. Find the curl of $\mathbf{v}(x, y, z) = x^2y \mathbf{i} + y^2x \mathbf{j} + xyz \mathbf{k}$.
141. Find the curl of $\mathbf{v}(x, y, z) = \cosh x \mathbf{i} + \sinh y \mathbf{j} + \ln xy \mathbf{k}$.
142. Find the curl of $\mathbf{v}(x, y, z) = e^x \cos y \mathbf{i} + e^x \sin y \mathbf{j} + z \mathbf{k}$.
143. Verify Stokes's Theorem if S is the portion of the sphere $x^2 + y^2 + z^2 = 1$ for which $z \geq 0$ and $\mathbf{v}(x, y, z) = (2x - y) \mathbf{i} - yz^2 \mathbf{j} - y^2z \mathbf{k}$.
144. Use Stokes's Theorem to evaluate $\oint_C (z - y)dx + (x - z)dy + (y - x)dz$ where C is the boundary, in the xy -plane, of the surface given by $z = 4 - (x^2 + y^2)$, $z \geq 0$.
145. Use Stokes's Theorem to evaluate $\oint_C y^2 dx + x^2 dy - (x + z)dz$ where C is the triangle in the xy -plane with vertices $(0, 0, 0)$, $(1, 0, 0)$, and $(1, 1, 0)$ with a counterclockwise orientation looking down the positive z -axis.
146. Use Stokes's Theorem to evaluate $\oint_C -3y dx + 3x dy + z dz$ over the circle $x^2 + y^2 = 1$, $z = 1$ traversed counterclockwise.
147. Use Stokes's Theorem to evaluate $\oint_C z dx + x dy + y dz$ over the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$ traversed in a counterclockwise manner.
148. Use Stokes's Theorem to evaluate $\iint_S [(\nabla \times \mathbf{v}) \cdot \mathbf{n}] dS$ where $\mathbf{v}(x, y, z) = x^2 \mathbf{i} + z^2 \mathbf{j} - y^2 \mathbf{k}$ and S is that portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \geq 0$ and \mathbf{n} is the upper unit normal.
149. Use Stokes's Theorem to evaluate $\iint_S [(\nabla \times \mathbf{v}) \cdot \mathbf{n}] dS$ where $\mathbf{v}(x, y, z) = (z - y) \mathbf{i} - (z^2 + x) \mathbf{j} + (x^2 - y^2) \mathbf{k}$ and S is that portion of the sphere $x^2 + y^2 + z^2 = 4$ for which $z \geq 0$ and \mathbf{n} is the upper unit normal.
150. Use Stokes's Theorem to evaluate $\iint_S [(\nabla \times \mathbf{v}) \cdot \mathbf{n}] dS$ where $\mathbf{v}(x, y, z) = y \mathbf{k}$ and S is that portion of the ellipsoid $4x^2 + 4y^2 + z^2 = 4$ for which $z \geq 0$ and \mathbf{n} is the upper unit normal.
151. Use Stokes's Theorem to evaluate $\oint_C \sin z dx - \cos x dy + \sin y dz$ over the boundary of rectangle $0 \leq x \leq \pi$, $0 \leq y \leq 1$, $z = 2$, traversed in a counterclockwise manner.
152. Use Stokes's Theorem to evaluate $\oint_C (x + y) dx + (2x - 3) dy + (y + z) dz$ over the boundary of the triangle with vertices $(2, 0, 0)$, $(0, 3, 0)$, and $(0, 0, 6)$ traversed in a counterclockwise manner.
153. Use Stokes's Theorem to evaluate $\oint_C 4z dx - 2x dy + 2x dz$ where C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $z = y + 1$, traversed in a counterclockwise manner.
154. Use Stokes's Theorem to evaluate $\oint_C -yz dx + xz dy + xy dz$ over the circle $x^2 + y^2 = 2$, $z = 1$, traversed in a counterclockwise manner.
155. Use Stokes's Theorem to evaluate $\oint_C (4x - 2y) dx + yz^2 dy + y^2z dz$ over the circle $x^2 + y^2 = 4$, $z = 2$, traversed in a counterclockwise manner.

156. Use Stokes's Theorem to evaluate $\oint_C (e^{-x^2} - yz) dx + (e^{-y^2} + xz + 2x) dy + e^{-z} dz$ over the circle $x^2 + y^2 = 1$, $z = 1$, traversed in a counterclockwise manner.
157. Use Stokes's Theorem to evaluate $\oint_C xz dx + y^2 dy + x^2 dz$ where C is the intersection of the plane $x + y + z = 5$ and the cylinder $x^2 + \frac{y^2}{4} = 1$, traversed in a counterclockwise manner.

Answers to Chapter 17 Questions

1. (a) $7/3$ (b) $-29/6$
2. (a) $2e^2 - e^{-6} - 1$ (b) $-34/3$
3. (a) 2 (b) $17/2$
4. (a) $\frac{-2}{5\sqrt{5}} + \frac{2}{5} - \frac{7}{5\sqrt{5}} \frac{\mathbf{p}}{4} + \frac{7}{5\sqrt{5}} \tan^{-1} \frac{1}{\sqrt{5}} \approx -0.007326$
(b) $23/60 + 3 \ln 5/6$
5. -20
6. $2e^{-3} - \frac{1}{5}(e^4 - e^{-1})$
7. $-16/3 - 2\mathbf{p}$
8. $-8/3$
9. (a) $-13/18$ (b) $5/6$
10. (a) $e^2 + \frac{3}{2}e^{-2} + \frac{4}{3}e^{-3} - \frac{11}{6}$
(b) $1 + e - e^{-1}$
11. $13/3$
12. $1/2$
13. $\mathbf{F} = -2m \mathbf{j}$; work = $2m$
14. 0
15. 6
16. $-\mathbf{p}$
17. 172
18. 11
19. $-1/8$
20. $8 + \ln 2$
21. e
22. $2/3$
23. 1
24. $-11/3$
25. $52/3$
26. 9
27. $6\mathbf{p}$
28. $e^2/2 + 4/e - 9/2$
29. $23/6$
30. $2\mathbf{p}$
31. -72
32. $2 \sin 1 - \cos 1 + 1$
33. length = $10\mathbf{p}$, centroid $(0, 0, 4\mathbf{p})$
34. mass = $\frac{1}{2}ka^3$, center of mass $(2a/3, 2a/3)$,
 $I_x = I_y = \frac{1}{4}ka^5$
35. $L^2M/3$
36. center of mass $(0, -3/5, 9/8)$,
 $I_x = 192/35$, $I_y = 65/15$, $I_z = 128/105$
37. $52/3$
38. $2/15$
39. $4/3$
40. $-2/\mathbf{p} - \mathbf{p}/4$
41. 9
42. $7/15$
43. 4
44. $\mathbf{p}(\cosh 1 - 1)$
45. $1/3$
46. $3/20$
47. $128\mathbf{p}$
48. 1
49. $1/4$
50. $8/5$
51. 0
52. $2\mathbf{p}$
53. $1/6$

54. $\sqrt{2}-1$

55. $25\sqrt{17}\mathbf{p}$

56. $\frac{\mathbf{p}}{6}(5\sqrt{5}-1)$

57. $4\sqrt{6}\mathbf{p}$

58. $\sqrt{61}$

59. $\frac{\mathbf{p}}{6}(101\sqrt{101}-1)$

60. $4\mathbf{p}$

61. $\frac{\mathbf{p}}{6}(37\sqrt{37}-1)$

62. $\frac{\mathbf{p}\alpha^2}{6}(5\sqrt{5}-1)$

63. 4

64. 200

65. $\frac{27-5\sqrt{5}}{12}$

66. $\frac{3\sqrt{3}\mathbf{p}}{2}$

67. $\frac{\mathbf{p}}{3}+2\sqrt{3}-4$

68. $\frac{1}{12}(5\sqrt{5}-1)$

69. $\frac{1}{12}(5\sqrt{5}-1)$

70. $8\mathbf{p}-16$

71. \mathbf{p}

72. $4\mathbf{p}^3$

73. $9\mathbf{p}$

74. $2(17\sqrt{17}-1)$

75. $\frac{1}{9}(145\sqrt{145}-1)$

76. $\sqrt{3}(2-\cos 1-\sin 1)$

77. $\frac{\mathbf{p}^2}{6}(37\sqrt{37}-5\sqrt{5})$

78. $6\sqrt{14}$

79. $\frac{81\sqrt{17}}{2}\mathbf{p}$

80. $4\sqrt{3}$

81. $4-\sqrt{3}-\frac{2\mathbf{p}}{3}$

82. $\frac{1}{144}(17\sqrt{17}-1)$

83. $\frac{1}{54}(10\sqrt{10}-1)$

84. $\frac{\sqrt{3}}{4}\mathbf{p}$

85. $\frac{\sqrt{3}}{12}$

86. $5\mathbf{p}^2$

87. $265\mathbf{p}/6$

88. $9\mathbf{p}$

89. $2(17\sqrt{17}-1)$

90. $\sqrt{3}(2-\cos 1-\sin 1)$

91. $-32\mathbf{p}$

92. $36\mathbf{p}$

93. 36

94. 0

95. $8/3$

96. $4p$
97. 24
98. $\frac{p}{3}(2\sqrt{2}-1)$
99. 18
100. $81/2$
101. 0
102. 0
103. 0
104. $8p$
105. $36p$
106. -6
107. $2x + 2 ; 0$
108. $0 ; -(4x + 3) \mathbf{k}$
109. $0 ; 4x \mathbf{i} - 2y \mathbf{j} - 2z \mathbf{k}$
110. $-2y^2 + x ; -\mathbf{i} - z \mathbf{j} + 4xy \mathbf{k}$
111. $-1 ; 4x \mathbf{j}$
112. $0 ; -2 \mathbf{k}$
113. $1 ; \frac{1}{y} \mathbf{i} - \sin z \mathbf{j} - e^x \mathbf{k}$
114. $3y ; 3z \mathbf{i} - ye^x \mathbf{k}$
115. $2x + 3y^2 + 3z^2 ; -6xy \mathbf{k}$
116. $-1 ; \mathbf{j} + (3x^2 + 3y^2) \mathbf{k}$
117. $6(x + y + z)$
118. $4y^3z + 12x^2yz$
119. 8
120. $-\sin x - \sin y - \sin z$
121. $5xy$
122. $\sinh x + \cosh y$
123. $2e^x \cos y + 1$
124. $p/2$
125. 24
126. $4p$
127. $16p$
128. $5p/6$
129. $\frac{1}{2}$
130. $12pa^5/5$
131. $144p$
132. $108p$
133. 3
134. $10p$
135. $4096p$
136. $64p/5$
137. $100p$
138. 0
139. $256p/15$
140. $xz \mathbf{i} - yz \mathbf{j} + (y^2 - x^2) \mathbf{k}$
141. $1/y \mathbf{i} - 1/x \mathbf{j}$
142. $2e^x \sin y \mathbf{k}$
143. p
144. $8p$
145. $1/3$
146. $6p$
147. $3/2$
148. 0
149. $8p$
150. 0
151. 2
152. 12

153. $-4\mathbf{p}$

154. $4\mathbf{p}$

155. $8\mathbf{p}$

156. $4\mathbf{p}$

157. 0

CHAPTER 18

Elementary Differential Equations

18.1 Introduction

1. Classify the differential equation $\frac{d^2 y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$ as ordinary or partial and give its order.
2. Classify the differential equation $\frac{\partial z}{\partial x} = z + x\frac{dz}{dy}$ as ordinary or partial and give its order.
3. Classify the differential equation $y''' + 2(y'')^2 + y' = \cos x$ as ordinary or partial and give its order.
4. Classify the differential equation $(y')^4 + 3xy - 2y'' = x^2$ as ordinary or partial and give its order.
5. Classify the differential equation $\frac{\partial^2 y}{\partial t^2} - 4\frac{d^2 y}{dx^2} = 0$ as ordinary or partial and give its order.
6. Determine whether $y_1(x) = 3e^x, y_2(x) = 5e^{-x}$ are solutions of $y' + y = 0$.
7. Determine whether $y_1(x) = 1/x, y_2(x) = 2/x$ are solutions of $y' + y^2 = 0$.
8. Determine whether $u_1(x, t) = xe^t, u_2(x, t) = te^x$ are solutions of $x\frac{\partial u}{\partial x} = \frac{du}{dt}$.
9. For what values of C is $y = Ce^{-x}$ a solution of $y' + y = 0$ with side condition $y(3) = 2$?
10. For what values of C_1, C_2 is $y = C_1 \sin 2x + C_2 \cos 2x$ a solution of $y'' + 4y = 0$ with side conditions $y(0) = 0$ and $y'(0) = 1$?
11. For what values of r is $y = e^{rx}$ a solution of $y'' + y' - 6y = 0$?
12. For what values of r is $y = x^r$ a solution of $x^3 y'' - 2xy = 0$?

18.2 First-Order Linear Differential Equations; Numerical Methods

13. Find the general solution of $y' - 3y = 6$.
14. Find the general solution of $y' - 2xy = x$.
15. Find the general solution of $y' + \frac{4}{x}y = x^4$.
16. Find the general solution of $y' + \frac{2}{10+2x}y = 4$.
17. Find the general solution of $y' - y = -e^x$.
18. Find the general solution of $x \ln xy' + y = \ln x$.

19. Find the particular solution of $y' + 10y = 20$ determined by the side condition $y(0) = 2$.
20. Find the particular solution of $y' - y = -e^x$ determined by the side condition $y(0) = 3$.
21. Find the particular solution of $xy' - 2y = x^3 \cos 4x$ determined by the side condition $y(p) = 1$.
22. A 100-gallon mixing tank is full of brine containing 0.8 pounds of salt per gallon. Find the amount of salt present t minutes later if pure water is poured into the tank at the rate of 4 gallons per minute and the mixture is drawn off at the same rate.
23. Determine the velocity at time t and the terminal velocity of a 2 kg object dropped with a velocity 3 m/s, if the force due to air resistance is $-50v$ Newtons.
24. Use a suitable transformation to solve the Bernoulli equation $y' + xy = xy^2$.
25. Use a suitable transformation to solve the Bernoulli equation $xy' + y = x^3y^6$.
26. Find the general solution of $y' = y^2x^3$.
27. Find the general solution of $y' = \frac{x^2 + 7}{y^9 - 3y^4}$.
28. Find the general solution of $y' = y^2 + 1$.
29. Find the general solution of $x(y^2 + 1)y' + y^3 - 2y = 0$.
30. Find the particular solution of $e^x dx - y dy = 0$ determined by the side condition $y(0) = 1$.
31. Find the particular solution of $y' = y(x - 2)$ determined by the side condition $y(2) = 5$.
32. Verify that the equation $y' = \frac{y+x}{x}$ is homogeneous, then solve it.
33. Verify that the equation $y' = \frac{2y^4 + x^4}{xy^3}$ is homogeneous, then solve it.
34. Verify that the equation $y' = \frac{y}{x + \sqrt{xy}}$ is homogeneous, then solve it.
35. Verify that the equation of $[2x \sinh(y/x) + 3y \cosh(y/x)]dx - 3x \cosh(y/x)dy = 0$ is homogeneous, then solve it.
36. Find the orthogonal trajectories for the family of curves $x^2 + y^2 = C$.
37. Find the orthogonal trajectories for the family of curves $x^2 + y^2 = Cx$.

18.3 The Equation $y'' + ay' + by = 0$

38. Find the general solution of $y'' - 5y = 0$.
39. Find the general solution of $y'' - 60y' + 900y = 0$.
40. Find the general solution of $y'' - 6y' + 25y = 0$.

41. Find the general solution of $16y'' + 8y' + y = 0$.
42. Find the general solution of $2y'' - 5y' + 2y = 0$.
43. Find the general solution of $y'' + 4y' + 5y = 0$.
44. Solve the initial value problem $y'' + 25y = 0$, $y(0) = 3$, $y'(0) = 10$.
45. Solve the initial value problem $y'' + y' - 6y = 0$, $y(0) = 4$, $y'(0) = 13$.
46. Solve the Euler equation $x^2y'' + 5xy' + 4y = 0$.
47. Solve the Euler equation $x^2y'' - 5xy' + 25y = 0$.

18.4 The Equation $y'' + ay' + by = f(x)$

48. Find a particular solution of $y'' - y' - 2y = 4x^2$.
49. Find a particular solution of $y'' + 5y' + 6y = 3e^{-2x}$.
50. Find a particular solution of $y'' + 4y' + 8y = 16 \cos 4x$.
51. Find a particular solution of $y'' + 6y' + 9y = 16e^{-x} \cos 2x$.
52. Find the general solution of $y'' - y' - 2y = e^{2x}$.
53. Find the general solution of $y'' - 7y' = (3 - 36x)e^{4x}$.
54. Find the general solution of $y'' + 4y = 8x \sin 2x$.
55. Use variation of parameters to find a particular solution of $y'' - 2y' + y = e^x/x$.
56. Use variation of parameters to find a particular solution of $y'' + y = \sec x$.
57. Use variation of parameters to find a particular solution of $y'' + 4y = \sin^2 2x$.

18.5 Mechanical Vibrations

58. An object is in simple harmonic motion. Find an equation for the motion given that the period is $p/2$ and, at time $t = 0$, $x = 2$ and $v = 1$. What is the amplitude? What is the frequency?
59. An object is in simple harmonic motion. Find an equation for the motion given that the frequency is $4/p$ and, at time $t = 0$, $x = 3$ and $v = -3$. What is the amplitude? What is the period?
60. An object in simple harmonic motion passes through the central point $x = 0$ at time $t = 2$ and every 4 seconds thereafter. Find the equation of motion given that $v(0) = 3$.
61. Find an equation for the oscillatory motion given that the period is $3p/4$ and at time $t = 0$, $x = 2$ and $v = 5$.
62. Find an equation for the oscillatory motion given that the period is $5p/6$ and at time $t = 0$, $x = 1$ and $v = 4$.

Answers to Chapter 18 Questions

1. ordinary, order 2
2. partial, order 1
3. ordinary, order 3
4. ordinary, order 2
5. partial, order 2
6. y_1 is not, y_2 is
7. y_1 is, y_2 is not
8. u_1 is, u_2 is not
9. $C = 2e^3$
10. $C_1 = 1/2, C_2 = 0$
11. $r = 2, -3$
12. $r = 2, -1$
13. $y = Ce^{-3x}$
14. $y = Ce^{x^2} - \frac{1}{2}$
15. $y = \frac{C}{x^4} + \frac{1}{9}x^5$
16. $y = \frac{40x + 4x^2 + C}{10 + 2x}$
17. $y = (C - x)e^x$
18. $y = \frac{\ln^2 x + C}{2 \ln x}$
19. $y = 2$ (identically)
20. $y = (3 - x)e^x$
21. $y = \frac{1}{4}x^2 \sin 4x + \left(\frac{x}{p}\right)^2$
22. $80e^{-0.04t}$ pounds
23. $v = 0.392 + 2.608e^{-25t}$;
terminal velocity 0.392 m/s
24. $y = \frac{1}{1 + Ce^{x^2/2}}$
25. $y = \left(\frac{5}{2}x^3 + Cx^5\right)^{-1/5}$
26. $y = \frac{-4}{x^4 + C}$
27. $\frac{1}{10}y^{10} + \frac{3}{5}y^5 - \frac{1}{3}y^3 + 7x = C$
28. $y = \tan(x + C)$
29. $(y^2 - 2)^3 x^4 = Cy^2$
30. $y = \sqrt{2e^x - 1}, x > \ln \frac{1}{2}$
31. $y = 5e^{(x-2)^2/2}$
32. $y = x \ln |Cx|$
33. $x^8 = C(y^4 + x^4)$
34. $-2\sqrt{x/y} + \ln |y| = C$
35. $x^2 = C \sinh^3(y/x)$
36. $y = Kx$
37. $x^2 + y^2 = Ky$
38. $y = C_1 e^{\sqrt{5}x} + C_2 e^{-\sqrt{5}x}$
39. $y = (C_1 + C_2 x)e^{30x}$
40. $y = e^{3x}(C_1 \cos 4x + C_2 \sin 4x)$
41. $y = (C_1 + C_2 x)e^{-x/4}$
42. $y = C_1 e^{x/2} + C_2 e^{2x}$
43. $y = e^{-2x}(C_1 \cos x + C_2 \sin x)$
44. $y = 3 \cos 5x + 2 \sin 5x$
45. $y = 5e^{2x} - e^{-3x}$
46. $y = \frac{C_1 + C_2 \ln x}{x^2}$
47. $y = x^3[C_1 \cos(\ln x^4) + C_2 \sin(\ln x^4)]$
48. $y_p = -2x^2 + 2x - 3$

49. $y_p = 3xe^{-2x}$

50. $y_p = \frac{4}{5}\sin 4x - \frac{2}{5}\cos 4x$

51. $y_p = 2e^{-x} \sin 2x$

52. $y = C_1 e^{-x} + C_2 e^{2x} + \frac{1}{3} x e^{2x}$

53. $y = C_1 + C_2 e^{7x} + 3x e^{4x}$

54. $y = C_1 \cos 2x + C_2 \sin 2x + \frac{1}{2} x \sin 2x - x^2 \cos 2x$

55. $y_p = -x e^x + x e^x \ln |x|$

56. $y_p = (\ln |\cos x|) \cos x + x \sin x$

57. $y_p = \frac{1}{6} \cos^2 2x - \frac{1}{12} \sin^2 2x$

58. $x(t) = 2 \cos 4t + \frac{1}{4} \sin 4t$
amplitude = $\frac{\sqrt{65}}{4}$; frequency = 4

59. $x(t) = 3 \cos \frac{4}{p} t - \frac{3p}{4} \sin \frac{4}{p} t$
amplitude = $\frac{\sqrt{144 + 9p^2}}{4}$; period = $\pi^2/2$

60. $x(t) = \frac{6}{p} \sin \frac{p}{2} t$

61. $x(t) = 2 \cos \frac{8}{3} t - \frac{15}{8} \sin \frac{8}{3} t$

62. $x(t) = \cos \frac{12}{5} t + \frac{5}{3} \sin \frac{12}{5} t$