EXAMINATION COVERSHEET

Autumn 2022 Final Examination



THIS EXAMINATION CONTENT IS STRICTLY CONFIDENTIAL Students must comply with requirements stated in the Examination Policy & Procedures	
Student Number:	
First Name:	
Family Name:	
Date of Examination: (DD/MM/YY)	14/12/2022
Subject Code:	MATH 141
Subject Title:	Foundation of Engineering Mathematics
Time Permitted to Write Exam:	2 Hours
Total Number of Questions:	11 (6 MCQ's + 5 written questions)
Total Number of Pages (including this page):	9

INSTRUCTIONS TO STUDENTS FOR THE EXAM

- 1. Please note that subject lecturer/tutor will be unavailable during exams. *If there is a doubt in any of the exam questions i.e. problem solving etc. students should proceed by assuming values etc. Students should mention their assumption on the question paper.*
- 2. Answers must be written (and drawn) in black or blue ink
- 3. Any mistakes must be crossed out. Whitener and ink erasers must not be used.
- 4. Part A (MCQ): Answer ALL/ 6 questions. The marks for each question are shown next to each question. The total for Part A is 30 marks.
- 5. Part B (Written): Answer ALL/ 5 questions. The marks for each question are shown next to each question. The total for Part B is 70 marks.
- 6. Total marks: 100. This Exam is worth 35% of your final marks for MATH 141.

EXAMINATION MATERIALS/AIDS ALLOWED

Approved Calculators. The formula sheet will be provided by the instructor.

<u>Exam Unauthorised Items</u> - Students bringing these items to the examination room must follow the instructions of the invigilators with regards to these items.

- 7. Bags, including carrier bags, backpacks, shoulder bags and briefcases
- 8. Any form of electronic device including but not limited to mobile phones, smart watches, MP3 players, handheld computers and unauthorised calculators;
- 9. Calculator cases and covers, opaque pencil cases
- 10. Blank paper
- 11. Any written material

NOTE: The University does not guarantee the safe-keeping of students' personal items during examinations. Students concerned about the safety of their valuable items should make alternative arrangements for their care.

Part 1 MCQ 30% (circle your choice)

(5pts)Problem 1

Find the values of x and y in the equation

$$x(1+i)^{2} + y(2-i)^{2} = 3 + 10i.$$

x + y is equal to

- (a) 10 (b) 8 (c) 7 (d) 11 (e) 13

(5pts)Problem 2

The complex conjugate of z = x + iy is denoted by $\bar{z} = x - iy$. Solve the equation

$$z - 8 = i\left(7 - 2\bar{z}\right)$$

The modulus of the solution z is equal to

(a)
$$|z| = \sqrt{19}$$

(b)
$$|z| = 13\sqrt{7}$$

$$(c) |z| = \sqrt{7}$$

$$(d) |z| = \sqrt{13}$$

(a)
$$|z| = \sqrt{19}$$
 (b) $|z| = 13\sqrt{7}$ (c) $|z| = \sqrt{7}$ (d) $|z| = \sqrt{13}$ (e) $|z| = \sqrt{23}$

(5pts)Problem 3

The point of intersection of the line whose parametric equations are

$$\begin{cases} x = 2 - 2t \\ y = 3t \\ z = 1 + t \end{cases}$$

and the plane x + 2y - z = 7 is (a, b, c). a+b+c=

- (a) 7 (b) 6 (c) 5 (d) 4 (e) 0

(5pts)Problem 4

If the angle between the vectors $\langle 2, 1, -1 \rangle$ and $\langle 1, x, 0 \rangle$ is $\theta = \frac{\pi}{4}$, then x =

- (a) $1 \pm \sqrt{6}$ (b) $2 \pm \sqrt{6}$ (c) $1 \pm \sqrt{3}$ (d) $1 \pm \frac{\sqrt{3}}{2}$ (e) $1 \pm \frac{\sqrt{6}}{2}$

(5pts)Problem 5

Find the value of x for which the matrix A does not have an inverse.

$$A = \left(\begin{array}{ccc} 1 & -1 & -x \\ 0 & 1 & 3 \\ x & 0 & 0 \end{array}\right).$$

The sum of all possible values of x is

$$(a)$$
 4

$$(a) \ 4 \qquad (b) \ -6 \qquad (c) \ 3 \qquad (d) \ 0 \qquad (e) \ 5$$

$$(c)$$
 3

(5pts)Problem 6

Suppose two vectors \overrightarrow{a} and \overrightarrow{b} satisfy

$$\overrightarrow{a} \cdot \overrightarrow{b} = \sqrt{15}$$
 and $\overrightarrow{a} \times \overrightarrow{b} = \langle -2, 0, -1 \rangle$,

then the angle between \overrightarrow{a} and \overrightarrow{b} is

(a)
$$\frac{\pi}{6}$$

(b)
$$\frac{\pi}{3}$$

$$(c)$$
 $\frac{\pi}{2}$

(a)
$$\frac{\pi}{6}$$
 (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$ (e) $\frac{2\pi}{5}$

Part 2 Written 70%

(15pts)Problem 1

Consider the points $A\left(2,\ 0,\ -1\right),\ B\left(1,\ -1,\ 1\right),\ C\left(0,\ 3,\ -2\right)$ and $D\left(5,-2,\ -1\right).$

- (a) Find the equation of the plane (\mathcal{P}) containing the points A, B, and C.
- (b) Find the distance from D to (\mathcal{P}) .

(13pts)Problem 2

Find the equation of the plane that contains the two lines

$$L_1:$$

$$\begin{cases}
x = 1 + t \\
y = 1 - t \\
z = 2t
\end{cases}$$
 and $L_2:$

$$\begin{cases}
x = 2 - t \\
y = t \\
z = 2
\end{cases}$$

(13pts)Problem 3

Use the **Gauss elimination** method to solve the linear system

$$\begin{cases} 3x - y - 5z = 3\\ 4x - 4y - 3z = -4\\ x - 5z = 2 \end{cases}$$
 (Show your work)

(15pts)Problem 4

Consider the following matrices

$$A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 6 & -3 \end{bmatrix}, \qquad B = \begin{bmatrix} -1 & 1 & 2 \\ 4 & 0 & -5 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 2 & 0 \\ -3 & -1 \end{bmatrix}$$

- (a) Compute the following matrices, where possible.
 - 1. $A + B^T$,
- 2. *AC*

(b) Find the matrix X such that

$$\frac{3}{2}X + C = \left[\begin{array}{cc} 3 & -4 \\ 5 & 4 \end{array} \right].$$

(14pts)Problem 5

Use the cofactor expansion method to find the determinant of the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}.$$
 (Show your work)