Tutorial 1

MATH 141

1. Find each limit.

A. $\lim\limits_{\theta \to 0} \frac{\sin 2\theta}{\theta} = \lim\limits_{\theta \to 0} 2 \cdot \frac{\sin 2\theta}{2\theta}$

Recall

$\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$

$= 2 \cdot \lim\limits_{\theta \to 0} \frac{\sin 2\theta}{2\theta}$

$= 2 \cdot 1$

$= 2$

B. $\lim\limits_{y \to \infty} \frac{\sqrt{y^2+2}}{5y-6} = \lim\limits_{y \to \infty} \frac{\sqrt{y^2}}{5y}$

$= \lim\limits_{y \to \infty} \frac{|y|}{5y}$

$= \lim\limits_{y \to \infty} \frac{y}{5y}$ $\qquad y > 0$

$= \frac{1}{5}$

C. $\lim\limits_{t \to 1^+} \frac{|1-t|}{1-t} = \lim\limits_{t \to 1^+} \frac{-(1-t)}{1-t}$

Recall

$|f(x)| = \begin{cases} f(x) & f(x) \geq 0 \\ -f(x) & f(x) < 0 \end{cases}$

$t \approx 1.001$

$= \lim\limits_{t \to 1^+} (-1)$

$= -1$

2. Find each of these limits.

$$f(x) = \frac{x-2}{|x|-2}$$

- $\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{x-2}{-x-2}$ $x < 0 \Rightarrow |x| = -x$

$$= \lim_{x \to -\infty} \left(\frac{x}{-x} \right)$$

$$= \lim_{x \to -\infty} (-1)$$

$$= -1$$

- $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{x-2}{x-2}$ $x > 0 \Rightarrow |x| = x$

$$= \lim_{x \to \infty} (1)$$

$$= 1$$

- $\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} \frac{x-2}{-x-2}$ $x < 0$

$$= \frac{-2-2}{0^+}$$ $x \approx -2.001$

$$= \frac{-4}{0^+}$$

$$= -\infty$$

$$\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} \frac{x-2}{-x-2} \qquad x < 0$$

$$= \frac{-2-2}{0^-} \qquad x \approx -1.999$$

$$= \frac{-4}{0^-}$$

$$= +\infty$$

$$x \to 2 \qquad x > 0 \Rightarrow |x| = x$$

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{x-2}{x-2}$$

$$= \lim_{x \to 2} (1)$$

$$= 1$$

3. $$\lim_{x \to 2} \frac{1 - \frac{4}{x^2}}{1 - \frac{2}{x}} = \frac{1 - \frac{4}{4}}{1 - \frac{2}{2}}$$

$$= \frac{1-1}{1-1}$$

$$= \frac{0}{0} \quad \text{Indi. Form}$$

$$\lim_{x \to 2} \frac{\frac{x^2-4}{x^2}}{\frac{x-2}{x}} = \lim_{x \to 2} \frac{x(x^2-4)}{x^2(x-2)}$$

$$= \lim_{x \to 2} \frac{x\cancel{(x-2)}(x+2)}{x^2 \cancel{(x-2)}} = \lim_{x \to 2} \frac{x+2}{x} = \frac{2+2}{2}$$

$$= 2$$

$$\lim_{x \to \infty} \frac{x + \frac{2}{x}}{x - \frac{3}{x}} = \frac{\infty}{-\infty} \text{ or } \frac{-\infty}{\infty} \quad \text{Ind. Forms}$$

$$\lim_{x \to \infty} \frac{\frac{x^2+2}{x}}{\frac{x^2-3}{x}} = \lim_{x \to \infty} \frac{x^2+2}{x^2-3}$$

$$= \frac{2}{-3}$$

$$= \boxed{-\frac{2}{3}}$$

4. A. $\lim_{x \to \infty} \frac{2x^3-6}{x^k+3} = \lim_{x \to \infty} \frac{2x^3}{x^k}$

$= 0$ or 2 if $k \geq 3$.

B. $\lim_{x \to 2} \frac{x^2+kx-10}{x-2}$

For $k=3$

$x^2+kx-10 = x^2+3x-10$

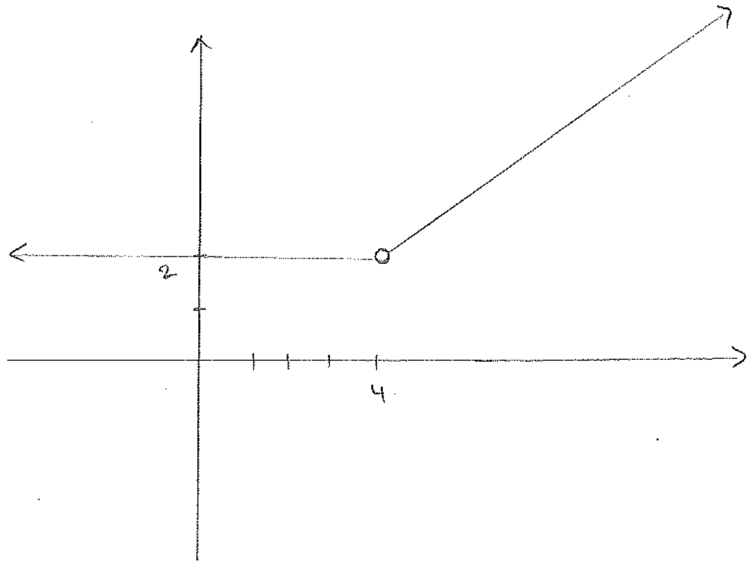
$= (x-2)(x+5)$

$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x+5)}{\cancel{x-2}}$

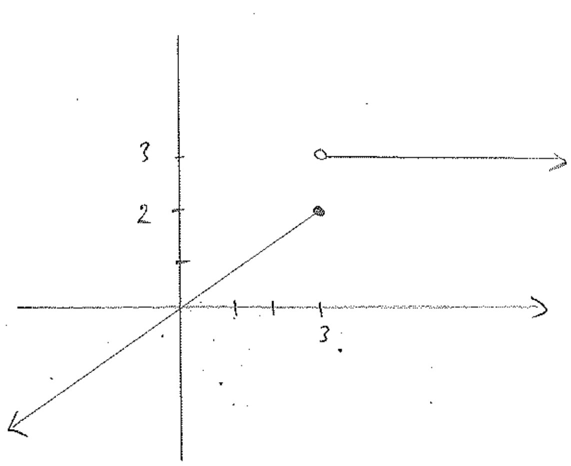
$= 2+5$

$= 7$

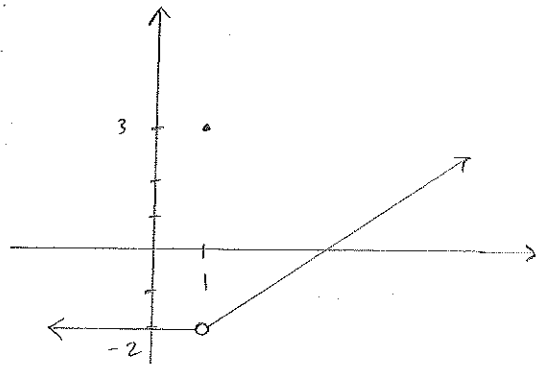
5. A. $f(4)$ is undefined and $\lim\limits_{x \to 4} f(x) = 2$.



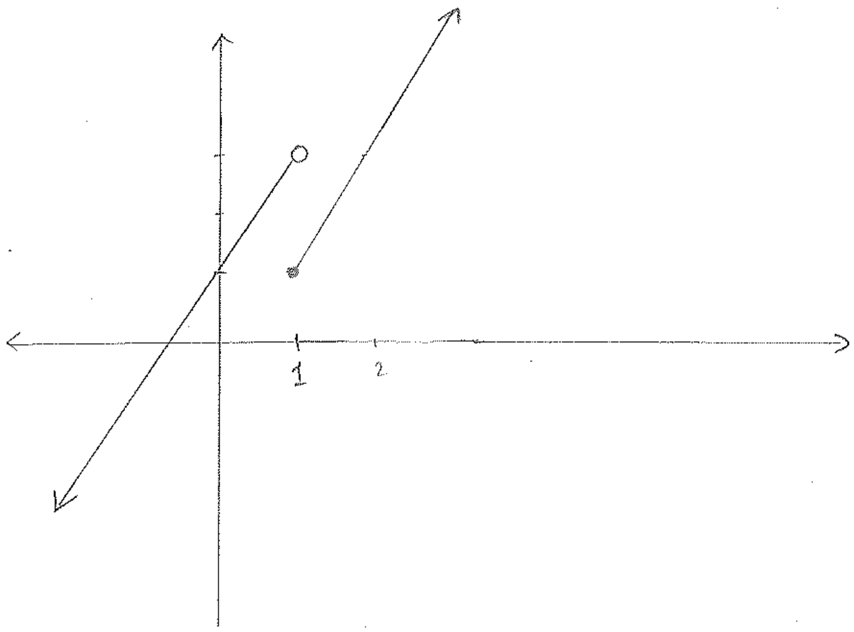
B. $f(3) = 2$ and $\lim\limits_{x \to 3} f(x)$ DNE.



C. $f(1) = 3$ and $\lim\limits_{x \to 1} f(x) = -2$



6. $$f(x) = \begin{cases} 2x+1, & x<1 \\ 1, & x=1 \\ 2x-1, & x>1 \end{cases}$$

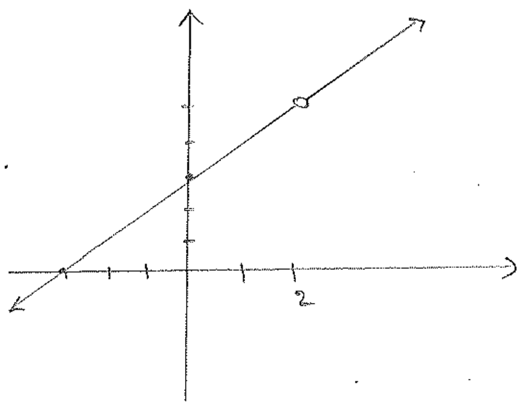


It is a jump Discontinuity.

7. $$f(x) = \frac{x^2+x-6}{x-2}$$

$$= \frac{(x+3)\cancel{(x-2)}}{\cancel{x-2}}$$

$$= x+3 \quad \text{if } x \neq 2$$



$$f(x) = \begin{cases} \dfrac{x^2+x-6}{x-2} & \text{if } x \neq 2 \\ 5 & \text{if } x = 2 \end{cases}$$

8. $f(x) = \begin{cases} \dfrac{x^2-x-2}{x+1} & x > -1 \\ A & x \leq -1 \end{cases}$

f is cont. at $x=-1$ iff

$\lim\limits_{x \to -1} f(x) = f(-1)$

OR $\lim\limits_{x \to -1^-} f(x) = \lim\limits_{x \to -1^+} f(x) = f(-1)$

$f(-1) = A$ $\quad \lim\limits_{\substack{x \to -1^+ \\ x > -1}} f(x) = \lim\limits_{x \to -1^+} \dfrac{x^2-x-2}{x+1}$

$= \lim\limits_{x \to -1^+} \dfrac{(x-2)(x+1)}{x+1}$

$= -1-2$

$= -3$

$\therefore \boxed{A = -3}$

9. $\lim\limits_{x \to 0^+} \sqrt{x} \cos \dfrac{1}{x^2}$

$-1 \leq \cos \dfrac{1}{x^2} \leq 1$

$-\sqrt{x} \leq \sqrt{x} \cos \dfrac{1}{x^2} \leq \sqrt{x}$

$\lim\limits_{x \to 0^+} -\sqrt{x} = \lim\limits_{x \to 0^+} \sqrt{x} = 0$. Hence $\lim\limits_{x \to 0^+} \sqrt{x} \cos \dfrac{1}{x^2} = 0$.