Problem 1

a) $\vec{PQ} = \langle 1, 2, -1 \rangle$

$\vec{PR} = \langle -2, 2, 3 \rangle$

b) $$\text{proj}_{\vec{PR}} \vec{PQ} = \left(\frac{\vec{PQ} \cdot \vec{PR}}{\|\vec{PR}\|^2} \right) \cdot \vec{PR}$$

$$= \left(\frac{-2 + 4 - 3}{(\sqrt{4+4+9})^2} \right) \cdot \vec{PR}$$

$$= \frac{-1}{17} \langle -2, 2, 3 \rangle$$

$$= \left\langle \frac{2}{17}, \frac{-2}{17}, \frac{-3}{17} \right\rangle$$

c) $$\text{Area of } \Delta = \frac{1}{2} \|\vec{PQ} \times \vec{PR}\|$$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -2 & 2 & 3 \end{vmatrix} = \langle 8, -1, 6 \rangle$$

$$\|\vec{PQ} \times \vec{PR}\| = \sqrt{8^2 + (-1)^2 + 6^2}$$

$$= \sqrt{64 + 36 + 1}$$

$$= \sqrt{101}$$

$$\therefore \text{Area of } \Delta = \frac{\sqrt{101}}{2}$$

Problem 2

a) $\text{proj}_{\vec{u}} \vec{v} = \left(\dfrac{\vec{v} \cdot \vec{u}}{\| \vec{u} \|^2} \right) \vec{u}$

$= \left(\dfrac{6 - 3 - 1}{9 + 4 + 1} \right) \langle 2, -3, 1 \rangle$

$= \dfrac{1}{7} \langle 2, -3, 1 \rangle$

$= \left\langle \dfrac{2}{7}, \dfrac{-3}{7}, \dfrac{1}{7} \right\rangle$

b) $\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 1 & -2 & 2 \end{vmatrix} = \langle -2, -5, -4 \rangle$

Unit vector $= \dfrac{\vec{u} \times \vec{v}}{\| \vec{u} \times \vec{v} \|}$

$\| \vec{u} \times \vec{v} \| = \sqrt{4 + 25 + 16} = \sqrt{45} = 3\sqrt{5}$

Unit vector $= \left\langle \dfrac{-2}{3\sqrt{5}}, \dfrac{-5}{3\sqrt{5}}, \dfrac{-4}{3\sqrt{5}} \right\rangle$

Vector having magnitude 7

$= 7 \left\langle \dfrac{-2}{3\sqrt{5}}, \dfrac{-5}{3\sqrt{5}}, \dfrac{-4}{3\sqrt{5}} \right\rangle$

$= \left\langle \dfrac{-14}{3\sqrt{5}}, \dfrac{-35}{3\sqrt{5}}, \dfrac{-28}{3\sqrt{5}} \right\rangle$

c) $\text{proj}_{\vec{v}}\,\vec{u} = \left(\dfrac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\right)\cdot\vec{v}$

$= \left(\dfrac{20-0-3}{25+9}\right)\langle 5,0,-3\rangle$

$= \dfrac{1}{2}\langle 5,0,-3\rangle$

$= \left\langle \dfrac{5}{2}, 0, \dfrac{-3}{2}\right\rangle = \vec{w}$

$\vec{u}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & 1 \\ 5 & 0 & -3 \end{vmatrix} = \langle 6, 17, 10\rangle$

$2\vec{u}\times\vec{v} - 3\vec{v} = \langle 12, 34, 20\rangle - \langle 15, 0, -9\rangle$

$= \langle -3, 34, 29\rangle$

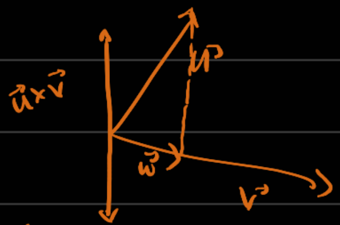
$\vec{w}\cdot[(2\vec{u}\times\vec{v}) - 3\vec{v}] = \left\langle \dfrac{5}{2}, 0, \dfrac{-3}{2}\right\rangle\cdot\langle -3, 34, 29\rangle$

$= \dfrac{-15}{2} + 0 - \dfrac{87}{2}$

$= -51$

UNNECESSARY

$\vec{w}\,(2\vec{u}\times\vec{v}) = 0$



$\vec{w}\perp(\vec{u}\times\vec{v})$

$\therefore$ Dot product $= 0$

$-3\left(\dfrac{\vec{u}\cdot\vec{v}}{\cancel{\|\vec{v}\|^2}}\right)\underbrace{\vec{v}\cdot\vec{v}} \Rightarrow \cancel{\|\vec{v}\|^2}$

$= -3\,\vec{u}\cdot\vec{v}$

d) $\|\vec{u}-\vec{v}\|^2 = (\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v})$

$= \vec{u}\cdot\vec{u} - 2\,\vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{v}$

$= \|\vec{u}\|^2 - 2\,\vec{u}\cdot\vec{v} + \|\vec{v}\|^2$

$= 9 - 2\left(\|\vec{u}\|\,\|\vec{v}\|\cos\theta\right) + 4$

$= 13 - 2\left(3\times 2\times\cos\dfrac{\pi}{3}\right)$

$= 13 - 2\left(6\times\dfrac{1}{2}\right)$

$= 7$

$\therefore \|u-v\| = \sqrt{7}$

Problem 3

a) $\vec{u} = \vec{AB}$

$= \langle -3, 0, 1 \rangle$

$$\begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases} \qquad \begin{aligned} x &= 7 - 3t \\ y &= 6 \\ z &= 4 + t \end{aligned}$$

b) $\frac{x - x_0}{a} = t \qquad \frac{y - y_0}{b} = t \qquad \frac{z - z_0}{c} = t$

$$\begin{cases} x = 1 + 5t \\ y = -3 + 3t \\ z = 2 + 2t \end{cases}$$

Problem 4

a) $R^2 = 1^2 + 1^2 + 1^2$

$R^2 = 3$

$$(x-1)^2 + (y-1)^2 + (z-1)^2 = 3$$

b) $x^2 - 4x + 4 - 4 + y^2 + 4y + 4 - 4 + z^2 = 8$

$(x-2)^2 - 4 + (y+2)^2 - 4 + z^2 = 8$

$(x-2)^2 + (y+2)^2 + z^2 = 16$

Centre $= (2, -2, 0)$

Radius $= 4$

$$x^2 - 4x + 4 - 4 + y^2 + z^2 - 6z + 9 - 9 = 3$$

$$(x-2)^2 + y^2 + (z-3)^2 - 4 - 9 = 3$$

$$(x-2)^2 + y^2 + (z-3)^2 = 16$$

Centre $= (2, 0, 3)$

Radius $= 4$

$$x^2 + y^2 + 2y + z^2 + 4z = 20$$

$$x^2 + y^2 + 2y + 1 - 1 + z^2 + 4z + 4 - 4 = 20$$

$$x^2 + (y+1)^2 - 1 + (z+2)^2 - 4 = 20$$

$$x^2 + (y+1)^2 + (z+2)^2 = 25$$

Centre $: (0, -1, -2)$

Radius $: 5$

Problem 5

$\vec{n_1} = \langle 1, -2, 1 \rangle$

$\vec{n_2} = \langle 2, 1, -1 \rangle$

$$\vec{u} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = \langle 1, 3, 5 \rangle$$

Let $z = 0$

$x - 2y = 10$

$2x + y = 0$

$x = 2y + 10$

$2(2y + 10) + y = 0$

$5y + 20 = 0$

$y = -4$

$x = -8 + 10$

$x = 2$

Parametric Equations

$$\begin{cases} x = 2 + t \\ y = -4 + 3t \\ z = 5t \end{cases}$$

Problem 6

$$D = \frac{|\vec{QP} \cdot \vec{n}|}{\|\vec{n}\|}$$

Let $Q = (0,0,5)$

$\vec{n} = \langle 2,-2,1 \rangle$

$\vec{QP} = \langle 1,-2,-2 \rangle$

$\vec{QP} \cdot \vec{n} = \langle 1,-2,-2 \rangle \cdot \langle 2,-2,1 \rangle$

$= 2 + 4 - 2$

$= 4$

$\|\vec{n}\| = \sqrt{4+4+1} = 3$

$\therefore D = \frac{4}{3}$ units

Problem 7

$x = -2 + 3t$

$y = \quad -2t$

$z = 1 + 4t$

$\vec{u} = \langle 3, -2, 4 \rangle$

$P(-2, 0, 1)$

$$D = \frac{\| \vec{PQ} \times \vec{u} \|}{\| \vec{u} \|}$$

$\vec{PQ} = \langle 5, -1, 3 \rangle$

$$\vec{PQ} \times \vec{u} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -1 & 3 \\ 3 & -2 & 4 \end{vmatrix} = \langle 2, -11, -7 \rangle$$

$\| \vec{PQ} \times \vec{u} \| = \sqrt{4 + 121 + 49} = \sqrt{174}$

$\| \vec{u} \| = \sqrt{9 + 4 + 16} = \sqrt{29}$

$$\therefore d = \sqrt{\frac{174}{29}}$$

$= 2.4495$ units