EXAMINATION COVERSHEET

Autumn 2023 Quiz 2

UNIVERSITY OF WOLLONGONG IN DUBAI

THIS EXAMINATION CONTENT IS STRICTLY CONFIDENTIAL Students must comply with requirements stated in the Examination Policy & Procedures	
Student Number:	
First Name:	
Family Name:	
Date of Examination: (DD/MM/YY)	**11/23/2023**
Subject Code:	**Math 141**
Subject Title:	**Foundation of Engineering Mathematics**
Time Permitted to Write Exam:	2 Hours
Total Number of Questions:	**4 (4 written questions)**
Total Number of Pages *(including this page)*:	**5**

INSTRUCTIONS TO STUDENTS FOR THE EXAM

1. Please note that subject lecturer/tutor will be unavailable during exams. *If there is a doubt in any of the exam questions i.e. problem solving etc. students should proceed by assuming values etc. Students should mention their assumption on the question paper.*
2. Answers must be written (and drawn) in black or blue ink
3. Any mistakes must be crossed out. Whitener and ink erasers must not be used.
4. Answer ALL/ 4 questions. The marks for each question are shown next to each question.
5. Total marks: 40.

($10pts$) **Problem 1**

Solve the quadratic equation and give the answer in the form $a + ib$

$$z^2 - (1-4i)\,z - (5-i) = 0.$$

Solution

$$z^2 - (1-4i)\,z - (5-i) = 0.$$

$$a = 1, \qquad b = -\,(1-4i) \quad \text{and } c = -\,(5-i)\,.$$

The solutions are given by

$$z = \frac{(1-4i) \pm \sqrt{(1-4i)^2 + 4\,(5-i)}}{2}$$

$$z = \frac{(1-4i) \pm \sqrt{5-12i}}{2}. \qquad \text{[5 points]}$$

Next, we need to find $\sqrt{5-12i}$. Put

$$\sqrt{5-12i} = a + ib \Leftrightarrow$$

$$\begin{aligned} (a+ib)^2 &= 5-12i \;\Leftrightarrow \\ a^2 + 2iab - b^2 &= 5-12i \Rightarrow \end{aligned}$$

$$\begin{cases} a^2 - b^2 = 5 \\ ab = -6 \end{cases} \Rightarrow a = 3 \text{ and } b = -2. \text{ (Using the sign convention)}$$

$$\sqrt{5-12i} = 3-2i. \qquad \text{[3 points]}$$

$$z_1 = \frac{(1-4i)+3-2i}{2} = 2-3i \text{ and } z_2 = \frac{(1-4i)-(3-2i)}{2} = -1-i. \qquad \text{[2 points]}$$

($10pts$) **Problem 2**
Which pairs from the following list of planes are perpendicular to one another?

$$x + \sqrt{3}z = 1, \qquad x + \sqrt{3}y = 2, \qquad \sqrt{3}x + y - z = 3$$

Solution
Denote by $\overrightarrow{n_1}$, $\overrightarrow{n_2}$ and $\overrightarrow{n_3}$ the normal vectors of these panes respectively.

$$\overrightarrow{n_1} = \left\langle 1,\ 0,\ \sqrt{3} \right\rangle, \quad \overrightarrow{n_2} = \left\langle 1,\ \sqrt{3},\ 0 \right\rangle, \quad \overrightarrow{n_3} = \left\langle \sqrt{3},\ 1\ -1 \right\rangle \qquad \text{[2 points]}$$

$$\overrightarrow{n_1} \cdot \overrightarrow{n_2} = \left\langle 1,\ 0,\ \sqrt{3} \right\rangle \cdot \left\langle 1,\ \sqrt{3},\ 0 \right\rangle = 1 \qquad \text{[2 points]}$$

$$\overrightarrow{n_1} \cdot \overrightarrow{n_3} = \left\langle 1,\ 0,\ \sqrt{3} \right\rangle \cdot \left\langle \sqrt{3},\ 1\ -1 \right\rangle = 0 \qquad \text{[2 points]}$$

$$\overrightarrow{n_2} \cdot \overrightarrow{n_3} = \left\langle 1,\ \sqrt{3},\ 0 \right\rangle \cdot \left\langle \sqrt{3},\ 1\ -1 \right\rangle = 2\sqrt{3} \qquad \text{[2 points]}$$

So the only two planes that are perpendicular are

$$x + \sqrt{3}z = 1 \ \text{ and } \ \sqrt{3}x + y - z = 3 \qquad \text{[2 points]}$$

($10pts$) **Problem 3**

(A) Let $\overrightarrow{u}$ and $\overrightarrow{v}$ be two vectors such that $\|\overrightarrow{u}\| = \frac{1}{2}$, $\|\overrightarrow{v}\| = 2$ and $\theta = \frac{\pi}{4}$ the angle between $\overrightarrow{u}$ and $\overrightarrow{v}$. Find $\|\overrightarrow{u} - \overrightarrow{v}\|$.

Solution of (A)

$$\begin{aligned}
\|\overrightarrow{u} - \overrightarrow{v}\|^2 &= (\overrightarrow{u} - \overrightarrow{v}) \cdot (\overrightarrow{u} - \overrightarrow{v}) \\
&= \|\overrightarrow{u}\|^2 - 2\overrightarrow{u} \cdot \overrightarrow{v} + \|\overrightarrow{v}\|^2 \\
&= \|\overrightarrow{u}\|^2 - 2\|\overrightarrow{u}\| \|\overrightarrow{v}\| \cos\frac{\pi}{4} + \|\overrightarrow{v}\|^2 \\
&= \frac{1}{4} - \sqrt{2}2 + 4 \\
&= \frac{17}{4} - \sqrt{2} = 2.8358
\end{aligned}$$

$$\|\overrightarrow{u} - \overrightarrow{v}\| = \sqrt{\frac{17}{4} - \sqrt{2}} \qquad = 1.6840 \qquad \text{[5 points]}$$

(B) A video store sells videos, tapes, CDs, and computer games. We define the quantity vector $\overrightarrow{q} = (q_1, q_2, q_3, q_4)$, where q_1, q_2, q_3, q_4 denote the quantities sold of each of the items, and the price vector $\overrightarrow{p} = (p_1, p_2, p_3, p_4)$, where $p_1, p_{2,}p_3, p_4$ denote the price per unit of each item. What does the dot product $\overrightarrow{p} \cdot \overrightarrow{q}$ represent?

Solution of (B)

The dot product is

$$\overrightarrow{p} \cdot \overrightarrow{q} = p_1q_1 + p_2q_2 + p_3q_3 + p_4q_4.$$

The quantity p_1q_1 represents the revenue received by the store for the videos, p_2q_2 represents the revenue for the tapes, and so on.

The dot product represents the total revenue received by the store for the sale of these four items. [5 points]

(10*pts*)**Problem** 4

Find parametric equations for the line of intersection of the planes $4x + 4y - 2z = 9$ and $2x + y + z = -3$.

Solution

Let $\overrightarrow{n_1}$ and $\overrightarrow{n_2}$ be the two normal vectors of the planes respectively.

$$\overrightarrow{n_1} = \langle 4, 4, -2 \rangle \,, \qquad \overrightarrow{n_2} = \langle 2, 1, -1 \rangle \,.$$

A direction vector of the line of intersection is

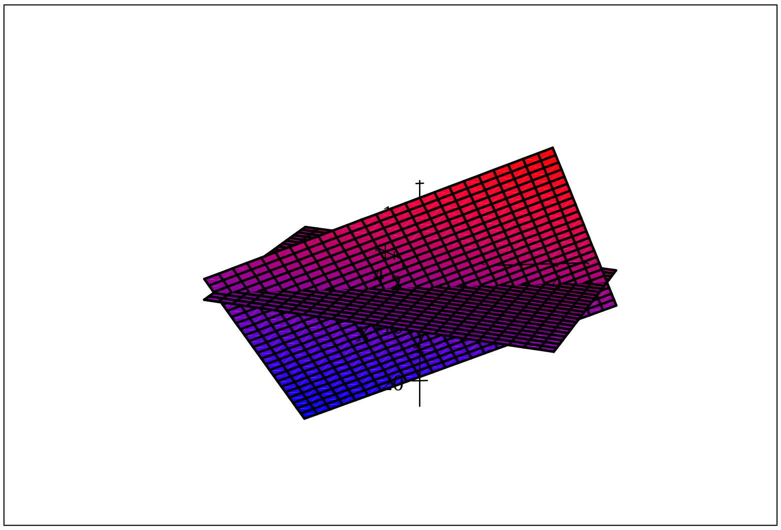
$$\overrightarrow{u} = \overrightarrow{n_1} \times \overrightarrow{n_2} = \begin{vmatrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\ 4 & 4 & -2 \\ 2 & 1 & 1 \end{vmatrix} = \langle 6,\ -8,\ -4 \rangle \qquad \text{[4 points]}\,.$$

Now to find a point on the line of intersection, we put $z = 0$ to get

$$\begin{cases} 4x + 4y = 9 \\ 2x + y = -3 \end{cases}$$

Solution is: $\left[x = -\frac{21}{4}, y = \frac{15}{2}\right]$. Thus $\left(-\frac{21}{4}, \frac{15}{2}, 0\right)$ is a point on the line of intersection.
The parametric equations are

$$\begin{cases} x = -\frac{21}{4} + 6t \\ y = \frac{15}{2} - 8t \\ z = -4t \end{cases} . \qquad \text{[6 points]}$$



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