prob. **1**

$$f(x) = -2x^3 + 9x^2 + 24$$

$$f'(x) = -6x^2 + 18x$$

$$f'(x) = 0 \Rightarrow -6x^2 + 18x = 0$$

$$6x(-x + 3) = 0$$

$$x = 0 \quad \text{or} \quad x = 3 \quad \text{C.Ns}$$

x	$-\infty$	0		3	∞
f'	$- - -$	0	$+ + +$	0	$- - -$
f	$\searrow$	R.min 24	$\nearrow$	R.max 51	$\searrow$

a) f is increasing on $(0, 3)$

b) f is decreasing on $(-\infty, 0)$ and $(3, \infty)$

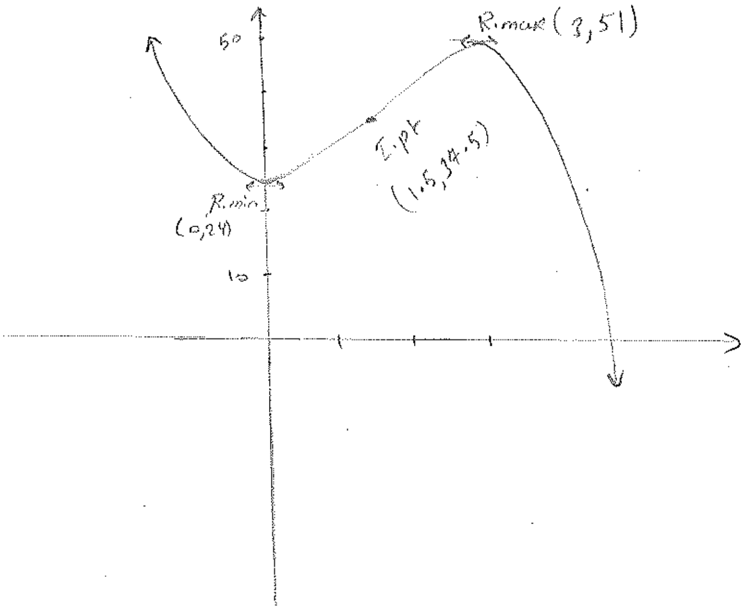
c) R.max at $x = 3$ $\quad (3, 51)$

d) R.min at $x = 0$ $\quad (0, 24)$

e) $f''(x) = -12x + 18$

$f''(x) = 0 \rightarrow -12x + 18 = 0$

$x = \frac{18}{12} = 1.5$



x	$-\infty$	1.5	∞
f''	$+ + + +$	0	$- - - -$
f	$\cup$		$\cap$

I.pt $= f(1.5)$

$= 37.5$

$(1.5, 37.5)$

prob. 2 $\quad f(x) = 12x^{2/3} - 16x$

Domain: $(-\infty, \infty)$

$f'(x) = 12\left(\frac{2}{3}\right)x^{-1/3} - 16$

$f'(x) = \frac{8}{x^{1/3}} - 16 \rightarrow f'(x) = \frac{8 - 16x^{1/3}}{x^{1/3}}$

$f'(x) = 0 \Rightarrow 8 - 16x^{1/3} = 0$

$x^{1/3} = \frac{8}{16} = \frac{1}{2} \rightarrow x = \frac{1}{8}$ C.N

$f'(x) =$ undefined $\Rightarrow x^{1/3} = 0 \rightarrow x = 0$

x	$-\infty$	0		$1/8$	∞
f'	$- - - -$		$+ + + +$	0	$- - - -$
f	$\searrow$		$\nearrow$		$\searrow$

R.min $= 0$ $\qquad$ R.max $= f\left(\frac{1}{8}\right) = 1$

prob. 3 $f(x) = x^3 - 12x + 8 \quad x \in [-4, 3]$

$f'(x) = 3x^2 - 12$

$f'(x) = 0 \rightarrow \quad 3x^2 - 12 = 0$

$3x^2 = 12$

$x^2 = 4 \rightarrow x = \pm 2$

x	$f(x)$
-4	$f(-4) = -8 \rightarrow$ Absolute min
-2	$f(-2) = 24 \rightarrow$ Absolute max.
2	$f(2) = -8 \rightarrow$ Absolute min
3	$f(3) = -1$

$f(x) = x^{4/3} - 3x^{1/3} \quad x \in [-1, 8]$

$f'(x) = \frac{4}{3}x^{1/3} - x^{-2/3}$

$f'(x) = \frac{4}{3}x^{1/3} - \frac{1}{x^{2/3}} \Rightarrow f'(x) = \frac{4x-3}{3x^{2/3}}$

$f'(x) = 0 \rightarrow \quad 4x - 3 = 0$

$x = 3/4$

$f'(x) =$ und. $\rightarrow \quad 3x^{2/3} = 0$

$x = 0$

x	$f(x)$
-1	$f(-1) = 4$
0	$f(0) = 0$
$3/4$	$f(3/4) = -2.04 \rightarrow$ Absolute min.
8	$f(8) = 10 \rightarrow$ Absolute max

prob. 4 $f(x) = x^4 - 24x^2$

$f'(x) = 4x^3 - 48x$

$f''(x) = 12x^2 - 48$

$f''(x) = 0 \longrightarrow 12x^2 - 48 = 0$

$x^2 = \frac{48}{12}$

$x^2 = 4 \longrightarrow x = \pm 2$

x	$-\infty$	-2		2	∞
f''	$+ + +$	0	$- - -$	0	$+ + + +$
f	$\cup$		$\cap$		$\cup$
		-80		-80	

I. pt $(-2, -80)$

I. pt $(2, -80)$

$f(x) = 4x^3 - 15x^2 - 18x + 10$

$f'(x) = 12x^2 - 30x - 18$

$f''(x) = 24x - 30$

$f''(x) = 0 \longrightarrow 24x - 30 = 0$

$24x = 30$

$x = \frac{30}{24} = \frac{5}{4}$

x	$-\infty$	$5/4 = 1.25$		$+\infty$
f''	$- - - -$	0	$+ + + +$	
f	$\cap$		$\cup$	
		-28.125		

I. pt $(1.25, -28.125)$

prob. 5 $\quad f(x) = x^3 + \frac{3}{2}x^2 - 6x + 12$

① Domain: $(-\infty, \infty)$

② $f'(x) = 0 \rightarrow \quad 3x^2 + 3x - 6 = 0$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$x = -2 \qquad x = 1 \quad \text{C.Ns.}$$

x	$-\infty$	-2		1		$+\infty$
f'	$+ + +$	0	$- - -$	0	$+ + +$	
f	↗		↘		↗	
		R.max 22		R.min 8.5		

R.max $(-2, 22)$

R.min $(1, 8.5)$

③ $f''(x) = 0 \longrightarrow \quad 6x + 3 = 0$

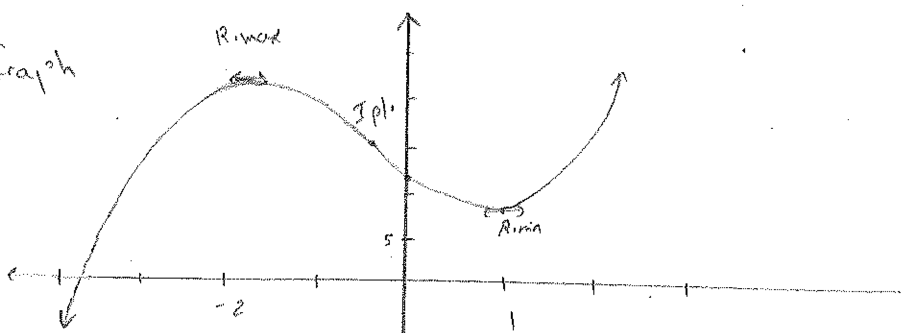
$$x = -3/6 \rightarrow x = -1/2$$

x	$-\infty$	$-1/2$		∞
f''	$- - -$	0	$+ + +$	
f	∩		∪	
		15.25		

I.pt $(-0.5, 15.25)$

④ y-int $\rightarrow x = 0 \quad y = 12 \quad (0, 12)$

⑤ Graph



$f(x) = x^{1/3}(x+4)$

OR $f(x) = x^{4/3} + 4x^{1/3}$

(1) Domain: $(-\infty, \infty)$

(2) $f'(x) = \frac{4}{3}x^{1/3} + \frac{4}{3} \cdot x^{-2/3}$

$f'(x) = \frac{4}{3}x^{1/3} + \frac{4}{3x^{2/3}}$

$f'(x) = \frac{4}{3}\left(x^{1/3} + \frac{1}{x^{2/3}}\right)$

$f'(x) = \frac{4}{3}\left(\frac{x+1}{x^{2/3}}\right)$

$f'(x) = 0 \rightarrow x + 1 = 0$

$x = -1$ C.N

$f'(x) = \text{und.} \rightarrow x^{2/3} = 0 \rightarrow x = 0$ C.N

x	$-\infty$	-1		0		∞
f'	$- - -$	0	$+ + +$	$\not\exists$	$+ + +$	
f	$+\infty$ ↘		↗		↗	∞
		Rmin -3				

$\boxed{\text{Rmin } (-1, -3)}$

(3) $f'(x) = \frac{4}{3}\left(x^{1/3} + x^{-2/3}\right)$

$f''(x) = \frac{4}{3}\left(\frac{1}{3}x^{-2/3} - \frac{2}{3}x^{-5/3}\right)$

$= \frac{4}{3} \cdot \frac{1}{3}\left(x^{-2/3} - 2x^{-5/3}\right)$

$= \frac{4}{9}\left(\frac{1}{x^{2/3}} - \frac{2}{x^{5/3}}\right)$

$= \frac{4}{9}\left(\frac{x}{x^{5/3}} - \frac{2}{x^{5/3}}\right)$

$= \frac{4}{9}\left(\frac{x-2}{x^{5/3}}\right)$

$f''(x) = 0 \rightarrow x = 2$

$f''(x) = \text{und.}\ \ x^{5/3} = 0 \rightarrow x = 0$

x	$-\infty$	0		2		∞
f''	$+ + +$	$\not\exists$	$- - -$	0	$+ + + +$	
f	$\cup$		$\cap$		$\cup$	
		0		7.6		

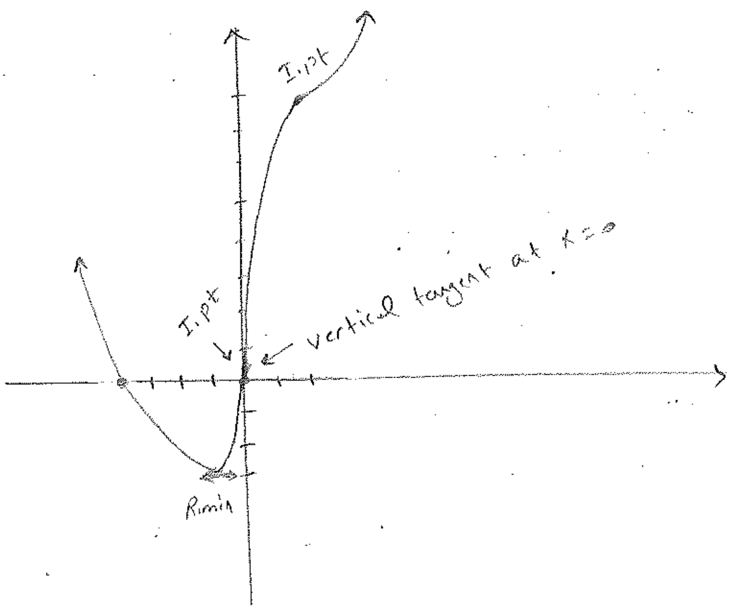
$\boxed{\begin{array}{l} \text{I.p: } (0,0) \\ \text{I.p: } (2, 7.6) \end{array}}$

(4) $x\text{-int} \rightarrow y = 0$

$x^{1/3}(x+4) = 0$

$x = 0$ or $x = -4$ $\boxed{(0,0)\ (-4,0}$

$y\text{-int} \rightarrow x = 0 \quad y = 0$



prob 6 Volume of the box = $l \times w \times h$ Surface Area = $10\,u^2$

$$V = w \times w \times h$$

$$\boxed{V = w^2 h}$$

$V_{max} = ?$ constraint $2w^2 + 4wh = 10$

$2w^2 + 4wh = 10 \Rightarrow 4wh = 10 - 2w^2$

$$h = \frac{10 - 2w^2}{4w}$$

$$h = \frac{5 - w^2}{2w}$$

substitute in $V = w^2 h$

$$V = w^2\left(\frac{5 - w^2}{2w}\right)$$

$$V = \frac{5}{2}w - \frac{w^3}{2}$$

we have $\dfrac{dV}{dw} = \frac{5}{2} - \frac{3}{2}w^2$ $\qquad \dfrac{dV}{dw} = V'(w)$

$V'(w) = 0 \Rightarrow \frac{5}{2} - \frac{3}{2}w^2 = 0$

$w^2 = \frac{5}{3} \Rightarrow w = \sqrt{5/3}$

$V''(w) = -3w \qquad V''(\sqrt{5/3}) < 0 \rightarrow$ we have

max. at $w = \sqrt{5/3}$

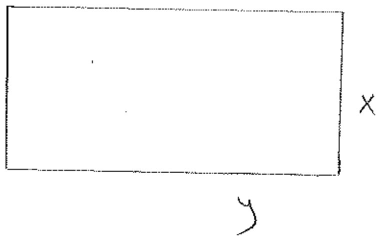
$$h = \frac{\sqrt{15}}{3}$$

$$V_{max} = \left(\sqrt{5/3}\right)^2 \cdot \left(\frac{\sqrt{15}}{3}\right) = \frac{5}{3} \cdot \frac{\sqrt{15}}{3}$$

$$= \frac{5\sqrt{15}}{9}$$

prob. 7

(10)



Area $= x \cdot y$ Area $=$ 100 square units

Perimeter $= 2x + 2y$

$\boxed{P = 2x + 2y}$ $P_{min} = ?$ constraint $x \cdot y = 100$

$x \cdot y = 100 \rightarrow y = \frac{100}{x}$

$P = 2x + 2\left(\frac{100}{x}\right)$

$P = \frac{2x^2 + 200}{x}$ or $P = 2x + \frac{200}{x}$

$P'(x) = 2 - \frac{200}{x^2}$ $P'(x) = 0 \rightarrow 2 - \frac{200}{x^2} = 0$

$\frac{200}{x^2} = \frac{2}{1}$

$x^2 = 100$

$x = 10$

$P''(x) = \frac{400}{x^3} \rightarrow P''(10) > 0$ so min at $x = 10$

$x = 10 \Rightarrow y = 10$ $\therefore P_{min} = 2(10) + 2(10)$

$= 40$ units.