EXAMINATION COVERSHEET
Autumn 2023 Quiz 1

UNIVERSITY OF WOLLONGONG IN DUBAI

THIS EXAMINATION CONTENT IS STRICTLY CONFIDENTIAL Students must comply with requirements stated in the Examination Policy & Procedures	
Student Number:	
First Name:	
Family Name:	
	Solution Key
Date of Examination: (DD/MM/YY)	**10/19/2023**
Subject Code:	**Math 141**
Subject Title:	**Foundation of Engineering Mathematics**
Time Permitted to Write Exam:	1 Hour
Total Number of Questions:	**5 written questions**
Total Number of Pages *(including this page)*:	**6**

INSTRUCTIONS TO STUDENTS FOR THE EXAM

1. Please note that subject lecturer/tutor will be unavailable during exams. *If there is a doubt in any of the exam questions i.e. problem solving etc. students should proceed by assuming values etc. Students should mention their assumption on the question paper.*
2. Answers must be written (and drawn) in black or blue ink
3. Any mistakes must be crossed out. Whitener and ink erasers must not be used.
4. Answer ALL/ 5 questions. The marks for each question are shown next to each question.
5. Total marks: 40.

(10pts) **Problem 1**

A) Evaluate the following limits

$$(a)\quad \lim_{x\to 0}\frac{\sin x - x}{x^2} \qquad\qquad (b)\quad \lim_{x\to -\infty}\frac{2x\,|x|+5x+1}{3x^2+x-2}$$

Solution

(a)

$$\begin{aligned}\lim_{x\to 0}\frac{\sin x - x}{x^2} &= \lim_{x\to 0}\frac{\cos x - 1}{2x}\\ &= \lim_{x\to 0}\frac{-\sin x}{2}\\ &= 0 \qquad (\mathbf{3pts})\end{aligned}$$

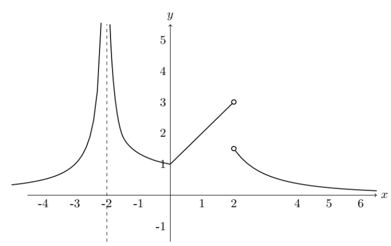
(b)

$$\lim_{x\to -\infty}\frac{2x\,|x|+5x+1}{3x^2+x-2}$$

Note that $x \to -\infty \Rightarrow x < 0$ and $|x| = -x$.

$$\begin{aligned}\lim_{x\to -\infty}\frac{2x\,|x|+5x+1}{3x^2+x-2} &= \lim_{x\to -\infty}\frac{-2x^2+5x+1}{3x^2+x-2}\\ &= \lim_{x\to -\infty}\frac{-2\cancel{x}^2}{3\cancel{x}^2}\\ &= -\frac{2}{3} = -0.666\,67 \qquad (\mathbf{3pts})\end{aligned}$$

B) Given the graph of the function $y = f(x)$ below find:



1. $\lim_{x\to -2^-} f(x) = +\infty \qquad (\mathbf{1pt})$

2. $\lim_{x\to -2^+} f(x) = +\infty \qquad (\mathbf{1pt})$

3. $\lim_{x\to 2^-} f(x) = 3 \qquad (\mathbf{1pt})$

4. $\lim_{x\to +\infty} f(x) = 0 \qquad (\mathbf{1pt})$

$(6pts)$**Problem 2**

Find $\dfrac{dy}{dx}$ for

$$(a) \quad y = (x^4 - x)^4 \qquad\qquad (b) \quad y^2x + yx^2 = 3$$

Solution

(a)

$$y = \left(x^4 - x\right)^4$$

$$\frac{dy}{dx} = 4\left(4x^3 - 1\right)\left(x^4 - x\right)^3 \qquad (\mathbf{3pts})$$

(b)

$$y^2x + yx^2 = 3$$

$$2yy'x + y^2 + y'x^2 + 2xy = 0$$
$$2yy'x + y'x^2 = -y^2 - 2xy$$
$$y' = -\frac{1}{2xy + x^2}\left(2xy + y^2\right) \qquad (\mathbf{3pts})$$

($8pts$) **Problem 3**

Find the absolute extrema of the function $f(x) = \frac{3}{4}x^4 + \frac{2}{3}x^3 - \frac{1}{2}x^2 - 2$ on the interval $[-1,\ 1]$.

Solution

$$f(x) = \frac{3}{4}x^4 + \frac{2}{3}x^3 - \frac{1}{2}x^2 - 2$$

$$\begin{aligned} f'(x) &= 3x^3 + 2x^2 - x \qquad\qquad (\mathbf{3pts}) \\ &= x(x+1)(3x-1) \end{aligned}$$

The critical numbers are $0,\ -1$ and $\frac{1}{3}$. **(3pts)**

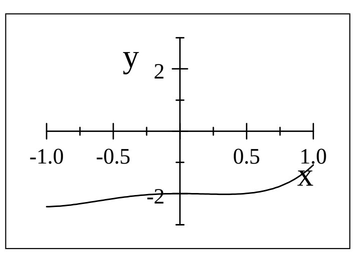
$$f(0) = -2$$

$$\begin{aligned} f(-1) &= \frac{3}{4} - \frac{2}{3} - \frac{1}{2} - 2 \\ &= -\frac{29}{12} = -2.4167 \end{aligned}$$

$$\begin{aligned} f(\frac{1}{3}) &= \frac{3}{4}\left(\frac{1}{3}\right)^4 + \frac{2}{3}\left(\frac{1}{3}\right)^3 - \frac{1}{2}\left(\frac{1}{3}\right)^2 - 2 \\ &= -\frac{655}{324} = -2.0216 \end{aligned}$$

$$\begin{aligned} f(1) &= \frac{3}{4} + \frac{2}{3} - \frac{1}{2} - 2 \\ &= -\frac{13}{12} = -1.0833 \end{aligned}$$

$$\begin{aligned} \text{Absolute Maximum} &= -\frac{13}{12} = -1.0833 \qquad (\mathbf{1pt}) \\ \text{Absolute Minimum} &= -\frac{29}{12} = -2.4167 \qquad (\mathbf{1pt}) \end{aligned}$$



(8*pts*)**Problem 4**
The length of a rectangle of constant area 800 square millimeters is increasing at the rate of 4 millmeters per second. What is the width of the rectangle at the moment the width is decreasing at the rate of 0.5 millimeter per second?

Solution
Let ℓ be the length and w the width.
The area
$$800 = \ell w \qquad \text{(related equation)} \qquad (\mathbf{2pts})$$
Differentiating,
$$0 = \frac{dw}{dt}\ell + \frac{d\ell}{dt}w \qquad \text{(related rate equation)} \qquad (\mathbf{2pts})$$
We are given that
$$\frac{d\ell}{dt} = 4.$$
We want to find w when $\frac{dw}{dt} = -0.5$.
From the related rate equation we obtain
$$-0.5\ell + 4w = 0. \qquad (\mathbf{2pts})$$
Now using the fact that
$$800 = \ell w \ \Rightarrow \ell = \frac{800}{w},$$
$-0.5\ell + 4w = 0$ becomes
$$-0.5\left(\frac{800}{w}\right) + 4w = 0 \Leftrightarrow \frac{1}{w}\left(4w^2 - 400.0\right) = 0$$
$$w^2 = 100, \qquad w = 10\ mm. \qquad (\mathbf{2pts})$$

$(8pts)$**Problem 5**
Use the second derivative test to find the local extrema of the function

$$f(x) = x(x-1)^3.$$

Solution

$$f(x) = x(x-1)^3.$$

$$\begin{aligned} f'(x) &= (x-1)^3 + 3x(x-1)^2 \qquad (\mathbf{2pts}) \\ &= (4x-1)(x-1)^2 \end{aligned}$$

The critical numbers are $[x = 1]$ and $\left[x = \frac{1}{4}\right]$. $\quad$ (**2pts**)

$$\begin{aligned} f''(x) &= 4(x-1)^2 + (2x-2)(4x-1) \\ &= 12x^2 - 18x + 6 \end{aligned}$$

$f''(1) = 12 - 18 + 6 = 0$. The test is not conclusif at $x = 1$. $\qquad$ (**2pts**)

$$\begin{aligned} f''(\frac{1}{4}) &= 4\left(\frac{1}{4} - 1\right)^2 + \left(\frac{1}{2} - 2\right)(1-1) \\ &= \frac{9}{4} > 0 \end{aligned}$$

$$\begin{aligned} f(\frac{1}{4}) &= \frac{1}{4}\left(\frac{1}{4} - 1\right)^3 \\ &= -\frac{27}{256} = -0.105\,47 \text{ is a local minimum.} \qquad (\mathbf{2pts}) \end{aligned}$$