

Math 141 Tutorial 3

Differentiation (continued)

1.

Express $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point $P(2, -1)$ for $x^2 - xy + y^2 = 7$.

2.

If the volume of an expanding cube is increasing at the rate of $4 \text{ m}^3/\text{min}$, how fast is its surface area increasing when the surface area is 24 m^2 ?

3.

Sand is falling into a conical pile so that the radius of the base of the pile is always equal to one-half of its altitude. If the sand is falling at a rate of 10 cubic feet per minute, how fast is the altitude of the pile

increasing when the pile is 5 feet deep? $V = \frac{1}{3}\pi r^2 h$

4.

Find the intervals on which $f(x) = 4x^3 - 15x^2 - 18x + 10$ increases and the intervals on which f decreases.

5.

$$\text{If } f'(x) = e^{(x-4)^3} (x^2 + x + 2)^3 (x - 2)^3 (x - 1),$$

Find the interval over which f is decreasing.

6.

Find the absolute extrema of the function on the interval $[-1, 2]$.

$$f(x) = x^4 - 2x^2 + 2$$

7.

Find the critical numbers and the local extreme values of $f(x) = 3x^5 - 5x^4$.

8.

Find the critical numbers and the local extreme values of $f(x) = 12x^{2/3} - 16x$.

9.

The function $f(x) = x^3 + ax^2 + bx + 1$ has a relative minimum at $x = -1$ and a relative maximum at $x = -3$. Find the open intervals in which f is concave up and down.