Part 1 MCQ (30%)

($6pts$) **Problem 1**

If f is an integrable function, then

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{6}{n}f\left(1+\frac{6i}{n}\right)=$$

(a) $\int_1^7 f(x)\,dx$

(b) $\int_1^6 f(x)\,dx$

(c) $\int_1^5 6f(x)\,dx$

(d) $\int_1^7 6f(x)\,dx$

(e) $\int_1^6 6f(x)\,dx$

($6pts$) **Problem 2**

If $\int_1^{10} f(x)\,dx = 5$ and $\int_2^1 f(x)\,dx = 3$, then $\int_2^{10}(1+f(x))\,dx =$

(a) 15

(b) 16

(c) 14

(d) 12

(e) 10

$(6pts)$ **Problem 3**

$$f(x) = \begin{cases} x+2 & \text{if} \quad -3 \le x \le -1 \\ x^2 & \text{if} \quad -1 < x \le 3 \end{cases}, \text{ then } \int_{-2}^{0} f(x)\, dx =$$

(a) $\frac{5}{6}$

(b) $\frac{7}{6}$

(c) $\frac{11}{6}$

(d) $\frac{13}{6}$

(e) $\frac{1}{6}$

$(6pts)$ **Problem 4**

A particle moves along a line so that its velocity at time t is $v(t) = t - t^3$ (measured in meters per second). The distance traveled, in meters, during the time period $0 \le t \le 2$ is equal to

(a) $\frac{3}{2}$

(b) $\frac{1}{2}$

(c) $\frac{7}{2}$

(d) $\frac{9}{2}$

(e) $\frac{5}{2}$

(6*pts*) **Problem 5**

$$\frac{d}{dx}\left[\int_{e^{-x}}^{e^x} \ln t \, dt\right] =$$

(a) $x\,(e^x - e^{-x})$

(b) 0

(c) $2x\,e^x$

(d) $e^x + e^{-x}$

(e) $x\,e^x - 1$

Part 2 Written Questions (70%)

$(15pts)$**Problem 1**
Evaluate

1. $\displaystyle\int \frac{\sin^3 x}{\sqrt{\cos x}} dx$

2. $\displaystyle\int_0^1 \left(x\sqrt{x} + \sqrt[3]{x}\right) dx$

(10pts)**Problem 2**

Use area of circles to find the average value of the function $f(x) = \sqrt{\pi^2 - x^2}$ on the interval $[-\pi,\ \pi]$.

$(10pts)$**Problem 3**
Evaluate the integrals

1. $\int \cos 2x \sin 3x dx$

2. $\int \sin 5x \sin 4x dx$

(15pts)**Problem 4**

(a) Find the modulus and the argument of the complex number

$$w = \frac{-9+3i}{1-2i}.$$

(b) Solve the equation

$$2z^2 - 2iz - 5 = 0$$

$(10pts)$**Problem 5**
Find x and y given that

$(a)\quad (x+iy)(2+i)=0$

$(b)\quad x(1+i)^2+y(2-i)^2=3+10i$

(10pts)**Problem 6**

Write the complex number in the form $a + ib$.

$$(a)\ z_1 = (2-i)^2 + \frac{7-4i}{2+i} - 8 \qquad\qquad (b)\quad z_2 = (1+i)^{10}$$