CHAPTER 12

Vectors

12.1 Cartesian Space Coordinates

1. Plot points $A(2, 7, 8)$ and $B(3, 9, 7)$ on a right-handed coordinate system. Then calculate the length of the line segment $\overline{AB}$ and find the midpoint.

2. Plot points $A(-3, -2, 4)$ and $B(9, 7, 2)$ on a right-handed coordinate system. Then calculate the length of the line segment $\overline{AB}$ and find the midpoint.

3. Plot points $A(-1, 1, 1)$ and $B(-1, 4, 4)$ on a right-handed coordinate system. Then calculate the length of the line segment $\overline{AB}$ and find the midpoint.

4. Find an equation for the plane through $(2, -1, -2)$ that is parallel to the xy-plane.

5. Find an equation for the plane through $(-3, 2, -1)$ that is perpendicular to the z-axis.

6. Find an equation for the plane through $(-2, -4, 3)$ that is parallel to the yz-plane.

7. Find an equation for the sphere centered at $(2, 1, 3)$ with radius 4.

8. Find an equation for the sphere that is centered at $(-4, 0, 6)$ and passes through $(2, 2, 3)$.

9. Find an equation for the sphere that is centered at $(5, 1, -4)$ and passes through $(3, -5, -1)$.

10. Find an equation for the sphere that has the line segment joining $(4, 3, 0)$ and $(2, 4, -4)$ as a diameter.

11. Find an equation for the sphere that is centered at $(-2, 1, 4)$ and is tangent to the plane $x = 2$.

12. The points $P(a, b, c)$ and $Q(3, 2, -1)$ are symmetric about the xy-plane. Find a, b, c.

13. The points $P(a, b, c)$ and $Q(-3, 2, -1)$ are symmetric about the yz-plane. Find a, b, c.

14. The points $P(a, b, c)$ and $Q(-3, -2, 1)$ are symmetric about the xz-plane. Find a, b, c.

15. The points $P(a, b, c)$ and $Q(1, 2, -4)$ are symmetric about the z-axis. Find a, b, c.

16. The points $P(a, b, c)$ and $Q(2, -1, 3)$ are symmetric about the plane $x = 2$. Find a, b, c.

17. The points $P(a, b, c)$ and $Q(-2, 1, -3)$ are symmetric about the plane $y = -3$. Find a, b, c.

18. The points $P(a, b, c)$ and $Q(4, 2, 2)$ are symmetric about the point $(0, 2, 1)$. Find a, b, c.

12.3 Vectors

19. Simplify $(3\,\mathbf{i} - \mathbf{j} + 2\,\mathbf{k}) - 2(\mathbf{i} - 2\,\mathbf{j} + \mathbf{k})$.

20. Simplify $2(\mathbf{i} - 3\,\mathbf{k}) - 3(2\,\mathbf{i} + \mathbf{j} - \mathbf{k})$.

21. Calculate the norm of the vector $4\,\mathbf{i} - 3\,\mathbf{j}$.

22. Calculate the norm of the vector $3\,\mathbf{i} - \mathbf{j} + \mathbf{k}$.

23. Calculate the norm of $2(2\,\mathbf{i} - \mathbf{j} + \mathbf{k}) - (-2\,\mathbf{i} - \mathbf{j})$.

24. Let $\mathbf{a} = (-2, 3, 5)$, $\mathbf{b} = (3, 5, -2)$, $\mathbf{c} = (2, 1, 2)$, $\mathbf{d} = (-3, 0, -1)$. Express $\mathbf{a} - 2\,\mathbf{b} + 2\,\mathbf{c} + 3\,\mathbf{d}$ as a linear combination of $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$.

25. Given that $\mathbf{a} = (1, 2, 5)$ and $\mathbf{b} = (-1, 0, 3)$, calculate
(a) $\|\mathbf{a}\|$
(b) $\|\mathbf{b}\|$
(c) $\|2\mathbf{a} - 3\mathbf{b}\|$
(d) $\|3\mathbf{a} + \mathbf{b}\|$

26. Find α given that $3\,\mathbf{i} + 2\,\mathbf{j}$ and $-2\,\mathbf{i} + \alpha\,\mathbf{j}$ have the same length.

27. Find the unit vector in the direction of $2\,\mathbf{i} - \mathbf{j} + 2\,\mathbf{k}$.

28. Given that $\mathbf{a} = 3\,\mathbf{i} - 5\,\mathbf{k}$ and $\mathbf{b} = -\mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k}$, find the unit vector in the direction of $\mathbf{a} - 2\,\mathbf{b}$.

29. Given that $\mathbf{a} = 2\,\mathbf{i} + 9\,\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 7\,\mathbf{j} + 8\,\mathbf{k}$, find the unit vector in the direction of $2\,\mathbf{a} + \mathbf{b}$.

30. Find the vector of norm 2 in the direction of $3\,\mathbf{i} + 4\,\mathbf{j} + 2\,\mathbf{k}$.

31. Find the vector of norm 2 parallel to $5\,\mathbf{i} - 12\,\mathbf{j} + \mathbf{k}$.

12.4 The Dot Product

32. Simplify $(2\,\mathbf{a} \bullet 2\,\mathbf{b}) + \mathbf{a} \bullet (\mathbf{a} + 2\,\mathbf{b})$.

33. Simplify $(\mathbf{a} - 2\,\mathbf{b}) \bullet \mathbf{c} + \mathbf{b} \bullet (\mathbf{a} - \mathbf{c}) - 2\,\mathbf{a} \bullet (\mathbf{b} - 3\mathbf{c})$.

34. Taking $\mathbf{a} = \mathbf{i} + 2\,\mathbf{j}$, $\mathbf{b} = 2\,\mathbf{i} - \mathbf{j} + 3\,\mathbf{k}$, $\mathbf{c} = -2\,\mathbf{j} + \mathbf{k}$, calculate:
(a) the three dot products $\mathbf{a} \bullet \mathbf{b}$, $\mathbf{a} \bullet \mathbf{c}$, $\mathbf{b} \bullet \mathbf{c}$
(b) the cosines of the angles between these vectors.
(c) the component of $\mathbf{a}$ (i) in the $\mathbf{b}$ direction, (ii) in the $\mathbf{c}$ direction
(d) the projection of $\mathbf{a}$ (i) in the $\mathbf{b}$ direction, (ii) in the $\mathbf{c}$ direction

35. Taking $\mathbf{a} = \mathbf{i} - 2\,\mathbf{j} + 4\,\mathbf{k}$ and $\mathbf{b} = 3\,\mathbf{i} - 2\,\mathbf{j} + \mathbf{k}$, $\mathbf{c} = -2\,\mathbf{j} - \mathbf{k}$, calculate:
(e) the three dot products $\mathbf{a} \bullet \mathbf{b}$, $\mathbf{a} \bullet \mathbf{c}$, $\mathbf{b} \bullet \mathbf{c}$
(f) the cosines of the angles between these vectors.
(g) the component of $\mathbf{a}$ (i) in the $\mathbf{b}$ direction, (ii) in the $\mathbf{c}$ direction
(h) the projection of $\mathbf{a}$ (i) in the $\mathbf{b}$ direction, (ii) in the $\mathbf{c}$ direction

36. Find the angle between the vectors $2\,\mathbf{i} + 2\,\mathbf{j} - \mathbf{k}$ and $\mathbf{i} + 2\,\mathbf{j} + \mathbf{k}$.

37. Find the angle between the vectors $2\,\mathbf{i} + 3\,\mathbf{j} + \mathbf{k}$ and $3\,\mathbf{i} + \mathbf{j} + 8\,\mathbf{k}$.

38. Find the direction angles of the vector $\sqrt{2}\,\mathbf{i} - \mathbf{j} + \mathbf{k}$.

39. Find the direction angles of the vector $\sqrt{3}\,\mathbf{i} - 2\,\mathbf{k}$.

40. Find the unit vectors $\mathbf{u}$ that are perpendicular to both $\mathbf{i} + 2\,\mathbf{j} + \mathbf{k}$ and $\mathbf{i} - 2\,\mathbf{j} + 2\,\mathbf{k}$.

41. Find the cosine of the angle between $\mathbf{u} = 2\mathbf{i} - 2\,\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = -\mathbf{i} + 4\,\mathbf{j} + 2\,\mathbf{k}$.

42. A 100 Newton force is applied along a rope making a 30° angle with the horizontal to pull a box a distance of 5 meters along the ground. What is the work done?

43. Find the work done by the force $\mathbf{F} = 3\,\mathbf{i} + 5\,\mathbf{j} + 2\,\mathbf{k}$ in moving an object from the point $P(2, 0, 2)$ to the point $Q(1, 4, 5)$.

12.5 The Cross Product

44. Calculate $(\mathbf{i} - \mathbf{k}) \times (\mathbf{j} + \mathbf{k})$.

45. Calculate $(\mathbf{j} \times \mathbf{k}) \bullet \mathbf{j}$.

46. Calculate $(\mathbf{k} \times \mathbf{j}) \times \mathbf{i}$.

47. Calculate $(\mathbf{i} - 4\,\mathbf{j} - 2\,\mathbf{k}) \times (2\,\mathbf{i} + \mathbf{j})$.

48. Calculate $[(\mathbf{i} + 2\,\mathbf{j} - \mathbf{k}) \times (\mathbf{i} + \mathbf{j} + \mathbf{k})] \times (\mathbf{i} + 2\,\mathbf{j} + 2\,\mathbf{k})$.

49. Calculate $(3\,\mathbf{i} - 4\,\mathbf{j} - 4\,\mathbf{k}) \times [(2\,\mathbf{i} - 6\,\mathbf{j}) \times (\mathbf{i} - 2\,\mathbf{j} + 2\,\mathbf{k})]$.

50. Calculate $(\mathbf{i} - \mathbf{j}) \bullet [(3\,\mathbf{i} - 4\,\mathbf{j}) \times (\mathbf{i} - 2\,\mathbf{j} + 2\,\mathbf{k})]$.

51. Calculate $(2\,\mathbf{i} + 3\,\mathbf{j} - 4\,\mathbf{k}) \bullet [(-\mathbf{i} + \mathbf{j} + 2\,\mathbf{k}) \times (\mathbf{i} - \mathbf{j} + \mathbf{k})]$.

52. Calculate $(3\,\mathbf{i} + 2\,\mathbf{k}) \times [(2\,\mathbf{i} + 2\,\mathbf{j} - \mathbf{k}) \times (\mathbf{i} - 2\,\mathbf{j} + 3\,\mathbf{k})]$.

53. Use a cross product to find the area of triangle PQR, $P(1, 2, 3)$, $Q(-1, 0, 1)$, $R(2, -2, -1)$.

54. Use a cross product to find the area of triangle PQR, $P(1, 1, 1)$, $Q(2, -1, 3)$, $R(2, 3, -4)$.

55. Find the volume of the parallelepiped with edges determined by $3\,\mathbf{i} - 4\,\mathbf{j} - \mathbf{k}$, $\mathbf{i} - 2\,\mathbf{j} + 2\,\mathbf{k}$, $\mathbf{i} + \mathbf{j}$.

56. Find the volume of the parallelepiped with vertices $A(0, 0, 0)$, $B(1, -1, 1)$, $C(2, 1, -2)$ and $D(-1, 2, -1)$.

57. Find the volume of the parallelepiped with edges determined by $\mathbf{i} + 2\,\mathbf{k}$, $4\,\mathbf{i} + 6\,\mathbf{j} + 2\,\mathbf{k}$, $3\,\mathbf{i} + 3\,\mathbf{j} - 6\,\mathbf{k}$.

58. Find the volume of the parallelepiped with edges determined by $2\,\mathbf{i} + \mathbf{k}$, $3\,\mathbf{i} + 2\,\mathbf{j} + 5\,\mathbf{k}$, $-\mathbf{i} + 2\,\mathbf{k}$.

59. Find the area of the triangle with vertices $P(1, -2, 3)$, $Q(2, 4, 1)$, $R(2, 0, 1)$.

60. Find the area of the triangle with vertices $P(1, 2, 1)$, $Q(2, 4, 3)$, $R(5, -1, 4)$.

12.6 Lines

61. Which of the points $P(-1, 3, -1)$, $Q(3, 2, -1)$, $R(3, 0, -2)$ lie on the line l: $\mathbf{r}(t) = (2\,\mathbf{i} + \mathbf{j}) + t(3\,\mathbf{i} - 2\,\mathbf{j} + \mathbf{k})$?

62. Determine whether the lines are parallel.
l_1: $\mathbf{r}_1(t) = (\mathbf{i} - 2\,\mathbf{k}) + t(\mathbf{i} - 2\,\mathbf{j} - 3\mathbf{k})$
l_2: $\mathbf{r}_2(u) = (3\,\mathbf{i} + 2\,\mathbf{j} - 3\,\mathbf{k}) + u(\mathbf{i} + 2\,\mathbf{j} - \mathbf{k})$

63. Find a vector parametrization for the line that passes through $P(2, 3, 3)$ and is parallel to the line $\mathbf{r}(t) = (2\,\mathbf{i} - \mathbf{j}) + t\,\mathbf{k}$.

64. Find a vector parametrization for the line that passes through the origin and $P(3, 1, 8)$.

65. Find a vector parametrization for the line that passes through $P(4, 0, 5)$ and $Q(2, 3, 1)$.

66. Find a vector parametrization for the line that passes through $P(3, 3, 1)$ and $Q(4, 0, 2)$.

67. Find a set of scalar parametric equations for the line that passes through $P(1, 4, 6)$ and $Q(2, -1, 3)$.

68. Find a set of scalar parametric equations for the line that passes through $P(-3, -1, 0)$ and $Q(-1, 2, 1)$.

69. Find a set of scalar parametric equations for the line that passes through $P(4, -2, -1)$ and is perpendicular to the xy-plane.

70. Find a set of scalar parametric equations for the line that passes through $P(-1, 2, -3)$ and is perpendicular to the xz-plane.

71. Give a vector parametrization for the line that passes through $P(1, -2, 3)$ and is parallel to the line $3(x - 2) = 2(y + 2) = 5z$.

72. Find the point where l_1 and l_2 intersect and give the angle of intersection:
l_1: $x_1(t) = 3 - t$, $y_1(t) = 5 + 3t$, $z_1(t) = -1 - 4t$
l_2: $x_2(u) = 8 + 2u$, $y_2(u) = -6 - 4u$, $z_2(u) = 5 + u$.

73. Where does the line that passes through (1, 4, 2) and is parallel to $3\,\mathbf{i} + 2\,\mathbf{j} - 2\,\mathbf{k}$ intersect the xy-plane?

74. Where does the line that passes through (3, 5, –1) and is parallel to $\mathbf{i} - \mathbf{j} + \mathbf{k}$ intersect the xz-plane?

75. Find scalar parametric equations for all lines that are perpendicular to the line $x(t) = 5 + 2t$, $y(t) = -5t$, $z(t) = -t$ and intersect the line at the point $P(-3, 2, 2)$.

76. Find the distance from $P(4, -3, 1)$ to the line through the origin parallel to $4\,\mathbf{i} - 3\,\mathbf{j} + \mathbf{k}$.

77. Find the distance from $P(3, -4, 1)$ to the line $\mathbf{r}(t) = 2\,\mathbf{i} - \mathbf{j} + t(\mathbf{i} - 2\,\mathbf{j} + 2\,\mathbf{k})$.

78. Find the standard vector parametrization for the line through $P(-1, 2, 4)$ parallel to $\mathbf{i} - 2\,\mathbf{j} + 3\,\mathbf{k}$.

79. Find the cosine of the angle between the lines $x_1(t) = 2 + t$, $y_1(t) = 3 + t$, $z(t) = -1 + 2t$ and $x_2(u) = 2 + 2u$, $y_2(u) = 3 - u$, $z_2(u) = -1 + 3u$.

80. Find the cosine of the angle between the line $x(t) = 2t$, $y(t) = 3t$, $z(t) = t$ and the y-axis.

12.7 Planes

81. Which of the points $P(-2, 3, -1)$, $Q(2, 3, 4)$, $R(3, 4, 1)$ lie on the plane $2(x - 2) + 3(y - 2) - 2(z + 3) = 0$?

82. Which of the points $P(4, 1, 0)$, $Q(2, 1, -3)$, $R(4, 1, -2)$, $S(0, 2, -1)$ lie on the plane $\mathbf{N} \bullet (\mathbf{r} - \mathbf{r}_0) = 0$ if $\mathbf{N} = 2\,\mathbf{i} - 4\,\mathbf{j} + \mathbf{k}$ and $\mathbf{r}_0 = \mathbf{i} + 2\,\mathbf{j} - 3\,\mathbf{k}$?

83. Write an equation for the plane that passes through the point $P(2, 1, 3)$ and is perpendicular to $3\,\mathbf{i} + \mathbf{j} - 5\,\mathbf{k}$.

84. Write an equation for the plane that passes through the point $P(5, -2, -1)$ and is perpendicular to the plane $3x - y + 6z + 8 = 0$.

85. Find the unit normals for the plane $3x + 3y - 5z - 6 = 0$.

86. Write the equation of the plane $5x - 3y - 2z - 1 = 0$ in intercept form.

87. Where does the plane $4x + 3y - 2z + 4 = 0$ intersect the coordinate axes?

88. Find the angle between the planes $3(x - 1) - 2(y - 5) + 2(z + 1) = 0$ and $2x + 5(y - 1) + (z + 4) = 0$.

89. Find the angle between the planes $x - 2y + 3z = 5$ and $2x + y - z = 7$.

90. Determine whether or not the vectors are coplanar: $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$, $\mathbf{i} - 2\mathbf{j}$, $4\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

91. Find an equation in x, y, z for the plane that passes through the points $P_1(1, 1, 1)$, $P_2(2, 4, 3)$, $P_3(-1, -2, -1)$.

92. Find a set of scalar parametric equations for the line formed by the two intersecting planes: P_1: $3x - 2y + z = 0$, P_2: $8x + 2y + z - 11 = 0$.

93. Let l be the line determined by P_1, P_2, and let p be the plane determined by Q_1, Q_2, Q_3. Where, if anywhere, does l intersect p?
$P_1(2, 5, -2)$, $P_2(1, -2, 2)$; $Q_1(2, 1, -4)$, $Q_2(1, 2, 3)$, $Q_3(-1, 2, 1)$.

94. Find an equation in x, y, z for the plane that passes through $(1, 2, -3)$ and is perpendicular to the line $x(t) = 1 + 2t$, $y(t) = 2 + t$, $z(t) = -3 - 5t$.

95. Find an equation in x, y, z for the plane that passes through $(2, 1, 5)$ and the line $x(t) = -1 + 3t$, $y(t) = -2$, $z(t) = 2 + 4t$.

96. Find a vector equation for the line through $(1, 1, 1)$ that is parallel to the line of intersection of the planes $3x - 4y + 2z - 2 = 0$ and $4x - 3y - z - 5 = 0$.

97. Find parametric equations for the line through $(2, 0, -3)$ that is parallel to the line of intersection of the planes $x + 2y + 3z + 4 = 0$ and $2x - y - z - 5 = 0$.

98. Find an equation for the plane that passes through $(3, 0, 1)$ and is perpendicular to the line $x(t) = 2t$, $y(t) = 1 - t$, $z(t) = 4 - 3t$.

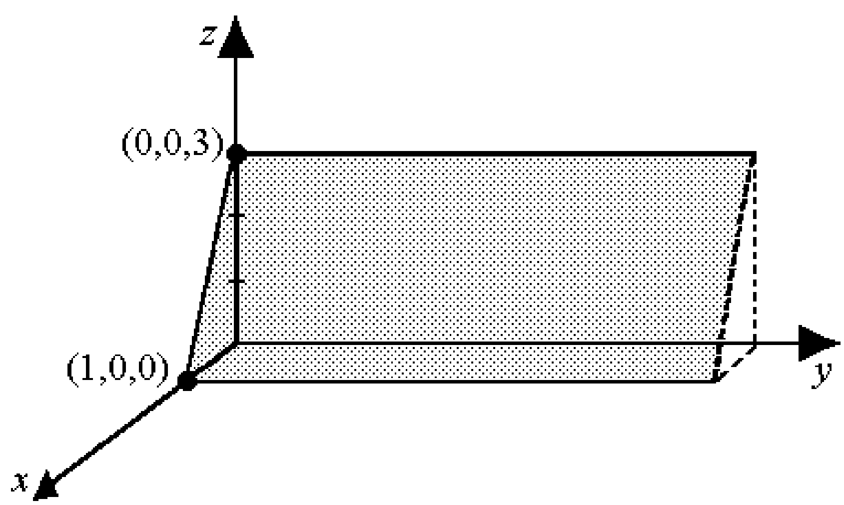
99. Find an equation for the plane that contains the point $(-2, 1, 1)$ and the line $\mathbf{r}(t) = 2\mathbf{i} + \mathbf{j} + \mathbf{k} + t(-\mathbf{i} + 4\mathbf{j} + 4\mathbf{k})$.

100. Find an equation for the plane that contains $P_1(1, 1, 1)$ and $P_2(-1, 2, 1)$ and is parallel to the line of intersection of the planes $2x + y - z - 4 = 0$ and $3x - y + z - 2 = 0$.

101. Find an equation for the plane that contains $P_1(3, 1, 2)$ and $P_2(-1, 2, -1)$ and is parallel to the line of intersection of the planes $2x - y - z - 2 = 0$ and $3x + 2y - 2z - 4 = 0$.

102. Sketch the graph of $20x + 12y + 15z - 60 = 0$.

103. Find the equation of the plane pictured below.



Answers to Chapter 12 Questions

1. $\sqrt{6}; \left(\frac{5}{2}, 8, \frac{15}{2}\right)$
2. $\sqrt{229}; \left(3, \frac{5}{2}, 3\right)$
3. $3\sqrt{2}; \left(-1, \frac{5}{2}, \frac{5}{2}\right)$
4. $z = -2$
5. $z = -1$
6. $x = -2$
7. $(x-2)^2 + (y-1)^2 + (z-3)^2 = 16$
8. $(x+4)^2 + y^2 + (z-6)^2 = 49$
9. $(x-5)^2 + (y-1)^2 + (z+4)^2 = 49$
10. $(x-3)^2 + (y-7/2)^2 + (z+2)^2 = 21/4$
11. $(x+2)^2 + (y-1)^2 + (z-4)^2 = 16$
12. (3, 2, 1)
13. (3, 2, –1)
14. (–3, 2, 1)
15. (–1, –2, –4)
16. (2, –1, 3)
17. (–2, –7, –3)
18. (–4, 2, 1)
19. $\mathbf{i} + 3\mathbf{j}$
20. $-4\mathbf{i} - 3\mathbf{j} - 3\mathbf{k}$
21. 5
22. $\sqrt{11}$
23. $\sqrt{73}$
24. $-13\mathbf{i} - 5\mathbf{j} + 10\mathbf{k}$
25. (a) $\sqrt{30}$ (b) $\sqrt{10}$ (c) $\sqrt{42}$ (d) $2\sqrt{91}$
26. ± 3
27. $\frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$
28. $\frac{5}{\sqrt{162}}\mathbf{i} - \frac{4}{\sqrt{162}}\mathbf{j} - \frac{11}{\sqrt{162}}\mathbf{k}$
29. $\frac{1}{\sqrt{30}}\mathbf{i} + \frac{5}{\sqrt{30}}\mathbf{j} + \frac{2}{\sqrt{30}}\mathbf{k}$
30. $\frac{6}{\sqrt{29}}\mathbf{i} + \frac{8}{\sqrt{29}}\mathbf{j} + \frac{4}{\sqrt{29}}\mathbf{k}$
31. $\frac{10}{\sqrt{170}}\mathbf{i} - \frac{24}{\sqrt{170}}\mathbf{j} + \frac{2}{\sqrt{170}}\mathbf{k}$
32. $\mathbf{a} \bullet \mathbf{a} + 6\mathbf{a} \bullet \mathbf{b}$
33. $-\mathbf{a} \bullet \mathbf{b} + 7\mathbf{a} \bullet \mathbf{c} - 3\mathbf{b} \bullet \mathbf{c}$
34. (a) 0, –4, 5
    (b) $\cos(\mathbf{a}, \mathbf{b}) = 0$; $\cos(\mathbf{a}, \mathbf{c}) = -4/5$
    $\cos(\mathbf{b}, \mathbf{c}) = \frac{\sqrt{70}}{14}$
    (c) (i) 0; (ii) $\frac{-4}{\sqrt{5}}$
    (d) (i) 0; (ii) $\frac{8}{5}\mathbf{j} + \frac{4}{5}\mathbf{k}$
35. (a) 11, –8, –5
    (b) $\cos(\mathbf{a}, \mathbf{b}) = \frac{11}{\sqrt{21}\sqrt{14}}$; $\cos(\mathbf{a}, \mathbf{c}) = \frac{-8}{\sqrt{21}\sqrt{5}}$
    $\cos(\mathbf{b}, \mathbf{c}) = \frac{-5}{\sqrt{14}\sqrt{5}}$
    (c) (i) $\frac{11}{\sqrt{14}}$; (ii) $\frac{-8}{\sqrt{5}}$
    (d) (i) $\left(\frac{33}{14}, \frac{-22}{14}, \frac{1}{14}\right)$; (ii) $\left(0, \frac{-16}{5}, \frac{8}{5}\right)$
36. ≈ 47.12° or 0.8225 radians
37. ≈ 58.12° or 1.014 radians
38. $\pi/4, 2\pi/3, \pi/3$
39. $\pi/6, 0, \pi/3$

40. $\frac{6}{\sqrt{53}}\mathbf{i} - \frac{1}{\sqrt{53}}\mathbf{j} - \frac{4}{\sqrt{53}}\mathbf{k}$ or
$\frac{-6}{\sqrt{53}}\mathbf{i} + \frac{1}{\sqrt{53}}\mathbf{j} + \frac{4}{\sqrt{53}}\mathbf{k}$

41. $\frac{-8}{3\sqrt{21}}$

42. $250\sqrt{3}$ Joules

43. $W = 23$

44. $\mathbf{i} - \mathbf{j} + \mathbf{k}$

45. 0

46. 0

47. $2\mathbf{i} - 4\mathbf{j} + 9\mathbf{k}$

48. $-2\mathbf{i} - 7\mathbf{j} + 8\mathbf{k}$

49. $-12\mathbf{i} + 6\mathbf{j} - 54\mathbf{k}$

50. –2

51. 15

52. $14\mathbf{i} + 26\mathbf{j} - 21\mathbf{k}$

53. $5\sqrt{2}$

54. $\frac{1}{2}\sqrt{101}$

55. 17

56. 4

57. 54

58. 10

59. $2\sqrt{5}$

60. $\frac{\sqrt{290}}{2}$

61. $P(-1, 3, -1)$

62. No

63. $\mathbf{r}(t) = 2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k} + t\mathbf{k}$

64. $\mathbf{r}(t) = t(3\mathbf{i} + \mathbf{j} + 8\mathbf{k})$

65. $\mathbf{r}(t) = 4\mathbf{i} + 5\mathbf{k} + t(-2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$

66. $\mathbf{r}(t) = 3\mathbf{i} + 3\mathbf{j} + \mathbf{k} + t(\mathbf{i} - 3\mathbf{j} + \mathbf{k})$

67. $x(t) = 1 + t$; $y(t) = 4 - 5t$; $z(t) = 6 - 3t$

68. $x(t) = -3 + 2t$; $y(t) = -1 + 3t$; $z(t) = t$

69. $x(t) = 4$; $y(t) = -2$; $z(t) = -1 + t$

70. $x(t) = -1$; $y(t) = 2 + t$; $z(t) = -3$

71. $\mathbf{r}(t) = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + t(10\mathbf{i} + 15\mathbf{j} + 6\mathbf{k})$

72. $(4, 2, 3)$; $q = \cos^{-1}\left(\frac{-3}{\sqrt{273}}\right)$

73. (4, 6, 0)

74. (8, 0, 4)

75. $x(t) = -3 + t\mathbf{a}$
$y(t) = 2 + t\mathbf{b}$
$z(t) = 2 + t(2\mathbf{a} - 5\mathbf{b})$; $\mathbf{a}, \mathbf{b} \in \Re$

76. 0

77. $\sqrt{2}$

78. $\mathbf{r}(t) = -\mathbf{i} + 2\mathbf{j} + 4\mathbf{k} + t(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$

79. $\frac{7}{2\sqrt{21}}$

80. $\frac{3}{\sqrt{14}}$

81. R

82. S

83. $3x + y - 5z + 8 = 0$

84. $3x + y + 6z - 7 = 0$

85. $\left(\frac{3}{\sqrt{43}}, \frac{3}{\sqrt{43}}, \frac{-5}{\sqrt{43}}\right), \left(\frac{-3}{\sqrt{43}}, \frac{-3}{\sqrt{43}}, \frac{5}{\sqrt{43}}\right)$

86. $\frac{x}{1/5} + \frac{y}{-1/3} + \frac{z}{-1/2} = 1$

87. $x = -1$, $y = -4/3$, $z = 2$

88. $\cos^{-1}\dfrac{2}{\sqrt{510}} \approx 84.92°$

89. $\cos^{-1}\dfrac{3}{2\sqrt{21}} \approx 70.89°$

90. No

91. $2y - 3z + 1 = 0$

92. $x = 1 - 4t,\ y = 3/2 + 5t,\ z = 22t$

93. $\left(\dfrac{92}{61}, \dfrac{95}{61}, \dfrac{-2}{61}\right)$

94. $2x + y - 5z - 19 = 0$

95. $10x - 3y - 9z + 28 = 0$

96. $\mathbf{i} + \mathbf{j} + \mathbf{k} + t(10\mathbf{i} + 11\mathbf{j} + 7\mathbf{k})$

97. $x = 2 + t,\ y = 7t,\ z = -3 - 5t$

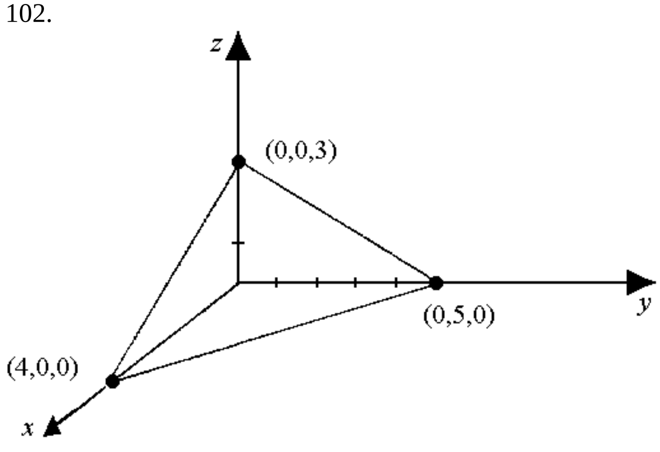
98. $2x - y - 3z - 3 = 0$

99. $y - z = 0$

100. $x + 2y - 2z - 1 = 0$

101. $5x + 8y - 4z - 15 = 0$

102.



103. $3x + z = 3$