Tutorial Sheet

1. Find each limit

A. $\lim\limits_{\theta \to 0} \dfrac{\sin 2\theta}{\theta}$

$= \lim\limits_{\theta \to 0} \sin 2\theta \times \dfrac{1}{\theta} \times \dfrac{2\theta}{2\theta}$

$= \lim\limits_{\theta \to 0} \dfrac{2\theta}{\theta}$

$= 2$

B. $\lim\limits_{y \to \infty} \dfrac{\sqrt{y^2+2}}{5y-6}$

$= \lim\limits_{y \to \infty} \dfrac{\sqrt{y^2}}{5y}$

$= \lim\limits_{y \to \infty} \dfrac{|y|}{5y}$

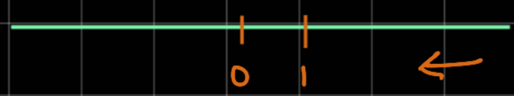
$= \lim\limits_{y \to \infty} \dfrac{\cancel{y}}{5\cancel{y}}$

$= \dfrac{1}{5}$

C. $\lim\limits_{t \to 1^+} \dfrac{|1-t|}{1-t}$

$= \lim\limits_{t \to 1^+} \dfrac{-(1-t)}{1-t}$

$= -1$



$t > 1$

$\therefore 1-t \Rightarrow -ve$

2. Find each of these limits $f(x) = \dfrac{x-2}{|x|-2}$

A. $\lim_{x \to -\infty} f(x)$

$= \lim_{x \to -\infty} \dfrac{x-2}{|x|-2}$

$= \lim_{x \to -\infty} \dfrac{x-2}{-x-2}$

$= \lim_{x \to -\infty} \dfrac{x}{-x}$

$= \dfrac{-1}{\ }$

B. $\lim_{x \to \infty} f(x)$

$= \lim_{x \to \infty} \dfrac{x-2}{|x|-2}$

$= \lim_{x \to \infty} \dfrac{x-2}{x-2}$

$= \dfrac{1}{\ }$

C. $\lim_{x \to -2^-} f(x)$

$= \lim_{x \to -2^-} \dfrac{x-2}{|x|-2}$

$= \lim_{x \to -2^-} \dfrac{x-2}{-x-2}$

$= \lim_{x \to -2^-} \dfrac{x-2}{-x-2}$

$= \dfrac{-2-2}{0}$

x	$-\infty$	-2		2	∞
$x-2$	$-$		$-$		$+$
$-x-2$	$+$		$-$		$-$
$f(x)$	$-$		$+$		$-$

$$\therefore \lim_{x \to -2^-} f(x) = -\infty$$

D. $\displaystyle\lim_{x \to -2^+} f(x)$

$$= \lim_{x \to -2^+} \frac{x-2}{|x|-2}$$

$$= \lim_{x \to -2^+} \frac{x-2}{-x-2}$$

$$= \lim_{x \to -2^+} \frac{x-2}{-x-2}$$

$$= \frac{-2-2}{0}$$

x	$-\infty$	-2		2	∞
$x-2$	$-$		$-$		$+$
$x+2$	$+$		$-$		$-$
$f(x)$	$-$		$+$		$-$

$$\therefore \lim_{x \to -2^+} f(x) = +\infty$$

E. $\lim_{x \to 2} f(x)$

$$= \lim_{x \to 2} \frac{x-2}{|x|-2}$$

LHL $= \lim_{x \to 2^-} \frac{x-2}{|x|-2}$

$$= \lim_{x \to 2^-} \frac{x-2}{x-2}$$

$$= 1$$

RHL $= \lim_{x \to 2^+} \frac{x-2}{|x|-2}$

$$= \lim_{x \to 2^+} \frac{x-2}{x-2}$$

$$= 1$$

$$\therefore \lim_{x \to 2} \frac{x-2}{|x|-2} = 1$$

3.

A. $\lim_{x \to 2} \frac{1-\frac{4}{x^2}}{1-\frac{2}{x}}$

$$= \lim_{x \to 2} \frac{\cancel{\left(1-\frac{2}{x}\right)}\left(1+\frac{2}{x}\right)}{\cancel{1-\frac{2}{x}}}$$

$$= \lim_{x \to 2} 1+\frac{2}{x}$$

$$= 1+\frac{2}{2}$$

$$= 1+1$$

$$= 2$$

B. $\lim_{x \to 0} \frac{x+\frac{2}{x}}{x-\frac{3}{x}}$

$$= \lim_{x \to 0} \frac{x^2+2}{x^2-3}$$

$$= -\frac{2}{3}$$

4.

A. $$\lim_{x\to\infty} \frac{2x^3-6}{x^k+3}$$

$$= \lim_{x\to\infty} \frac{2x^3}{x^k}$$

$$= \lim_{x\to\infty} \frac{2}{x^{k-3}}$$

For limit to exist

k - 3 must be positive

$k - 3 \geq 0$

$k \geq 3$

@ $k=3$

$$\lim_{x\to\infty} \frac{2}{x^{3-3}}$$

$$= \lim_{x\to\infty} \frac{2}{1}$$

$$= \underline{2}$$

@ $k=4$

$$\lim_{x\to\infty} \frac{2}{x^{4-3}}$$

$$= \lim_{x\to\infty} \frac{2}{x}$$

$$= \frac{2}{\infty}$$

$$= \underline{0}$$

B. $$\lim_{x\to 2} \frac{x^2+kx-10}{x-2}$$

Substitute with 2

$$\lim_{x\to 2} \frac{4+2k-10}{2-2}$$

$$= \lim_{x\to 2} \frac{2k-6}{0}$$

$$\frac{\text{constant}}{0} = \pm\infty \Rightarrow (2k-6) \text{ must be zero}$$

$2k-6=0 \Rightarrow k=3$

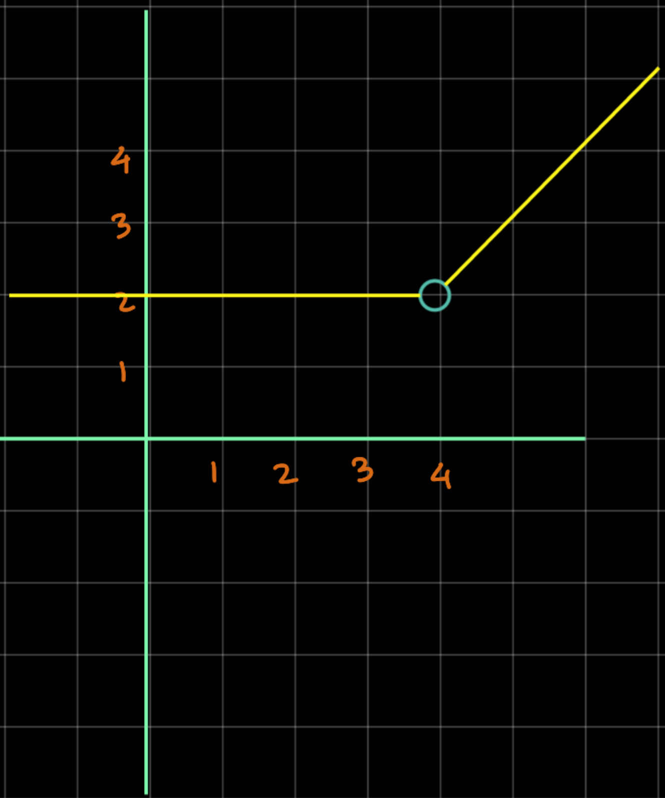
$$\lim_{x \to 2} \frac{x^2 + 3x - 10}{x-2}$$

$$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x+5)}{\cancel{(x-2)}}$$

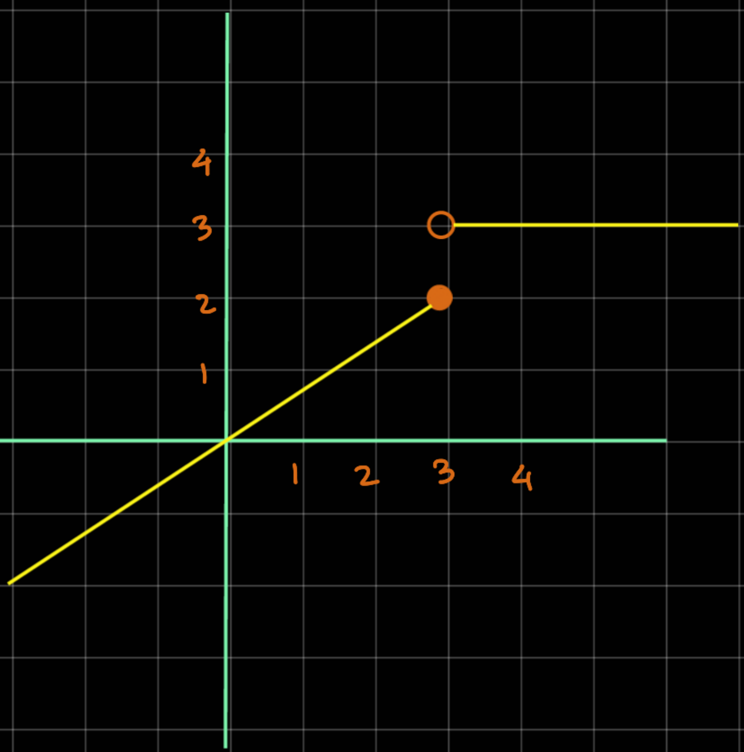
$$= \lim_{x \to 2} x+5$$

$$= 7$$

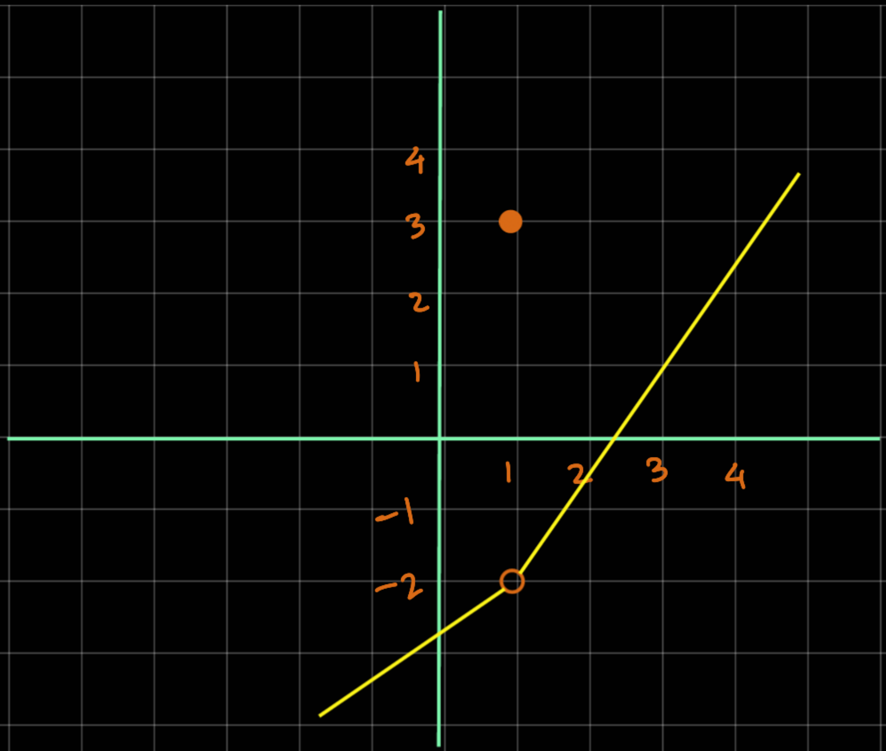
5. A.



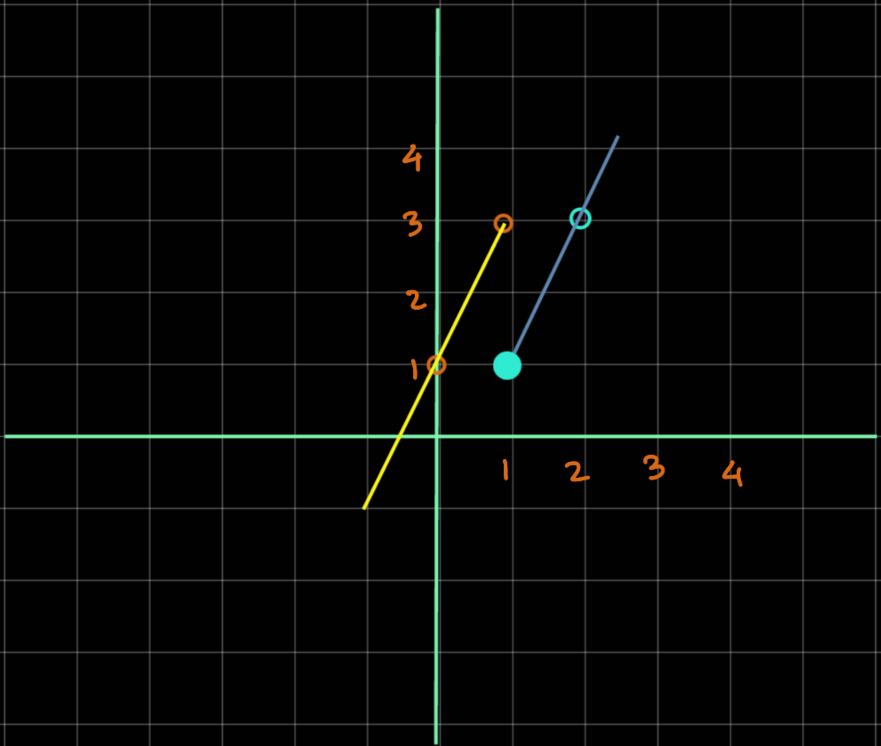
B.



C.



6. A.



$y = 2x + 1$

$y = 2x - 1$

7. $\lim_{x \to 2} \dfrac{x^2 + x + 6}{x - 2}$

$= \lim_{x \to 2} \dfrac{\cancel{(x-2)}(x+3)}{\cancel{(x-2)}}$

$= \lim_{x \to 2} x + 3$

$= \underline{5}$

$$f(x) = \begin{cases} \dfrac{x^2 + x - 6}{x} & \text{if } x \neq 2 \\ 5 & \text{if } x = 2 \end{cases}$$

8. $$\lim_{x \to -1^-} A = A$$

$$\lim_{x \to -1^+} f(x)$$

$$= \lim_{x \to -1^+} \frac{x^2 - x - 2}{x+1}$$

$$= \lim_{x \to -1^+} \frac{(x-2)\cancel{(x+1)}}{\cancel{(x+1)}}$$

$$= \lim_{x \to -1^+} (x-2)$$

$$= -1 - 2$$

$$= -3$$

$\because$ f is continuous

$\therefore$ $A = -3$

9. Pinching Theorem

If $f(x)$ is squeezed by two other functions

$$g(x) \leq f(x) \leq h(x)$$

$$\lim_{x \to c} g(c) = \lim_{x \to c} h(x) = \lim_{x \to c} f(x)$$

$$1 \le \cos \frac{1}{x^2} \le 1$$

$$-\sqrt{x} \le \sqrt{x} \cos \frac{1}{x^2} \le \sqrt{x}$$

$$\lim_{x \to 0^+} -\sqrt{x} \qquad \lim_{x \to 0^-} \sqrt{x}$$

$$= 0 \qquad = 0$$

Using pinching theorem

$$\therefore \lim_{x \to 0} \sqrt{x} \cos \frac{1}{x^2} = 0$$