Faculty of Engineering and Information Sciences

UNIVERSITY OF WOLLONGONG IN DUBAI

**MATH 141,** Quiz 1, Spring 2020, Duration: 60 minutes

Name:____________________, ID Number:____________________


Time Allowed: 1 Hour
Total Number of Questions: 5
Total Number of Pages (incl. this page): 6
EXAM UNAUTHORISED ITEMS
Students bringing these items to the examination room shall be required to leave the items at the front of the room or outside the examination room. The University does not guarantee the safe-keeping of students' personal items during examinations. Students concerned about the safety of their valuable items should make alternative arrangements for their care.

1. Bags, including carry bags, backpacks, shoulder bags and briefcases
2. Any form of electronic device including but not limited to mobile phones, smart watches, laptops, iPads, MP3 players, handheld computers and electronic dictionaries,
3. Calculator cases and covers
4. blank paper
5. Any written material

DIRECTIONS TO CANDIDATES

1. Total marks: 40
2. All questions are compulsory.
3. Answer all questions on the given exam paper sheets.
4. Write your name and Id number on the papers provided for rough work.

$(8pts)$ **Problem 1**

Evaluate the following limits

$$(a) \quad \lim_{x\to-\infty} \frac{2x^4+100x-1}{3x^4-100} \qquad (b) \quad \lim_{x\to 0} \frac{x^2-4x}{-3x^3+4x^2-2x}$$

$$(c) \quad \lim_{x\to 0} \frac{\sqrt{4+x}-2}{2x} \qquad (d) \quad \lim_{x\to 0} \frac{5x^3+2x^2}{1-\cos x}$$

**Solution**

$(a)$

$$\lim_{x\to-\infty} \frac{2x^4+100x-1}{3x^4-100} = \lim_{x\to-\infty} \frac{2x^4}{3x^4} = \frac{2}{3} \qquad (\mathbf{2pts})$$

$(b)$

$$\begin{aligned} \lim_{x\to 0} \frac{x^2-4x}{-3x^3+4x^2-2x} &= \lim_{x\to 0} \frac{\cancel{x}(x-4)}{\cancel{x}(-3x^2+4x-2)} \\ &= \lim_{x\to 0} \frac{(x-4)}{(-3x^2+4x-2)} = \frac{-4}{-2} = 2 \qquad (\mathbf{2pts}) \end{aligned}$$

$(c)$

$$\begin{aligned} \lim_{x\to 0} \frac{\sqrt{4+x}-2}{2x} &= \lim_{x\to 0} \frac{\left(\sqrt{4+x}-2\right)\left(\sqrt{4+x}+2\right)}{2x\left(\sqrt{4+x}+2\right)} \\ &= \lim_{x\to 0} \frac{\cancel{4}+x-\cancel{4}}{2x\left(\sqrt{4+x}+2\right)} \\ &= \lim_{x\to 0} \frac{\cancel{x}}{2\cancel{x}\left(\sqrt{4+x}+2\right)} \\ &= \lim_{x\to 0} \frac{1}{2\left(\sqrt{4+x}+2\right)} \\ &= \frac{1}{8} \qquad (\mathbf{2pts}) \end{aligned}$$

$(d)$

$$\begin{aligned} \lim_{x\to 0} \frac{5x^3+2x^2}{1-\cos x} &= \lim_{x\to 0} \frac{15x^2+4x}{\sin x} \qquad (\text{ L'Hospitals Rule}) \\ &= \lim_{x\to 0} \frac{30x+4}{\cos x} \\ &= 4 \qquad (\mathbf{2pts}) \end{aligned}$$

$(8pts)$**Problem 2**

Find $\dfrac{dy}{dx}$ for

$$(a) \quad y = x^3 (\ln x) \qquad\qquad (b) \quad y = \frac{e^{3x}}{x+1}$$

**Solution**

$(a)$

$$y = x^3 (\ln x)$$

$$\begin{aligned} \frac{dy}{dx} &= 3x^2 \ln x + \left(\frac{1}{x}\right) x^3 \\ &= 3x^2 \ln x + x^2 \qquad (\mathbf{4pts}) \end{aligned}$$

$(b)$

$$\begin{aligned} \frac{dy}{dx} &= \frac{3e^{3x}(x+1) - e^{3x}}{(x+1)^2} \\ &= \frac{e^{3x}(3x+2)}{(x+1)^2} \qquad (\mathbf{4pts}) \end{aligned}$$

($8pts$) **Problem 3**
If $G(x) = f(g(x))$ where $f(-4){=}\ 5,\ \ f\ '(-4) = 2,\ \ f\ '(-2) = 3,\ \ \ g(2) = -4,\ \ g'(2) = 7$, find $G'(2)$.

**Solution**

$$G'(x) = g'(x) \cdot f\ '(g(x)) \qquad \textbf{(2pts)}$$

$$\begin{aligned} G'(2) &= g'(2) \cdot f\ '(g(2)) && \textbf{(2pts)} \\ &= (7)\, f\ '(-4) && \textbf{(2pts)} \\ &= (7)\,(2) \\ &= 14 && \textbf{(2pts)} \end{aligned}$$

($8pts$)**Problem 4**
Use implicit differentiation to find the equation for the tangent line at the point $P(1,-1)$ for $2\ln x - 2xy + 2y^2 = 4$ .
**Solution**

Differentiate implicitly to get

$$\begin{aligned} 4yy' - 2xy' &= 2y - \frac{2}{x} \\ y'\left(4y - 2x\right) &= 2y - \frac{2}{x} \\ \frac{dy}{dx} = y' &= \frac{y - \frac{1}{x}}{2y - x}. \qquad \textbf{(4pts)} \end{aligned}$$

The slope at $P(1,-1)$ is

$$m = \frac{(-1) - \frac{1}{1}}{2\left(-1\right) - 1} = \frac{2}{3}. \qquad \textbf{(2pts)}$$

The equation of the tangent line is

$$\begin{aligned} y &= \frac{4}{3}\left(x - 1\right) - 1 \\ &= \frac{4}{3}x - \frac{7}{3}. \qquad \textbf{(2pts)} \end{aligned}$$

($8pts$)**Problem 5**

Air is escaping from a spherical balloon at the rate of 2 cm $^3$ per minute. How fast is the surface area shrinking when the radius is 1 cm? V= 4/3 $\pi r^3$ and S = $4\pi r^2$ where V is the volume and S is the surface area, r is the radius.

**Solution**
We have

$$V = \frac{4\pi}{3}r^3, \quad \frac{dV}{dt} = -2$$

We would like to find

$$\frac{dS}{dt} =? \quad \text{when } r = 1. \qquad (\mathbf{2pts})$$

The corresponding related equation is

$$S = 4\pi r^2.$$

The related rate equation is

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}. \qquad (\mathbf{2pts})$$

To find $\frac{dr}{dt}$, we use $\frac{dV}{dt} = -2$. i.e.

$$4\pi r^2 \frac{dr}{dt} = -2.$$

When $r = 1$,

$$\frac{dr}{dt} = -\frac{2}{4\pi} = -\frac{1}{2\pi}. \qquad (\mathbf{2pts})$$

Thus

$$\begin{aligned} \frac{dS}{dt} &= 8\pi (1) \left(-\frac{1}{2\pi}\right) \\ &= -4 \\ &= -4 \; cm^2/\min. \qquad (\mathbf{2pts}) \end{aligned}$$