

(12pts)**Problem 1**

A) Find the area of the largest rectangle that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 9 - x^2$.

B) A cylindrical can is to be made to hold 16π cm³ of laban. If r is the radius and h is the height of the can, then find the dimensions that will minimize the cost of the metal to manufacture the can.

(12pts)**Problem 2**

Use definite integral to find the limit

$$\lim_{n \rightarrow \infty} \frac{2}{n} \sum_{i=1}^n \sqrt{1 - \left(-1 + \frac{2i}{n}\right)^2}.$$

(12pts)**Problem 3**

A) If $\int_1^7 f(x)dx = 7$ and $\int_1^3 2f(x)dx = 6$, find $\int_3^7 f(x)dx$.

B) If $G(x) = \int_1^{e^x} (\ln t)^2 dt$, find $G'(\ln x)$.

(12pts)**Problem 4**

Evaluate the following integrals

1. $\int \frac{\cos^3 x}{\sin x} dx$

2. $\int_0^1 -2x^3 \sqrt{1-x^2} dx$

(10pts)**Problem 5**

A) If f is continuous on $[0, 3]$ and $\int_0^3 f(t)dt = 5$, find $\int_0^3 f(3-x)dx$.

B) Find the average value of $f(x) = 2|x| + 1$ on the interval $[-2, 2]$.

(10pts)**Problem 6**

A particle moves along a line so that its velocity at time t is $v(t) = t - t^2$. Find the distance traveled by the particle during the time period $0 \leq t \leq 2$.

(10pts)**Problem 7**

Find the absolute extrema of the function

$$F(x) = \int_1^x t^3 (2t + 1) dt \quad \text{on the interval } [-1, 2].$$

(10pts)**Problem 8**

Write the following complex numbers in the form $a + ib$ and find the modulus of each number. (Show your work)

$$z_1 = \left(\frac{1+i}{2-i} \right)^2$$

$$z_2 = \frac{2+5i}{1-i} + \frac{2-5i}{1+i}$$

(12pts)**Problem 9**

Solve the equation

$$2z^2 + (2 + 3i)z + 2i - 1 = 0$$