UNIVERSITY OF WOLLONGONG IN DUBAI

Math 141 Tutorial 2

Problem 1

If $f(x) = \begin{cases} \ln x & \text{if} \quad 0 < x \leq 1 \\ ax^2 + b & \text{if} \quad 1 < x \leq 5 \end{cases}$,

is continuous and $f(2) = 3$ then $2a + b =$

(a) 1
(b) -1
(c) 0
(d) 3
(e) 2

Problem 2

If $\lim_{h \to 0} \dfrac{e^{-3+h} - e^{-3}}{h} = f'(a)$ then

(a) $f(x) = e^x$ and $a = -3$
(b) $f(x) = e^{-x}$ and $a = 3$
(c) $f(x) = e^{-3}$ and $a = 1$
(d) $f(x) = e^{-3+x}$ and $a = -3$
(e) $f(x) = e^{-3+x}$ and $a = 3$

Problem 3

Suppose that $f(x)$ and $g(x)$ are differentiable functions and that $h(x) = f(x)g(x)$. You are given the following table of values:

$h(1)$	24
$g(1)$	6
$f'(1)$	-2
$h'(1)$	20

Using the table, find $g'(1)$.

Problem 4

An equation of the tangent line to the curve of $y = \dfrac{\sqrt{x} - 2x}{x}$ at $x = 4$ is

(a) $y = -\dfrac{x}{16} - \dfrac{5}{4}$

(b) $y = \dfrac{x}{16} - \dfrac{7}{4}$

(c) $y = \dfrac{x}{32} - \dfrac{13}{8}$

(d) $y = -\dfrac{5x}{16} - \dfrac{5}{4}$

(e) $y = -x + \dfrac{5}{2}$

Problem 5

If $g(x) = \sqrt{1 + 3f(x)}$, where $f(2) = 5$, $f'(2) = 16$, then the slope of the normal line to the curve of g at $x = 2$ is

(a) $-\dfrac{1}{6}$

(b) $-\dfrac{1}{8}$

(c) $-\dfrac{1}{3}$

(d) $-\dfrac{1}{4}$

(e) $-\dfrac{1}{16}$

Problem 6

Which of the following statements is (are) true?

(1) If $\lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$ exists, then f is differentiable at a

(2) If $\lim\limits_{x \to a} f(x)$ exists, then f is differentiable at a

(3) If $\lim\limits_{x \to a} f(x)$ exists, then f is continuous at a

(4) If f is differentiable at a, then $\lim_{x \to a} f(x) = f(a)$

Problem 7

If $f(x) = x + 72^x$, then $f'(1) =$

(a) $1 + 72 \ln 72$

(b) $1 + 8 \ln 8 + 9 \ln 9$

(c) $1 + 9 \ln 8 + 8 \ln 9$

(d) $1 + \frac{8}{9} \ln 72$

(e) $1 + \frac{9}{8} \ln 72$

Problem 8

$$\lim_{h \to 0} \frac{\sin\left(\frac{\pi}{3} + h\right) \cos\left(\frac{\pi}{3} + h\right) - \frac{\sqrt{3}}{4}}{h} =$$

Problem 9

Let $f(x) = (2x^2 + 3x - 1)\, e^x$, then $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$

Problem 10

If $y = (1+x)(1+x^2)(1+x^3)(1+x^4)(1+x^5),\ x > 0$, then $\frac{dy}{dx}\Big|_{x=1} =$

Problem 11

Use l'Hospital's rule to find the following limits:

$$\lim_{x \to 0} \frac{e^x - x - 1}{\cos x - 1}, \qquad \lim_{x \to a} \frac{x - a}{\ln x - \ln a}, \qquad \lim_{x \to 0} \left(\frac{e^x}{e^x - 1} - \frac{1}{x} \right)$$

Problem 12

Find the equations for the tangent and normal at the point $P(-1, -1)$ for $2x^2 - 3xy + 3y^2 = 2$.