(10pts) **Problem 4**
Solve the quadratic equation and give the answer in the form $a+ib$

$$z^2 - z + 8 + 2\,(z+1)\,i = 0.$$

Solution

$$\begin{aligned} z^2 - z + 8 + 2\,(z+1)\,i &= 0 \Leftrightarrow \\ z^2 - (1-2i)\,z + 8 + 2i &= 0 \end{aligned}$$

$$a = 1, \qquad b = -\,(1-2i) \quad \text{and } c = 8 + 2i.$$

The solutions are given by

$$z = \frac{(1-2i) \pm \sqrt{(1-2i)^2 - 4\,(8+2i)}}{2}$$

$$z = \frac{(1-2i) \pm \sqrt{-35-12i}}{2}. \qquad\qquad [5 \text{ points}]$$

Next, we need to find $\sqrt{-35-12i}$. Put

$$\sqrt{-35-12i} = a + ib \Leftrightarrow$$

$$\begin{aligned} (a+ib)^2 &= -35 - 12i \quad \Leftrightarrow \\ a^2 + 2iab - b^2 &= -35 - 12i \Rightarrow \end{aligned}$$

$$\left\{ \begin{array}{c} a^2 - b^2 = -35 \\ ab = -6 \end{array} \right. \Rightarrow a = -1 \text{ and } b = 6. \ \ \text{(Using the sign convention)}$$

$$\sqrt{-35-12i} = -1 + 6i. \qquad\qquad [3 \text{ points}]$$

$$z_1 = \frac{(1-2i) + (-1+6i)}{2} = 2i \ \ \text{and} \ \ z_2 = \frac{(1-2i) - (-1+6i)}{2} = 1 - 4i. \qquad [2 \text{ points}]$$

(10pts) **Problem 2**

Find the area of the triangle with vertices $P(4,3,6)$, $Q(-2,0,8)$, $R(1,5,0)$ and the equation of the plane containing the points $P,\ Q$ and R.

Solution

$$\overrightarrow{PQ} = \langle -6,\ -3,\ 2\rangle \quad \text{and} \quad \overrightarrow{PR} = \langle -3,\ 2,\ -6\rangle \qquad [2 \text{ points}]$$

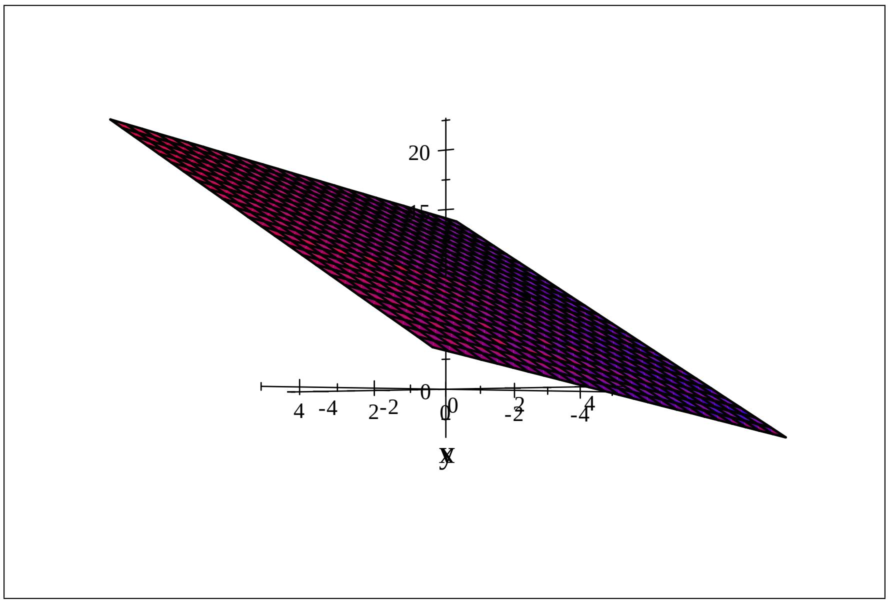
$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\ -6 & -3 & 2 \\ -3 & 2 & -6 \end{vmatrix} = \langle 14,\ -42,\ -21\rangle. \qquad [3 \text{ points}]$$

$$\begin{aligned} Area &= \frac{1}{2}\left\|\overrightarrow{PQ} \times \overrightarrow{PR}\right\| \\ &= \frac{1}{2}\sqrt{(14)^2 + (-42)^2 + (-21)^2} \\ &= \frac{\sqrt{2401}}{2} = \frac{49}{2} = 24.5. \qquad [2 \text{ points}] \end{aligned}$$

The equation of the plane containing the points $P,\ Q$ and R is

$$14(x-4) - 42(y-3) - 21(z-6) = 0 \Leftrightarrow$$

$$14x - 42y - 21z = -196. \qquad [3 \text{ points}]$$



(10pts) **Problem 3**

(A) Let $\overrightarrow{u}$ and $\overrightarrow{v}$ be two unit vectors and $\theta = \dfrac{2\pi}{3}$ the angle between $\overrightarrow{u}$ and $\overrightarrow{v}$. Find $\|\overrightarrow{u} + 2\overrightarrow{v}\|$.

Solution of (A)

$$\begin{aligned}
\|\overrightarrow{u} + 2\overrightarrow{v}\|^2 &= (\overrightarrow{u} + 2\overrightarrow{v}) \cdot (\overrightarrow{u} + 2\overrightarrow{v}) \\
&= \|\overrightarrow{u}\|^2 + 4\overrightarrow{u} \cdot \overrightarrow{v} + 4\|\overrightarrow{v}\|^2 \\
&= \|\overrightarrow{u}\|^2 + 4\|\overrightarrow{u}\|\,\|\overrightarrow{v}\| \cos\frac{2\pi}{3} + 4\|\overrightarrow{v}\|^2 \\
&= 1 + 4\left(\frac{-1}{2}\right) + 4 \\
&= 3
\end{aligned}$$

$$\|\overrightarrow{u} + 2\overrightarrow{v}\| = \sqrt{3} \qquad \text{[5 points]}$$

(B) Find a value of c for which $\overrightarrow{a} = \langle 3, -2, 5\rangle$ and $\overrightarrow{b} = \langle 2, 4, c\rangle$ will be perpendicular.

Solution of (B)

$\overrightarrow{a} = \langle 3, -2, 5\rangle$ and $\overrightarrow{b} = \langle 2, 4, c\rangle$ are perpendicular if and only if

$$\begin{aligned}
\langle 3, -2, 5\rangle \cdot \langle 2, 4, c\rangle &= 0 \Leftrightarrow \qquad \text{[3 points]} \\
6 - 8 + 5c &= 0
\end{aligned}$$

$$c = \frac{2}{5} \qquad \text{[2 points]}$$

$(10pts)$**Problem 7**
Find parametric equations for the line of intersection of the planes $4x + 4y - 2z = 9$ and $2x + y + z = -3$.

Solution
Let $\overrightarrow{n_1}$ and $\overrightarrow{n_2}$ be the two normal vectors of the planes respectively.

$$\overrightarrow{n_1} = \langle 4, 4, -2 \rangle \,, \qquad \overrightarrow{n_2} = \langle 2, 1, -1 \rangle \,.$$

A direction vector of the line of intersection is

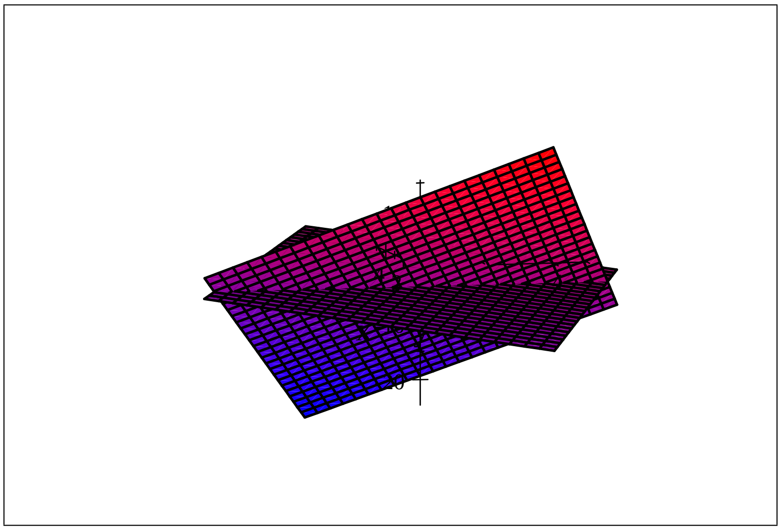
$$\overrightarrow{u} = \overrightarrow{n_1} \times \overrightarrow{n_2} = \begin{vmatrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\ 4 & 4 & -2 \\ 2 & 1 & 1 \end{vmatrix} = \langle 6, \ -8, \ -4 \rangle \qquad [4 \text{ points}] \,.$$

Now to find a point on the line of intersection, we put $z = 0$ to get

$$\begin{cases} 4x + 4y = 9 \\ 2x + y = -3 \end{cases}$$

Solution is: $\left[x = -\frac{21}{4}, y = \frac{15}{2}\right]$. Thus $\left(-\frac{21}{4}, \frac{15}{2}, 0\right)$ is a point on the line of intersection.
The parametric equations are

$$\begin{cases} x = -\frac{21}{4} + 6t \\ y = \frac{15}{2} - 8t \\ z = -4t \end{cases} . \qquad [6 \text{ points}]$$



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