Problem 1

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt[3]{\left(1+\frac{2i}{n}\right)} \;\frac{2}{n}$$

$a = 1$

$$i\left(\frac{b-a}{n}\right) = \frac{2i}{n}$$

$b - a = 2$

$b - 1 = 2$

$b = 3$

$$x = \left(1 + \frac{2i}{n}\right)$$

$f(x) = \sqrt[3]{x}$

$$\int_1^3 \sqrt[3]{x}\, dx$$

$$= \int_1^3 x^{1/3}\, dx$$

$$= \left.\frac{3x^{4/3}}{4}\right|_1^3$$

$$= \frac{3}{4}\left[3^{4/3} - 1^{4/3}\right] \qquad = \frac{3}{4}\left[3\sqrt[3]{3} - 1\right]$$

$$= \frac{3}{4}(3.32675) \qquad = \frac{9\sqrt[3]{3} - 3}{4}$$

$= 2.4951$

Additional Problem

Use definite integrals to evaluate

a) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left(2 + \frac{2i}{n}\right)^2 \frac{4}{n}$

$a = 2$

$\cancel{i}\left(\frac{b-a}{\cancel{n}}\right) = \frac{2\cancel{i}}{\cancel{n}}$

$b - a = 2$

$b - 2 = 2$

$b = 4$

$x = a + i\left(\frac{b-a}{n}\right)$

$f(x) = 2x^2$

$\int_2^4 2x^2 \, dx$

$= 2 \int_2^4 x^2 \, dx$

$= \frac{2}{3}\left[x^3\right]_2^4$

$= \frac{2}{3}\left[4^3 - 2^3\right]$

$= \frac{2 \times 56}{3}$

$= \frac{112}{3}$

Additional Problem

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i}{n}\right)^2 \frac{5}{n}$$

$a = 0$

$i \left(\frac{b-a}{\cancel{n}}\right) = \frac{2\cancel{i}}{\cancel{n}}$

$b - a = 2$

$b = 2$

$\frac{5}{\cancel{n}} = \frac{2}{\cancel{n}} \times x$

$x = 5/2$

$f(x) = \frac{5x^2}{2}$

$\int_0^2 \frac{5}{2} x^2 \, dx$

$\frac{5}{2} \int_0^2 x^2 \, dx$

$\frac{5}{2} \left(\frac{x^3}{3}\right)_0^2$

$\frac{5}{6} (8)$

$\frac{40}{6} = \frac{20}{3}$

Problem 2

Average of $f(x)$ on $[a, b]$ is defined as

$$\bar{f} = \frac{1}{b-a} \int_a^b f(x)\, dx$$

$$\bar{f} = \frac{1}{5-1} \int_1^5 f(x)\, dx$$

$$\int_1^5 f(x)\, dx = 3 \times \frac{1}{2} \times 1 \times 1 + 4(1 \times 1)$$

$$= \frac{3}{2} + 4$$

$$= \frac{11}{2}$$

$$\bar{f} = \frac{1}{4} \times \frac{11}{2}$$

$$= \frac{11}{8}$$

Problem 3

$$\int_0^7 f(x)\, dx = \int_0^3 f(x)\, dx + \int_3^7 f(x)\, dx$$

$$= 3 + 1$$

$$= 4$$

$$\int_1^3 f(x)\,dx = \int_0^3 f(x)\,dx - \int_0^1 f(x)\,dx$$

$$= 3 - 5$$

$$= -2$$

$$\int_1^7 f(x)\,dx = \int_1^3 f(x)\,dx + \int_3^7 f(x)\,dx$$

$$= -2 + 1$$

$$= -1$$

$$\int_3^0 f(x)\,dx = -\int_0^3 f(x)\,dx$$

$$= -3$$

Problem 4

$$\int \frac{x^2-4}{\sqrt[3]{x^2}}\,dx$$

$$= \int (x^2-4)\,x^{-2/3}\,dx$$

$$= \int \left(x^{\frac{4}{3}} - 4x^{-2/3}\right)dx$$

$$= \frac{3x^{7/3}}{7} - 12x^{1/3} + c$$

$$\int (x+1)\sqrt{x}\,dx$$

$$= \int \left(x^{3/2} + x^{1/2}\right)dx$$

$$= \frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + c$$

$$\int (\sqrt{x}+2)^2 \, dx$$

$$= \int \left(x + 4 + 4\sqrt{x}\right) dx$$

$$= \frac{x^2}{2} + 4x + \frac{8x^{3/2}}{3} + C$$

Problem 5

a) $$-\frac{1}{2x^{3/2}} \sin\left(\frac{1}{x}\right) + \frac{1}{x^2} \sin\left(\frac{1}{x^2}\right)$$

b) $$f(t) = \frac{3}{2 + t\sqrt{2}}$$

$v(x) = x^2 + x$ $\quad$ $u(x) = \sqrt{x}$

$v'(x) = 2x + 1$ $\quad$ $u'(x) = \dfrac{1}{2\sqrt{x}}$

$$f(v) = \frac{3}{2 + (x^2+x)(\sqrt{2})} \qquad f(u) = \frac{3}{2 + \sqrt{2x}}$$

$$H'(x) = \frac{3(2x+1)}{2 + (\sqrt{2})(x^2+x)} - \frac{1}{2\sqrt{x}} \times \frac{3}{2 + \sqrt{2x}}$$

$$H'(2) = \frac{3(4+1)}{2 + \sqrt{2}(4+2)} - \frac{3}{(2\sqrt{x})(2 + \sqrt{2x})}$$

$$= \frac{15}{2 + 6\sqrt{2}} - \frac{3}{8\sqrt{2}}$$

$$= 1.17$$

Problem 6

$$\int_0^{\pi/4} \sec^2 x \, dx$$

$$= \tan x \Big|_0^{\pi/4}$$

$$= \tan \pi/4 - 0$$

$$= 1$$

$$\int_0^{\pi/6} \sec x \tan x \, dx$$

$$= \sec x \Big|_0^{\pi/6}$$

$$= \frac{1}{\cos \pi/6} - \frac{1}{\cos 0}$$

$$= \frac{2}{\sqrt{3}} - 1$$

$$\int_1^3 f(x)\, dx \Rightarrow f(x) = \begin{cases} (x+1)^2 & 1 \le x \le 2 \\ 3 - x^2 & 2 < x \le 3 \end{cases}$$

$$\int_1^2 x^2 + 2x + 1 \, dx$$

$$= \frac{x^3}{3} + x^2 + x \Big|_1^2$$

$$= \left(\frac{8}{3} + 4 + 2\right) - \left(\frac{1}{3} + 1 + 1\right)$$

$$= \frac{7}{3} + 6 - 2$$

$$= \frac{7}{3} + 4$$

$$\int_2^3 3 - x^2 \, dx$$

$$= 3x - \frac{x^3}{3} \Big|_2^3$$

$$= 9 - 9 - 6 + \frac{8}{3}$$

$$= \frac{8}{3} - 6$$

$\frac{7}{3} + 4 + \frac{8}{3} - 6$

$= \frac{15}{3} - 2$

$= 5 - 2$

$= 3$

Problem 7

Displacement $= \int_0^3 v(t)\, dt$

Displacement = $\int$ velocity

Displacement = Final Pos - Init pos

Total Distance $= \int_0^3 |v(t)|\, dt$

Total dist travelled = $\int$ velocity

$s(3) = \int_0^3 (t^2 - t - 2)\, dt$

$= \left[\frac{t^3}{3} - \frac{t^2}{2} - 2t\right]_0^3$

$= 9 - \frac{9}{2} - 6$

$= \frac{9}{2} - \frac{12}{2}$

$= -3/2$

Total dist. travelled $= \int_0^3 |t^2 - t - 2|\, dt$

$t^2 - t - 2 = 0$

$(t-2)(t+1) = 0$

$t = -1$ & $t = 2$

x	$-\infty$		-1		0		2		3		∞
t^2-t-2		$+$	0	$-$		$-$	0	$+$		$+$	

$$\int_0^2 -t^2+t+2\,dt \quad + \quad \int_2^3 t^2-t-2\,dt$$

$$= -\frac{t^3}{3}+\frac{t^2}{2}+2t\Bigg|_0^2 \quad + \quad \frac{t^3}{3}-\frac{t^2}{2}-2t\Bigg|_2^3$$

$$=\left(\frac{-8}{3}+2+4\right)+\left[9-\frac{9}{2}-4-\left(\frac{8}{3}-2-4\right)\right]$$

$=$

Problem 8

$u = x+1 \Rightarrow x = u-1$

$\frac{du}{dx} = 1$

$du = dx$

$\int \frac{(u-1)^2}{\sqrt{u}} \, du$

$= \int \frac{(u^2 - 2u + 1)}{\sqrt{u}} \, du$

$= \int \left(u^{3/2} - 2u^{1/2} + u^{-1/2} \right) du$

$= \frac{2}{5}(x+1)^{5/2} - \frac{4}{3}(x+1)^{3/2} + 2(x+1)^{1/2} + C$

$\int \frac{x-2}{(x^2-4x+4)^2} \, dx$

$u = x^2 - 4x + 4$

$\frac{du}{dx} = 2x - 4$

$du = 2x - 4 \, dx$

$= 2(x-2)\,dx$

$(x-2)\,dx = \frac{du}{2}$

$\frac{1}{2} \int u^{-2} \, du$

$= \frac{1}{2} \times \frac{u^{-1}}{-1}$

$= \frac{-1}{2u}$

$$= \frac{-1}{2(x^2-4x+4)}$$

$$\int_0^1 \frac{1}{\sqrt{x+1}}\, dx$$

$u = x+1$ at $x=0$ $u=1$

$du = dx$ $x=1$ $u=2$

$$\int_1^2 \frac{1}{\sqrt{u}}\, du$$

$$\int_1^2 u^{-1/2}\, du$$

$$= 2u^{1/2} \Big|_1^2$$

$$= 2\sqrt{2} - 2$$

$$= 2(\sqrt{2}-1)$$

$$\int_1^2 (x^2+1)\sqrt{2x^3+6x}\, dx$$

$u = 2x^3+6x$ at $x=1$ $u=8$

$\frac{du}{dx} = 6x^2+6$ $x=2$ $u=28$

$du = 6(x^2+1)\, dx$

$(x^2+1)\, dx = \frac{1}{6}\, du$

$$\frac{1}{6}\int_8^{28} u^{1/2}\, du$$

$$= \frac{1}{\cancel{6}_3} \times \frac{\cancel{2}}{3}\left[u^{3/2}\right]_8^{28}$$

$$= \frac{1}{9}\left[28^{3/2} - 8^{3/2}\right] = 13.948$$