Math Midterm Aut 21

1\. a) $$\lim_{x \to 3} \frac{\sqrt{2x+3} - 3}{x - 3}$$

$$= \lim_{x \to 3} \frac{\sqrt{2x+3} - 3}{x-3} \times \frac{\sqrt{2x+3} + 3}{\sqrt{2x+3} + 3}$$

$$= \lim_{x \to 3} \frac{2x+3-9}{(x-3)(\sqrt{2x+3}+3)}$$

$$= \lim_{x \to 3} \frac{2x-6}{(x-3)(\sqrt{2x+3}+3)}$$

$$= \lim_{x \to 3} \frac{2\cancel{(x-3)}}{\cancel{(x-3)}(\sqrt{2x+3}+3)}$$

$$= \lim_{x \to 3} \frac{2}{\sqrt{2x+3}+3}$$

$$= \frac{2}{3+3}$$

$$= \frac{1}{3}$$

1\. b) $$\lim_{x \to -1^+} \frac{2|x| - 2}{x+1}$$

when $x \to -1^+$, $x < 0$

$\therefore |x| = -x$

$$\lim_{x \to -1^+} \frac{-2x-2}{x+1}$$

$$= \lim_{x \to -1^+} \frac{-2\cancel{(x+1)}}{\cancel{(x+1)}}$$

$$= \frac{-2}{2}$$

1. c) $\lim_{x \to -\infty} \dfrac{3|x| - 1}{2x+7}$

when $x \to -\infty$, $x < 0$

$\therefore |x| = -x$

$\lim_{x \to -\infty} \dfrac{-3x - 1}{2x+7}$

Using L'Hopital's Rule

$\lim_{x \to -\infty} \dfrac{d/dx(-3x-1)}{d/dx(2x+7)}$

$= \dfrac{-3}{2}$

2.

$f(2) = 3$

$\therefore 4a + b = 3$

function is continuous at $x = 1$

$\therefore a + b = \ln 1$

$a + b = 0$

$4a + b = 3$ — (i)

$a + b = 0$ — (ii)

(i) - (ii)

$3a = 3$

$a = 1$

Substituting in (ii)

$1 + b = 0$

$b = -1$

3

$$f(x) = \frac{\ln x + 1}{x^2 + 1}$$

$$f'(x) = \frac{(x^2+1)\left(\frac{1}{x}\right) - (\ln x + 1)(2x)}{(x^2+1)^2}$$

$$= \frac{x + \frac{1}{x} - 2x \ln x - 2x}{(x^2+1)^2}$$

$$f'(1) = \frac{1 + 1 - 2 \ln 1 - 2}{(1+1)^2}$$

$$= \frac{2 - 2 - 0}{2}$$

$$= 0$$

$\therefore$ Slope of tangent line $= f'(1) = 0$

$f(1) = 1/2$

Eq of line

$$y = m(x - x_1) + y_1$$

$$y = 0(x - 1) + \frac{1}{2}$$

$$y = \frac{1}{2}$$

4 A)
$$y \sin x + y^3 = 2x + 1$$

Differentiating both sides

$$y \cos x + y' \sin x + 3y^2 y' = 2$$

$$y'(\sin x + 3y^2) = 2 - y \cos x$$

$$y' = \frac{2 - y \cos x}{\sin x + 3y^2}$$

B) $\frac{dr}{dt} = 3$ cm/sec $\qquad \frac{dh}{dt} = 2$ cm/sec

$V = \pi r^2 h$

$$\frac{dV}{dt} = \pi \left[2rh \frac{dr}{dt} + r^2 \frac{dh}{dt} \right]$$

@ $r = 1$, $h = 4$

$$\frac{dV}{dt} = \pi \left[2(1)(4)(3) + (1)^2(2) \right]$$

$$= \pi [24 + 2]$$

$$= 26\pi \text{ cm}^3/\text{sec}$$

$$= 81.681 \text{ cm}^3/\text{sec}$$

5) $g(x) = \sqrt{x}\,(x-3)$

$$g(x) = x\sqrt{x} - 3\sqrt{x}$$

$$= x^{3/2} - 3x^{1/2}$$

$$g'x = \frac{3}{2} x^{1/2} - \frac{3}{2} x^{-1/2}$$

$$= \frac{3}{2} \left(x^{1/2} - x^{-1/2} \right)$$

$$= \frac{3}{2} \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right)$$

$g'(x) = 0$

$$\frac{3}{2} \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right) = 0$$

$$\sqrt{x} - \frac{1}{\sqrt{x}} = 0$$

$$\sqrt{x} = \frac{1}{\sqrt{x}}$$

$x = 1$

when $x = 0$, $g'(0)$ is undefined

$\therefore$ $x = 0$ and $x = 1$ are critical points

Both are within $[0, 4]$

$g(0) = 0$

$g(1) = 1 - 3 = -2$

$g(4) = 8 - 6 = 2$

$\therefore$ Absolute maximum $= 2$ at $x = 1$

Absolute minimum $= -2$ at $x = 4$

6) $A = lb$ $\quad\quad$ $P = 2(l+b)$

$A = 50 \text{ cm}^2$

$50 = lb$

$l = \frac{50}{b}$

$P = 2(l+b)$

$= 2\left(\frac{50}{b} + b\right)$

$P' = 2\left(\frac{-50}{b^2} + 1\right)$

$P' = 0$

$2\left(-\frac{50}{b^2} + 1\right) = 0$

$\frac{-50}{b^2} + 1 = 0$

$\frac{50}{b^2} = 1$

$b^2 = 50$

$b = \pm\sqrt{50}$

b cannot be negative as measurements cannot be negative

$\therefore\ b = \sqrt{50} = 5\sqrt{2}$

$p'' = 2\left(\frac{100}{b^3}\right) > 0$

$b = 5\sqrt{2} = 7.0711$ cm

$l = \frac{50}{b} = 7.0711$ cm

$\therefore\ P = 2(l+b)$

$= 28.284$ cm

7. $f(x) = 2x - 3x^{2/3}$

$f'(x) = 2 - \frac{3\times 2}{3} x^{-1/3}$

$= 2 - \frac{2}{\sqrt[3]{x}}$

$f'(x) = 0$

$2 - \frac{2}{\sqrt[3]{x}} = 0$

$2 = \frac{2}{\sqrt[3]{x}}$

$\sqrt[3]{x} = 1$

$x = 1$

at $x = 0$, $f'(0)$ is undefined

$\therefore$ critical points are $x = 0$ and $x = 1$

$$f''(x) = -\frac{2}{3} x^{-1/3 - 1}$$

$$= \frac{2}{3} x^{-4/3} > 0$$

8)