

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

$$A = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & -1 & 0 & 1 \end{array} \right]$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -5 & -2 & 1 \end{array} \right]$$

$$R_2 \rightarrow R_2 / -5$$

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 2/5 & -1/5 \end{array} \right]$$

$$R_1 \rightarrow R_1 - 2R_2$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 1/5 & 2/5 \\ 0 & 1 & 2/5 & -1/5 \end{array} \right]$$

$$\therefore \vec{A} = \frac{1}{5} \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

Problem 6

$$A = \begin{bmatrix} -3 & 2 & 2 & 3 \\ 0 & 0 & -2 & 0 \\ 0 & -2 & -2 & 0 \\ 3 & 0 & 2 & h \end{bmatrix}$$

$$R_1 \rightarrow R_4 + R_1$$

$$A = \begin{bmatrix} 0 & 2 & 4 & h+3 \\ 0 & 0 & -2 & 0 \\ 0 & -2 & -2 & 0 \\ 3 & 0 & 2 & h \end{bmatrix}$$

$$\det(A) = 0$$

$$\det(A) = 3(-1)^{4+1} \begin{vmatrix} 2 & 4 & h+3 \\ 0 & -2 & 0 \\ -2 & -2 & 0 \end{vmatrix}$$

$$= -3 [(h+3)(-4)]$$

$$= 12(h+3)$$

$$= 12h + 36$$

$$12h + 36 = 0$$

$$\underline{h = -3}$$

Problem 8

$$1. |A| = 9 \quad |B| = 4$$

$$|-3A| = (-3)^3 \times 9$$

$$= -243$$

$$2. |BA^{-1}| = \frac{|B|}{|A|} = \frac{4}{9}$$

$$3. 9 \times 2 = 18$$

$$4. (-2) \times 9 = -18$$

Properties of Determinant

Let $A = n \times n$ matrix

$$|A^T| = |A|$$

$$|AB| = |A||B|$$

$$|A^{-1}| = \frac{1}{|A|}$$

$$|kA| = k^n |A|$$

$$|A^k| = |A|^k$$

$$|B| = k|A| \quad \text{when Row Operation: Multiply by Scalar}$$

$$|B| = -|A| \quad \text{when Row Operation: Row Swap}$$

$$|B| = |A| \quad \text{when Row Operation: Pivoting (Adding multiple of row to another)}$$

