Problem 1

$f(x) = -2x^3 + 9x^2 + 24$

$f'(x) = -6x^2 + 18x$

$f'(x) = 0$

$-6x^2 + 18x = 0$

$-6x(x-3) = 0$

$-6x = 0 \qquad x - 3 = 0$

$x = 0 \qquad x = 3$

x	$-\infty$	0		3		∞
$f'(x)$	$-$		$+$		$-$	
$f(x)$	$\downarrow$		$\uparrow$		$\downarrow$	

$\therefore$ f is decreasing on $(-\infty, 0) \cup (3, \infty)$

f is increasing on $(0, 3)$

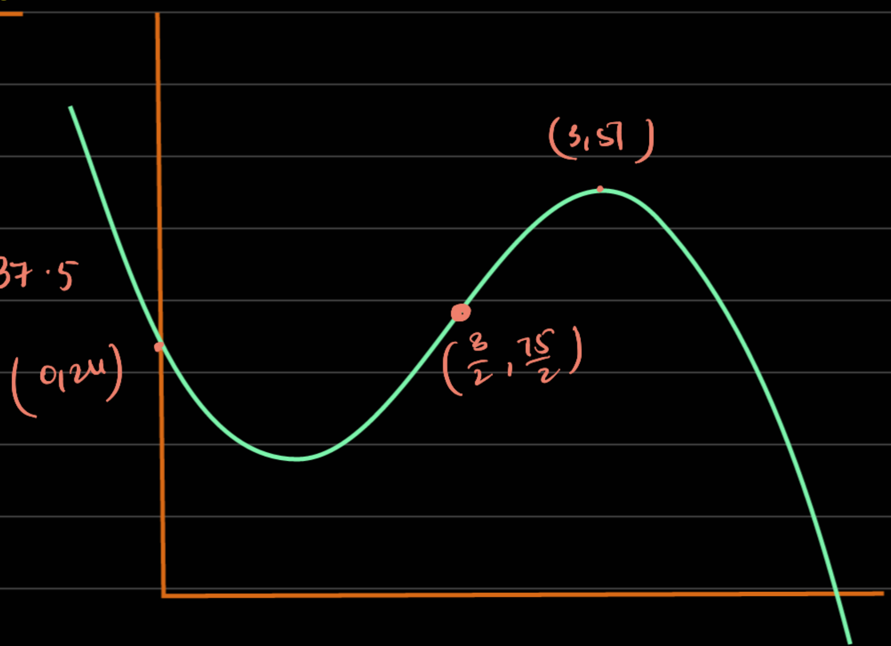
$f(0) = 24 \Rightarrow$ Relative minimum

$f(3) = 51 \Rightarrow$ Relative maximum

$f''(x) = -12x + 18$

x	$-\infty$	$3/2$		∞
$f''(x)$	$+$		$-$	
$f(x)$	$\cup$		$\cap$	

Inflection point $\Rightarrow f(3/2) = 37.5$



Problem 2

$$f(x) = 12x^{2/3} - 16x$$

$$f'(x) = \frac{12 \times 2}{3} x^{-1/3} - 16$$

$$= \frac{8}{\sqrt[3]{x}} - 16$$

$$f'(x) = 0$$

$$\frac{8}{\sqrt[3]{x}} - 16 = 0$$

$$\frac{8}{\sqrt[3]{x}} = 16$$

$$\sqrt[3]{x} = \frac{1}{2}$$

$$x = \frac{1}{8}$$

at $x = 0$, $f'(0)$ is undefined

$\therefore$ $x = 0$ is a critical number

$\therefore$ Critical numbers are $0, 1/8$

x	$-\infty$	0	$1/8$	∞
$f'(x)$	$-$	‖	$+$	$-$
$f(x)$	$\downarrow$	‖	$\uparrow$	$\downarrow$

$$f(0) = 0$$

$$f(1/8) = 3.241$$

Problem 3

a) $f'(x) = 3x^2 - 12$

$f'(x) = 0$

$3(x^2 - 4) = 0$

$x^2 - 4 = 0$

$x = \pm 2$

$f(-4) = -8$

$f(-2) = 24$

$f(2) = -8$

$f(3) = -1$

Absolute maximum = 24 at $x = -2$

Absolute minimum = -8 at $x = -4$ and $x = 2$

b) $f'(x) = \frac{4}{3} x^{1/3} - x^{-2/3}$

$f'(x) = 0$

$$\frac{4}{3}\sqrt[3]{x} - \frac{1}{\sqrt[3]{x^2}} = 0$$

$$\frac{4}{3}\sqrt[3]{x} = \frac{1}{\sqrt[3]{x^2}}$$

$$\frac{4}{3} = \frac{1}{x}$$

$$x = 3/4$$

at $x = 0$, $f'(0)$ is undefined

$\therefore$ $x = 0$ is a critical point

$\therefore$ critical points are $3/4$, 0

$f(-1) = 4$

$f(0) = 0$

$f(3/4) = -2.044$

$f(8) = 10$

Absolute maximum = 10 at $x = 8$

Absolute minimum = -2.044 at $x = 3/4$

Problem 4

a) $f'(x) = 4x^3 - 48x$

$f'(x) = 0$

$4x^3 - 48x = 0$

$4x(x^2 - 12) = 0$

$4x = 0$ $\quad\quad$ $x^2 - 12 = 0$

$x = 0$ $\quad\quad$ $x = \pm 2\sqrt{3}$

$f''(x) = 12x^2 - 48$

$f''(x) = 0$

$12x^2 - 48 = 0$

$12(x^2 - 4) = 0$

$x^2 - 4 = 0$

$x = \pm 2$

$\therefore$ Points of inflection are $x = -2$ and $x = 2$

x	$-\infty$ to -2	-2 to 2	2 to ∞
$f''(x)$	+	−	+
$f(x)$	∪	∩	∪

$\therefore$ Concave up in $(-\infty, -2)$ & $(2, \infty)$ $\quad$ Concave down in $(-2, 2)$

b) $f'(x) = 12x^2 - 30x - 18$

$-24 = \frac{24}{1} \quad \frac{12}{2}$

$f''(x) = 24x - 30$

$f''(x) = 0$

$24x - 30 = 0$

$x = \frac{30}{24}$

$= \frac{5}{4}$

$\therefore$ Inflection point is $x = 5/4$

x	$-\infty$	$5/4$	∞
$f''(x)$	$-$	$\|\|$	$+$
$f(x)$	$\cap$	$\|\|$	$\cup$

$\therefore$ Concave up in $(-\infty, 5/4)$

Concave down in $(5/4, \infty)$

Problem 5

$f(x) = x^3 + \frac{3}{2}x^2 - 6x + 12$

$f'(x) = 3x^2 + 3x - 6$

$f'(x) = 0$

$3x^2 + 3x - 6 = 0$

$3(x^2 + x - 2) = 0$

$x^2 + x - 2 = 0$

$(x-1)(x+2) = 0$

$x = 1$ OR $x = -2$

x	$-\infty$	-2		1	∞
$f'(x)$	$+$		$-$		$+$
$f(x)$	$\uparrow$		$\downarrow$		$\uparrow$

$f(-2) = 22$

$f(1) = 8.5$

$f''(x) = 6x + 3$

$f''(x) = 0$

$6x + 3 = 0$

$2x + 1 = 0$

$x = -1/2$

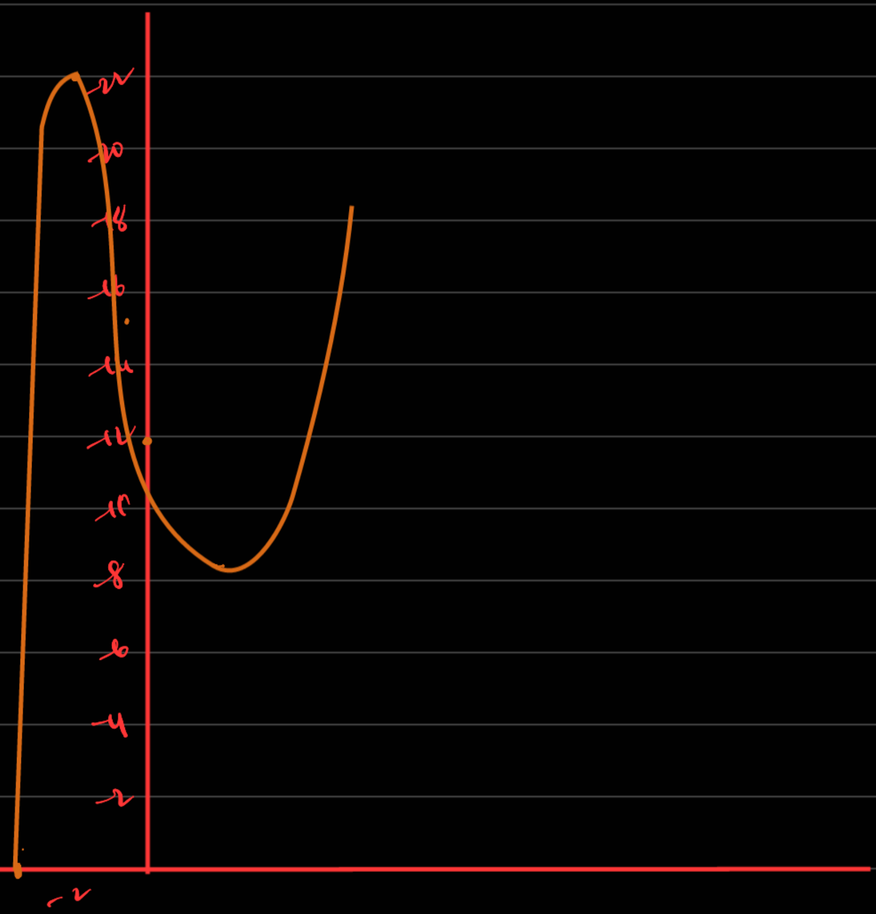
x	$-\infty$	$-1/2$	∞
$f''(x)$	$-$		$+$
$f(x)$	$\cap$		$\cup$

$f(-1/2) = 15.25$

Inflection point $(-1/2, 15.25)$

y intercept $= (0, 12)$

x intercept $= (-3.85, 0)$



$$f'(x) = \frac{4}{3} x^{1/3} + \frac{4}{3} x^{-2/3}$$

$$= \frac{4}{3} x^{1/3} + \frac{4}{3x^{2/3}}$$

$$f'(x) = 0$$

$$\cancel{\frac{4}{3}} x^{1/3} = \frac{-\cancel{4}}{\cancel{3} x^{2/3}}$$

$$x^{1/3} = \frac{-1}{x^{2/3}}$$

$$\sqrt[3]{x} = \frac{-1}{\sqrt[3]{x^2}}$$

$$x = -1$$

at $x = 0$, $f'(0)$ is undefined

$\therefore$ Critical numbers are -1 and 0

x	$-\infty$		-1		0		∞
$f'(x)$		$-$		$+$		$+$	
$f(x)$		$\downarrow$		$\uparrow$		$\uparrow$	

$f(-1) = -3$ Relative min.

$$f''(x) = \frac{4}{3} \times \frac{1}{3} x^{-2/3} + \frac{4}{3}\left(\frac{-2}{3}\right) x^{-5/3}$$

$$= \frac{4}{9\sqrt[3]{x^2}} - \frac{8}{9x\sqrt[3]{x^2}}$$

$$= \frac{4x - 8}{9x\sqrt[3]{x^2}}$$

$f''(x) = 0$

$$\frac{4x-8}{9x\sqrt[3]{x^2}} = 0$$

$4x - 8 = 0$ $\qquad$ $9x^{5/3} = 0$

$x = 2$ $\qquad$ $x^{5/3} = 0$

$x = 0$

$\therefore$ Inflection points are $x = 0$ & $x = 2$

$f(0) = 0$

$f(2) = 7.5595$

x	$-\infty$	0	2	∞
$f''(x)$	+	−	+	
$f(x)$	∪	∩	∪	

Inflection points $(0,0)$ & $(2, 7.6)$

x - intercept $= (-4, 0)$ & $(0,0)$

y - intercept $= (0,0)$

Problem 6

$V = w^2 h$

$A = 2w^2 + 4wh = 10$

$h = \frac{10 - 2w^2}{4w} = \frac{5 - w^2}{2w}$

$V = w\cancel{2}\left(\frac{5 - w^2}{2\cancel{w}}\right)$

$= \frac{5w}{2} - \frac{w^3}{2}$

$\frac{dV}{dw} = \frac{5}{2} - \frac{3}{2}w^2$

$\frac{dV}{dw} = 0$

$\frac{5}{\cancel{2}} = \frac{3}{\cancel{2}}w^2$

$w^2 = \frac{5}{3}$

$w = \pm\sqrt{\frac{5}{3}}$

$\because$ width cannot be negative

$w = \sqrt{\frac{5}{3}}$

$\frac{d^2V}{dw^2} = -3w$

$V''\left(\sqrt{\frac{5}{3}}\right) = -\sqrt{15} < 0$

V is max at $w = \sqrt{\frac{5}{3}}$

$$h = \frac{5 - w^2}{2w}$$

$$h = \frac{5 - 5/3}{2\sqrt{\frac{5}{3}}} = \frac{\sqrt{15}}{3}$$

$$V = \frac{5\sqrt{15}}{9} \ m^3$$

Problem 7

$P = 2x + 2y$

$A = xy = 100$

$y = \frac{100}{x}$

$$P = \frac{200}{x} + 2x$$

$$P'(x) = \frac{-200}{x^2} + 2$$

$P'(x) = 0$

$\frac{200}{x^2} = 2$

$x^2 = 100$

$x = 10$

$$P''(x) = \frac{400}{x^3}$$

$$P''(10) = \frac{400}{10^3} > 0$$

$\therefore$ P is minimum at $x = 10$

$y = \frac{100}{x} = 10$ units $\qquad \therefore P_{min} = 40$ units