Problem 1(a) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
Write down the vector projection of **b** along **a**. (Hint: Use projections.)

Problem 1(a) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
Write down the vector projection of $\mathbf{b}$ along $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have $|\mathbf{a}| = \sqrt{9 + 36 + 4}$

Problem 1(a) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
Write down the vector projection of $\mathbf{b}$ along $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have $|\mathbf{a}| = \sqrt{9 + 36 + 4} = \sqrt{49}$

Problem 1(a) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle$.
Write down the vector projection of $\mathbf{b}$ along $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have $|\mathbf{a}| = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$.

Problem 1(a) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle$, $\quad \mathbf{b} = \langle 1, 2, 3 \rangle$.
Write down the vector projection of $\mathbf{b}$ along $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have $|\mathbf{a}| = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$.
- Then

$$\mathbf{n} = \frac{\mathbf{a}}{|\mathbf{a}|}$$

Problem 1(a) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle$.
Write down the vector projection of $\mathbf{b}$ along $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have $|\mathbf{a}| = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$.
- Then

$$\mathbf{n} = \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{1}{7}\mathbf{a}$$

Problem 1(a) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle$, $\qquad \mathbf{b} = \langle 1, 2, 3 \rangle$.
Write down the vector projection of $\mathbf{b}$ along $\mathbf{a}$. (Hint: Use projections.)

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- Then

$$\mathbf{n} = \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{1}{7}\mathbf{a} = \text{ unit vector parallel to } \mathbf{a}.$$

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- So,

$$\mathbf{proj}_{\mathbf{a}}\mathbf{b} = (\mathbf{b} \cdot \mathbf{n})\mathbf{n}$$

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Problem 1(a) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle$, $\mathbf{b} = \langle 1, 2, 3 \rangle$.
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$$\frac{1}{49}\langle 1, 2, 3 \rangle \cdot \langle 3, 6, -2 \rangle \langle 3, 6, -2 \rangle$$

Problem 1(a) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle$, $\qquad \mathbf{b} = \langle 1, 2, 3 \rangle$.
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$$\frac{1}{49}\langle 1, 2, 3 \rangle \cdot \langle 3, 6, -2 \rangle \langle 3, 6, -2 \rangle = \frac{9}{49}\langle 3, 6, -2 \rangle.$$

Problem 1(b) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
Write $\mathbf{b}$ as a sum of a vector parallel to $\mathbf{a}$ and a vector orthogonal to $\mathbf{a}$. (Hint: Use projections.)

Problem 1(b) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle$.
Write $\mathbf{b}$ as a sum of a vector parallel to $\mathbf{a}$ and a vector orthogonal to $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have

$$\mathbf{b} = \langle 1, 2, 3 \rangle$$

Problem 1(b) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
Write $\mathbf{b}$ as a sum of a vector parallel to $\mathbf{a}$ and a vector orthogonal to $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have

$$\mathbf{b} = \langle 1, 2, 3 \rangle = \langle 1, 2, 3 \rangle - \frac{9}{49}\langle 3, 6, -2 \rangle + \frac{9}{49}\langle 3, 6, -2 \rangle$$

Problem 1(b) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
Write $\mathbf{b}$ as a sum of a vector parallel to $\mathbf{a}$ and a vector orthogonal to $\mathbf{a}$. (Hint: Use projections.)

Solution:

- We have

$$\mathbf{b} = \langle 1, 2, 3 \rangle = \langle 1, 2, 3 \rangle - \frac{9}{49}\langle 3, 6, -2 \rangle + \frac{9}{49}\langle 3, 6, -2 \rangle$$

$$= \frac{1}{49}\langle 22, 44, 165 \rangle + \frac{9}{49}\langle 3, 6, -2 \rangle.$$

Problem 1(b) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle$, $\mathbf{b} = \langle 1, 2, 3 \rangle$.
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$$= \frac{1}{49}\langle 22, 44, 165 \rangle + \frac{9}{49}\langle 3, 6, -2 \rangle.$$

- Here

$$\frac{9}{49}\langle 3, 6, -2 \rangle \ \textbf{parallel to} \ \ \mathbf{a} = \langle 3, 6, -2 \rangle$$

and

$$\frac{1}{49}\langle 22, 44, 165 \rangle \ \textbf{orthogonal to } \mathbf{a} = \langle 3, 6, -2 \rangle.$$

$\square$

Problem 1(b) Continuation - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
Write $\mathbf{b}$ as a sum of a vector parallel to $\mathbf{a}$ and a vector orthogonal to $\mathbf{a}$. (Hint: Use projections.)

Problem 1(b) Continuation - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle$, $\mathbf{b} = \langle 1, 2, 3 \rangle$.
Write $\mathbf{b}$ as a sum of a vector parallel to $\mathbf{a}$ and a vector orthogonal to $\mathbf{a}$. (Hint: Use projections.)

Solution:

- Why so? All we did was to write

$$\mathbf{b} = \mathbf{b} - (\mathbf{b} \cdot \mathbf{n})\mathbf{n} + (\mathbf{b} \cdot \mathbf{n})\mathbf{n}$$

where $\mathbf{n} = \frac{\mathbf{a}}{7}$, $\mathbf{n}^2 = 1$.

Problem 1(b) Continuation - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle$.
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- Of course this is the same as

$$\mathbf{b} = (\mathbf{b} - \mathbf{proj}_{\mathbf{a}}\mathbf{b}) + \mathbf{proj}_{\mathbf{a}}\mathbf{b}.$$

Problem 1(b) Continuation - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle$, $\qquad \mathbf{b} = \langle 1, 2, 3 \rangle$.
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  That is, we write $\mathbf{b}$ as $\mathbf{proj}_{\mathbf{a}}\mathbf{b}$ plus “the rest”.

Problem 1(b) Continuation - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle$, $\quad \mathbf{b} = \langle 1, 2, 3 \rangle$.
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  That is, we write $\mathbf{b}$ as $\mathbf{proj}_{\mathbf{a}}\mathbf{b}$ plus “the rest”. But “the rest” is orthogonal to $\mathbf{n}$ (and to $\mathbf{a}$), since

  $(\mathbf{b} - (\mathbf{b} \cdot \mathbf{n})\mathbf{n}) \cdot \mathbf{n}$

Problem 1(b) Continuation - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle$, $\qquad \mathbf{b} = \langle 1, 2, 3 \rangle$.
Write $\mathbf{b}$ as a sum of a vector parallel to $\mathbf{a}$ and a vector orthogonal to $\mathbf{a}$. (Hint: Use projections.)

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- Of course this is the same as

$$\mathbf{b} = (\mathbf{b} - \mathbf{proj_a b}) + \mathbf{proj_a b}.$$

  That is, we write $\mathbf{b}$ as $\mathbf{proj_a b}$ plus “the rest”. But “the rest” is orthogonal to $\mathbf{n}$ (and to $\mathbf{a}$), since

$$(\mathbf{b} - (\mathbf{b} \cdot \mathbf{n})\mathbf{n}) \cdot \mathbf{n} = \mathbf{b} \cdot \mathbf{n} - (\mathbf{b} \cdot \mathbf{n})(\mathbf{n} \cdot \mathbf{n})$$

Problem 1(b) Continuation - Spring 2009

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$$\mathbf{b} = \mathbf{b} - (\mathbf{b}\cdot\mathbf{n})\mathbf{n} + (\mathbf{b}\cdot\mathbf{n})\mathbf{n}$$

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$$(\mathbf{b} - (\mathbf{b}\cdot\mathbf{n})\mathbf{n})\cdot\mathbf{n} = \mathbf{b}\cdot\mathbf{n} - (\mathbf{b}\cdot\mathbf{n})(\mathbf{n}\cdot\mathbf{n}) = 0, \text{ as } \mathbf{n}\cdot\mathbf{n} = 1.$$

$\square$

Problem 1(c) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle$.
Let θ be the angle between $\mathbf{a}$ and $\mathbf{b}$. Find $\cos\theta$.

Problem 1(c) - Spring 2009

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Let θ be the angle between $\mathbf{a}$ and $\mathbf{b}$. Find $\cos\theta$.

Solution:

$$\cos(\theta) = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

Problem 1(c) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
Let θ be the angle between $\mathbf{a}$ and $\mathbf{b}$. Find $\cos\theta$.

Solution:

$$\cos(\theta) = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \frac{\langle 3, 6, -2 \rangle \cdot \langle 1, 2, 3 \rangle}{|\langle 3, 6, -2 \rangle||\langle 1, 2, 3 \rangle|}$$

Problem 1(c) - Spring 2009

Given $\mathbf{a} = \langle 3, 6, -2 \rangle, \qquad \mathbf{b} = \langle 1, 2, 3 \rangle.$
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$\square$

Problem 2(a) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
Let **L** be the line which passes through the points $A = (-1, 7, 5)$ and $B = (3, 2, 2)$. Find the parametric equations for **L**.

Problem 2(a) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
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Solution:

- To get **parametric equations** for **L** you need a point through which the line passes and a vector parallel to the line.

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- To get **parametric equations** for **L** you need a point through which the line passes and a vector parallel to the line. For example, take the point to be A and the vector to be $\overrightarrow{AB}$.

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Solution:

- To get **parametric equations** for **L** you need a point through which the line passes and a vector parallel to the line. For example, take the point to be A and the vector to be $\overrightarrow{AB}$.
- The vector equation of **L** is

$$\mathbf{r}(t) = \overrightarrow{OA} + t\overrightarrow{AB}$$

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- The vector equation of **L** is

$$\mathbf{r}(t) = \overrightarrow{OA} + t\overrightarrow{AB} = \langle -1, 7, 5 \rangle + t \langle 4, -5, -3 \rangle$$

Problem 2(a) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
Let **L** be the line which passes through the points $A = (-1, 7, 5)$ and $B = (3, 2, 2)$. Find the parametric equations for **L**.

Solution:

- To get **parametric equations** for **L** you need a point through which the line passes and a vector parallel to the line. For example, take the point to be A and the vector to be $\overrightarrow{AB}$.
- The vector equation of **L** is

$$\mathbf{r}(t) = \overrightarrow{OA} + t\overrightarrow{AB} = \langle -1, 7, 5\rangle + t\,\langle 4, -5, -3\rangle = \langle -1 + 4t, 7 - 5t, 5 - 3t\rangle,$$

where O is the origin.

Problem 2(a) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
Let **L** be the line which passes through the points $A = (-1, 7, 5)$ and $B = (3, 2, 2)$. Find the parametric equations for **L**.

Solution:

- To get **parametric equations** for **L** you need a point through which the line passes and a vector parallel to the line. For example, take the point to be A and the vector to be $\overrightarrow{AB}$.
- The vector equation of **L** is

$$\mathbf{r}(t) = \overrightarrow{OA} + t\overrightarrow{AB} = \langle -1, 7, 5\rangle + t\,\langle 4, -5, -3\rangle \; = \langle -1 + 4t, 7 - 5t, 5 - 3t\rangle\,,$$

where O is the origin.

- The **parametric equations** are:

$$\left|\begin{array}{l} x = -1 + 4t \\ y = 7 - 5t, \qquad t \in \mathbb{R}. \\ z = 5 - 3t \end{array}\right.$$

$\square$

Problem 2(b) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
A, B and C are three of the four vertices of a parallelogram, while CA and CB are two of the four edges. Find the fourth vertex.

Problem 2(b) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
A, B and C are three of the four vertices of a parallelogram, while CA and CB are two of the four edges. Find the fourth vertex.

Solution:

Denote the fourth vertex by D.

Problem 2(b) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
A, B and C are three of the four vertices of a parallelogram, while CA and CB are two of the four edges. Find the fourth vertex.

Solution:

Denote the fourth vertex by D. Then

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{CB}$$

Problem 2(b) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
A, B and C are three of the four vertices of a parallelogram, while CA and CB are two of the four edges. Find the fourth vertex.

Solution:

Denote the fourth vertex by D. Then

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{CB} = \langle -1, 7, 5 \rangle + \langle 2, 0, -1 \rangle$$

Problem 2(b) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
A, B and C are three of the four vertices of a parallelogram, while CA and CB are two of the four edges. Find the fourth vertex.

Solution:

Denote the fourth vertex by D. Then

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{CB} = \langle -1, 7, 5 \rangle + \langle 2, 0, -1 \rangle = \langle 1, 7, 4 \rangle ,$$

where O is the origin.

Problem 2(b) - Spring 2009

Given $A = (-1, 7, 5)$, $B = (3, 2, 2)$ and $C = (1, 2, 3)$.
A, B and C are three of the four vertices of a parallelogram, while CA and CB are two of the four edges. Find the fourth vertex.

Solution:

Denote the fourth vertex by D. Then

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{CB} = \langle -1, 7, 5 \rangle + \langle 2, 0, -1 \rangle = \langle 1, 7, 4 \rangle ,$$

where O is the origin. That is,

$$D = (1, 7, 4).$$

$\square$

Problem 3(a) - Spring 2009

Consider the points $P(1,3,5)$, $Q(-2,1,2)$, $R(1,1,1)$ in $\mathbb{R}^3$. Find an equation for the plane containing P, Q and R.

Problem 3(a) - Spring 2009

Consider the points $P(1,3,5)$, $Q(-2,1,2)$, $R(1,1,1)$ in $\mathbb{R}^3$. Find an equation for the plane containing P, Q and R.

Solution:

Since a plane is determined by its normal vector $\mathbf{n}$ and a point on it, say the point P, it suffices to find $\mathbf{n}$.

Problem 3(a) - Spring 2009

Consider the points $P(1,3,5)$, $Q(-2,1,2)$, $R(1,1,1)$ in $\mathbb{R}^3$. Find an equation for the plane containing P, Q and R.

Solution:

Since a plane is determined by its normal vector **n** and a point on it, say the point P, it suffices to find **n**. Note that:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR}$$

Problem 3(a) - Spring 2009

Consider the points $P(1,3,5)$, $Q(-2,1,2)$, $R(1,1,1)$ in $\mathbb{R}^3$. Find an equation for the plane containing P, Q and R.

Solution:

Since a plane is determined by its normal vector **n** and a point on it, say the point P, it suffices to find **n**. Note that:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -2 & -3 \\ 0 & -2 & -4 \end{vmatrix}$$

Problem 3(a) - Spring 2009

Consider the points $P(1,3,5)$, $Q(-2,1,2)$, $R(1,1,1)$ in $\mathbb{R}^3$. Find an equation for the plane containing P, Q and R.

Solution:

Since a plane is determined by its normal vector $\mathbf{n}$ and a point on it, say the point P, it suffices to find $\mathbf{n}$. Note that:

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Problem 3(a) - Spring 2009

Consider the points $P(1,3,5)$, $Q(-2,1,2)$, $R(1,1,1)$ in $\mathbb{R}^3$. Find an equation for the plane containing P, Q and R.

Solution:

Since a plane is determined by its normal vector $\mathbf{n}$ and a point on it, say the point P, it suffices to find $\mathbf{n}$. Note that:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -2 & -3 \\ 0 & -2 & -4 \end{vmatrix} = \langle 2, -12, 6 \rangle = 2\langle 1, -6, 3 \rangle.$$

Problem 3(a) - Spring 2009

Consider the points $P(1,3,5)$, $Q(-2,1,2)$, $R(1,1,1)$ in $\mathbb{R}^3$. Find an equation for the plane containing P, Q and R.

Solution:

Since a plane is determined by its normal vector **n** and a point on it, say the point P, it suffices to find **n**. Note that:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -2 & -3 \\ 0 & -2 & -4 \end{vmatrix} = \langle 2, -12, 6 \rangle = 2\langle 1, -6, 3 \rangle.$$

So the **equation of the plane** is:

$$(x-1) - 6(y-3) + 3(z-5) = 0. \quad \square$$

Problem 3(b) - Spring 2009

Consider the points $P(1,3,5)$, $Q(-2,1,2)$, $R(1,1,1)$ in $\mathbb{R}^3$.
Find the area of the triangle with vertices P, Q, R.

Problem 3(b) - Spring 2009

Consider the points $P(1, 3, 5)$, $Q(-2, 1, 2)$, $R(1, 1, 1)$ in $\mathbb{R}^3$. Find the area of the triangle with vertices P, Q, R.

Solution:

The area of the triangle $\boldsymbol{\Delta}$ with vertices P, Q, R can be found by taking the area of the parallelogram spanned by $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ and dividing it by 2.

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$$\textbf{Area}(\mathbf{\Delta}) = \frac{|\overrightarrow{PQ} \times \overrightarrow{PR}|}{2}$$

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$$\textbf{Area}(\boldsymbol{\Delta}) = \frac{|\overrightarrow{PQ} \times \overrightarrow{PR}|}{2} = \frac{1}{2}\left|2\langle 1,-6,3\rangle\right|$$

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$$= \sqrt{1 + 36 + 9}$$

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$$= \sqrt{1+36+9} = \sqrt{46}.$$

$\square$

Problem 4 - Spring 2009

Find parametric equations for the line of intersection of the planes $x + y + 3z = 1$ and $x - y + 2z = 0$.

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- A vector $\mathbf{v}$ parallel to the line is the cross product of the normal vectors of the planes:

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- Setting $z = 0$, we obtain the equations $x + y = 1$ and $x - y = 1$ and find such a point $(\frac{1}{2}, \frac{1}{2}, 0)$. Therefore **parametric equations** for $\mathbf{L}$ are:

$$\left| \begin{array}{l} x = \frac{1}{2} + 5t \\ y = \frac{1}{2} + t \\ z = -2t. \end{array} \right.$$

$\square$

Problem 6(a) - Spring 2009

Consider the sphere **S** in $\mathbb{R}^3$ given by the equation

$$x^2 + y^2 + z^2 - 4x - 6z - 3 = 0.$$

Find its center **C** and its radius **R**.

Problem 6(a) - Spring 2009

Consider the sphere **S** in $\mathbb{R}^3$ given by the equation

$$x^2 + y^2 + z^2 - 4x - 6z - 3 = 0.$$

Find its center **C** and its radius **R**.

Solution:

- Completing the square we get

$$(x-2)^2 - 4 + y^2 + (z-3)^2 - 9 - 3 = 0$$

$$\iff$$

$$(x-2)^2 + y^2 + (z-3)^2 = 16.$$

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- This gives:

$$\mathbf{C} = (2,0,3) \qquad \mathbf{R} = 4$$

$\square$

Problem 6(b) - Spring 2009

What does the equation $x^2 + z^2 = 4$ describe in $\mathbb{R}^3$? Make a sketch.

Problem 6(b) - Spring 2009

What does the equation $x^2 + z^2 = 4$ describe in $\mathbb{R}^3$? Make a sketch.

Solution:

- This a (straight, circular) **cylinder** determined by the circle in the xz-plane of radius 2 and center $(0, 0)$ and parallel to the y-axis. $\square$

Problem 1(a) - Fall 2008

Find **parametric equations** for the line **L** which contains $A(1, 2, 3)$ and $B(4, 6, 5)$.

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- The vector equation of **L** is

$$\mathbf{r}(t) = \overrightarrow{OA} + t\overrightarrow{AB}$$

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$$\mathbf{r}(t) = \overrightarrow{OA} + t\overrightarrow{AB} = \langle 1, 2, 3\rangle + t\,\langle 3, 4, 2\rangle\ = \langle 1 + 3t, 2 + 4t, 3 + 2t\rangle\,,$$

where O is the origin.

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  where O is the origin.
- The **parametric equations** are:

$$\begin{aligned} x &= 1+3t \\ y &= 2+4t, \qquad t \in \mathbb{R}. \\ z &= 3+2t \end{aligned}$$

$\square$

Problem 1(b) - Fall 2008

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 10$ and $2x + y - z = 0$.

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- The vector part $\mathbf{v}$ of the line **L** of intersection is orthogonal to the normal vectors $\langle 1, -2, 1 \rangle$ and $\langle 2, 1, -1 \rangle$.

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- Choose $P \in \mathbf{L}$ so the z-coordinate of P is zero. Setting $z = 0$, we obtain:

  $$x - 2y = 10$$

  $$2x + y = 0.$$

Problem 1(b) - Fall 2008

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$$x - 2y = 10$$
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Solving, we find that $x = 2$ and $y = -4$.

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Problem 1(b) - Fall 2008

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 10$ and $2x + y - z = 0$.

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Solving, we find that $x = 2$ and $y = -4$. Hence, $P = \langle 2, -4, 0\rangle$ lies on the line **L**.
- The **parametric equations** are:
$$x = 2 + t$$
$$y = -4 + 3t$$
$$z = 0 + 5t = 5t.$$

Problem 2(a) - Fall 2008

Find an **equation of the plane** which contains the points $P(-1, 0, 1)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

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Solution:

Method 1

- Consider the vectors $\overrightarrow{PQ} = \langle 2,-2,0\rangle$ and $\overrightarrow{PR} = \langle 3,0,-2\rangle$ which lie parallel to the plane.

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- Then consider the normal vector:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR}$$

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- So the **equation of the plane** is given by:

$$\langle 4,4,6\rangle \cdot \langle x+1, y, z-1\rangle = 4(x+1) + 4y + 6(z-1) = 0.$$

Problem 2(a) - Fall 2008

Find an **equation of the plane** which contains the points $P(-1,0,1)$, $Q(1,-2,1)$ and $R(2,0,-1)$.

Problem 2(a) - Fall 2008

Find an **equation of the plane** which contains the points $P(-1,0,1)$, $Q(1,-2,1)$ and $R(2,0,-1)$.

Solution:

Method 2

- The plane consists of all the points $S(x,y,z) \in \mathbb{R}^3$, such that $\overrightarrow{PS}$, $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are in the same plane (coplanar).

Problem 2(a) - Fall 2008

Find an **equation of the plane** which contains the points $P(-1,0,1)$, $Q(1,-2,1)$ and $R(2,0,-1)$.

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- The plane consists of all the points $S(x,y,z) \in \mathbb{R}^3$, such that $\overrightarrow{PS}$, $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are in the same plane (coplanar).
- But this happens if and only if their box product is zero.

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- So the **equation of the plane** is:

$$\begin{vmatrix} x+1 & y & z-1 \\ 2 & -2 & 0 \\ 3 & 0 & -2 \end{vmatrix}$$

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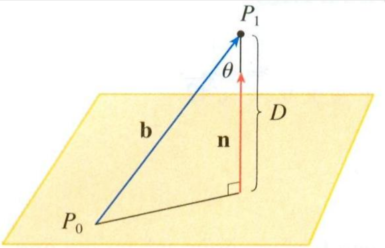
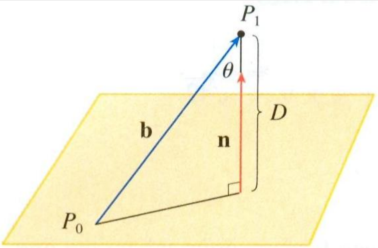
$\square$



Problem 2(b) - Fall 2008

Find the distance **D** from the point $(1, 6, -1)$ to the plane $2x + y - 2z = 19$.

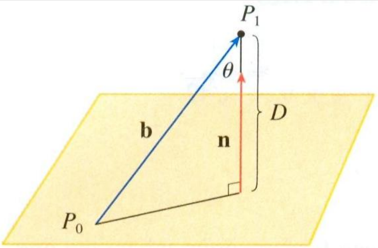


Problem 2(b) - Fall 2008

Find the distance **D** from the point $(1, 6, -1)$ to the plane $2x + y - 2z = 19$.

Solution:

- Recall the distance formula $\mathbf{D} = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$ from a point $P = (x_1, y_1, z_1)$ to a plane $ax + by + cz + d = 0$.



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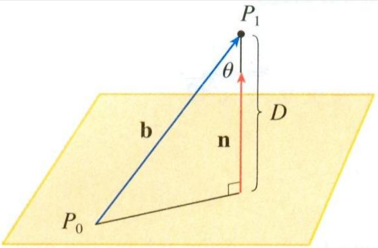
Solution:

- Recall the distance formula $\mathbf{D} = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$ from a point $P = (x_1, y_1, z_1)$ to a plane $ax + by + cz + d = 0$.
- In order to apply the formula, rewrite the equation of the plane in standard form: $2x + y - 2z - 19 = 0$.



Problem 2(b) - Fall 2008

Find the distance **D** from the point $(1, 6, -1)$ to the plane $2x + y - 2z = 19$.

Solution:

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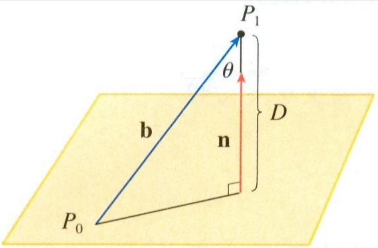

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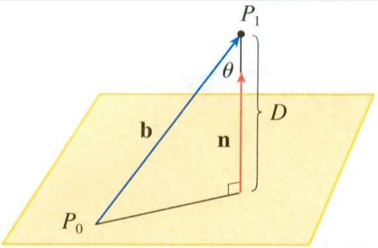

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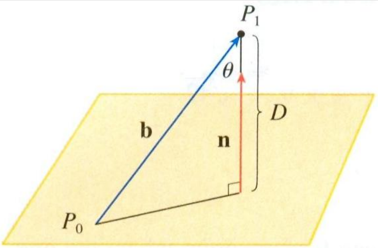

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Problem 2(c) - Fall 2008

Find the point Q in the plane $2x + y - 2z = 19$ which is closest to the point $(1, 6, -1)$. (Hint: You can use part b) of this problem to help find Q or first find the equation of the line **L** passing through Q and the point $(1, 6, -1)$ and then solve for Q.)

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Solution:

- The line **L** in the Hint passes through $(1, 6, -1)$ and is parallel to $\mathbf{n} = \langle 2, 1, -2 \rangle$.

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- So, **L** has **parametric equations**:

$$\begin{aligned} x &= 1 + 2t \\ y &= 6 + t, \qquad\qquad t \in \mathbb{R}. \\ z &= -1 - 2t \end{aligned}$$

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$$2(1 + 2t) + (6 + t) - 2(-1 - 2t) = 19 \iff 9t = 9 \iff t = 1.$$

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- Substituting $t = 1$ in the **parametric equations** of **L** gives the point $Q = (3, 7, -3)$. $\square$

Problem 3(a) - Fall 2008

Find the volume **V** of the **parallelepiped** such that the following four points $A = (3, 4, 0)$, $B = (3, 1, -2)$, $C = (4, 5, -3)$, $D = (1, 0, -1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A.

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Solution:

The **parallelepiped** is determined by its edges

$$\overrightarrow{AB} = \langle 0, -3, -2 \rangle, \quad \overrightarrow{AC} = \langle 1, 1, -3 \rangle, \quad \overrightarrow{AD} = \langle -2, -4, -1 \rangle.$$

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$$\mathbf{V} = \left|\begin{vmatrix} 0 & -3 & -2 \\ 1 & 1 & -3 \\ -2 & -4 & -1 \end{vmatrix}\right|$$

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Problem 3(b) - Fall 2008

Find the **center** and **radius** of the sphere
$x^2 - 4x + y^2 + 4y + z^2 = 8.$

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- Completing the square we get

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$$= (x-2)^2 - 4 + (y+2)^2 - 4 + z^2 = 8$$

$$\Longleftrightarrow$$

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- This gives:

Center $= (2, -2, 0)$ **Radius** $= 4$

$\square$

Problem 5(a) - Fall 2008

Consider the points $A(2, 1, 0)$, $B(3, 0, 2)$ and $C(0, 2, 1)$. Find the area of the triangle ABC. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

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$\square$

Problem 5(b) - Fall 2008

Three of the four vertices of a parallelogram are $P(0,-1,1)$, $Q(0,1,0)$ and $R(2,1,1)$. Two of the sides are PQ and PR. Find the coordinates of the fourth vertex.

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where O is the origin.

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where O is the origin. That is,

$$\mathbf{S} = (2,3,0).$$

$\square$

Problem 6(a) - Spring 2008

Find an **equation of the plane** through the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$.

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So the **equation of the plane** is:

$$-(x - 1) + (y - 2) + 2(z - 3) = 0.$$

□

Problem 6(b) - Spring 2008

Find the area of the triangle Δ with vertices at the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$.

Hint: the area of this triangle is related to the area of a certain parallelogram

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Solution:

Consider the points $A = (1, 2, 3)$, $B = (0, 1, 3)$ and $C = (2, 1, 4)$.

Problem 6(b) - Spring 2008

Find the area of the triangle $\boldsymbol{\Delta}$ with vertices at the points $A = (1, 2, 3)$, $B = (0, 1, 3)$, and $C = (2, 1, 4)$.
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Solution:

Consider the points $A = (1, 2, 3)$, $B = (0, 1, 3)$ and $C = (2, 1, 4)$. Then the area of the triangle $\boldsymbol{\Delta}$ with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2.

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$$= \frac{1}{2} |\langle -1, 1, 2 \rangle|$$

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$$= \frac{1}{2}|\langle -1, 1, 2\rangle| = \frac{1}{2}\sqrt{1+1+4} = \frac{1}{2}\sqrt{6}.$$

$\square$

Problem 7(a) - Spring 2008

Find the **parametric equations** of the line passing through the point $(2, 4, 1)$ that is perpendicular to the plane $3x - y + 5z = 77$.

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- The vector part of the line $\mathbf{L}$ is the normal vector $\mathbf{n} = \langle 3, -1, 5 \rangle$ to the plane.
- The **vector equation** of $\mathbf{L}$ is:

$$\mathbf{r}(t) = \langle 2, 4, 1 \rangle + t\mathbf{n}$$

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- The vector part of the line $\mathbf{L}$ is the normal vector $\mathbf{n} = \langle 3, -1, 5\rangle$ to the plane.
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$$\mathbf{r}(t) = \langle 2, 4, 1\rangle + t\mathbf{n}$$

$$= \langle 2, 4, 1\rangle + t\langle 3, -1, 5\rangle$$

Problem 7(a) - Spring 2008

Find the **parametric equations** of the line passing through the point $(2, 4, 1)$ that is perpendicular to the plane $3x - y + 5z = 77$.

Solution:

- The vector part of the line $\mathbf{L}$ is the normal vector $\mathbf{n} = \langle 3, -1, 5 \rangle$ to the plane.
- The **vector equation** of $\mathbf{L}$ is:

$$\mathbf{r}(t) = \langle 2, 4, 1 \rangle + t\mathbf{n}$$

$$= \langle 2, 4, 1 \rangle + t\langle 3, -1, 5 \rangle = \langle 2 + 3t, 4 - t, 1 + 5t \rangle.$$

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$$= \langle 2, 4, 1 \rangle + t\langle 3, -1, 5 \rangle = \langle 2 + 3t, 4 - t, 1 + 5t \rangle.$$

- The **parametric equations** are:

$$\begin{aligned} x &= 2 + 3t \\ y &= 4 - t \\ z &= 1 + 5t. \end{aligned}$$

$\square$

Problem 7(b) - Spring 2008

Find the intersection point of the line $\mathbf{L}(t) = \langle 2 + 3t, 4 - t, 1 + 5t \rangle$ in part (a) and the plane $3x - y + 5z = 77$.

Problem 7(b) - Spring 2008

Find the intersection point of the line $\mathbf{L}(t) = \langle 2+3t, 4-t, 1+5t \rangle$ in part (a) and the plane $3x - y + 5z = 77$.

Solution:

- By part (a), we have **L** has **parametric equations**:

$$\begin{aligned} x &= 2 + 3t \\ y &= 4 - t \\ z &= 1 + 5t. \end{aligned}$$

Problem 7(b) - Spring 2008

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Solution:

- By part (a), we have **L** has **parametric equations**:

$$x = 2 + 3t$$
$$y = 4 - t$$
$$z = 1 + 5t.$$

- Plug these t-values into equation of plane and solve for t:

Problem 7(b) - Spring 2008

Find the intersection point of the line $\mathbf{L}(t) = \langle 2+3t, 4-t, 1+5t\rangle$ in part (a) and the plane $3x - y + 5z = 77$.

Solution:

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$$\begin{aligned} x &= 2+3t \\ y &= 4-t \\ z &= 1+5t. \end{aligned}$$

- Plug these t-values into equation of plane and solve for t:

$$3(2+3t) - (4-t) + 5(1+5t) = 77,$$

$$6 + 9t - 4 + t + 5 + 25t = 77,$$

$$35t = 70;$$

Problem 7(b) - Spring 2008

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$$35t = 70; \quad \implies t = 2.$$

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$$35t = 70; \quad \Longrightarrow t = 2.$$

So $\mathbf{L}$ intersects the plane at time $t = 2$.

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Solution:

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$$\begin{aligned} x &= 2+3t \\ y &= 4-t \\ z &= 1+5t. \end{aligned}$$
- Plug these t-values into equation of plane and solve for t:
$$3(2+3t) - (4-t) + 5(1+5t) = 77,$$
$$6 + 9t - 4 + t + 5 + 25t = 77,$$
$$35t = 70; \quad \Longrightarrow t = 2.$$
So $\mathbf{L}$ intersects the plane at time $t = 2$.
- At $t = 2$, the **parametric equations** give the point:
$$\langle 2 + 3\cdot 2, 4 - 2, 1 + 5\cdot 2\rangle = \langle 8, 2, 11\rangle.$$

□

Problem 11(a) - Spring 2007

Find **parametric equations** for the line **L** which contains $A(7, 6, 4)$ and $B(4, 6, 5)$.

Problem 11(a) - Spring 2007

Find **parametric equations** for the line **L** which contains $A(7,6,4)$ and $B(4,6,5)$.

Solution:

- A vector parallel to the line **L** is:

$$\mathbf{v} = \overrightarrow{AB} = \langle 4-7, 6-6, 5-4, \rangle = \langle -3, 0, 1 \rangle.$$

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Solution:

- A vector parallel to the line **L** is:

$$\mathbf{v} = \overrightarrow{AB} = \langle 4-7, 6-6, 5-4, \rangle = \langle -3, 0, 1 \rangle.$$

- A point on the line is $A(7,6,4)$.

Problem 11(a) - Spring 2007

Find **parametric equations** for the line **L** which contains $A(7, 6, 4)$ and $B(4, 6, 5)$.

Solution:

- A vector parallel to the line **L** is:

$$\mathbf{v} = \overrightarrow{AB} = \langle 4 - 7, 6 - 6, 5 - 4, \rangle = \langle -3, 0, 1 \rangle.$$

- A point on the line is $A(7, 6, 4)$.
- Therefore **parametric equations** for the line **L** are:

$$\begin{aligned} x &= 7 - 3t \\ y &= 6 \\ z &= 4 + t. \end{aligned}$$

$\square$

Problem 11(b) - Spring 2007

Find the **parametric equations** for the **line L of intersection** of the planes $x - 2y + z = 5$ and $2x + y - z = 0$.

Problem 11(b) - Spring 2007

Find the **parametric equations** for the **line L of intersection** of the planes $x - 2y + z = 5$ and $2x + y - z = 0$.

Solution:

- A vector $\mathbf{v}$ parallel to the line is the cross product of the normal vectors of the planes:

Problem 11(b) - Spring 2007

Find the **parametric equations** for the **line L of intersection** of the planes $x - 2y + z = 5$ and $2x + y - z = 0$.

Solution:

- A vector $\mathbf{v}$ parallel to the line is the cross product of the normal vectors of the planes:

  $$\mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle$$

Problem 11(b) - Spring 2007

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$$\mathbf{v} = \langle 1, -2, 1 \rangle \times \langle 2, 1, -1 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

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- A point on **L** is any (x_0, y_0, z_0) that satisfies **both** of the plane equations.

Problem 11(b) - Spring 2007

Find the **parametric equations** for the **line L of intersection** of the planes $x - 2y + z = 5$ and $2x + y - z = 0$.

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Problem 11(b) - Spring 2007

Find the **parametric equations** for the **line $\mathbf{L}$ of intersection** of the planes $x - 2y + z = 5$ and $2x + y - z = 0$.

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- Therefore **parametric equations** for $\mathbf{L}$ are:

$$\begin{aligned} x &= 1 + t \\ y &= -2 + 3t \\ z &= 5t. \end{aligned}$$

□

Problem 12(a) - Spring 2007

Find an **equation of the plane** which contains the points $P(-1, 0, 2)$, $Q(1, -2, 1)$ and $R(2, 0, -1)$.

Problem 12(a) - Spring 2007

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Solution:

- A normal vector to the plane can be found by taking the cross product of *any* two vectors that lie **in** the plane.

Problem 12(a) - Spring 2007

Find an **equation of the plane** which contains the points $P(-1,0,2)$, $Q(1,-2,1)$ and $R(2,0,-1)$.

Solution:

- A normal vector to the plane can be found by taking the cross product of *any* two vectors that lie **in** the plane. Two vectors that lie in the plane are $\overrightarrow{PQ} = \langle 2,-2,-1\rangle$ and $\overrightarrow{PR} = \langle 3,0,-3\rangle$.

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- So the normal vector is

$$\mathbf{n} = \langle 2, -2, -1 \rangle \times \langle 3, 0, -3 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -2 & -1 \\ 3 & 0 & -3 \end{vmatrix} =$$

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Problem 12(a) - Spring 2007

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- A point on the plane is $P(-1,0,2)$.

Problem 12(a) - Spring 2007

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Solution:

- A normal vector to the plane can be found by taking the cross product of *any* two vectors that lie **in** the plane. Two vectors that lie in the plane are $\overrightarrow{PQ} = \langle 2,-2,-1\rangle$ and $\overrightarrow{PR} = \langle 3,0,-3\rangle$.
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- A point on the plane is $P(-1,0,2)$. Therefore,

$$6(x-(-1)) + 3(y-0) + 6(z-2) = 0,$$

Problem 12(a) - Spring 2007

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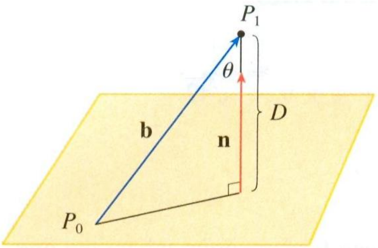
- A point on the plane is $P(-1, 0, 2)$. Therefore,

$$6(x - (-1)) + 3(y - 0) + 6(z - 2) = 0,$$

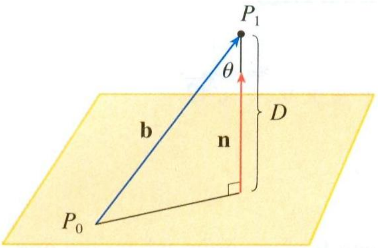
or simplified, $$6x + 3y + 6z - 6 = 0.$$

□



Problem 12(b) - Spring 2007

Find the distance **D** from the point $P_1 = (1, 0, -1)$ to the plane $2x + y - 2z = 1$.



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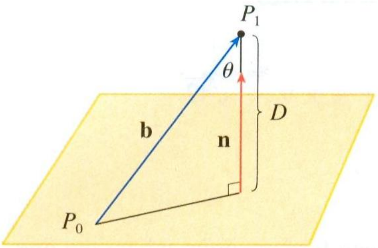
The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane.



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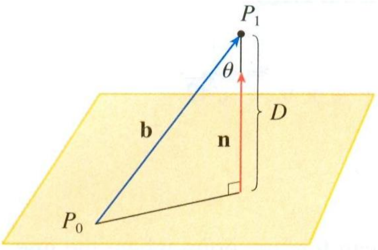
Solution:

The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from P_0 to $P_1 = (1, 0, -1)$ which is $\mathbf{b} = \langle 1, -1, -1 \rangle$.



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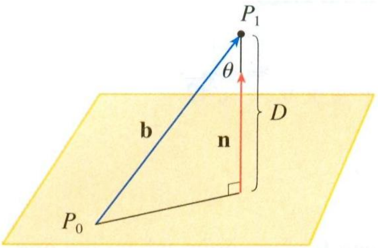

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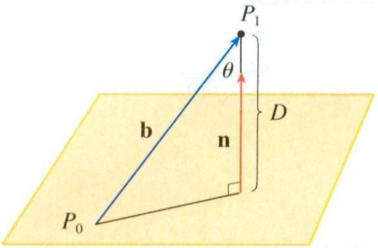
$$|\mathbf{comp_n\ b}| =$$



Problem 12(b) - Spring 2007

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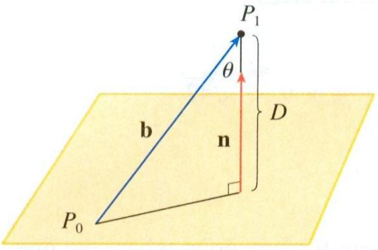
$$|\mathbf{comp}_{\mathbf{n}}\ \mathbf{b}| = \left| \mathbf{b} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right|$$



Problem 12(b) - Spring 2007

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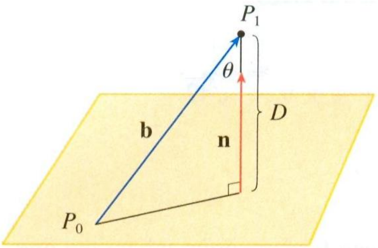
$$|\mathbf{comp_n\ b}| = \left| \mathbf{b} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right| = |\langle 1, -1, -1 \rangle \cdot \frac{1}{3} \langle 2, 1, -2 \rangle|$$



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The normal to the plane is $\mathbf{n} = \langle 2, 1, -2 \rangle$ and the point $P_0 = (0, 1, 0)$ lies on this plane. Consider the vector from P_0 to $P_1 = (1, 0, -1)$ which is $\mathbf{b} = \langle 1, -1, -1 \rangle$. The distance **D** from $(1, 0, -1)$ to the plane is equal to:

$$|\mathbf{comp_n\ b}| = \left| \mathbf{b} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right| = |\langle 1, -1, -1 \rangle \cdot \frac{1}{3} \langle 2, 1, -2 \rangle| = 1.$$

$\square$

Problem 12(c) - Spring 2007

Find the point P in the plane $2x + y - 2z = 1$ which is closest to the point $(1, 0, -1)$. (Hint: You can use part (b) of this problem to help find P or first find the equation of the line passing through P and the point $(1, 0, -1)$ and then solve for P.)

Problem 12(c) - Spring 2007

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Solution:

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Problem 12(c) - Spring 2007

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$$2(1 + 2t) + (t) - 2(-1 - 2t) = 1.$$

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- So the point on the plane closest to $(1, 0, -1)$ is $P = (\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3})$.

Problem 13(b) - Spring 2007

Find the **center** and **radius** of the sphere

$$x^2 + y^2 + 2y + z^2 + 4z = 20.$$

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- Rewriting, we have

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- Hence, the **center** is $\mathbf{C} = (0, -1, -2)$ and the **radius** is $r = 5$.

$\square$

Problem 15(a) - Spring 2008

Consider the points $A(2,1,0)$, $B(1,0,2)$ and $C(0,2,1)$. Find the area **A** of the triangle ABC. (Hint: If you know how to find the area of a parallelogram spanned by 2 vectors, then you should be able to solve this problem.)

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- The area of the parallelogram is

$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 2 \\ -2 & 1 & 1 \end{vmatrix}$$

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- So the area of the triangle ABC is

$$\mathbf{A} = \frac{\sqrt{27}}{2}.$$

$\square$

Problem 16 - Fall 2007

Find the **equation of the plane** containing the lines

$$x = 4 - 4t, \quad y = 3 - t, \quad z = 1 + 5t \quad \text{and}$$

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Solution:

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is orthogonal to both $\mathbf{v}_1$ and $\mathbf{v}_2$, it is the normal to the plane.

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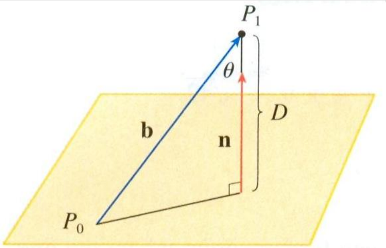
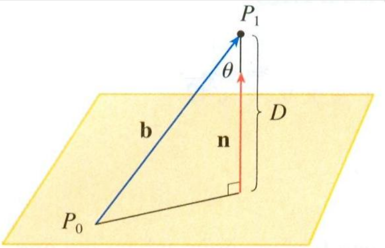
□



Problem 17 - Fall 2007

Find the distance **D** from the point $P_1 = (3, -2, 7)$ and the plane $4x - 6y - z = 5$.



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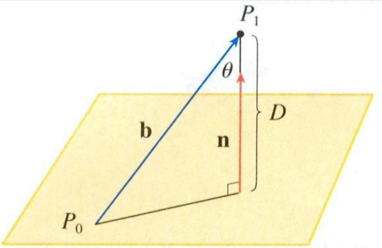
- Recall the distance formula $\mathbf{D} = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$ from a point $P = (x_1, y_1, z_1)$ to a plane $ax + by + cz + d = 0$.



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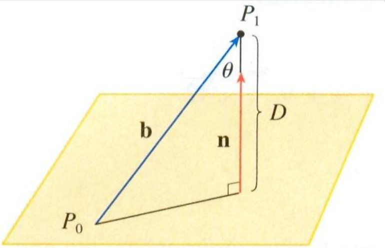

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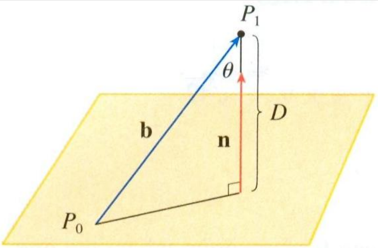
- So, the distance from $(3, -2, 7)$ to the plane is:



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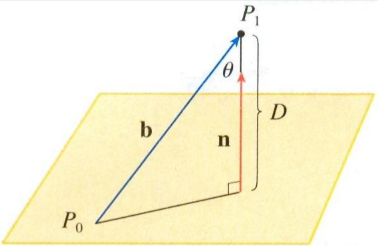
$$\mathbf{D} = \frac{|(4 \cdot 3) + (-6 \cdot -2) + (-1 \cdot 7) - 5|}{\sqrt{4^2 + (-6)^2 + (-1)^2}}$$



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- So, the distance from $(3, -2, 7)$ to the plane is:

$$\mathbf{D} = \frac{|(4 \cdot 3) + (-6 \cdot -2) + (-1 \cdot 7) - 5|}{\sqrt{4^2 + (-6)^2 + (-1)^2}} = \frac{12}{\sqrt{53}}.$$

Problem 18 - Fall 2007

Determine whether the lines $\mathbf{L}_1$ and $\mathbf{L}_2$ given below are **parallel**, **skew** or **intersecting**. If they intersect, find the point of intersection.

$$\mathbf{L}_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

$$\mathbf{L}_2 : \frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$$

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Solution:

- Rewrite these lines as vector equations:

$$\mathbf{L}_1(t) = \langle t, 2t+1, 3t+2 \rangle$$

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- Equating x and y-coordinates:

$$x = t = -4s + 3$$
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- Solving gives $s = 1$ and $t = -1$.

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- Equating x and y-coordinates:

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- Solving gives $s = 1$ and $t = -1$.
- $\mathbf{L}_1(-1) = \langle -1, -1, -1 \rangle \neq \langle -1, -1, 3 \rangle = \mathbf{L}_2(1)$.

Problem 18 - Fall 2007

Determine whether the lines $\mathbf{L}_1$ and $\mathbf{L}_2$ given below are **parallel**, **skew** or **intersecting**. If they intersect, find the point of intersection.

$$\mathbf{L}_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

$$\mathbf{L}_2 : \frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$$

Solution:

- Rewrite these lines as vector equations:

$$\mathbf{L}_1(t) = \langle t, 2t+1, 3t+2 \rangle$$

$$\mathbf{L}_2(s) = \langle -4s+3, -3s+2, 2s+1 \rangle$$

- Equating x and y-coordinates:

$$\begin{aligned} x = t &= -4s+3 \\ y = 2t+1 &= -3s+2. \end{aligned}$$

- Solving gives $s = 1$ and $t = -1$.
- $\mathbf{L}_1(-1) = \langle -1, -1, -1 \rangle \neq \langle -1, -1, 3 \rangle = \mathbf{L}_2(1)$. So these lines do **not intersect**.

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$$\mathbf{L}_1(t) = \langle t, 2t+1, 3t+2 \rangle$$
$$\mathbf{L}_2(s) = \langle -4s+3, -3s+2, 2s+1 \rangle$$
- Equating x and y-coordinates:
$$x = t = -4s + 3$$
$$y = 2t + 1 = -3s + 2.$$
- Solving gives $s = 1$ and $t = -1$.
- $\mathbf{L}_1(-1) = \langle -1, -1, -1 \rangle \neq \langle -1, -1, 3 \rangle = \mathbf{L}_2(1)$. So these lines do **not intersect**.
- Since the lines are clearly **not parallel** (the direction vectors $\langle 1, 2, 3 \rangle$ and $\langle -4, -3, 2 \rangle$ are **not parallel**), the lines are **skew**. $\square$

Problem 20(a) - Fall 2007

Three of the four vertices of a parallelogram are $P(0,-1,1)$, $Q(0,1,0)$ and $R(3,1,1)$. Two of the sides are PQ and PR. Find the area of the parallelogram.

Problem 20(a) - Fall 2007

Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(3, 1, 1)$. Two of the sides are PQ and PR. Find the area of the parallelogram.

Solution:

Consider the vectors $\overrightarrow{PQ} = \langle 0, 2, -1 \rangle$ and $\overrightarrow{PR} = \langle 3, 2, 0 \rangle$.

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Consider the vectors $\overrightarrow{PQ} = \langle 0, 2, -1 \rangle$ and $\overrightarrow{PR} = \langle 3, 2, 0 \rangle$. Then the area of the parallelogram spanned by $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ is:

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$$\textbf{Area}(\boldsymbol{\Delta}) = |\overrightarrow{PQ} \times \overrightarrow{PR}|$$

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$$\textbf{Area}(\boldsymbol{\Delta}) = |\overrightarrow{PQ} \times \overrightarrow{PR}| = \left\| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & -1 \\ 3 & 2 & 0 \end{vmatrix} \right\|$$

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$$\textbf{Area}(\boldsymbol{\Delta}) = |\overrightarrow{PQ} \times \overrightarrow{PR}| = \left\| \begin{array}{ccc} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & -1 \\ 3 & 2 & 0 \end{array} \right\|$$

$$= |\langle 2,-3,-6\rangle|$$

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$$= |\langle 2,-3,-6\rangle| = \sqrt{4+9+36} = 7$$

$\square$

Problem 20(b) - Fall 2007

Three of the four vertices of a parallelogram are $P(0,-1,1)$, $Q(0,1,0)$ and $R(3,1,1)$. Two of the sides are PQ and PR. Find the cosine of the angle between the vector PQ and PR.

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Three of the four vertices of a parallelogram are $P(0,-1,1)$, $Q(0,1,0)$ and $R(3,1,1)$. Two of the sides are PQ and PR. Find the cosine of the angle between the vector PQ and PR.

Solution:

- Note that:

$$\overrightarrow{PQ} = \langle 0, 2, -1 \rangle \qquad \overrightarrow{PR} = \langle 3, 2, 0 \rangle.$$

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Solution:

- Note that:

$$\overrightarrow{PQ} = \langle 0, 2, -1 \rangle \qquad \overrightarrow{PR} = \langle 3, 2, 0 \rangle.$$

- By our formula for dot products:

$$\cos\theta = \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{|\overrightarrow{PQ}||\overrightarrow{PR}|}$$

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- By our formula for dot products:

$$\cos\theta = \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{|\overrightarrow{PQ}||\overrightarrow{PR}|} = \frac{\langle 0, 2, -1 \rangle \cdot \langle 3, 2, 0 \rangle}{\sqrt{5}\sqrt{13}}$$

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- By our formula for dot products:

$$\cos\theta = \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{|\overrightarrow{PQ}||\overrightarrow{PR}|} = \frac{\langle 0, 2, -1 \rangle \cdot \langle 3, 2, 0 \rangle}{\sqrt{5}\sqrt{13}} = \frac{4}{\sqrt{5}\sqrt{13}}.$$

$\square$

Problem 20(c) - Fall 2007

Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(3, 1, 1)$. Two of the sides are PQ and PR. Find the coordinates of the fourth vertex.

Problem 20(c) - Fall 2007

Three of the four vertices of a parallelogram are $P(0, -1, 1)$, $Q(0, 1, 0)$ and $R(3, 1, 1)$. Two of the sides are PQ and PR. Find the coordinates of the fourth vertex.

Solution:

Denote the fourth vertex by $\mathbf{S}$.

Problem 20(c) - Fall 2007

Three of the four vertices of a parallelogram are $P(0,-1,1)$, $Q(0,1,0)$ and $R(3,1,1)$. Two of the sides are PQ and PR. Find the coordinates of the fourth vertex.

Solution:

Denote the fourth vertex by **S**. Then

$$\overrightarrow{O\mathbf{S}} = \overrightarrow{OQ} + \overrightarrow{PR} = \langle 0,1,0\rangle + \langle 3,2,0\rangle = \langle 3,3,0\rangle\,,$$

where O is the origin.

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$$\overrightarrow{O\mathbf{S}} = \overrightarrow{OQ} + \overrightarrow{PR} = \langle 0,1,0\rangle + \langle 3,2,0\rangle = \langle 3,3,0\rangle\,,$$

where O is the origin. That is,

$$\mathbf{S} = (3,3,0).$$

□

Problem 21(a) - Fall 2007

Let **C** be the parametric curve

$$x = 2 - t^2, \;\; y = 2t - 1, \;\; z = \ln t.$$

Determine the point(s) of **intersection** of **C** with the xz-plane.

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Solution:

- The points of intersection of **C** with the xz-plane correspond to the points where the y-coordinate of **C** is 0.
- When $y = 0$, then $0 = 2t - 1$ or $t = \frac{1}{2}$.

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- When $y = 0$, then $0 = 2t - 1$ or $t = \frac{1}{2}$.
- Hence,

$$\langle 2 - (\frac{1}{2})^2, 2 \cdot \frac{1}{2} - 1, \ln \frac{1}{2} \rangle = \langle 1\frac{3}{4}, 0, -\ln 2 \rangle$$

is the unique point of the **intersection** of **C** with *xz*-plane.

□

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$$\langle 2 - (\frac{1}{2})^2, 2 \cdot \frac{1}{2} - 1, \ln \frac{1}{2} \rangle = \langle 1\frac{3}{4}, 0, -\ln 2 \rangle$$

is the unique point of the **intersection** of **C** with xz-plane.

$\square$

Problem 22(a) - Fall 2006

Find **parametric equations** for the line **r** which contains $A(2, 0, 1)$ and $B(-1, 1, -1)$.

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Solution:

- Note that $\overrightarrow{AB} = \langle -3, 1, -2 \rangle$ and the **vector equation** is:

$$\mathbf{r}(t) = \vec{A} + t\overrightarrow{AB}$$

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- The **parametric equations** are:

$$\begin{aligned} x &= 2 - 3t \\ y &= t \\ z &= 1 - 2t. \end{aligned}$$

□

Problem 22(b) - Fall 2006

Determine whether the lines $\mathbf{L}_1 : x = 1 + 2t,\ y = 3t,\ z = 2 - t$ and $\mathbf{L}_2 : x = -1 + s,\ y = 4 + s,\ z = 1 + 3s$ are **parallel**, **skew** or **intersecting**.

Problem 22(b) - Fall 2006

Determine whether the lines $\mathbf{L}_1 : x = 1 + 2t,\ y = 3t,\ z = 2 - t$ and $\mathbf{L}_2 : x = -1 + s,\ y = 4 + s,\ z = 1 + 3s$ are **parallel**, **skew** or **intersecting**.

Solution:

- Vector part of line $\mathbf{L}_1$ is $\mathbf{v}_1 = \langle 2, 3, -1 \rangle$ and for line $\mathbf{L}_2$ is $\mathbf{v}_2 = \langle 1, 1, 3 \rangle$.

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Solution:

- Vector part of line $\mathbf{L}_1$ is $\mathbf{v}_1 = \langle 2, 3, -1 \rangle$ and for line $\mathbf{L}_2$ is $\mathbf{v}_2 = \langle 1, 1, 3 \rangle$. Clearly, $\mathbf{v}_1$ is not a scalar multiple of $\mathbf{v}_2$ and so these lines are **not parallel**.

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- If these lines intersect, then for some values of t and s:

$$x = 1 + 2t = -1 + s$$

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- If these lines intersect, then for some values of t and s:

$$x = 1 + 2t = -1 + s \implies 2t = -2 + s,$$

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- If these lines intersect, then for some values of t and s:

$$x = 1 + 2t = -1 + s \implies 2t = -2 + s,$$

$$y = 3t = 4 + s \implies 3t = 4 + s.$$

Solving yields: $\quad t = 6 \ \textbf{and} \ s = 14.$

Problem 22(b) - Fall 2006

Determine whether the lines **L_1** $: x = 1 + 2t,\ y = 3t,\ z = 2 - t$ and **L_2** $: x = -1 + s,\ y = 4 + s,\ z = 1 + 3s$ are **parallel**, **skew** or **intersecting**.

Solution:

- Vector part of line **L_1** is $\mathbf{v}_1 = \langle 2, 3, -1 \rangle$ and for line **L_2** is $\mathbf{v}_2 = \langle 1, 1, 3 \rangle$. Clearly, $\mathbf{v}_1$ is not a scalar multiple of $\mathbf{v}_2$ and so these lines are **not parallel**.
- If these lines intersect, then for some values of t and s:

$$x = 1 + 2t = -1 + s \implies 2t = -2 + s,$$

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Solving yields: $$t = 6 \ \textbf{and} \ s = 14.$$

Plugging these values into $z = 2 - t = 1 + 3s$ yields the inequality $-4 \neq 43$, which means the z-coordinates are never equal and the lines do **not intersect**.

Problem 22(b) - Fall 2006

Determine whether the lines $\mathbf{L}_1 : x = 1 + 2t,\ y = 3t,\ z = 2 - t$ and $\mathbf{L}_2 : x = -1 + s,\ y = 4 + s,\ z = 1 + 3s$ are **parallel**, **skew** or **intersecting**.

Solution:

- Vector part of line $\mathbf{L}_1$ is $\mathbf{v}_1 = \langle 2, 3, -1\rangle$ and for line $\mathbf{L}_2$ is $\mathbf{v}_2 = \langle 1, 1, 3\rangle$. Clearly, $\mathbf{v}_1$ is not a scalar multiple of $\mathbf{v}_2$ and so these lines are **not parallel**.
- If these lines intersect, then for some values of t and s:

$$x = 1 + 2t = -1 + s \implies 2t = -2 + s,$$

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  Solving yields: $$t = 6 \ \textbf{and} \ s = 14.$$

  Plugging these values into $z = 2 - t = 1 + 3s$ yields the inequality $-4 \neq 43$, which means the z-coordinates are never equal and the lines do **not intersect**.
- Thus, the lines are **skew**.

$\square$

Problem 23(a) - Fall 2006

Find an **equation of the plane** which contains the points $P(-1, 2, 1)$, $Q(1, -2, 1)$ and $R(1, 1, -1)$.

Problem 23(a) - Fall 2006

Find an **equation of the plane** which contains the points $P(-1,2,1)$, $Q(1,-2,1)$ and $R(1,1,-1)$.

Solution:

- Consider the vectors $\overrightarrow{PQ} = \langle 2,-4,0\rangle$ and $\overrightarrow{PR} = \langle 2,-1,-2\rangle$ which are parallel to the plane.

Problem 23(a) - Fall 2006

Find an **equation of the plane** which contains the points $P(-1, 2, 1)$, $Q(1, -2, 1)$ and $R(1, 1, -1)$.

Solution:

- Consider the vectors $\overrightarrow{PQ} = \langle 2, -4, 0 \rangle$ and $\overrightarrow{PR} = \langle 2, -1, -2 \rangle$ which are parallel to the plane.
- The normal vector to the plane is:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR}$$

Problem 23(a) - Fall 2006

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Solution:

- Consider the vectors $\overrightarrow{PQ} = \langle 2,-4,0\rangle$ and $\overrightarrow{PR} = \langle 2,-1,-2\rangle$ which are parallel to the plane.
- The normal vector to the plane is:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & 0 \\ 2 & -1 & -2 \end{vmatrix}$$

Problem 23(a) - Fall 2006

Find an **equation of the plane** which contains the points $P(-1,2,1)$, $Q(1,-2,1)$ and $R(1,1,-1)$.

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- Consider the vectors $\overrightarrow{PQ} = \langle 2,-4,0\rangle$ and $\overrightarrow{PR} = \langle 2,-1,-2\rangle$ which are parallel to the plane.
- The normal vector to the plane is:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & 0 \\ 2 & -1 & -2 \end{vmatrix} = 8\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}.$$

Problem 23(a) - Fall 2006

Find an **equation of the plane** which contains the points $P(-1, 2, 1)$, $Q(1, -2, 1)$ and $R(1, 1, -1)$.

Solution:

- Consider the vectors $\overrightarrow{PQ} = \langle 2, -4, 0 \rangle$ and $\overrightarrow{PR} = \langle 2, -1, -2 \rangle$ which are parallel to the plane.
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- Since $P(-1, 2, 1)$ lies on the plane, the **equation of the plane** is:

Problem 23(a) - Fall 2006

Find an **equation of the plane** which contains the points $P(-1, 2, 1)$, $Q(1, -2, 1)$ and $R(1, 1, -1)$.

Solution:

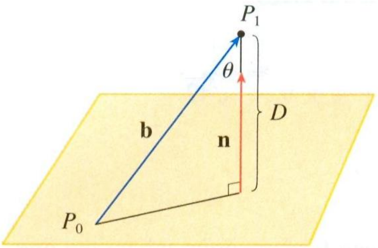
- Consider the vectors $\overrightarrow{PQ} = \langle 2, -4, 0 \rangle$ and $\overrightarrow{PR} = \langle 2, -1, -2 \rangle$ which are parallel to the plane.
- The normal vector to the plane is:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & 0 \\ 2 & -1 & -2 \end{vmatrix} = 8\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}.$$

- Since $P(-1, 2, 1)$ lies on the plane, the **equation of the plane** is:

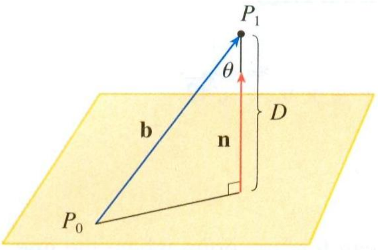
$$\langle 8, 4, 6 \rangle \cdot \langle x+1, y-2, z-1 \rangle = 8(x+1) + 4(y-2) + 6(z-1) = 0.$$

$\square$



Problem 23(b) - Fall 2006

Find the distance **D** from the point $(1, 2, -1)$ to the plane $2x + y - 2z = 1$.



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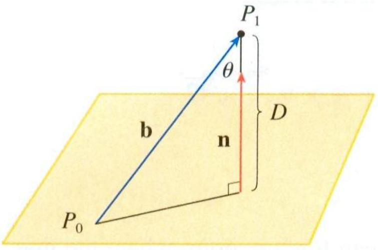

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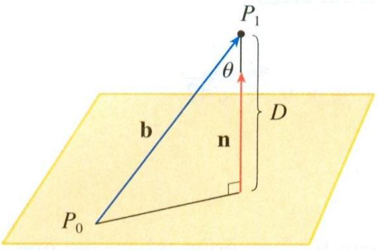

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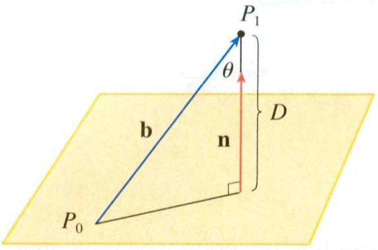

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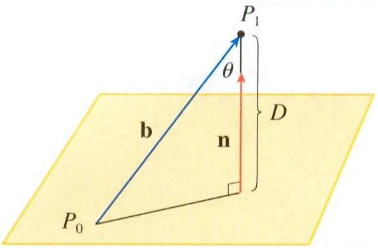

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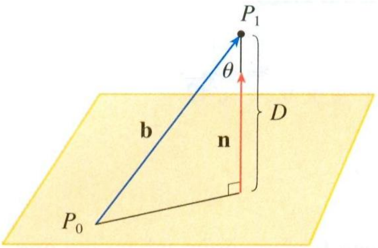

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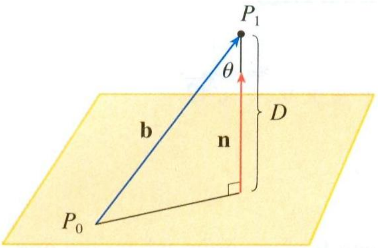
$$|\mathbf{comp}_{\mathbf{n}}\ \mathbf{b}| = \left| \mathbf{b} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right| = |\langle 1, 1, -1 \rangle \cdot \frac{1}{3} \langle 2, 1, -2 \rangle|$$



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□

Problem 26(b) - Fall 2006

Find the **center** and **radius** of the sphere $x^2 + y^2 + z^2 + 6z = 16$.

Problem 26(b) - Fall 2006

Find the **center** and **radius** of the sphere $x^2 + y^2 + z^2 + 6z = 16$.

Solution:

- Complete the square in order to put the equation in the form:

$$(x - x_0)^2 + (y - y_0) + (z - z_0)^2 = r^2.$$

Problem 26(b) - Fall 2006

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Solution:

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- We get:

$$x^2 + y^2 + (z^2 + 6z) = x^2 + y^2 + (z^2 + 6z + 9) - 9 = 16.$$

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- This gives the equation

$$(x - 0)^2 + (y - 0)^2 + (z + 3)^2 = 25 = 5^2.$$

Problem 26(b) - Fall 2006

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- This gives the equation

$$(x - 0)^2 + (y - 0)^2 + (z + 3)^2 = 25 = 5^2.$$

Hence, the **center** is $\mathbf{C} = (0, 0, -3)$ and the **radius** is $r = 5$.

□

Problem 27

Consider the line **L** through points $A = (2, 1, -1)$ and $B = (5, 3, -2)$. Find the **intersection** of the line **L** and the plane given by $2x - 3y + 4z = 13$.

Problem 27

Consider the line **L** through points $A = (2, 1, -1)$ and $B = (5, 3, -2)$. Find the **intersection** of the line **L** and the plane given by $2x - 3y + 4z = 13$.

Solution:

- The vector part of **L** is $\overrightarrow{AB} = \langle 3, 2, -1 \rangle$ and the point A is on the line.

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- The **vector equation** of **L** is:

$$\mathbf{L} = \vec{A} + t\overrightarrow{AB}$$

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- So, the **point of intersection** is:

  $$\mathbf{L}(-4) = \langle 2 - 12, 1 - 8, -1 - (-4) \rangle$$

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$\square$

Problem 29

Consider the parallelogram with vertices A, B, C, D such that B and C are adjacent to A. If $A = (2, 5, 1)$, $B = (3, 1, 4)$, $D = (5, 2, -3)$, find the point C.

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After drawing a picture, the point C is easily seen to be:

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$$\overrightarrow{OA} + \overrightarrow{BD} = \langle 2, 5, 1 \rangle + \langle 2, 1, -7 \rangle = \langle 4, 6, -6 \rangle,$$

where O is the origin. $\square$

Problem 30(a)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the **orthogonal projection** $\mathbf{proj}_{\vec{AB}}(\overrightarrow{AC})$ of the vector $\overrightarrow{AC}$ onto the vector $\overrightarrow{AB}$.

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- We just plug in the vectors $\mathbf{a} = \overrightarrow{AB} = \langle -1, -1, 2\rangle$ and $\mathbf{b} = \overrightarrow{AC} = \langle -2, 1, 1\rangle$ into the formula:

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- We just plug in the vectors $\mathbf{a} = \overrightarrow{AB} = \langle -1,-1,2\rangle$ and $\mathbf{b} = \overrightarrow{AC} = \langle -2,1,1\rangle$ into the formula:

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- Plugging in, we get:

$$\mathbf{proj}_{\overrightarrow{AB}}(\overrightarrow{AC}) = \frac{\langle -1,-1,2\rangle \cdot \langle -2,1,1\rangle}{\langle -1,-1,2\rangle \cdot \langle -1,-1,2\rangle}\,\langle -1,-1,2\rangle = \frac{1}{2}\langle -1,-1,2\rangle.$$

$\square$

Problem 30(b)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the area of triangle ABC.

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Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Then the area of the triangle $\boldsymbol{\Delta}$ with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2.

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$$= \frac{1}{2}|\langle -3, -3, -3 \rangle|$$

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$$= \frac{1}{2}|\langle -3, -3, -3\rangle| = \frac{1}{2}\sqrt{9+9+9}$$

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Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the area of triangle ABC.

Solution:

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Then the area of the triangle $\boldsymbol{\Delta}$ with these vertices can be found by taking the area of the parallelogram spanned by $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and dividing by 2. Thus:

$$\textbf{Area}(\boldsymbol{\Delta}) = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2} = \frac{1}{2}\left\| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 2 \\ -2 & 1 & 1 \end{vmatrix} \right\|$$

$$= \frac{1}{2}|\langle -3, -3, -3\rangle| = \frac{1}{2}\sqrt{9+9+9} = \frac{1}{2}\sqrt{27}.$$

□

Problem 30(c)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the distance **d** from the point C to the line **L** that contains points A and B.

Problem 30(c)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the distance **d** from the point C to the line **L** that contains points A and B.

Solution:

- From the figure drawn on the blackboard, we see that the distance **d** from C to **L** is the absolute value of the scalar projection of $\overrightarrow{AC}$ in the direction

$$\mathbf{v} = \overrightarrow{AC} - \mathbf{proj}_{\overrightarrow{AB}} \overrightarrow{AC}.$$

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$$\mathbf{v} = \overrightarrow{AC} - \mathbf{proj}_{\overrightarrow{AB}}\, \overrightarrow{AC}.$$

- The vector **v** lies in the plane containing A, B, C and is perpendicular to $\overrightarrow{AB}$.

Problem 30(c)

Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. Find the distance $\mathbf{d}$ from the point C to the line $\mathbf{L}$ that contains points A and B.

Solution:

- From the figure drawn on the blackboard, we see that the distance $\mathbf{d}$ from C to $\mathbf{L}$ is the absolute value of the scalar projection of $\overrightarrow{AC}$ in the direction

$$\mathbf{v} = \overrightarrow{AC} - \mathbf{proj}_{\overrightarrow{AB}} \overrightarrow{AC}.$$

- The vector $\mathbf{v}$ lies in the plane containing A, B, C and is perpendicular to $\overrightarrow{AB}$.
- Hence,

$$\mathbf{d} = |\mathbf{comp}_{\mathbf{v}} \overrightarrow{AC}|.$$

Problem 30(c)

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- Hence,

$$\mathbf{d} = |\mathbf{comp}_{\mathbf{v}}\overrightarrow{AC}|.$$

- Next, you the student, do the algebraic calculation of $\mathbf{d}$. □

Problem 31

Find **parametric equations** for the line **L** of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 1$.

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Solution:

- The vector part **v** of the line **L** of intersection is orthogonal to the normal vectors $\langle 1, -2, 1 \rangle$ and $\langle 2, 1, 1 \rangle$.

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$$\mathbf{v} = \langle 1, -2, 1\rangle \times \langle 2, 1, 1\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

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- Choose $\mathbf{P} \in \mathbf{L}$ so the z-coordinate of $\mathbf{P}$ is zero.

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Solving, we find that $x = \frac{3}{5}$ and $y = -\frac{1}{5}$.

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- The **parametric equations** are:

$$\begin{aligned} x &= \tfrac{3}{5} - 3t \\ y &= -\tfrac{1}{5} + t \\ z &= 5t. \end{aligned}$$

□

Problem 32

Let **L_1** denote the line through the points $(1, 0, 1)$ and $(-1, 4, 1)$ and let **L_2** denote the line through the points $(2, 3, -1)$ and $(4, 4, -3)$. Do the lines **L_1** and **L_2** **intersect**? If not, are they **skew** or **parallel**?

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Solution:

- The vector equations of the lines are:

$$\mathbf{L}_1(t) = \langle 1,0,1\rangle + t\langle -2,4,0\rangle$$

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- Equating y-coordinates with $s = -1$, we find $4t = 3 - 1 \Longrightarrow t = \frac{1}{2}$.

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- Equating y-coordinates with $s = -1$, we find $4t = 3 - 1 \implies t = \frac{1}{2}$.
- Equating x-coordinates with $s = -1$ and $t = \frac{1}{2}$, we find:
$$\mathbf{L}_1(\frac{1}{2}) = \langle 0, 2, 1\rangle = \mathbf{L}_2(-1).$$

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- Equating x-coordinates with $s = -1$ and $t = \frac{1}{2}$, we find:

$$\mathbf{L}_1(\frac{1}{2}) = \langle 0, 2, 1\rangle = \mathbf{L}_2(-1).$$

- Hence, the lines **intersect**.

□

Problem 33(a)

Find the volume **V** of the **parallelepiped** such that the following four points $A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3, -3)$, $D = (1, 0, -1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A.

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Find the volume **V** of the **parallelepiped** such that the following four points $A = (1, 4, 2)$, $B = (3, 1, -2)$, $C = (4, 3, -3)$, $D = (1, 0, -1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A.

Solution:

The volume **V** is equal to the absolute value of the determinant of the matrix with rows $\overrightarrow{AB} = \langle 2, -3, -4 \rangle$, $\overrightarrow{AC} = \langle 3, -1, -5 \rangle$, $\overrightarrow{AD} = \langle 0, -4, -3 \rangle$.

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$$\mathbf{V} = \left\| \begin{matrix} 2 & -3 & -4 \\ 3 & -1 & -5 \\ 0 & -4 & -3 \end{matrix} \right\|$$

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$$\mathbf{V} = \left\| \begin{matrix} 2 & -3 & -4 \\ 3 & -1 & -5 \\ 0 & -4 & -3 \end{matrix} \right\|$$

$$= |2 \cdot (-17) + -(-3) \cdot (-9) + (-4) \cdot (-12)|$$

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$$= |2 \cdot (-17) + -(-3) \cdot (-9) + (-4) \cdot (-12)| = |-13| = 13.$$

$\square$

Problem 33(b)

Find an **equation of the plane** through
$A = (1, 4, 2), B = (3, 1, -2), C = (4, 3, -3)$.

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Find an **equation of the plane** through
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$$\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC}$$

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- Since $A = (1, 4, 2)$, is on the plane, then the **equation of the plane** is given by:

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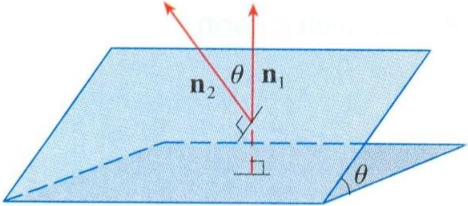
- Since $A = (1, 4, 2)$, is on the plane, then the **equation of the plane** is given by:

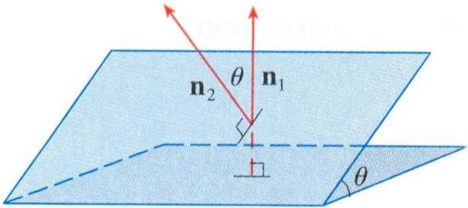
$$11(x-1) - 2(y-4) + 7(z-2) = 0.$$

$\square$



Problem 33(c)

Find the angle between the plane through
$A = (1, 4, 2),\ B = (3, 1, -2),\ C = (4, 3 - 3)$ and the xy-plane.



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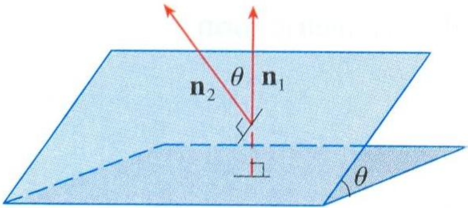
- The normal vectors of these planes are $\mathbf{n}_1 = \langle 0, 0, 1 \rangle$, $\mathbf{n}_2 = \langle 11, -2, 7 \rangle$.



Problem 33(c)

Find the angle between the plane through
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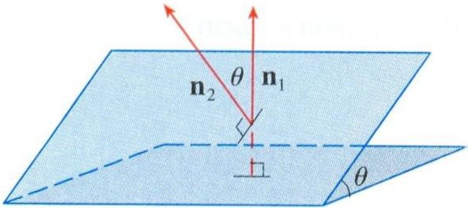

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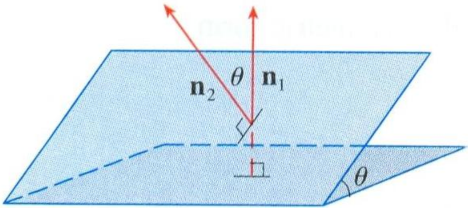

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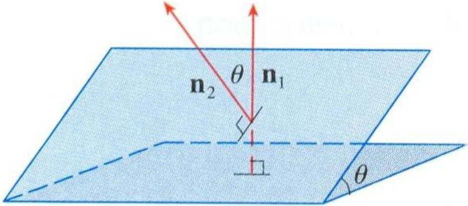

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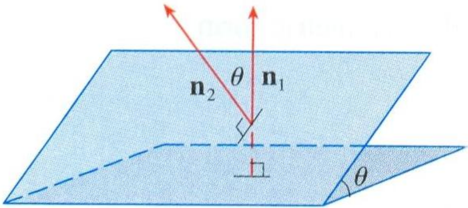

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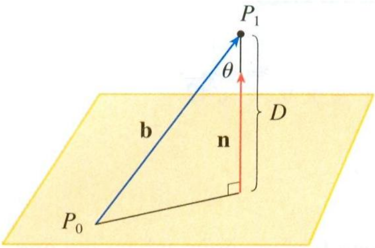
- $$\theta = \cos^{-1}\left(\frac{1}{\sqrt{174}}\right).$$

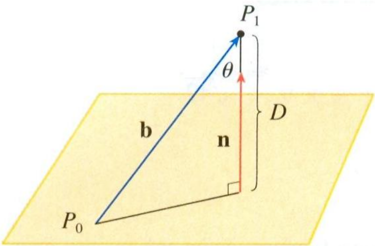
□



Problem 37

Find the distance **D** between the given parallel planes

$$z = 2x + y - 1, \qquad -4x - 2y + 2z = 3.$$



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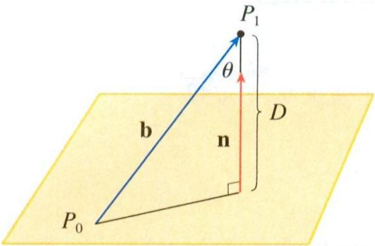

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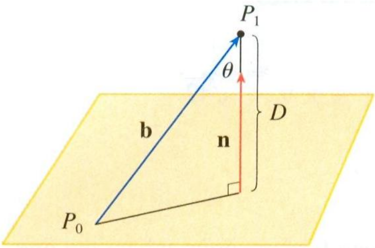

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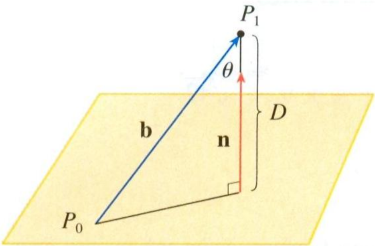

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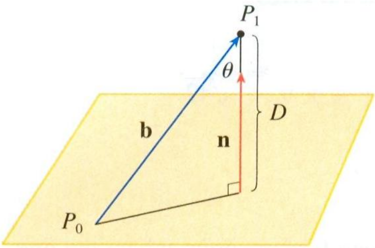

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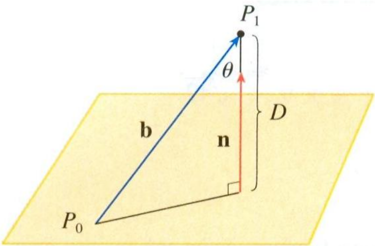

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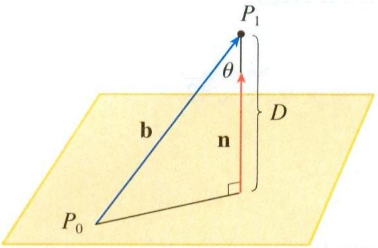

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Problem 41

Find an **equation of the plane** that passes through the point $P(1, 1, 0)$ and contains the line given by **parametric equations** $x = 2 + 3t$, $y = 1 - t$, $z = 2 + 2t$.

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- To find a normal vector to the plane, take cross products:

$$\mathbf{n} = \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 2 \\ 1 & 0 & 2 \end{vmatrix} = \langle -2, -4, 1 \rangle.$$

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- Since $(1,1,0)$ is on the plane, the **equation of the plane** is:

$$\langle -2, -4, 1 \rangle \cdot \langle x - 1, y - 1, z \rangle = -2(x-1) - 4(y-1) + z = 0.$$

$\square$