Problem 1

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

$$\lim_{x \to 1^-} \ln(x) = \lim_{x \to 1^+} ax^2 + b$$

$$\ln(1) = a + b$$

$$0 = a + b \quad (1)$$

$f(2) = 3$

$f(2) = a(2)^2 + b$

$= 4a + b$

$4a + b = 3$

$3a + a + b = 3$

$3a = 3$

$a = 1$

Substituting in (1)

$a + b = 0$

$1 + b = 0$

$b = -1$

$2a + b = 2(1) - 1$

$= 1$

Problem 2

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

$$\lim_{h \to 0} \frac{e^{-3+h} - e^{-3}}{h} = f'(a)$$

$$\boxed{\begin{array}{l} \therefore \quad a = -3 \\ \& \ f(x) = e^{x} \end{array}}$$

Problem 3

$$h(x) = f(x)\, g(x)$$

$$h(1) = f(1)\, g(1)$$

$$24 = f(1)\, 6$$

$$f(1) = 4$$

$$h'(1) = f(1)\, g'(1) + f'(1)\, g(1)$$

$$20 = 4\, g'(1) + (-2)\, 6$$

$$32 = 4\, g'(1)$$

$$g'(1) = 8$$

Problem 4

$$y = \frac{\sqrt{x} - 2x}{x}$$

$$y = \frac{\sqrt{x}}{x} - \frac{2\cancel{x}}{\cancel{x}}$$

$$= \frac{1}{\sqrt{x}} - 2$$

$$\frac{dy}{dx} = \frac{-1}{2x\sqrt{x}}$$

@ $x = 4$

$$\frac{dy}{dx} = \frac{-1}{8 \times 2}$$

$$= \frac{-1}{16}$$

$$y = m(x - x_1) + y_1$$

$$= \frac{-1}{16}(x-4) - \frac{3}{2}$$

$$= \frac{-x}{16} + \frac{1}{4} - \frac{3}{2}$$

$$y = \frac{-x}{16} - \frac{5}{4}$$

Problem 5

$$g(x) = \sqrt{1 + 3f(x)}$$

$$g'(x) = \frac{1}{2\sqrt{1+3f(x)}} \times 3f'(x)$$

@ $x = 2$

$$g'(2) = \frac{1}{2\sqrt{1+3f(2)}} \times 3f'(2)$$

$$= \frac{1}{2\sqrt{16}} \times 3 \times 16$$

$$= \frac{48}{8}$$

$$= 6$$

$$\boxed{\text{Slope of normal} \times \text{Slope of tangent} = -1}$$

$$\text{Slope of normal} = \frac{-1}{\text{Slope of tangent}} = \frac{-1}{6}$$

Problem 6

1. True
2. False
3. False
4. True

Problem 7

$f(x) = x + 72^x$

$f'(x) = 1 + 72^x \cdot \ln 72$

$f'(1) = 1 + 72 \ln 72$

Problem 8

$$\lim_{h \to 0} \frac{\sin\left(\frac{\pi}{3}+h\right)\cos\left(\frac{\pi}{3}+h\right) - \frac{\sqrt{3}}{4}}{h} = f'(a)$$

$f(x) = \sin x \cdot \cos x$

$a = \frac{\pi}{3}$

$f'(x) = \cos^2 x - \sin^2 x$

$f'(a) = \cos^2 \frac{\pi}{3} - \sin^2 \frac{\pi}{3}$

$= \frac{1}{4} - \frac{3}{4}$

$= \frac{-1}{2}$

Problem 9

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$$

$$f(x) = e^x(2x^2+3x-1)$$

$$f'(x) = e^x(4x+3) + e^x(2x^2+3x-1)$$

$$= e^x[4x+3+2x^2+3x-1]$$

$$= e^x[2x^2+7x+2]$$

Problem 10

$$y = (1+x)(1+x^2)(1+x^3)(1+x^4)(1+x^5)$$

$$\ln y = \ln(1+x) + \ln(1+x^2) + \ln(1+x^3) + \ln(1+x^4) + \ln(1+x^5)$$

$$\frac{y'}{y} = \frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{3x}{1+x^3} + \frac{4x}{1+x^4} + \frac{5x}{1+x^5}$$

$$\frac{y'(1)}{y(1)} = \frac{1}{2} + \frac{2}{2} + \frac{3}{2} + \frac{4}{2} + \frac{5}{2}$$

$$\frac{y'(1)}{32} = \frac{15}{2}$$

$$y'(1) = \frac{15 \times \overset{16}{\cancel{32}}}{\cancel{2}}$$

$$y'(1) = 240$$

Problem 11

$$\lim_{x\to 0} \frac{e^x - x - 1}{\cos x - 1}$$

$$= \lim_{x\to 0} \frac{e^x - 1}{-\sin x}$$

$$= \lim_{x\to 0} \frac{e^x}{-\cos x}$$

$$= \frac{e^0}{-\cos 0} = -1$$

$$\lim_{x\to a} \frac{x-a}{\ln x - \ln a}$$

$$= \lim_{x\to a} \frac{1}{\frac{1}{x}}$$

$$= \lim_{x\to a} x$$

$$= a$$

$$\lim_{x\to 0} \left(\frac{e^x}{e^x - 1} - \frac{1}{x} \right)$$

$$= \lim_{x\to 0} \left(\frac{e^x}{e^x} - \frac{1}{x^2} \right)$$

$$=$$

Problem 12

$$2x^2 - 3xy + 3y^2 = 2$$

$$4x - 3[xy' + y] + 6yy' = 0$$

$$4x - 3xy' - 3y + 6yy' = 0$$

$$y'[6y - 3x] = 3y - 4x$$

$$y' = \frac{3y - 4x}{6y - 3x}$$

@ $(-1, -1)$

$$y' = \frac{-3+4}{-6+3} = \frac{-1}{3}$$

Slope of tangent $= -\frac{1}{3}$

Slope of normal $= \frac{-1}{-1/3} = 3$

Equation of tangent

$y = m(x - x_1) + y_1$

$= -\frac{1}{3}(x+1) - 1$

$= \underline{-\frac{x}{3} - \frac{4}{3}}$

Equation of normal

$y = m(x - x_1) + y_1$

$= 3(x+1) - 1$

$= \underline{3x - 2}$