UNIVERSITY
OF WOLLONGONG
IN DUBAI
Faculty of Engineering
and Information Sciences

MATH142

Essentials of Engineering Mathematics

First-Order ODEs

I. Separable
II. Homogeneous
III. Linear
IV. Exact
V. Bernoulli

V. Bernoulli Equations

Definition: A Bernoulli equation is a DE of the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

where n is any real number other than 0 or 1.

Steps for Solving a Bernoulli Equation

1. Write the DE in the following form

$$\boldsymbol{\frac{dy}{dx} + P(x)y = Q(x)y^n} \qquad (I)$$

2. Divide (I) by $\boldsymbol{y^n}$ (or multiply (I) by $\boldsymbol{y^{-n}}$)

$$\boldsymbol{y^{-n}\frac{dy}{dx} + P(x)y^{1-n} = Q(x)} \qquad (II)$$

3. Let $\boldsymbol{v = y^{1-n}} \Longrightarrow \frac{dv}{dx} = (1-n)y^{-n}\frac{dy}{dx}$ (Differentiate v w.r.t x)

$$y^{-n}\frac{dy}{dx} = \frac{1}{(1-n)}\frac{dv}{dx}$$

4. Substitute $v = y^{1-n}$ and $y^{-n}\frac{dy}{dx} = \frac{1}{(1-n)}\frac{dv}{dx}$ in (II) , we get:

$$\frac{1}{(1-n)}\frac{dv}{dx} + P(x)v = Q(x) \text{ or } \frac{dv}{dx} + (1-n)P(x)v = (1-n)Q(x) \qquad (III)$$

5. Equation (III) is a linear equation in v . We proceed and solve for v.

6. Substitute back v by y^{1-n} and solve for y in terms of x if possible.

Example 1: Solve $\frac{dy}{dx} + y = e^x y^{-2}$

Example 2: Solve $x\frac{dy}{dx} + y = x^3y^2$