UNIVERSITY OF WOLLONGONG IN DUBAI

Faculty of Engineering and Information Sciences

MATH142

Tutorial 9

Problem 1

In each part, find a formula for the general term of the sequence.

a) $1, \frac{1}{5}, \frac{1}{25}, \frac{1}{125}, \ldots$

b) $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \ldots$

c) $\frac{1}{\sqrt{\pi}}, \frac{4}{\sqrt[3]{\pi}}, \frac{9}{\sqrt[4]{\pi}}, \frac{16}{\sqrt[5]{\pi}}, \ldots$

d) $1, \frac{4}{5}, \frac{6}{8}, \frac{8}{11}, \frac{10}{14}, \frac{12}{17}, \ldots$

Problem 2

State whether the sequence converges as $n \longrightarrow \infty$, if it does, find the limit.

a) $a_n = \frac{1-n^2}{3n^2+9n+7}$

b) $a_n = \frac{ln\ n}{n}$

c) $a_n = n^2 e^{-n}$

d) $a_n = ln\left(\frac{2n}{4n+1}\right)$

e) $a_n = \frac{2+cosn}{\sqrt{n}}$

f) $a_n = n\ ln\left(1+\frac{1}{n}\right)$

Problem 3

Consider the sequence given by $a_1 = 1,\ a_2 = 4,\ \ a_{n+1} = 2a_n - a_{n-1}$.

a) Write the first six terms of the sequence.

b) Find a formula for the general term a_n .

Problem 4

Show that the sequence $-2, 6, -18, 54, \ldots$ is a geometric sequence and find its nth term

Problem 5

Show that the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots, \frac{n}{n+1}, \dots$ is strictly increasing sequence.

Problem 6

Determine whether each series converges, and if so find its sum.

a) $\sum_{k=0}^{\infty} \frac{2^{k+3}}{3^k}$

b) $\sum_{n=1}^{\infty} 5.7^{1-n}$

c) $\sum_{k=1}^{\infty} 3^{2k}.5^{1-k}$

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Tutorial 10

Problem 1

Find the sum of the series.

a) $\sum_{k=1}^{\infty} \frac{1}{k^2+7k+12}$

b) $\sum_{k=1}^{\infty} ln\left(\frac{k}{k+1}\right)$

Problem 2

Use the Divergence Test to determine the convergence and divergence of the following series.

a) $\sum_{k=1}^{\infty} \frac{k}{k+1}$

b) $\sum_{k=1}^{\infty} \left(\frac{k}{k+1}\right)^k$

c) $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{\frac{n}{2}}$

d) $\sum_{k=1}^{\infty}\frac{k}{e^k}$

Problem 3

Use the P-series Test to determine the convergence and divergence of the following series.

a) $\sum_{n=1}^{\infty} 2n^{\frac{-5}{3}}$

b) $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{k^2}}$

Problem 4

Show that the Integral Test applies, and use the integral test to determine whether the following series converge or diverge.

a) $\sum_{n=1}^{\infty} ne^{-n^2}$

b) $\sum_{n=1}^{\infty} \frac{ln\, n}{n^2}$

Problem 5

Use the Direct Comparison Test to determine the convergence and divergence of the following series.

a) $\sum_{k=1}^{\infty} \frac{ln\ k}{k^3}$

b) $\sum_{k=1}^{\infty} \frac{2+\sin k}{k^2}$

Problem 6

Use the Limit Comparison Test to determine whether the series converges or diverges.

a) $\sum_{k=1}^{\infty} \frac{3k+4}{k^3}$

b) $\sum_{n=1}^{\infty}\left(\sqrt[n]{2}-1\right)$

Problem 7

Use the Alternating Series Test to show that the following series converges.

a) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n+2}{n^2+n}$

b) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^3+4}$

Problem 8

Approximate the sum of the following series by its first 5 terms. $\sum_{k=1}^{\infty} \frac{(-1)^{n+1}}{3^n}$

Problem 9

Use the sum of the first ten terms to approximate. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

Problem 10

Determine how many terms should be used to estimate the sum of the entire series with an error of less than 0.001.

a) $\sum_{n=1}^{\infty}(-1)^n \frac{1}{8n^2+1}$

b) $\sum_{n=1}^{\infty} \frac{1}{n^3}$

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Tutorial 11

Problem 1

Use the sum of the first ten terms to approximate. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

Problem 2

Determine how many terms should be used to estimate the sum of the entire series with an error of less than 0.001.

a) $\sum_{n=1}^{\infty}(-1)^n \frac{1}{8n^2+1}$

b) $\sum_{n=1}^{\infty} \frac{1}{n^3}$

Problem 3

Determine whether the series converges absolutely or conditionally.

a) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n+3}{n(n+1)}$

b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{nlnn}$

c) $\sum_{n=1}^{\infty} \frac{(-1)^{\frac{n(n+1)}{2}}}{2^n}$

Problem 4

Use the Ratio Test to determine whether the following series converge or diverge.

a) $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n!}$

b) $\sum_{n=1}^{\infty}(-1)^n \frac{(2n-1)!}{3^n}$

c) $\sum_{n=0}^{\infty} \frac{(-1)^n n!}{e^n}$

Problem 5

Use the Root Test to determine whether the following series converge or diverge.

a) $\sum_{n=0}^{\infty} \frac{12^n}{n^n}$

b) $\sum_{n=1}^{\infty}\left(\frac{4n-5}{2n+1}\right)^{n}$

c) $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n^2}$

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Tutorial 12

Problem 1

Find a power series representation for each of the following functions.

a) $f(x) = \frac{9}{x+3}$

b) $f(x) = \frac{x}{2+x^2}$

Problem 2

Find the radius and interval of convergence of the following series.

a) $\sum_{n=0}^{\infty}(-1)^n \frac{x^n}{3^n(n+1)}$

b) $\sum_{k=0}^{\infty} \frac{x^k}{k!}$

c) $\sum_{n=1}^{\infty} \frac{(x-2)^n}{ln(n+4)}$

d) $\sum_{k=0}^{\infty} k!\, x^k$

Problem 3

Find the Maclaurin series for the given function.

$f(x) = 2sin(2x)cos(2x)$

Problem 4

Use a known series to find a power series in x that has the given function as its sum.

a) $xsin(x^3)$

b) $\frac{ln(1+x)}{x}$

c) $\frac{x - tan^{-1}x}{x^3}$

Problem 5

Find the sum of the following series.

a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n+1}(2n+1))}$

b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{100^{n+1}(n+1))}$

c) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{10^{2n+1}(2n+1))}$