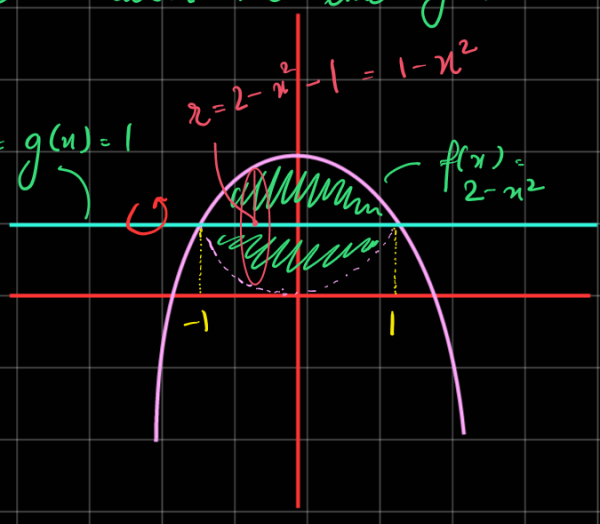
Volume by Disc Method

$$V = \int \text{Area of slice} \; \begin{matrix} dx \\ \text{or} \\ dy \end{matrix}$$

the slice should be chosen perpendicular to the axis of revolution

Example

Use the disc method to find the volume of the solid formed by revolving the region bounded by $f(x) = 2 - x^2$ and $g(x) = 1$ about the line $y = 1$



$$2 - x^2 = 1$$
$$x^2 - 1 = 0$$
$$x = \pm 1$$

$$V = \int_{-1}^{1} \pi (1 - x^2)^2 \, dx$$

$$= \pi \int_{-1}^{1} (1 - 2x^2 + x^4) \, dx$$

$$= \pi \left[x - \frac{2x^3}{3} + \frac{x^5}{5} \right]_{-1}^{1}$$

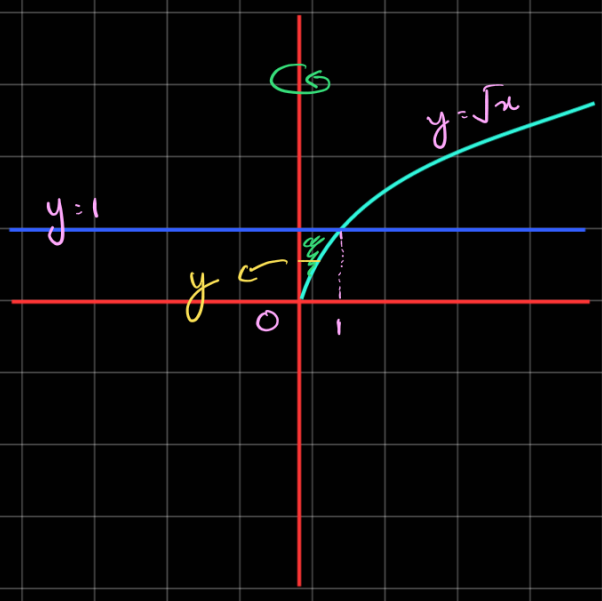
$$= \pi \left[\left(1 - \frac{2}{3} + \frac{1}{5} \right) - \left(-1 + \frac{2}{3} - \frac{1}{5} \right) \right]$$

$$= \pi \left[\frac{15}{15} - \frac{10}{15} + \frac{3}{15} + \frac{15}{15} - \frac{10}{15} + \frac{3}{15} \right]$$

$= \frac{16\pi}{15}$

Example

Use the disc method to find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$, $x = 0$ and $y = 1$ about the y-axis.



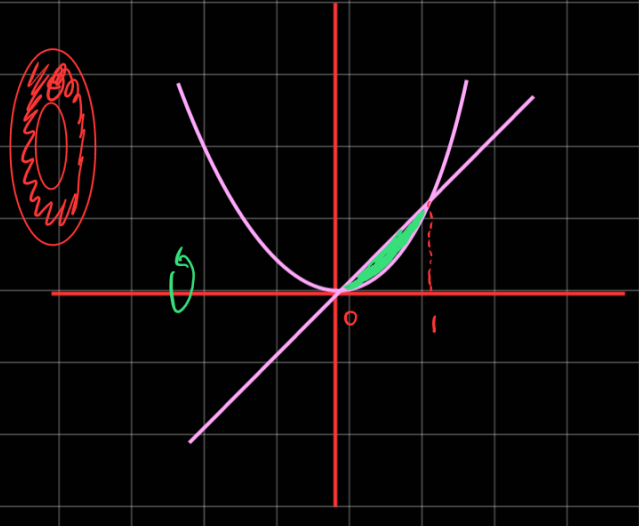
$$V = \int_0^1 \pi (y^2)^2 \, dy$$

$$= \pi \left[\frac{y^5}{5} \right]_0^1$$

$$= \frac{\pi}{5}$$

Example → Washer Method

Find the volume of the solid obtained by rotating the region enclosed by the curve $y = x$ and $y = x^2$ about the x-axis.



$x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0$ OR $x = 1$

$$V = \int_0^1 \pi x^2 - \pi x^4 \, dx$$

$$= \pi \int_0^1 x^2 - x^4 \, dx$$

$$= \pi \left[\frac{x^3}{3} - \frac{x^5}{5} \right]_0^1$$

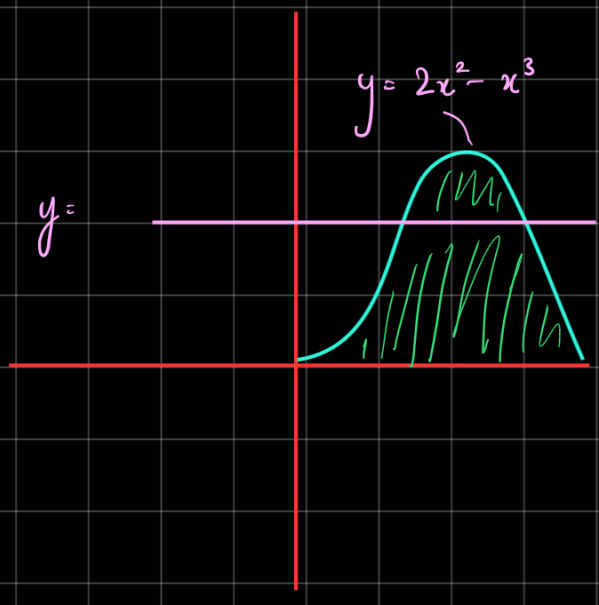
$$= \pi \left[\frac{1}{3} - \frac{1}{5} \right]$$

$$= \frac{2\pi}{15}$$

Shell Method

Example — Shell Method

Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = 2x^2 - x^3$, $y = 0$ between $x = 0$ and $x = 2$.



Cylindrical Shell Method

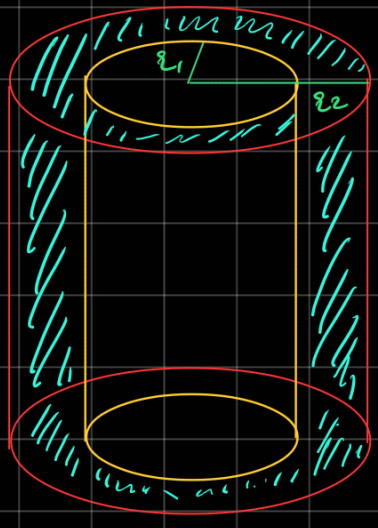
$$V = \pi r_2^2 h - \pi r_1^2 h$$

$$= \pi (r_2^2 - r_1^2) h$$

$$= \pi (r_2 + r_1)(r_2 - r_1) h$$

$$= 2\pi \underbrace{\frac{(r_2 + r_1)}{2}} h \underbrace{(r_2 - r_1)}_{\Delta r}$$

$$= 2\pi r h \Delta x$$



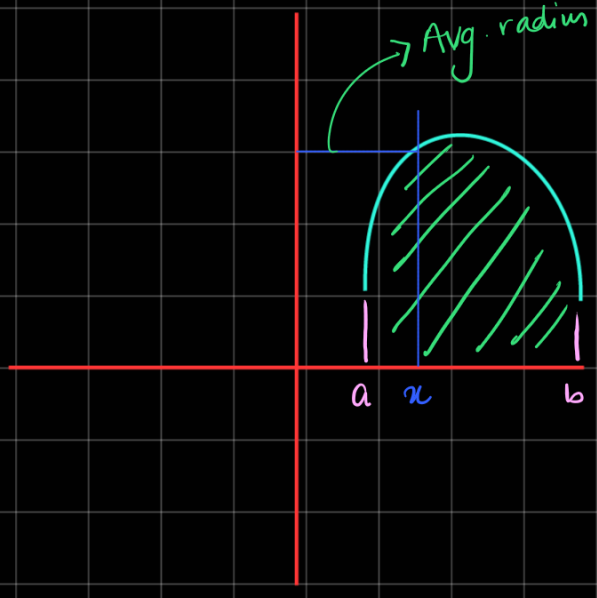
$$V = \int_a^b 2\pi \cdot \text{Avg Radius} \cdot \text{height} \cdot \begin{matrix} dx \\ \text{OR} \\ dy \end{matrix}$$

Slice is parallel to axis of rotation !!

Avg radius = Distance from variable to axis of rotation

$$V = 2\pi \int_a^b x\, f(x)\, dx$$

$$V = 2\pi \int_a^b (x+1)\, f(x)\, dx \quad \text{if rotation about } x = -1$$



Example

Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = 2x^2 - x^3$, $y = 0$ between $x = 0$ and $x = 2$.

$$V = \int_0^2 2\pi x\,(2x^2 - x^3)\, dx$$

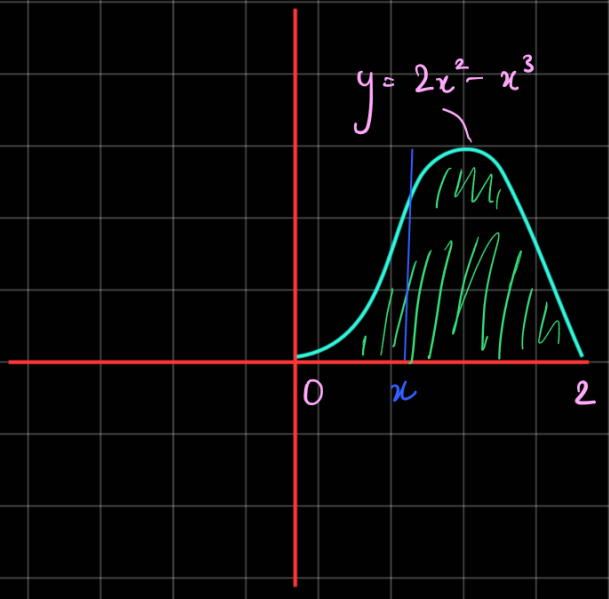
$$= 2\pi \int_0^2 (2x^3 - x^4)\, dx$$

$$= 2\pi \left[\frac{2x^4}{4} - \frac{x^5}{5} \right]_0^2$$

$$= 2\pi \left[8 - 6.4 \right]$$

$$= 3.2\pi$$

$$= \frac{16\pi}{5}$$



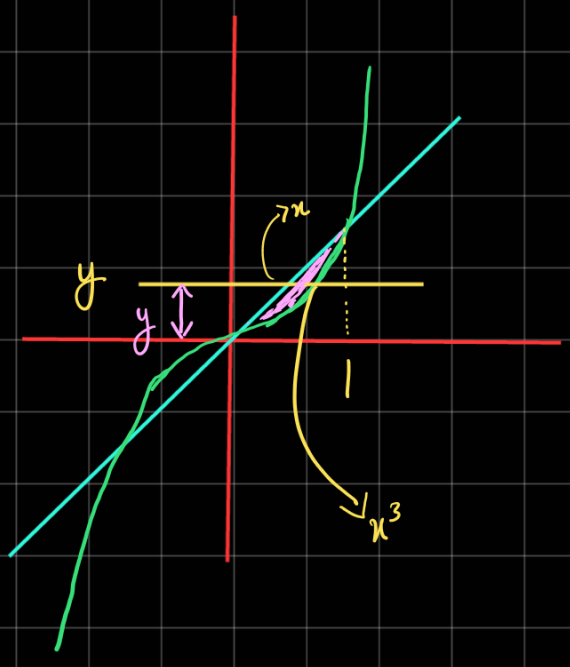
Example

Use the shell method to setup (DO NOT EVALUATE) the integral for finding the volume of the solid obtained by rotating the region in the first quadrant bounded by $y = x$ and $y = x^3$

A) about the x-axis

B) about the line $x = 1$

A) $V = \int_0^1 2\pi y \left(\sqrt[3]{y} - y\right) dy$



B) $V = \int_0^1 2\pi (1-x)(x-x^3)\, dx$