UNIVERSITY
OF WOLLONGONG
IN DUBAI

Faculty of Engineering
and Information Sciences

Tutorial 1-Math142

IVP

(8) $\boxed{(x^3 - y)\,dx + x\,dy = 0 \quad y(1) = 3}$

$$\frac{dy}{dx} + p(x)y = \varphi(x)$$

$$(x^3 - y) + x\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} + \frac{x^3 - y}{x} = 0$$

$$\frac{dy}{dx} + x^2 - \frac{y}{x} = 0$$

(I) $\boxed{\frac{dy}{dx} - \frac{1}{x}y = -x^2}$

$$\mu(x) = e^{\int -\frac{1}{x}\,dx}$$

$$= e^{-\ln x} \qquad x > 0$$

$$= \cancel{e^{\ln}} x^{-1}$$

$$= x^{-1}$$

$$p(x) = -\frac{1}{x}$$

(I) $\xrightarrow{x^{-1}}$ $x^{-1}\frac{dy}{dx} - x^{-2}y = -x$

$$\frac{d}{dx}\left(x^{-1} \cdot y\right) = -x$$

$$\int d(x^{-1}y) = \int -x \, dx$$

$$x^{-1}y = -\frac{x^2}{2} + C$$

$$\boxed{y = -\frac{x^3}{2} + Cx}$$

$y(1) = 3$

$3 = -\frac{1}{2} + C$

$3 + \frac{1}{2} = C$

$\frac{7}{2} = C$

$$y = -\frac{x^3}{2} + \frac{7}{2}x$$

$$\boxed{y = \frac{-x^3 + 7x}{2}}$$

(9) Show that the following equation is exact and solve it.

$$\underbrace{\left[2\cos(2x+y)-x^2\right]}_{M(x,y)}dx+\left[\underbrace{\cos(2x+y)+e^y}_{N(x,y)}\right]dy=0$$

$$\frac{\partial M}{\partial y}=-2\sin(2x+y) \qquad \frac{\partial N}{\partial x}=-2\sin(2x+y)$$

$$\frac{\partial f}{\partial x}=2\cos(2x+y)-x^2$$

$$f(x,y)=\int\left(2\cos(2x+y)-x^2\right)dx+h(y)$$

$$=\sin(2x+y)-\frac{x^3}{3}+h(y)$$

$$\frac{\partial f}{\partial y} = \cancel{\cos(2x+y)} + h'(y) = \cancel{\cos(2x+y)} + e^y$$

$$h'(y) = e^y \Rightarrow h(y) = e^y$$

$$f(x,y) = \sin(2x+y) - \frac{x^3}{3} + e^y$$

$$\boxed{\sin(2x+y) - \frac{x^3}{3} + e^y = c}$$

10 $\boxed{(x^2-3y^2)\,dx + 2xy\,dy = 0}$

show that the equat- is not exact and solve it by finding the right integrating factor.

$$\frac{\partial M}{\partial y} = -6y \qquad \frac{\partial N}{\partial x} = 2y \qquad \text{Not equal}$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{-6y-2y}{2xy} = \frac{-8y}{2xy}$$

$$= -\frac{4}{x}$$

$$= f(x)$$

$$\mu(x) = e^{\int -\frac{4}{x}\,dx}$$

$$= e^{-4\ln x}$$

$$= \cancel{e^{\ln x^{-4}}} \qquad (x>0)$$

$$= x^{-4}$$

$$(x^{-2} - 3x^{-4}y^2)\,dx + 2x^{-3}y\,dy = 0$$

$$\frac{\partial f}{\partial x} = x^{-2} - 3x^{-4}y^2$$

$$f(x,y) = \int (x^{-2} - 3x^{-4}y^2)\,dx + h(y)$$

$$= \frac{x^{-1}}{-1} - \frac{3x^{-3}}{-3}y^2 + h(y)$$

$$= -x^{-1} + x^{-3}y^2 + h(y)$$

$$\frac{\partial f}{\partial y} = \cancel{2yx^{-3}} + h'(y) = \cancel{2x^{-3}y}$$

$$h'(y) = 0 \rightarrow h(y) = c_1$$

$$f(x,y) = -x^{-1} + x^{-3}y^2 + C_1$$

$$f(x,y) = C_2$$

$$-x^{-1} + x^{-3}y^2 + C_1 = C_2$$

$$\boxed{-x^{-1} + x^{-3}y^2 = C}$$

$$x^{-3}y^2 = x^{-1} + C$$

$$\boxed{y^2 = x^2 + Cx^3}$$