Tutorial 8

Q1.

(a). Approximate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ by using the sum of the first ten terms. Estimate the error involved in this approximation.

(b) How many terms are required to ensure that the value of the sum is accurate within 0.0005.

Q2. Determine whether the series converges absolutely or conditionally.

1. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n+3}{n(n+1)}$

2. $\sum_{n=2}^{\infty}\frac{(-1)^n}{nlnn}$

Q3. Determine whether the series converges or diverges. Justify your answer by citing a relevant test.

1. $\sum_{n=1}^{\infty} \frac{e^n}{n!}$

2. $\sum_{n=1}^{\infty} \left(\frac{2n}{13n+1} \right)^n$

3. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+20}$

Q4. Find the interval of convergence for

1. $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n} (x+1)^n$

2. $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

Q5. Find the interval of convergence for

1. $\sum_{n=1}^{\infty} \frac{n}{3^n} (x-1)^n$

2. $\sum_{n=1}^{\infty} \frac{x^n}{2n!}$

Q6. Find the power series representation of $f(x) = \dfrac{x^2}{4+x^3}$ and the corresponding interval of convergence.

Q7. A) Find the Maclaurin series of

$$f(x) = \frac{x^2}{3-x}.$$ (Show your work)

B) Find the sum of the series

$$\frac{\pi}{2}\sum_{n=0}^{\infty}(-1)^n\frac{\pi^{2n}}{(2n+1)!}.$$ (Show your work)

Q8. Find the Maclaurin series of $f(x) = x\cos(x^3)$.

Q 9. Find a power series representation for the following functions. (Your answer should include the interval of convergence)

1. $g(x) = \cos\sqrt{x}$

2. $h(x) = \tan^{-1}\left(\frac{x}{5}\right)$

3. $g(x) = e^{\sqrt{\frac{x}{2}}}$