Math 142 Midterm Formulas

UOWD

Common Integrals

Polynomials

$$\int dx = x + c \qquad \int k\,dx = k\,x + c \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$$

$$\int \frac{1}{x}dx = \ln|x| + c \qquad \int x^{-1}\,dx = \ln|x| + c \qquad \int x^{-n}dx = \frac{1}{-n+1}x^{-n+1} + c,\ n \neq 1$$

$$\int \frac{1}{ax+b}dx = \frac{1}{a}\ln|ax+b| + c \qquad \int x^{\frac{p}{q}}dx = \frac{1}{\frac{p}{q}+1}x^{\frac{p}{q}+1} + c = \frac{q}{p+q}x^{\frac{p+q}{q}} + c$$

Trig Functions

$$\int \cos u\,du = \sin u + c \qquad \int \sin u\,du = -\cos u + c \qquad \int \sec^2 u\,du = \tan u + c$$

$$\int \sec u \tan u\,du = \sec u + c \qquad \int \csc u \cot u du = -\csc u + c \qquad \int \csc^2 u\,du = -\cot u + c$$

$$\int \tan u\,du = \ln|\sec u| + c \qquad \int \cot u\,du = \ln|\sin u| + c$$

$$\int \sec u\,du = \ln|\sec u + \tan u| + c \qquad \int \sec^3 u\,du = \frac{1}{2}\left(\sec u \tan u + \ln|\sec u + \tan u|\right) + c$$

$$\int \csc u\,du = \ln|\csc u - \cot u| + c \qquad \int \csc^3 u\,du = \frac{1}{2}\left(-\csc u \cot u + \ln|\csc u - \cot u|\right) + c$$

Exponential/Logarithm Functions

$$\int \mathbf{e}^u\,du = \mathbf{e}^u + c \qquad \int a^u\,du = \frac{a^u}{\ln a} + c \qquad \int \ln u\,du = u\ln(u) - u + c$$

$$\int \mathbf{e}^{au}\sin(bu)\,du = \frac{\mathbf{e}^{au}}{a^2+b^2}\left(a\sin(bu) - b\cos(bu)\right) + c \qquad \int u\mathbf{e}^u du = (u-1)\mathbf{e}^u + c$$

$$\int \mathbf{e}^{au}\cos(bu)\,du = \frac{\mathbf{e}^{au}}{a^2+b^2}\left(a\cos(bu) + b\sin(bu)\right) + c \qquad \int \frac{1}{u\ln u}du = \ln|\ln u| + c$$

Inverse Trig Functions

$$\int \frac{1}{\sqrt{a^2-u^2}}du = \sin^{-1}\left(\frac{u}{a}\right) + c \qquad \int \sin^{-1}u\,du = u\sin^{-1}u + \sqrt{1-u^2} + c$$

$$\int \frac{1}{a^2+u^2}du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + c \qquad \int \tan^{-1}u\,du = u\tan^{-1}u - \frac{1}{2}\ln\left(1+u^2\right) + c$$

$$\int \frac{1}{u\sqrt{u^2-a^2}}du = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + c \qquad \int \cos^{-1}u\,du = u\cos^{-1}u - \sqrt{1-u^2} + c$$

Miscellaneous

$$\int \frac{1}{a^2-u^2}\,du = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + c \qquad \int \frac{1}{u^2-a^2}\,du = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + c$$

$$\int \sqrt{a^2+u^2}\,du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln\left|u+\sqrt{a^2+u^2}\right| + c$$

$$\int \sqrt{u^2-a^2}\,du = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right| + c$$

$$\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right) + c$$

$$\int \sqrt{2au-u^2}\,du = \frac{u-a}{2}\sqrt{2au-u^2} + \frac{a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + c$$

Standard Integration Techniques

Note that all but the first one of these tend to be taught in a Calculus II class.

u Substitution

Given $\int_a^b f\left(g(x)\right)g'(x)\,dx$ then the substitution $u = g(x)$ will convert this into the integral, $\int_a^b f\left(g(x)\right)g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$.

Integration by Parts

The standard formulas for integration by parts are,

$$\int u\,dv = uv - \int v\,du \qquad \int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

Choose u and dv and then compute du by differentiating u and compute v by using the fact that $v = \int dv$.

Trig Substitutions

If the integral contains the following root use the given substitution and formula.

$$\sqrt{a^2-b^2x^2} \quad \Rightarrow \quad x = \frac{a}{b}\sin\theta \quad \text{and} \quad \cos^2\theta = 1-\sin^2\theta$$

$$\sqrt{b^2x^2-a^2} \quad \Rightarrow \quad x = \frac{a}{b}\sec\theta \quad \text{and} \quad \tan^2\theta = \sec^2\theta - 1$$

$$\sqrt{a^2+b^2x^2} \quad \Rightarrow \quad x = \frac{a}{b}\tan\theta \quad \text{and} \quad \sec^2\theta = 1+\tan^2\theta$$

<u>Partial Fractions</u>

If integrating $\int \frac{P(x)}{Q(x)}dx$ where the degree (largest exponent) of $P(x)$ is smaller than the degree of $Q(x)$ then factor the denominator as completely as possible and find the partial fraction decomposition of the rational expression. Integrate the partial fraction decomposition (P.F.D.). For each factor in the denominator we get term(s) in the decomposition according to the following table.

Factor in $Q(x)$	Term in P.F.D	Factor in $Q(x)$	Term in P.F.D
$ax+b$	$\frac{A}{ax+b}$	$(ax+b)^k$	$\frac{A_1}{ax+b}+\frac{A_2}{(ax+b)^2}+\cdots+\frac{A_k}{(ax+b)^k}$
ax^2+bx+c	$\frac{Ax+B}{ax^2+bx+c}$	$(ax^2+bx+c)^k$	$\frac{A_1x+B_1}{ax^2+bx+c}+\cdots+\frac{A_kx+B_k}{(ax^2+bx+c)^k}$

Formula: If $f'(x)$ is continuous on $[a,b]$, then the *arc length of the curve* $y=f(x)$ on the interval $[a,b]$ is given by

$$s=\int_a^b \sqrt{1+(f'(x))^2}\,dx.$$

Formula: If $f'(x)$ is continuous on $[a,b]$, then the *surface area of a solid of revolution* obtained by rotating the curve $y=f(x)$

1. Around the y-axis on the interval $[a,b]$ is given by (provided that $x\geq 0$)

$$S=\int_a^b 2\pi x\sqrt{1+(f'(x))^2}\,dx.$$

2. Around the x-axis on the interval $[a,b]$ is given by (provided that $y=f(x)\geq 0$)

$$S=\int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx.$$