Part A MCQ (30%)

(5*pts*) **Problem 1**
If a_n is the sequence given by

$$\ln\left(\frac{2}{1}\right),\ \ln\left(\frac{3}{2}\right),\ \ln\left(\frac{4}{3}\right),...$$

$$\text{Evaluate } \lim_{n\to\infty} a_n.$$

(*a*) a_n converges to 1

(*b*) a_n converges to $\ln 2$

(*c*) a_n converges to 0

(*d*) a_n converges to $\ln 3$

(*e*) a_n diverges

(5*pts*) **Problem 2**
The sum of the geometric series

$$4 - 1 + \frac{1}{4} - \frac{1}{16} + ...$$

is

(*a*) $\frac{17}{16}$

(*b*) $\frac{19}{4}$

(*c*) $\frac{145}{16}$

(*d*) $\frac{14}{3}$

(*e*) $\frac{16}{5}$

(5pts) **Problem 3**
The series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n}}$$

(a) converges absolutely

(b) converges conditionally

(c) diverges

(d) is a convergent geometric series

(e) is a divergent telescoping series

(5pts) **Problem 4**

The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{2^n x^n}{n+1}$ is

(a) 2

(b) $\frac{1}{2}$

(c) 1

(d) ∞

(e) 0

(5*pts*) **Problem 5**

The power series representation of the function $\dfrac{1}{4-x^2}$ is equal to

$$(a) \quad \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{4^n}, \quad |x| < 2$$

$$(b) \quad \sum_{n=0}^{\infty} \frac{x^{2n}}{4^{n+1}}, \quad |x| < 2$$

$$(c) \quad \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+2}}{4^{n+1}}, \quad |x| < 2$$

$$(d) \quad \sum_{n=0}^{\infty} \frac{x^{2n+1}}{2^{n+1}}, \quad |x| < 2$$

$$(e) \quad \sum_{n=0}^{\infty} \frac{x^{4n}}{2^{n+1}}, \quad |x| < 2$$

(5*pts*) **Problem 6**

The coefficient of x^3 in Maclaurin series of the function $f(x) = \ln(1-x)$ equal to

$$(a) \quad \frac{-1}{3}$$

$$(b) \quad \frac{-1}{6}$$

$$(c) \quad \frac{5}{6}$$

$$(d) \quad \frac{1}{2}$$

$$(e) \quad -1$$

Part B Written Questions (70%)

(15pts)**Problem 1**
Find the interval of convergence of the following power series

1. $\sum_{n=1}^{\infty} \frac{x^n}{n2^n}$

2. $\sum_{n=0}^{\infty} \frac{(x+2)^n}{n!}$.

(15pts)**Problem 2**
Solve the initial value problem for the separable equation below

$$\frac{dy}{dx} = 3x^2y^2, \qquad y(0) = \frac{1}{2}.$$

(20*pts*)**Problem 3**
Show that the differential equation is exact and solve the equation.

$$(\cos y + y\cos x)\,dx + (\sin x - x\sin y)\,dy = 0.$$

(20*pts*)**Problem 4**
Solve the initial value problem for the Bernoulli equation below

$$x\frac{dy}{dx} - 2y = 4x^3y^{1/2}, \qquad y(1) = 0.$$