The Solution Key

Faculty of Engineering and Information Sciences

UNIVERSITY OF WOLLONGONG IN DUBAI

MATH 142, Quiz 1, Winter 2022, Duration: 110 minutes

Name:____________________, ID Number:____________________

Time Allowed: 1 hour 50 minutes
Total Number of Questions: 6
Total Number of Pages (incl. this page): 7

EXAM UNAUTHORISED ITEMS

Students bringing these items to the examination room shall be required to leave the items at the front of the room or outside the examination room. The University does not guarantee the safe-keeping of students' personal items during examinations. Students concerned about the safety of their valuable items should make alternative arrangements for their care.

1. Bags, including carry bags, backpacks, shoulder bags and briefcases
2. Any form of electronic device including but not limited to mobile phones, smart watches, laptops, iPads, MP3 players, handheld computers and electronic dictionaries,
3. Calculator cases and covers
4. blank paper
5. Any written material

DIRECTIONS TO CANDIDATES

1. Total marks: 40
2. All questions are compulsory.
3. Answer all questions on the given exam paper sheets.
4. Write your name and Id number on the papers provided for rough work.

(6pts) Problem 1.
Find the area enclosed by the graphs of

$$x = y^2 + 2 \text{ and } \quad y = x - 8.$$

Solution
<u>Method 1 without graphing</u>

$$x = y^2 + 2 \text{ and } \quad y = x - 8 \Leftrightarrow x = y + 8.$$

$$\begin{aligned} A &= \int_a^b \left| \left(y^2+2\right) - (y+8) \right| dy \\ &= \int_a^b \left| y^2 - y - 6 \right| dy \qquad [\textbf{2pts}] \end{aligned}$$

To find a and b, we set

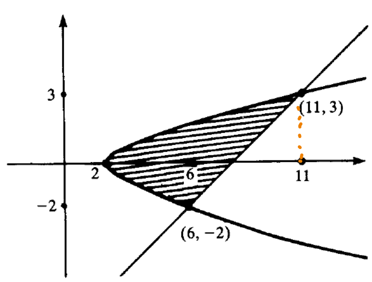
$$y^2 - y - 6 = 0 \text{ and solve to get}$$

$$a = -2 \text{ and } b = 3.$$

equation Now using the fact that the quadratic $y^2 - y - 6$ has the opposite sign of $+1$ inside the roots, we have

$$\begin{aligned} A &= \int_{-2}^{3} -\left(y^2 - y - 6\right) dy \\ &= \frac{125}{6} = 20.833 \qquad [\textbf{4pts}] \end{aligned}$$

<u>Method 2 with graphing</u>



[**2pts**]

$$\begin{aligned} A &= \int_{-2}^{3} Right - Left\ dy \\ &= \int_{-2}^{3} \left[(y+8) - \left(y^2+2\right)\right] dy \\ &= \int_{-2}^{3} \left(-y^2 + y + 6\right) dy = \frac{125}{6} = 20.833 \qquad [\textbf{4pts}] \end{aligned}$$

(6pts) Problem 2
Find the arclength of the function

$$f(x)=\frac{x^4}{8}+\frac{1}{4x^2}$$

from $x=1$ to $x=2$.
Solution

$$L=\int_a^b\sqrt{1+f\,'(x)^2}dx$$

$$f\,'(x)=\frac{x^3}{2}-\frac{1}{2x^3}$$

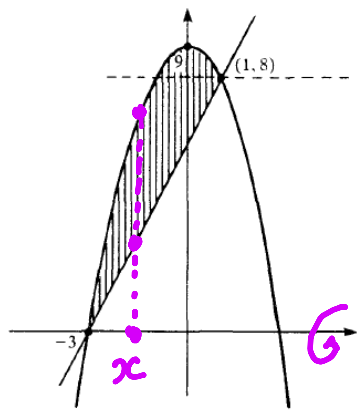
$$\begin{aligned}f\,'(x)^2 &= \left(\frac{x^3}{2}-\frac{1}{2x^3}\right)^2\\ &= \frac{1}{4x^6}+\frac{1}{4}x^6-\frac{1}{2}\end{aligned}$$

$$\begin{aligned}1+f\,'(x)^2 &= \frac{1}{4x^6}+\frac{1}{4}x^6+\frac{1}{2}\\ &= \left(\frac{x^3}{2}+\frac{1}{2x^3}\right)^2\end{aligned}$$

$$\begin{aligned}L &= \int_1^2\sqrt{\left(\frac{x^3}{2}+\frac{1}{2x^3}\right)^2}dx \qquad [\mathbf{3pts}]\\ &= \int_1^2\left(\frac{x^3}{2}+\frac{1}{2x^3}\right)dx\\ &= \frac{33}{16}=2.0625. \qquad [\mathbf{3pts}]\end{aligned}$$

(6pts) Problem 3
Sketch the region bounded by $y = 9 - x^2,\ y = 2x + 6$, and use the disc method to find the volume of the solid generated by revolving the region about the x-axis.

Solution

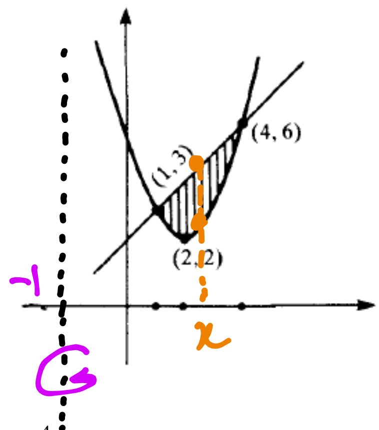


[**2pts**]

$$
\begin{aligned}
V &= \int_{-3}^{1} \left[\pi\left(9 - x^2\right)^2 - \pi\left(2x + 6\right)^2\right] dx \\
&= \pi \int_{-3}^{1} \left[\left(9 - x^2\right)^2 - \left(2x + 6\right)^2\right] dx \\
&= \pi \int_{-3}^{1} \left(x^4 - 22x^2 - 24x + 45\right) dx \\
&= \frac{1792}{15}\pi = 375.32 \qquad \textbf{[4pts]}
\end{aligned}
$$

(6pts) Problem 4
Sketch the region bounded by $y = x^2 - 4x + 6,\ y = x + 2$, and use the shell method to find the volume of the solid generated by revolving the region about the line $x = -1$.

Solution



[**2pts**]

$$
\begin{aligned}
V &= \int_1^4 2\pi\,(x+1)\left[(x+2) - \left(x^2 - 4x + 6\right)\right] dx \\
&= \int_1^4 2\pi\,(x+1)\left(-x^2 + 5x - 4\right) dx \\
&= 2\pi \int_1^4 \left(-x^3 + 4x^2 + x - 4\right) dx \\
&= \frac{63}{2}\pi = 98.96 \qquad \text{[}\mathbf{4pts}\text{]}
\end{aligned}
$$

(8pts) Problem 5
Use trigonometric substitution to evaluate the integral

$$\int \frac{\sqrt{1+x^2}dx}{x}.$$

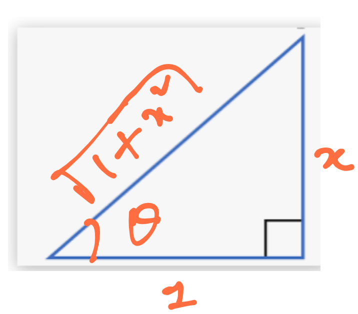
Solution
Put

$$x = \tan\theta$$

$$dx = \sec^2\theta d\theta \qquad [\textbf{2pts}]$$

The integral becomes

$$\begin{aligned}
\int \frac{\sqrt{1+x^2}dx}{x} &= \int \frac{\sec\theta}{\tan\theta}\sec^2\theta d\theta. \\
&= \int \frac{\sec\theta}{\tan\theta}\left(1+\tan^2\theta\right) d\theta \\
&= \int \left(\frac{1}{\sin\theta} + \sec\theta\tan\theta\right) d\theta \\
&= \int (\csc\theta + \sec\theta\tan\theta)\, d\theta \\
&= \ln|\csc\theta - \cot\theta| + \sec\theta + C \qquad [\textbf{3pts}]
\end{aligned}$$



$\tan\theta = \frac{x}{1}$

$$\cos\theta = \frac{1}{\sqrt{1+x^2}}, \quad \sin\theta = \frac{x}{\sqrt{1+x^2}} \text{ and } \cot\theta = \frac{1}{\tan\theta} = \frac{1}{x}$$

$$\int \frac{\sqrt{1+x^2}dx}{x} = \ln\left|\frac{\sqrt{1+x^2}}{x} - \frac{1}{x}\right| + \sqrt{1+x^2} + C \qquad [\textbf{3pts}]$$

(8pts) Problem 6
Use partial fraction decomposition to evaluate the integrals

$$\int \frac{x}{(x+2)(x+3)} dx$$

Solution

$$\begin{aligned}
\frac{x}{(x+2)(x+3)} &= \frac{A}{x+2} + \frac{B}{x+3} \\
A &= -2 \text{ and } B = 3 \\
\int \frac{x}{(x+2)(x+3)} dx &= \int \left(\frac{-2}{x+2} + \frac{3}{x+3} \right) dx \\
&= 3 \ln |x+3| - 2 \ln |x+2| + C \\
&= \ln \frac{|x+3|^3}{(x+2)^2} + C
\end{aligned}$$

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