

$$\int_a^b \sqrt{1 + [f'(x)]^2} \, dx$$

$$f(x) = \frac{2}{3}(x^2+1)^{3/2}$$

$$f'(x) = \frac{2}{3} \times \frac{3}{2} \times 2x(x^2+1)^{1/2} = 2x(x^2+1)^{1/2}$$

$$[f'(x)]^2 = 4x^2(x^2+1) \\ = 4x^4 + 4x^2$$

$$1 + [f'(x)]^2 = 4x^4 + 4x^2 + 1 \\ = 4x^4 + 2x^2 + 2x^2 + 1 \\ = 2x^2(2x^2+1) + 1(2x^2+1) \\ = (2x^2+1)^2$$

$$\int_1^4 \sqrt{(2x^2+1)^2} \, dx$$

$$\int_1^4 2x^2 + 1$$

$$= \left. \frac{2x^3}{3} + x \right|_1^4$$

$$= \frac{128}{3} + 4 - \frac{2}{3} - 1$$

$$= \frac{126}{3} + 3$$

$$= \frac{126+9}{3}$$

$$\begin{array}{r} 45 \\ 3 \overline{) 135} \end{array}$$

