Math 142 Midterm Review-UOWD

Problem 1

Find the area of the region bounded by $y = x^3 - 3x$ and $y = x$.

Problem 2
Consider the curve given by

$$x^{2/3} + y^{2/3} = 4, \qquad 1 \leq x \leq 8.$$

(a) Find the arclength of the curve.
(b) Find the area of the surface obtained by rotating the curve about the x-axis.
Hint: Use implicit differentiation to find $\dfrac{dy}{dx}$.

Problem 3

Find an equation of the tangent to the curve

$$x = t^2 - 2t y = t^3 - 3t \qquad \text{when} \qquad t = -2$$

Problem 4

Find the arc length of the spiral defined by

$$x = e^t \cos t \qquad y = e^t \sin t \qquad 0 \leq t \leq 2\pi$$

Problem 5
Find the arclength of the polar curve

$$r = e^{\theta/2} \quad \text{from} \quad \theta = 0 \quad \text{to} \quad \theta = 2\pi.$$

Problem 6
Sketch the region bounded by $y = x^3$, $x = 1$ and $y = 0$ and use the **disc metho**d to find the volume of the solid obtained by rotating the region about the line $y = -1$.

Problem 7
Sketch the region bounded by $y = x^3$, $x = 1$, $x = 2$, and $y = x - 1$ and use the **cylindrical shell** method to find the volume of the solid generated by rotating the region about the x-axis.

Problem 8
Evaluate the following integrals

$$(1) \quad \int_0^{\pi} x \cos(3x - \pi)\, dx, \qquad (2) \quad \int \ln\left(x^2 + 1\right) dx$$

Problem 9
Evaluate the following integral

$$\int \frac{x^4 + 2x^3 + 3x^2 + 2}{x\left(x^2 + 5x + 6\right)} dx$$

Problem 10
Evaluate the integral

$$\int \frac{dx}{(4 - x^2)\sqrt{4 - x^2}}$$

Problem 11
Evaluate the integral

$$\int \frac{dx}{x\left(x^2 + 1\right)^2}$$