(14pts)**Problem 1.**
Determine convergence or divergence of the following improper integrals,

1. $\displaystyle\int_0^3 \frac{dx}{(x-1)^{2/3}}$

2. $\displaystyle\int_2^\infty \frac{dx}{x\,(\ln x)^3}$

(15pts)**Problem 2.**
Solve the following differential equation

$$\frac{dy}{dx} = \frac{(y+3)(x-1)}{(y-2)(x+4)}$$

(15pts)**Problem 3.**
Show that the equation is linear and solve the initial value problem

$$x\frac{dy}{dx} - 2y = x^2, \qquad y(1) = 3$$

$$y(1) = 3 \text{ gives } C = 3$$

(14pts)**Problem 4.**
Show that the differential equation is exact and solve the equation.

$$\left(1 + 2x - y^3\right) dx + \left(2y - 3xy^2\right) dy = 0$$

(14pts)**Problem 5.**
Show that the differential equation is homogeneous and solve it.

$$(x - 2y)dx + xdy = 0.$$

(14pts)**Problem 6**
Solve the following Bernoulli differential equation

$$x\frac{dy}{dx} + y = x^2y^2$$

(14pts)**Problem 7**
Show that the differential equation is not exact, find the special integrating factor, make it exact and solve the equation.

$$\left(y^2 - x\right) dx + 2ydy = 0.$$