

**(5pts) Problem 1.**

If

$$L = \int_e^{\infty} \frac{dx}{x (\ln x)^2},$$

then

- (a)  $L = 2e$       (b)  $L = 1$       (c)  $L = \infty$       (d)  $L = -1$       (e)  $L = 0$

**(5pts) Problem 2.** If

$$L = \int_{-2}^2 \frac{dx}{x+1},$$

then

- (a)  $L = \frac{8}{9}$       (b)  $L = \frac{1}{2} \ln 3$       (c)  $L = 0$       (d)  $L = \ln 3$       (e)  $L = -\infty$

**(5pts) Problem 3.** Evaluate the improper integral

$$L = \int_0^{\infty} x e^{-x^2} dx$$

then

- (a)  $\frac{1}{2}$       (b) 1      (c)  $2e$       (d) divergent      (e)  $e$

**(5pts) Problem 4.** Evaluate the improper integral

$$L = \int_0^2 \frac{dx}{x-1}$$

then

- (a) 0      (b) diverges      (c) 4      (d) -2      (e)  $e$

**(12pts)Problem 5.**

Show that the equation is separable and find the general solution.

$$2\frac{dy}{dx} = (y^2 - 1) \sin x.$$

**(12pts)Problem 6.**

Solve the initial value problem for the linear equation below

$$\frac{dy}{dx} = -\frac{1}{x}y + \sin x, \quad y(\pi) = 1.$$

**(14pts)Problem 7.**

Show that the differential equation is exact and find the general solution.

$$(xe^{2y} - x^2) dx + (x^2e^{2y} + e^y) dy = 0.$$

**(14pts)Problem 8.**

Show that the differential equation is NOT exact and transform it into an exact equation.

$$(x^3y - y) dx - xdy = 0$$

**(14pts)Problem 9.**

(a) Solve the homogeneous equation

$$\frac{dy}{dx} = \frac{x^2 + y^2}{xy}.$$

(b) Find an explicit solution of the initial value problem

$$\frac{dy}{dx} = \frac{x^2 + y^2}{xy}, \quad y(e) = 2e.$$

**(14pts)Problem 10.**

Find the general solution of the Bernoulli equation

$$3(1+x^2)\frac{dy}{dx} = 2xy(y^3-1).$$