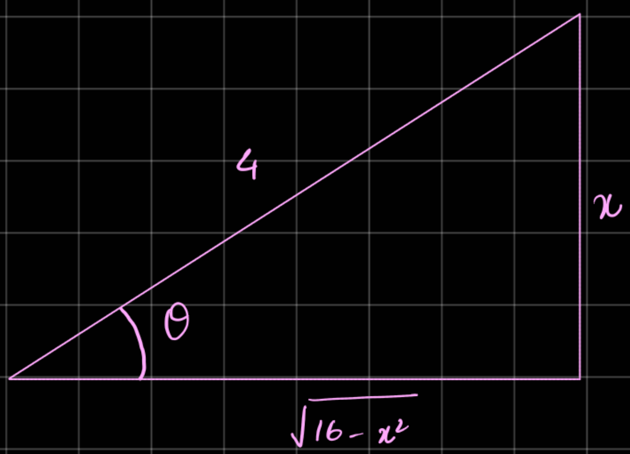
2. $x = a \sin\theta$

$dx = a \cos\theta \, d\theta$

$\sqrt{16 - x^2} = \sqrt{16 \cos^2\theta}$

$= 4\cos\theta$



$$\int \frac{x^2}{\sqrt{16 - x^2}} \, dx$$

$$= \int \frac{16 \sin^2\theta}{4 \cancel{\cos\theta}} \; 4 \cancel{\cos\theta} \, d\theta$$

$$= 16 \int \sin^2\theta \, d\theta$$

$$= 8 \int 1 - \cos 2\theta \, d\theta + c$$

$$= 8 \left[\theta - \frac{1}{2} \sin 2\theta \right] + c$$

$$= 8\theta - 4 \sin 2\theta + c$$

$$= 8 \sin^{-1}\left(\frac{x}{4}\right) - 8 \sin\theta \cos\theta + c$$

$$= 8 \sin^{-1}\left(\frac{x}{4}\right) - \frac{x\sqrt{16 - x^2}}{2} + c$$

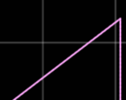
4. $$\int \frac{\sqrt{x^2 - 4}}{x}$$

$x = 2 \sec\theta$

$dx = 2 \sec\theta \tan\theta \, d\theta$

$\sqrt{x^2 - 4} = 2 \tan\theta$

$$\int \frac{\overset{2}{\cancel{4}} \tan^2\theta \, \cancel{\sec\theta} \, d\theta}{\cancel{2} \, \cancel{\sec\theta}}$$



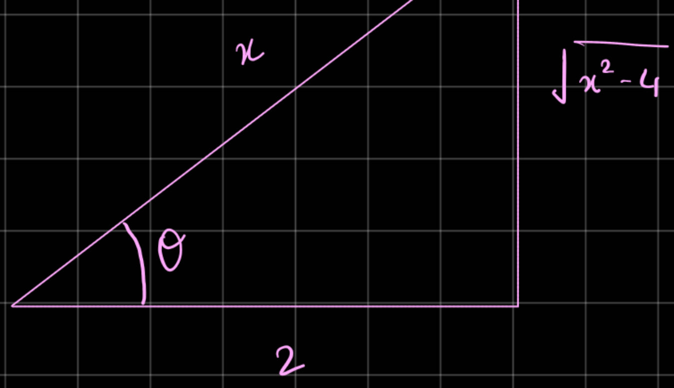
$2 \int \tan^2\theta \, d\theta$

$= 2\int \tan^2\theta \, d\theta$

$= 2\int \sec^2\theta - 1 \, d\theta$

$= 2(\tan\theta - \theta) + C$

$= \sqrt{x^2-4} - 2\cos^{-1}\left(\frac{2}{x}\right) + C$



Tutorial 3

1. $\displaystyle\int \frac{3x-4}{x^2-2x+1} \, dx$

$$\frac{3x-4}{x^2-2x+1} = \frac{3x-4}{(x-1)^2}$$

$$\frac{3x-4}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$$

$$3x - 4 = A(x-1) + B$$
$$= Ax - A + B$$

$$Ax = 3x$$
$$\underline{A = 3}$$

$$-A + B = -4$$
$$B = -4 + A$$
$$= -4 + 3$$
$$\underline{B = -1}$$

$$\int \frac{3x-4}{(x-1)^2} = \int \frac{3}{x-1} \, dx - \int \frac{dx}{(x-1)^2}$$

$$= 3\ln|x-1| + \frac{1}{\quad} + C$$

$x-1$

5. ii)

$$\int \frac{3x^3 - 3x^2 + 4}{x^2 - x}\, dx$$

$$\begin{array}{r|l} & 3x \\ \hline x^2 - x & 3x^3 - 3x^2 + 4 \\ & \underline{(-)\,3x^3 \;\overset{-}{(+)}\; 3x^2} \\ & 4 \end{array}$$

$$\int \frac{3x^3 - 3x^2 + 4}{x^2 - x} = \int \left(3x + \frac{4}{x^2 - x}\right) dx$$

$$\frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$$

$$1 = Ax - A + Bx$$

$$0x = Ax + Bx \qquad -A = 1$$

$$A + B = 0 \qquad A = -1$$

$$A = -B$$

$$-B = -1$$

$$B = 1$$

$$\int \frac{dx}{x^2 - x} = \int \frac{-1}{x} + \frac{1}{x-1}\, dx$$

$$\int \frac{3x^3 - 3x^2 + 4}{x^2 - x} = \int 3x\, dx + 4\int \frac{dx}{x^2 - x}$$

$$= \frac{3x^2}{2} + 4\left[-1\int \frac{dx}{x} + \int \frac{dx}{x-1}\right]$$

$$= \frac{3x^2}{2} - 4\ln|x| + 4\ln|x-1| + C$$

7. $\int \frac{x-3}{x^3+3x}$

$$\frac{x-3}{x^3+3x} = \frac{x-3}{x(x^2+3)}$$

$$\frac{x-3}{x(x^2+3)} = \frac{A}{x} + \frac{Bx+C}{x^2+3}$$

$$x-3 = A(x^2+3) + Bx^2 + Cx$$
$$= Ax^2 + 3A + Bx^2 + Cx$$

$Cx = x$ $3A = -3$ $A+B=0$

$C = 1$ $A = -1$ $B = 1$

$$\int \frac{x-3}{x^3+3x}\,dx = \int\left(\frac{-1}{x} + \frac{x+1}{x^2+3}\right)dx$$

$$= -\ln|x| + \frac{1}{2}\int \frac{2x}{x^2+3}\,dx + \int \frac{dx}{x^2+3}$$

$$= -\ln|x| + \frac{1}{2}\ln|x^2+3| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$$

2. i) $\int \frac{\sqrt{x}\,dx}{x-4}$

$u = \sqrt{x}$

$u^2 = x$

$2u\,du = dx$

$$\int \frac{2u^2\,du}{u^2-4}$$

$$2\int \frac{u^2-4+4}{u^2-4}\,du$$

$$= 2\int \left(\frac{u^2-4}{u^2-4} + \frac{4}{u^2-4}\right) du$$

$$= 2\int 1 \, du + 8\int \frac{1}{u^2-4} \, du$$

3. $$\int_0^{12} \frac{9}{\sqrt{12-x}} \, dx$$

$$= \lim_{t \to 12^-} \int_0^t \frac{9}{\sqrt{12-x}} \, dx$$

$$= \lim_{t \to 12^-} 9 \int_0^t (12-x)^{-1/2} \, du$$

$$= \lim_{t \to 12^-} -18\sqrt{12-x} \Big|_0^t$$

$$= \lim_{t \to 12^-} -18\sqrt{12-t} + 18\sqrt{12}$$

$$= \lim_{t \to 12^-} -18\sqrt{12-12} + 18\sqrt{12}$$

$$= \underline{18\sqrt{12}} = 62.354$$

Converges at $18\sqrt{12}$