

Part 1 MCQ (30%)

Directions: Circle the letter that corresponds to the correct answer. There is only one correct answer for each question. You do not need to show your work.

(6pts)**Problem 1**

$$\int x^3 \ln x dx$$

is equal to:

- (a) $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$
- (b) $\ln x - \frac{1}{16}x^4 + C$
- (c) $\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C$
- (d) $x^3 \ln x - \frac{1}{9}x^4 + C$
- (e) $\frac{1}{4}x^4 \ln x - x^3 + C$

(6pts)**Problem 2** Let

$$F(x) = \int e^x \cos x dx \quad (\text{ without the arbitrary constant } C).$$

$F(0)$ is equal to

- (a) \sqrt{e}
- (b) 0
- (c) e
- (d) $\frac{1}{2}$
- (e) $\frac{1}{4}$

(6pts)**Problem 3**

If the region enclosed by the curves $y = x$ and $y = x^2$ is rotated about the line $x = -1$, then the volume of the obtained solid is

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{4}$

(c) π

(d) $\frac{3\pi}{2}$

(e) $\frac{5\pi}{2}$

(6pts)**Problem 4**

The area of the region enclosed by the curves $y = \frac{1}{x}$, $y = 0$, $x = -3$, $x = -2$

(a) 1

(b) $\ln\left(\frac{3}{2}\right)$

(c) 3

(d) $\ln 3$

(e) $\ln 2$

(6pts)**Problem 5**

The length of the curve

$$F(x) = \int_{-2}^x \sqrt{3t^4 - 1} dt, \quad -2 \leq x \leq -1$$

is equal to

(a) $\frac{\sqrt{3}}{2}$

(b) $\frac{\sqrt{3}}{3}$

(c) $\sqrt{3}$

(d) $\frac{3\sqrt{3}}{4}$

(e) $\frac{7\sqrt{3}}{3}$

Hint : $\frac{d}{dx} \int_a^x f(t) dt = f(x)$

Part 2 Written Questions (70%)

(10pts) **Problem 1**

Find the area of the region enclosed by the curves $y = x^2 - 4x + 5$ and $y = 2x - 3$.

(10pts)**Problem 2**

Find the area of the surface obtained by rotating the graph of

$$f(x) = 2\sqrt{x+1} \ , \quad 0 \leq x \leq 1$$

about the x-axis.

(10pts)**Problem 3**

Sketch the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 9$, and use the disc method to find the volume of the solid generated by revolving the region about the y-axis.

(10pts)**Problem 4**

Sketch the region bounded by $y = x^3$ and $y = x$, and use the shell method to find the volume of the solid generated by revolving the region about the x-axis.

(10pts)**Problem 5**

Evaluate the integral

$$\int \frac{(x+1)(x+3)}{(2+x)^3 - (2+x)^2} dx$$

(10pts)**Problem 6**

Use trigonometric substitution to evaluate

$$\int \frac{4dx}{x^2 (x^2 + 4)}$$

(10pts)**Problem 7**

Evaluate the integral

$$\int \frac{\sqrt{x}}{2(1+\sqrt{x})} dx$$