1. Find the area of the region bounded by graphs of $f(x) = 3 - x^2$ and $g(x) = -x + 1$ between $x = 0$ and $x = 2$

Intersection points

$$3 - x^2 = -x + 1$$

$$x^2 - x - 2 = 0$$

$$x^2 - 2x + x - 2 = 0$$

$$x(x-2) + 1(x-2) = 0$$

$$(x-2)(x+1) = 0$$

$$x = -1 \text{ OR } x = 2$$

$$\text{Area} = \left| \int_0^2 [3 - x^2 + x - 1] \, dx \right|$$

$$= \int_0^2 [-x^2 + x + 2] \, dx \Big|$$

$$= \left| \left[-\frac{x^3}{3} + \frac{x^2}{2} + 2x \right]_0^2 \right|$$

$$= \left| \frac{-8}{3} + 2 + 4 \right|$$

$$= \frac{10}{3} \text{ sq units}$$

2. Find the area of the region enclosed by the curves $y = x^3 - 2x$, $y = 2x$

$$x^3 - 2x = 2x$$

$$x^3 - 4x = 0$$

$$x(x^2 - 4) = 0$$

$x(x+2)(x-2) = 0$

$x = -2,\ x = 0,\ x = 2$

$$\text{Area} = \left| \int_{-2}^{0} (x^3 - 4x)\,dx \right| + \left| \int_{0}^{2} (x^3 - 4x)\,dx \right|$$

$$= \left| \left[\frac{x^4}{4} - 2x^2 \right]_{-2}^{0} \right| + \left| \left[\frac{x^4}{4} - 2x^2 \right]_{0}^{2} \right|$$

$$= |-(4-8)| + |4-8|$$

$$= |4| + |-4|$$

$$= \underline{8}$$

3. Find the area of the region bounded by $y = x^3 - 3x^2 + 2x$ and the x-axis. Set up the integral only.

$x^3 - 3x^2 + 2x = 0$

$x(x^2 - 3x + 2) = 0$

$x[x^2 - 2x - x + 2] = 0$

$x[x(x-2) - 1(x-2)] = 0$

$x(x-2)(x-1) = 0$

$x = 0,\ x = 1,\ x = 2$

$$\text{Area} = \left| \int_{0}^{1} (x^3 - 3x^2 + 2x)\,dx \right| + \left| \int_{1}^{2} (x^3 - 3x^2 + 2x)\,dx \right|$$

4. Find the area bounded by the graphs of $x = y^2$ and $x = 2 - y^2$

$y^2 = 2 - y^2$

$2y^2 - 2 = 0$

$(\sqrt{2}y + \sqrt{2})(\sqrt{2}y - \sqrt{2}) = 0$

$2(y^2 - 1) = 0$

$y^2 - 1 = 0$

$y^2 = 1$

$y = -1$ OR 1

$\sqrt{2}y + \sqrt{2} = 0$ $\quad\quad$ $\sqrt{2}y - \sqrt{2} = 0$

$y = -1$ $\quad\quad$ $y = 1$

$$\text{Area} = \left| \int_{-1}^{1} (2y^2 - 2)\, dy \right|$$

$$= \left| \left[\frac{2y^3}{3} - 2y \right]_{-1}^{1} \right|$$

$$= \left| \frac{2}{3} - 2 - \left(\frac{-2}{3} + 2 \right) \right|$$

$$= \left| \frac{2}{3} - 2 + \frac{2}{3} - 2 \right|$$

$$= \left| \frac{4}{3} - 4 \right|$$

$$= \left| \frac{4 - 12}{3} \right|$$

$$= \left| -\frac{8}{3} \right|$$

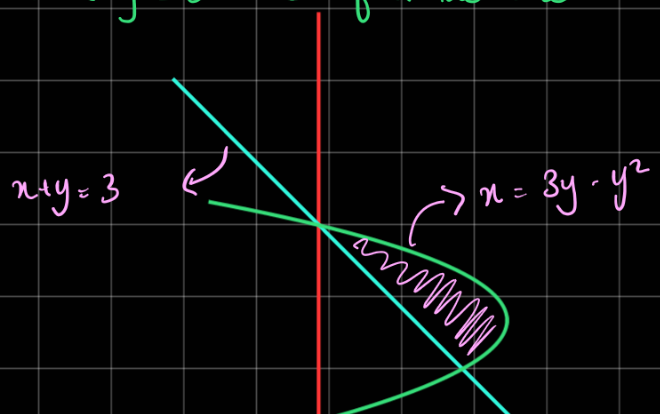
$$= \frac{8}{3}$$

5. Sketch the region bounded by $x = 3y - y^2$ and $x + y = 3$ and find the area.

$3y - y^2 = 3 - y$

$y^2 - 4y + 3 = 0$

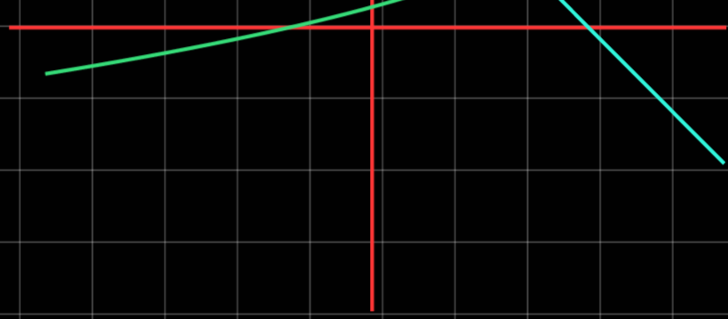
$y^2 - 3y - y + 3 = 0$



$y(y-3)-1(y-3)=0$

$(y-3)(y-1)=0$

$y=1$ or $y=3$



$$\text{Area} = \left| \int_1^3 \left(3y - y^2 - 3 + y\right) dy \right|$$

$$= \left| \int_1^3 \left[y^2 - 4y + 3\right] dy \right|$$

$$= \left| \left[\frac{y^3}{3} - 2y^2 + 3y\right]_1^3 \right|$$

$$= \left| [9 - 18 + 9] - \left[\frac{1}{3} - 2 + 3\right] \right|$$

$$= \frac{4}{3}$$

6. Find the arc length of the curve $f(x) = x^{2/3}$, $x \in [0, 8]$. Set up the integral, do not evaluate

$$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$$

$$[f'(x)]^2 = \frac{4}{9\sqrt[3]{x^2}}$$

$$L = \int_0^8 \sqrt{1 + \frac{4}{9\sqrt[3]{x^2}}}\, dx$$

7. Given $f(x) = \frac{x^3}{6} + \frac{1}{2x}$ and $x \in [1, 3]$. Find the arc length.

$$f'(x) = \frac{x^2}{2} - \frac{1}{2x^2}$$

$[f'(x)]^2 = x^4$ …

$$[f'(x)]^2 = \frac{x^4}{4} - \cancel{2}\left(\frac{x^2}{2}\right)\left(\frac{1}{\cancel{2x^2}}\right) + \frac{1}{4x^4}$$

$$= \frac{x^4}{4} - \frac{1}{2} + \frac{1}{4x^4}$$

$$1 + [f'(x)]^2 = \frac{x^4}{4} + \frac{1}{2} + \frac{1}{4x^4}$$

$$= \left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^2$$

$$\sqrt{1 + [f'(x)]^2} = \left(\frac{x^2}{2} + \frac{1}{2x^2}\right)$$

$$\frac{1}{2x^2} = \frac{x^{-2+1}}{2(-1)}$$

$$= \frac{-1}{2x}$$

$$L = \int_1^3 \left(\frac{x^2}{2} + \frac{1}{2x^2}\right) dx$$

$$= \left[\frac{x^3}{6} - \frac{1}{2x}\right]_1^3$$

$$= \left[\frac{9}{2} - \frac{1}{6}\right] - \left[\frac{1}{6} - \frac{1}{2}\right]$$

$$= \frac{9}{2} - \frac{1}{6} - \frac{1}{6} + \frac{1}{2}$$

$$= 5 - \frac{2}{6}$$

$$= 5 - \frac{1}{3}$$

$$= \frac{14}{3}$$

8. Find the arc length of $f(x) = \frac{1}{2}\left(e^x + e^{-x}\right)$ on the interval $[0, 2]$

$$f'(x) = \frac{1}{2} e^x - \frac{1}{2} e^{-x}$$

$$= \frac{1}{2}\left(e^{x} - e^{-x}\right)$$

$$1 + \left[f'(x)\right]^2 = 1 + \left[\frac{1}{4}\left(e^{2x} - 2(\cancel{e^{x}})(\cancel{e^{-x}}) + e^{-2x}\right]\right.$$

$$= 1 + \left[\frac{1}{4}\left(e^{2x} + e^{-2x} - 2\right)\right]$$

$$= 1 + \left[\frac{e^{2x}}{4} + \frac{e^{-2x}}{4} - \frac{2}{4}\right]$$

$$= \frac{e^{2x}}{4} + \frac{e^{-2x}}{4} + \frac{2}{4}$$

$$= \left(\frac{1}{2}\left(e^{x} + e^{-x}\right)\right)^2$$

$$L = \int_0^2 \left(\frac{1}{2} e^{x} + e^{-x}\right) dx$$

$$= \frac{1}{2}\int_0^2 \left(e^{x} + e^{-x}\right) dx$$

$$= \frac{1}{2}\left[e^{x} - e^{-x}\right]_0^2$$

$$= \frac{1}{2}\left[e^{2} - \frac{1}{e^{2}} - 2e^{0}\right]$$

$$= \underline{2.62}$$