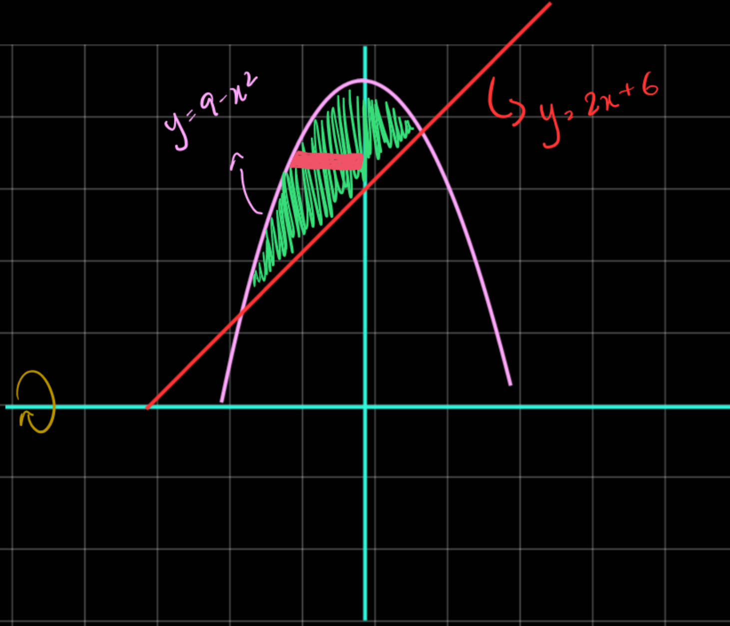
7.

$y = 9 - x^2$

$y = 2x + 6$



$9 - x^2 = 2x + 6$

$9 - x^2 - 2x - 6 = 0$

$x^2 + 2x - 3 = 0$

$x^2 + 3x - x - 3 = 0$

$x(x-1) + 3(x-1) = 0$

$(x-1)(x+\ ) = 0$

$x = 1$ OR $x = -3$

$$V = \int_{-3}^{1} (\pi R^2 - \pi r^2)\, dx$$

$$= \pi \int_{-3}^{1} \left[(9-x^2)^2 - (2x+6)^2 \right] dx$$

$a^2 - b^2 = (a+b)(a-b)$

$$= \pi \int_{-3}^{1} \left[(9 - x^2 + 2x + 6)(9 - x^2 - 2x - 6) \right] dx$$

$$= \pi \int_{-3}^{1} \left[(-x^2 + 2x + 15)(-x^2 - 2x + 3) \right] dx$$

$$= \pi \int_{-3}^{1} \left[x^4 + \cancel{2x^3} - 3x^2 - \cancel{2x^3} - 4x^2 + 6x - 15x^2 - 30x + 45 \right] dx$$

$$= \pi \int_{-3}^{1} \left[x^4 - 22x^2 - 24x + 45 \right] dx$$

$$= \pi \left[\frac{x^5}{5} - \frac{22x^3}{3} - 12x^2 + 45 \right]_{-3}^{1}$$

9.

$$V = \int_{a}^{b} (\pi R^2 - \pi r^2)\, dy$$

$y = 2\sqrt{x}$

$x = \frac{y^2}{4}$

$x = 4$

$4 = \frac{y^2}{4}$

$y^2 = 16$

$y = \pm 4$

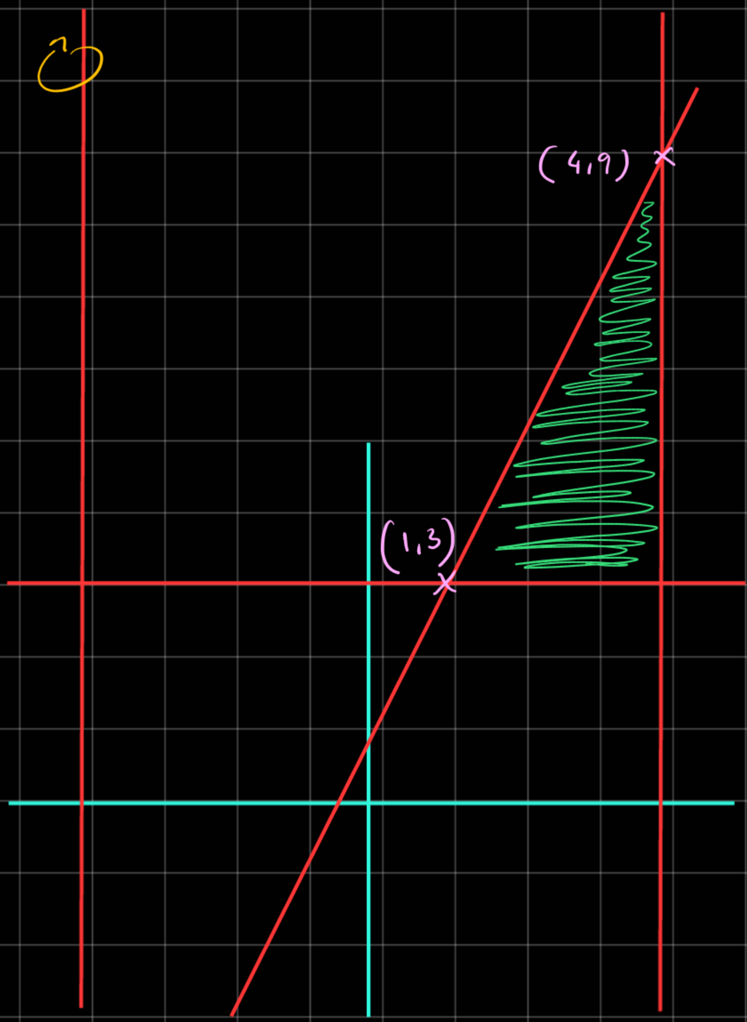
$$V = \int_0^4 \pi(4)^2 - \pi\left(\frac{y^2}{4}\right)^2 \, dy$$

$$= \pi \int_0^4 \left(16 - \frac{y^4}{16}\right) dy$$

$$= \pi \left[16y - \frac{y^5}{80}\right]_0^4$$

$$= \pi\left[64 - \frac{1024}{80}\right]$$

$$= \underline{160.85}$$



8. $$V = 2\pi \int_a^b \left(\text{Avg Radius} \times \text{Shell height}\right) dx$$

$$V = 2\pi \int_1^4 (x+4)(2x+1-3) \, dx$$

$$= 2\pi \int_1^4 (2x^2 - 2x + 8x - 8) \, dx$$

$$= 2\pi \int_1^4 [2x^2 + 6x - 8] \, dx$$

$$= 4\pi \left[\frac{x^3}{3} + \frac{3x^2}{2} - 4x\right]_1^4$$

$$= 4\pi \left[21 + \frac{45}{2} - 12 \right]$$

$$= 2\pi \left[42 + 45 - 24 \right]$$

$$= 2\pi \times 63$$

$$= 126\pi$$

$$= \underline{395.84}$$

10.

$$0 = 1 - x^2$$

$$x = \pm 1$$

$$V = 2\pi \int_{-1}^{1} (x+1)(1-x^2)\,dx$$

$$= 2\pi \int_{-1}^{1} (x - x^3 + 1 - x^2)\,dx$$

$$= 2\pi \int_{-1}^{1} (-x^3 - x^2 + x + 1)\,dx$$

$$= 2\pi \left[\frac{-x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2} + x \right]_{-1}^{1}$$

$$= 2\pi \left[\frac{1}{4} - \frac{1}{3} + \frac{1}{2} + 1 \right] - 2\pi \left[\frac{-1}{4} + \frac{1}{3} + \frac{1}{2} - 1 \right]$$

$$= 2\pi \left[\frac{2}{4} - \frac{2}{3} + 2 \right]$$

$$= 2\pi \left[\frac{6}{12} - \frac{8}{12} + \frac{24}{12} \right]$$

$$= \frac{2\pi}{12} \left[22 \right]$$

$$= \frac{\cancel{\cancel{44}}^{\,11}\pi}{\cancel{\cancel{12}}\,3}$$

$= \frac{\pi\pi}{3}$

$= \underline{11.52}$

Tutorial 2

1.

1. $u = x - 1$ $\quad u' = 1\,dx$

$dv = e^{-x}$ $\quad v = -e^{-x}$

$$-(x-1)e^{-x} + \int_0^1 e^{-x}\,dx$$

$$= \left[-(x-1)e^{-x} - e^{-x}\right]_0^1$$

$$= \frac{-1}{e} - \Big[$$

LIATE → Exponential

Trigonometric

Algebraic

Inverse Trigonometric

Logarithmic

$u \Rightarrow$ function whose category appears first

2. $u = \ln x$ $\quad u' = \frac{1}{x}\,dx$

$dv = x^{11}$ $\quad v = \frac{x^{12}}{12}$

$$\frac{x^{12}\ln x}{12} - \int \frac{x^{12}}{12} \times \frac{1}{x}\,dx$$

$$\frac{x^{12}\ln x}{12} - \frac{1}{12}\int x^{11}\,dx$$

$$= \frac{x^{12}\ln x}{12} - \frac{x^{12}}{12^2} + C$$

$$= \frac{x^{12}\ln x}{12} - \frac{x^{12}}{144} + C$$

$$= \frac{x^{12}}{12}\left(\ln x - \frac{1}{12}\right) + C$$

3. $\displaystyle\int_0^{\pi/4} x \sin(2x)\, dx$

$u = x \qquad u' = 1$

$dv = \sin 2x \qquad v = -\dfrac{\cos 2x}{2}$

$-\dfrac{x\cos 2x}{2} + \displaystyle\int \dfrac{\cos 2x}{2}\, dx$

$= \left[-\dfrac{x\cos 2x}{2} + \dfrac{1}{4}\sin 2x\right]_0^{\pi/4}$

$= \dfrac{1}{4}$

2. $\displaystyle\int \cos(\ln x)\, dx$

using substitution

$X = \ln x$

$dX = \dfrac{1}{x}\, dx \Rightarrow dX = x\, dx$

$e^X = e^{\ln x}$

$x = e^X$

$A = \displaystyle\int \cos X\, e^X\, dX$

$u = \cos X \qquad u' = \sin X$

$dv = e^X \qquad v = e^X$

$A = \cos X\, e^X - \displaystyle\int e^X \sin X\, dX$

$u = e^X \qquad u' = e^X$

$dv = \sin X \qquad v = -\cos X$

$A = \cos x\, e^{x} + e^{x} \sin x - \int e^{x} \cos x \, dx$

$2A = e^{x} (\cos x + \sin x)$

$A = \frac{1}{2} e^{x} (\cos x + \sin x)$

$= \frac{1}{2} e^{\ln x} (\cos(\ln x) + \sin(\ln x))$

$= \frac{x}{2} [\cos(\ln x) + \sin(\ln x)]$

$\therefore \int \cos(\ln x)\, dx = \frac{x}{2} [\cos(\ln x) + \sin(\ln x)]$