

## Faculty of Engineering and Information Sciences



**MATH 142,**      Quiz 1,      Spring 2020,      Duration: 60 minutes

Name: \_\_\_\_\_,

ID Number: \_\_\_\_\_



Time Allowed: 1 Hour

Total Number of Questions: 6

Total Number of Pages (incl. this page): 7

### EXAM UNAUTHORISED ITEMS

Students bringing these items to the examination room shall be required to leave the items at the front of the room or outside the examination room. The University does not guarantee the safe-keeping of students' personal items during examinations. Students concerned about the safety of their valuable items should make alternative arrangements for their care.

1. Bags, including carry bags, backpacks, shoulder bags and briefcases
2. Any form of electronic device including but not limited to mobile phones, smart watches, laptops, iPads, MP3 players, handheld computers and electronic dictionaries,
3. Calculator cases and covers
4. blank paper
5. Any written material

### DIRECTIONS TO CANDIDATES

1. Total marks: 40

2. All questions are compulsory.
3. Answer all questions on the given exam paper sheets.
4. Write your name and Id number on the papers provided for rough work.

**(6pts) Problem 1.**

Find the area of the region bounded by the graphs of

$$f(x) = 2 - x^2 \quad \text{and} \quad g(x) = x.$$

**Solution**

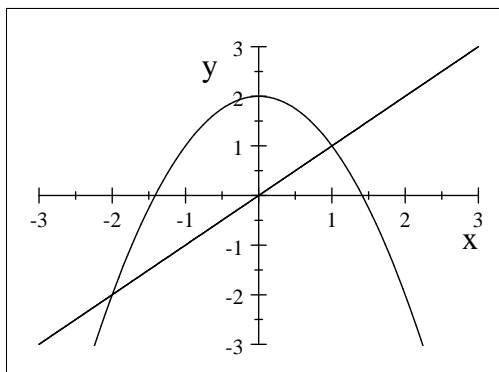
$$A = \int_a^b |f(x) - g(x)| dx.$$

$$2 - x^2 = x \Rightarrow x = -2 \text{ or } x = 1.$$

$$a = -2 \text{ and } b = 1. \quad \textbf{(2pts)}$$

$$\begin{aligned} A &= \int_{-2}^1 |-x^2 - x + 2| dx \\ &= \int_{-2}^1 (-x^2 - x + 2) dx \quad (\text{opposite sign of a inside of the root}) \quad \textbf{(2pts)} \\ &= \frac{9}{2} = 4.5. \quad \textbf{(2pts)} \end{aligned}$$

or by graphing without using absolute value



You see from the graph that the parabola is on top of the line between -2 and 1. So,

$$\begin{aligned} A &= \int_{-2}^1 [(2 - x^2) - x] dx \\ &= \int_{-2}^1 (-x^2 - x + 2) dx \\ &= \frac{9}{2} = 4.5. \end{aligned}$$

**(6pts) Problem 2**

Find the arclength of the function

$$f(x) = \frac{1}{2} (e^x + e^{-x})$$

on the interval  $[0, 2]$ .**Solution**

$$L = \int_0^2 \sqrt{1 + [f'(x)]^2} dx \quad (2pts)$$

$$f'(x) = \frac{1}{2} (e^x - e^{-x})$$

$$\begin{aligned} [f'(x)]^2 &= \left[ \frac{1}{2} (e^x - e^{-x}) \right]^2 \\ &= \frac{1}{4e^{2x}} + \frac{1}{4}e^{2x} - \frac{1}{2} \end{aligned}$$

$$\begin{aligned} 1 + [f'(x)]^2 &= \frac{1}{4e^{2x}} + \frac{1}{4}e^{2x} + \frac{1}{2} \\ &= \left[ \frac{1}{2} (e^x + e^{-x}) \right]^2. \end{aligned} \quad (2pts)$$

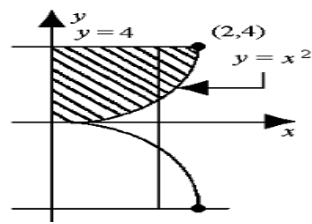
Hence,

$$\begin{aligned} L &= \int_0^2 \sqrt{\left[ \frac{1}{2} (e^x + e^{-x}) \right]^2} dx \\ &= \int_0^2 \frac{1}{2} (e^x + e^{-x}) dx \\ &= \frac{1}{2} (e^2 + e^{-2}) \\ &= 3.7622 \end{aligned} \quad (2pts)$$

**(6pts) Problem 3**

Sketch the region bounded by  $y = x^2$ ,  $y = 4$ , and  $x = 0$ , and use the disc method to find the volume of the solid generated by revolving the region about the  $x$ -axis.

**Solution**



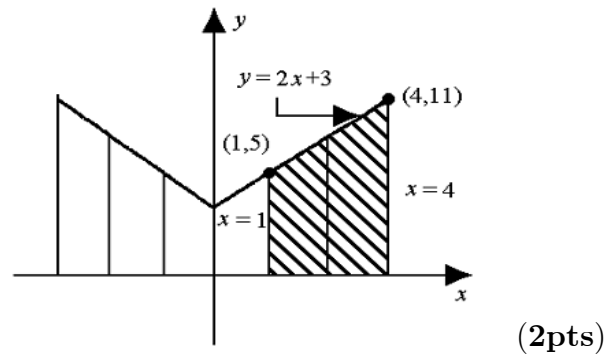
**(2pts)**

$$\begin{aligned} V &= \int_0^2 \left[ \pi 4^2 - \pi (x^2)^2 \right] dx && \textbf{(2pts)} \\ &= \int_0^2 (16\pi - \pi x^4) dx \\ &= \frac{128}{5} \pi = 80.425 && \textbf{(2pts)} \end{aligned}$$

**(6pts) Problem 4**

Sketch the region bounded by  $y = 2x + 3$ ,  $x = 1$ , and  $x = 4$ , and  $y = 0$ , and use the shell method to find the volume of the solid generated by revolving the region about the y-axis.

**Solution**



$$\begin{aligned} V &= \int_1^4 2\pi \cdot (x) \cdot (2x + 3) dx && \text{(2pts)} \\ &= 129\pi = 405.27. && \text{(2pts)} \end{aligned}$$

**(8pts) Problem 5**

Use trigonometric substitution to evaluate the integral

$$\int \frac{dx}{x^2\sqrt{4-x^2}}.$$

**Solution**

Put

$$x = 2 \sin \theta, \quad dx = 2 \cos \theta d\theta \quad \textbf{(2pts)}$$

$$\begin{aligned} \int \frac{dx}{x^2\sqrt{4-x^2}} &= \int \frac{2 \cos \theta d\theta}{4 \sin^2 \theta 2 \cos \theta} \\ &= \int \frac{d\theta}{4 \sin^2 \theta} \quad \textbf{(4pts)} \\ &= \frac{1}{4} \int \csc^2 \theta d\theta \\ &= -\frac{1}{4} \cot \theta + C \\ &= -\frac{1}{4} \frac{\sqrt{4-x^2}}{x} + C \quad \textbf{(2pts)} \end{aligned}$$

**(8pts) Problem 6**

Use partial fraction decomposition to evaluate the integrals

$$1. \int \frac{(2x+6) dx}{(x-1)(x-2)^2}$$

**Solution**

1.

$$\frac{x+3}{(x-1)(x-2)^2} = \frac{A}{x-1} + \frac{B_1}{x-2} + \frac{B_2}{(x-2)^2} \quad \textbf{(3pts)}$$

$\Rightarrow$

$$\frac{x+3}{(x-1)(x-2)^2} = \frac{4}{x-1} - \frac{4}{x-2} + \frac{5}{(x-2)^2} \quad \textbf{(3pts)}$$

Hence,

$$\begin{aligned} 2 \int \frac{(x+3) dx}{(x-1)(x-2)^2} &= \int \frac{8}{x-1} - \frac{8}{x-2} + \frac{10}{(x-2)^2} dx \\ &= 8 \ln|x-1| - 8 \ln|x-2| - \frac{10}{x-2} + C. \quad \textbf{(2pts)} \end{aligned}$$