

EXAMINATION COVERSHEET

Winter 2023 Final Examination



UNIVERSITY
OF WOLLONGONG
IN DUBAI

THIS EXAMINATION CONTENT IS STRICTLY CONFIDENTIAL
Students must comply with requirements stated in the Examination Policy & Procedures

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| Student Number: | |
| First Name: | |
| Family Name: | |
| | |
| Date of Examination: (DD/MM/YY) | 01/04/2023 |
| | |
| Subject Code: | Math 142 |
| Subject Title: | Essentials of Engineering Mathematics |
| Time Permitted to Write Exam: | 2 Hours |
| Total Number of Questions: | 11 (6 MCQ's + 5 written questions) |
| Total Number of Pages (including this page): | 9 |

INSTRUCTIONS TO STUDENTS FOR THE EXAM

1. Please note that subject lecturer/tutor will be unavailable during exams. *If there is a doubt in any of the exam questions i.e. problem solving etc. students should proceed by assuming values etc. Students should mention their assumption on the question paper.*
2. Answers must be written (and drawn) in black or blue ink
3. Any mistakes must be crossed out. Whitener and ink erasers must not be used.
4. Part A (MCQ): Answer ALL/ 6 questions. The marks for each question are shown next to each question. The total for Part A is 30 marks.
5. Part B (Written): Answer ALL/ 5 questions. The marks for each question are shown next to each question. The total for Part B is 70 marks.)
6. Total marks: 100. This Exam is worth 40% of your final marks for MATH 142.

EXAMINATION MATERIALS/AIDS ALLOWED

Approved Calculator and Formula Sheet

Exam Unauthorised Items – Students bringing these items to the examination room must follow the instructions of the invigilators with regards to these items.

7. Bags, including carrier bags, backpacks, shoulder bags and briefcases
8. Any form of electronic device including but not limited to mobile phones, smart watches, MP3 players, handheld computers and unauthorised calculators;
9. Calculator cases and covers, opaque pencil cases
10. Blank paper
11. Any written material

NOTE: The University does not guarantee the safe-keeping of students' personal items during examinations. Students concerned about the safety of their valuable items should make alternative arrangements for their care.

Part 1 MCQ's 30% (Circle Your Choice)

(5pts) Problem 1

Evaluate the improper integral

$$I = \int_6^8 \frac{4}{\sqrt{x-6}} dx$$

A) $I = 8$

B) $I = 8\sqrt{2}$

C) $I = 6$

D) $I = \infty$

E) $I = 2\sqrt{2}$

(5pts) Problem 2

Evaluate the improper integral

$$I = \int_{10}^{\infty} \frac{1}{x \ln x} dx$$

A) $I = 10 \ln 10$

B) $I = 100$

C) $I = \sqrt{10}$

D) $I = 10\sqrt{10}$

E) $I = \infty$

(5pts) Problem 3

Consider the differential equation

$$\frac{dy}{dx} = \frac{-2xy}{x^2 + y^2}$$

Which of the following is TRUE.

- A) The differential equation is linear
- B) The differential equation is separable
- C) The differential equation is homogeneous
- D) The differential equation is exact
- E) None of the above is true

(5pts) Problem 4

Let a_n be the sequence given by

$$\ln \frac{2}{1}, \ln \frac{3}{2}, \ln \frac{4}{3}, \dots$$

$\lim_{n \rightarrow \infty} a_n$ is equal to

- A) 0
- B) 1
- C) 2
- D) 3
- E) 4

(5pts) Problem 5

If

$$S = \frac{9}{10} + \frac{9}{10^2} + \frac{9}{10^3} + \dots,$$

then

A) $S = \infty$

B) $S = 1$

C) $S = 10000$

D) $S = 0.0001$

E) $S = 90909$

(5pts) Problem 6

If

$$\mathcal{L} = \sum_{n=1}^{\infty} \frac{1}{4n^2 - 1},$$

then

A) $\mathcal{L} = \frac{1}{4}$

B) $\mathcal{L} = \frac{1}{2}$

C) $\mathcal{L} = \infty$

D) $\mathcal{L} = 4$

E) $\mathcal{L} = 7$

Part 2 Written Questions (70%)

(16pts) Problem 1

Determine convergence or divergence of the following series.

$$(A) \sum_{n=0}^{\infty} (-1)^{n+1} \frac{2^n}{n!} \qquad (B) \sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{2n+1}$$

$$(C) \sum_{n=0}^{\infty} \frac{n+1}{\sqrt{n^2+n+1}} \qquad (D) \sum_{n=0}^{\infty} \frac{7}{n\sqrt{n}}$$

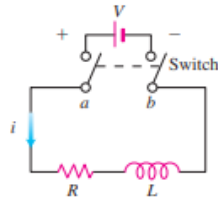
(14pts) Problem 2

Find the interval of convergence of the following power series

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}.$$

(12pts) Problem 3

The diagram in the Figure below represents an electrical circuit whose total resistance is a constant R ohms and whose self-inductance, shown as a coil, is L henries, also a constant. There is a switch whose terminals at a and b can be closed to connect a constant electrical source of V volts.



Ohm's Law, $V = RI$, has to be modified for such a circuit. The modified form is a linear differential equation given by

$$L \frac{di}{dt} + Ri = V$$

where i is the intensity of the current in amperes and t is the time in seconds. By solving this equation, we can predict how the current will flow after the switch is closed. If the switch is closed at time $t = 0$ ($i = 0$),

How will the current flow as a function of time if $\frac{R}{L} = 3$ and $\frac{V}{L} = 5$?

(14pts) Problem 4

Show that the differential equation is exact and solve the initial value problem

$$(\cos x - x \sin x + y^2) dx + 2xy dy = 0, \quad y(\pi) = 1.$$

(14pts) Problem 5

Show that the equation is Bernoulli and solve it.

$$\frac{dy}{dx} = y(xy^3 - 1)$$

Hint: $\int -3xe^{-3x} dx = \frac{1}{3}e^{-3x}(3x + 1) + C$