UNIVERSITY OF WOLLONGONG IN DUBAI

Faculty of Engineering and Information Sciences

Tutorial 5

Question 1

State whether the following equations are separable or not.

a. $\frac{dy}{dx} = y^2 x e^{3x+4y}$

$= y^2 x e^{3x} . e^{4y}$

$= x e^{3x} . y^2 e^{4y}$

$= g(x) . h(y) \longrightarrow$ separable

b. $3xy + (x-3)y' = 0$

$(x-3)\frac{dy}{dx} = -3xy$

$\frac{dy}{dx} = \frac{-3xy}{x-3}$

$\frac{dy}{dx} = \frac{-3x}{x-3} . y$

$= g(x) . h(y)$

separable

c. $\frac{dy}{dx} = y + sinx$

$\neq g(x) . h(y)$ not separable

Question 2

Show that the differential equation is separable and solve the equation.

$$(1+x)dy - ydx = 0$$

$$(1+x)\,dy = y\,dx$$

$$\frac{dy}{dx} = \frac{y}{1+x}$$

$$\frac{dy}{dx} = \frac{1}{1+x}\cdot y$$

$$\int \frac{dy}{y} = \int \frac{1}{1+x}\cdot dx$$

$$\ln|y| = \ln|1+x| + C_1$$

$$\ln|y| - \ln|1+x| = C_1$$

$$\ln\left|\frac{y}{1+x}\right| = C_1$$

$$\left|\frac{y}{1+x}\right| = e^{C_1}$$

$$\frac{y}{1+x} = \pm e^{C_1}$$

$$\frac{y}{1+x} = C$$

$$\boxed{y = C(1+x)}$$

Question 3

Show that the differential equation is separable and solve the equation.

$$2xy + 6x + (x^2 - 4)y' = 0$$

$$(x^2-4)\frac{dy}{dx} = -2xy - 6x$$

$$\frac{dy}{dx} = \frac{-2xy-6x}{x^2-4}$$

$$\frac{dy}{dx} = \frac{-2x(y+3)}{x^2-4}$$

$$\int \frac{dy}{y+3} = \int \frac{-2x}{x^2-4}\,dx$$

$$\ln|y+3| = -\ln|x^2-4| + C_1$$

$$\ln|y+3| + \ln|x^2-4| = C_1$$

$$\ln|(y+3)(x^2-4)| = C_1$$

$$|(y+3)(x^2-4)| = e^{C_1}$$

$$(y+3)(x^2-4) = \pm e^{C_1}$$

$$(y+3)(x^2-4) = C$$

$$y+3 = \frac{C}{x^2-4}$$

$$\boxed{y = \frac{C}{x^2-4} - 3}$$

Question 4

Solve the IVP (Initial value problem)

$$(e^{2y}-y)cos\frac{dy}{dx} = e^{y}sin2x \qquad\qquad y(0)=0$$

$$\frac{dy}{dx} = \frac{e^{y}\sin 2x}{(e^{2y}-y)\cos x}$$

$$= \frac{e^{y}\,2\sin x\,\cancel{\cos x}}{(e^{2y}-y)\,\cancel{\cos x}}$$

$$= \frac{e^{y}}{e^{2y}-y}\cdot 2\sin x$$

$$\frac{dy}{\frac{e^{y}}{e^{2y}-y}} = 2\sin x\,dx$$

$$\int \frac{e^{2y}-y}{e^{y}}\,dy = \int 2\sin x\,dx$$

$$\int y e^{-y}\,dy$$

D		I
y	$+$	e^{-y}
1	$-$	$-e^{-y}$
0		e^{-y}

$$\int (e^{y}-ye^{-y})\,dy = \int 2\sin x\,dx$$

$$e^{y}-(-ye^{-y}-e^{-y}) = -2\cos x + C$$

$$e^{y}+ye^{-y}+e^{-y} = -2\cos x + c$$

$$y(0)=0$$

$$e^{0}+0+e^{0} = -2\cos 0 + C$$

$$C = 4$$

$$\therefore\; e^{y}+ye^{-y}+e^{-y} = -2\cos x + 4$$

Question 5

Solve the linear equation $\frac{1}{x}\frac{dy}{dx} - 4y = 1$

$$\frac{dy}{dx} - 4xy = x \rightarrow \frac{dy}{dx} + P(x)y = \varphi(x)$$

$$P(x) = -4x \qquad \varphi(x) = x$$

$$\mu = e^{\int P(x)dx} = e^{\int -4x\,dx} = e^{-2x^2}$$

$$y = \frac{1}{e^{-2x^2}}\left[\frac{-1}{4}\int -4x \cdot e^{-2x^2}dx + C\right]$$

$$= e^{2x^2}\left[-\frac{1}{4}e^{-2x^2} + C\right]$$

$$= -\frac{1}{4} + Ce^{2x^2}$$

Question 6

Solve the linear equation $x\frac{dy}{dx} - 4y = x^6e^x$

① $\frac{dy}{dx} - \frac{4y}{x} = \frac{x^6e^x}{x}$

$P(x) = -\frac{4}{x}$ $\qquad Q(x) = \frac{x^6e^x}{x}$

$= x^5e^x$

② $\mu = e^{\int P(x)\,dx}$

$= e^{\int -\frac{4}{x}dx} = e^{-4\ln x}, \quad x > 0$

$= \cancel{e}^{\ln x^{-4}} = x^{-4}$

③ $y = x^4\left[\int (x^{-4})\, x^5e^x \, dx + c\right]$

$y = x^4\left[\int x\, e^x \,dx + c\right]$

D		I
x	$+$	e^x
1	$-$	e^x
0		e^x

$y = x^4\left[xe^x - e^x + c\right]$

$y = x^5e^x - x^4e^x + x^4C$

Question 7

Solve the IVP $y' + y = x$ $\quad y(0) = 4$

(1) $\frac{dy}{dx} + y = x$

$P(x) = 1 \quad Q(x) = x$

$\mu = e^{\int P(x)dx} = e^{\int 1 dx} = e^{x}$

(2) $y = \frac{1}{e^x}\left[\int (e^x)(x)\,dx + c\right]$

D	I
x +	e^x
1 −	e^x
0	e^x

$y = \frac{1}{e^x}\left[xe^x - e^x + C\right]$

$y = x - 1 + \frac{C}{e^x}$

$4 = 0 - 1 + \frac{C}{1}$

$4 = -1 + C$

$5 = C$

$\therefore \boxed{y = x - 1 + \frac{5}{e^x}}$