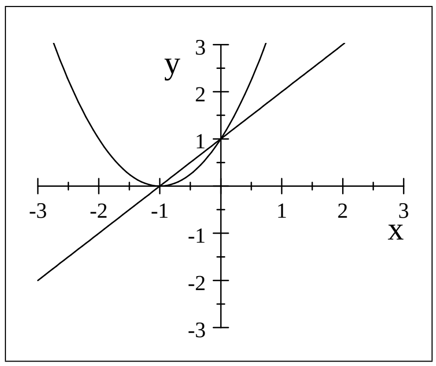
(8pts)Problem 1

Find the area of the region bounded by the curves

$$f(x) = x^2 + 2x + 1 \text{ and } \quad g(x) = x + 1$$

<u>Solution</u>



$$A = \int_a^b \left[(x+1) - \left(x^2 + 2x + 1\right)\right] dx. \quad [2 \textbf{ points}]$$

a and b are obtained fron the intersecting points.

$$x^2 + 2x + 1 = x + 1 \Leftrightarrow x^2 + x = 0 \Rightarrow x = 0 \text{ or } x = -1 \qquad [\textbf{3 points}]$$

$$\begin{aligned} A &= \int_{-1}^{0} \left(-x^2 - x\right) dx \\ &= \frac{1}{6}. \end{aligned} \qquad [\textbf{3 points}]$$

(8pts)Problem 2

Find the arc length of the graph of

$$f(x) = \frac{x^3}{6} + \frac{1}{2x}$$

on the interval $\left[\frac{1}{2}, 1\right]$.

<u>Solution</u>

$$L = \int_{1/2}^{1} \sqrt{1 + [f\,'(x)]^2}\,dx. \qquad [2\ \textbf{points}]$$

$$f\,'(x) = \frac{x^2}{2} - \frac{1}{2x^2}$$

$$\begin{aligned}
[f\,'(x)]^2 + 1 &= \left(\frac{x^2}{2} - \frac{1}{2x^2}\right)^2 + 1 \\
&= \frac{x^4}{4} - \frac{1}{2} + \frac{1}{4x^4} + 1 \\
&= \frac{x^4}{4} + \frac{1}{2} + \frac{1}{4x^4} \\
&= \left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^2. \qquad [2\ \textbf{points}]
\end{aligned}$$

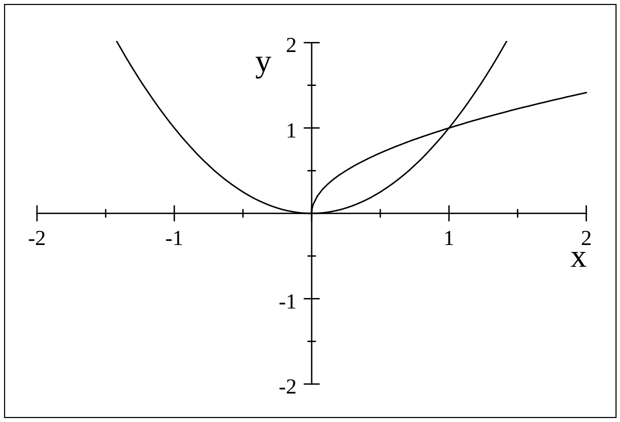
Hence,

$$\begin{aligned}
L &= \int_{1/2}^{1} \left(\frac{x^2}{2} + \frac{1}{2x^2}\right) dx \\
&= \frac{31}{48} = 0.645\,83 \qquad [4\ \textbf{points}]
\end{aligned}$$

(8pts) Problem 3

Use the disc (washer) method to find the volume of the solid formed by revolving the region bounded by the graphs of
$y = \sqrt{x}$ and $y = x^2$ about the x-axis.

Solution



[2 **points**]

$$V = \int_a^b \pi \left[\left(\sqrt{x}\right)^2 - \left(x^2\right)^2 \right] dx \qquad [4 \textbf{ points}]$$

$$\sqrt{x} = x^2 \Rightarrow a = 0 \text{ and } b = 1.$$

$$\begin{aligned} V &= \int_0^1 \pi \left(x - x^4\right) dx \\ &= \frac{3}{10}\pi \ . \end{aligned} \qquad [2 \textbf{ points}]$$

(8pts)Problem 4.
Find the area of the surface generated by revolving the parametric curve

$$x = \frac{1}{2}t^2 \quad \text{and} \quad y = \frac{1}{3}(2t+1)^{3/2}, \qquad 0 \le t \le 1.$$

about the y-axis.

Solution

$$\text{Area } = 2\pi \int_0^1 \frac{1}{2}t^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \qquad [2 \textbf{ points}]$$

$$\left(\frac{dx}{dt}\right)^2 = t^2, \qquad \left(\frac{dy}{dt}\right)^2 = 2t+1$$

$$\begin{aligned}
\text{Area} &= 2\pi \int_0^1 \frac{1}{2}t^2 \sqrt{t^2+2t+1}dt \\
&= 2\pi \int_0^1 \frac{1}{2}t^2 \sqrt{(t+1)^2}dt \\
&= 2\pi \int_0^1 \frac{1}{2}t^2 \ (t+1)\, dt \qquad [4 \textbf{ points}] \\
&= \frac{7}{12}\pi = 1.8326 \qquad [2 \textbf{ points}]
\end{aligned}$$

(8pts)Problem 5.
Find the slope of the line that is tangent to the polar curve

$$r = 3\sin\theta$$

at $\theta = \dfrac{\pi}{2}$.

Solution
We have

$$x = 3\sin\theta\cos\theta = \frac{3}{2}\sin 2\theta \quad \text{and} \qquad y = 3\sin^2\theta \qquad [2 \textbf{ points}]$$

$$\begin{aligned} \text{Slope} &= \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} \\ &= \frac{6\cos\theta\sin\theta}{3\cos 2\theta} \qquad [4 \textbf{ points}] \end{aligned}$$

The slope at $\theta = \dfrac{\pi}{2}$ is

$$\text{Slope } = \frac{0}{-1} = 0. \qquad [2 \textbf{ points}]$$