1. $2\frac{dy}{dx} = (y-1)e^x$

$\frac{dy}{y-1} = e^x \, dx$

$\int \frac{dy}{y-1} = \int \frac{e^x}{2} \, dx$

$\ln|y-1| = \frac{e^x}{2} + c$

$y-1 = e^{\frac{e^x}{2}+c}$

$y = 1 + e^{\frac{e^x}{2}+c}$

3. $\frac{dy}{dx} + \frac{y}{x} = \sin x$

$IF = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

$y = \frac{1}{IF} \int Q(x) \, IF \, dx$

$= \frac{1}{x} \int x \sin x \, dx$

$\frac{dy}{dx} = \frac{(y+3)(x-1)}{(y-2)(x+4)} = \left(\frac{y+3}{y-2}\right)\left(\frac{x-1}{x+4}\right)$

$\int \frac{y-2}{y+3} \, dy = \int \frac{x-1}{x+4} \, dx$

$$\int \frac{y+3-5}{y+3}\,dy = \int \frac{x+4-5}{x+4}\,dx$$

$$\int 1 - \frac{5}{y+3}\,dy = \int 1 - \frac{5}{x+4}\,dx$$

$$y - 5\ln|y+3| = x - 5\ln|x+4| + c$$

$$y - 5\ln|y+3| - x + 5\ln|x-4| = c$$

$$c = y - x + 5\ln\left|\frac{x+4}{y+3}\right|$$

$$x\frac{dy}{dx} - 2y = x^2$$

$$\frac{dy}{dx} - \frac{2y}{x} = x$$

$$\frac{dy}{dx} - P(x)\,y = Q(x)$$

$$IF = e^{\int P(x)\,dx}$$

$$= e^{-\int 2/x\,dx} = e^{-\ln x^2}$$

$$= -x^2$$

$$y = \frac{1}{IF}\int Q\,IF\,dx$$

$$= x^2 \int x\,x^{-2}\,dx$$

$$= x^2 \int \frac{1}{x}\,dx$$

$$= x^2(\ln x + c)$$

$$= x^2 \ln x + x^2 c$$

$y(1) = 3$

$3 = 1(0 + c)$

$c = 3$

$y = x^2 \ln x + 3x^2$

$(1 + 2x - y^3)\,dx + (2y - 3xy^2)\,dy = 0$

$M_y = -3y^2$

$N_x = -3y^2$

$M_y = N_x = -3y^2$

$\therefore$ eq is exact

$f_x = 1 + 2x - y^3$

$f(x,y) = \int 1 + 2x - y^3 \, dx + c(y)$

$= x + x^2 - xy^3 + c(y)$

$f_y = -3xy^2 + c'(y)$

$2y - \cancel{3xy^2} = \cancel{-3xy^2} + c(y)$

$c'(y) = 2y$

$c(y) = y^2$

$f(x,y) = x + x^2 - xy^3 + y^2 + c$

$x + x^2 - xy^3 + y^2 = c$

$$\sum_{n=1}^{\infty} \frac{n!}{3^n}$$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(n+1)!}{3^{n+1}} \times \frac{3^n}{n!} \right|$$

$$= \lim_{n\to\infty} \left| \frac{n+1}{3} \right|$$

$$= \infty > 1$$

$\therefore$ series diverges by ratio test

$$\sum_{n=1}^{\infty} \left(\frac{3n+1}{\pi n+3} \right)^n$$

Root Test

$$\lim_{n\to\infty} |a_n|^{1/n}$$

$$= \lim_{n\to\infty} \left| \frac{3n+1}{\pi n+3} \right|$$

$$= \lim_{n\to\infty} \frac{3\cancel{n}}{\pi \cancel{n}}$$

$$= \lim_{n\to\infty} 3/\pi$$

$$= 3/\pi < 1$$

$\therefore$ Converges by root test

$$S - S_N \leq \int \frac{1}{} \leq 0.009$$

$$\int \frac{}{n^4}$$

$$\frac{1}{3n^3} \leq 0.009$$

$$\frac{1}{n^3} \leq 0.027$$

$$n^3 \geq 37.037$$

$$n \geq \sqrt[3]{37.037}$$

$$n \geq 3.3$$

$$n \geq 4$$

$$\frac{dy}{dx} = \frac{-2xy}{x^2+y^2}$$

$$(x^2+y^2)dy + 2xy\,dx = 0$$

$$M_y = 2x$$

$$N_x = 2x$$

$$M_y = N_x$$

$\therefore$ Eq is exact

$$\frac{dy}{dx} = \frac{-2xy}{x^2+y^2}$$

$$= \left(\frac{x(-2y)}{x^2\left(1+\left(\frac{y}{x}\right)^2\right)}\right)$$

$$= \frac{1}{x}\left(\frac{-2y}{1+(y/x)^2}\right)$$

$$= \frac{-2(y/x)}{}$$

$(1+(9/n))$

$\Rightarrow$ Homogeneous

$$\lim_{n \to \infty} \ln \left| \frac{n+1}{n} \right|$$

$$= \ln \left| \frac{n}{n} \right|$$

$$= \ln(1)$$

$$= 0$$

$$S = \frac{9}{10} \left(\frac{1}{10} \right)^{n-1}$$

$$S = \frac{a}{1-r} = \frac{9/10}{1-1/10} = \frac{9/10}{9/10} = 1$$

$$\alpha = \sum_{n=1}^{\infty} \frac{1}{4n^2-1}$$

$$= \sum_{n=1}^{\infty} \frac{1}{(2n+1)(2n-1)}$$

$1 = A(2n+1) + B(2n-1)$

@ $n = -1/2$ @ $n = 1/2$

$A = 1/2$

$B = -1/2$

$$\sum_{n=1}^{\infty} \frac{1}{2(2n-1)} - \frac{1}{2(2n+1)}$$

$$= \lim_{N \to \infty} \frac{1}{2} \sum_{n=1}^{N} \left(\frac{1}{2n-1} - \frac{1}{2n+1} \right)$$

$$= \lim_{n\to\infty} \frac{1}{2}\left(\frac{1}{1} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{5}} \cdots \cancel{+} \frac{1}{2n+1}\right)$$

$$= \lim_{n\to\infty} \frac{1}{2}\left(1 - \frac{1}{2n+1}\right)$$

$$= \frac{1}{2}(1-0)$$

$$= 1/2$$

$$\sum_{n=0}^{\infty} (-1)^{n+1} \frac{2^n}{n!}$$

Ratio test

$$\lim_{n\to\infty} \left| \frac{\cancel{(-1)^{n+1}}(-1)\cancel{2^n} \times 2}{(n+1)\cancel{n!}} \times \frac{\cancel{n!}}{\cancel{(-1)^{n+1}}\,\cancel{2^n}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{2}{n+1} \right|$$

$$= 0 < 1$$

$\therefore$ Converges by ratio test

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{2n+1}$$

Using AST

$b_n = \dfrac{1}{2n+1}$

$b_n > 0 \;\therefore$

$b_{n+1} > b_n$

$$\lim_{n\to\infty} \frac{1}{2n+1} = 0$$

$$= \sum_{n=0} \frac{(-1)^{n+1}}{2n+1} \quad \text{converges by AST}$$

$$\sum_{n=0}^{\infty} \frac{n+1}{\sqrt{n^2+n+1}}$$

Using divergence test

$$\lim_{n\to\infty} \frac{n+1}{\sqrt{n^2+n+1}}$$

$$= \lim_{n\to\infty} \frac{n}{\sqrt{n^2}}$$

$$= \lim_{n\to\infty} \frac{n}{n}$$

$$= 1 \neq 0$$

$\therefore$ Diverges by divergence test

$$\sum_{n=0}^{\infty} \frac{7}{n^{3/2}}$$

$$p = 3/2 > 1$$

$\therefore$ converges by p-series

$$L\frac{di}{dt} + Ri = V$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}$$

$$\frac{di}{dt} + 3i = 5$$

$$IF = e^{\int P(t)\,dt} = e^{\int 3\,dt} = e^{3t}$$

$$i = \frac{1}{e^{3t}} \int 5e^{3t}\, dt$$

$$= \frac{5}{e^{3t}} \int e^{3t}\, dt$$

$$= \frac{5e^{-3t}}{3} \left[e^{3t} + c\right]$$

$$= \frac{5}{3} + ce^{-3t}$$

@ $t=0$, $i=0$

$$0 = \frac{5}{3} + c$$

$$c = -5/3$$

$$i = \frac{5}{3} - \frac{5}{3e^{3t}}$$

$$(\cos x - x\sin x + y^2)\, dx + 2xy\,dy = 0$$

$$M_y = 2y$$
$$N_x = 2y$$

$$M_y = N_x$$

$\therefore$ Eq is exact

$$f_y = 2xy$$

$$f(x,y) = \int 2xy\, dy + c(x)$$

$$= xy^2 + c(x)$$

$f_x = y^2 + C'(x)$

$\cos x - x\sin x + \cancel{y^2} = \cancel{y^2} + C'(x)$

$C'(x) = \cos x - x\sin x$

$C(x) = \int (\cos x - x\sin x)\,dx$

$= \sin x - \int x\sin x\,dx$

$= \sin x - [-x\cos x + \sin x]$

$= \sin x + x\cos x - \sin x$

$= x\cos x$

D	I
x	$\sin x$
1	$-\cos x$
0	$-\sin x$

$f(x,y) = c$

$xy^2 + x\cos x = c$

$y(\pi) = 1$

$\pi(1)^2 + \pi\cos\pi = c$

$xy^2 + x\cos x = 0$

$\frac{dy}{dx} = y(xy^3 - 1)$

$\frac{dy}{dx} = xy^4 - y$

$\frac{dy}{dx} + y = xy^4$

$y^{-4}y' + y^{-3} = x$

$u = y^{-3}$

$du = -3y^{-4}\,dy$

$$\frac{du}{dx} = -3y \quad \frac{dy}{dx}$$

$$y^{-4}\frac{dy}{dx} = -\frac{1}{3}\frac{du}{dx}$$

$$-\frac{1}{3}\frac{du}{dx} + u = x$$

$$\frac{du}{dx} - 3u = -3x$$

$$IF = e^{\int -3\,dx} = e^{-3x}$$

$$u = e^{3x}\int (-3x)e^{-3x}\,dx$$

$$= e^{3x}\left(\frac{1}{3}e^{-3x}(3x+1) + c\right)$$

$$= e^{3x}\left(\frac{\cancel{3}x e^{-3x}}{\cancel{3}} + \frac{e^{-3x}}{3} + c\right)$$

$$u = x + \frac{1}{3} + ce^{3x}$$

$$= \frac{1}{3}(3x+1) + ce^{3x}$$

$$y^{-3} = \frac{1}{3}(3x+1) + ce^{3x}$$

$$y = \frac{1}{\sqrt[3]{\frac{(3x+1)}{3} + ce^{3x}}}$$