MATH142

Essentials of Engineering Mathematics

First-Order ODEs

I. Separable
II. Homogeneous
III. Linear
IV. Exact
V. Bernoulli

I. Separable Equations

Definition: A first-order ODE $\frac{dy}{dx} = f(x, y)$ is separable if it can be written in the form $\frac{dy}{dx} = g(x).h(y)$

Example 1: State whether the following equations are separable or not.

a. $\frac{dy}{dx} = y^2 x e^{3x+4y}$

b. $3xy + (x-3)y' = 0$

c. $\frac{dy}{dx} = y + sinx$

Steps for Solving Separable Equations

1. Write the DE as

$$\frac{dy}{dx} = f(x, y) = g(x).h(y)$$

2. Separate the variables along with their corresponding differentials

$$\frac{dy}{h(y)} = g(x)dx$$

3. Integrate both sides

$$\int \frac{dy}{h(y)} = \int g(x)dx \Rightarrow H(y) = G(x) + C$$

4. Solve for y in terms of x, if possible

Example 2: Solve the following differential equation.

$$(1 + x)d - ydx = 0$$

Example 3: Show that the differential equation $2xy + 6x + (x^2 - 4)y' = 0$ is separable and solve the equation.

Example 4: Solve the IVP (Initial value problem)

$$(e^{2y} - y)cos\frac{dy}{dx} = e^{y}sin2x \qquad y(0) = 0$$