Math 1441 Exam 2 Sprg 19

1. $$\int_e^{\infty} \frac{dx}{x(\ln x)^2}$$

$$= \lim_{t\to\infty} \int_e^{t} \frac{dx}{x(\ln x)^2}$$

$u = \ln x$ $\quad\quad$ $x = e \quad u = \ln e = 1$

$du = \frac{1}{x}\,dx$ $\quad\quad$ $x = t \quad u = \ln t$

$$= \lim_{t\to\infty} \int_1^{\ln t} \frac{du}{u^2}$$

$$= \lim_{t\to\infty} \left[-\frac{1}{u}\right]_1^{\ln t}$$

$$= \lim_{t\to\infty} \left[\frac{-1}{\ln t} + 1\right]$$

$$= \frac{-1}{\infty} + 1$$

$$= \underline{\underline{1}}$$

2. $$\int_{-2}^{2} \frac{dx}{x+1}$$

$$= \int_{-2}^{-1} \frac{dx}{x+1} + \int_{-1}^{2} \frac{dx}{x+1}$$

$$\int^{-1} dx$$

$$\int_{-2} \frac{dx}{x+1}$$

$$= \lim_{t \to -1} \int_{-2}^{t} \frac{dx}{x+1}$$

$$= \left[\ln|x+1|\right]_{-2}^{t}$$

$$= \ln|t+1| - \ln|-1|$$

$$= -\infty$$

3. $$\int_{0}^{\infty} x e^{-x^2}$$

$$= \lim_{t \to \infty} \int_{0}^{t} x e^{-x^2}\, dx$$

$$= \lim_{t \to \infty} \frac{-1}{2} \int_{0}^{t} -2x e^{-x^2}\, du$$

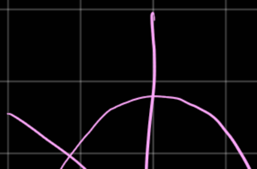
$$\lim_{t \to \infty} \frac{-1}{2} e^{-x^2} \Big|_{0}^{t}$$

$$= \frac{-1}{2}\left[e^{-t^2} - 1\right]$$

$$= \frac{-1}{2}\left[0 - 1\right]$$

$$= 1/2$$

Exam 1 Review



1.

$y = 3 - x^2$

$y = 1 - x$

$$\int_0^2 (3 - x^2 - 1 + x)\,dx$$

$$= \int_0^2 (2 - x^2 + x)\,dx$$

$$= \left(2x - \frac{x^3}{3} + \frac{x^2}{2}\right)_0^2$$

$$= \frac{10}{3}$$

1.

$x = 3y - y^2$

$x = 3 - y$

$3y - y^2 = 3 - y$

$3y - y^2 - 3 + y = 0$

$-y^2 + 4y - 3 = 0$

$y^2 - 4y + 3 = 0$

$y(y-3) - 1(y-3) = 0$

$y = 3$ or $y = 1$

$$\text{Area} = \int_1^3 3y - y^2 - 3 + y \quad dy$$

$$= \int_1^3 -y^2 + 4y - 3 \quad dy$$

$$= -\frac{y^3}{3} + 2y^2 - 3y \Big|_1^3$$

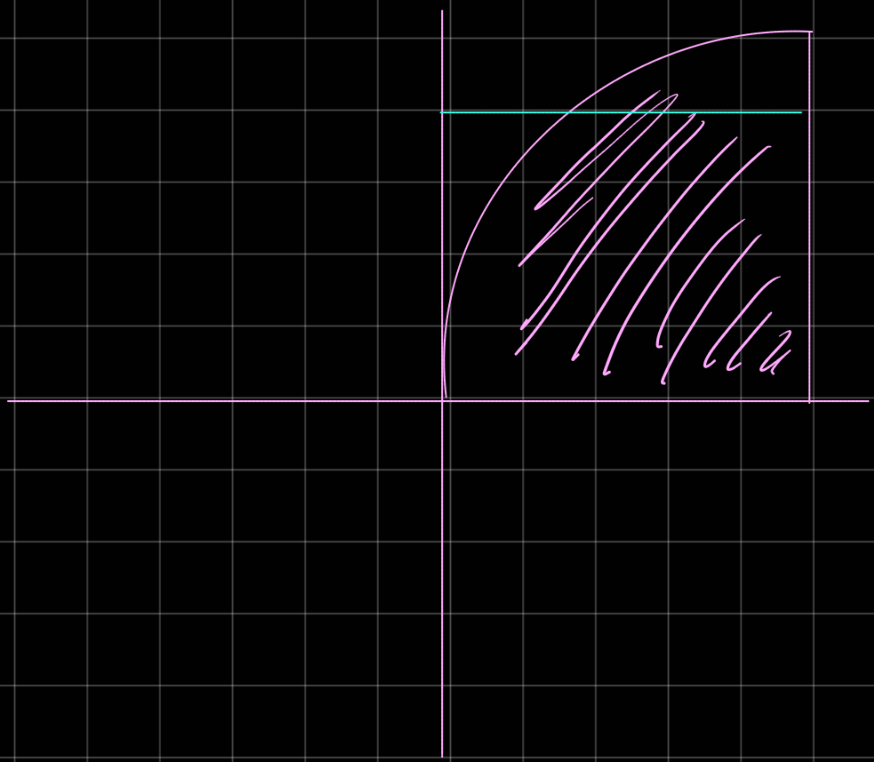
$$= \frac{4}{3}$$

2. $$V = \int_0^4 \pi (4-x)^2 \, dx$$

$$= \pi \int_0^4 16 + x^2 - 8x \, dx$$

$$= \pi \int_0^4 16x + \frac{x^3}{3} - 4x^2$$

$$= 67.021 \text{ or } \frac{64\pi}{3}$$



2.

$y^2 = 4x$

$x = 4$

$y^2 = 16$

$y = \pm 4$

$$V = \pi \int_{-4}^{4} 16 - \left(\frac{y^2}{4}\right)^2 \, dy$$

$$= \pi \int_0^4 16 - \frac{y^4}{16} \, dy$$

$$= \pi \left[16y - \frac{y^5}{80}\right]_0^4$$

$$= \pi\left(64 - \frac{64}{5}\right)$$

$$= \frac{256\pi}{5}$$

3. $y = \sqrt{x+1}$ $\quad x=0 \quad y=0 \quad x=3$

$$V = 2\pi\int_0^3 x\sqrt{x+1}\,dx$$

$u = x+1 \qquad x=0 \quad u=1$

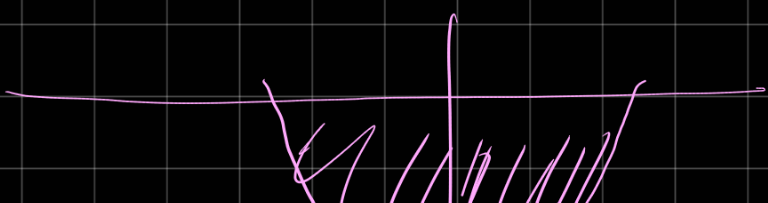
$du = 1 \qquad x=3 \quad u=4$

$$V = 2\pi\int_1^4 (u-1)\sqrt{u}\,du$$

$$= 2\pi\int_1^4 u^{3/2} - u^{1/2}\,du$$

$$= 2\pi\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right)\Bigg|_1^4$$

$$= \frac{232}{15}\pi$$

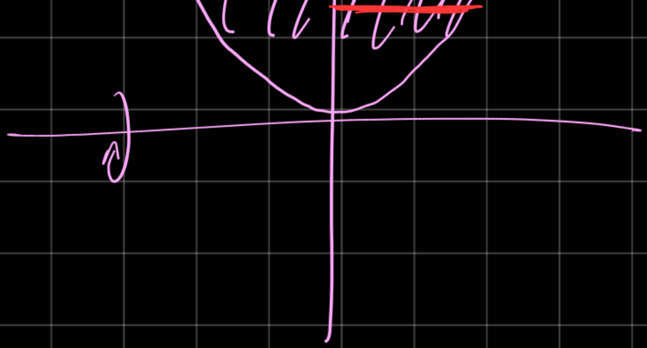


2.

$x^2 = 4$

$x = \pm 2$

$0 \leq x \leq 2$



$$V = 2\pi \int_0^2 y\left(\sqrt{y}\right)\, dx$$

$$= 2\pi \frac{2}{5} y^{5/2} \Big|_0^2$$

$$= \frac{128}{5}\pi$$

$$\int 9\sqrt{x}\ \ln x\ dx$$

$u = \ln x \qquad u' = 1/x$

$v' = \sqrt{x} \qquad v = \frac{2}{3} x^{3/2}$

$$I = \frac{2}{3} x^{3/2} \ln x - \int \frac{2}{3} x^{1/2}\ dx$$

$$= 9\left(\frac{2}{3} x^{3/2} \ln x - \frac{4}{9} x^{3/2} + C\right)$$

$$= 6x^{3/2} \ln x - 4x^{3/2} + C$$

$3 - y^2 = y + 1$

$0 = y^2 + y + 1 - 3$

$= y^2 + 2y - y - 2 = 0$

$y(y+2) - 1(y+2) = 0$

$y = 1 \quad y = -2$

$$\int_{-2} y^2+y-2$$

$$= \frac{y^3}{3} + \frac{y^2}{2} - 2y \Big|_{-2}^{1}$$

$$= \left(\frac{1}{3} + \frac{1}{2} - 2\right) - \left(\frac{-8}{3} + \frac{4}{2} + 4\right)$$

$$= 9/2$$

$y = 7/x^2$ axis $x = -1/2$

$$V = 2\pi \int_1^5 \frac{7}{x^2}\left(x + 1/2\right)\,dx$$

$$= 2\pi \int_1^5 \left(\frac{7}{x} + \frac{7}{2x^2}\right)dx$$

$$= 2\pi \left(7\ln x - \frac{7}{2x}\right)\Big|_1^5$$

$$= 88.3797$$