2.2. $$\int_0^{\infty} \frac{e^x}{1+e^x}\,dx$$

$$= \lim_{t\to\infty} \int_0^{t} \frac{e^x}{1+e^x}\,dx$$

$$= \lim_{t\to\infty} \Big[\ln|1+e^x|\Big]_0^t$$

$$= \lim_{t\to\infty} \ln|1+e^t| - \ln 2$$

$$= \lim_{t\to\infty} \ln|1+e^{\infty}| - \ln 2$$

$$= \infty$$

Diverges

6.1. $$\lim_{t\to\infty} \int_0^{t} \frac{x}{1+x^2}\,dx$$

$$= \lim_{t\to\infty} \frac{1}{2}\Big[\ln|x^2+1|\Big]_0^t$$

$$= \lim_{t\to\infty} \frac{1}{2} \ln|t^2+1|$$

$$= \ln|\infty|$$

$$= \infty$$

Diverges

2. $$\int_{-2}^{14} \frac{dx}{\sqrt[4]{x+2}}$$

$$= \lim_{t\to -2^+} \int_t^{14} (x+2)^{-1/4}\,dx$$

$$= \lim_{t \to -2^+} \frac{4}{3} \left[(x+2)^{3/4} \right]_t^{14}$$

$$= \lim_{t \to -2^+} \frac{4}{3} \left[16^{3/4} - (t+2)^{3/4} \right]$$

$$= \lim_{t \to -2^+} \frac{32}{3} - (-2+2)^{3/4}$$

$$= \frac{32}{3}$$

$\therefore$ converges at $\frac{32}{3}$

Tutorial 4

1. $$\frac{dx}{dt} = 3e^t$$

$$\frac{dy}{dt} = -5e^{-t}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$= \frac{-5e^{-t}}{3e^t}$$

@ $t = 0$

$$\frac{dy}{dx} = \frac{-5}{3}$$

$x = 3e^t = 3$

$y = -5e^{-t} = 5$

$y = m(x - x_1) + y_1$

$y = \frac{-5}{3}(x - 3) + 5$

$$3$$

$$= -\frac{5x}{3} + 10$$

2. $r = 4\cos\theta - 5\sin\theta$

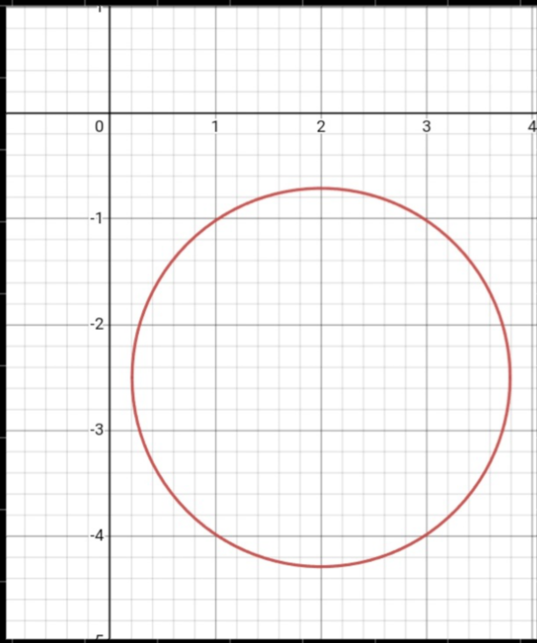
$r^2 = 4r\cos\theta - 5r\sin\theta$

$x^2 + y^2 = 4x - 5y$

$x^2 - 4x + y^2 + 5y = 0$

$(x-2)^2 + \left(y + \frac{5}{2}\right)^2 - 4 - \frac{25}{4} = 0$

$(x-2)^2 + \left(y + \frac{5}{2}\right) = \frac{41}{4}$



3. $S = 2\pi \int_a^b x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$\frac{dx}{dt} = t \qquad \frac{dy}{dt} = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{2}} (2t+1)^{1/2} \times \cancel{2}$

$= (2t+1)^{1/2}$

$$S = \pi \int_0^1 t^2 \sqrt{t^2 + (2t+1)}\, dt$$

$$= \pi \int_0^1 t^2 \sqrt{t^2 + 2t + 1}\, dt$$

$$= \pi \int_0^1 t^2 \sqrt{(t+1)^2}\, dt$$

$$= \pi \int_0^1 t^2 (t+1)\, dt$$

$$= \pi \int_0^1 t^3 + t^2\, dt$$

$$= \left[\frac{t^4}{} \quad \frac{t^3}{}\right]_{}^{1}$$

$$= \pi \left[\frac{t^4}{4} + \frac{t^3}{3} \right]_0$$

$$= \pi \left(\frac{1}{4} + \frac{1}{3} \right)$$

$$= \underline{1.8326}$$

4. $x = r\cos\theta = 3\sin\theta\cos\theta = \frac{3}{2}\sin 2\theta$

$y = r\sin\theta = 3\sin^2\theta$

$$\frac{dy}{d\theta} = 6\sin\theta\cos\theta$$

$$\frac{dx}{d\theta} = \frac{3}{\cancel{2}} \times \cos 2\theta \times \cancel{2}$$

$$= 3\cos 2\theta$$

$$m = \frac{dy/d\theta}{dx/d\theta} = \frac{6\sin\theta\cos\theta}{3\cos 2\theta}$$

@ $\theta = \pi/2$

$$= \frac{6\sin(\pi/2)\cos(\pi/2)}{3\cos(\pi)}$$

$$= \underline{0}$$

5. $\frac{dx}{dt} = e^t - 1$ $\qquad \frac{dy}{dt} = 4e^{t/2} \times \frac{1}{\cancel{2}}$

$$= 2e^{t/2}$$

$\left(\frac{dx}{dt}\right)^2 = e^{2t} + 1 - 2e^t$ $\qquad \left(\frac{dy}{dt}\right)^2 = 4e^t$

$$S = 2\pi \int^1 4e^{t/2} \sqrt{e^{2t} + 1 - 2e^t + 4e^t}\, dt$$

$$= 8\pi \int_0^1 e^{t/2} \sqrt{e^{2t} + 1 + 2e^t}$$

$$= 8\pi \int_0^1 e^{t/2} \sqrt{(e^t+1)^2} \, dt$$

$$= 8\pi \int_0^1 e^{t/2} (e^t + 1) \, dt$$

$$= 8\pi \int_0^1 e^{3t/2} + e^{t/2} \, dt$$

$$= 8\pi \left[\frac{2}{3} e^{3t/2} + \frac{1}{2} e^{t/2} \right]_0^1$$

$$= \underline{90.9445}$$

6. $x = r\cos\theta$

$= e^{2\theta} \cos\theta$

$y = r\sin\theta$

$= e^{2\theta} \sin\theta$

$$\frac{dx}{dt} = 2e^{2\theta}\cos\theta - e^{2\theta}\sin\theta$$

$$\frac{dy}{dt} = 2e^{2\theta}\sin\theta + e^{2\theta}\cos\theta$$

$$\frac{dy}{dx} = \frac{2e^{2\theta}\sin\theta + e^{2\theta}\cos\theta}{2e^{2\theta}\cos\theta - e^{2\theta}\sin\theta}$$

@ $\theta = 0$

$$\frac{dy}{dx} = \frac{1}{2}$$

$$x = e^0 \cdot \cos 0 = 1$$

$$y = e^0 \cdot \sin 0 = 0$$

$$y = \frac{x}{2} - \frac{1}{2}$$

7. Arc Length (Polar Curve)

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$L = \int_0^{\ln 2} \sqrt{e^{2\theta} + e^{2\theta}}\, d\theta$$

$$L = \int_0^{\ln 2} \sqrt{2e^{2\theta}}\, d\theta$$

$$= \int_0^{\ln 2} e^{\theta}\sqrt{2}\, d\theta$$

$$= \sqrt{2}\left[e^{\theta}\right]_0^{\ln 2}$$

$$= \sqrt{2}\,[2 - 1]$$

$$= \sqrt{2}$$