MATH142

Essentials of Engineering Mathematics

First-Order ODEs

I. Separable
II. Homogeneous
III. Linear
IV. Exact
V. Bernoulli

II. Homogeneous Equations

Definition: A first-order ODE $\frac{dy}{dx} = f(x, y)$ is homogeneous if it can be written in the form $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$

Example 1: State whether the following equations are homogeneous or not.

a. $\frac{dy}{dx} = \frac{x^2 - 3y^2}{2xy}$

b. $\frac{dy}{dx} = \frac{x+y}{x-y}$

c. $(y^4 + 3xy)dx - x^2dy = 0$

Steps for Solving Homogeneous Equations

1. Write the DE as

   $$\frac{dy}{dx} = f(x, y) = F\left(\frac{y}{x}\right) \qquad (I)$$

2. Let $v = \frac{y}{x} \implies y = xv$

3. $y = xv \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ (differentiate y w.r.t x)

4. Substitute $\frac{y}{x} = v$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in (I) we get:

   $$v + x\frac{dv}{dx} = F(v) \qquad (II)$$

5. Equation (II) is separable. Proceed and solve for v.

6. Substitute back $v = \frac{y}{x}$ and find y in terms of x if possible.

Example 2: Solve the following differential equation.

$$xy' = \frac{y^2}{x} + y$$

Example 3: Show that the differential equation is homogeneous and solve it.

$$(x^2 - 3y^2)dx + 2xydy = 0$$

Example 4: Solve $\frac{dy}{dx} = \frac{x-2y}{x}$