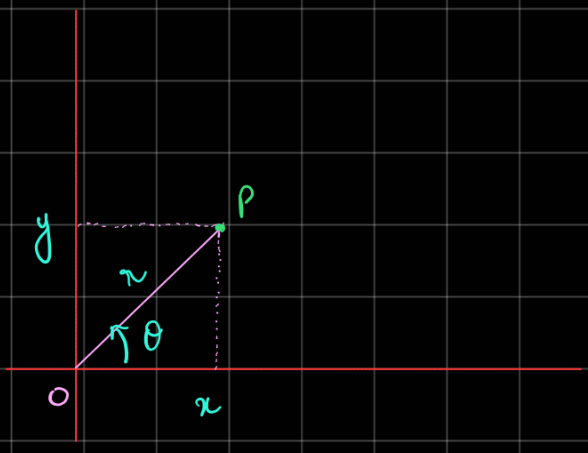
Polar $\longrightarrow$ Rectangular Conversion

$\cos\theta = \frac{x}{r}$ $\qquad$ $\sin\theta = \frac{y}{r}$

$x = r\cos\theta$ $\qquad$ $y = r\sin\theta$



If you know r and θ, then

$x = r\cos\theta$

$y = r\sin\theta$

Example

Find the rectangular coordinates of the point $\left(\frac{3}{2}, \frac{\pi}{4}\right)$ given in polar coordinates

$r = 3/2 \qquad \theta = \pi/4$

$x = r\cos\theta = \frac{3}{2} \times \frac{1}{\sqrt{2}} = \frac{3}{2\sqrt{2}}$

$y = r\sin\theta = \frac{3}{2} \times \frac{1}{\sqrt{2}} = \frac{3}{2\sqrt{2}}$

If you know the rectangular coordinates x and y, then

$r^2 = x^2 + y^2 \longrightarrow r = \pm\sqrt{x^2+y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Example

Find the polar coordinates of the points $(-1,1)$ and $(0,2)$ given in rectangular coordinates.

$(-1,1)$

$r = \sqrt{(-1)^2 + (1)^2} = \pm\sqrt{2}$

$\theta = \tan^{-1}\left(\frac{1}{-1}\right) = \frac{3\pi}{4}$ OR $\frac{7\pi}{4}$

If $r = \sqrt{2}$, $\theta = 3\pi/4$

If $r = -\sqrt{2}$, $\theta = 7\pi/4$

$(-1,1)$ $\left(\sqrt{2}, \frac{3\pi}{4}\right)$ $\left(-\sqrt{2}, \frac{7\pi}{4}\right)$

$(0,2)$

$r = \pm\sqrt{4} = \pm 2$

$\theta = \tan^{-1}\left(\frac{2}{0}\right) \Rightarrow \theta = \frac{\pi}{2}$ OR $\theta = \frac{3\pi}{2}$

If $r = 2$, $\theta = \pi/2$

If $r = -2$, $\theta = 3\pi/2$

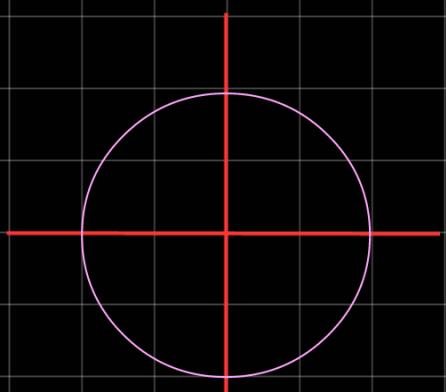
Polar Curve

In polar coordinates, the graph of $r = f(\theta)$ is called a polar curve.

Example

Sketch the following polar curves

1. $r = 2$



$r = 2$

$r^2 = 4$

$x^2 + y^2 = 4$

Circle centered at $(0,0)$ with radius 2.

2. $\theta = \pi/3$

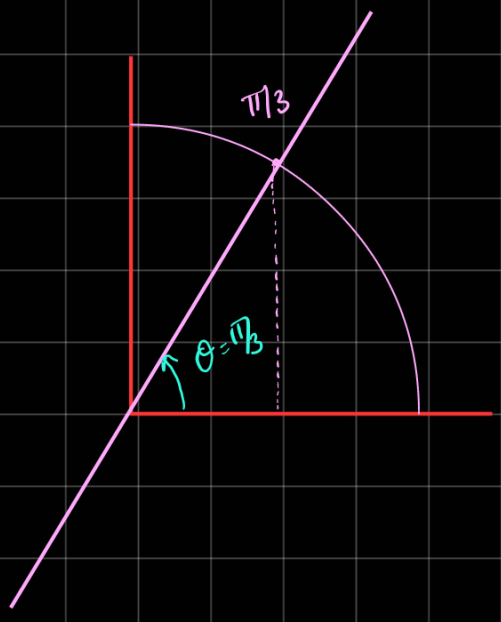
$\tan\theta = \tan\frac{\pi}{3}$

$= \sqrt{3}$

$\frac{y}{x} = \sqrt{3}$

$y = x\sqrt{3}$

$\theta = \pi/3$ is the line $y = x\sqrt{3}$



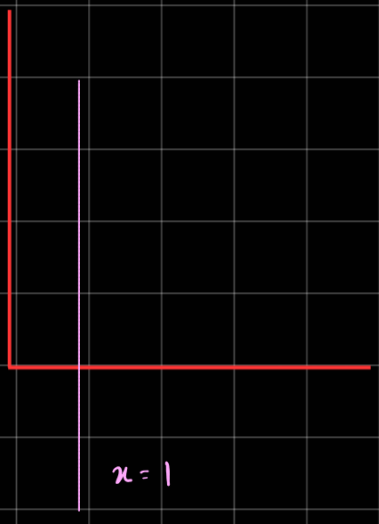
3. $r = \sec\theta$

$r = \frac{1}{\cos\theta}$

$r\cos\theta = 1$

$x = 1$

$r = \sec\theta$ is the vertical line $x = 1$



4. $r = \sin\theta$

$r^2 = r\sin\theta$

$x^2 + y^2 = y$

$x^2 + y^2 - y = 0$

$x^2 + \left(y - \frac{1}{2}\right)^2 - \frac{1}{4} = 0$

$x^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}$

Circle at $(0, \frac{1}{2})$ with radius $\frac{1}{2}$

Circle centered at $\left(0, \frac{1}{2}\right)$ with radius $\frac{1}{2}$

5. $r = \cos\theta + 2\sin\theta$

$r^2 = r\cos\theta + 2r\sin\theta$

$x^2 + y^2 = x + 2y$

$x^2 - x + y^2 - 2y = 0$

$\left(x - \frac{1}{2}\right)^2 + (y-1)^2 - \frac{1}{4} - 1 = 0$

$\left(x - \frac{1}{2}\right)^2 + (y-1)^2 = \frac{5}{4}$

Circle centered at $\left(\frac{1}{2}, 1\right)$ with radius $\left(\frac{\sqrt{5}}{2}\right)$