Faculty of Engineering and Information Sciences

UNIVERSITY
OF WOLLONGONG
IN DUBAI

MATH 142,   Quiz 1,   Spring 2018,   Duration: 60 minutes

Name:_____,   ID Number:_____

Time Allowed: 1 Hour
Total Number of Questions: 6
Total Number of Pages (incl. this page): 7

EXAM UNAUTHORISED ITEMS
Students bringing these items to the examination room shall be required to leave the items
at the front of the room or outside the examination room. The University does not guarantee
the safe-keeping of students' personal items during examinations. Students concerned about
the safety of their valuable items should make alternative arrangements for their care.
1.   Bags, including carry bags, backpacks, shoulder bags and briefcases
2.   Any form of electronic device including but not limited to mobile phones, smart
watches, laptops, iPads, MP3 players, handheld computers and electronic dictionaries,
3.   Calculator cases and covers
4.   blank paper
5.   Any written material

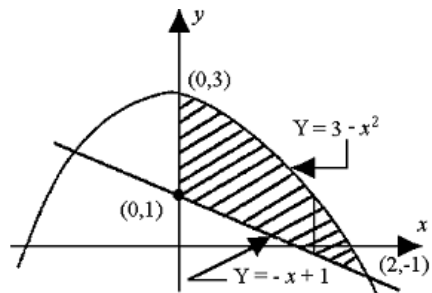DIRECTIONS TO CANDIDATES
1.   Total marks: 40
2.   All questions are compulsory.
3.   Answer all questions on the given exam paper sheets.
4.   Write your name and Id number on the papers provided for rough work.

(*6pts*) **Problem 1**

Find the area of the region bounded by the graphs of

$$f(x) = 3 - x^2 \quad \text{and} \quad g(x) = -x + 1.$$
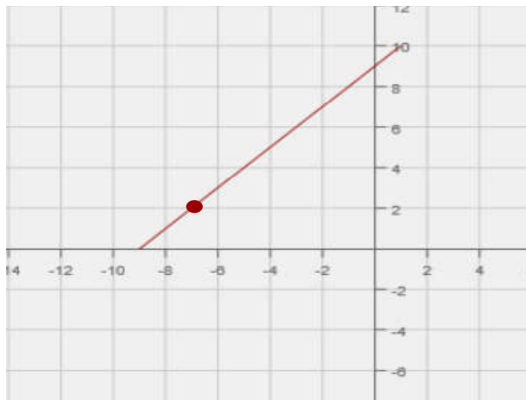
Solution

$$
\begin{aligned}
A &= \int_{-1}^{2} \left[(3 - x^2) - (-x + 1) \right] dx \quad \textbf{(3pts)} \\
&= \int_{-1}^{2} \left(-x^2 + x + 2 \right) dx \\
&= \frac{9}{3} = 4.5. \qquad \textbf{(2pts)}
\end{aligned}
$$

2

(8*pts*)**Problem 2**

(i) Sketch the region bounded by $x + 9 = y$, $y = 0$, and $x = 0$, and use the disc method to **set up the integral** for the volume of the solid generated by revolving the region about the x-axis.
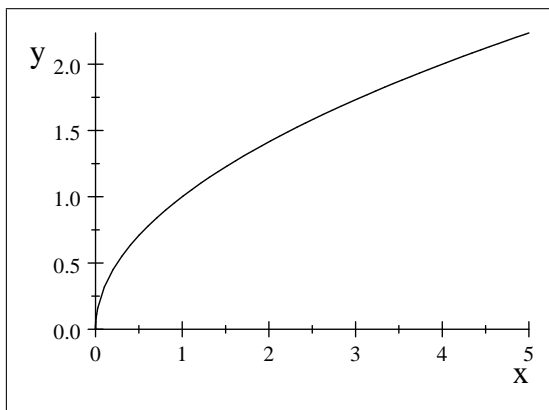
Solution

(**2pts**)

$$V = \int_{-9}^{0} \pi \left(x + 9\right)^2 dx \qquad (\textbf{2pts})$$

(ii) Sketch the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$, and use the shell method to **set up the integral** for the volume of the solid generated by revolving the region about the line $x = -2$.

Solution

(**2pts**) (1)

$$V = \int_{0}^{9} 2\pi \left(x + 2\right) \sqrt{x}\, dx \qquad (\textbf{2pts})$$

(*8pts*) **Problem 3**

Use trigonometric substitution to evaluate the integral

$$\int \frac{2dx}{x^2\sqrt{x^2+4}}.$$

Solution

Here do the substitution

$$x = 2\tan\theta \quad \text{with} \quad \frac{-\pi}{2} < \theta < \frac{\pi}{2}. \qquad \textbf{(2pts)}$$

$$dx = 2\sec^2\theta d\theta$$

$$
\begin{aligned}
x^2 + 4 &= 4\left(1 + \tan^2\theta\right) \\
&= 4\sec^2\theta. \qquad \textbf{(2pts)}
\end{aligned}
$$

$$
\begin{aligned}
\int \frac{2dx}{x^2\sqrt{x^2+4}} &= 2\int \frac{2\sec^2\theta d\theta}{4\tan^2\theta\left(2\sec\theta\right)} \\
&= \frac{2}{4}\int \frac{\sec\theta d\theta}{\tan^2\theta} \\
&= \frac{1}{2}\int \frac{1}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}d\theta \\
&= \frac{1}{2}\int \sin^{-2}\theta\cos\theta d\theta \\
&= -\frac{1}{2\sin\theta} + C \qquad \textbf{(2pts)}
\end{aligned}
$$

Now using the triangle, we get

$$
\begin{aligned}
\int \frac{2dx}{x^2\sqrt{x^2+4}} &= -\frac{1}{2\sin\theta} + C \\
&= -\frac{\sqrt{x^2+4}}{2x} + C \qquad \textbf{(2pts)}
\end{aligned}
$$

4

(*8pts*)**Problem 4**

Use partial fraction decomposition to evaluate the integral

$$\int \frac{(x^2 - 6)\, dx}{x\,(x-1)^2}.$$

Solution

The partial fraction decomposition gives

$$\frac{x^2 - 6}{x\,(x-1)^2} = -\frac{6}{x} + \frac{7}{x-1} - \frac{5}{(x-1)^2}. \qquad \textbf{(3pts)}$$

Thus,

$$
\begin{aligned}
\int \frac{(x^2 - 6)\, dx}{x\,(x-1)^2} &= \int \left[-\frac{6}{x} + \frac{7}{x-1} - \frac{5}{(x-1)^2} \right] dx \\
&= -6\ln|x| + 7\ln|x-1| + \frac{5}{x-1} + C. \qquad \textbf{(5pts)}
\end{aligned}
$$

(*6pts*)**Problem 5**

Find the arc length of the curve $f(x) = \frac{2}{3}(x+1)^{\frac{3}{2}}$, $x \in [1,2]$.

Solution

$$L = \int_1^2 \sqrt{1 + [f'(x)]^2}\, dx$$

$$f'(x) = (x+1)^{1/2}$$

$$
\begin{aligned}
L &= \int_1^2 \sqrt{2+x}\, dx && \textbf{(3pts)}\\
&= \frac{16}{3} - 2\sqrt{3} = 1.869\,2 && \textbf{(3pts)}
\end{aligned}
$$

(*4pts*)**Problem 6**
Evaluate the integral
$$\int_0^{\frac{\pi}{2}} x \sin x dx$$

Solution
Using integration by parts,
$$\int_0^{\frac{\pi}{2}} x \sin x dx = \sin x - x \cos x \big|_0^{\pi/2} = 1$$