Essentials of Engineering Mathematics

First-Order ODEs

    I.    Separable
   II.    Homogeneous
 III.    Linear
 IV.    Exact
  V.    Bernoulli

### III.    Linear Equations

**Definition**: A first-order DE of the form $a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$ , $a_1(x) \neq 0$ is said to be a linear equation in the variable $y$.

The **standard form** of a linear first-order ODE is $\frac{dy}{dx} + P(x)y = Q(x)$

### Steps for Solving a Linear Equation

1. Write the DE in standard form

$$\frac{dy}{dx} + P(x)y = Q(x) \quad (I)$$

2. Find the integrating factor $\mu(x) = e^{\int P(x)dx}$

3. Multiply equation $(I)$ by $\mu(x)$

$$\mu(x)\frac{dy}{dx} + \mu(x)P(x)y = \mu(x)Q(x) \quad (II)$$

4. Equation $(II)$ can be expressed as $\frac{d}{dx}[\mu(x)y] = \mu(x)Q(x)$ or

$$d[\mu(x)y] = \mu(x)Q(x)dx \quad (III)$$

5. Integrate both sides of $(III)$ to get $y = \frac{1}{\mu(x)}\left[\int \mu(x)Q(x)dx + C\right]$

Example 1: Solve $\frac{1}{x}\frac{dy}{dx} - 4y = 1$

Example 2: Solve $x\frac{dy}{dx} - 4y = x^6 e^x$

Example 3: Solve the IVP $y' + y = x$ $\quad y(0) = 4$