5. $\frac{1}{x}\frac{dy}{dx} - 4y = 1$

$\frac{dy}{dx} - 4xy = x$

$P(x) = -4x$

$IF = e^{\int -4x\,dx}$

$= e^{-2x^2}$

$u = -2x^2$

$du = -4x \quad dx$

$-\frac{1}{4}\,du = x\,dx$

$y = \frac{1}{e^{-2x^2}}\int xe^{-2x^2}\,dx$

$= e^{2x^2}\int xe^{-2x^2}\,dx$

$= e^{2x^2}\left[-\frac{1}{4}e^{-2x^2} + c\right]$

$y = -\frac{1}{4} + ce^{2x^2}$

6. $x\frac{dy}{dx} - 4y = x^6e^x$

$\frac{dy}{dx} - \frac{4}{x}y = x^5e^x$

$P(x) = -\frac{4}{x}$

$IF = e^{\int P(x)\,dx}$

$= e^{\int \frac{-4}{x}dx}$

$= e^{-4\ln x}$

x^{-4}

$$y = \frac{1}{x^{-4}} \int \frac{x^5 e^x}{x^4}\, dx$$

$$= x^4 \int x e^x \, dx$$

$$= x^4 \left[x e^x - e^x + c \right]$$

$$= \underline{x^5 e^x - x^4 e^x + x^4 c}$$

7. $\dfrac{dy}{dx} + y = x$ $\qquad x = 0,\ y = 4$

$P(x) = 1$

$IF = e^{\int 1\, dx}$

$= e^x$

$$y = e^{-x} \int x e^x \, dx + c$$

$$= e^{-x} \left[x e^x - e^x + c \right]$$

$$= x - 1 + e^{-x} c$$

$4 = 0 - 1 + c$

$c = 5$

$\therefore\ y = \underline{x - 1 + 5e^{-x}}$

Tutorial 6

4. $\cos x - 2xy + (e^y - x^2)\, y' = 0$ $\qquad y(1) = 4$

$(\cos x - 2xy)\,dx + (e^y - x^2)\,dy = 0$

$M_y = -2x$

$N_x = -2x$

$\therefore$ Equation is exact

$df(x,y) = f_x\,dx + f_y\,dy$

$f_x = \cos x - 2xy$

$f(x,y) = \int \cos x - 2xy \; dx + c(y)$

$= \sin x - x^2 y + c(y)$

$e^y - \cancel{x^2} = -\cancel{x^2} + c'(y)$

$c'(y) = e^y$

$c(y) = e^y$

$f(x,y) = \sin x - x^2 y + e^y = c$

$x = 1, \; y = 4$

$c = \sin(1) - (1)^2(4) + e^4$

$c = \sin(1) - 4 + e^4$

$=$

$f(x,y) = \sin(x) - x^2 y + e^y +$

5 $(x + \sin y)\,dx + (x\cos y - 2y)\,dy = 0$

5. $(x + \sin y)\,dx + (x\cos y - 2y)\,dy = 0$

$M_y = \cos y$

$N_x = \cos y$

$M_y = N_x$

$\therefore$ Equation is exact

$f_x = x + \sin y$

$f(x,y) = \int x + \sin y \; dx + c(y)$

$= \frac{x^2}{2} + x\sin y + c(y)$

$\cancel{x\cos y} - 2y = \cancel{x\cos y} + c'(y)$

$c'(y) = -2y$

$c(y) = -y^2$

$f(x,y) = \frac{x^2}{2} + \sin y - y^2$

$f(x,y) = c$

$\frac{x^2}{2} + \underline{\sin y - y^2} = c$

6. $\left(\frac{y}{x^2} + 1\right) dx + \left(\frac{1}{x}\right) dy = 0$

$M_y = \frac{1}{x^2}$

$N_x = -\frac{1}{x^2}$

$M_y \neq N_x$

Special Integrating Factor

$\therefore$ Equation is not exact

$$\frac{M_y - N_x}{N(x,y)} = \frac{\frac{1}{x^2} + \frac{1}{x^2}}{\frac{1}{x}}$$

$$= \frac{2}{x^2} \times x$$

$$= \frac{2}{x} = f(x)$$

1. $\frac{M_y - N_x}{N(x,y)} = f(x)$

OR

2. $\frac{N_x - M_y}{M(x,y)} = g(y)$

$$IF = e^{\int f(x)\,dx}$$

$$= e^{\ln x^2}$$

$$= x^2$$

$$(y + x^2)\,dx + x\,dy = 0$$

$$f_x = x^2 + y$$

$$f(x,y) = \int x^2 + y\,dx + c(y)$$

$$= \frac{x^3}{3} + xy + c(y)$$

$$\cancel{x} = \cancel{x} + c'(y)$$

$$c'(y) = 0$$

$$c(y) = c$$

$$f(x,y) = \frac{x^3}{3} + xy + c$$

$$f(x,y) = c$$

$$c = \frac{x^3}{3} + xy$$