Week 2 Workshop Example Solutions

Example 7:

For $t \le 0$, the current through R_3-C path is zero. Voltage divider rule: v(0) = 5 V×20/50 = 2 V.
At $t = \infty$, the current through R_3-C path is zero. Current through R_4-R_2 path is i_2 = 0.2 mA:
Current divider rule: 0.5 mA×20/(20 + 10 + 20) = 0.2 mA

$v(\infty) = R_2 i_2 = 20\ \text{k}\Omega \times 0.2\ \text{mA} = 4\ \text{V}$

For $0 \le t$, $R_{eq} = R_3 + [R_2 \,||\, (R4 + R5)] = 8\ \text{k}\Omega + [20\ \text{k}\Omega \,||\, 30\ \text{k}\Omega] = 20\ \text{k}\Omega$
For $0 \le t$, $\tau = R_{eq}C = 20{,}000 \times 2 \times 10^{-8} = 0.4$ ms, $1/\tau = 2500$ (1/s)
For $0 \le t$, v(t) = [4 + (2 – 4)exp(–2500t)] u(t) V = [4 – 2exp(–2500t)] u(t) V
For $0 \le t$, i(t) = C dv(t)/dt = 0.1exp(-2500t)] u(t) mA

$v_a(t) = R_3 i + v(t)$ = [4 – 1.2exp(-2500t)] u(t) V

Example 8:

The voltage v_a can be obtained by applying the voltage divider rule:

$v_a = V_s \times R_2/(R_1 + R_2) = 0.5\ \text{V} \times 2/5 = 0.2\ \text{V}$

Summing the currents leaving node *b*, we obtain

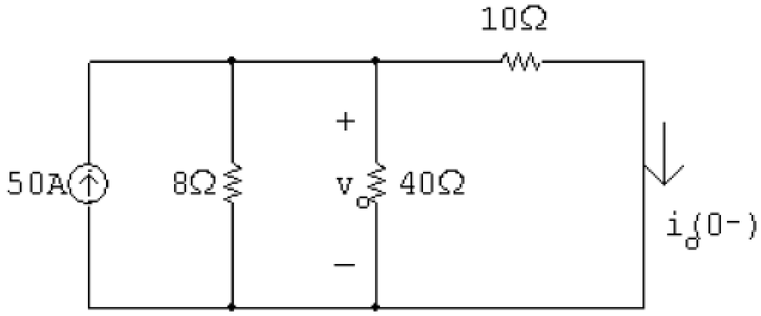
$$0.01 \times 0.2 + \frac{v(t)}{1000} + 1 \times 10^{-6}\frac{dv(t)}{dt} = 0 \Rightarrow \frac{dv(t)}{dt} + 1000v(t)$$

At $t = \infty$, dv(t)/dt = 0, $1000v(\infty) = -2000 \Rightarrow v(\infty) = -2$ V
$1/\tau = 1000$ (1/s), $\tau = 1/1000 = 1$ ms
The voltage across the capacitor is

$$v(t) = \left[v(\infty) + (v(0) - v(\infty))e^{-\frac{t}{\tau}}\right]u(t) = [-2 + 2e^{-1000t}]u(t)\text{V}$$

Example 12:

[a] $t < 0$



KVL equation at the top node:

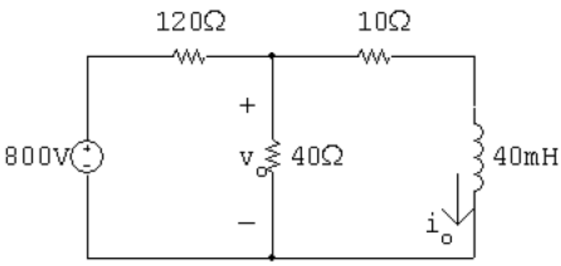
$$50 = \frac{v_o}{8} + \frac{v_o}{40} + \frac{v_o}{10}$$

Multiply by 40 and solve:

$$2000 = (5 + 1 + 4)v_o; \qquad v_o = 200\,\text{V}$$

$$\therefore \quad i_o(0^-) = \frac{v_o}{10} = 200/10 = 20\,\text{A}$$

$t > 0$



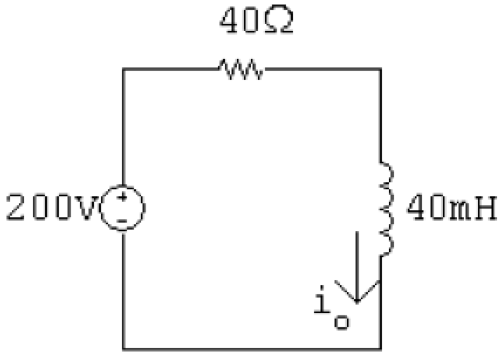
Use voltage division to find the Thévenin voltage:

$$V_{\text{Th}} = v_o = \frac{40}{40 + 120}(800) = 200\,\text{V}$$

Remove the voltage source and make series and parallel combinations of resistors to find the equivalent resistance:

$$R_{\text{Th}} = 10 + 120\|40 = 10 + 30 = 40\,\Omega$$

The simplified circuit is:



$$\tau = \frac{L}{R} = \frac{40 \times 10^{-3}}{40} = 1\,\text{ms}; \qquad \frac{1}{\tau} = 1000$$

$$i_o(\infty) = \frac{200}{40} = 5\,\text{A}$$

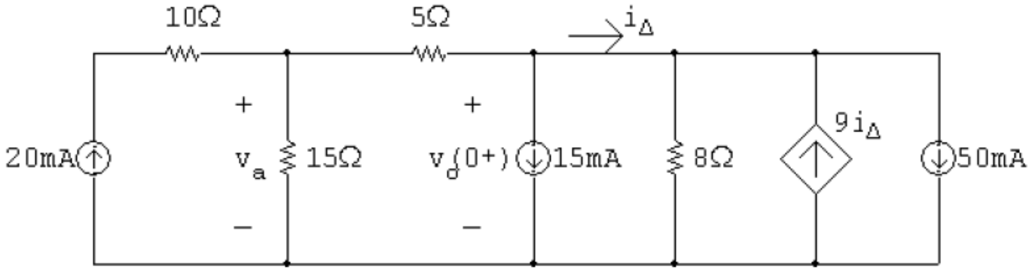
$$\therefore \quad i_o = i_o(\infty) + [i_o(0^+) - i_o(\infty)]e^{-t/\tau}$$

$$= 5 + (20 - 5)e^{-1000t} = 5 + 15e^{-1000t}\,\text{A}, \qquad t \geq 0$$

$$\begin{aligned} \text{[b]} \quad v_o &= 10i_o + L\frac{di_o}{dt} \\ &= 10(5 + 15e^{-1000t}) + 0.04(-1000)(15e^{-1000t}) \\ &= 50 + 150e^{-1000t} - 600e^{-1000t} \\ v_o &= 50 - 450e^{-1000t}\,\text{V}, \qquad t \geq 0^+ \end{aligned}$$

Example 13:

$t > 0$; calculate $v_o(0^+)$



$$\frac{v_a}{15} + \frac{v_a - v_o(0^+)}{5} = 20 \times 10^{-3}$$

$$\therefore \quad v_a = 0.75v_o(0^+) + 75 \times 10^{-3}$$

$$15 \times 10^{-3} + \frac{v_o(0^+) - v_a}{5} + \frac{v_o(0^+)}{8} - 9i_\Delta + 50 \times 10^{-3} = 0$$

$$13v_o(0^+) - 8v_a - 360i_\Delta = -2600 \times 10^{-3}$$

$$i_\Delta = \frac{v_o(0^+)}{8} - 9i_\Delta + 50 \times 10^{-3}$$

$$\therefore \quad i_\Delta = \frac{v_o(0^+)}{80} + 5 \times 10^{-3}$$

$$\therefore \quad 360i_\Delta = 4.5v_o(0^+) + 1800 \times 10^{-3}$$

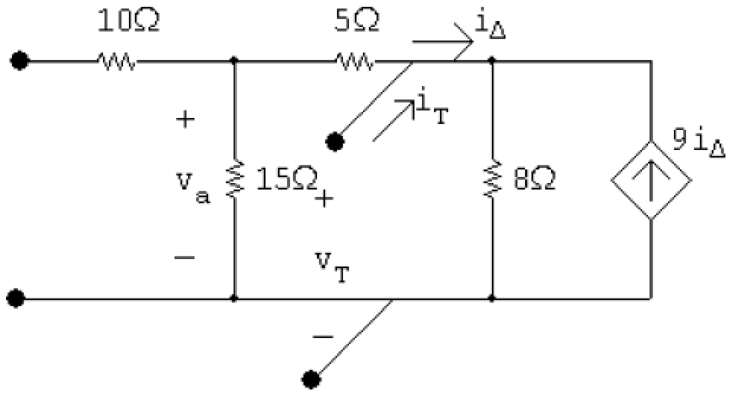
$$8v_a = 6v_o(0^+) + 600 \times 10^{-3}$$

$$\therefore \quad 13v_o(0^+) - 6v_o(0^+) - 600 \times 10^{-3} - 4.5v_o(0^+) - 1800 \times 10^{-3} = -2600 \times 10^{-3}$$

$$2.5v_o(0^+) = -200 \times 10^{-3}; \qquad v_o(0^+) = -80\,\text{mV}$$

$$v_o(\infty) = 0$$

Find the Thévenin resistance seen by the 4 mH inductor:



$$i_T = \frac{v_T}{20} + \frac{v_T}{8} - 9i_\Delta$$

$$i_\Delta = \frac{v_T}{8} - 9i_\Delta \qquad \therefore \quad 10i_\Delta = \frac{v_T}{8}; \qquad i_\Delta = \frac{v_T}{80}$$

$$i_T = \frac{v_T}{20} + \frac{10v_T}{80} - \frac{9v_T}{80}$$

$$\frac{i_T}{v_T} = \frac{1}{20} + \frac{1}{80} = \frac{5}{80} = \frac{1}{16}\,\text{S}$$

$$\therefore \quad R_{\text{Th}} = 16\Omega$$

$$\tau = \frac{4 \times 10^{-3}}{16} = 0.25\,\text{ms}; \qquad 1/\tau = 4000$$

$$\therefore \quad v_o = 0 + (-80 - 0)e^{-4000t} = -80e^{-4000t}\,\text{mV}, \qquad t \geq 0^+$$