Week 1 Workshop Example Solutions

***Example 6:***

We do not know currents $I_5$, $I_6$ through $V_{s2}$.

The currents $I_5$, $I_6$ flow in opposite direction.

Thus, we have $I_6 = -I_5$.

KCL at node 1: $I_1 + I_2 + I_5 = 0$ (1)

KCL at node 2: $I_3 + I_4 + I_6 = 0$ (2)

Equation (2) can be rewritten as

$I_3 + I_4 - I_5 = 0$ (3)

Adding Equations (1) and (3), we obtain

$I_1 + I_2 + I_3 + I_4 = 0$ (4)

Equation (4) is the sum of currents leaving nodes 1 and 2. Since $I_5 + I_6 = 0$, $I_5$ and $I_6$ are not included in the sum.

Since $V_2$ is 4 V higher than $V_1$, the constraint equation is given by

$V_2 = V_1 + 4$ (5)

The supernode consisting of nodes 1 and 2

is shown in Figure as an ellipse.

When writing a node equation for a supernode,

sum the currents leaving the supernode,

ignoring currents inside the supernode.

From Equation (4), we have

$$\frac{V_1 - 8}{2000} + \frac{V_1}{1000} + \frac{V_2 - 8}{1000} + \frac{V_2}{2000} = 0$$

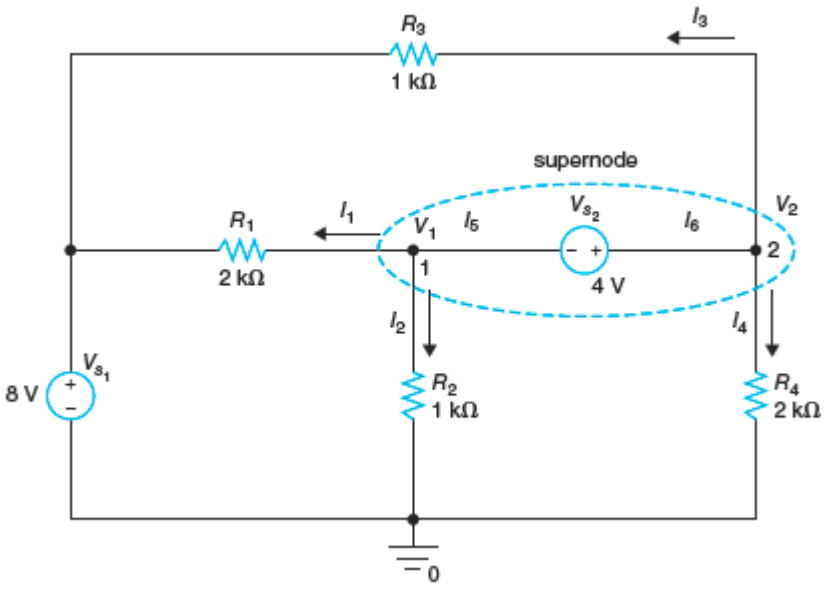
Multiply by 2000:

$V_1 - 8 + 2V_1 + 2V_2 - 16 + V_2 = 0 \Rightarrow 3V_1 + 3V_2 = 24$ (6)

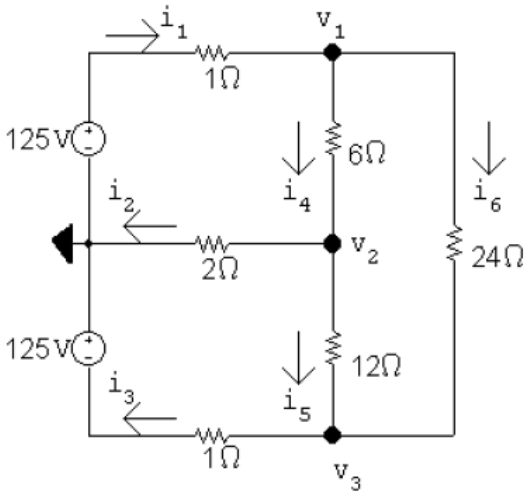
(5) $\Rightarrow$ (6): $3V_1 + 3V_1 + 12 = 24 \Rightarrow 6V_1 = 12 \Rightarrow V_1 = 2$ V (7)

(7) $\Rightarrow$ (5)

$V_2 = 6$ V


$R_3$
1 kΩ
$I_3$
supernode
$R_1$
2 kΩ
$I_1$
$V_1$
$I_5$
$V_{s_2}$
$I_6$
$V_2$
1
2
4 V
$I_2$
$I_4$
$V_{s_1}$
8 V
$R_2$
1 kΩ
$R_4$
2 kΩ
0

***Example 7:***




$$\begin{aligned}
\frac{v_1 - 125}{1} + \frac{v_1 - v_2}{6} + \frac{v_1 - v_3}{24} &= 0 \\
\frac{v_2 - v_1}{6} + \frac{v_2}{2} + \frac{v_2 - v_3}{12} &= 0 \\
\frac{v_3 + 125}{1} + \frac{v_3 - v_2}{12} + \frac{v_3 - v_1}{24} &= 0
\end{aligned}$$

In standard form:

$$\begin{aligned}
v_1\left(\frac{1}{1} + \frac{1}{6} + \frac{1}{24}\right) + v_2\left(-\frac{1}{6}\right) + v_3\left(-\frac{1}{24}\right) &= 125 \\
v_1\left(-\frac{1}{6}\right) + v_2\left(\frac{1}{6} + \frac{1}{2} + \frac{1}{12}\right) + v_3\left(-\frac{1}{12}\right) &= 0 \\
v_1\left(-\frac{1}{24}\right) + v_2\left(-\frac{1}{12}\right) + v_3\left(\frac{1}{1} + \frac{1}{12} + \frac{1}{24}\right) &= -125
\end{aligned}$$

Solving, $v_1 = 101.24$ V; $v_2 = 10.66$ V; $v_3 = -106.57$ V

Thus, $i_1 = \dfrac{125 - v_1}{1} = 23.76$ A $\quad i_4 = \dfrac{v_1 - v_2}{6} = 15.10$ A

$i_2 = \dfrac{v_2}{2} = 5.33$ A $\quad i_5 = \dfrac{v_2 - v_3}{12} = 9.77$ A

$i_3 = \dfrac{v_3 + 125}{1} = 18.43$ A $\quad i_6 = \dfrac{v_1 - v_3}{24} = 8.66$ A

$$\sum P_{\text{dev}} = 125i_1 + 125i_3 = 5273.09 \text{ W}$$

$$\sum P_{\text{dis}} = i_1^2(1) + i_2^2(2) + i_3^2(1) + i_4^2(6) + i_5^2(12) + i_6^2(24) = 5273.09 \text{ W}$$

***Example 8:***

$$\frac{v_1 + 40}{12} + \frac{v_1}{25} + \frac{v_1 - v_2}{20} + 5 = 0$$

$$\left[\frac{v_2 - v_1}{20}\right] - 5 + \frac{v_2 - v_1}{40} + -7.5 = 0$$

$$\frac{v_3}{40} + \frac{v_3 - v_2}{40} + 7.5 = 0$$

Solving, $v_1 = -10$ V; $v_2 = 132$ V; $v_3 = -84$ V; $i_{40\text{V}} = \dfrac{-10 + 40}{12} = 2.5$ A

$$p_{5\text{A}} = 5(v_1 - v_2) = 5(-10 - 132) = -710 \text{ W} \quad \text{(del)}$$

$$p_{7.5\text{A}} = (-84 - 132)(7.5) = -1620 \text{ W} \quad \text{(del)}$$

$$p_{40\text{V}} = -(40)(2.5) = -100 \text{ W} \quad \text{(del)}$$

$$p_{12\Omega} = (2.5)^2(12) = 75 \text{ W}$$

$$p_{25\Omega} = \frac{v_1^2}{25} = \frac{10^2}{25} = 4 \text{ W}$$

$$p_{20\Omega} = \frac{(v_1 - v_2)^2}{20} = \frac{142^2}{20} = 1008.2 \text{ W}$$

***Example 9:***

$$i_o = \frac{v_2}{40}$$

$$-5i_o + \frac{v_1}{20} + \frac{v_1 - v_2}{5} = 0 \qquad \text{so} \quad 10v_1 - 13v_2 + 0v_3 = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{40} + \frac{v_2 - v_3}{10} \qquad \text{so} \quad -8v_1 + 13v_2 - 4v_3 = 0$$

$$\frac{v_3 - v_2}{10} + \frac{v_3 - 11.5i_o}{5} + \frac{v_3 - 96}{4} = 0 \qquad \text{so} \quad 0v_1 - 63v_2 + 220v_3 = 9600$$

Solving, $v_1 = 156$ V; $\quad v_2 = 120$ V; $\quad v_3 = 78$ V

$$i_o = \frac{v_2}{40} = \frac{120}{40} = 3 \text{ A}$$

$$i_3 = \frac{v_3 - 11.5i_o}{5} = \frac{78 - 11.5(3)}{5} = 8.7 \text{ A}$$

$$i_g = \frac{78 - 96}{4} = -4.5 \text{ A}$$

$$p_{5i_o} = -5i_o v_1 = -5(3)(156) = -2340 \text{ W(dev)}$$

$$p_{11.5i_o} = 11.5i_o i_3 = 11.5(3)(8.7) = 300.15 \text{ W(abs)}$$

$$p_{96V} = 96(-4.5) = -432 \text{ W(dev)}$$

$$\sum p_{\text{dev}} = 2340 + 432 = 2772 \text{ W}$$

CHECK

$$\sum p_{\text{dis}} = \frac{156^2}{20} + \frac{(156 - 120)^2}{5} + \frac{120^2}{40} + \frac{(120 - 78)^2}{50}$$

$$+(8.7)^2(5) + (4.5)^2(4) + 300.15 = 2772 \text{ W}$$

$$\therefore \quad \sum p_{\text{dev}} = \sum p_{\text{dis}} = 2772 \text{ W}$$

***Example 14:***

$$230 - 115 = 7i_1 - 1i_2 - 2i_3$$

$$0 = -1i_1 + 10i_2 - 3i_3$$

$$115 - 460 = -2i_1 - 3i_2 + 10i_3$$

$$\text{Solving, } i_1 = 4.4 \text{ A}; \quad i_2 = -10.6 \text{ A}; \quad i_3 = -36.8 \text{ A}$$

$$p_{230} = -230i_1 = -1012 \text{ W(del)}$$

$$p_{115} = 115(i_1 - i_3) = 4738 \text{ W(abs)}$$

$$p_{460} = 460i_3 = -16{,}928 \text{ W(del)}$$

$$\therefore \quad \sum p_{\text{dev}} = 17{,}940 \text{ W}$$

***Example 17:***

$$-20 + 4i_1 + 9i_2 - 90 + 6i_2 + 1i_1 = 0; \quad i_1 - i_2 = 6$$

$$\text{Solving, } i_1 = 10 \text{ A}; \quad i_2 = 4 \text{ A}$$

$$p_{20V} = -20i_1 = -200 \text{ W (diss)}$$

$$p_{4\,\Omega} = (10)^2(4) = 400 \text{ W}$$

$$p_{1\,\Omega} = (10)^2(1) = 100 \text{ W}$$

$$p_{9\,\Omega} = (4)^2(9) = 144 \text{ W}$$

$$p_{6\,\Omega} = (4)^2(6) = 96 \text{ W}$$

$$v_o = 9(4) - 90 + 6(4) = -30 \text{ V}$$

$$p_{6A} = 6v_o = -180 \text{ W}$$

$$p_{90V} = -90i_2 = -360 \text{ W}$$

$$\sum p_{\text{dev}} = 200 + 180 + 360 = 740 \text{ W}$$

$$\sum p_{\text{diss}} = 400 + 100 + 144 + 96 = 740 \text{ W}$$

Thus the total power dissipated is 740 W.

***Example 18:***

$$-4i_d + 10(i_e - i_d) + 5(i_e - i_x) = 0$$

$$5(i_x - i_e) + 10(i_d - i_e) - 240 + 40(i_x - 19) = 0$$

$$i_d - i_x = 2i_b = 2(i_e - i_x)$$

Solving, $i_d = 10$ A; $i_e = 18$ A; $i_x = 26$ A

$i_a = 19 - i_x = -7$ A; $i_b = i_e - i_x = -8$ A; $i_c = i_e - i_d = 8$ A;

$v_{\text{a}} = 40i_{\text{a}} = -280$ V; $v_{\text{b}} = 5i_b + 40i_a = -320$ V

$$p_{19\text{A}} = -19v_a = 5320 \text{ W}$$

$$p_{4i_d} = -4i_d i_e = -720 \text{ W}$$

$$p_{2i_a} = -2i_b v_b = -5120 \text{ W}$$

$$p_{240\text{V}} = -240i_d = -2400 \text{ W}$$

$$p_{40\Omega} = (7)^2(40) = 1960 \text{ W} =$$

$$p_{5\Omega} = (8)^2(5) = 320 \text{ W}$$

$$p_{10\Omega} = (8)^2(10) = 640 \text{ W}$$

$$\sum P_{\text{gen}} = 720 + 5120 + 2400 = 8240 \text{ W}$$

$$\sum P_{\text{diss}} = 5320 + 1960 + 320 + 640 = 8240 \text{ W}$$

***Example 19:***

$$\frac{v_1 - 24}{2} + \frac{v_1 - 6i_\Delta}{4} + \frac{v_1 - v_2}{5} = 0$$

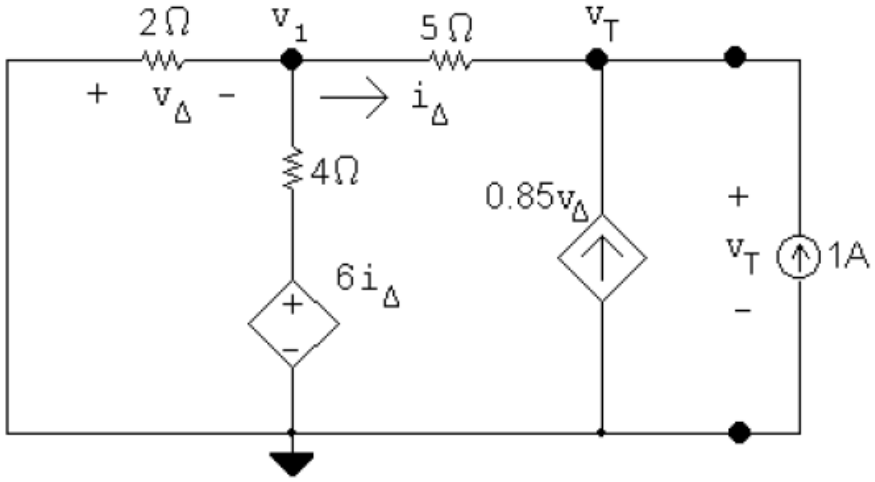
$$\frac{v_2 - v_1}{5} - 0.85v_\Delta = 0$$

Constraint equations:

$$i_\Delta = \frac{v_1 - v_2}{5}; \qquad v_\Delta = 24 - v_1$$

Solving, $v_2 = 84\ \text{V} = v_{\text{Th}}$

Thévenin resistance using a test source:




$$\frac{v_1}{2} + \frac{v_1 - 6i_\Delta}{4} + \frac{v_1 - v_T}{5} = 0$$

$$\frac{v_T - v_1}{5} - 0.85v_\Delta - 1 = 0$$

$$i_\Delta = \frac{v_1 - v_T}{5}; \qquad v_\Delta = -v_1$$

Solving, $v_T = 10$

$$R_{\text{Th}} = \frac{v_T}{1} = 10\,\Omega$$

$$\therefore \quad R_o = R_{\text{Th}} = 10\,\Omega$$