Week 8 Workshop Example Solutions

Example 1:

- **Coupled coils: $V_1 = j4\,I_1 - j5\,I_2$ (1), $V_2 = j5\,I_1 - j9\,I_2$ (2)**
- **The voltage across R_2: $V_2 = 9I_2$ (3)**
- **Substitute (3) into (2): $9I_2 = j5\,I_1 - j9\,I_2 \Rightarrow I_1 = [(9 + j9)/(j5)]\,I_2 = (1.8 - j1.8)\,I_2$ (4)**
- **Sum the voltage drops around mesh 1: $-20 + 3I_1 + V_1 = 0$ (5)**
- **Substitute (1) into (5): $-20 + 3I_1 + j4\,I_1 - j5\,I_2 = 0 \Rightarrow (3 + j4)\,I_1 - j5\,I_2 = 20$ (6)**
- **Substitute (4) into (6): $(3 + j4)(1.8 - j1.8)\,I_2 - j5\,I_2 = 20 \Rightarrow I_2 = 1.4911 + j0.3787$**

  $= 1.5385\angle 14.25^o$ A, (4): $I_1 = (1.8 - j1.8)\,I_2 = 3.3657 - j2.0024 = 3.9163\angle -30.75^o$ A
- **(3): $V_2 = 9I_2 = 13.4201 + j3.4083 = 13.8462\angle 14.25^o$ V**
- **(5): $V_1 = 20 - 3I_1 = 9.9030 + j3.4083$**

  $= 11.5825\angle 31.2409^o$ V

Problem 2:

- $\mathbf{V}_1 = j10\,\mathbf{I}_1 + j6\,\mathbf{I}_2 + j8\,\mathbf{I}_3$ (1), $\mathbf{V}_2 = j6\,\mathbf{I}_1 + j12\,\mathbf{I}_2 + j7\,\mathbf{I}_3$ (2), $\mathbf{V}_3 = j8\,\mathbf{I}_1 + j7\,\mathbf{I}_2 + j15\,\mathbf{I}_3$ (3)
- $\mathbf{V}_1 = 60 - 5\,\mathbf{I}_1$ (4), $\mathbf{V}_2 = -8\,\mathbf{I}_2$ (5), $\mathbf{V}_3 = -12\,\mathbf{I}_3$ (6)
- Solve Equations (1) – (6) give the following results
- $\mathbf{I}_1 = 6.4802\angle -41.8064^o$ A
- $\mathbf{I}_2 = 1.9375\angle 155.9193^o$ A
- $\mathbf{I}_3 = 2.0376\angle 170.7976^o$ A
- $\mathbf{V}_1 = 41.8522\angle 31.0698^o$ V
- $\mathbf{V}_2 = 15.5002\angle -24.0807^o$ V
- $\mathbf{V}_3 = 24.4515\angle -9.2024^o$ V

Problem 3:

$Z_L = 16 - j24 = 28.84\angle -56^\circ\ \Omega$

Z_L REFERRED to PRIMARY : $\frac{Z_L}{n^2} = 1.8\angle -56^\circ$

$I_1 = \frac{240\angle 0}{2\angle 0 + 1.8\angle -56} = 71.55\angle 26.57^\circ$

$I_2 = \frac{-I_1}{4} = -17.89\angle 26.57^\circ = 17.89\angle -153.43^\circ$

Complex power $= VI\angle \theta_v - \theta_i = 17.7\angle -26.57^\circ$

Problem 4:

- $\mathbf{V}_2 = n\mathbf{V}_1$, $\mathbf{I}_2 = \mathbf{I}_1/n$, $\mathbf{V}_1 = \mathbf{V}_2/n$, $\mathbf{I}_1 = n\mathbf{I}_2$.
- The input impedance looking into the transformer is given by

  $\mathbf{Z}_{in} = \mathbf{Z}_L/n^2 = 125 + j75\ \Omega$
- The current $\mathbf{I}_1$ is given by $\mathbf{I}_1 = \mathbf{V}_s/(\mathbf{Z}_s + \mathbf{Z}_{in}) = 0.3331 - j0.2109 = 0.3942\angle -32.3474^\circ$ A
- $\mathbf{V}_1 = \mathbf{Z}_{in}\times\mathbf{I}_1 = 57.4544 - j1.3878 = 57.4712\angle -1.3837^\circ$ V
- $\mathbf{V}_2 = n\mathbf{V}_1 = 11.4909 - j0.2776 = 11.4942\angle -1.3837^\circ$ V
- $\mathbf{I}_2 = \mathbf{I}_1/n = 1.6653 - j1.05472$

  $= 1.9712\angle -32.3474^\circ$ A

Problem 5:

a) We begin by constructing the phasor domain equivalent circuit. The voltage source becomes $2500\angle 0^\circ$ V; the 5 mH inductor converts to an impedance of $j2\ \Omega$; and the 125 μH inductor converts to an impedance of $j0.05\ \Omega$. The phasor domain equivalent circuit is shown in Fig. 9.46.

It follows directly from Fig. 9.46 that

$$2500\angle 0^\circ = (0.25 + j2)\mathbf{I}_1 + \mathbf{V}_1,$$

and

$$\mathbf{V}_1 = 10\mathbf{V}_2 = 10[(0.2375 + j0.05)\mathbf{I}_2].$$

Because

$$\mathbf{I}_2 = 10\mathbf{I}_1$$

we have

$$\mathbf{V}_1 = 10(0.2375 + j0.05)10\mathbf{I}_1$$
$$= (23.75 + j5)\mathbf{I}_1.$$

Therefore

$$2500\angle 0^\circ = (24 + j7)\mathbf{I}_1,$$

or

$$\mathbf{I}_1 = 100\angle -16.26^\circ \text{ A}.$$

Thus the steady-state expression for i_1 is

$$i_1 = 100\cos(400t - 16.26^\circ) \text{ A}.$$

b) $\mathbf{V}_1 = 2500\angle 0^\circ - (100\angle -16.26^\circ)(0.25 + j2$

$$= 2500 - 80 - j185$$
$$= 2420 - j185 = 2427.06\angle -4.37^\circ \text{ V}.$$

Hence

$$v_1 = 2427.06\cos(400t - 4.37^\circ) \text{ V}.$$

c) $\mathbf{I}_2 = 10\mathbf{I}_1 = 1000\angle -16.26^\circ$ A.

Therefore

$$i_2 = 1000\cos(400t - 16.26^\circ) \text{ A}.$$

d) $\mathbf{V}_2 = 0.1\mathbf{V}_1 = 242.71\angle -4.37^\circ$ V,

giving

$$v_2 = 242.71\cos(400t - 4.37^\circ) \text{ V}.$$

Problem 6

$$\mathbf{I}_1 = \frac{\mathbf{V}_s}{Z_1 + 2s^2 Z_2} = \frac{25 \times 10^3 \underline{/0^\circ}}{1500 + j6000 + (25)^2(4 - j14.4)}$$

$$= 4 + j3 = 5\underline{/36.87^\circ} \text{ A}$$

$$\mathbf{V}_1 = \mathbf{V}_s - Z_1 \mathbf{I}_1 = 25{,}000\underline{/0^\circ} - (4 + j3)(1500 + j6000)$$

$$= 37{,}000 - j28{,}500$$

$$\mathbf{V}_2 = -\frac{1}{25}\mathbf{V}_1 = -1480 + j1140 = 1868.15\underline{/142.39^\circ} \text{ V}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{Z_2} = \frac{1868.15\underline{/142.39^\circ}}{4 - j14.4} = 125\underline{/216.87^\circ} \text{ A}$$