Week 5 Workshop Example Solutions

***Problem 1:***

What is the instantaneous voltage across a 2-$\mu$F capacitor when the current through it is $I = 4 \sin(10^6 t + 25°)$ A?

**Soln 7.**

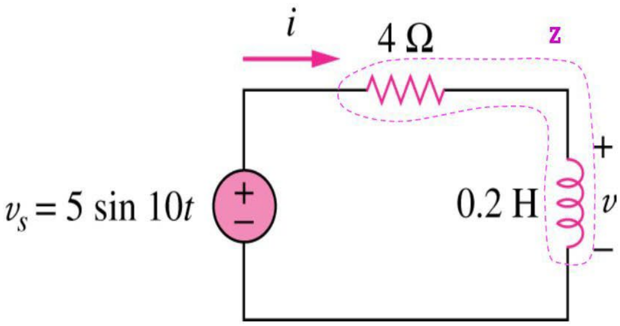
$$\mathbf{Z} = \frac{1}{j\omega C} = \frac{1}{j(10^6)(2\times 10^{-6})} = -j0.5$$

$$\mathbf{V} = \mathbf{IZ} = (4\angle \cancel{25}°)(0.5\angle -90°) = 2\angle -65°$$

Therefore $v(t) = \underline{\mathbf{2\sin(10^6 t - 65°)\ V}}$.

***Problem 2:***

*Determine v(t) and i(t)*




$$\mathbf{V}_s = 5\angle 0^\circ, \ \ \omega = 10$$

$$\mathbf{Z} = 4 + j\omega L = 4 + j2$$

$$\mathbf{I} = \mathbf{V}_s / \mathbf{Z} = \frac{5\angle 0^\circ}{4+j2} = \frac{(5\angle 0^\circ)(4-j2)}{(4+j2)(4-j2)} = \frac{5(4-j2)}{16+4} = 1 - j0.5 = 1.118\angle -26.57^\circ$$

$$\mathbf{V} = j\omega L\mathbf{I} = j2\mathbf{I} = (2\angle 90^\circ)(1.118\angle -26.57^\circ) = 2.236\angle 63.43^\circ$$

*therefore,*

$$v(t) = 2.236\sin(10t + 63.43^\circ)\text{V}$$

$$i(t) = 1.118\sin(10t - 26.57^\circ)\text{A}$$

Problem 3:

A series *RL* circuit is connected to a 110-V ac source. If the voltage across the resistor is 85 V, find the voltage across the inductor.

**Soln 9.**

$$110 = \sqrt{v_R^2 + v_L^2}$$

$$v_L = \sqrt{110^2 - v_R^2}$$

$$v_L = \sqrt{110^2 - 85^2} = \underline{\mathbf{69.82\ V}}$$

Problem 4:

*Let* $\mathbf{Z}_1$ = *impedance of the* 0.5-*H inductor in parallel with the* 10-Ω *resistor*
*and* $\mathbf{Z}_2$ = *impedance of the* (1/20)-*F capacitor*

$$\mathbf{Z}_1 = 10 \,||\, j5 = \frac{(10)(j5)}{(10+j5)} = \frac{(10)(j5)}{(10+j5)}\frac{(10-j5)}{(10-j5)} = \frac{(j500+250)}{(100+25)} = 2+j4 \quad \text{and } \mathbf{Z}_2 = -j2$$

$$\mathbf{V}_0 = \mathbf{Z}_2/(\mathbf{Z}_1+\mathbf{Z}_2)\mathbf{V}_s$$

$$\mathbf{V}_0 = \frac{-j2}{2+j4-j2}(10\angle 75^\circ) = \frac{-j(10\angle 75^\circ)}{1+j} = \frac{10\angle(75^\circ-90^\circ)}{\sqrt{2}\angle 45^\circ}$$

$$\mathbf{V}_0 = 7.071\angle -60^\circ \qquad \text{and } v_0(t) = 7.071\cos(10t-60^\circ)\text{V}$$

Problem 5:

$$\mathbf{V}_g = 40\underline{/-15^\circ}\,\text{V}; \qquad \mathbf{I}_g = 40\underline{/-68.13^\circ}\,\text{mA}$$

$$Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = 1000\underline{/53.13^\circ}\,\Omega = 600 + j800\,\Omega$$

$$Z = 600 + j\left(3.2\omega - \frac{0.4 \times 10^6}{\omega}\right)$$

$$\therefore \quad 3.2\omega - \frac{0.4 \times 10^6}{\omega} = 800$$

$$\therefore \quad \omega^2 - 250\omega - 125{,}000 = 0$$

Solving,

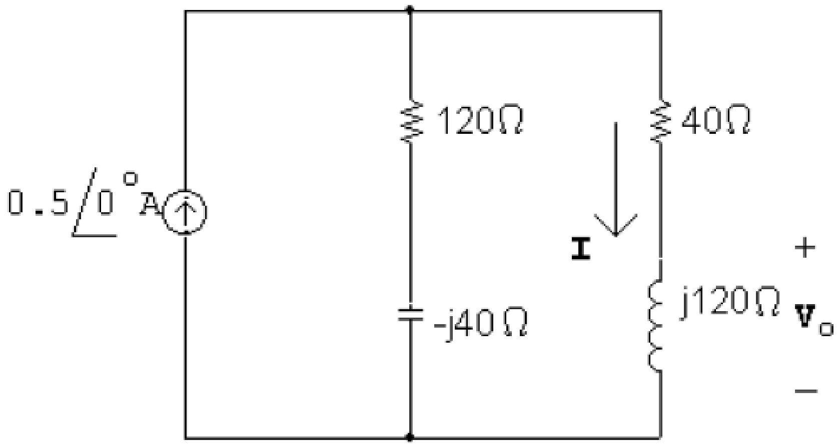
$$\omega = 500\,\text{rad/s}$$

Problem 6

$$Z_L = j(2000)(60 \times 10^{-3}) = j120\,\Omega$$

$$Z_C = \frac{-j}{(2000)(12.5 \times 10^{-6})} = -j40\,\Omega$$

Construct the phasor domain equivalent circuit:



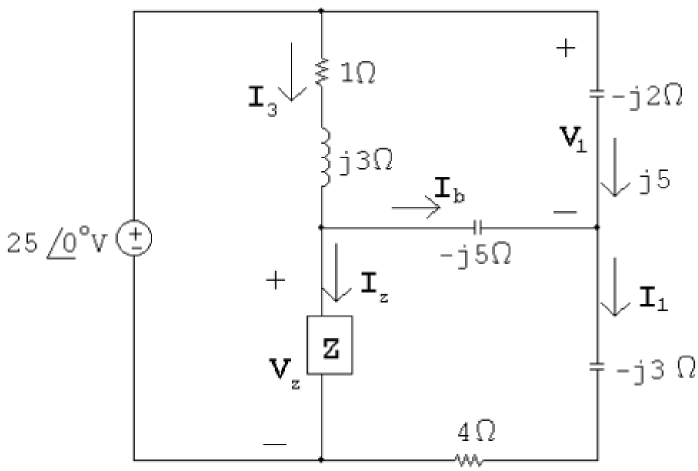

Using current division:

$$\mathbf{I} = \frac{(120 - j40)}{120 - j40 + 40 + j120}(0.5) = 0.25 - j0.25\,\mathrm{A}$$

$$\mathbf{V}_o = j120\mathbf{I} = 30 + j30 = 42.43\underline{/45^\circ}$$

$$v_o = 42.43\cos(2000t + 45^\circ)\,\mathrm{V}$$

Problem 7:




$$\mathbf{V}_1 = j5(-j2) = 10\,\mathrm{V}$$

$$-25 + 10 + (4 - j3)\mathbf{I}_1 = 0 \quad \therefore \quad \mathbf{I}_1 = \frac{15}{4 - j3} = 2.4 + j1.8\,\mathrm{A}$$

$$\mathbf{I}_\mathrm{b} = \mathbf{I}_1 - j5 = (2.4 + j1.8) - j5 = 2.4 - j3.2\,\mathrm{A}$$

$$\mathbf{V}_Z = -j5\mathbf{I}_2 + (4 - j3)\mathbf{I}_1 = -j5(2.4 - j3.2) + (4 - j3)(2.4 + j1.8) = -1 - j12\,\mathrm{V}$$

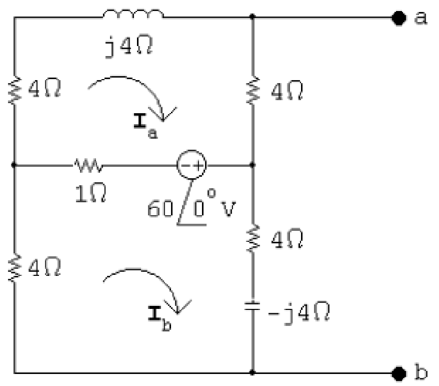
$$-25 + (1 + j3)\mathbf{I}_3 + (-1 - j12) = 0 \quad \therefore \quad \mathbf{I}_3 = 6.2 - j6.6\,\mathrm{A}$$

$$\mathbf{I}_Z = \mathbf{I}_3 - \mathbf{I}_2 = (6.2 - j6.6) - (2.4 - j3.2) = 3.8 - j3.4\,\mathrm{A}$$

$$Z = \frac{\mathbf{V}_Z}{\mathbf{I}_Z} = \frac{-1 - j12}{3.8 - j3.4} = 1.42 - j1.88\,\Omega$$

Problem 8:

Open circuit voltage:




$$(9 + j4)\mathbf{I}_a - \mathbf{I}_b = -60\underline{/0^\circ}$$

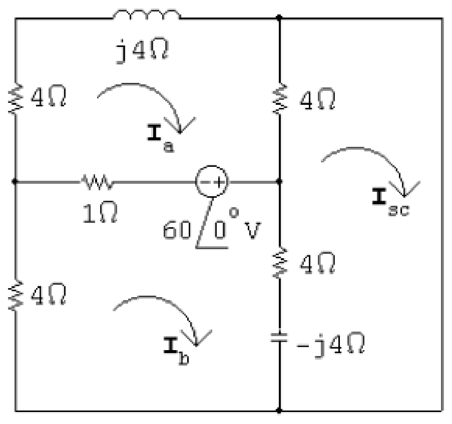
$$-\mathbf{I}_a + (9 - j4)\mathbf{I}_b = 60\underline{/0^\circ}$$

Solving,

$$\mathbf{I}_a = -5 + j2.5\,\text{A}; \qquad \mathbf{I}_b = 5 + j2.5\,\text{A}$$

$$\mathbf{V}_{\text{Th}} = 4\mathbf{I}_a + (4 - j4)\mathbf{I}_b = 10\underline{/0^\circ}\,\text{V}$$

Short circuit current:




$$(9 + j4)\mathbf{I}_a - 1\mathbf{I}_b - 4\mathbf{I}_{sc} = -60$$

$$-1\mathbf{I}_a + (9 - j4)\mathbf{I}_b - (4 - j4)\mathbf{I}_{sc} = 60$$

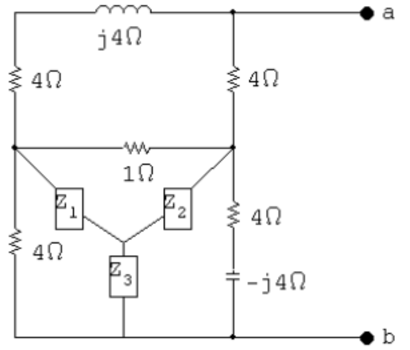
$$-4\mathbf{I}_a - (4 - j4)\mathbf{I}_b + (8 - j4)\mathbf{I}_{sc} = 0$$

Solving,

$$\mathbf{I}_{sc} = 2.07\underline{/0^\circ}$$

$$Z_{\mathrm{Th}} = \frac{\mathbf{V}_{\mathrm{Th}}}{\mathbf{I}_{\mathrm{sc}}} = \frac{10\underline{/0^\circ}}{2.07\underline{/0^\circ}} = 4.83\,\Omega$$

Alternate calculation for $Z_{\mathrm{Th}}$:

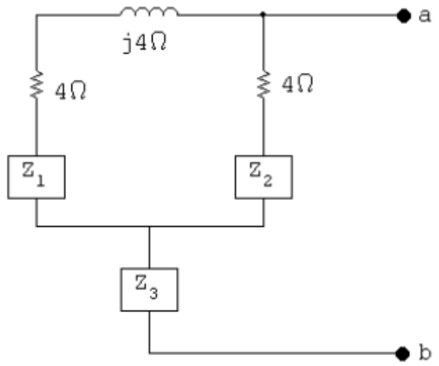



$$\sum Z = 4 + 1 + 4 - j4 = 9 - j4$$

$$Z_1 = \frac{4}{9 - j4}$$

$$Z_2 = \frac{4 - j4}{9 - j4}$$

$$Z_3 = \frac{16 - j16}{9 - j4}$$




$$Z_{\mathrm{a}} = 4 + j4 + \frac{4}{9 - j4} = \frac{56 + j20}{9 - j4}$$

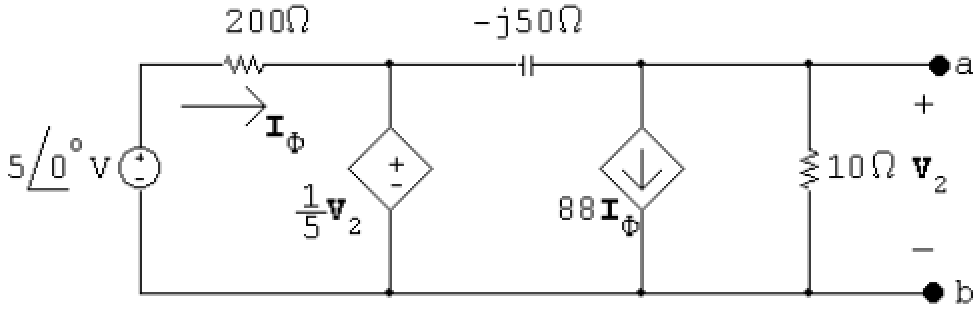
$$Z_{\mathrm{b}} = 4 + \frac{4 - j4}{9 - j4} = \frac{40 - j20}{9 - j4}$$

$$Z_{\mathrm{a}} \| Z_{\mathrm{b}} = \frac{2640 - j320}{884 - j384}$$

$$Z_3 + Z_{\mathrm{a}} \| Z_{\mathrm{b}} = \frac{16 - j16}{9 - j4} + \frac{2640 - j320}{884 - j384} = 4.83\,\Omega$$

Problem 9:

Open circuit voltage:



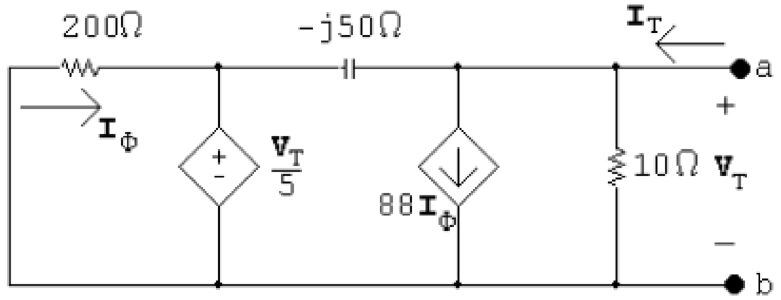

$$\frac{\mathbf{V}_2}{10} + 88\mathbf{I}_\phi + \frac{\mathbf{V}_2 - \frac{1}{5}\mathbf{V}_2}{-j50} = 0$$

$$\mathbf{I}_\phi = \frac{5 - (\mathbf{V}_2/5)}{200}$$

Solving,

$$\mathbf{V}_2 = -66 + j88 = 110\underline{/126.87^\circ}\ \mathrm{V} = \mathbf{V}_{\mathrm{Th}}$$

Find the Thévenin equivalent impedance using a test source:




$$\mathbf{I}_T = \frac{\mathbf{V}_T}{10} + 88\mathbf{I}_\phi + \frac{0.8\mathbf{V}_t}{-j50}$$

$$\mathbf{I}_\phi = \frac{-\mathbf{V}_T/5}{200}$$

$$\mathbf{I}_T = \mathbf{V}_T\left(\frac{1}{10} - 88\frac{\mathbf{V}_T/5}{200} + \frac{0.8}{-j50}\right)$$

$$\therefore \qquad \frac{\mathbf{V}_T}{\mathbf{I}_T} = 30 - j40 = Z_{\text{Th}}$$

$$\mathbf{I}_{\text{N}} = \frac{\mathbf{V}_{\text{Th}}}{Z_{\text{Th}}} = \frac{-66 + j88}{30 - j40} = -2.2 + j0\,\text{A}$$

The Norton equivalent circuit: