Week 4 Workshop Example Solutions

Example 4:

[a] Let i be the current in the direction of the voltage drop $v_o(t)$. Then by hypothesis

$$i = i_f + B_1' e^{-\alpha t} \cos \omega_d t + B_2' e^{-\alpha t} \sin \omega_d t$$

$$i_f = i(\infty) = 0, \qquad i(0) = \frac{V_g}{R} = B_1'$$

Therefore $\quad i = B_1' e^{-\alpha t} \cos \omega_d t + B_2' e^{-\alpha t} \sin \omega_d t$

$$L\frac{di(0)}{dt} = 0, \qquad \text{therefore} \quad \frac{di(0)}{dt} = 0$$

$$\frac{di}{dt} = \left[(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\alpha B_2' + \omega_d B_1')\sin\omega_d t\right] e^{-\alpha t}$$

$$\text{Therefore} \quad \omega_d B_2' - \alpha B_1' = 0; \qquad B_2' = \frac{\alpha}{\omega_d} B_1' = \frac{\alpha}{\omega_d}\frac{V_g}{R}$$

Therefore

$$v_o = L\frac{di}{dt} = -\left\{L\left(\frac{\alpha^2 V_g}{\omega_d R} + \frac{\omega_d V_g}{R}\right)\sin\omega_d t\right\} e^{-\alpha t}$$

$$= -\left\{\frac{LV_g}{R}\left(\frac{\alpha^2}{\omega_d} + \omega_d\right)\sin\omega_d t\right\} e^{-\alpha t}$$

$$= -\frac{V_g L}{R}\left(\frac{\alpha^2 + \omega_d^2}{\omega_d}\right) e^{-\alpha t}\sin\omega_d t$$

$$= -\frac{V_g L}{R}\left(\frac{\omega_o^2}{\omega_d}\right) e^{-\alpha t}\sin\omega_d t$$

$$= -\frac{V_g L}{R\omega_d}\left(\frac{1}{LC}\right) e^{-\alpha t}\sin\omega_d t$$

$$v_o = -\frac{V_g}{RC\omega_d} e^{-\alpha t}\sin\omega_d t\,\mathrm{V}, \quad t \geq 0$$

[b] $$\frac{dv_o}{dt} = -\frac{V_g}{\omega_d RC}\{\omega_d \cos \omega_d t - \alpha \sin \omega_d t\}e^{-\alpha t}$$

$$\frac{dv_o}{dt} = 0 \quad \text{when} \quad \tan \omega_d t = \frac{\omega_d}{\alpha}$$

$$\text{Therefore} \quad \omega_d t = \tan^{-1}(\omega_d/\alpha) \quad (\text{smallest } t)$$

$$t = \frac{1}{\omega_d} \tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

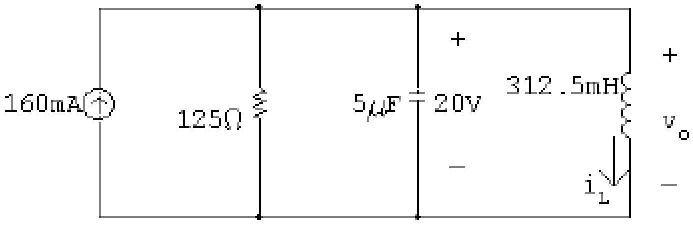
Example 8:

$t < 0$:

$$v_o(0^-) = v_o(0^+) = \frac{625}{781.25}(25) = 20\,\text{V}$$

$$i_\text{L}(0^-) = i_\text{L}(0^+) = 0$$

$t > 0$:



$$-160\times 10^{-3} + \frac{20}{125} + i_\text{C}(0^+) + 0 = 0; \qquad \therefore \quad i_\text{C}(0^+) = 0$$

$$\frac{1}{2RC} = \frac{1}{2(125)(5\times 10^{-6})} = 800\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(312.5\times 10^{-3})(5\times 10^{-6})} = 64\times 10^4$$

$$\therefore \quad \alpha^2 = \omega_o^2 \quad \text{critically damped}$$

[a] $v_o = V_f + D_1' te^{-800t} + D_2' e^{-800t}$

$$V_f = 0$$

$$\frac{dv_o(0)}{dt} = -800D_2' + D_1' = 0$$

$$v_o(0^+) = 20 = D_2'$$

$$D_1' = 800D_2' = 16{,}000\,\text{V/s}$$

$$\therefore \quad v_o = 16{,}000te^{-800t} + 20e^{-800t}\,\text{V}, \quad t \geq 0^+$$

[b] $i_{\rm L} = I_f + D_3' t e^{-800t} + D_4' e^{-800t}$

$$i_{\rm L}(0^+) = 0; \qquad I_f = 160\,\text{mA}; \qquad \frac{di_{\rm L}(0^+)}{dt} = \frac{20}{312.5 \times 10^{-3}} = 64\ \text{A/s}$$

$$\therefore \quad 0 = 160 + D_4'; \qquad D_4' = -160\,\text{mA};$$

$$-800D_4' + D_3' = 64; \qquad D_3' = -64\,\text{A/s}$$

$$\therefore \quad i_{\rm L} = 160 - 64{,}000te^{-800t} - 160e^{-800t}\,\text{mA} \qquad t \geq 0$$