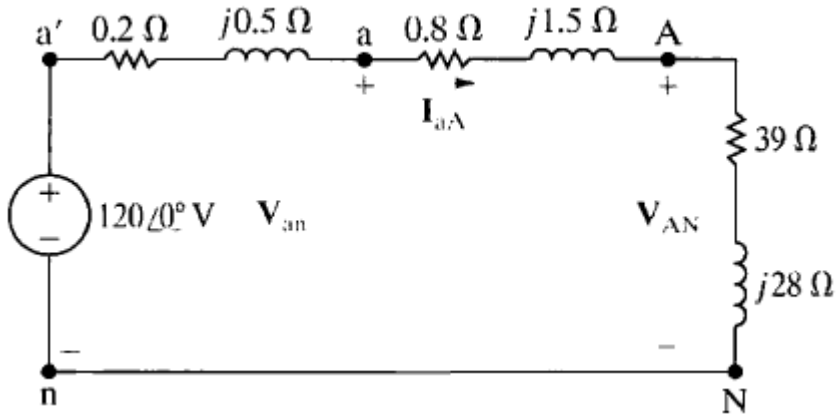
Week 6 Workshop Example Solutions

Example 1:

- $\mathbf{Z}_Y = 50 + j25 = 55.9017\angle 26.5651^\circ\ \Omega$, $V_P = V_L/\quad = 508.0682$ V
- $\mathbf{V}_{an} = 508.0682\angle 0^\circ$, $\mathbf{V}_{bn} = 508.0682\angle -120^\circ$, $\mathbf{V}_{cn} = 508.0682\angle -240^\circ$
- $I_L = V_P/|\mathbf{Z}_Y| = 9.0886$ A, $\mathbf{I}_A = I_L\angle -\theta_Y = 9.0886\angle -26.5651^\circ$ A, $\mathbf{I}_B = 9.0886\angle -146.5651^\circ$ A
- $\mathbf{I}_C = 9.0886\angle 93.4349^\circ$ A, $\mathbf{V}_{ab} = 880\angle 30^\circ$ V, $\mathbf{V}_{bc} = 880\angle -90^\circ$ V, $\mathbf{V}_{ca} = 880\angle -210^\circ$ V
- $|\mathbf{S}_L| = \quad V_L I_L = 13852.8883$ VA
- $P_L = |\mathbf{S}_L|\cos(\theta_Y) = 12{,}390.4$ W
- $Q_L = |\mathbf{S}_L|\sin(\theta_Y) = 6195.2$ VAR
- $pf = \cos(\theta_Y) = 0.8944272$

Problem 2:



The a-phase line current is

$$\mathbf{I}_{aA} = \frac{120\angle 0^\circ}{(0.2 + 0.8 + 39) + j(0.5 + 1.5 + 28)}$$

$$= \frac{120\angle 0^\circ}{40 + j30}$$

$$= 2.4\angle -36.87^\circ \text{ A}.$$

For a positive phase sequence,

$$\mathbf{I}_{bB} = 2.4 \angle -156.87^\circ \text{ A},$$
$$\mathbf{I}_{cC} = 2.4 \angle 83.13^\circ \text{ A}.$$

c) The phase voltage at the A terminal of the load is

$$\mathbf{V}_{AN} = (39 + j28)(2.4 \angle -36.87^\circ)$$
$$= 115.22 \angle -1.19^\circ \text{ V}.$$

For a positive phase sequence,

$$\mathbf{V}_{BN} = 115.22 \angle -121.19^\circ \text{ V},$$
$$\mathbf{V}_{CN} = 115.22 \angle 118.81^\circ \text{ V}.$$

d) For a positive phase sequence, the line voltages lead the phase voltages by 30°; thus

$$\mathbf{V}_{AB} = (\sqrt{3} \angle 30^\circ)\mathbf{V}_{AN}$$
$$= 199.58 \angle 28.81^\circ \text{ V},$$
$$\mathbf{V}_{BC} = 199.58 \angle -91.19^\circ \text{ V},$$
$$\mathbf{V}_{CA} = 199.58 \angle 148.81^\circ \text{ V}.$$

e) The phase voltage at the a terminal of the source is

$$\mathbf{V}_{an} = 120 - (0.2 + j0.5)(2.4 \angle -36.87^\circ)$$
$$= 120 - 1.29 \angle 31.33^\circ$$
$$= 118.90 - j0.67$$
$$= 118.90 \angle -0.32^\circ \text{ V}.$$

For a positive phase sequence,

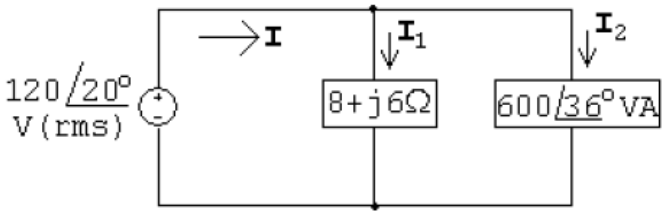
$$\mathbf{V}_{bn} = 118.90 \angle -120.32^\circ \text{ V},$$
$$\mathbf{V}_{cn} = 118.90 \angle 119.68^\circ \text{ V}.$$

f) The line voltages at the source terminals are

$$\mathbf{V}_{ab} = (\sqrt{3} \angle 30^\circ)\mathbf{V}_{an}$$
$$= 205.94 \angle 29.68^\circ \text{ V},$$
$$\mathbf{V}_{bc} = 205.94 \angle -90.32^\circ \text{ V},$$
$$\mathbf{V}_{ca} = 205.94 \angle 149.68^\circ \text{ V}.$$

Problem 3:

The a-phase of the circuit is shown below:



$$\mathbf{I}_1 = \frac{120\underline{/20^\circ}}{8+j6} = 12\underline{/-16.87^\circ}\text{A (rms)}$$

$$\mathbf{I}_2^* = \frac{600\underline{/36^\circ}}{120\underline{/20^\circ}} = 5\underline{/16^\circ}\text{A (rms)}$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 12\underline{/-16.87^\circ} + 5\underline{/-16^\circ} = 17\underline{/-16.61^\circ}\text{ A (rms)}$$

$$S_\text{a} = \mathbf{V}\mathbf{I}^* = (120\underline{/20^\circ})(17\underline{/16.61^\circ}) = 2040\underline{/36.61^\circ}\text{ VA}$$

$$S_\text{T} = 3S_\text{a} = 6120\underline{/36.61^\circ}\text{ VA}$$

Problem 4:

The complex power of the source per phase is $S_s = 20{,}000\underline{/(\cos^{-1} 0.6)} = 20{,}000\underline{/53.13^\circ} = 12{,}000 + j16{,}000$ kVA. This complex power per phase must equal the sum of the per-phase impedances of the two loads:

$$S_s = S_1 + S_2 \quad \text{so} \quad 12{,}000 + j16{,}000 = 10{,}000 + S_2$$

$$\therefore \quad S_2 = 2000 + j16{,}000 \text{ VA}$$

$$\text{Also,} \quad S_2 = \frac{|V_{\text{rms}}|^2}{Z_2^*}$$

$$|V_{\text{rms}}| = \frac{|\text{V}_{\text{load}}|}{\sqrt{3}} = 120 \text{ V (rms)}$$

$$\text{Thus,} \quad Z_2^* = \frac{|V_{\text{rms}}|^2}{S_2} = \frac{(120)^2}{2000 + j16{,}000} = 0.11 - j0.89\,\Omega$$

$$\therefore \quad Z_2 = 0.11 + j0.89\,\Omega$$

Problem 5:

$$\textbf{[a]}\ \mathbf{I}_{\mathrm{AB}} = \frac{13{,}200\underline{/0^\circ}}{100 - j75} = 105.6\underline{/36.87^\circ}\ \mathrm{A\ (rms)}$$

$$\mathbf{I}_{\mathrm{BC}} = 105.6\underline{/156.87^\circ}\ \mathrm{A\ (rms)}$$

$$\mathbf{I}_{\mathrm{CA}} = 105.6\underline{/-83.13^\circ}\ \mathrm{A\ (rms)}$$

$$\textbf{[b]}\ \mathbf{I}_{\mathrm{aA}} = \sqrt{3}\underline{/-30^\circ}\mathbf{I}_{\mathrm{AB}} = 182.9\underline{/66.87^\circ}\ \mathrm{A\ (rms)}$$

$$\mathbf{I}_{\mathrm{bB}} = 182.9\underline{/-173.13^\circ}\ \mathrm{A\ (rms)}$$

$$\mathbf{I}_{\mathrm{cC}} = 182.9\underline{/-53.13^\circ}\ \mathrm{A\ (rms)}$$

$$\textbf{[c]}\ \mathbf{I}_{\mathrm{ba}} = \mathbf{I}_{\mathrm{AB}} = 105.6\underline{/36.87^\circ}\ \mathrm{A\ (rms)}$$

$$\mathbf{I}_{\mathrm{cb}} = \mathbf{I}_{\mathrm{BC}} = 105.6\underline{/156.87^\circ}\ \mathrm{A\ (rms)}$$

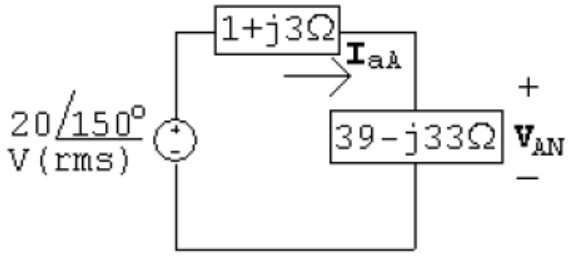
$$\mathbf{I}_{\mathrm{ac}} = \mathbf{I}_{\mathrm{CA}} = 105.6\underline{/-83.13^\circ}\ \mathrm{A\ (rms)}$$

Problem 6

[a] $\mathbf{V}_{an} = \mathbf{V}_{cn} - \underline{/120^\circ} = 20\underline{/-210^\circ} = 20\underline{/150^\circ}\,\text{V (rms)}$

$Z_y = Z_\Delta/3 = 39 - j33\,\Omega$

The a-phase circuit is



[b] $\mathbf{I}_{aA} = \dfrac{20\underline{/150^\circ}}{40 - j30} = 0.4\underline{/-173.13^\circ}\,\text{A (rms)}$

[c] $\mathbf{V}_{AN} = (39 + j33)\mathbf{I}_{aA} = 20.44\underline{/146.63^\circ}\,\text{V (rms)}$

$\mathbf{V}_{AB} = \sqrt{3}\underline{/30^\circ}\mathbf{V}_{AN} = 35.39\underline{/176.63^\circ}\,\text{A (rms)}$