Week 6 Workshop Example Solutions

Example 1:

$$H(s) = \frac{\omega_c}{s + \omega_c} = \frac{12{,}700}{s + 12{,}700}$$

$$H(j\omega) = \frac{12{,}700}{12{,}700 + j\omega}$$

Problem 2:

$$Z_L = j\omega L = j0L = 0 \quad \text{so it is a short circuit.}$$

$$\text{At } \omega = 0, \quad V_o = V_i$$

$$Z_L = j\omega L = j\infty L = \infty \quad \text{so it is an open circuit.}$$

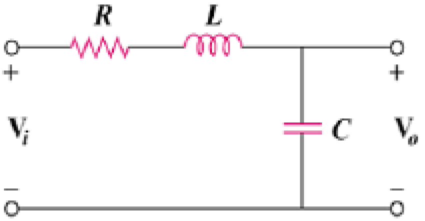
$$\text{At } \omega = \infty, \quad V_o = 0$$

$$H(s) = \frac{V_o}{V_i} = \frac{R}{R + sL} = \frac{R/L}{s + R/L}$$

$$H(s) = \frac{V_o}{V_i} = \frac{R \| R_L}{R \| R_L + sL} = \frac{\frac{R}{L}\left(\frac{R_L}{R + R_L}\right)}{s + \frac{R}{L}\left(\frac{R_L}{R + R_L}\right)}$$

Problem 3:

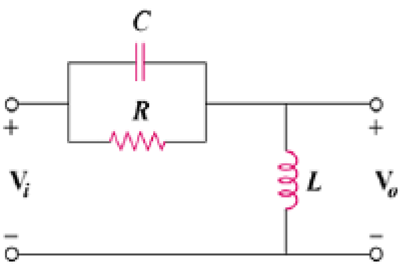
(a)



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1/j\omega C}{R + j\omega L + 1/j\omega C}$$

$$= \frac{1}{1 + j\omega RC - \omega^2 LC}$$

(b)



$$\mathrm{R} \parallel \frac{1}{\mathrm{j\omega C}} = \frac{\mathrm{R}}{1 + \mathrm{j\omega RC}}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega L}{j\omega L + R/(1 + j\omega RC)} = \frac{j\omega L\,(1 + j\omega RC)}{R + j\omega L\,(1 + j\omega RC)}$$

$$= \frac{j\omega L - \omega^2 RLC}{R + j\omega L - \omega^2 RLC}$$

Problem 4:

Refer to notes from the lecture session

Problem 5:

Refer to notes from the lecture session

Problem 6

Refer to notes from the lecture session

Problem 7:

Sketch the Bode plots for

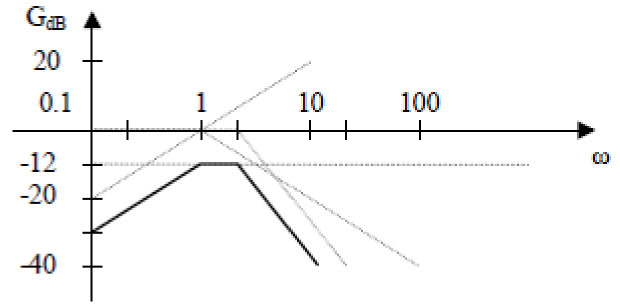
$$G(s) = \frac{s}{(s+2)^2(s+1)}, \qquad s = j\omega$$

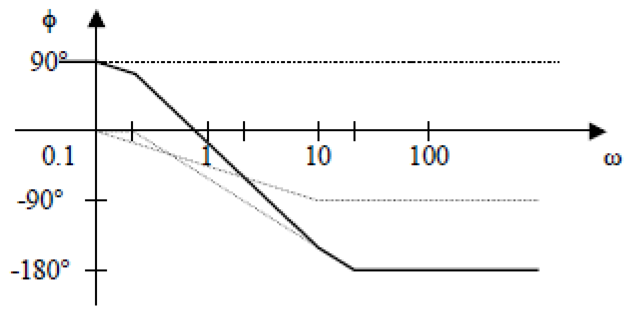
$$\mathbf{G}(\omega) = \frac{(1/4)\, j\omega}{(1+j\omega)(1+j\omega/2)^2}$$

$$G_{dB} = -20\log_{10}4 + 20\log_{10}|j\omega| - 20\log_{10}|1+j\omega| - 40\log_{10}|1+j\omega/2|$$

$$\phi = 90° - \tan^{-1}\omega - 2\tan^{-1}\omega/2$$

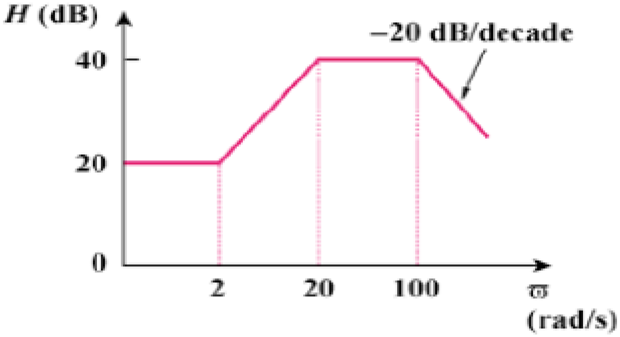
Magnitude and phase plots





Problem 8:

Find the transfer function $\mathbf{H}(\omega)$ *with the Bode magnitude plot shown*



$$20 = 20\log_{10} k \longrightarrow k = 10$$

$$\text{A zero of slope } +20\ dB/dec\ at\quad \omega = 2 \rightarrow \quad 1 + j\omega/2$$

$$\text{A pole of slope } -20\ dB/dec\quad \omega = 20 \longrightarrow \frac{1}{1 + j\omega/20}$$

$$\text{A pole of slope } -20\ dB/dec\ at\ \omega = 100 \longrightarrow \frac{1}{1 + j\omega/100}$$

Hence

$$\mathbf{H}(\omega) = \frac{10(1 + j\omega/2)}{(1 + j\omega/20)(1 + j\omega/100)}$$