Week 10 Workshop Example Solutions

Problem 1:

$$v_i = \frac{4A}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n} \sin n\omega_0 (t + T/4)$$

$$= \frac{4A}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \left(\frac{1}{n} \sin \frac{n\pi}{2} \right) \cos n\omega_0 t$$

$$\omega_0 = \frac{2\pi}{4\pi} \times 10^3 = 500 \text{ rad/s}; \qquad \frac{4A}{\pi} = 60$$

$$v_i = 60 \sum_{n=1,3,5,\ldots}^{\infty} \left(\frac{1}{n} \sin \frac{n\pi}{2} \right) \cos 500nt \text{ V}$$

From the circuit

$$\mathbf{V}_o = \frac{\mathbf{V}_i}{R + j\omega L} \cdot j\omega L = \frac{j\omega}{R/L + j\omega} \mathbf{V}_i = \frac{j\omega}{1000 + j\omega} \mathbf{V}_i$$

$$\mathbf{V}_{i1} = 60\underline{/0^\circ} \text{ V}; \qquad \omega = 500 \text{ rad/s}$$

$$\mathbf{V}_{i3} = -20\underline{/0^\circ} = 20\underline{/180^\circ} \text{ V}; \qquad 3\omega = 1500 \text{ rad/s}$$

$$\mathbf{V}_{i5} = 12\underline{/0^\circ} \text{ V}; \qquad 5\omega = 2500 \text{ rad/s}$$

$$\mathbf{V}_{o1} = \frac{j500}{1000 + j500}(60\underline{/0^\circ}) = 26.83\underline{/63.43^\circ} \text{ V}$$

$$\mathbf{V}_{o3} = \frac{j1500}{1000 + j1500}(20\underline{/180^\circ}) = 16.64\underline{/-146.31^\circ} \text{ V}$$

$$\mathbf{V}_{o5} = \frac{j2500}{1000 + j2500}(12\underline{/0^\circ}) = 11.14\underline{/21.80^\circ} \text{ V}$$

$$\therefore \qquad v_o = 26.83 \cos(500t + 63.43^\circ) + 16.64 \cos(1500t - 146.31^\circ)$$

$$+11.14 \cos(2500t + 21.80^\circ) + \ldots \text{ V}$$

Problem 2:

[a] $v(t)$ is even and has both half- and quarter-wave symmetry, therefore $a_v = 0$, $b_k = 0$ for all k, $a_k = 0$ for k-even; for odd k we have

$$a_k = \frac{8}{T}\int_0^{T/4} V_m \cos k\omega_0 t\,dt = \frac{4V_m}{\pi k}\sin\left(\frac{k\pi}{2}\right)$$

$$v(t) = \frac{4V_m}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\left[\frac{1}{n}\sin\frac{n\pi}{2}\right]\cos n\omega_0 t\ \mathrm{V}$$

[b] $v(t)$ is even and has both half- and quarter-wave symmetry, therefore $a_v = 0$, $b_k = 0$ for k-even, $a_k = 0$ for all k; for k-odd we have

$$a_k = \frac{8}{T}\int_0^{T/4}\left(\frac{4V_p}{T}t - V_p\right)\cos k\omega_0 t\,dt = \frac{-8V_p}{\pi^2 k^2}$$

Therefore $\quad v(t) = \dfrac{-8V_p}{\pi^2}\displaystyle\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n^2}\cos n\omega_0 t\ \mathrm{V}$