MapReduce and PageRank

Question 1:

Suppose our input data to a map-reduce operation consists of integer values (the keys are not important). The map function takes an integer i and produces the list of pairs (p,i) such that p is a prime divisor of i. For example, map(12) = [(2,12),(3,12)].

The reduce function is addition. That is, reduce(p,[i_1,i_2,...,i_k]) is (p,i_1+i_2+...+i_k).

Compute the output, if the input is the set of integers 15, 21, 24, 30, 49.

Sol:

1. Prime number : 2,3,5,11,----

map (15) : [3,15], [5,15]

map (21) : [3,21], [7,21]

map (27) : [2,27), [3,24]

map (30) : [2,30], [3,30], [5,30]

map (47) : [7,49]

by combining all common elements pairs i.e, compare left element and add rightmost to get the solution.

reduce (2,54]

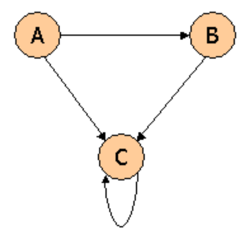
reduce (3,90)

reduce (5,45)

reduce (7,70)

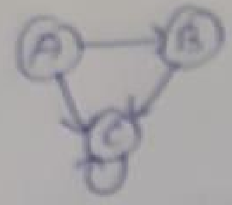
Question 2:

Consider three Web pages with the following links:



Suppose we compute PageRank with a β of 0.7, and we introduce the additional constraint that the sum of the PageRanks of the three pages must be 3, to handle the problem that otherwise any multiple of a solution will also be a solution. Compute the PageRanks *a*, *b*, and *c* of the three pages A, B, and C, respectively.

(2)



$$M = \begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 0 & 0 \\ B & 1/2 & 0 & 0 \\ C & 1/2 & 1 & 1 \end{array}$$

page rank equation:

$$r^2 = \beta M \cdot r^2 + (1-\beta)\left(1/N\right)_N \qquad \beta = 0.7 = 7/10$$

$$r^2 = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\beta M = 7/10 \begin{bmatrix} 0 & 0 & 0 \\ 1/2 & 0 & 0 \\ 1/2 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 7/20 & 0 & 0 \\ 7/20. & 7/10 & 7/10 \end{bmatrix}$$

$$\beta M \cdot r^2 = \begin{bmatrix} 0 & 0 & 0 \\ 7/20 & 0 & 0 \\ 7/20 & 7/10 & 7/10 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.1155 \\ 0.35 \times 0.35 + 0.7 \times 0.33 + 0.9 \times 0.33 \end{bmatrix} = \begin{bmatrix} 0. \\ 0.1155 \\ 0.5775 \end{bmatrix}$$

$$(1-\beta)\left[\frac{1}{N}\right]_N$$

$$(1-0.7)\begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$r^1 = \begin{bmatrix} 0 \\ 0.1155 \\ 0.5775 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.2155 \\ 0.6775 \end{bmatrix}$$

$$r^2 = \beta M x^2 + (1-\beta)[1/N]$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0.35 & 0 & 0 \\ 0.35 & 0.9 & 0.9 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.2155 \\ 0.6775 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.2155 \\ 0.6775 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 \\ 0.125 \\ 0.7601 \end{bmatrix}$$

$$r^3 = \beta M x^2 + (1-\beta)[1/N]$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0.35 & 0 & 0 \\ 0.35 & 0.7 & 0.7 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.125 \\ 0.701 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.035 \\ 0.66157 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 \\ 0.135 \\ 0.76157 \end{bmatrix}$$

$$r^4 = \beta M r^3 + (1-\beta)[1/N]$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0.35 & 0 & 0 \\ 0.35 & 0.7 & 0.7 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.135 \\ 0.76157 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.035 \\ 0.6626 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 \\ 0.135 \\ 0.7626 \end{bmatrix}$$

$$r^5 = \beta M x^4 + (1-\beta)[1/N]$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0.35 & 0 & 0 \\ 0.35 & 0.7 & 0.7 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.135 \\ 0.7626 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.055 \\ 0.6633 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

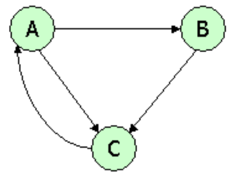
$$= \begin{bmatrix} 0.1 \\ 0.125 \\ 0.7633 \end{bmatrix}$$

After 5th iter

pgrank $\begin{bmatrix} 0.1 \\ 0.135 \\ 0.7633 \end{bmatrix} \times 3$

$a = 0.3$

Question 3:



Suppose we compute PageRank with β=0.85. Write the equations for the PageRanks *a*, *b*, and *c* of the three pages A, B, and C, respectively.

Sol:

Q-3

Formula

$A = \beta * C + (1-\beta)\, 1/3$

$B = \beta * A/2 + (1-\beta)\, 1/3$

$C = \beta * (A/2 + B) + (1-\beta)\, 1/3$

Since $\beta = 0.85$

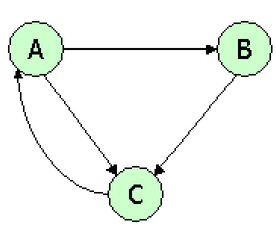
$A = 0.85C + (1-0.85)\, 1/3$ $B = 0.85 \times 0.5A + (1-0.85)\, 1/3$

$\boxed{A = 0.85C + 0.05}$ $\boxed{B = 0.425A + 0.05}$

$C = 0.85\,[0.5A + 0.85] + 0.05$

$\boxed{C = 0.425A + 0.85B + 0.05}$

Question 4:



Assuming no "taxation," compute the PageRanks *a*, *b*, and *c* of the three pages A, B, and C, using iteration, starting with the "0th" iteration where all three pages have rank *a* = *b* = *c* = 1. Compute as far as the 5th iteration, and also determine what the PageRanks are in the limit.

SOl:

Q-4 Formula = $A = C$

$B = A/2$

$C = A/2 + B$

At 0^{th} iteration

$A = 1 \quad B = 1 \quad C = 1$

1^{th} iteration

$A = 1 \quad B = 1/2 \quad C = 3/2$

2^{nd} iteration

$A = 3/2 \quad B = 1/2 \quad C = 1/2 + 1/2 = 1$

3^{rd} iteration

$A = 1 \quad B = 3/2 \times 1/2 = 3/4 \quad C = 3/4 + 1/2$

$C = 5/4$

4^{th} iteration

$A = 5/4 \quad B = 1/2 \quad C = 1/2 + 3/4$

$C = 5/4$

At 5^{th} iteration

$A = 5/4 \quad B = 5/8 \quad C = 5/8 + 1/2$

$C = 9/8$

$\therefore$ pagerank at 8^{th} iteration are

$A = 5/4$

$B = 5/8$

$C = 9/8$