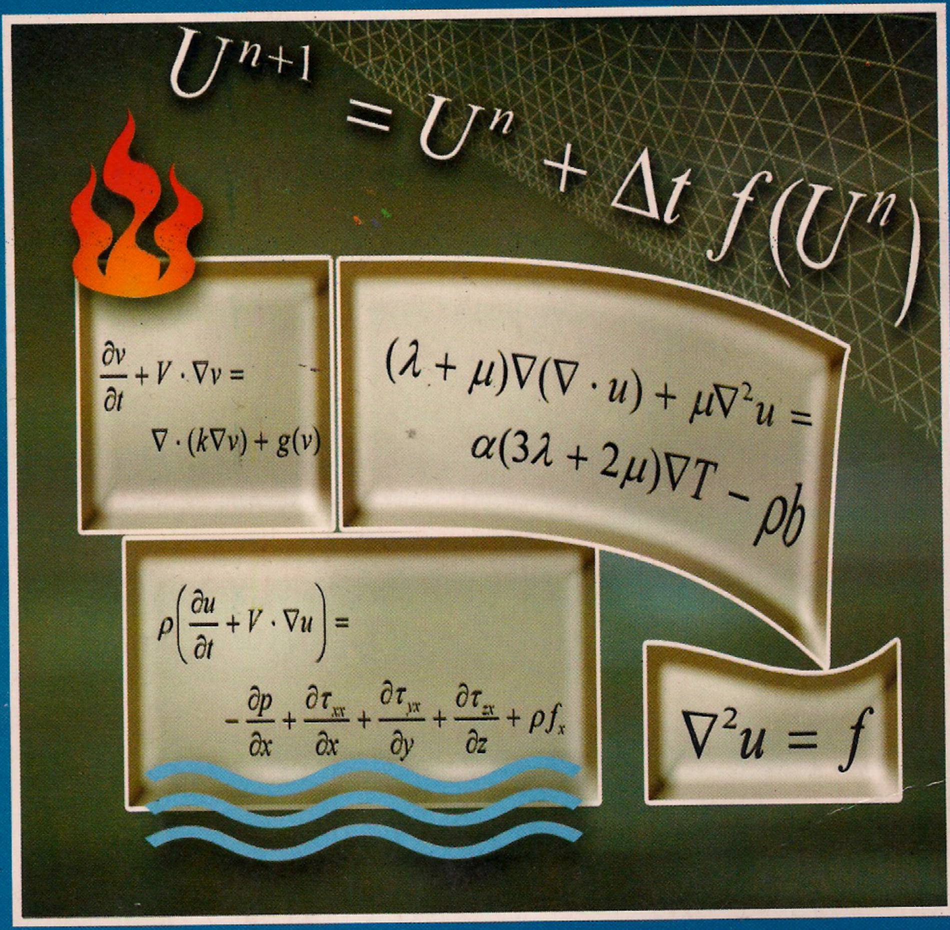
# NUMERICAL METHODS



## EXERCISE 1.1

1. The law of machine is $P = aW + b$, where $P$ is the effort and $W$, the load in *lb*. Sketch a graph showing the relation between $P$ and $W$, given

| | | | | | |
|---|---|---|---|---|---|
| $P$ | 60 | 75 | 100 | 125 | 145 |
| $W$ | 225 | 300 | 430 | 560 | 600 |

Find $P$ when $W = 500$.

2. $R$ is the resistance to motion of a train at speed $V$. Find a law of the type $R = aV^2 + b$ connecting $R$ and $V$ using the following data

| | | | | | |
|---|---|---|---|---|---|
| $R$ kg/ton | 8 | 10 | 15 | 21 | 30 |
| $V$ (km/hr) | 10 | 20 | 30 | 40 | 50 |

3. The resistance $R$ of a carbon filament lamp was measured at various values of voltage $V$ and the following observations were made.

| $V$ | 62 | 70 | 78 | 84 | 92 |
|---|---|---|---|---|---|
| $R$ | 73 | 70.7 | 69.2 | 67.8 | 66.3 |

*(Ranchi B.Tech 1986)*

Assuming a law of the form $R = a/V + b$, find by graphical method the best values of $a$ and $b$.

4. $\mu$, the co-efficient of friction between a belt and pulley and $v$, the velocity of the belt in ft/min, are connected as shown in the following table:

| $v$ | 500 | 1000 | 2000 | 4000 | 6000 |
|---|---|---|---|---|---|
| $\mu$ | 0.29 | 0.33 | 0.38 | 0.45 | 0.51 |

The probable law is $\mu = a + b\sqrt{v}$. Test graphically the accuracy of this law and if it is true, find the values of $a$ and $b$.

5. Fit a curve of the form $y = ae^{bx}$ to the following data:

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $y$ | 14 | 27 | 40 | 55 | 68 | 300 |

6. The following observations are corresponding to pressure and specific volume of dry saturated steam. Fit a curve of the form $PV^n = k$ by graphical method.

| $V$ | 38.4 | 20 | 8.51 | 4.44 | 3.03 | 2.31 |
|---|---|---|---|---|---|---|
| $P$ | 10 | 20 | 50 | 100 | 150 | 200 |

## ANSWERS

1. $P = 0.21W + 12,\ P = 117$
2. $a = 0.0085,\ b = 7.35$
3. $a = 1120,\ b = 55.1$
4. $a = 0.2\ \ b = 0.0044$
5. $y = 7.943\, e^{0.5419x}$
6. $PV^{1.073} = 501$

## EXERCISE 1.2

1. The weights of a calf taken at weekly intervals are given below. Fit a straight line using the method of group averages.

| Age in weeks | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| weight | 52.5 | 58.7 | 65 | 70.2 | 75.4 | 81.1 | 87.2 | 95.5 | 102.2 | 108.4 |

2. Fit a curve of the form $y = ax^n$ to the following set of observations by the method of group averages:

| $x$ | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|---|
| $y$ | 1.06 | 1.33 | 1.52 | 1.68 | 1.81 | 1.91 | 2,01 | 2.11 |

3. Fit a curve of the form $y = ab^x$ using the method of group averages for the following data:

| $x$ | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|
| $y$ | 7.32 | 8.24 | 9.20 | 10.19 | 11.01 | 12.05 |

4. Convert the equation $y = b/x(x - a)$ to a linear form and hence, determine $a$ and $b$ which will fit the following data using the method of group averages:

| $x$ | 8 | 10 | 15 | 20 | 30 | 40 |
|---|---|---|---|---|---|---|
| $y$ | 13 | 14 | 15.4 | 16.3 | 17.2 | 17.8 |

5. The following data represent test values obtained while testing a centrifugal pump. Assuming the relation to be $H = a + bQ + cQ^2$, where $Q$ is the discharge in liter per second and $H$, head in meter of water, find the relation by the method of group averages.

| $Q$ | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 | 5.5 | 6 |
|---|---|---|---|---|---|---|---|---|---|
| $H$ | 18 | 17.8 | 17.5 | 17 | 15.8 | 14.8 | 13.3 | 11.7 | 9 |

*(M.U, B.E., 1971)*

6. The temperature $\theta$ of a vessel of cooling water and the time $t$ in minutes since the beginning of observation are connected by the law of the form $\theta = ae^{bt} + c$. The corresponding values of $t$ and $\theta$ are given by:

| $t$ | 0 | 1 | 2 | 3 | 5 | 7 | 10 | 15 | 20 |
|---|---|---|---|---|---|---|---|---|---|
| $\theta$ | 52.2 | 48.8 | 46.0 | 43.5 | 39.7 | 36.5 | 33.0 | 28.7 | 26.0 |

Find the best values of $a$, $b$ and $c$ using the method of group averages.

7. Fit a curve of the form $y = a + bx^c$ to the following data using the method of group averages.

| $x$ | 1 | 2 | 4 | 6 | 10 | 16 |
|---|---|---|---|---|---|---|
| $y$ | 15 | 45 | 165 | 364 | 1004 | 2564 |

8. Fit a curve of the form $y = a + bc^x$ to the following data using the method of group averages.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | 2.4 | 3.2 | 3.7 | 5.1 | 7.8 | 13.2 | 23.6 | 44.8 | 87 |

## *ANSWERS*

1. $y = 46.048 + 6.104\,x$
2. $y = 0.4851\,x^{0.3354}$
3. $y = (6.7468)\,(1.0505)^x$
4. $a = 0.2039,\ b = 0.051$
5. $H = 1.58 + 2.1\,Q - 0.5\,Q^2$
6. $a = 29.5393,\ b = -0.09968,\ c = 21.98$
7. $y = 5 + 10x^2$
8. $y = 2.26 + (0.3)\,(2.07)^x$

# EXERCISE 1.3

1. A simply supported beam carries a concentrated load $P$ (lb) at its midpoint. Corresponding to various values of $P$, the maximum deflection $Y$ (in) is measured. The data are given below. Find a law of the type $Y = a + bP$ by the method of least squares.

| $P$ | 100 | 125 | 140 | 160 | 180 | 200 |
|---|---|---|---|---|---|---|
| $Y$ | 0.45 | 0.55 | 0.60 | 0.70 | 0.80 | 0.85 |

(*Shivaji B.E., 1984*)

2. In the following table, $y$ is the weight of potassium bromide which will dissolve in 100 gm of water at temperature $x$°C. Find a linear law between $x$ and $y$ using least square method.

| $x$(°C) | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 |
|---|---|---|---|---|---|---|---|---|
| $y$(gm) | 53.5 | 59.5 | 65.2 | 70.6 | 75.5 | 80.2 | 85.5 | 90 |

3. By the method of least squares, find the curve $y = ax + bx^2$ that best fits the following data :

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 1.8 | 5.1 | 8.9 | 14.1 | 19.8 |

4. Find the parabola of the form $y = a + bx + cx^2$ which fits most closely with the following observations by the method of least squares.

| $x$ | $-3$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 4.63 | 2.11 | 0.67 | 0.09 | 0.63 | 2.15 | 4.58 |

(*Kerala B.E., 1985*)

5. By the method of least squares, fit a second degree curve $y = a+bx+cx^2$ to the following data :

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | 2 | 6 | 7 | 8 | 10 | 11 | 11 | 10 | 9 |

6. By the method of least squares, fit a parabola $y = a + bx + cx^2$ to the following data.

| $x$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| $y$ | 3.07 | 12.85 | 31.47 | 57.38 | 91.29 |

(*Mangalore B.E.. 1985*)

7. Fit an equation of the form $y = ae^{bx}$ to the following data by the method of least squares.

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $y$ | 1.65 | 2.7 | 4.5 | 7.35 |

8. The voltage $v$ across a capacitor at time $t$ seconds is given by the following table. Use the principle of least squares to fit a curve of the form $v = a\,e^{bt}$ to the data:

| $t$ | 0 | 2 | 4 | 6 | 8 |
|---|---|---|---|---|---|
| $v$ | 150 | 63 | 28 | 12 | 5.6 |

9. Fit a curve of the form $y = ae^{bx}$ to the following data in least square sense:

| $x$ | 0 | 2 | 4 |
|---|---|---|---|
| $y$ | 5.012 | 10 | 31.62 |

10. Fit a curve of the form $y = ax^b$ to the data given below in square sense:

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 7.1 | 27.8 | 62.1 | 110 | 161 |

11. Fit a curve of the form $y = ax^b$ in least square sense to the following observations:

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 0.5 | 2 | 4.5 | 8 | 12.5 |

(*Calicut B.E., 1988*)

12. Fit a curve of the form $y = ab^x$ in least square sense to the data given below:

| $x$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $y$ | 144 | 172.8 | 207.4 | 248.8 | 298.5 |

(*Karnataka B.E., 1993*)

13. Fit a curve of the form $y = ab^x$ in least square sense to the data given below:

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $y$ | 4 | 11 | 35 | 100 |

14. Fit a straight line $y = ax + b$ and also a parabola $y = ax^2 + bx + c$ to the following set of observations:

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 1 | 5 | 10 | 22 | 38 |

Calculate the sum of squares of the residuals in each case and test which curve is more suitable to the data.

## *ANSWERS*

1. $Y = 0.004P + 0.048$
2. $y = 54.35 + 0.5184x$
3. $y = 1.37x + 0.53x^2$
4. $y = 1.243 - 0.004x + 0.22x^2$
5. $y = -1 + 3.55x - 0.27x^2$
6. $y = 0.34 - 0.78x + 0.99x^2$
7. $y = e^{0.5x}$
8. $v = 146.3\, e^{-0.4118t}$
9. $y = 4.642\, e^{0.46x}$
10. $y = 7.173\, x^{1.952}$
11. $y = 0.5012\, x^{1.9977}$
12. $y = 99.86\,(1.2)^x$
13. $y = 1.33\,(2.95)^x$
14. $y = 9.1\,x - 3$ ; $y = 2.2x^2 + 0.3x + 1.4$

    $E_1 = 70.7$, $E_2 = 2.5$, $E_2 < E_1$, parabola is the best curve of fit.

$$27.14 = 10a + 30.3333b + 102.5c$$
$$101.14 = 30.3333a + 102.5b + 369.05c$$

Solving, we get

$$a = 1.399,\ b = -1.7856 \text{ and } c = 0.6567$$

$\therefore$ From (i) the required parabola is

$$y = 1.399 - 1.7856x - 0.6567x^2$$

## EXERECISE 1.4

1. Use the method of moments to fit a straight line to the data given below :

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 1 | 3 | 5 | 7 | 9 |
| $y$ | 1.5 | 2.8 | 4.0 | 4.7 | 6.0 |

(*M.K.U.,1976*)

2. Fit a parabola of the form $y = ax^2 + bx + c$ to the data

| | | | | |
|---|---|---|---|---|
| $x$ | 1 | 2 | 3 | 4 |
| $y$ | 1.7 | 1.8 | 2.3 | 3.2 |

by the method of moments. (*Coimbatore, B.E.,1988*)

## ANSWERS

1 $y = 1.1845 + 0.5231x$

2. $y = 0.74x^2 + 0.063x + 1.53$

## EXERCISE 2.1

1. Solve $x^3 + 6x + 20 = 0$, one root being $-2$.
2. Solve $x^3 - 12x^2 + 39x - 28 = 0$, whose roots are in arithmetic progression. *(M.U., B.E. 1995)*
3. Solve $x^4 - 2x^3 - 21x^2 + 22x + 40 = 0$, whose roots are in arithmetic progression.
4. Solve $27x^3 + 42x^2 - 28x - 8 = 0$, the roots of which are in geometric progression. *(M.U., B.E. 1994)*
5. Solve $x^4 + 15x^3 + 70x^2 + 120x + 64 = 0$, whose roots are in geometric progression.
6. Solve the equation $6x^3 - 11x^2 - 3x + 2 = 0$ whose roots are in harmonic progression.

7. Solve $15x^4 - 8x^3 - 14x^2 + 8x - 1 = 0$, whose roots are in harmonic progression.
8. Solve $x^3 - 8x^2 + 9x + 18 = 0$ given that two of its roots are in the ratio 1 : 2.
9. The equation $x^4 - 4x^3 + px^2 + 4x + q = 0$ has two pairs of equal roots. Find the values of $p$ and $q$.
10. Solve the equation $x^4 - 8x^3 + 14x^2 + 8x - 15 = 0$, given that the sum of two of the roots is equal to the sum of the other two.
11. Solve $x^4 - 8x^3 + 23x^2 - 28x + 12 = 0$, given that the difference of two roots is equal to the difference of the other two.
12. Solve the equation $x^4 - 8x^3 + 7x^2 + 36x - 36 = 0$, given that product of two roots is negative of the product of the remaining two.
13. Solve $x^3 - 4x^2 - 20x + 48 = 0$, given that the relationship between two roots, $\alpha$ and $\beta$, is $\alpha + 2\beta = 0$.
14. Find the conditions in which the cubic $x^3 + px^2 + qx + r = 0$ should have its roots in

    (i) arithmetical progression (*M.U B.E., 1993, 1994*) (ii) geometrical progression and (iii) harmonic progression.
15. Solve the equation $3x^3 - 4x^2 + x + 88 = 0$, given that $2 - i\sqrt{7}$ is a root.
16. Solve $3x^5 - 4x^4 - 42x^3 + 56x^2 + 27x - 36 = 0$, given that $\sqrt{2} + \sqrt{5}$, $-\sqrt{2} - \sqrt{5}$ are two roots.

## *ANSWERS*

1. $1 \pm 3i, 2$
2. 1, 4, 7
3. $-4, -1, 2, 5$
4. $-2/9, -2/3, -2$
5. $-2, -4, -1, -8$
6. $-1/2, 2, 1/3$
7. $-1, 1, 1/3, 1/5$
8. $3, 6, -1$
9. $p = 2, q = 1$
10. $-1, 1, 3, 5$
11. 1, 2, 2, 3
12. $3, -2, 1, 6$
13. $-7, 2, 6$
14. (i) $2p^3 - 9pq + 27r = 0$ (ii) $p^3r = q^3$ (iii) $2q^3 - 9pqr + 27r^2 = 0$
15. $2 \pm i\sqrt{7}, -8/3$
16. $\sqrt{2} \pm \sqrt{5}, -\sqrt{2} \pm \sqrt{5}, 4/3$

## EXERCISE 2.2

1. If $\alpha$, $\beta$, and $\gamma$ are the roots of the $x^3 + px + q = 0$, then find
   (i) $\Sigma\alpha^3$ (*M.U.. B.E., 1994*) (ii) $\Sigma\alpha^2\beta$ and
   (iii) $\Sigma\alpha^4$
2. If $\alpha$, $\beta$ and $\gamma$ are the roots of $x^3 + px^2 + qx + r = 0$, find the values of
   (i) $\Sigma\, 1/\alpha$ (ii) $\Sigma\, \alpha^3$
   (iii) $\Sigma\, \alpha^3\beta$ (iv) $\Sigma\, (\beta^2 + \beta\gamma + \gamma^2)$
   (v) $\Sigma\, (\beta + \gamma - \alpha)^3$, and (vi) $\Sigma\, (\alpha^2 + \beta\gamma)/(\beta + \gamma)$
3. If $\alpha$, $\beta$ and $\gamma$ are the roots of $x^3 + px^2 + qx + \mathrm{r} = 0$, form the equation whose roots are
   (i) $\alpha^2, \beta^2, \gamma^2$ (ii) $\alpha\beta, \beta\gamma, \alpha\gamma$
   (iii) $\alpha(\beta + \gamma), \beta(\gamma + \alpha), \gamma(\alpha + \beta)$ and
   (iv) $\alpha + 1/\beta\gamma, \beta + 1/\alpha\gamma, \gamma + 1/\alpha\beta$.
4. If $\alpha$, $\beta$ and $\gamma$ are the roots of the equation $x^2 + px + q = 0$, obtain the equation whose roots are
   (i) $\alpha + \beta - \gamma, \beta + \gamma - \alpha, \gamma + \alpha - \beta$ (*M.U., B.E., 1993*)
   (ii) $(\alpha + \beta)(\gamma + \alpha), (\beta + \gamma)(\alpha + \beta), (\gamma + \alpha)(\beta + \gamma)$
5. If $\alpha$, $\beta$, and $\gamma$ are the roots of $x^3 - 7x + 6 = 0$, form an equation whose roots are $(\beta - \gamma)^2, (\gamma - \alpha)^2, (\alpha - \beta)^2$. (*Raipur B.E., 1987*)
6. If $\alpha$, $\beta$, and $\gamma$ are the roots of $2x^3 + 3x^2 - x - 1 = 0$, obtain an equation whose roots are $(1 - \alpha)^{-1}, (1 - \beta)^{-1}, (1 - \gamma)^{-1}$. (*Kerala B. Tech, 1988*)
7. If $\alpha$, $\beta$, and $\gamma$ are the roots of $x^3 - 3x + 1 = 0$, form the equation whose roots are $\dfrac{(\alpha - 2)}{(\alpha + 2)}, \dfrac{(\beta - 2)}{(\beta + 2)}, \dfrac{(\gamma - 2)}{(\gamma + 2)}$.
8. If $\theta$ is a root of $x^3 + x^2 - 2x - 1 = 0$, then prove that $\theta^2 - 2$ is also a root. (*M.U, B.E., 1993*)
9. If $\alpha$, $\beta$, and $\gamma$ are the roots of $x^3 + 2x^2 + 3x + 3 = 0$, prove that
   $$\frac{\alpha^2}{(\alpha + 1)^2} + \frac{\beta^2}{(\beta + 1)^2} + \frac{\gamma^2}{(\gamma + 1)^2} = 13.$$
10. Find the equation whose roots are $-3$ times those of $x^4 - 3x^3 + x^2 - 6x + 4 = 0$.
11. Find the equation whose roots are with opposite signs to those of $x^5 - 4x^4 + 3x^3 - 5x^2 + x - 11 = 0$.
12. Find the equation whose roots are reciprocal of the roots of $x^5 - 11x^4 + 7x^3 - 8x^2 + 6x - 13 = 0$.

13. Diminish by 3 the roots of $x^4 + 3x^3 - 2x^2 - 4x - 3 = 0$.

14. Diminish the equation $x^4 - 8x^3 + 19x^2 - 12x + 2 = 0$ by 2 and hence solve it. *(M.U., B.E., 1996)*

15. Increase the roots of $3x^4 + 2x^3 - 10x^2 + 15x - 9 = 0$ by 4.

16. Find the equation whose roots are the roots of the eqution $x^3 - 4x^2 - 3x - 2 = 0$ increased by 2. *(M.U., B.E., 1995)*

17. Remove the second term in $x^4 - 8x^3 - x^2 + 68x + 60 = 0$ and solve it.

18. Diminish the roots of the equation $x^4 - 4x^3 - 7x^2 + 22x + 24 = 0$ by 1 and solve it.

19. Increase the roots of the equation $x^4 - 2x^3 - 10x^2 + 6x + 21 = 0$ by 2 and solve it.

20. Solve $x^3 - 4x^2 + 5x - 2 = 0$, given that it has a double root.

## *ANSWERS*

1. (i) $-3q$ (ii) $3q$ (iii) $2p^2$

2. (i) $-(q/r)$ (ii) $pq - 3r - p^3$ (iii) $p^2q - 2q^2 - pr$ (iv) $2p^2 - 3q$ (v) $24r - p^3$

3. (i) $y^3 + (2q - p^2)y^2 + (q^2 - 2pr)y - r^2 = 0$
(ii) $y^3 - qy^2 + pry - r^2 = 0$
(iii) $y^3 - 2qy^2 + (pr + q^2)y + (r^2 - prq) = 0$
(iv) $y^3 + 2py^2 + (p^2 + q)y + pq - r = 0$

4. (i) $y^2 - 2py + 4q = 0$ (ii) $y^3 - py^2 - q^2 = 0$

5. $y^3 - 42y^2 + 441y - 400 = 0$

6. $3y^3 - 11y^2 + 9y - 2 = 0$

7. $y^3 + 33y^2 + 27y + 3 = 0$

10. $y^4 + 9y^3 + 9y^2 + 162y + 324 = 0$

11. $y^5 + 4y^4 + 3y^3 + 5y^2 + y + 11 = 0$

12. $13y^5 - 6y^4 + 8y^3 - 7y^2 + 11y - 1 = 0$

13. $y^4 + 15y^3 + 79y^2 + 173y + 129 = 0$

14. $y^4 + 5y^2 + 6 = 0$, $2 \pm \sqrt{3}$, $2 \pm \sqrt{2}$

15. $3y^4 - 46y^3 + 254y^2 - 577y + 411 = 0$

16. $y^3 - 10y^2 + 31y - 32 = 0$

17. $-1, -2, 5, 6$

18. $-2, -3, 2, 3$

19. $1 \pm 2\sqrt{2}$, $\pm\sqrt{3}$

20. $1, 1, 2$

## EXERCISE 2.3

1. Solve $x^5 + 4x^4 + x^3 + x^2 + 4x + 1 = 0$
2. Solve $x^5 + x^4 + x^3 + x^2 + x + 1 = 0$
3. Solve $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$
   *(M.U, B.E., 1987, 1990, 1996)*
4. Solve $2x^6 - 9x^5 + 10x^4 - 3x^3 + 10x^2 - 9x + 2 = 0$
5. Solve $x^4 - 2x^3 + 3x^2 - 2x + 1 = 0$ *(M.U, B.E., 1986)*
6. Solve $2x^4 + x^3 - 6x^2 + x + 2 = 0$ *(M.U, B.E., 1986)*
7. Solve $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$
8. Solve $x^4 + 6x^3 - 5x^2 + 6x + 1 = 0$ *(M.U, B.E., 1988)*
9. Solve $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$ *(M.U, B.E., 1991)*
10. Solve $x^5 - x^4 + x^3 - x^2 + x - 1 = 0$
11. Solve $x^6 + 2x^5 + 2x^4 - 2x^2 - 2x - 1 = 0$ *(M.U, B.E., 1994)*
12. Solve $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$ *(M.U, B.E., 1986)*
13. Solve $3x^6 + x^5 - 27x^4 + 27x^2 - x - 3 = 0$
14. Show that the equation $x^4 - 3x^3 + 4x^2 - 2x + 1 = 0$ transforms into a reciprocal equation by diminishing the root by 1. Hence solve it.
    *(M.U, B.E., 1990)*
15. Show that $x^4 - 10x^3 + 23x^2 - 6x - 15 = 0$ can be transformed into a reciprocal equation by diminishing the roots by 2. Hence solve it.
    *(M.U, B.E., 1993, Coimbatore B.E., 1988)*

## *ANSWERS*

1. $-1, \dfrac{1 \pm \sqrt{3}i}{2}, -2 \pm \sqrt{3}i$

2. $-1, \dfrac{-1 \pm \sqrt{3}i}{2}, \dfrac{-1 \pm \sqrt{3}i}{2}$

3. $-1, 2, 1/2, -3, -1/3$

4. $2, 1/2, \dfrac{3 \pm \sqrt{5}}{2}, \dfrac{-1 \pm \sqrt{3}i}{2}$

5. $\dfrac{1 \pm \sqrt{3}i}{2}, \dfrac{1 \pm \sqrt{3}i}{2}$

6. $1, 1, -2, -1/2$

7. $2 \pm \sqrt{3}, 3 \pm 3\sqrt{2}$

8. $\dfrac{-7 \pm 3\sqrt{5}}{2}, \dfrac{1 \pm \sqrt{3}i}{2}$

9. $1, 2, 1/2, -3, -1/3$

10. $1, \dfrac{1 \pm \sqrt{3}i}{2}, \dfrac{-1 \pm \sqrt{3}i}{2}$

11. $\pm 1, \dfrac{-1 \pm \sqrt{3}i}{2}, \dfrac{-1 \pm \sqrt{3}i}{2}$

12. $\pm 1, 2, 1/2, 3, 1/3$

13. $\pm 1, -3, -1/3, \dfrac{3 \pm \sqrt{5}}{2}$

14. $\dfrac{\sqrt{5} + 3 \pm \sqrt{-10 + 2\sqrt{5}}}{4}, \dfrac{-\sqrt{5} + 3 \pm \sqrt{-10 + 2\sqrt{5}}}{4}$

15. $\dfrac{9 \pm \sqrt{21}}{2} \; \dfrac{1 \pm \sqrt{5}}{2}$

## EXERCISE 3.1

1. Find a root of the following equations correct to three decimal places, using the Bisection method.

   (i) $x^3 - x^2 + x - 7 = 0$

   (ii) $x^3 - 2x - 5 = 0$ (*Bangalore, B.E., 1989*)

   (iii) $x^3 - 3x - 5 = 0$ (*Mysore, B.E., 1987*)

   (iv) $x^3 - 4x - 9 = 0$ (*S. Gujarat B.E., 1990*)

   (v) $x^4 - x - 10 = 0$ (*B.U, B.E., 1995*)

   (vi) $x - \cos x = 0$

   (vii) $3x - e^x = 0$

   (viii) $3x = \sqrt{(1 + \sin x)}$

   (ix) $x \log_{10} x - 1.2 = 0$

2. Using Bisection method find the negative root of $x^3 - 4x + 9 = 0$, correct to three decimal places.

3. Find a root of the following equations correct to three decimal places, using Iteration method.

(i) $x^3 + x^2 - 100 = 0$ (ii) $x = \frac{1}{2} + \sin x$

(iii) $3x - 6 = \log_{10} x$ (iv) $xe^x - \cos x = 0$

(v) $\sin x = e^x - 3x$ (vi) $2x - 7 - \log_{10} x = 0$

(vii) $1 - x + \dfrac{x^2}{(2!)^2} - \dfrac{x^3}{(3!)^2} + \dfrac{x^4}{(4!)^2} - \dfrac{x^5}{(5!)^2} + \ldots = 0$

4. Find a negative root of $x^3 - 2x + 5 = 0$, correct to three decimal places, using Successive Approximation method.

5. Find a root of the following equations correct to four decimal places using the method of False Position (*regula false* method).

(i) $x^3 - 4x - 9 = 0$ (ii) $x^3 + 2x^2 + 10x - 20 = 0$

(iii) $x^3 - 4x - 1 = 0$ (iv) $x^6 - x^4 - x^3 - 1 = 0$

(v) $xe^x = 2$ (vi) $e^x \sin x = 1$

(vii) $x = \cos x$ (viii) $x \tan x = -1$ in $(2.5, 3)$

(ix) $x \log_{10} x = 1.2$

### *ANSWERS*

1. (i) 2.105 (ii) 2.095 (iii) 2.280 (iv) 2.706 (v) 1.813
(vi) 0.739 (vii) 0.619 (viii) 0.392 (ix) 2.740

2. −2.706

3. (i) 4.331 (ii) 1.497 (iii) 2.108 (iv) 0.518 (v) 0.360
(vi) 3.789 (vii) 1.445

4. −2.095

5. (i) 2.7065 (ii) 1.3688 (iii) 0.2541 (iv) 1.7365 (v) 0.8526
(vi) 0.5885 (vii) 0.7391 (viii) 2.7981 (ix) 2.7406

## EXERCISE 3.2

Using Newton–Raphson method, find a root correct to three decimal places of the following :

1. $x^3 - 3x^2 + 7x - 8 = 0$ (*M.U, B.E., 1992*)
2. $x^3 - 3x - 5 = 0$ (*Kerala B.Tech 1989*)
3. $x^3 - 5x + 3 = 0$ (*Gulbarga B.E, 1993*)
4. $x^4 - x - 10 = 0$
5. $x^4 - x - 13 = 0$
6. $e^x = 1 + 2x$
7. $xe^x - \cos x = 0$ (*Gujarat B.E., 1990; B.U, B.E., 1995*)
8. $e^x \sin x = 1$
9. $x^x = 1000$
10. $3x - 1 = \cos x$ (*B.R, B.E., 1993*)
11. $\sin x = 1 - x$
12. $x^2 + 4 \sin x = 0$
13. $2x \tan x = 1$
14. $x(1 - \log_e x) = 0.5$ (*M.U, B.E., 1987*)
15. $3x - e^x + \sin x = 0$
16. $x \sin x + \cos x = 0$ near $x = \pi$ (*Karnataka B.E., 1993*)

17. Find the interative formulae for finding $1/\sqrt{N}$, $3\sqrt{N}$, $4\sqrt{N}$ where $N$ is a positive real number, using Newton's method. Hence evaluate $1/\sqrt{17}$, $3\sqrt{10}$, $4\sqrt{25}$.

18. Find a negative root of the following equations using Newton's method.

    (i) $x^3 - x^2 + x + 100 = 0$ (ii) $x^3 - 21x + 3500 = 0$

19. Find by Horner's method the root of the following equations correct to three decimal places.

    (i) $x^3 + 3x^2 - 12x - 11 = 0$ (ii) $x^3 + x^2 + x - 100 = 0$

    (iii) $x^3 - 6x - 13 = 0$ (iv) $x^3 - 3x + 1 = 0$

    (v) $x^3 - 30 = 0$ (vi) $x^4 + x^3 - 4x^2 - 16 = 0$

20. A sphere of pine wood, 2 metres in diameter, floating in water sinks to the depth of $h$ metre, given by the equation $h^3 - 3h^2 + 2.5 = 0$. Find $h$ correct to two decimal places using Horner's method.

21. Find a negative root of $x^3 - 2x + 5 = 0$ correct to two decimal places using Horner's method.

22. Find all the roots of the following equations by Graeffe's method squaring thrice.

    (i) $x^3 - 4x^2 + 5x - 2 = 0$ (ii) $x^3 - 2x^2 - 5x + 6 = 0$

    (iii) $x^3 - 5x^2 - 17x + 20 = 0$ *(M.U, B.E., 1991)*

    (iv) $x^3 - 9x^2 + 18x - 6 = 0$

    (v) $x^3 - x - 1 = 0$

## ANSWERS

| | | |
|---|---|---|
| 1. 1.674 | 2. 2.279 | 3. 1.834 |
| 4. 1.856 | 5. 1.961 | 6. 1.256 |
| 7. 0.518 | 8. 0.589 | 9. 3.592 |
| 10. 0.607 | 11. 0.511 | 12. −1.934 |
| 13. 0.653 | 14. 0.187 | 15. 0.360 |
| 16. 2.798 | 17. 0.24246, 2.15466, 2.236 | |
| 18. (i) −4.264 | (ii) −16.56 | |
| 19. (i) 2.769 | (ii) 4.264 | (iii) 3.177 |
| (iv) 1.532 | (v) 3.107 | (vi) 2.231 |
| 20. 1.17 | 21. −2.094 | |
| 22. (i) 2,1,1 | | (ii) 3, −2, 1 |
| (iii) 7.018, −2.974, 0.958 | | (iv) 6.3, 2.3, 0.4 |
| (v) 1.3247, −0.6624, ± 0.5622$i$ | | |

## EXERCISE 4.1

Solve the following equations by Gauss elimination method

1. $3x + 4y - z = 8,\ -2x + y + z = 3,\ x + 2y - z = 2$
2. $x - y + z = 1,\ -3x + 2y - 3z = -6,\ 2x - 5y + 4z = 5$
3. $10x + y + z = 12,\ 2x + 10y + z = 13,\ 2x + 2y + 10z = 14$

   *(M.U, B.E., 1991)*
4. $2x - y + 2z = 2,\ x + 10y - 3z = 5,\ x - y - z = 3$

   *(Ranchi, B. Tech, 1987)*
5. $10x_1 + x_2 + x_3 = 18.141, x_1 + x_2 + 10x_3 = 38.139, x_1 + 10x_2 + x_3 = 28.140$

   *(M.U, B.E., 1991)*
6. $x + y + z = 6.6,\ x - y + z = 2.2,\ x + 2y + 3z = 15.2$

   *(North Bengal B Tech, 1987)*
7. $2x + 4y + 2z = 15,\ 2x + y + 2z = -5,\ 4x + y - 2z = 0$

   *(M.U, B.E., 1989)*

8. $2x + y + z = 10,\ 3x + 2y + 3z = 18,\ x + 4y + 9z = 16$

(*Bangalore, B.E., 1990*)

9. $2x_1 + 2x_2 + x_3 = 12,\ 3x_1 + 2x_2 + 2x_3 = 8,\ 5x_1 + 10x_2 - 8x_3 = 10$

(*S.Gujarat, B.E., 1990*)

10. $x + 2y - 12z + 8w = 27,\ 5x + 4y + 7z - 2w = 4,$

$-3x + 7y + 9z + 5w = 11,\ 6x - 12y - 8z + 3w = 49$

(*M.U, B.E., 1987*)

Solve the following equations by Gauss - Jordan method

11. $2x - 3y + z = -1,\ x + 4y + 5z = 25,\ 3x - 4y + z = 2$

(*M.U, B.E., 1993*)

12. $2x + y + z = 12,\ 3x + 2y + 3z = 24,\ x + 4y + 9z = 34$

13. $10x + y + z = 12,\ x + 10y + z = 12,\ x + y + 10z = 12$

(*Bhopal B.E., 1991*)

14. $x + 2y + z = 8,\ 2x + 3y + 4z = 20,\ 4x + 3y + 2z = 16$

(*Punjab, B.E., 1987*)

15. $4x - y - z = -7,\ x - 5y + z = -10,\ x + 2y + 6z = 9$

16. $2x + 2y - z + t = 4,\ 4x + 3y - z + 2t = 6,$
$8x + 5y - 3z + 4t = 12,\ 3x + 3y - 2z + 2t = 6$

17. $5x_1 + x_2 + x_3 + x_4 = 4\ ;\ x_1 + 7x_2 + x_3 + 4x_4 = 2$
$x_1 + x_2 + 6x_3 + x_4 = -5\ ;\ x_1 + x_2 + x_3 + x_4 = -6$

**Find the inverse of the following matrices using Gauss elimination method.**

18. $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ 19. $\begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ 20. $\begin{bmatrix} 3 & -1 & 10 & 2 \\ 5 & 1 & 20 & 3 \\ 9 & 7 & 39 & 4 \\ 1 & -2 & 2 & 1 \end{bmatrix}$

## *ANSWERS*

| | | | |
|---|---|---|---|
| 1. | 1, 2, 3 | 2. | $-2, 3, 6$ |
| 3. | 1, 1, 1 | 4. | $2, 0, -1$ |
| 5 | 1.234, 2.348, 3.455 | 6., | 1.2, 2.2, 3.2 |
| 7. | $-3.0556, 6.6667, -2.778$ | 8. | $7, -9, 5$ |

## EXERCISE 4.2

Solve the following equations by Factorisation (or Triangularisation) method.

1. $3x + y + 2z = 16;\ 2x - 6y + 8z = 24;\ 5x + 4y - 3z = 2$
2. $3x + 2y + 7z = 32;\ 2x + 3y + z = 40;\ 3x + 4y - z = 56$
3. $10x + y + z = 12;\ 2x + 10y + z = 13;\ x + y + 5z = 7$

   *(M.U, B.E., 1991)*
4. $28x + 4y - z = 32;\ x + 3y + 10z = 24;\ 2x + 17y + 4z = 35$
5. $2x - y + z = 0.3; -4x + 3y - 2z = -1.4;\ 3x - 8y + 3z = 0.1$
6. $10x + 7y + 8z + 7w = 32;\ 7x + 5y + 6z + 5w = 23,$
   $8x + 6y + 10z + 9w = 33\ ;\ 7x + 5y + 9z + 10w = 31$

Solve the following equations by Crout's method

7. $x + 3y + 8z = 4, x + 4y + 3z = -2;\ x + 3y + 4z = 1$ *(M.U, B.E., 1991)*
8. $2x - 6y + 8z = 24, 5x + 4y - 3z = 2;\ 3x + y + 2z = 16$*(M.U, B.E., 1993)*
9. $10x + y + 2z = 13; 3x + 10y + z = 14, 2x + 3y + 10z = 15$

   *(Madurai, B.E., 1987)*
10. $9x - 2y + z = 50,\ x + 5y - 3z = 18,\ -2x + 2y + 7z = 19$
11. $10x + y + z = 12,\ 2x + 10y + z = 13,\ 2x + 2y + 10z = 14$

    *(Coimbatore, B.Tech., 1988)*

12. $3x + y + 2z = 3,\ 2x - 3y - z = -3,\ x - 2y + z = 4$

13. $10x_1 + 9x_2 + 6x_3 + x_4 = 26,\ 11x_1 + 6x_2 - x_3 + 2x_4 = 18$
$x_1 - 7x_2 + 3x_3 + 6x_4 = 3,\ 7x_1 + x_2 + x_3 + x_4 = 10$

14. $5x + y + z + w = 4,\ x + 7y + z + 4w = 12,$
$x + y + 6z + w = -5,\ x + y + z + 4w = -6$

Find the inverse of the following matrices using Crout's method.

(15) $\begin{bmatrix} -2 & 4 & 8 \\ -4 & 18 & -16 \\ -6 & 2 & -20 \end{bmatrix}$

(16) $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

(17) $\begin{bmatrix} 13 & 14 & 6 & 4 \\ 8 & -1 & 13 & 9 \\ 6 & 7 & 3 & 2 \\ 9 & 5 & 16 & 11 \end{bmatrix}$

## *ANSWERS*

(1) 1, 3, 5

(2) 7, 9, − 1

(3) 1, 1, 1

(4) 0.996, 1.5070, 1.8485

(5) 1.6, − 0.8, − 3.71

(6) 1, 1, 1, 1

(7) $\frac{19}{4}, -\frac{9}{4}, \frac{3}{4}$

(8) 1, 3, 5

(9) 1, 1, 1

(10) 6.13, 4.31, 3.23

(11) 1, 1, 1

(12) 1, 2, − 1

(13) 1, 1, 1, 1

(14) 1, 2, − 1, − 2

(15) $\frac{1}{190}\begin{bmatrix} -41 & 12 & -26 \\ 2 & 11 & -8 \\ 12.5 & -2.5 & -2.5 \end{bmatrix}$

(16) $\frac{1}{12}\begin{bmatrix} -5 & 3 & 4 \\ -7 & 3 & -8 \\ 1 & -3 & 4 \end{bmatrix}$

(17) $\begin{bmatrix} 1 & 0 & -2 & 0 \\ -5 & 1 & 11 & -1 \\ 287 & -67 & -630 & 65 \\ -416 & 97 & 913 & -94 \end{bmatrix}$

## EXERCISE 4.3

Solve the following system of linear equations by (i) Gauss and (ii) Gauss Seidel iteration method.

1. $2x + y + z = 4,\ x + 2y + z = 4;\ x + y + 2z = 4$
2. $8x + y + z = 8;\ 2x + 4y + z = 4;\ x + 3y + 5z = 5$
3. $5x + 2y + z = 12,\ x + 4y + 2z = 15,\ x + 2y + 5z = 20$

4. $9x + 2y + 4z = 20,\ x + 10y + 4z = 6,\ 2x - 4y + 10z = -15$
5. $54x + y + z = 110,\ 2x + 15y + 6z = 72,\ -x + 6y + 27z = 85$ *(M.U, B.E., 1993)*
6. $28x - 4y - z = 32,\ x + 3y + 10z = 24,\ 2x + 17y + 4z = 35$
7. $5x - y + z = 10,\ 2x + 4y = 12,\ x + y + 5z = -1$ *(Bangalore, B.E., 1990)*
8. $10x_1 - 5x_2 - 2x_3 = 3,\ 4x_1 - 10x_2 + 3x_3 = -3,\ x_1 + 6x_2 + 10x_3 = -3$
9. $10x + 2y + z = 9,\ 2x + 20y - 2z = -44,\ -2x + 3y + 10z = 22$
10. $10x_1 + 7x_2 + 8x_3 + 7x_4 = 32,\ 7x_1 + 5x_2 + 6x_3 + 5x_4 = 23;$
    $8x_1 + 6x_2 + 10x_3 + 9x_4 = 33,\ 7x_1 + 5x_2 + 9x_3 + 10x_4 = 31$

Solve by relaxation method the following equations:

11. $9x + 2y + z = 50,\ x + 5y - 3z = 18,\ -2x + 2y + 7z = 19$
12. $3x + 9y - 2z = 11,\ 4x + 2y + 13z = 24,\ 4x - 4y + 3z = -8$ *(M.U, B.E., 1993)*
13. $4.215x - 1.212y + 1.105z = 3.216$
    $-2.120x + 3.505y - 1.632z = 1.247$
    $1.122x - 1.313y + 3.986z = 2.112$
14. $10x - 2y - 3z = 305,\ -2x + 10y - 2z = 154,\ -2x - y + 10z = 120$
15. $8x_1 + x_2 + x_3 + x_4 = 14;\ 2x_1 + 10x_2 + 3x_3 + x_4 = -8$
    $x_1 - 2x_2 - 20x_3 + 3x_4 = 111,\ 3x_1 + 2x_2 + 2x_3 + 19x_4 = 53$

## *ANSWERS*

1. $x = 1, y = 1, z = 1$
2. $x = 0.876, y = 0.919, z = 0.574$
3. $x = 0.996, y = 1.95, z = 3.16$
4. $x = 2.733, y = 0.986, z = -1.652$
5. $x = 1.926, y = 3.573, z = 2.425$
6. $x = 0.994, y = 1.507, z = 1.849$
7. $x = 2.556, y = 1.722, z = -1.055$
8. $x_1 = 0.342, x_2 = 0.285, x_3 = -0.505$
9. $x = 1.013, y = -1.996, z = 3.001$
10. $x_1 = 1, x_2 = 1, x_3 = 1, x_4 = 1$
11. $x = 6.13, y = 4.31, z = 3.23$
12. $x = 1.35, y = 2.103, z = 2.845$
13. $x = 0.943, y = 1.239, z = 0.673$
14. $x = 32, y = 26, z = 21$
15. $x_1 = 2, x_2 = 0, x_3 = -5, x_4 = 3$

## EXERCISE 5.1

1. Tabulate the forward differences for the given data:

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | 1 | 8 | 27 | 64 | 125 | 216 | 343 | 512 | 729 |

2. Form a table of backward differences of the function

$$f(x) = x^3 - 3x^2 - 5x - 7 \text{ for } x = -1, 0, 1, 2, 3, 4, 5.$$

3. Form the difference table of $f_x = x^4 - 5x^3 + 6x^2 + x - 2$ for the values of $x = -3, -2, -1, 0, 1, 2, 3$. Extend the table in both directions to give $f_{-4}, f_{-5}, f_4, f_5$.

4. Show that

   (i) $y_3 = y_2 + \Delta y_1 + \Delta^2 y_0 + \Delta^3 y_0$

   (ii) $\nabla^2 y_8 = y_8 - 2y_7 + y_6$ (iii) $\delta^2 y_5 = y_6 - 2y_5 + y_4$

5. If $y_0 = 3$, $y_1 = 12$, $y_2 = 81$, $y_3 = 2000$, $y_4 = 100$ show that $\Delta^4 y_0 = -7459$.

6. If the interval of differencing is unity, prove that

   (i) $\Delta \sin x = 2\sin\frac{1}{2} \cos\left(x + \frac{1}{2}\right)$

   (ii) $\Delta f(x) = \dfrac{-\Delta f(x)}{f(x) f(x+1)}$

   (iii) $\Delta \tan^{-1}\left(\dfrac{n-1}{n}\right) = \tan^{-1} \dfrac{1}{2n^2}$

   (iv) $\Delta \dfrac{2^x}{x!} = \dfrac{2^x(1-x)}{(x+1)!}$

(v) $\Delta[x(x+1)(x+2)(x+3)] = 4(x+1)(x+2)(x+3)$

(vi) $\Delta^2\left[\frac{5x+12}{x^2+5x+6}\right] = \frac{10x+32}{(x+2)(x+3)(x+4)(x+5)}$

(vii) $\Delta^n e^x = (e-1)^n e^x$

(viii) $\Delta^n(1/x) = \frac{(-1)^n n!}{x(x+1)x+2)\ldots(x+n)}$

7. If $h$ is the interval of differencing, prove that

(i) $\Delta^2\cos 2x = -4\sin^2 h\cos 2(x+h)$ (*Kerala B.E., 1989, M.U. B.E., 1996*)

(ii) $\Delta^3 a^{cx+d} = (a^{ch}-1)^3 a^{cx+d}$

(iii) $\Delta^n\sin(ax+b) = 2\sin(ah/2)^n\sin\left(ax+b+\frac{nah+n\pi}{2}\right)$

8. Show that

(i) $\Delta^3[(1-x)(1-2x)(1-3x)] = -36$ if $h = 1$.

(ii) $\Delta^{10}[(1-x)(1-2x^2)(1-3x^3)(1-4x^4)] = 24\times 2^{10}\times 10!$ if $h = 2$.

9. Find the seventh term of the sequence 2, 9, 28, 65, 126, . . . and also find the general term.

10. Evaluate $\Delta^2 f(x)$ if $f(x)$ is

(i) $\frac{1}{x(x+4)x+8)}$ (ii) $\frac{1}{(3x+1)(3x+4)(3x+7)}$

11. Find $\Delta^3 f(x)$ if $f(x)$ is $(3x+1)(3x+4)(3x+7)\ldots(3x+19)$

12. Express the following in factorial notation.

(i) $f(x) = 2x^3 - 3x^2 + 3x - 10$

(ii) $f(x) = x^3 - 2x^2 + x - 1$

(iii) $f(x) = 3x^4 - 4x^3 + 6x^2 + 2x + 1$

(iv) $f(x) = x^4 - 3x^3 - 5x^2 + 6x - 7$ and get their successive forward differences.

13. Obtain the function whose first difference is $x^3 + 3x^2 + 5x + 12$.

14. Express the following in factorial notation taking $h = 2$ and find their differences of second order.

(i) $f(x) = 7x^4 + 12x^3 - 6x^2 + 5x - 3$

(ii) $f(x) = x^3 - 3x^2 + 5x + 7$

15. Prepare a forward difference table for values $(x_i, y_i)$, i = 1, 2, 3, . . 7. Indicate the propogation error $\varepsilon$ introduced in the tabulated value of $y_4$.

16. The value of a polynomial of degree 5 are tabulated below. If $f(3)$ is known to be an error, find its correct value.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 1 | 2 | 33 | 254 | 1025 | 3126 | 7777 |

17. A polynomial function is given by the following table:

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 0 | 3 | 14 | 39 | 84 | 155 | 258 |

Form a difference table and explain how the correctness of the arithmetic may be checked.

18. Find $y_6$ if $y_0 = 9, y_1 = 18, y_2 = 20, y_3 = 24$ and the third differences are constant.

19. Assuming that the following values of $y_x$ belong to a polynomial of degree 4, compute the next three values.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 1 | –1 | 1 | –1 | 1 | – | – | – |

20. Find the missing term in the following table:

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 2 | 4 | 8 | – | 32 | 64 | 128 |

21. Find and correct a single error in $y$ in the following table:

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 0 | 0 | 1 | 6 | 24 | 60 | 120 | 210 |

22. With the usual notations prove that

(i) $E\Delta = \Delta E$

(ii) $E\nabla = \nabla E = \Delta$

(iii) $E = (\Delta / \delta)^2$ *(M.U., B.E., 1996)*

(iv) $\nabla = 1 - (1 + \nabla)^{-1}$

(v) $\Delta = \delta E^{1/2}$; $\nabla = \delta E^{-1/2}$

(vi) $(1 + \Delta)(1 - \nabla) = 1$

(vii) $\Delta^2 = (1+\Delta)\delta^2$ (viii) $\mu\,\delta = \frac{1}{2}\Delta E^{-1} + \frac{1}{2}\Delta$

(ix) $E^{\frac{1}{2}} = \mu + \frac{1}{2}\delta,\ E^{-\frac{1}{2}} = \mu - \frac{1}{2}\delta$ (x) $\delta = \Delta(1+\Delta)^{-\frac{1}{2}} = \nabla(1-\nabla)^{-\frac{1}{2}}$

(xi) $\mu\delta = \frac{1}{2}(\Delta + \nabla)$ (xii) $\mu = \dfrac{2+\Delta}{2\sqrt{1+\Delta}}$

(xiii) $\dfrac{\Delta^2}{E^2} = E^{-2} - 2E^{-1} + 1$ (xiv) $\mu^2 = 1 + \dfrac{1}{4}\delta^2$

*(Coimbatore B.E., 1985)*

(xv) $E = \sum_{i=o}^{\infty} \nabla_i$

(xvi) $\nabla^2 = h^2D^2 - h^3D^3 + \dfrac{7}{12}h^4D^4 + \cdots$ *(Madurai, B.E, 1989)*

(xvii) $\dfrac{\Delta}{\nabla} - \dfrac{\nabla}{\Delta} = E - E^{-1}$ *(M.U, B.E, 1997)*

(xviii) $(1+\Delta)(1-\nabla) = 1$ *(M.U, B.E, 1997)*

23) Show the following

(i) $\nabla^3 y_2 = \nabla^3 y_5$ (ii) $\sum_{k=0}^{n-1} \Delta^2 f_k = \Delta f_n - \Delta f_0$

(iii) $(\Delta + \nabla)^2 (x^2 + x) = 8$ (iv) $(\Delta^2 E^{-1})\, x^3 = 6x$

(v) $\dfrac{\Delta^2}{E}\sin(x+h) + \dfrac{\Delta^2 \sin(x+h)}{E\sin(x+h)} = 2(\cosh - 1)\,[\sin(x+h) + 1]$

(vi) $\Delta f_k^2 = (f_k + f_{k+1})\,\Delta f_k$

(vii) $\dfrac{\Delta^2 x^2}{E(x + \log x)} = \dfrac{2}{x + 1 + \log(x+1)}$

24) Use the method of separation of symbols to prove that

$$(u_1 - u_0) - x(u_2 - u_1) + x^2(u_3 - u_2) - \cdots$$

$$= \frac{\Delta u_0}{1+x} - x\frac{\Delta^2 u_0}{(1+x)^2} + x^2\frac{\Delta^3 u_0}{(1+x)^3} - \cdots$$

25) $u_x = u_{x-1} + \Delta u_{x-2} + \Delta^2 u_{x-3} + \cdots + \Delta^n u_{x-n}$

26) $y_x = y_n - {}^{n-x}C_1\,\Delta y_{n-1} + {}^{n-x}C_2\,\Delta^2 y_{n-2} - \cdots + (-1)^{n-x}\Delta^{n-x} y_x$

27) $u_0 + \frac{u_1 x}{1!} + \frac{u_2 x^2}{2!} + \cdots = e^x \left(u_0 + x\,\Delta\, u_0 + \frac{x^2}{2!}\,\Delta^2 u_0 + \cdots\right)$

28) $u_0 + {}^xC_1\,\Delta u_1 + {}^xC_2\,\Delta^2 u_2 + \cdots = u_x + {}^xC_1\,\Delta^2 u_{x-1} + {}^xC_2\,\Delta^4 u_{x-2} + \cdots$

29) Sum the series to $n$ terms:

(i) 1.2.3 + 2.3.4 + 3.4.5 + $\cdots$

(ii) 4.5.6. + 5.6.7 + 6.7.8 + $\cdots$

(iii) 2.5 + 5.8 + 8.11 + $\cdots$

30) Using the method of finite differences, find the sum to $n$ terms of the series whose $n$th term is $n(n-1)(n-2)$.

31) Using the method of finite differences, find the sum of the first

(i) $n$ squares and (ii) $n$ cubes.

32) Sum the series using the identity of Example 5.15.

(i) $5 + \frac{4x}{1!} + \frac{5x^2}{2!} + \frac{14x^3}{3!} + \frac{37x^4}{4!} + \cdots$

(ii) $1 + \frac{4x}{1!} + \frac{10x^2}{2!} + \frac{20x^3}{3!} + \frac{35x^4}{4!} + \cdots$

33) Using Montmort's theorem, sum the series

$1.3 + 3.5x + 5.7x^2 + 7.9x^3 + \cdots$

## *ANSWERS*

9) 344, $(n+1)^3 + 1$

10) (i) $\Delta^2 f(x) = \dfrac{192}{x(x+4)(x+8)(x+12)(x+16)}$

(ii) $\Delta^2 f(x) = \dfrac{108}{(3x+1)(3x+4)(3x+7)(3x+10)(3x+13)}$

11) $\Delta^3 f(x) = 459270\,(3x+19)(3x+16)(3x+13)(3x+10)$

12) (i) $f(x) = 2x^{(3)} + 3x^{(2)} + 2x^{(1)} - 10$

(ii) $f(x) = x^{(3)} + x^{(2)} - 1$;

(iii) $f(x) = 3x^{(4)} + 14x^{(3)} + 15x^{(2)} + 7x^{(1)} + 1$;

(iv) $f(x) = x^{(4)} + 9x^{(3)} + 11x^{(2)} + 5x^{(1)} - 7$

13) $f(x) = \frac{1}{4} x^{(4)} + 2x^{(3)} + \frac{9}{2} x^{(2)} + 12x^{(1)}$ + constant

14) (i) $f(x) = 7x^{(4)} + 96x^{(3)} + 262x^{(2)} + 97x^{(1)} - 3$

$\Delta f(x) = 56x^{(3)} + 576x^{(2)} + 1048x^{(1)} + 194$

$\Delta^2 f(x) = 336x^{(2)} + 2304x^{(1)} + 2096$

(ii) $f(x) = x^{(3)} + 3x^{(2)} + 3x^{(1)} + 7$

$\Delta f(x) = 6x^{(2)} + 12x^{(1)} + 6$

$\Delta^2 f(x) = 24x^{(1)} + 24$

15) $f(x) = 244$, error $= -10$

18) $y_6 = 138$

19) 31, 129, 351

20) 16.1

21) Error is at $x = 2$; $y(2) = 0$

29) (i) $\frac{1}{4} n(n+1)\ (n+2)\ (n+4)$

(ii) $\frac{1}{4}[\ (n+6)\ (n+5)\ (n+4)\ (n+3) - 360]$

(iii) $n\ (3n^2 + 6n + 1)$

30) $\frac{1}{4}(n+1)\ (n)(n-1)\ (n-2)$

31) (i) $\frac{n(n+1)(2n+1)}{6}$ (ii) $\frac{n^2(n+1)^2}{4}$

32) (i) $e^x\ (x^3 + x^2 - x + 5)$ (ii) $e^x \left(1+3x+\frac{3x^2}{2}+\frac{x^3}{6}\right)$

33) $\frac{3+6x-x^2}{(1-x)^3}$