B·CA - III rd sem

Q.1 Find the order and degree of the following differential equations.

(i) $\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2+4y=0$

(ii) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+3y=0$

(iii) $\frac{d^3y}{dx^3}-3\left(\frac{dy}{dx}\right)^6+2y=x^2$

(iv) $\left[1+\frac{d^2y}{dx^2}\right]^{\frac{3}{2}}=a\frac{d^2y}{dx^2}$

(v) $y'+(y'')^2=(x+y'')^2$

(vi) $\frac{d^3y}{dx^3}-\left(\frac{dy}{dx}\right)^{\frac{1}{2}}=0$

(vii) $y=2\left(\frac{dy}{dx}\right)^2+4x\frac{dx}{dy}$

Solution

(i) $\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2+4y=0$

Highest order derivative is $\frac{d^2y}{dx^2}$

$\therefore$ order = 2

Power of the highest order derivative $\frac{d^2y}{dx^2}$ is 1.

$\therefore$ Degree =1

(ii) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+3y=0$

Highest order derivative is $\frac{d^2y}{dx^2}$

$\therefore$ order =2

Power of the highest order derivative $\frac{d^2y}{dx^2}$ is 1.

$\therefore$ degree = 1

(iii) $\frac{d^3y}{dx^3}-3\left(\frac{dy}{dx}\right)^6+2y=x^2$

$\therefore$ order =3, Degree =1

(iv) $\left[1+\frac{d^2y}{dx^2}\right]^{\frac{3}{2}}=a\frac{d^2y}{dx^2}$

Here we eliminate the radical sign.

Squaring both sides, we get

$$\left[1+\frac{d^2y}{dx^2}\right]^3=a^2\left(\frac{d^2y}{dx^2}\right)^2$$

$\therefore$ Order=2, degree =3

(v) $y' + (y'')^2 = (x + y'')^2$

$$y' + (y'')^2 = x^2 + 2xy'' + (y'')^2$$

$$y' = x^2 + 2xy'' \Rightarrow \frac{dy}{dx} = x^2 + 2x\frac{d^2y}{dx^2}$$

$\therefore$ Order=2, degree=1

(vi) $\dfrac{d^3y}{dx^3} - \left(\dfrac{dy}{dx}\right)^{\frac{1}{2}} = 0$

Here we eliminate the radical sign.

For this write the equation as

$$\frac{d^3y}{dx^3} = \left(\frac{dy}{dx}\right)^{\frac{1}{2}}$$

Squaring both sides, we get

$$\left(\frac{d^3y}{dx^3}\right)^2 = \frac{dy}{dx}$$

$\therefore$ Order=3, degree=2

(vii) $y = 2\left(\dfrac{dy}{dx}\right)^2 + 4x\dfrac{dx}{dy}$

$$y = 2\left(\frac{dy}{dx}\right)^2 + 4x\frac{1}{\left(\frac{dy}{dx}\right)}$$

$$y\frac{dy}{dx} = 2\left(\frac{dy}{dx}\right)^3 + 4x$$

$\therefore$ order=1, degree=3

Q.2 **Solve the LDE = dy/dx = [1/(1+x^3)] – [3x^2/(1 + x^2)]y**
Solution:
The above mentioned equation can be rewritten as dy/dx + [3x^2/(1 + x^2)] y = 1/(1+x^3)
Comparing it with dy/dx + Py = O, we get

$P = 3x^2/1+x^3$
$Q= 1/1 + x^3$
Let's figure out the integrating factor(I.F.) which is,
$e^{\int Pdx}$

$$I.F = e^{\int \frac{3x^2}{1+x^3}dx} = e^{ln(1+x^3)}$$

$\Rightarrow$I.F. = $1 + x^3$
Now, we can also rewrite the L.H.S as:
d(y × I.F)/dx,
$\Rightarrow d(y \times (1 + x^3))\ dx = [1/(1 + x^3)] \times (1 + x^3)$
Integrating both the sides w. r. t. x, we get,
$\Rightarrow y \times (1 + x^3) = x$
$\Rightarrow y = x/(1 + x^3)$
$\Rightarrow y = [x/(1 + x^3) + C$

Q.3 **Solve the following differential equation:**
dy/dx + (sec x)y = 7
Solution:
Comparing the given equation with dy/dx + Py = Q
We see, P = sec x, Q = 7
Now lets find out the integrating factor using the formula

$$e^{\int Pdx} = I.F$$

$$e^{\int secdx} = I.F.$$

$$I.F. = e^{ln|secx + tanx|} = secx + tanx$$

Now we can also rewrite the L.H.S as
d(y × I.F)/dx},
i.e . d(y × (sec x + tan x))
$\Rightarrow$d(y × (sec x + tan x))/dx = 7(sec x + tan x)
Integrating both the sides w. r. t. x, we get,

$$\int d(y\times(secx + tanx)) = \int 7(secx + tanx)dx$$

$$\Rightarrow y\times(secx + tanx) = 7(ln|secx + tanx| + log|secx|)$$

$$y = \frac{7(ln|secx + tanx| + log|secx|}{(secx + tanx)} + c$$

Q.4 Solve: $x^2 y'' - 6xy' - 18y = 0$
Solution: Given second order Cauchy-Euler equation is:
$x^2 y'' - 6xy' - 18y = 0$
Let $y = x^r$ and substitute in the given differential equation.
$x^2 y'' - 6xy' - 18y = 0$
$x^2[x^r]'' - 6x[x^r]' - 18(x^r) = 0$

$x^2 [r(r-1)x^{r-2}] - 6x [r x^{r-1}] - 18x^r = 0$
$(r^2 - r)x^r - 6r x^r - 18x^r = 0$
$(r^2 - r - 6r - 18)x^r = 0$
$\Rightarrow r^2 - 7r - 18 = 0$
$\Rightarrow r^2 + 2r - 9r - 18 = 0$
$\Rightarrow r(r + 2) - 9(r + 2) = 0$
$\Rightarrow (r + 2)(r - 9) = 0$
$\Rightarrow r = -2, r = 9$
So, r = -2 and 9 are the two distinct possible values of r.
Therefore, the set of fundamental solution is $\{x^{-2}, x^9\}$ and the general solution is $y = c_1x^{-2} + c_2x^9$.

Q.5 Solve the differential equation $x^2 y'' - 7xy' + 16y = 0$ for $x > 0$.
Solution: Given second order Cauchy-Euler equation is:
$x^2 y'' - 7xy' + 16y = 0$
Let $y = x^r$ and substitute in the given differential equation.
$x^2 y'' - 7xy' + 16y = 0$
$x^2[x^r]'' - 7x[x^r]' + 16(x^r) = 0$
$x^2 [r(r-1)x^{r-2}] - 7x [r x^{r-1}] + 16x^r = 0$
$(r^2 - r)x^r - 7r x^r + 16x^r = 0$
$(r^2 - r - 7r + 16)x^r = 0$
$\Rightarrow (r - 4)^2 = 0$
$\Rightarrow r = 4$
Therefore, the solution of the given equation is $y_1(x) = x^4$.
Since we got only one solution, we cannot simply write out the general solution. Let's perform the reduction of order method.
Now, let $y(x) = x^4 u(x)$
Let us differentiate the above equation.
$y'(x) = [x^4u]' = 4x^3 u + x^4 u'$
Again differentiating, we get;
$y''(x) = [4x^3 u + x^4 u']' = 12x^2 u + 8x^3 u' + x^4 u''$
Now, take the actual equation, i.e.,
$x^2 y'' - 7xy' + 16y = 0$
Substitute y(x), y'(x) and y"(x) in this equation, we get;
$x^2[12x^2 u + 8x^3 u' + x^4 u''] - 7x[4x^3 u + x^4 u'] + 16[x^4 u] = 0$
$12x^4 u + 8x^5 u' + x^6u'' - 28x^4 u - 7x^5 u' + 16x^4 u = 0$
$x^6u'' + [8x^5 - 7x^5]u' + [12x^4 - 28x^4 + 16x^4]u = 0$
$x^6u'' + x5 u' = 0$
Let $u' = v$.
So, $x^6v' + x^5v = 0$
$x^6(dv/dx) + x^5v = 0$
$x^6(dv/dx) = -x^5v$
$(1/v) (dv/dx) = -x^5/x^6$
$(1/v) (dv/dx) = -1/x$
Integrating on both sides, we get;

$\int (1/v)(dv/dx)\,dx = \int(-1/x)\,dx$
$\ln|v| = -\ln|x| + c_0$
So, $v = \pm e^{-\ln|x|+c0} = c_2/x$
Now,
$u(x) = \int u'(x)\,dx = \int v(x)\,dx = \int c_2/x\,dx = c_2 \ln|x| + c_1$
Hence, the general solution of the differential equation is:
$y(x) = x^4 u(x) = x^4 [c_2 \ln|x| + c_1] = c_1x^4 + c_2x^4 \ln|x|$

UNIT-II

If $z_1 = a + ib$ and $z_2 = c + id$ are two complex numbers such that;

- $(a + ib) + (c + id) = (a + c) + i(b + d)$
- $(a + ib) - (c + id) = (a - c) + i(b - d)$
- $(a + ib).(c + id) = (ac - bd) + i(ad + bc)$
- $(a + ib) / (c + id) = [(ac + bd)/ (c^2 + d^2)] + i[(bc - ad) / (c^2 + d^2)]$

Learn more about complex numbers here.

Q.6 **Suppose $z = (2 - i)^2 + [(7 - 4i)/(2 + i)] - 8$, express z in the form of $x + iy$ such that x and y are real numbers.**
Solution: Given,
$z = (2 - i)^2 + [(7 - 4i)/(2 + i)] - 8$
$= (2)^2 - 2(2)(i) + (i^2) + [(7 - 4i)(2 - i)/ (2 + i)(2 - i)] - 8$
$= (4 - 4i - 1) + [(14 - 7i - 8i + 4i^2)/ (4 - i2)] - 8$
$= (3 - 4i) + [(14 - 15i - 4)/(4 + 1)] - 8$ {since $i^2 = -1$}
$= (3 - 4i) + [(10 - 15i)/5] - 8$
$= (3 - 4i) + (2 - 3i) - 8$
$= -3 - 7i$
This is of the form $x + iy$ such that $x = -3$ and $y = -7$.

Q.7. If $z_1 = 2 + 8i$ and $z_2 = 1 - i$, then find $|z_1/z_2|$.
Solution:
Given,
$z_1 = 2 + 8i$ and $z_2 = 1 - i$
$z_1/z_2 = (2 + 8i)/(1 - i)$
$= (2 + 8i)(1 + i)/ (1 - i)(1 + i)$
$= [2 + 2i + 8i + 8i^2]/ [1 - i^2]$
$= (2 + 10i - 8)/ (1 + 1)$ {since $i^2 = -1$}
$= (-6 + 10i)/2$
$= -3 + 5i$
Now, $|z_1/z_2| = \sqrt{[(-3)^2 + (5)^2]}$
$= \sqrt{(9 + 25)}$
$= \sqrt{34}$

Q.8 If $|z^2 - 1| = |z^2| + 1$, then show that z lies on an imaginary axis.
Solution:
Let $z = x + iy$ be the complex number.
Now, $z^2 = z.z = (x + iy)(x + iy)$

$= x^2 + ixy + ixy + (iy)^2$
$= x^2 + 2ixy - y^2$ {since $i^2 = -1$}
$z^2 - 1 = x^2 + 2ixy - y^2 - 1$
$= (x^2 - y^2 - 1) + i(2xy)$
Thus, $|z^2 - 1| = \sqrt{[(x^2 - y^2 - 1)^2 + (2xy)^2]}$
$= \sqrt{[(x^2 - y^2 - 1)^2 + 4x^2y^2]}$
$|z|^2 + 1 = [\sqrt{(x^2 + y^2)}]^2 + 1$
$= x^2 + y^2 + 1$
Given that,
$|z^2 - 1| = |z^2| + 1$
So, $\sqrt{[(x^2 - y^2 - 1)^2 + 4x^2y^2]} = x^2 + y^2 + 1$
Squaring on both sides, we get;
$(x^2 - y^2 - 1)^2 + 4x^2y^2 = (x^2 + y^2 + 1)^2$
$[x^2 - (y^2 + 1)]^2 + 4x^2y^2 = [x^2 + (y^2 + 1)]^2$
$[x^2 - (y^2 + 1)]^2 - [x^2 + (y^2 + 1)]^2 + 4x^2y^2 = 0$
As we know, $(a - b)^2 - (a + b)^2 = -4ab$,
$-4x^2(y^2 + 1) + 4x^2y^2 = 0$
$-4x^2y^2 - 4x^2 + 4x^2y^2 = 0$
$4x^2 = 0$
$x = 0$
Therefore, z lies on the y-axis.

9. Find the relation between a and b if $z = a + ib$ if $|(z - 3)/(z + 3)| = 2$.

Solution:

Given,

$z = a + ib$

$|(z - 3)/(z + 3)| = 2$

$|(a + ib - 3)/(a + ib + 3)| = 2$

$|(a - 3) + ib| = 2|(a + 3) + ib|$

$\sqrt{}[(a - 3)2 + b2] = 2\sqrt{}[(a + 3)2 + b2]$

Squaring on both sides, we get;

$(a - 3)2 + b2 = 4[(a + 3)2 + b2]$

$a2 - 6a + 9 + b2 = 4(a2 + 6a + 9 + b2)$

$4a2 + 24a + 36 + 4b2 - a2 + 6a - 9 - b2 = 0$

$3a2 + 30a + 27 + 3b2 = 0$

$a2 + 10a + 9 + b2 = 0$

$(a2 + 10a + 25) + (b2 + 9 - 25) = 0$

$(a + 5)2 + b2 = 16$

$(a + 5)2 + b2 = 42$

Q.10 If $|z + 1| = z + 2 (1 + i)$, then find z.

Solution:

Let $z = x + iy$ be the complex number.

Given,

$|z + 1| = z + 2 (1 + i)$

$\Rightarrow |x + iy + 1| = x + iy + 2 (1 + i)$

We know,

$|z| = \sqrt{}(x2 + y2)$

$\sqrt{}[(x + 1)2 + y2] = (x + 2) + i(y + 1)$

Comparing real and imaginary parts,

$\Rightarrow \sqrt{}((x + 1)2 + y2) = x + 2$

And $0 = y + 2$

$\Rightarrow y = -2$

Substituting the value of y in $\sqrt{}((x + 1)2 + y2) = x + 2$, we get;

$(x + 1)2 + (-2)2 = (x + 2)2$

$x2 + 2x + 1 + 4 = x2 + 4x + 4$

$\Rightarrow 2x = 1$

$\Rightarrow x = ½$

Therefore, $z = x + iy = ½ - 2i$.

UNIT-3

Q.11 De-Moivres theorem statement and proof ?

Solution: De Moivre's theorem is one of the fundamental theorem of complex numbers which is used to solve various problems of complex numbers. This theorem is also widely used for solving trigonometric functions of multiple angles. DeMoivre's Theorem is also called "De Moivre's Identity" and "De Moivre's Formula". **De Moivre's Theorem Statement**
De Moivre's Theorem is a special theorem of complex numbers which is used to expand the complex number raised to any integer. De Moivre's Formula states that for all real values of x,

$$(\cos x + i.\sin x)n = \cos(nx) + i.\sin(nx)$$

De Moivre's Formula
De Moivre's Formula for complex numbers is, for any real value of x,

$$(\cos x + i.\sin x)n = \cos(nx) + i.\sin(nx)$$

Also, we know that,
$\cos x + i.\sin x = eix$
Now,

$$(eix)n = einx$$

Note, "n" in the above formula is an integer, and "i" is an imaginary number iota. Such that,
$i = \sqrt{(-1)}$
De Moivre's Formula is shown in the image added below,
De Moivre's Theorem Proof
DeMoivre's Theorem can be proved with the help of mathematical induction as follows:
$P(n) = (\cos x + i.\sin x)n = \cos(nx) + i.\sin(nx)$ □ (1)
Step 1: For $n = 1$,
$(\cos x + i \sin x)1 = \cos(1x) + i \sin(1x) = \cos(x) + i \sin(x)$,
Which is true. thus, P(n) is true for n = 1.
Step 2: Assume P(k) is true
$(\cos x + i.\sin x)k = \cos(kx) + i.\sin(kx)$ □ (2)
Step 3: Now we have to prove P(k+1) is true.
$(\cos x + i.\sin x)k+1 = (\cos x + i.\sin x)k(\cos x + i \sin x)$
$= [\cos (kx) + i.\sin (kx)].[\cos x + i.\sin x]$ □ [Using (1)]
$= \cos (kx).\cos x - \sin (kx).\sin x + i [\sin (kx).\cos x + \cos (kx).\sin x)$
$= \cos \{(k+1)x\} + i.\sin \{(k+1)x\}$
$= \cos \{(k+1)x\} + i.\sin \{(k+1)x\}$
Thus, P(k+1) is also true and by the principle of mathematical induction P(n) is true.

Q.12 **Simplify [(cos x + i.sin x)/(sin x + i.cos x)]8**
Solution:
$= [(\cos x + i.\sin x)/(\sin x + i.\cos x)]8$
$= (\cos x + i.\sin x)8/i8(\cos x - i.\sin x)8$
Using De Movire's Theorem and taking the value of $i8 = 1$
$= (\cos 8x + i.\sin 8x)/(\cos 8x - i.\sin 8x)$
Rationalising we get,
$= (\cos 8x + i.\sin 8x)2 / (\cos2 8x - i2.\sin2 8x)$
$= \cos 16x + i.\sin 16x$
Since, $[(\cos2 8x - i2.\sin2 8x)$
$= (\cos2 8x + \sin2 8x) = 1]$

Q.13 Find the square root of complex number **z=3+4i**

Solution: Given, z=3+4i

(x+iy)2=3+4i

x2−y2+i(2xy)=3+4i

⇒x2−y2=3 and 2xy=4

⇒x2−y2=3 and xy=2

∴x=2 and y=1

⇒3+4i=2+i

Q.14 Find the square root of complex number −4−3i.
Solution
Correct option is A)
Let −4−3i=a+ib
−4−3i=a2−b2+2aib
a2−b2=−4,ab=2−3
ab=2−3⇒b=2a−3
a2−b2=−4⇒a2−4a29=−4
4a4−9=−16a2
4a4+16a2−9=0
4a4+18a2−2a2−9=0
2a2(2a2+9)−1(2a2+9)=0
a=±21,a=±i23
b=±23,b=±21
Therefore, −4−3i=±21(1+3i)
or ±21(3+i)

Q.15 Find the square root of complex number −21−20i.
Solution:
Correct option is B)
Let −21−20i=x+iy

−21−20i=(x+iy)2=x2−y2+2ixy
Equating real and imaginary parts we get
x2−y2=−21 ------(1)
2xy=−20 ------(2)
Therefore, (x2+y2)2=(x2−y2)2+4x2y2=(−21)2+(−20)2=841

$x^2+y^2=\sqrt{841}=29$ ------(3)

adding (1) and (3) we get

$2x^2=8$

$x=\pm\sqrt{4}=\pm 2$

Substituting x in (2) we get

$y=\mp 5$

Therefore, the square root of $-21-20i$ is $\pm(2-5i)$

UNIT-4

Q.16 Find the prime factorisation of the following numbers:

(i) 25600

(ii) 51000

(iii) 3700

(iv) 20000

Solution:

(i) 25600

Factors	Prime Factors
$25600 = 12800 \times 2$	2
$12800 = 6400 \times 2$	2, 6400

$\therefore$Prime Factorisation of 25600 = $2 \times 2 \times 6400$

(ii) 51000

Factors	Prime Factors
$51000 = 25500 \times 2$	2
$25500 = 12750 \times 2$	2, 12750

$\therefore$Prime Factorisation of 51000 = $2 \times 2 \times 12750$

(iii) 3700

Factors	Prime Factors
$3700 = 1850 \times 2$	2, 1850

∴Prime Factorisation of 3700 = 2 × 1850

(iv) 20000

Factors	Prime Factors
20000 = 10000 × 2	2
10000 = 5000 × 2	2, 5000

∴Prime Factorisation of 20000 = 2 × 2 × 5000

Q.17 Find the prime factorisation of the following numbers:

(i) 312

(ii) 420

(iii) 7040

(iv) 6000

Solution:

(i) 312

Factors	Prime Factors
312 = 156 × 2	2
156 = 2 × 78	2, 78

∴Prime Factorisation of 312 = 2 × 2 × 78

(ii) 420

Factors	Prime Factors
420 = 210 × 2	2, 210

(iii) 7040

Factors	Prime Factors
7040 = 3520 × 2	2
3520 = 1760 × 2	2
1760 = 880 × 2	2
880 = 440 × 2	2
440 = 220 × 2	2
220 = 110 × 2	2
110 = 55 × 2	2

$55 = 11 \times 5$	5, 11

∴Prime factorisation of $7040 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 11$.

(iv) 6000

Factors	Prime Factors
$6000 = 3000 \times 2$	2
$3000 = 1500 \times 2$	2, 1500

∴Prime factorisation of $6000 = 2 \times 2 \times 1500$.

Q.18 Rahul and Rohan have 45 marbles together. After losing 5 marbles each, the product of the number of marbles they both have now is 124. How to find out how many marbles they had to start with.

Solution: Say, the number of marbles Rahul had be x.

Then the number of marbles Rohan had = 45 – x.

The number of marbles left with Rahul after losing 5 marbles = x – 5

The number of marbles left with Rohan after losing 5 marbles = 45 – x – 5 = 40 – x

The product of number of marbles = 124

(x – 5) (40 – x) = 124

40x – x2 – 200 + 5x = 124

– x2 + 45x – 200 = 124

x2 – 45x + 324 = 0

This represents the quadratic equation. Hence by solving the given equation for x, we get;

x = 36 and x = 9

So, the number of marbles Rahul had is 36 and Rohan had is 9 or vice versa.

Q.19 **Solve the equation x2+4x-5=0.**

Solution:

x2 + 4x – 5 = 0

x2-1x+5x-5 = 0

x(x-1)+5(x-1) =0

(x-1)(x+5) =0

Hence, (x-1) =0, and (x+5) =0

x-1 = 0

x = 1

similarly, x+5 = 0

x =-5.

Therefore,

x=-5 & x=1

Q.20 **Solve the quadratic equation 2x2+ x – 300 = 0 using factorisation.**

Solution: 2x2+ x – 300 = 0

2x2 – 24x + 25x – 300 = 0

2x (x – 12) + 25 (x – 12) = 0

(x – 12)(2x + 25) = 0

So,

x-12=0; x=12

(2x+25) = 0; x=-25/2 = -12.5

Therefore, 12 and -12.5 are two roots of the given equation.
Also, read Factorisation.
Q.21 **Find the roots of the equation 2x2 – 5x + 3 = 0 using factorisation.**
Solution: Given,
2x2 – 5x + 3 = 0
2x2 – 2x-3x+3 = 0
2x(x-1)-3(x-1) = 0
(2x-3) (x-1) = 0
So,
2x-3 = 0; x = 3/2
(x-1) = 0; x=1
Therefore, 3/2 and 1 are the roots of the given equation.

UNIT-5

Polynomial Definition
A polynomial p(x) in one variable x is an algebraic expression in x of the form:
p(x) = anxn + an–1xn – 1 + . . . + a2x2 + a1x + a0.
where a0, a1 , a2, ... an are constants and an ≠ 0.
a0, a1, a2, ... an are respectively the coefficients of x0, x1, x2 , ... xn.
Each of anxn, an–1 xn–1, …. a0, with an ≠ 0, is called a term of the polynomial p(x).
The highest power of the variable in a polynomial is called the degree of that polynomial.

Polynomial with –	Name of the polynomial
One term	Monomial
Two terms	Binomial
Three terms	Trinomial
Degree 1	Linear polynomial
Degree 2	Quadratic polynomial
Degree 3	Cubic polynomial

Q.22 **Find the value of $x^3 + y^3 – 12xy + 64$, when $x + y = – 4$.**
Solution:
Given,
$x + y = -4$
Or
$x + y + 4 = 0$….(i)
Let the given polynomial be:
$p(x) = x^3 + y^3 – 12xy + 64$
$= x^3 + y^3 + 4^3 – 3(4xy)$ {since $64 = 4^3$}
We know that is $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

That means if $x + y + 4 = 0$, $x^3 + y^3 + 4^3 = 3(x)(y)(4)$ {from (i)}
So, $p(x) = 3(x)(y)(4) – 3(4xy)$
$= 12xy – 12xy$
$= 0$

Q.23 **Without actual division, prove that 2x4 – 5x3 + 2x2 – x + 2 is divisible by x2 – 3x + 2. [Hint: Factorise x2 – 3x + 2]**

Solution:

Let p(x) = 2x4 – 5x3 + 2x2 – x + 2
Let us factorise x2 – 3x + 2.
x2 – 3x + 2 = x2 – 2x – x + 2
= x(x – 2) -1(x – 2)
= (x – 1)(x – 2)
Hence, 1 and 2 are the zeroes of x2 – 3x + 2.
Now, substitute x = 1 and x = 2 in p(x).
p(1) = 2(1)4 – 5(1)3 + 2(1)2 -1 + 2
= 2 – 5 + 2 – 1 + 2
= 6 – 6
= 0
p(2) = 2(2)4 – 5(2)3 + 2(2)2 – 2 + 2
= 2(16) – 5(8) + 2(4)
= 32 – 40 + 8
= 40 – 40
= 0
Therefore, p(x) = 2x4 – 5x3 + 2x2 – x + 2 is divisible by x2 – 3x + 2.

Q.24 **Find the zeroes of the polynomial 4x2 – 3x – 1 by the factorisation method and verify the relation between the zeroes and the coefficients of the polynomial.**

Solution:

Let the given polynomial be:
p(x) = 4x2 – 3x – 1
= 4x2 -4x + x – 1
= 4x(x – 1) + 1(x – 1)
= (4x + 1)(x – 1)
Thus, x = -1/4 and x = 1 are the zeroes of the given polynomial.
Now,
Sum of zeroes = (-1/4) + 1 = (-1 + 4)/4 = 3/4 = -coefficient of x/coefficient of x2
Product of zeroes = (-1/4)(1) = -1/4 = constant term/coefficient of x2
Hence, the relation between the zeroes and the coefficients of the polynomial is verified.

Q.25 **Find a quadratic polynomial whose sum and product respectively of the zeroes are 21/8 and 5/16. Also, find the zeroes of the polynomial by factorisation.**

Solution:

Given,
Sum of zeroes = 21/8
Product of zeroes = 5/16
A quadratic polynomial with the sum and product of zeroes is given by:
p(x) = x2 – (sum of zeros) x + (product of roots)
= x2 – (21/8)x + (5/16)

Consider p(x) = 0 to solve for the zeroes.

⇒x2 – (21/8)x + (5/16) = 0

⇒16x2 – 42x + 5 = 0

⇒16x2 – 40x – 2x + 5 = 0

⇒8x(2x – 5) – 1(2x – 5) = 0

⇒(2x – 5) (8x – 1) = 0

⇒(2x – 5) = 0, (8x – 1) = 0

⇒2x = 5, 8x = 1

⇒x = 5/2, x = 1/8

Therefore, the two zeroes of the polynomial are 5/2 and 1/8.

Q.26 **For which values of a and b, are the zeroes of q(x) = x3 + 2x2 + a, also the zeroes of the polynomial p(x) = x5 – x4 – 4x3 + 3x2 + 3x + b? Which zeroes of p(x) are not the zeroes of q(x)?**

Solution:

Given,

p(x) = x5 – x4 – 4x3 + 3x2 + 3x + b

q(x) = x3 + 2x2 + a

$$-3x \quad +2$$

$$+2x^2 + a \;\big)\; -x^4 \quad -4x^3 \quad +3x^2 \quad +3x \quad +b$$

$$-$$

$$x^5 \quad +2x^4 \quad +0x^3 \quad +ax^2$$

$$-3x^4 \quad -4x^3 \quad +x^2(3-a) \quad +3x \quad +b$$

$$-$$

$$-3x^4 \quad -6x^3 \quad +0x^2 \quad -3ax$$

$$2x^3 \quad +x^2(3-a) \quad +x(3a+3) \quad +b$$

$$-$$

$$2x^3 \quad +4x^2 \quad +0x \quad +2a$$

$$x^2(-a-1) \quad +x(3a+3) \quad + \quad 2a+b$$

By division algorithm of polynomials,

Dividend = (divisor) (quotient) + remainder

⇒p(x) = g(x).q(x) + r(x)

r(x) = -(a + 1)x2 + 3(1 + a)x + b – 2a = 0

and

g(x) = x2 – 3x + 2

By comparing the coefficients,

– (a + 1) = 0 , (1 + a) = 0

⇒a = – 1

Also, b – 2a = 0

b = 2a = 2(- 1) = – 2

⇒b = – 2

q(x) = x3 + 2x2 – 1

Equating q(x) = 0 to get the zeroes.
x3 + 2x2 – 1 = 0
x3 + x2 + x2 – 1 = 0
x2(x + 1) + (x2 – 1) = 0
x2(x + 1) + (x + 1)(x – 1) = 0
(x + 1)(x2 + x – 1) = 0
g(x) = x2 – 3x + 2
Equating g(x) = 0 to get the zeroes.
x2 – 3x + 2 = 0
x2 – (x + 2x) + 2 = 0
x(x – 1) – 2(x – 1) = 0
(x – 1)(x – 2) = 0
Dividend = (divisor) (quotient) + remainder
⇒p(x) = g(x).q(x) + r(x) P(x) = x5 – x4 – 4x3 + 3x – 2 = q(x).g(x) + 0
= (x3 + 2x2 – 1) (x2 – 3x + 2)
= (x + 1) (x2 + x – 1) (x – 1) (x – 2)
Therefore, 1 and 2 are the zeroes of the polynomial p(x) that are not in q(x).

Q.27 What are the properties and operations that define a finite field?

A finite field, also known as a Galois field, is a field with a finite number of elements. It has two fundamental operations: addition and multiplication. The properties that define it include closure, associativity, commutativity, distributivity, existence of identity and inverse elements.

Closure means for any two elements in the field, their sum or product is also in the field. Associativity implies (a+b)+c=a+(b+c) and (ab)c=a(bc). Commutativity ensures a+b=b+a and ab=ba. Distributivity links both operations through a(b+c)=ab+ac.

Identity elements are special members of the field. For addition, there's '0' such that a+0=a; for multiplication, there's '1' such that a*1=a. Every element has an additive inverse (-a), satisfying a+(-a)=0, and a multiplicative inverse (1/a), fulfilling a*(1/a)=1, except for 0 which lacks a multiplicative inverse.

Q.28 Can you describe an algorithm for constructing a finite field of a given order?

A finite field, also known as a Galois Field, can be constructed using the following algorithm. Given an order 'p' which is a prime number, we start by defining a set of integers from 0 to p-1. The addition and multiplication operations are then defined modulo p. This results in a finite field GF(p).

For non-prime orders, say q=p^n where p is prime and n>1, we first construct the field GF(p) as above. Then, we find a polynomial of degree n that is irreducible over GF(p). The elements of GF(q) are then represented as polynomials of degree less than n, with coefficients in GF(p). Addition and multiplication are performed modulo this irreducible polynomial.

Q.29 How can you use the Chinese Remainder Theorem in the context of finite fields?

The Chinese Remainder Theorem (CRT) is a useful tool in finite fields, particularly for simplifying computations. In the context of finite fields, CRT can be used to break down complex problems into simpler ones by working modulo several smaller primes instead of one large prime. This allows us to perform calculations more efficiently.

For instance, consider a finite field F_p where p is a prime number. If we want to solve an equation in this field, it might be easier to first solve it in several smaller fields F_q1, F_q2,...,F_qn and then use CRT to combine these solutions into a solution in F_p.

This approach is especially beneficial when dealing with polynomial equations over finite fields. Instead of solving the polynomial equation over the entire field, we can solve it over several subfields and then use CRT to find the overall solution.

Q.30 How can we use finite fields in image and signal processing?

Finite fields, also known as Galois fields, are used in image and signal processing to perform operations on data. They provide a mathematical framework for error detection and correction algorithms which is crucial in digital communications. In image processing, finite fields can be applied in various techniques such as error diffusion, halftoning, and image compression. For instance, the JPEG2000 standard uses finite field arithmetic in its wavelet transform coding scheme. Similarly, in signal processing, they play an integral role in convolutional codes used for error control in wireless communication systems. Finite fields allow these processes to be more efficient and reliable by reducing errors and improving quality of transmission or storage.