CSE 318
Artificial Intelligence Sessional

Report on Assignment3 - Local Search

Section - A1

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August 2023

1 Introduction

1.1 The max-cut problem

Given an undirected graph, $G = (V, U)$, where V is the set of vertices and U is the set of edges, and weights w_{uv} associated with each edge $(u, v)\epsilon U$, the maximum cut (MAX-CUT) problem consists in finding a nonempty proper subset of vertices $S \subset V (S \neq \phi)$, such that the weight of the cut (S, S'), given by $w(S, S') = \sum_{u\epsilon S, v\epsilon S'} w_{uv}$, is maximized.

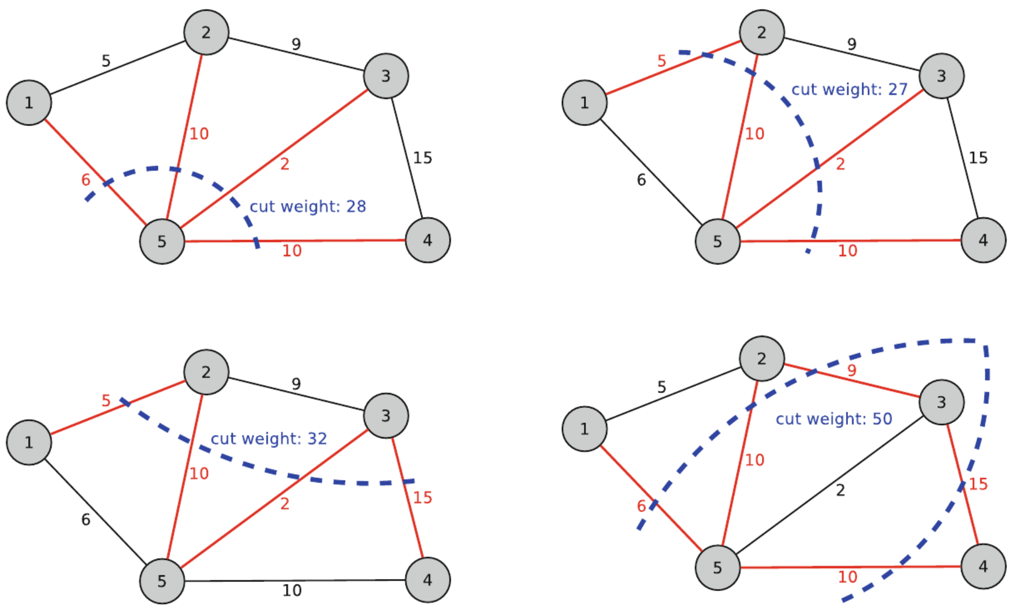

Figure 1: Example of the maximum cut problem on a graph with five vertices and seven edges. Four cuts are shown. The maximum cut is $(S, S') = (\{1, 2, 4\}, \{3, 5\})$ and has a weight $w(S, S') = 50$.

1.2 The tasks

In this assignment, our focus has been on three distinctive algorithmic strategies: randomized, greedy, and semi-greedy construction phases. By analyzing key metrics such as average iterations, average local optima, and GRASP cost(in case of semi-greedy construction), we are able to gain insights of these approaches. This investigation sheds light on their convergence characteristics, adaptability, and efficiency.

2 Analysis

The implementation and analysis of randomized, greedy, and semi-greedy construction phases in solving the max-cut problem have provided valuable insights into their performance. The obtained results, including the average number of iterations, average local optima, and GRASP cost, offer a comprehensive understanding of how these algorithms approach and address the problem.

2.1 Simple Randomized or Randomized-1

The randomized construction phase involves creating initial solutions using a randomized approach.

Initially, both partitions X and Y are empty. For each vertex $v \epsilon V$, place v in partition X or partition Y with uniform randomness, i.e., with probability $\frac{1}{2}$. The procedure terminates when all vertices are placed either in X or Y.

This means that the solutions are generated by making random decisions without adhering to any strict heuristic rules. The higher average number of iterations indicates that the randomized algorithm might require more iterations to converge or find locally optimal solutions. As the construction phase is random, sometimes it may outperform greedy and semi-greedy in terms of local optima(the max-cut cost).

2.2 Simple Greedy or Greedy-1

The greedy construction phase relies on a deterministic heuristic that makes locally optimal decisions at each step. Hence, it was run for only one iteration(local search followed by construction). The lower average number of iterations suggests that the greedy algorithm converges faster, indicating that the heuristic choices are effective at reaching solutions quickly. But there is an issue in this approach. Greedy approach follows a specific path in the solution space, always moving towards the best solution, it may get stuck in local optima. Hence, the average best value is lower(in most cases) compared to GRASP best value which uses Semi-greedy construction phase.

2.3 Semi-greedy-1

The semi-greedy construction phase combines aspects of both randomization and greedy heuristics. This could mean that the algorithm balances exploration and exploitation to some extent(Here $\alpha = 0.6$). The average number of iterations and local optima falls between the randomized and greedy approaches, showcasing a trade-off between exploration and convergence speed. The average best value might also fall in between the other two methods since semi-greedy construction aims to strike a balance between the two extremes, potentially resulting in moderate-quality initial solutions.

3 Summary

Construction	**Initial Solution Quality**	**Efficiency**	**Limitations**
Simple Randomized	Random	Bad	Inconsistent Performance
Simple Greedy	Good	Good	Trap in Local Optima
Semi-greedy	Moderate	Moderate	Both to some extent

Table 1: A simple Comparision among Three Construction Algorithms

4 Conclusion

In conclusion, the choice of construction phase significantly influences the behavior and performance of the GRASP algorithm in solving the max-cut problem. The randomized approach offers wide exploration potential but might require more iterations and optimization steps to reach competitive solutions. The greedy approach provides quick convergence but might get trapped in local optima. The semi-greedy approach finds a middle ground between the two, aiming to balance exploration and exploitation. The average best value of local search, which summarizes the quality of initial solutions, underscores the importance of the construction phase's impact on the overall algorithm's efficiency and effectiveness.

Besides, the best value of GRASP(over 50 iterations) is quite close(Mean of deviation $10.87\%, \sigma = 8.8\%$) to those of Best Known Value(given in the specification), hence showing the merit of GRASP. As there is a bit of randomization, these values may differ from workstations to workstations as well as number of iterations.