

# OPTIMIZING THE PAKISTAN SUPER LEAGUE SCHEDULE BY INTEGER PROGRAMMING

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## 1 INTRODUCTION

The purpose of this paper is to optimize the Pakistan Super League (PSL) Schedule through the use of Integer programming. Scheduling games for any professional sport is an extremely complex process. A scheduler must adhere to multiple scheduling rules, while ensuring that the players play their allotment of games (Grabau, 2012). In scheduling professional games, several constraints must be met as part of the rules. These constraints also exist to enhance the overall experience for the program's customers: players, coaches, and parents. Other constraints are also fundamental, because the games would be chaotic without them; for example, only one game may be played on a field at a time. PSL is a professional Twenty20 cricket league contested during February and March every year by 6 teams from 6 different cities in Pakistan. It was launched in 2015 with 5 teams initially and has been a great success afterwards. This objective of this paper is therefore to examine and reproduce the scheduling model proposed by Grabau (2012) based on the data and situation of the professional Twenty20 cricket league. It then presents an integer programming approach to solve this problem. A final optimized schedule will first be generated using the model, followed by a more schedules based different scenarios. All models are reproduced through programming language and computer optimization tool, followed by a visualized calendar view representation.

## 2 OPTIMIZATION MODEL

The proposed model is based on Grabau (2012). With some of the constraints in the reference paper tailored for the softball game situation, we shortlisted the constraints applicable to the context of the PSL and utilized the adjusted model to generate results schedules and scenarios.

The mathematical representation of the model is as follows.

$$\text{Min } \sum_i \sum_j \sum_k \sum_w \sum_d \sum_t \sum_f X_{ijkwdtf} \quad \forall i \text{ in Divisions, } j \text{ in Away Teams}[i], k \text{ in Home Teams}[i], w \text{ in Weeks, } d \text{ in Days, } t \text{ in Times, } f \text{ in Fields} \quad (0)$$

ST

$$X_{ijkwdtf} = 0 \quad \forall i \text{ in Divisions, } j \text{ in Away Teams}[i], w \text{ in Weeks, } d \text{ in Days, } t \text{ in Times, } f \text{ in Fields} \quad (0)$$

$$\sum_j \sum_w \sum_d \sum_t \sum_f X_{ijkwdtf} \geq \text{Away Games} \quad \forall i \text{ in Divisions, } k \text{ in Home Teams}[i] \quad (1a)$$

$$\sum_k \sum_w \sum_d \sum_t \sum_f X_{ijkwdtf} \geq \text{Home Games} \quad \forall i \text{ in Divisions, } j \text{ in Away Teams}[i] \quad (1b)$$

$$\sum_k \sum_d \sum_t \sum_f (X_{ijkwdtf} + X_{ikjwdtf}) \geq 1 \quad \forall i \text{ in Divisions, } j \text{ in Away Teams}[i], w \text{ in Weeks} \quad (2a)$$

$$\sum_k \sum_d \sum_t \sum_f (X_{ijkwdtf} + X_{ikjwdtf}) \leq 2 \quad \forall i \text{ in Divisions, } j \text{ in Away Teams}[i], w \text{ in Weeks} \quad (2b)$$

$$\sum_k \sum_t \sum_f (X_{ijkwdtf} + X_{ikjwdtf}) \leq 1 \quad \forall i \text{ in Divisions, } j \text{ in Away Teams}[i], w \text{ in Weeks, } d \text{ in Days} \quad (3)$$

$$\sum_t \sum_j \sum_k X_{ijkwdtf} \leq 1 \quad \forall w \text{ in Weeks, } d \text{ in Days, } t \text{ in Times, } f \text{ in Fields} \quad (4)$$

$$\sum_w \sum_d \sum_t \sum_f (X_{ijkwdtf} + X_{ikjwdtf}) \leq \text{maxReplays}[i] \quad \forall i \text{ in Divisions, } j \text{ in Away Teams}[i], k \text{ in Home Teams} \quad (5)$$

$$\sum_k \sum_t \sum_f (X_{ijkwdtf} + X_{ikjwdtf} + X_{ijkw(d+1)tf} + X_{ikjw(d+1)tf}) \leq 1 \quad \forall i \text{ in Divisions, } j \text{ in Away Teams}[i], w \text{ in Weeks, } d \text{ in Days} \in \{\text{Saturday}\} \quad (6)$$

$$\sum_d \sum_t \sum_f (X_{ijkwdtf} + X_{ikjwdtf}) \leq 1 \quad \forall i \text{ in Divisions, } j \text{ in Away Teams}[i], k \text{ in Home Teams}[i], w \text{ in Weeks} \quad (7)$$

**Objective Function (O):** To minimize the number of games in the schedule. Each team is scheduled to play for a minimum number of games based on Constraint (1), which specifies that each team should play a certain number of home and away games. The objective function's aim is to formulate a schedule to keep any games, in addition to those enforced by Constraint (1), to a minimum. By minimizing the total number of games played, while establishing a lower bound instead of a strict equality on the number of home and away games, flexibility is built into the program to allow extra games to be played only if necessary (Grabau, 2012). The schedules generated would provide the teams with benefits such as lower costs and fairer seasons, while at the same time complying with the format of the league as well as the rules and regulations dictated by the International Cricket Council.

**Constraints (0-7):**

- (0) A team may not play itself. This makes sense in the context of a competition.
- (1) Each team should play a specified number of away and home games.
- (2) Each team should play at least one game, but no more than two, per week.
- (3) Each team can play only one game per day.
- (4) Only one game can be played on a field at a time.
- (5) Each team should play the same team as few times as possible.
- (6) Teams should not play on consecutive nights.
- (7) Do not schedule a team to play the same team twice in the same week.

### 3 NUMERICAL EXAMPLE

We have used Python, which is a high level and well-supported programming language for mathematical modeling. The reason Python program being used is because it has special features that make it much easier to build our optimization model. In addition to that, we have chosen ‘Gurobi’ as the mathematical optimization solver. We also used ‘PyCharm’ as an integrated development environment (IDE) to edit, modify, and execute the Python code. After executing the Python code, we got the output that was exported into an excel worksheet. To show the output of the model, we have used the Tableau Desktop Reader (Public Edition), which is a free software to visualize the data.

We have taken different data sources to collect the necessary information for the PSL cricket tournament. At first, we have reviewed the online sports website e.g., ESPN Cricinfo to get the most updated news on how many teams are playing and to understand the structure of this tournament. Then we have gathered the team records and stadiums information with match scorecard from the website: (<https://stats.espncricinfo.com>). We have also seen the timetable of the PSL matches from (<https://www.pslmatches.com>). Finally, we have found the full match schedule of the fifth edition of the Pakistan Super League (PSL) that was held in February and March 2020 from this website: (<https://www.sportskeeda.com>)

The data set is a specific type of (T20 Cricket) sports that has been taken on their teams, fields, on which days those matches will be played on, as well as time. Matches are being played on a weekly basis at a (home/away) fields. There is a given time when the matches will be played between 2 different teams in a division. This is the final output that has been exported into an excel worksheet where matches are being played over 5 weeks among different teams in several stadiums. Each team is playing at least once but no more than two times in a week. Also, each team is playing one match per day and one match can be played in a stadium at a time. We have seen that none of the teams are playing on consecutive days. Hence, there is no schedule of a team to play the same team twice in the same week.

TABLE 1 Final Output of Scheduling the T20 PSL Tournament (Excel)

Week	Days	Time (hrs.)	Match	Fields
1	Sunday	21	Lahore Qalandars vs Islamabad United	Peshawar Cricket Stadium (Peshawar)
1	Wednesday	16	Lahore Qalandars vs Karachi Kings	Multan Cricket Stadium (Multan)
1	Wednesday	20	Peshawar Zalmi vs Multan Sultans	National Stadium (Karachi)
1	Saturday	21	Multan Sultans vs Quetta Gladiators	Multan Cricket Stadium (Multan)
2	Sunday	18	Lahore Qalandars vs Karachi Kings	Peshawar Cricket Stadium (Peshawar)
2	Monday	16	Multan Sultans vs Quetta Gladiators	Quetta Cricket Stadium (Quetta)
2	Thursday	21	Quetta Gladiators vs Peshawar Zalmi	Multan Cricket Stadium (Multan)
2	Saturday	16	Karachi Kings vs Islamabad United	Rawalpindi Cricket Stadium (Islamabad)
3	Monday	16	Karachi Kings vs Islamabad United	Quetta Cricket Stadium (Quetta)
3	Tuesday	18	Quetta Gladiators vs Peshawar Zalmi	Multan Cricket Stadium (Multan)
3	Saturday	19	Karachi Kings vs Lahore Qalandars	Gaddafi Stadium (Lahore)
3	Saturday	22	Multan Sultans vs Peshawar Zalmi	Multan Cricket Stadium (Multan)
4	Monday	22	Lahore Qalandars vs Islamabad United	Rawalpindi Cricket Stadium (Islamabad)
4	Wednesday	15	Quetta Gladiators vs Multan Sultans	Gaddafi Stadium (Lahore)
4	Friday	20	Karachi Kings vs Islamabad United	Gaddafi Stadium (Lahore)
4	Saturday	18	Peshawar Zalmi vs Multan Sultans	Gaddafi Stadium (Lahore)
5	Monday	20	Islamabad United vs Karachi Kings	Rawalpindi Cricket Stadium (Islamabad)
5	Monday	17	Peshawar Zalmi vs Multan Sultans	Gaddafi Stadium (Lahore)
5	Wednesday	19	Karachi Kings vs Lahore Qalandars	Rawalpindi Cricket Stadium (Islamabad)
5	Saturday	17	Quetta Gladiators vs Multan Sultans	National Stadium (Karachi)

TABLE 2 Final Solution of the Optimization Model - T20 PSL Tournament (Python)

```
Explored 1 nodes (209 simplex iterations) in 2.48 seconds
Thread count was 4 (of 4 available processors)
```

Solution count 1: 20

Optimal solution found (tolerance 1.00e-04)

```
Best objective 2.0000000000000e+01, best bound 2.0000000000000e+01, gap 0.0000%
```

x Division 1:	Islamabad United vs Karachi Kings (Week): 5	on Monday (Time): 20 hrs.(Field): Rawalpindi Cricket Stadium (Islamabad)
x Division 1:	Karachi Kings vs Islamabad United (Week): 2	on Saturday (Time): 16 hrs.(Field): Rawalpindi Cricket Stadium (Islamabad)
x Division 1:	Karachi Kings vs Islamabad United (Week): 3	on Monday (Time): 16 hrs.(Field): Quetta Cricket Stadium (Quetta)
x Division 1:	Karachi Kings vs Islamabad United (Week): 4	on Friday (Time): 20 hrs.(Field): Gaddafi Stadium (Lahore)
x Division 1:	Karachi Kings vs Lahore Qalandars (Week): 3	on Saturday (Time): 19 hrs.(Field): Gaddafi Stadium (Lahore)
x Division 1:	Karachi Kings vs Lahore Qalandars (Week): 5	on Wednesday (Time): 19 hrs.(Field): Rawalpindi Cricket Stadium (Islamabad)
x Division 1:	Lahore Qalandars vs Islamabad United (Week): 1	on Sunday (Time): 21 hrs.(Field): Peshawar Cricket Stadium (Peshawar)
x Division 1:	Lahore Qalandars vs Islamabad United (Week): 4	on Monday (Time): 22 hrs.(Field): Rawalpindi Cricket Stadium (Islamabad)
x Division 1:	Lahore Qalandars vs Karachi Kings (Week): 1	on Wednesday (Time): 16 hrs.(Field): Multan Cricket Stadium (Multan)
x Division 1:	Lahore Qalandars vs Karachi Kings (Week): 2	on Sunday (Time): 18 hrs.(Field): Peshawar Cricket Stadium (Peshawar)
x Division 2:	Multan Sultans vs Peshawar Zalmi (Week): 3	on Saturday (Time): 22 hrs.(Field): Multan Cricket Stadium (Multan)
x Division 2:	Multan Sultans vs Quetta Gladiators (Week): 1	on Saturday (Time): 21 hrs.(Field): Multan Cricket Stadium (Multan)
x Division 2:	Multan Sultans vs Quetta Gladiators (Week): 2	on Monday (Time): 16 hrs.(Field): Quetta Cricket Stadium (Quetta)
x Division 2:	Peshawar Zalmi vs Multan Sultans (Week): 1	on Wednesday (Time): 20 hrs.(Field): National Cricket Stadium (Karachi)
x Division 2:	Peshawar Zalmi vs Multan Sultans (Week): 4	on Saturday (Time): 18 hrs.(Field): Gaddafi Stadium (Lahore)
x Division 2:	Peshawar Zalmi vs Multan Sultans (Week): 5	on Monday (Time): 17 hrs.(Field): Gaddafi Stadium (Lahore)
x Division 2:	Quetta Gladiators vs Multan Sultans (Week): 4	on Wednesday (Time): 15 hrs.(Field): Gaddafi Stadium (Lahore)
x Division 2:	Quetta Gladiators vs Multan Sultans (Week): 5	on Saturday (Time): 17 hrs.(Field): National Cricket Stadium (Karachi)
x Division 2:	Quetta Gladiators vs Peshawar Zalmi (Week): 2	on Thursday (Time): 21 hrs.(Field): Multan Cricket Stadium (Multan)
x Division 2:	Quetta Gladiators vs Peshawar Zalmi (Week): 3	on Tuesday (Time): 18 hrs.(Field): Multan Cricket Stadium (Multan)

This is the final schedule for the T20 PSL tournament. There are a total of 6 teams playing with each other in 6 stadiums for a duration of 5 weeks. Matches are being played 7 days a week on different times. The first match during week 1 will be played between Lahore Qalandars vs Islamabad United at 2100 hours in Peshawar Cricket Stadium on Sunday and Lahore Qalandars vs Karachi Kings as well as Peshawar Zalmi vs Multan Sultans will be playing at 1600 hours and 2000 hours respectively in Multan Cricket Stadium on Wednesday. Multan Sultans vs Quetta Gladiators will be playing at 2100 hours in National Stadium (Karachi) on Saturday. During week 2, Lahore Qalandars vs Karachi Kings will be playing at 1800 hours in Peshawar Cricket Stadium on Sunday. Multan Sultans vs Quetta Gladiators will be playing with each other at 1600 hours in Quetta Cricket Stadium on Monday. Thereafter, Quetta Gladiators vs Peshawar Zalmi will be playing at 2100 hours in Multan Cricket Stadium on Thursday. The last match during week 2 will be played between Karachi Kings vs Islamabad United at 1600 hours in Rawalpindi Cricket Stadium on Saturday.

TABLE 3 Final Schedule of the T20 PSL Tournament during Week 1 & 2

		Week / Fields						
		1			2			
Days	Match	Multan Cricket Stadium (Multan)	National Stadium (Karachi)	Peshawar Cricket Stadium (Peshawar)	Multan Cricket Stadium (Multan)	Peshawar Cricket Stadium (Peshawar)	Quetta Cricket Stadium (Quetta)	Rawalpindi Cricket Stadium (Islamabad)
Sunday	Lahore Qalandars vs Islamabad United Lahore Qalandars vs Karachi Kings							
Monday	Islamabad United vs Karachi Kings							
	Karachi Kings vs Islamabad United							
	Lahore Qalandars vs Islamabad United							
	Multan Sultans vs Quetta Gladiators							
	Peshawar Zalmi vs Multan Sultans							
Tuesday	Quetta Gladiators vs Peshawar Zalmi							
Wednesday	Karachi Kings vs Lahore Qalandars							
	Lahore Qalandars vs Karachi Kings							
	Peshawar Zalmi vs Multan Sultans							
	Quetta Gladiators vs Multan Sultans							
Thursday	Quetta Gladiators vs Peshawar Zalmi							
Friday	Karachi Kings vs Islamabad United							
Saturday	Karachi Kings vs Islamabad United							
	Karachi Kings vs Lahore Qalandars							
	Multan Sultans vs Peshawar Zalmi							
	Multan Sultans vs Quetta Gladiators							
	Peshawar Zalmi vs Multan Sultans							
	Quetta Gladiators vs Multan Sultans							

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The first match during week 3 will be played between Karachi Kings vs Islamabad United at 1600 hours in Quetta Cricket Stadium on Monday whereas Quetta Gladiators vs Peshawar Zalmi will be playing at 1800 hours in Multan Cricket Stadium on Tuesday. Karachi Kings vs Lahore Qalandars and Multan Sultans vs Peshawar Zalmi will be playing at 1900 hours and 1800 hours respectively on Saturday in two different stadiums.

During week 4, Lahore Qalandars vs Islamabad United will be playing at 2200 hours in Rawalpindi Cricket Stadium on Monday. 3 other matches will be played in Gaddafi Stadium (Lahore) among Quetta Gladiators vs Multan Sultans, Karachi Kings vs Islamabad United, and Peshawar Zalmi vs Multan Sultans teams at 1500 hours, 1800 hours and 2000 hours on Wednesday, Friday and Saturday respectively. During week 5, Islamabad United vs Karachi Kings will be playing at 2000 hours in Rawalpindi Cricket Stadium and Peshawar Zalmi vs Multan Sultans will be playing at 1700 hours in Gaddafi Stadium (Lahore) on Monday. After that, Karachi Kings vs Lahore Qalandars will be playing at 1900 hours in Rawalpindi Cricket Stadium on Wednesday. The last match will be played between Quetta Gladiators vs Multan Sultans at 1700 hours in National Stadium (Karachi) on Saturday.

TABLE 4 Final Schedule of the T20 PSL Tournament during Week 3, 4 and 5

Sports Scheduling - T20 PSL									
		Week / Fields							
Days	Match	Gaddafi Stadium (Lahore)	Multan Cricket Stadium (Multan)	Quetta Cricket Stadium (Quetta)	Gaddafi Stadium (Lahore)	Rawalpindi Cricket Stadium (Islamabad)	Gaddafi Stadium (Lahore)	National Stadium (Karachi)	Rawalpindi Cricket Stadium (Islamabad)
Sunday	Lahore Qalandars vs Islamabad United Lahore Qalandars vs Karachi Kings								
Monday	Islamabad United vs Karachi Kings Karachi Kings vs Islamabad United Lahore Qalandars vs Islamabad United Multan Sultans vs Quetta Gladiators Peshawar Zalmi vs Multan Sultans								
Tuesday	Quetta Gladiators vs Peshawar Zalmi								
Wednesday	Karachi Kings vs Lahore Qalandars Lahore Qalandars vs Karachi Kings Peshawar Zalmi vs Multan Sultans Quetta Gladiators vs Multan Sultans								
Thursday	Quetta Gladiators vs Peshawar Zalmi								
Friday	Karachi Kings vs Islamabad United								
Saturday	Karachi Kings vs Islamabad United Karachi Kings vs Lahore Qalandars Multan Sultans vs Peshawar Zalmi Multan Sultans vs Quetta Gladiators Peshawar Zalmi vs Multan Sultans Quetta Gladiators vs Multan Sultans								
		15 20 Time (hrs.)	15 20 Time (hrs.)	15 20 Time (hrs.)	15 20 Time (hrs.)	15 20 Time (hrs.)	15 20 Time (hrs.)	15 20 Time (hrs.)	15 20 Time (hrs.)

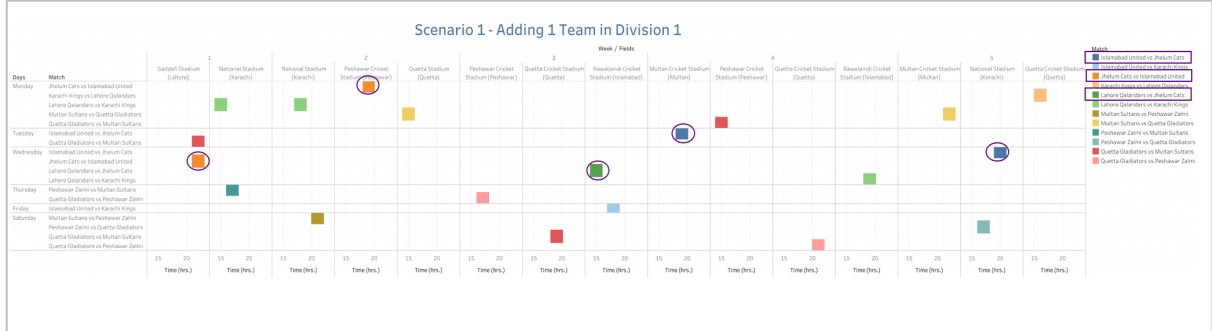
## 4 DISCUSSION

After examining the base-case scenario, where we had 3 teams in each division, we wanted to test our model under different scenarios. Since PSL started with 5 teams and another team was added because of the growing popularity, it deemed right to test the model by adding more teams to the divisions. We tried the model under 3 following scenarios:

- 1) Adding 1 team in Division 1
- 2) Adding 1 team in both Divisions
- 3) Adding 2 teams in Division 1

When we add 1 team in Division 1 (Jhelum Cats), we get the following output.

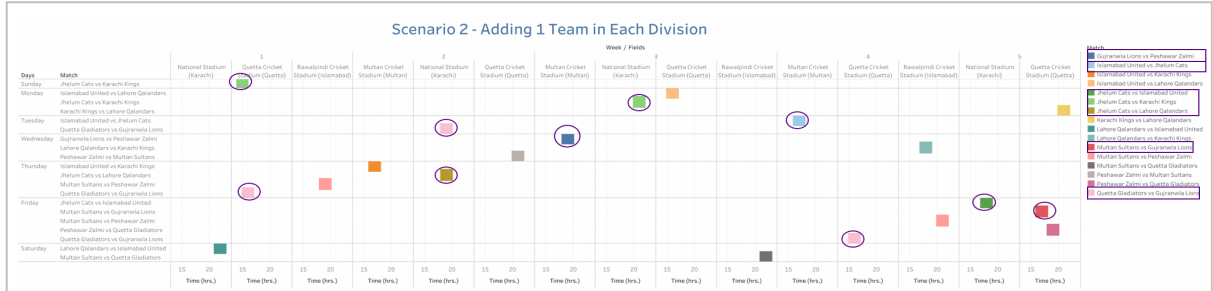
TABLE 4 Schedule of Scenario 1



When Jhelum Cats is added to Division 1, the model outputs 5 matches over 5 weeks where Jhelum Cats plays games. They play one game in each week.

When we add 1 team in both Divisions (Jhelum Cats and Gujranwala Lions), we get the following output.

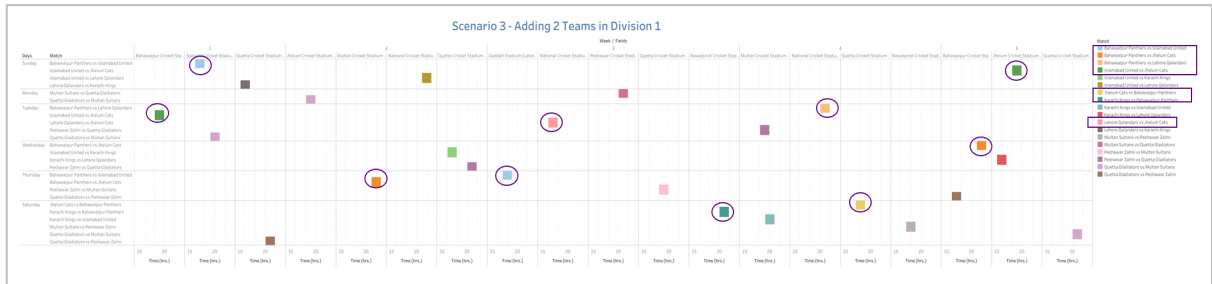
TABLE 5 Schedule of Scenario 2



The model outputs 10 matches over 5 weeks where Jhelum Cats and Gujranwala Lions play.

When we add 2 teams in Division 1 (Jhelum Cats and Bahawalpur Panthers), we get the following output.

TABLE 6 Schedule of Scenario 3



The model outputs 10 matches over the timespan of 5 weeks where Jhelum Cats and Bahawalpur panthers play. Bahawalpur and Jhelum each play 1 game in week 1, Bahawalpur and Jhelum play 1 game in week 2, Bahawalpur plays against Jhelum and both of these teams also play one more game each both in week 3 and week 4, And Bahawalpur plays once in week 5 whereas, Jhelum plays twice.



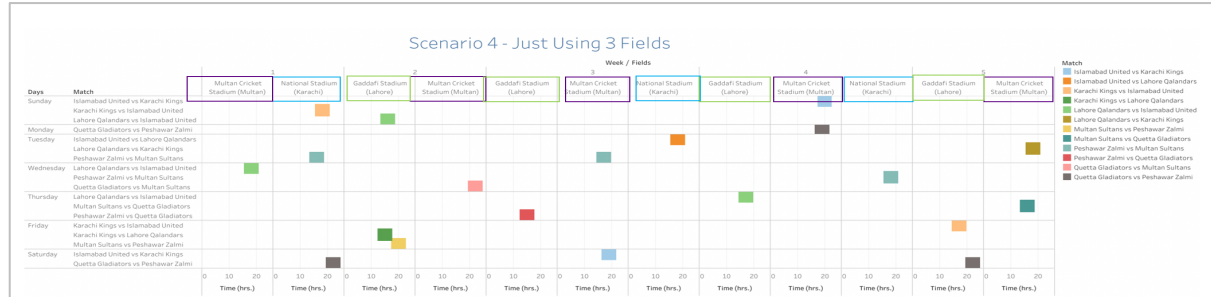
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We also wanted to change the number of fields that we could use in PSL in case the security situation did not allow for all fields to be used. For this, we looked at two more scenarios:

- 1) Just using 3 fields
- 2) Just using 1 field

When we use 3 fields (Multan Cricket Stadium, National Stadium (Karachi) and Gaddafi Stadium (Lahore)), we get the following output.

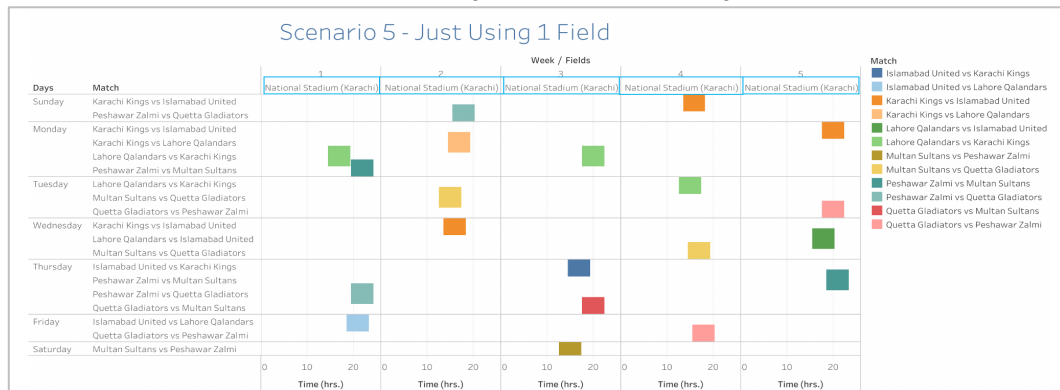
TABLE 7 Schedule of Scenario 4



In the output, there are a total of 20 games out of which 8 games are scheduled to be played in Multan, 5 in Karachi and 7 in Lahore.

When we use 1 field (National Stadium (Karachi)), we get the following output.

TABLE 8 Schedule of Scenario 5



The model shows an output where all 20 games are played in Karachi keeping in mind the constraints we have defined.

## 5 SUMMARY & FUTURE WORK

The model was able to successfully schedule PSL with all 7 constraints and our 5 scenarios. As we add more teams and fields to the model, the model becomes more complex and it takes longer for the computer to generate the outputs.

As future work, it would be interesting to schedule the ICC World Cup which takes place every 4 years where 10 international teams from around the globe participate. This event has a great economic significance for countries where the event is hosted. For e.g., during the last World Cup which took place in the United Kingdom, it provided a 350 Million Pounds economy boost to the country. Scheduling the World Cup would be a step up from

scheduling a national cricket league (PSL) as we can take into account more complex constraints. Even though a lot of people are able to watch the games in the stadiums, millions join via their TVs and the internet. Taking into account the ad-revenue for different advertisers, we would want to schedule most of the games where most of online and TV audience could be catered. Moreover, it would be interesting to group the rival teams together (for e.g., Pakistan vs India & Australia and England) and have the play matches on weekends which could allow more people to watch the games live.

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