Faster, More Accurate Quantum State Preparation

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The Problem: Quantum State Preparation

Loading classical discretized data from an arbitrary function into quantum computers is a critical bottleneck stalling breakthroughs in the application of quantum algorithms and machine learning. Traditional methods can be expensive in terms of resources and time, usually on an exponential scale. Recent works in this area have harnessed some sophisticated methods to drastically reduce this quantum gate complexity, but the work is far from adequate, as the classical subroutines involved in such state preparation methods still impose exponential time overheads.

$$|\psi_3\rangle = C_0\,|000\rangle + C_1\,|001\rangle + \cdots + C_7\,|111\rangle$$

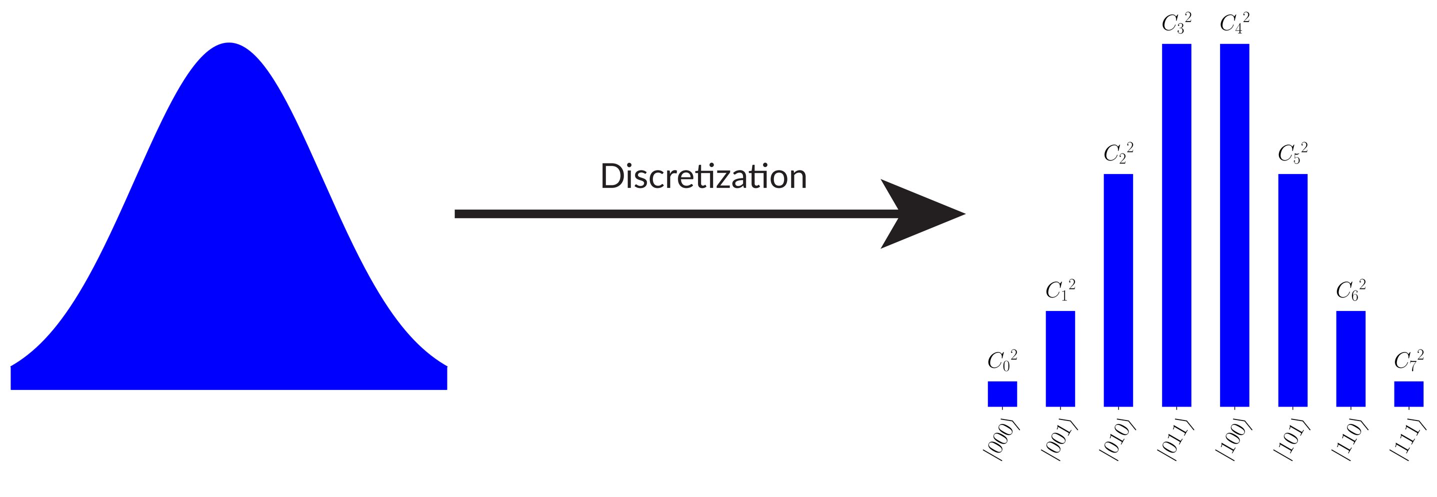

Figure 1. Initializing a 3-qubit quantum register with a normal distribution

Grover–Rudolph: The $O(2^n)$ brute-force solution

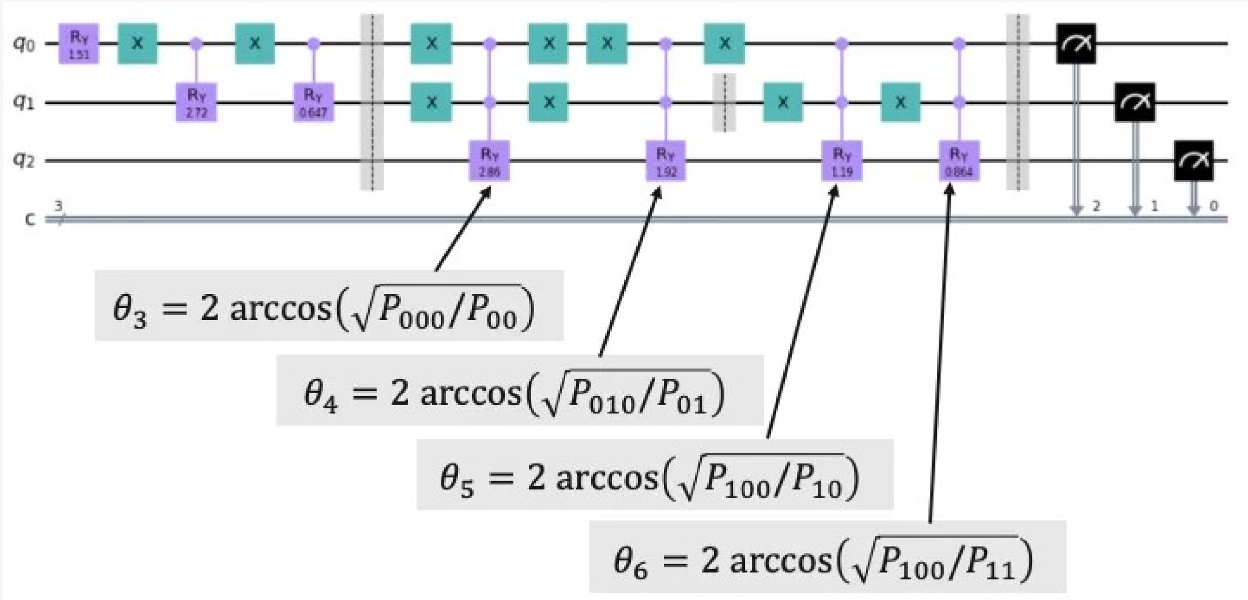

Figure 2. Quantum circuit for the Grover–Rudolph algorithm (source: [1])

This technique[2] divides the total region of a function in range $[x_{\min}, x_{\max}]$ into 2^{k-1} sub-regions where $k \in \{1, 2, \ldots, n\}$, here n is the number of qubits. To calculate the angle required for lth division where $l \in \{0, 1, 2, \ldots, 2^{k-1} - 1\}$ we define $\delta_k := (x_{\max} - x_{\min})/(2^k - 1)$ and we take the ratio of total area of that sub-division and half of the sub-division using this formula.

$$\theta_l^{k-1}(l) = 2\cos^{-1}\sqrt{\frac{\int_{x_{\min}+l\delta_k}^{x_{\min}+(l+1/2)\delta_k} f(x)\,\mathrm{d}x}{\int_{x_{\min}+l\delta_k}^{x_{\min}+(l+1)\delta_k} f(x)\,\mathrm{d}x}} \tag{1}$$

As we can see that the time and gate complexity for this circuit grows exponentially with increases in n, implying a total time complexity of $O(2^n)$.

Our solution: $O(2^{k_f(\epsilon)} + n)$

We limit our investigations to functions $f : [0, 1] \to \mathbb{R}^+$ for which $\partial_x^2 \log f(x)$ don't contain singularities on the allowed interval. Under these smoothness conditions, the technique proposed by Marin-Sanchez *et al.*[3] modifies the Grover-Rudolph algorithm[2] in the following way: It starts from an upper bound on the allowed infidelity ϵ_f in the final quantum state, and finds $k_0 = k_0(\epsilon_f)$ which serves as the separation point such that all the angles θ_l^{k-1} for $l \in \{0, 1, \ldots, 2^{k-1} - 1\}$ at the k-qubit block with $k \geq k_0$ can be clustered into a single angle $\bar{\theta}^{k-1}$. This is possible due to their proven theorem that any pair of angles in such a block differ by no more than a certain small constant for a given function. However, they don't say anything about the exact way to compute those $k - 1$ values.

We, on the other hand, start out from an allowed upper bound θ_f on the difference between pairs of clustered angles and discover k_c such that the k-qubit blocks with $k \geq k_0$ can be clustered into a single angle $\pi/2$. This simplifies the implementation from an engineering perspective, as well as possibly make it faster.

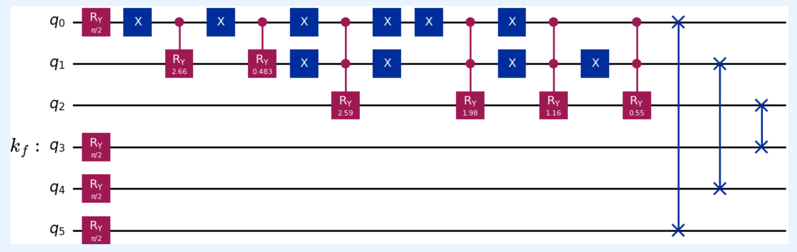

Figure 3. Quantum circuit for our technique

Deriving k_f

Let's begin with the observation that to achieve $\theta_l^k = \pi/2$ for all $l \in \{0, 1, \ldots, 2^{k-1} - 1\}$ from equation (1), the areas of two subdivisions must be equal. In other words, in that particular division, $\Delta f(x)$ must approach 0. Now, let's introduce an error term, ϵ_f, ensuring that the derivative of $f(x)$ in any region $[x, x + \Delta x]$ does not vary by more than an infinitesimal fraction ϵ_f. Then, we proceed to solve for Δx.

$$\Delta x = \frac{1}{2^{k_f - 1}} \Rightarrow k_f = \left\lceil \log_2 \frac{1}{\Delta x} \right\rceil + 1.$$

$$|f'(x + \Delta x) - f'(x)| \leq \epsilon_f. \tag{2}$$

Assuming $f(x)$ is at least twice differentiable, the Taylor expansion of f' around $x + \Delta x$ is

$$f'(x + \Delta x) = f'(x) + f''(x)\,\Delta x + \frac{f'''(\xi)}{2}(\Delta x)^2$$

$$\Rightarrow \left|f'(x + \Delta x) - f'(x)\right| = \left|f''(x)\,\Delta x + \frac{f'''(\xi)}{2}(\Delta x)^2\right| \tag{3}$$

for some $\xi \in (x, x + \Delta x)$. For Δx sufficiently small, the term involving $\frac{f'''(\xi)}{2}(\Delta x)^2$ can be made arbitrarily small compared to ϵ_f. Thus, from (2) and (3), we can approximate

$$|f''(x)\,\Delta x| \leq \epsilon_f$$

$$\Rightarrow \Delta x \leq \frac{\epsilon_f}{f''(x)}.$$

Hence, the target Δx such that the derivative $f'(x)$ does not vary more than an infinitesimal fraction ϵ_f over any interval $[x, x + \Delta x]$ is given by

$$\Delta x = \frac{\epsilon_f}{\sup_{x \leq \xi \leq x + \Delta x} |f''(\xi)|}$$

where $\sup_{x \leq \xi <= x + \delta x} |f''(\xi)|$ is the supremum of the absolute value of the second derivative over the interval $[x, x + \Delta x]$.

Results

Marin-Sanchez *et al.*'s technique versus ours

All results are for $n = 18$ qubits and (in case of our version) $k_f(\epsilon = 0.05) = 9$.

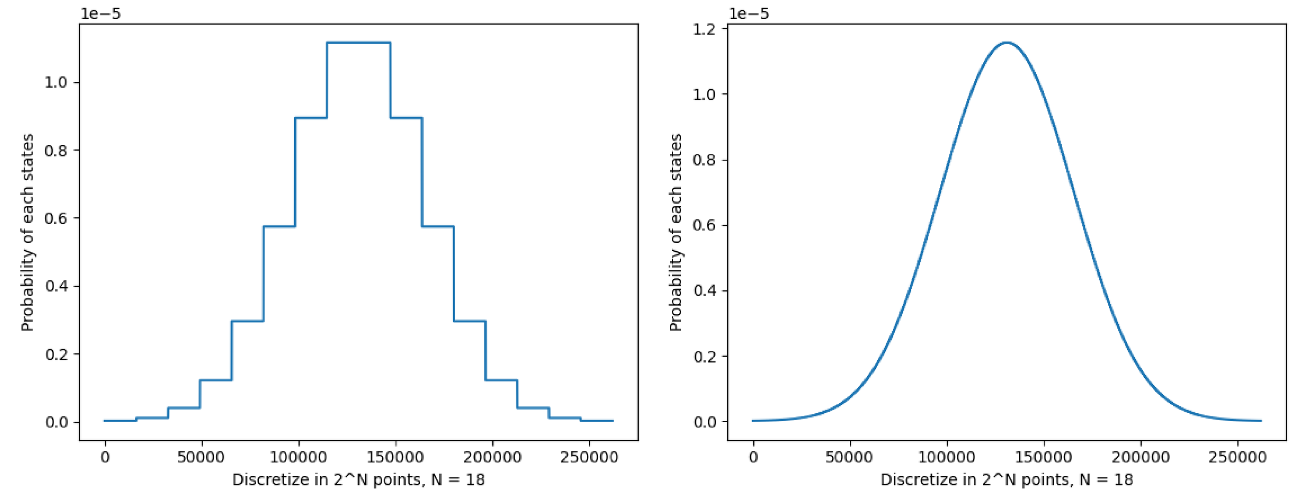

Figure 4. Normal distribution

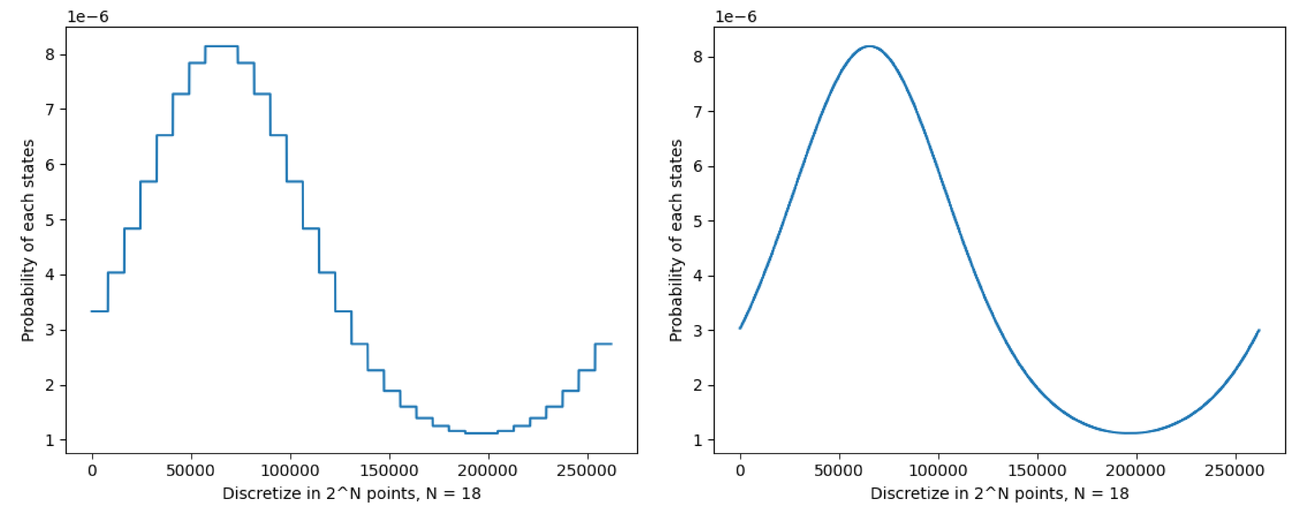

Figure 5. $p(x) = e^{\sin x}$

Fidelity

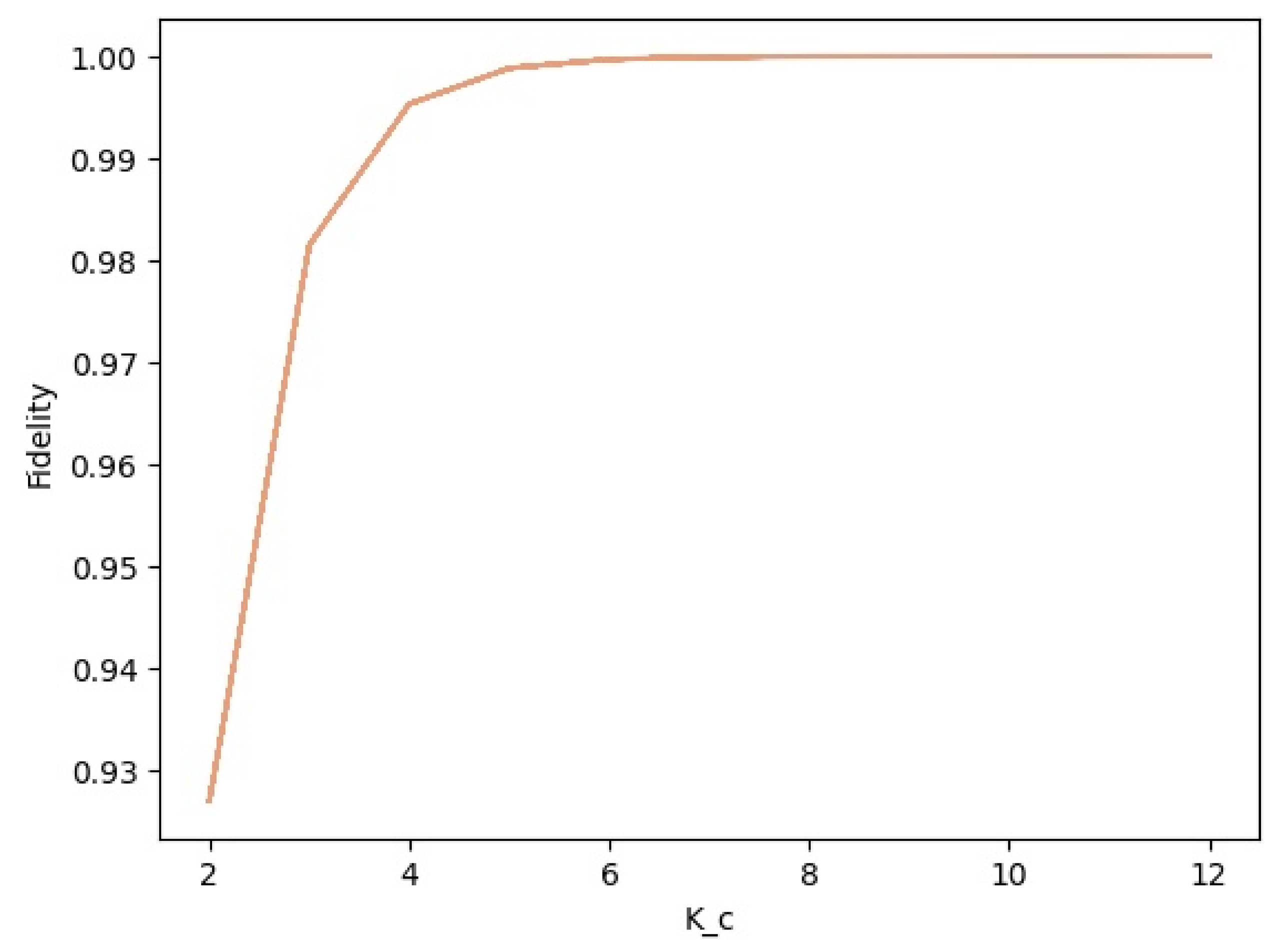

Figure 6. Fidelity for different k_c

References

[1] Y. Nakamura, "Systematic preparation of arbitrary probability distribution with a quantum computer."

[2] L. Grover and T. Rudolph, "Creating superpositions that correspond to efficiently integrable probability distributions."

[3] G. Marin-Sanchez, J. Gonzalez-Conde, and M. Sanz, "Quantum algorithms for approximate function loading," vol. 5, no. 3, p. 033114. Publisher: American Physical Society.