

# Faster, More Accurate Quantum State Preparation

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## The Problem

Loading classical discretized data from an arbitrary function into quantum computers is a critical bottleneck stalling breakthroughs in the application of quantum algorithms and machine learning. Traditional methods can be expensive in terms of resources and time, usually on an exponential scale. Recent works in this area have harnessed some sophisticated methods to drastically reduce this quantum gate complexity, but the work is far from adequate, as the classical subroutines involved in such state preparation methods still impose exponential time overheads.

$$|\psi_3\rangle = C_0 |000\rangle + C_1 |001\rangle + \dots + C_7 |111\rangle$$

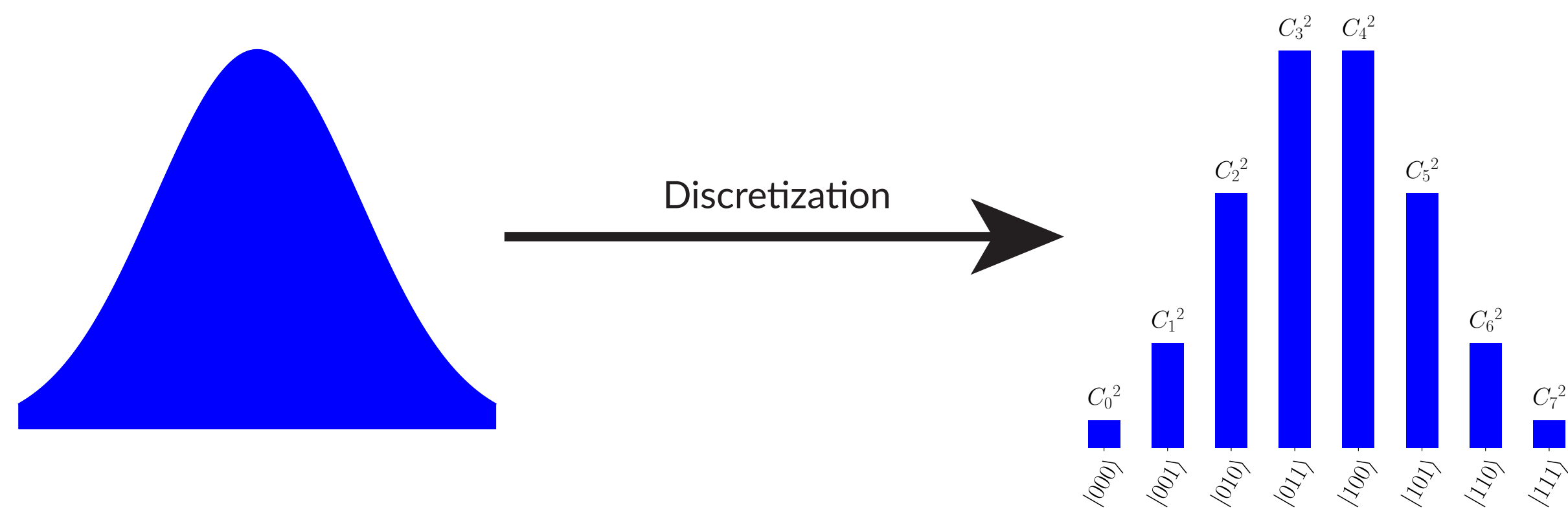


Figure 1. Initializing a 3-qubit quantum register with a normal distribution

## Grover-Rudolph: The $O(2^n)$ brute-force solution

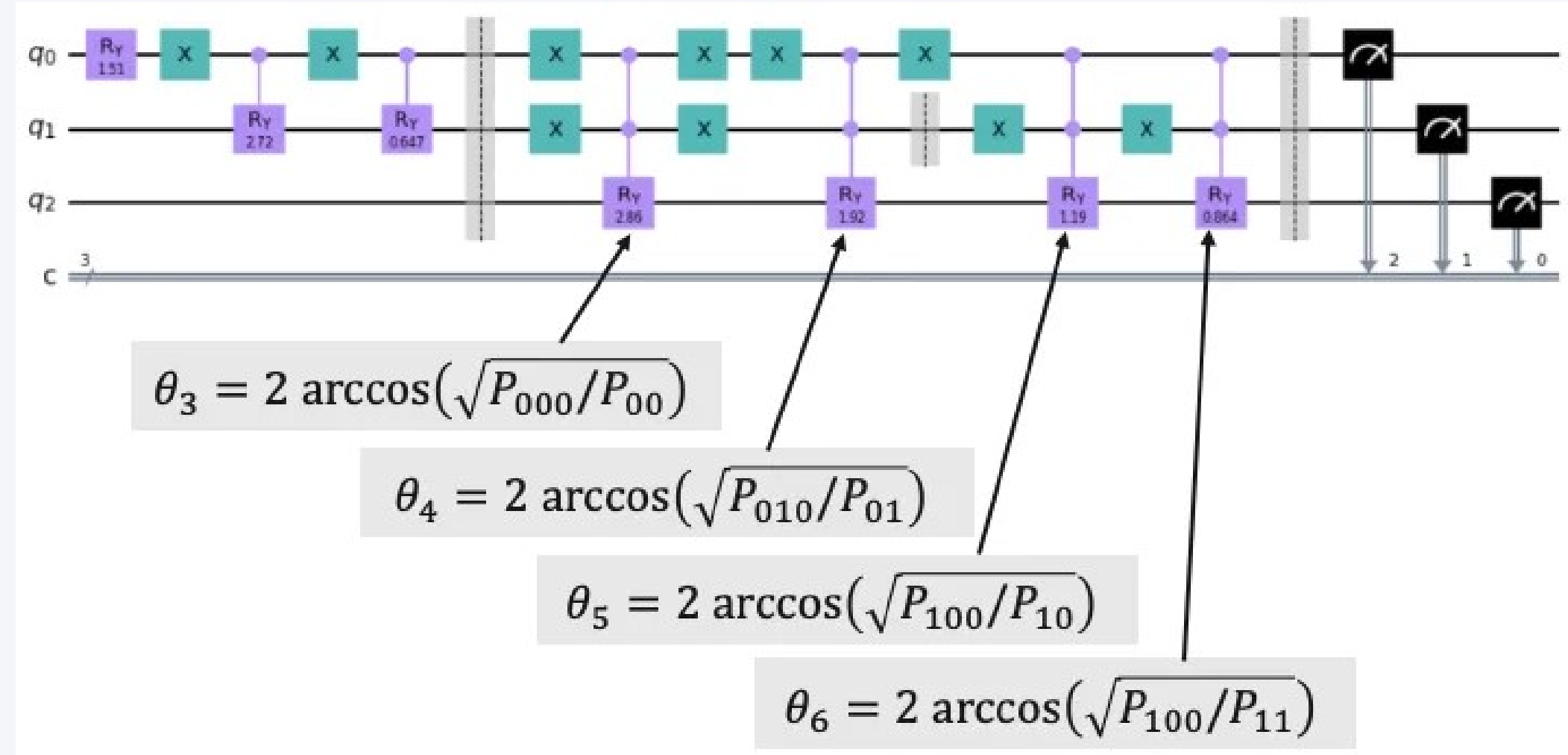


Figure 2. Quantum circuit for the Grover-Rudolph algorithm (source: [3])

This technique[1] divides the total region of a function in range  $[x_{\min}, x_{\max}]$  into  $2^{k-1}$  sub-regions where  $k \in \{1, 2, \dots, N\}$ , here  $N$  is the number of qubits. To calculate the angle required for  $l$ th division where  $l \in \{0, 1, 2, \dots, 2^{k-1} - 1\}$  we define  $\delta_k := (x_{\max} - x_{\min}) / (2^k - 1)$  and we take the ratio of total area of that sub-division and half of the sub-division using this formula.

$$\theta_l^{k-1}(l) = 2 \arccos \sqrt{\frac{\int_{x_{\min}}^{x_{\min} + (l+1/2)\delta_k} f(x) dx}{\int_{x_{\min}}^{x_{\min} + (l+1)\delta_k} f(x) dx}} \quad (1)$$

As we can see that the time and gate complexity for this circuit grows exponentially with respect to  $n$  and the final complexity reaches to  $O(2^n)$ .

## Our solution: $O(2^{k_f(\epsilon)} + n)$

We limit our investigations to functions  $f : [0, 1] \rightarrow \mathbb{R}^+$  for which  $\partial_x^2 f(x)$  don't contain singularities on the allowed interval. Under these smoothness conditions, the technique proposed by Marin-Sanchez et al.[2] starts from an upper bound on the allowed infidelity  $\epsilon_f$  in the final quantum state, and finds  $k_0 = k_0(\epsilon_f)$  which serves as the separation point such that all the angles  $\theta_l^{k-1}$  for  $l = 0, 1, \dots, 2^{k-1} - 1$  at the  $k$ -qubit block with  $k \geq k_0$  can be clustered into a single angle  $\bar{\theta}^{k-1}$ . This is possible due to their proven theorem that any pair of angles in such a block differ by no more than a certain small constant for a given function. However, they don't say anything about the exact way to compute those  $k-1$  values.

We, on the other hand, start out from an allowed upper bound  $\theta_f$  on the difference between pairs of clustered angles and discover  $k_c$  such that the  $k$ -qubit blocks with  $k \geq k_0$  can be clustered into a single angle  $\pi/2$ . This simplifies the implementation from an engineering perspective, as well as possibly make it faster.

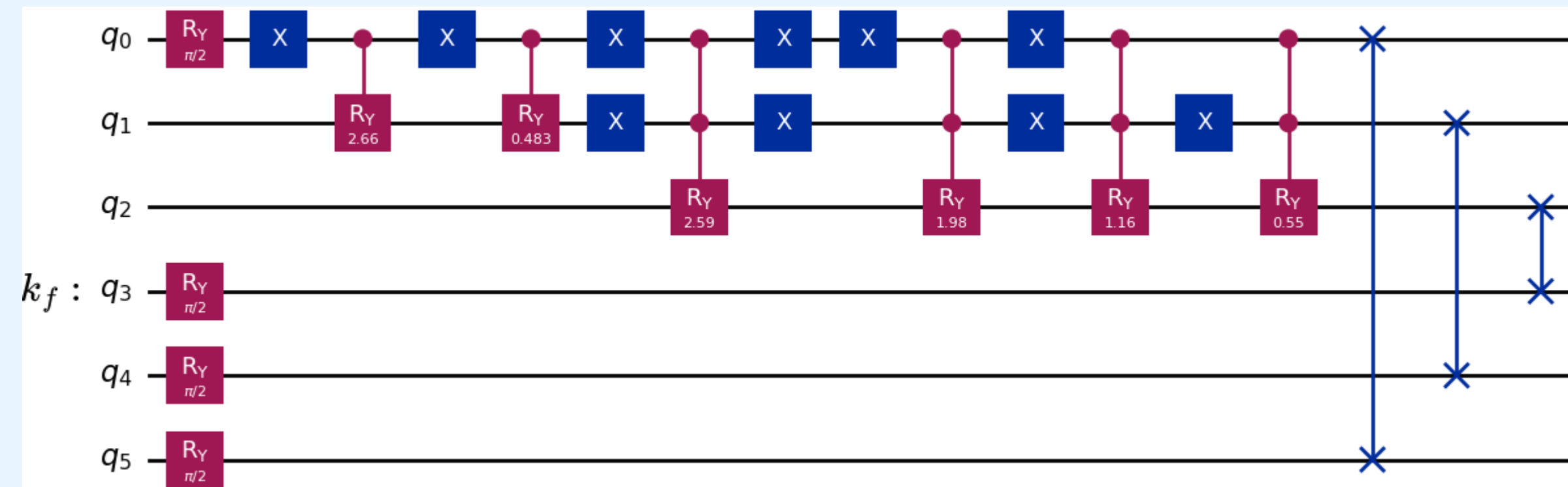


Figure 3. Quantum circuit for our technique

## Deriving $k_f$

Let's begin with the observation that to achieve  $\theta_l^k = \pi/2$  for all  $l \in \{0, 1, 2, \dots, 2^{k-1}\}$  from equation (1), the areas of two subdivisions must be equal. In other words, in that particular division,  $\Delta f(x)$  must approach 0. Now, let's introduce an error term,  $\epsilon_f$ , ensuring that the derivative of  $f(x)$  in any region  $[x, x + \Delta x]$  does not vary by more than an infinitesimal fraction  $\epsilon_f$ . Then, we proceed to solve for  $\Delta x$ .

$$\Delta x = \frac{1}{2^{k_f-1}} \Rightarrow k_f = \left\lceil \log_2 \frac{1}{\Delta x} \right\rceil + 1.$$

$$|f'(x + \Delta x) - f'(x)| \leq \epsilon_f$$

Taylor expansion:

$$f'(x + \Delta x) = f'(x) + f''(x) \Delta x + \frac{f'''(\xi)}{2} \Delta x^2$$

$$|f'(x + \Delta x) - f'(x)| = \left| f''(x) \Delta x + \frac{f'''(\xi)}{2} \Delta x^2 \right|$$

$$|f''(x) \Delta x| \leq \epsilon_f$$

$$\Delta x \leq \frac{\epsilon_f}{f''(x)}$$

Hence, the target  $\Delta x$  such that the derivative  $f'(x)$  does not vary more than an infinitesimal fraction  $\epsilon_f$  over any interval  $[x, x + \Delta x]$  is given by

$$\Delta x = \frac{\epsilon_f}{\sup_{x \leq \xi \leq x + \Delta x} |f''(\xi)|}$$

where  $\sup_{x \leq \xi \leq x + \Delta x} |f''(\xi)|$  is the supremum of the absolute value of the second derivative over the interval  $[x, x + \Delta x]$ .

## Results

### Marin-Sanchez et al's technique versus ours

All results are for  $n = 18$  qubits and (in case of our version)  $k_f(\epsilon = 0.05) = 9$ .

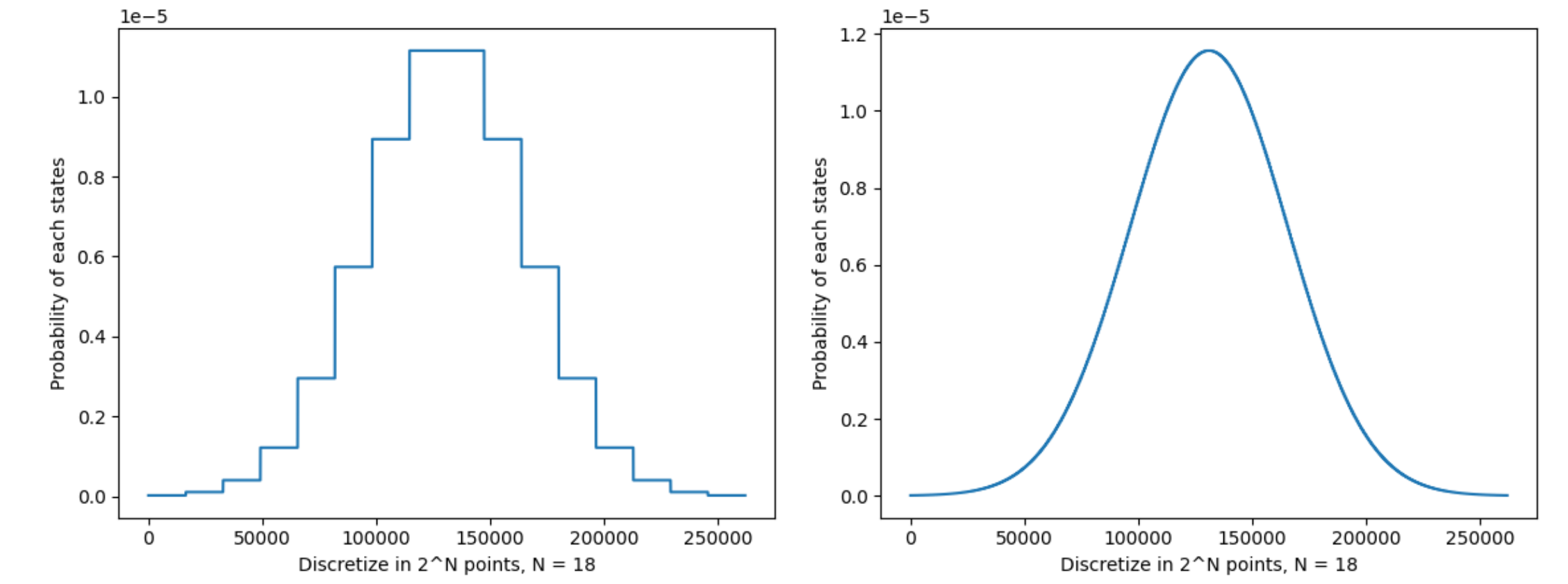


Figure 4. Normal distribution

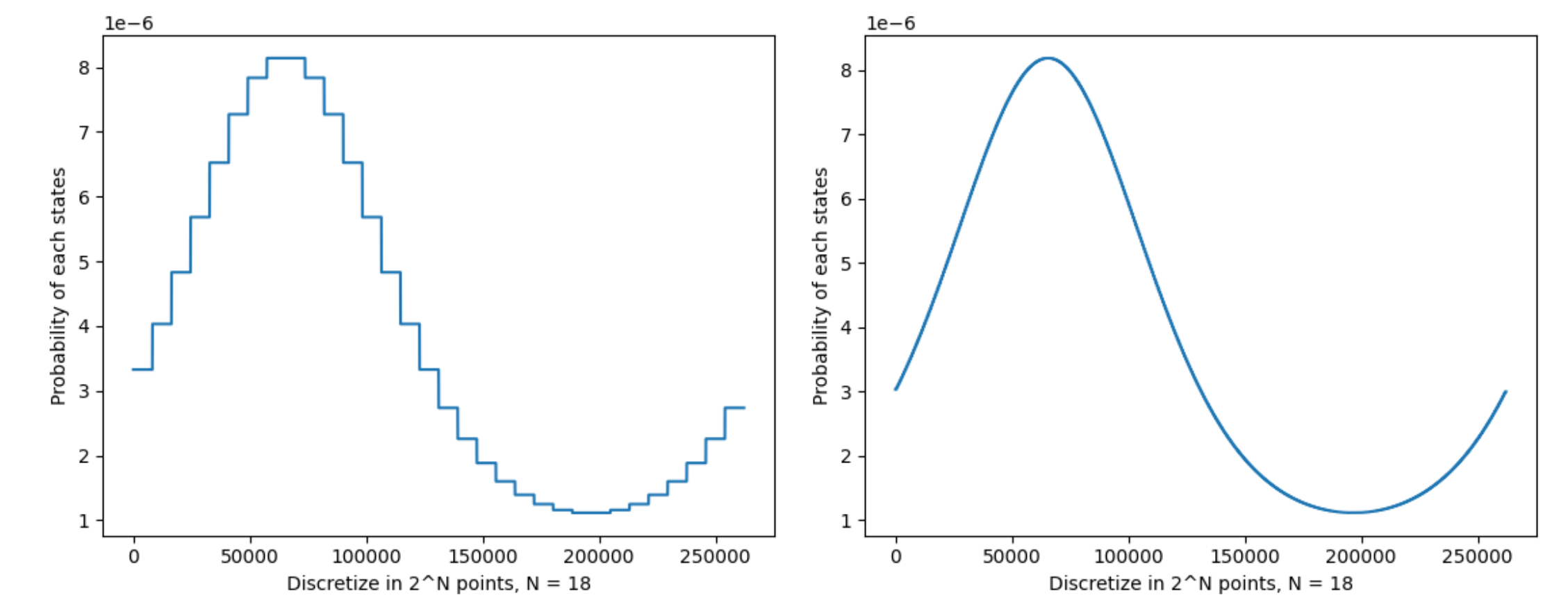


Figure 5.  $p(x) = e^{\sin x}$

### Fidelity

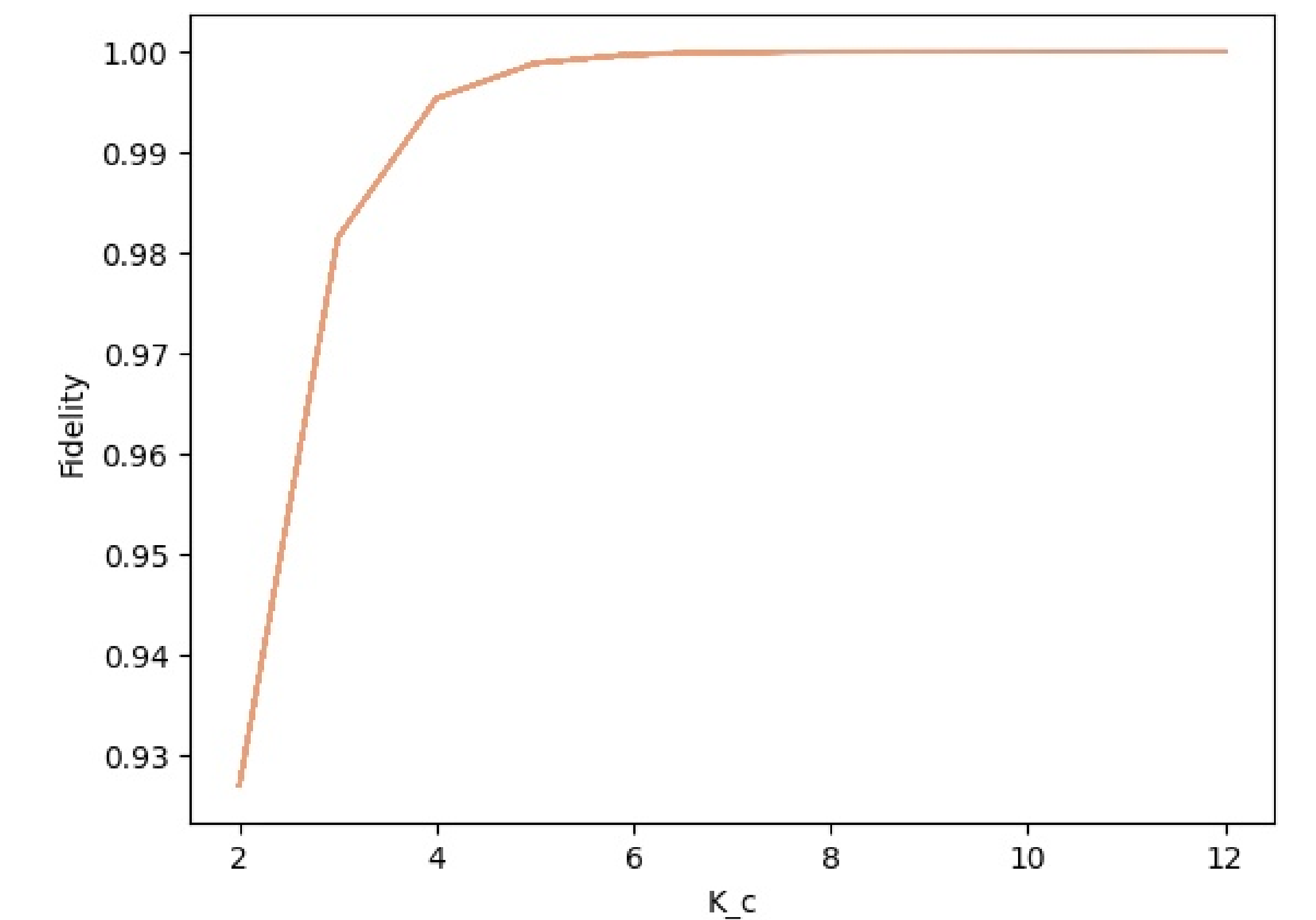


Figure 6. Fidelity for different  $k_c$

## References

- [1] Lov Grover and Terry Rudolph. Creating superpositions that correspond to efficiently integrable probability distributions.
- [2] Gabriel Marin-Sanchez, Javier Gonzalez-Conde, and Mikel Sanz. Quantum algorithms for approximate function loading. 5(3):033114. Publisher: American Physical Society.
- [3] Yuma Nakamura. Systematic preparation of arbitrary probability distribution with a quantum computer.