

Problem A.1: $x_1(t) = \cos \frac{3\pi}{10} t + \frac{1}{2} \cos \frac{\pi}{10} t$

$$\frac{W_{01}}{W_{02}} = \frac{\frac{3\pi}{10}}{\frac{\pi}{10}} = \frac{3\pi}{10} \times \frac{10}{\pi} = 3 \quad W_0 = \frac{\text{GCF}}{\text{LCM}} = \frac{\pi}{10} \quad T_0 = 20$$

$$\rightarrow x_1(t) = \frac{1}{2} e^{j\frac{3\pi}{10}t} + \frac{1}{2} e^{-j\frac{3\pi}{10}t} + \frac{1}{4} e^{j\frac{\pi}{10}t} + \frac{1}{4} e^{-j\frac{\pi}{10}t}$$

$$x_1(t) = \sum_{n=-\infty}^{\infty} D_n e^{-jn\frac{\pi}{10}t}$$

$$D_n = \frac{1}{20} \int \left[\frac{1}{2} e^{j\frac{3\pi}{10}t} + \frac{1}{2} e^{-j\frac{3\pi}{10}t} + \frac{1}{4} e^{j\frac{\pi}{10}t} + \frac{1}{4} e^{-j\frac{\pi}{10}t} \right] e^{-jn\frac{\pi}{10}t} dt$$

$$= \frac{1}{20} \int \frac{1}{2} e^{j\frac{3\pi}{10}t} e^{-jn\frac{\pi}{10}t} dt + \frac{1}{20} \int \frac{1}{2} e^{-j\frac{3\pi}{10}t} e^{-jn\frac{\pi}{10}t} dt$$

$$+ \frac{1}{20} \int \frac{1}{4} e^{j\frac{\pi}{10}t} e^{-jn\frac{\pi}{10}t} dt + \frac{1}{20} \int \frac{1}{4} e^{-j\frac{\pi}{10}t} e^{-jn\frac{\pi}{10}t} dt$$

$$= \frac{1}{40} \int_{T_0} e^{j\frac{\pi}{10}t(3-n)} dt + \frac{1}{40} \int_{T_0} e^{-j\frac{\pi}{10}t(3+n)} dt$$

$$+ \frac{1}{80} \int_{T_0} e^{j\frac{\pi}{10}t(1-n)} dt + \frac{1}{80} \int_{T_0} e^{-j\frac{\pi}{10}t(1+n)} dt$$

$$D_3 = \frac{1}{40} (20) \quad D_{-3} = \frac{1}{40} (20) \quad D_1 = \frac{1}{80} (20) \quad D_{-1} = \frac{1}{80} (20)$$

$$= \frac{1}{2} \quad = \frac{1}{2} \quad = \frac{1}{4} \quad = \frac{1}{4}$$

$$\therefore x_1(t) = \underbrace{\frac{1}{2}}_{D_3} e^{j\frac{3\pi}{10}t} + \underbrace{\frac{1}{2}}_{D_{-3}} e^{-j\frac{3\pi}{10}t} + \underbrace{\frac{1}{4}}_{D_1} e^{j\frac{\pi}{10}t} + \underbrace{\frac{1}{4}}_{D_{-1}} e^{-j\frac{\pi}{10}t}$$