Department of Electrical
& Computer Engineering
Faculty of Engineering & Architectural Science

Course Title:	Signals and Systems I
Course Number:	ELE 532
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Instructor:	Mahdi Shamsi

Assignment/Lab Number:	Lab 3
Assignment/Lab Title:	Fourier Series Analysis Using MATLAB

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*By signing above you attest that you have contributed to this written lab report and confirm that all work you have contributed to this lab report is your own work. Any suspicion of copying or plagiarism in this work will result in an investigation of Academic Misconduct and may result in a "0" on the work, an "F" in the course, or possibly more severe penalties, as well as a Disciplinary Notice on your academic record under the Student Code of Academic Conduct, which can be found online at: http://www.ryerson.ca/senate/current/pol60.pdf

Lab Assignment Answers

A.Lab Assignment

Problem A.1: Given the periodic signal x1(t):

$$x_1(t) = \cos\frac{3\pi}{10}t + \frac{1}{2}\cos\frac{\pi}{10}t,$$

derive an expression for the Exponential Fourier Series coefficients Dn.

The solution to this problem is at the end of the report in a written solution.

Problem A.2: Repeat Problem A.1 for the periodic signals x2(t) and x3(t) shown in Figure 1.

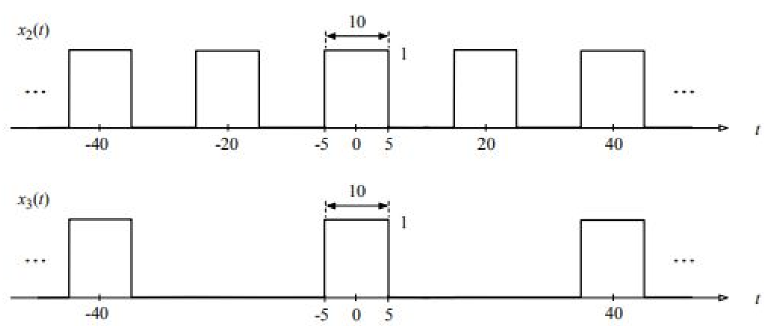

Figure 1: Periodic functions $x_2(t)$ and $x_3(t)$.

The solution to this problem is also at the end of the report in a written solution.

Problem A.3: Now that you have an expression for Dn, write a MATLAB function that generates Dn for a user specified range of values of n.

The following code was used to generate Dn values for a specified range of values of n. In here, as shown a function was used to find the three values D_n1, D_n2, Dn_3 for the respective signals shown above in the previous two questions.

```
%A.3:

function[D_n1,D_n2,D_n3] = ELE532_Assignment_3(n)
% This function provides the Dn expressions

D_n1 = (1/2)*(abs(n)==3)+(1/4)*(abs(n)==1);
D_n2 = (1/2).*(sinc(pi*n/2));
D_n3 = (1/4).*(sinc(pi*n/4));

end
```

Figure 1A: Code for generating Dn for Problem A.3.

Problem A.4: Generate and plot the magnitude and phase spectra of x1(t), x2(t) and x3(t) (using the stem command) from their respective Dn sets for the following index ranges: (a) $-5 \le n \le 5$; (b) $-20 \le n \le 20$; (c) $-50 \le n \le 50$; (d) $-500 \le n \le 500$. Note: You can use the MATLAB commands abs and angle to determine the magnitude and phase of a complex number.

The following code was used from the Lathi Textbook of Chapter 6. The code was changed for each specific range of n to plot the magnitude and phase spectra of x1(t), x2(t) and x3(t). As shown in the following figures, the plots can be seen for each range after the code that was used.

```
%A.4:
% -5<=n<=5
subplot(1,6,1);
stem(n1,abs(D_nx1(n1)),'.k');
xlabel('n1');
ylabel('|D_nx1|');
subplot(1,6,2);
stem(n1,angle(D_nx1(n1)),'.k');
xlabel('n1');
ylabel('\angle D_nx1 [rad]');
subplot(1,6,3);
stem(n1,abs(D_nx2(n1)),'.k');
xlabel('n1');
ylabel('|D_nx2|');
subplot(1,6,4);
stem(n1,angle(D_nx2(n1)),'.k');
xlabel('n1');
ylabel('\angle D_nx2 [rad]');
subplot(1,6,5);
stem(n1,abs(D_nx3(n1)),'.k');
xlabel('n1');
ylabel('|D_nx3|');
subplot(1,6,6);
stem(n1,angle(D_nx3(n1)),'.k');
xlabel('n1');
ylabel('\angle D_nx3 [rad]');
```

Figure 1B: Code for the plot of the range -5<=n<=5.

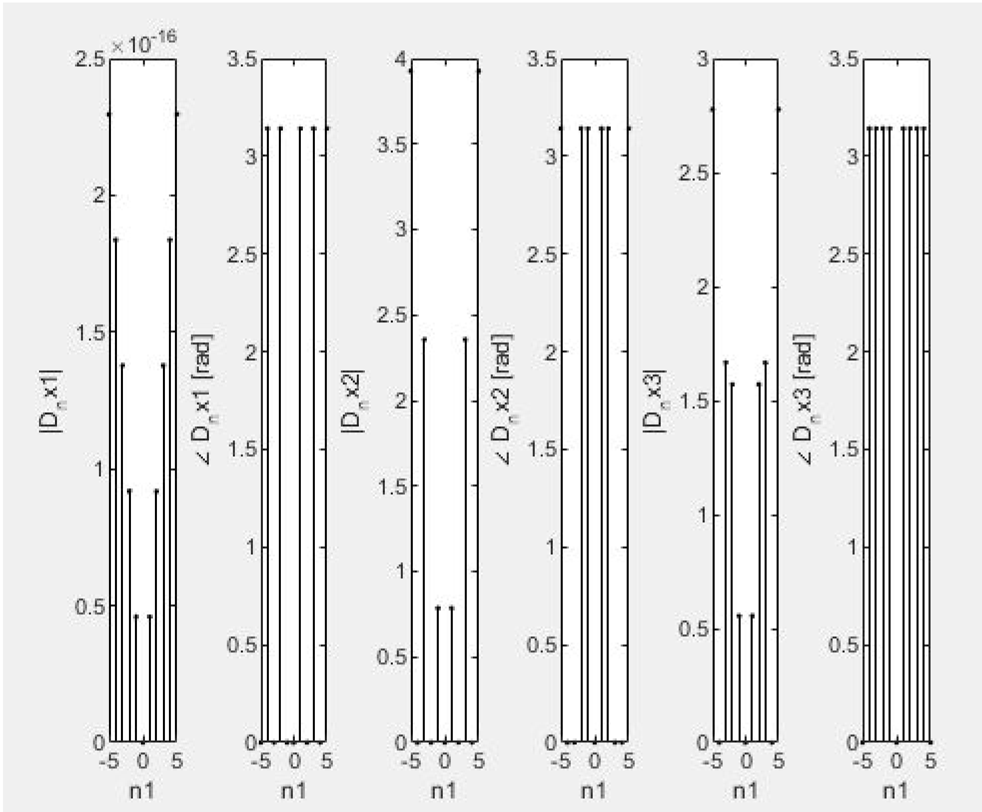

Figure 1C: Plot for the range of -5<=n<=5.

```
%-20<=n<=20
figure;
subplot(1,6,1);
stem(n2,abs(D_nx1(n2)),'.k');
xlabel('n');
ylabel('|D_nx1|');
subplot(1,6,2);
stem(n2,angle(D_nx1(n2)),'.k');
xlabel('n');
ylabel('\angle D_nx1 [rad]');
subplot(1,6,3);
stem(n2,abs(D_nx2(n2)),'.k');
xlabel('n');
ylabel('|D_nx2|');
subplot(1,6,4);
stem(n2,angle(D_nx2(n2)),'.k');
xlabel('n');
ylabel('\angle D_nx2 [rad]');
subplot(1,6,5);
stem(n2,abs(D_nx3(n2)),'.k');
xlabel('n');
ylabel('|D_nx3|');
subplot(1,6,6);
stem(n2,angle(D_nx3(n2)),'.k');
xlabel('n');
ylabel('\angle D_nx3 [rad]');
```

Figure 1D: Code for the plot of the range -20<=n<=20.

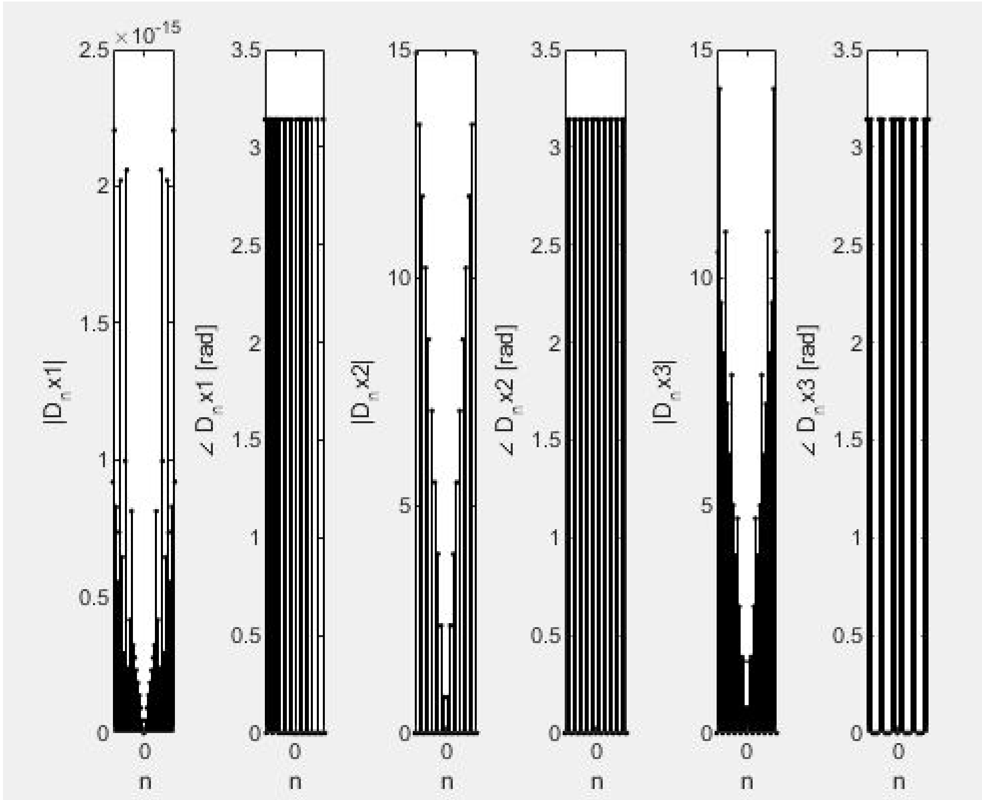

Figure 1E: Plot for the range of -20<=n<=20.

```
%-50<=n<=50
figure;
subplot(1,6,1);
stem(n3,abs(D_nx1(n3)),'.k');
xlabel('n');
ylabel('|D_nx1|');
subplot(1,6,2);
stem(n3,angle(D_nx1(n3)),'.k');
xlabel('n');
ylabel('\angle D_nx1 [rad]');
subplot(1,6,3);
stem(n3,abs(D_nx2(n3)),'.k');
xlabel('n');
ylabel('|D_nx2|');
subplot(1,6,4);
stem(n3,angle(D_nx2(n3)),'.k');
xlabel('n');
ylabel('\angle D_nx2 [rad]');
subplot(1,6,5);
stem(n3,abs(D_nx3(n3)),'.k');
xlabel('n');
ylabel('|D_nx3|');
subplot(1,6,6);
stem(n3,angle(D_nx3(n3)),'.k');
xlabel('n');
ylabel('\angle D_nx3 [rad]');
```

Figure 1F: Code for the plot of the range -50<=n<=50.

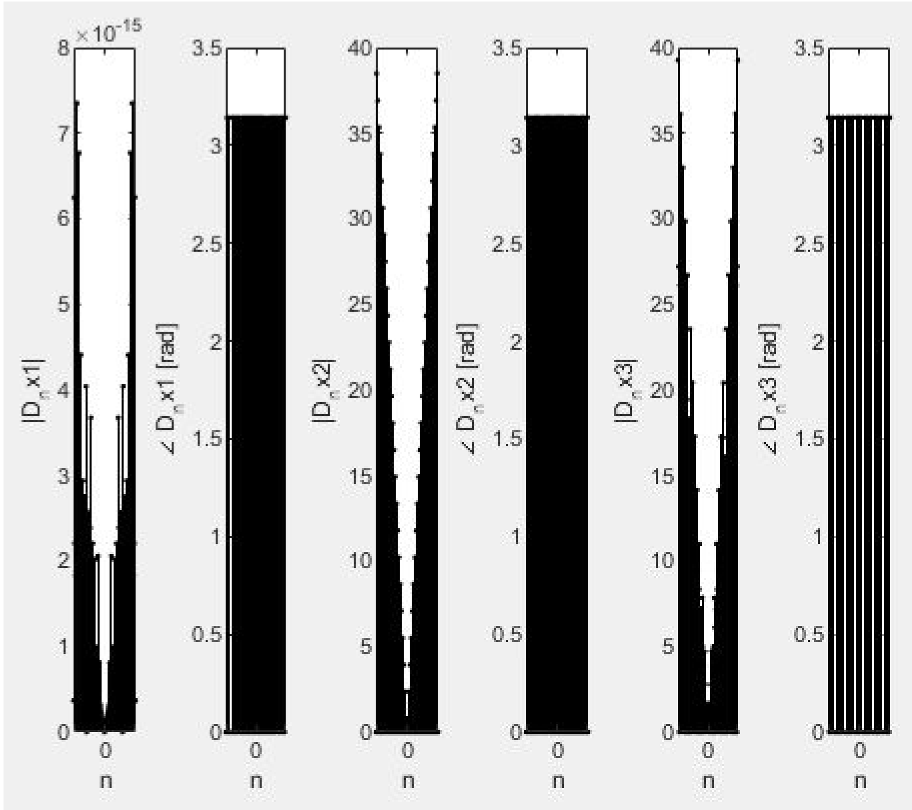

Figure 1G: Plot for the range of -50<=n<=50.

```
%-500<=n<=500
figure;
subplot(1,6,1);
stem(n4,abs(D_nx1(n4)),'.k');
xlabel('n');
ylabel('|D_nx1|');
subplot(1,6,2);
stem(n4,angle(D_nx1(n4)),'.k');
xlabel('n');
ylabel('\angle D_nx1 [rad]');
subplot(1,6,3);
stem(n4,abs(D_nx2(n4)),'.k');
xlabel('n');
ylabel('|D_nx2|');
subplot(1,6,4);
stem(n4,angle(D_nx2(n4)),'.k');
xlabel('n');
ylabel('\angle D_nx2 [rad]');
subplot(1,6,5);
stem(n4,abs(D_nx3(n4)),'.k');
xlabel('n');
ylabel('|D_nx3|');
subplot(1,6,6);
stem(n4,angle(D_nx3(n4)),'.k');
xlabel('n');
```

Figure 1H: Code for the plot of the range -500<=n<=500.

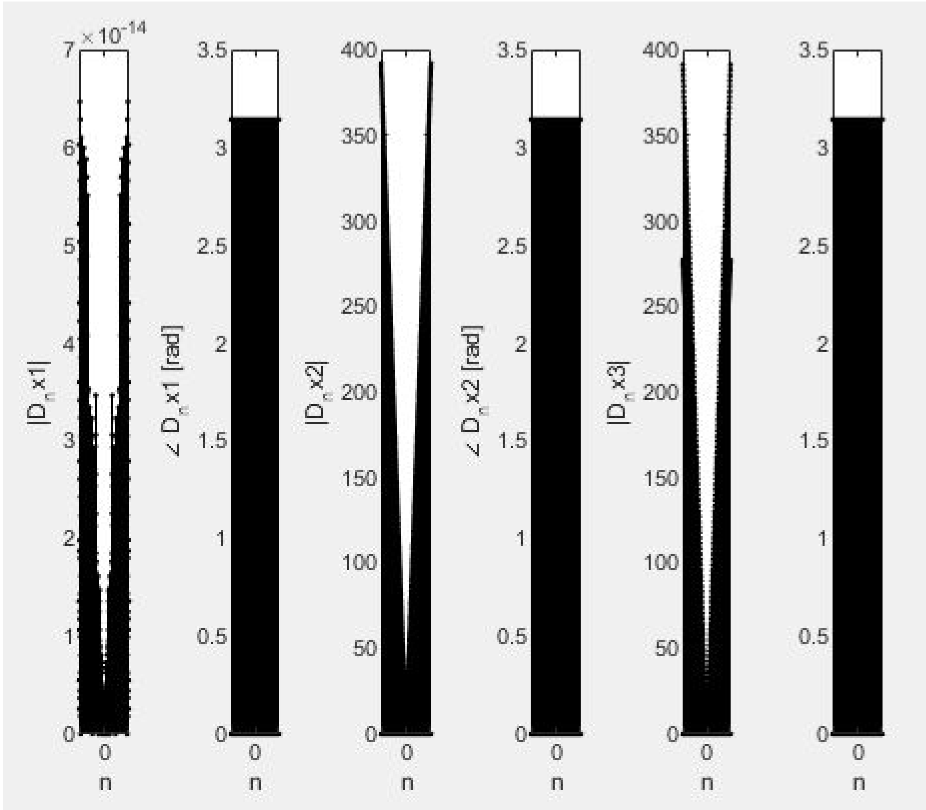

Figure 1I: Plot for the range of -500<=n<=500.

Problem A.5: Write a MATLAB function that takes a MATLAB generated Dn set and reconstructs the original time-domain signal from which the Fourier coefficients had been derived. For example, given the set of truncated Fourier coefficients {Dn, n = 0, ±1, . . . , ±20}, your code should reconstruct the time-domain signal from this set using Equation (1). Note: Use the time variable t defined with the MATLAB command t=[-300:1:300].

The following code in the next figure was used to reconstruct the original time-domain signal from the Fourier coefficients that was derived in the previous Problems. The following code also is used to plot the reconstructed signals as shown.

```
%A.5 and plotting code for A.6:
t= -300:1:300;
x1 = zeros(size(t));
for n= -20:20
 x1 = x1+ D_nx1(n)*exp(1j*pi/10*n*t);
end
figure;
plot(t,x1);
x2 = zeros(size(t));
for n= -20:20
 x2 = x2+ D_nx2(n)*exp(1j*pi/10*n*t);
end
figure;
plot(t,x2);
x3 = zeros(size(t));
for n= -20:20
 x3 = x3+ D_nx3(n)*exp(1j*pi/20*n*t);
end
figure;
plot(t,x3);
```

Figure 1J: Code for the reconstruction of the signals from derived Fourier coefficients.

Problem A.6: Reconstruct the time-domain signals x1(t), x2(t) and x3(t) with the Fourier coefficient sets you generated in Problem A.4. Plot each reconstructed signal.

The following graphs show the reconstructed signals from the code above.

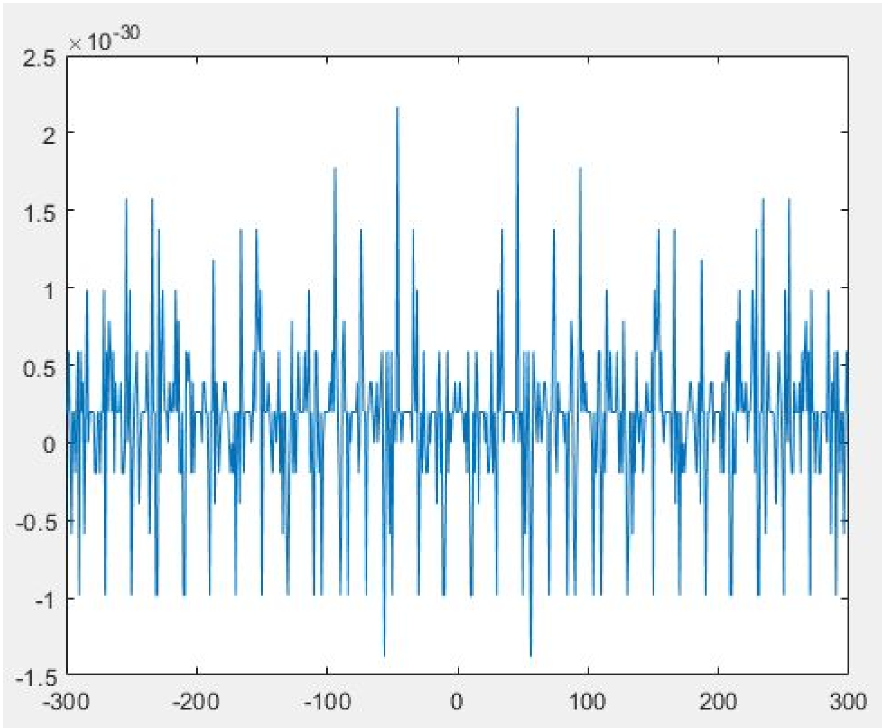

Figure 1K: Plot of the reconstructed signal x1(t).

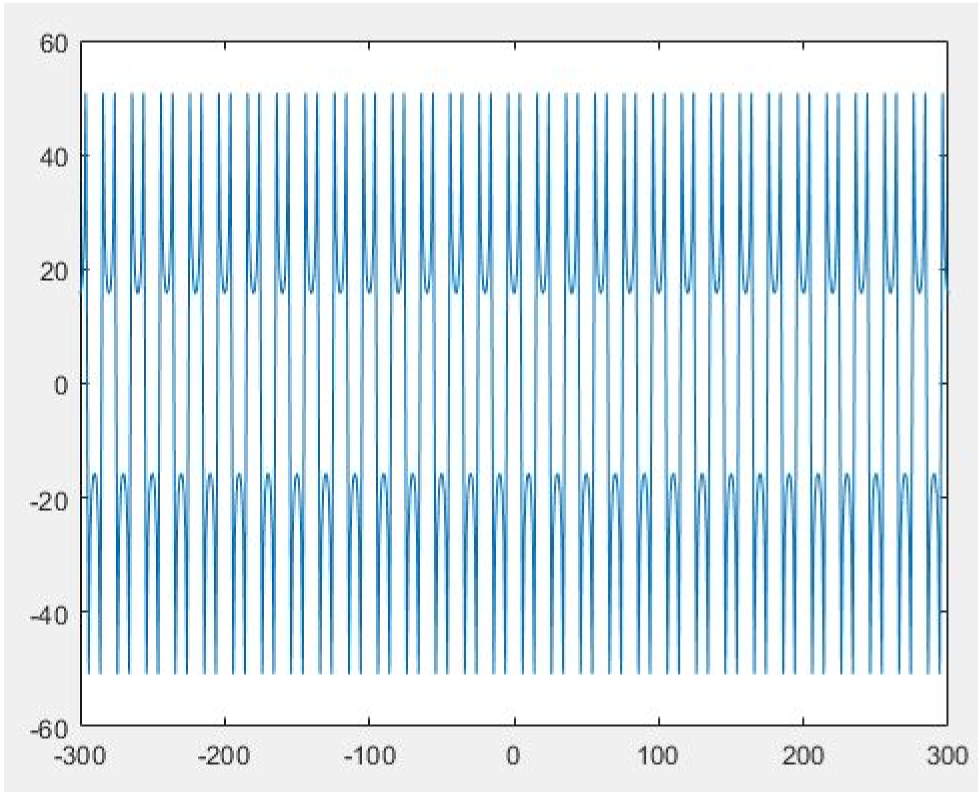

Figure 1L: Plot of the reconstructed signal x2(t).

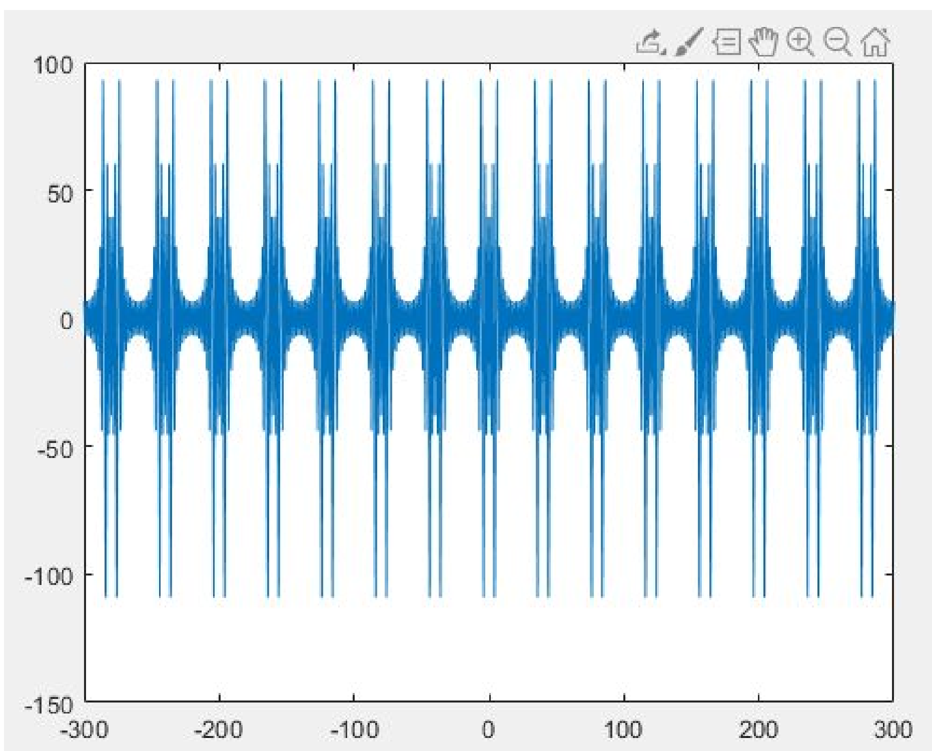

Figure 1M: Plot of the reconstructed signal x3(t).

B. Discussion

Problem B.1: Determine the fundamental frequencies of x1(t), x2(t) and x3(t).

The fundamental frequencies of x1(t), x2(t) and x3(t) are as follows:

x1(t): 1/20 Hz   x2(t): 10/π Hz x3(t): 20/π Hz

Problem B.2: What is the main difference between the Fourier coefficients of x1(t) and x2(t)?

The fourier coefficients obtained from x1(t) are only given a finite set, D3, D-3, D1, D-1 which are D3 = ½, D-3 = ½, D1 = ¼, D-1 = ¼. This is different from the fourier coefficients of x2(t) which can be found using Dn = sin(nπ/2)/nπ. This provided fourier coefficients for x2(t) such as D1 = 1/π, D2 = 0, D3 = -1/3π, D-1 = 1/π, D-3 = -1/3π. As such, there are an infinite amount of fourier coefficients for x2(t).

Problem B.3: Signals x2(t) and x3(t) have the same rectangular pulse shape but different periods. How are these characteristics reflected in their respective Fourier coefficients?

As the periods of the x2(t) and x3(t) are different in that the period of x2(t) = 20, whereas the period for x3(t) = 40, this affects the fourier coefficients sinc function. The fourier coefficient of x2(t) can be expressed as Dn = sin(π/2)/π whereas the fourier coefficient of x3(t) is Dn = 10sin(nπ/4)/nπ. Thus, this resulted in different sin functions of the fourier coefficients.

Problem B.4: The Fourier coefficient D0 represents the DC value of the signal. Let x4(t) be the periodic waveform shown in Figure 2. Derive D0 of x4(t) from D0 of x2(t).

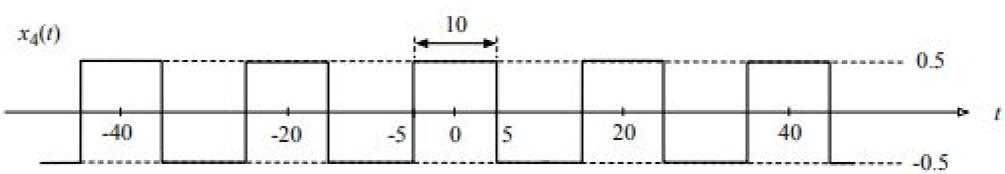

Figure 2: Periodic function $x_4(t)$.

In this case, there is a shift in the signal x4(t). The shift is vertically down by 0.5. Thus, 0.5 - 0.5 would make D0 = 0. This can also be seen as the top and bottom portions of the signals cancel each other, thus making D0 = 0. The integration of D0 is as follows:

D0 = 1/20 * $\int_{-5}^{3} dt - \int_{5}^{1.5} dt = 0$ → Thus D0 = 0.

Problem B.5: Using the results of Problem A.6, explain how the reconstructed signal changes as you increase the number of Fourier coefficients used in the reconstruction. Discuss for both x1(t) and x2(t).

In x1(t), the reconstructed signal will not change. This is due to the signal having a finite number of fourier coefficients as opposed to x2(t), which has an infinite number of fourier coefficients. Thus, as the number of fourier coefficients of x2(t) increases, the reconstructed signal will appear to be more accurate. This was previously shown in the generated plots of the reconstructed signal for x2(t).

Problem B.6: How many Fourier coefficients do you need to perfectly reconstruct the periodic waveforms discussed in this lab experiment?

A perfect reconstructed signal can be produced for x1(t). This is because it has a finite number of fourier coefficients. As opposed to x2(t) or x3(t), there are an infinite amount of fourier coefficients, thus it requires an infinite number of fourier coefficients to obtain the most perfect reconstructed signal for x2(t) and x3(t).

Problem B.7: Let x(t) be an arbitrary periodic signal. Instead of storing x(t) on a computer, we consider storing the corresponding Fourier coefficients. When we need to access x(t), we read the Fourier coefficients stored on the computer hard drive and reconstruct the signal. Is this a viable scenario? Explain your answer.

In periodic signals, there are an infinite number of fourier coefficients. As an infinite number of fourier coefficients cannot be stored inside MATLAB, this is not a possible scenario. However, a finite number of fourier coefficients can be stored, thus this is feasible. As the number of the fourier coefficients for x(t) is an infinite amount, this method may not always work and thus, this can cause errors to reconstruct the signal.

ELE532 Lab Assignment 3

Problem A.1: $x_1(t) = \cos\frac{3\pi}{10}t + \frac{1}{2}\cos\frac{\pi}{10}t$

$$\frac{w_{01}}{w_{02}} = \frac{\frac{3\pi}{10}}{\frac{\pi}{10}} = \frac{3\cancel{\pi}}{\cancel{10}} \times \frac{\cancel{10}}{\cancel{\pi}} = 3 \qquad w_0 = \frac{GCF}{LCM} = \frac{\pi}{10} \quad \Big| \quad T_0 = 20$$

$$\rightarrow x_1(t) = \frac{1}{2}e^{j\frac{3\pi}{10}t} + \frac{1}{2}e^{-j\frac{3\pi}{10}t} + \frac{1}{4}e^{j\frac{\pi}{10}t} + \frac{1}{4}e^{-j\frac{\pi}{10}t}$$

$$x_1(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\frac{\pi}{10}t}$$

$$D_n = \frac{1}{20}\int_{20}\left[\frac{1}{2}e^{j\frac{3\pi}{10}t} + \frac{1}{2}e^{-j\frac{3\pi}{10}t} + \frac{1}{4}e^{j\frac{\pi}{10}t} + \frac{1}{4}e^{-j\frac{\pi}{10}t}\right]e^{-jn\frac{\pi}{10}t}dt$$

$$= \frac{1}{20}\int_{T_0}\frac{1}{2}e^{j\frac{3\pi}{10}t}e^{-jn\frac{\pi}{10}t}dt + \frac{1}{20}\int_{T_0}\frac{1}{2}e^{-j\frac{3\pi}{10}t}e^{-jn\frac{\pi}{10}t}dt$$

$$\frac{1}{20}\int_{T_0}\frac{1}{4}e^{j\frac{\pi}{10}t}e^{-jn\frac{\pi}{10}t}dt + \frac{1}{20}\int_{T_0}\frac{1}{4}e^{-j\frac{\pi}{10}t}e^{-jn\frac{\pi}{10}t}dt$$

$$= \frac{1}{40}\int_{T_0} e^{j\frac{\pi}{10}t(3-n)}dt + \frac{1}{40}\int_{T_0} e^{-j\frac{\pi}{10}t(3+n)}dt$$

$$+ \frac{1}{80}\int_{T_0} e^{j\frac{\pi}{10}t(1-n)}dt + \frac{1}{80}\int_{T_0} e^{-j\frac{\pi}{10}t(1+n)}dt$$

$$D_3 = \frac{1}{40}(20) = \frac{1}{2} \qquad D_{-3} = \frac{1}{40}(20) = \frac{1}{2} \qquad D_1 = \frac{1}{80}(20) = \frac{1}{4} \qquad D_{-1} = \frac{1}{80}(20) = \frac{1}{4}$$

$$\therefore x_1(t) = \underbrace{\frac{1}{2}}_{D_3}e^{j\frac{3\pi}{10}t} + \underbrace{\frac{1}{2}}_{D_{-3}}e^{-j\frac{3\pi}{10}t} + \underbrace{\frac{1}{4}}_{D_1}e^{j\frac{\pi}{10}t} + \underbrace{\frac{1}{4}}_{D_{-1}}e^{-j\frac{\pi}{10}t}$$

Problem A.2:

$x_2(t)$: $T_0 = 20 \rightarrow W_0 = \frac{2\pi}{T_0} = \frac{2\pi}{20} = \frac{\pi}{10}$

$$x_2(t) = \sum_{n=-\infty}^{\infty} D_n e^{j\frac{n\pi}{10}t}$$

$$D_n = \frac{1}{20}\int_{T_0} x_2(t) e^{-j\frac{n\pi}{10}t} dt$$

$$D_n = \frac{1}{20}\int_{-5}^{5} e^{-j\frac{n\pi}{10}t} dt = \frac{1}{20}\int_{-5}^{5} e^{-j\frac{n\pi}{10}t} dt$$

$$= \left[\frac{1}{20}\left(-\frac{10}{jn\pi}\right) e^{-j\frac{n\pi}{10}t}\right]_{t=-5}^{5} = \left[-\frac{e^{-j\frac{n\pi}{10}t}}{j2n\pi}\right]_{-5}^{5}$$

$$= -\frac{e^{-j\frac{n\pi}{2}}}{j2n\pi} + \frac{e^{j\frac{n\pi}{2}}}{j2n\pi} = \frac{1}{n\pi}\left[\frac{e^{j\frac{n\pi}{2}}}{j2} - \frac{e^{-j\frac{n\pi}{2}}}{j2}\right]$$

$$\boxed{D_n = \frac{\sin\left(\frac{\pi}{2}n\right)}{n\pi}}$$

$D_1 = \frac{\sin\left(\frac{\pi}{2}\right)}{\pi} = \frac{1}{\pi}$, $D_2 = 0$, $D_3 = -\frac{1}{3\pi}$, $D_{-1} = \frac{1}{\pi}$, $D_{-3} = -\frac{1}{3\pi}$

are the plugged in values for D_n.

Problem A.2:

$x_3(t)$: $T_0 = 40 \rightarrow W_0 = \frac{2\pi}{40} = \frac{\pi}{20}$

$$x_3(t) = \sum_{n=-\infty}^{\infty} D_n e^{j\frac{n\pi}{20}t}$$

$$D_n = \frac{1}{40}\int_{T_0} x_3(t) e^{-j\frac{n\pi}{20}t} dt$$

$$D_n = \frac{10}{40}\int_{-5}^{5} e^{-j\frac{n\pi}{20}t} dt = \frac{1}{4}\int_{-5}^{5} e^{-j\frac{n\pi}{20}t} dt$$

$$= \left[\frac{1}{4}\left(\frac{-20}{jn\pi}\right) e^{-j\frac{n\pi}{20}t}\right]_{t=-5}^{5} = \left[\frac{-5}{jn\pi} e^{-j\frac{n\pi}{20}t}\right]_{t=-5}^{5}$$

$$= \frac{-5}{jn\pi} e^{-j\frac{n\pi}{4}} + \frac{5}{jn\pi} e^{j\frac{n\pi}{4}} = \frac{10}{n\pi}\left[\frac{e^{j\frac{n\pi}{4}}}{j2} - \frac{e^{-j\frac{n\pi}{4}}}{j2}\right]$$

$$\boxed{D_n = \frac{10\sin\left(\frac{\pi}{4}n\right)}{n\pi}}$$