UNIVERSITY OF WATERLOO

Value Iteration for Learning Concurrently Executable Robotic Control Tasks

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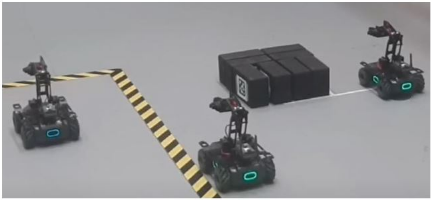
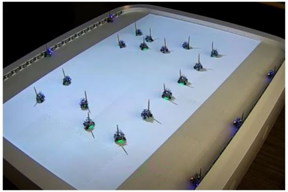
Motivation


Modern robots are required to do complex tasks and possibly multiple at the same time.

- Let's use RL to learn several control tasks for a robotic system to execute.
  - RL lets us generalize to possibly complex control tasks.
- Let's combine and execute each of these tasks together.
  - Preferably in a way that lets us swap out tasks and/or reorder priorities.
- How do we know that tasks will not interfere with each other?

Assumptions

- Assume that our robotic system is control-affine:

$$\dot{x} = f(x) + g(x)u, \;\; x \in \mathbb{R}^n, \;\; u \in \mathbb{R}^p$$

- Assume that each RL task we learn is encoded with a "cost-to-go"/value function of the form:

$$J_i(x) \approx \min_{u(\cdot)} \int_t^{\infty} q_i(x(\tau))) + \|u(\tau)\|^2 d\tau \qquad\qquad q_i \text{ is P.S.D}$$

Key Related Works

Related Work - Combining Learned Tasks Using a Min-Norm Controller

- Treat learned value functions as Control Lyapunov Functions
- Make progress on each task using constrained optimization problem

$$
\begin{aligned}
\min_{u \in \mathcal{U}, \delta \in \mathbb{R}^N} \quad & \|u\|^2 + \kappa \|\delta\|^2 \\
\text{s.t.} \quad & L_f J_1(x) + L_g J_1(x) u \leq -\sigma_1(x) + \delta_1 \\
& \vdots \\
& L_f J_N(x) + L_g J_N(x) u \leq -\sigma_N(x) + \delta_N \\
& K\delta \geq 0
\end{aligned}
$$

Note that: $L_f J_i(x) = \frac{\partial J_i}{\partial x} f(x)$, $L_g J_i(x) = \frac{\partial J_i}{\partial x} g(x)$.

[1] G. Notomista, "A Constrained-Optimization Approach to the Execution of Prioritized Stacks of Learned Multi-robot Tasks," in Distributed Autonomous Robotic Systems, 2024, pp. 479–493

Related Work - Value Iteration for Continuous Action Spaces

Assume continuous, control-affine dynamics and cost function as mentioned previously.

$$\dot{x} = f(x) + g(x)u, \; x \in \mathbb{R}^n, \; u \in \mathbb{R}^p$$
$$J_i(x) \approx \min_{u(\cdot)} \int_t^\infty q_i(x(\tau))) + \|u(\tau)\|^2 d\tau$$

Use expression from solved HJB equation as "optimal input" at each iteration.

$$u^\star = -\frac{1}{2}(L_g J_i(x))^\top = -\frac{1}{2} g(x)^\top \left(\frac{\partial J_i}{\partial x}\right)^\top$$

2] M. Lutter, S. Mannor, J. Peters, D. Fox, and A. Garg, "Value Iteration in Continuous Actions, States and Time," in Proceedings of the 38th International Conference on Machine Learning, Jul. 2021, vol. 139, pp. 7224–7234.

Related Work - Independence and Orthogonality for Robotic Control Tasks

Come back to this. Finish the other slides first.

How do we know that tasks are compatible with each other?

$$
\begin{aligned}
\min_{u\in\mathcal{U},\delta\in\mathbb{R}^N} \quad & \|u\|^2 \\
\text{s.t.} \quad & L_f J_1(x) + L_g J_1(x) u \leq -\sigma_1(x) \\
& \vdots \\
& L_f J_N(x) + L_g J_N(x) u \leq -\sigma_N(x)
\end{aligned}
$$

We do not.

Sometimes they are compatible

$J_1 \rightarrow$ Learn to avoid circular region

$J_2 \rightarrow$ Learn to go to some point

<ADD “ANIMATION">

Sometimes they are compatible

$J_1 \to \text{Learn to form a triangle}$

$J_2 \to \text{Learn to send one robot to a point}$

<ADD “ANIMATION">

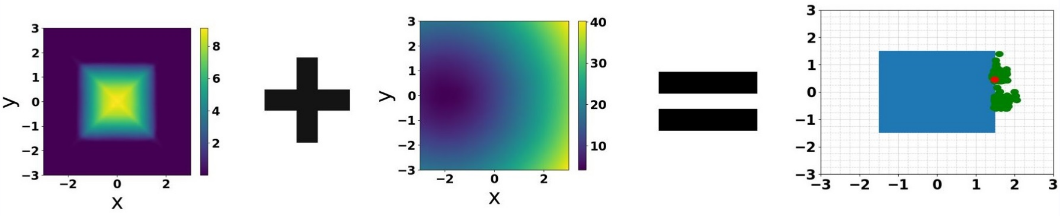
Sometimes they are NOT compatible

$J_1 \rightarrow$ Learn to avoid square-shaped region

$J_2 \rightarrow$ Learn to go to a point

<ADD "ANIMATION">

Sometimes they are NOT compatible



- The above are heat maps of the two value functions.
- Combining them results in the trajectory on the right.

Definitions of *Independence* and *Orthogonality*

$$
\begin{aligned}
\min_{u\in\mathcal{U},\delta\in\mathbb{R}^N} \quad & \|u\|^2 \\
\text{s.t.} \quad & L_f J_1(x) + \color{red}{L_g J_1(x)}\color{black} u \leq -\sigma_1(x) \\
& \vdots \\
& L_f J_N(x) + \color{red}{L_g J_N(x)}\color{black} u \leq -\sigma_N(x)
\end{aligned}
$$

Definitions of *Independence* and *Orthogonality*

$$J_1, \dots J_N \text{ are } \textit{independent} \text{ at } x \in \mathcal{X} \Leftrightarrow L_g J_1(x)^\top, \dots, L_g J_N(x)^\top \text{ are linearly independent}$$

$$J_1, \dots J_N \text{ are } \textit{orthogonal} \text{ at } x \in \mathcal{X} \Leftrightarrow \langle L_g J_i(x)^\top, L_g J_j(x)^\top \rangle = 0 \;\; \forall \; i, j \in \{1, \dots, N\}$$

Introducing “Interference" Input Cost

$$\tilde{J}_{N+1} \approx \min_{u(\cdot)} \int_t^\infty e^{-\beta\tau} \left(q_{N+1}(x) + \|u\|^2 + \sum_{i=1}^{N} (L_g \tilde{J}_i(x) u)^2 \lambda_i \right) d\tau$$

In Proposition 2, we show that by picking large enough values of λ_i and successfully fitting the cost functional, we can make the new task, J_{N+1} independent to previously trained tasks $J_1, \ldots, J_N$.

Math Expressions

Integrals and Other Expressions

$$\iint_{\partial\Omega} f(x)\mathrm{d}x \in \mathbb{C} \tag{1}$$

$$E = mc^2 \tag{2}$$

$$F = ma \tag{3}$$

m	Mass
c	Speed of light

Theorems, Lemmas, ...

Theorem

The following statement is correct

$$\frac{\partial f(\vec{x})}{\partial x_i} = \sum_{l=1}^{L} \cos\left(l\frac{2\pi}{L} + 0\right) \tag{4}$$

Elements

Typography

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The theme provides sensible defaults to
\emph{emphasize} text, \alert{accent} parts
or show \textbf{bold} results.
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becomes

The theme provides sensible defaults to *emphasize* text, accent parts or show **bold** results.

Font feature test

- Regular
- *Italic*
- Small Caps
- **Bold**
- ***Bold Italic***
- **Bold Small Caps**
- `Monospace`
- *`Monospace Italic`*
- **`Monospace Bold`**
- ***`Monospace Bold Italic`***

Lists

Items

- Milk
- Eggs
- Potatoes

Enumerations

1. First,
2. Second and
3. Last.

Descriptions

PowerPoint Meeh.

Beamer Yeeeha.

Tables

Table 1: Largest cities in the world (source: Wikipedia)

City	Population
Mexico City	20,116,842
Shanghai	19,210,000
Peking	15,796,450
Istanbul	14,160,467

Blocks

Three different block environments are pre-defined and may be styled with an optional background color.

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Example
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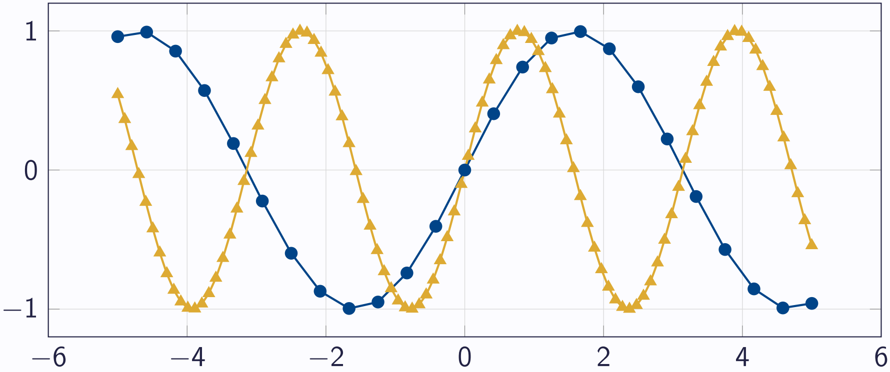
Line plots



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The best way to do this is to include the `appendixnumberbeamer` package in your preamble and call `\appendix` before your backup slides.

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