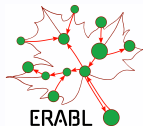UNIVERSITY OF
WATERLOO

Ecological and Resilient
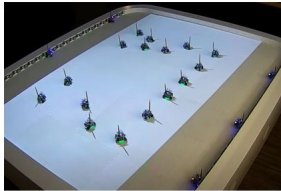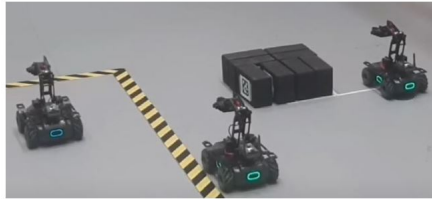Autonomous roBots
Laboratory
ERABL

Value Iteration for Learning Concurrently Executable Robotic Control Tasks

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Modern robots are required to do complex tasks and possibly multiple at the same time.

- Let's use RL to learn several control tasks for a robotic system to execute.
  - RL lets us generalize to possibly complex control tasks.
- Let's combine and execute each of these tasks together.
  - Preferably in a way that lets us swap out tasks and/or reorder priorities.
- How do we know that tasks will not interfere with each other?

Assumptions

- Assume that our robotic system is control-affine:

$$\dot{x} = f(x) + g(x)u, \ x \in \mathbb{R}^n, \ u \in \mathbb{R}^p$$

- Assume that each RL task we learn is encoded with a "cost-to-go"/value function of the form:

$$J_i(x) \approx \min_{u(\cdot)} \int_t^\infty e^{-\beta\tau} \left(q_i(x(\tau)) + \|u(\tau)\|^2 \right) d\tau \qquad q_i \text{ is P.S.D}$$

Key Related Works

Related Work - Combining Learned Tasks Using a Min-Norm Controller

- Treat learned value functions as Control Lyapunov Functions
- Make progress on each task using constrained optimization problem

$$\min_{u \in \mathcal{U}, \delta \in \mathbb{R}^N} \quad \|u\|^2 + \kappa \|\delta\|^2$$

$$\text{s.t.} \quad L_f J_1(x) + L_g J_1(x)u \leq -\sigma_1(x) + \delta_1$$

$$\vdots$$

$$L_f J_N(x) + L_g J_N(x)u \leq -\sigma_N(x) + \delta_N$$

$$K\delta \geq 0$$

Note that: $L_f J_i(x) = \frac{\partial J_i}{\partial x} f(x)$, $L_g J_i(x) = \frac{\partial J_i}{\partial x} g(x)$.

[1] G. Notomista, "A Constrained-Optimization Approach to the Execution of Prioritized Stacks of Learned Multi-robot Tasks," in Distributed Autonomous Robotic Systems, 2024, pp. 479–493

Related Work - Value Iteration for Continuous Action Spaces

Assume continuous, control-affine dynamics and cost function as mentioned previously.

$$\dot{x} = f(x) + g(x)u, \ x \in \mathbb{R}^n, \ u \in \mathbb{R}^p$$

$$J_i(x) \approx \min_{u(\cdot)} \int_t^\infty q_i(x(\tau))) + \|u(\tau)\|^2 d\tau$$

Use expression from solved HJB equation as "optimal input" at each iteration.

$$u^\star = -\frac{1}{2}(L_g J_i(x))^\top = -\frac{1}{2}g(x)^\top \left(\frac{\partial J_i}{\partial x}\right)^\top$$

[2] M. Lutter, S. Mannor, J. Peters, D. Fox, and A. Garg, "Value Iteration in Continuous Actions, States and Time," in Proceedings of the 38th International Conference on Machine Learning, Jul. 2021, vol. 139, pp. 7224–7234.

Related Work - Independence and Orthogonality for Robotic Control Tasks

- Work in [3] defines notions of independence and orthogonality between Jacobian-based tasks to analyze how multi-joint robot arms can achieve multiple tasks at the same time.
- Work in [4] define these notions for Extended Set-Based Tasks.

[3] Gianluca Antonelli. 2009. Stability Analysis for Prioritized Closed-Loop Inverse Kinematic Algorithms for Redundant Robotic Systems. IEEE Transactions on Robotics 25, 5 (2009), 985–994

[4] Gennaro Notomista, Mario Selvaggio, María Santos, Siddharth Mayya, Francesca Pagano, Vincenzo Lippiello, and Cristian Secchi. 2023. Beyond Jacobian-based tasks: Extended set-based tasks for multi-task execution and prioritization. (2023).
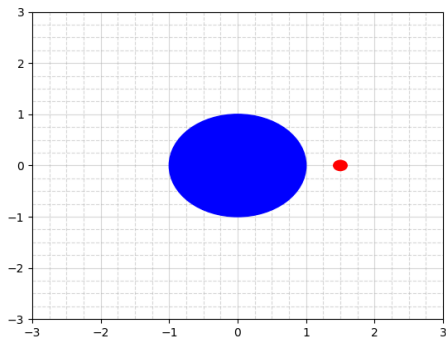
$$\min_{u \in \mathcal{U}, \delta \in \mathbb{R}^N} \quad \|u\|^2$$

$$\text{s.t.} \quad L_f J_1(x) + L_g J_1(x)u \leq -\sigma_1(x)$$

$$\vdots$$

$$L_f J_N(x) + L_g J_N(x)u \leq -\sigma_N(x)$$

We do not.

Sometimes they are compatible
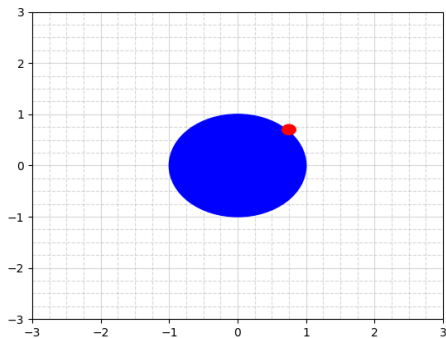
$J_1 \rightarrow$ Learn to avoid circular region

$J_2 \rightarrow$ Learn to go to some point

$J_1 \rightarrow$ Learn to avoid circular region

$J_2 \rightarrow$ Learn to go to some point

Sometimes they are compatible

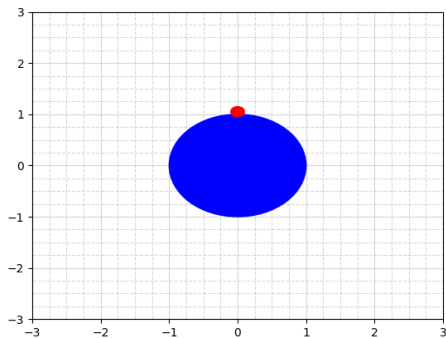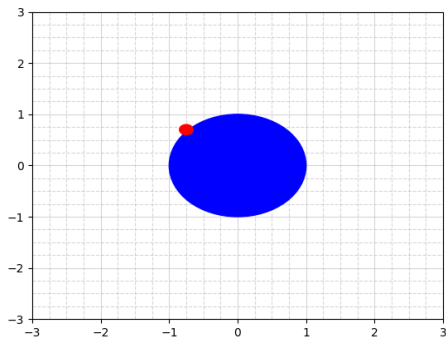$J_1 \rightarrow$ Learn to avoid circular region

$J_2 \rightarrow$ Learn to go to some point

Sometimes they are compatible

$J_1 \rightarrow$ Learn to avoid circular region

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Sometimes they are compatible

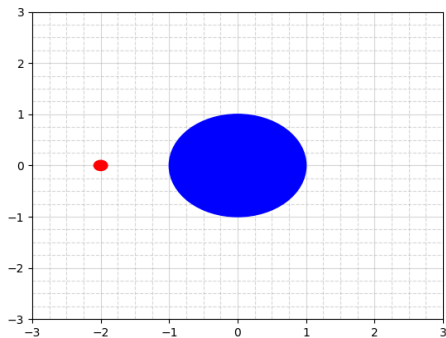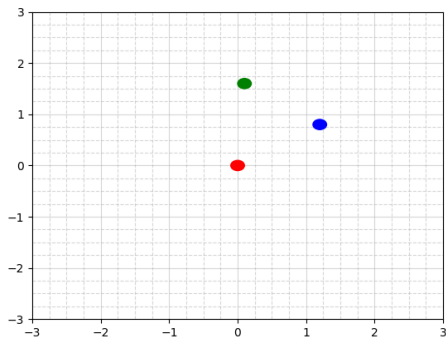$J_1 \rightarrow$ Learn to avoid circular region

$J_2 \rightarrow$ Learn to go to some point

Sometimes they are compatible

$J_1 \rightarrow$ Learn to form a triangle

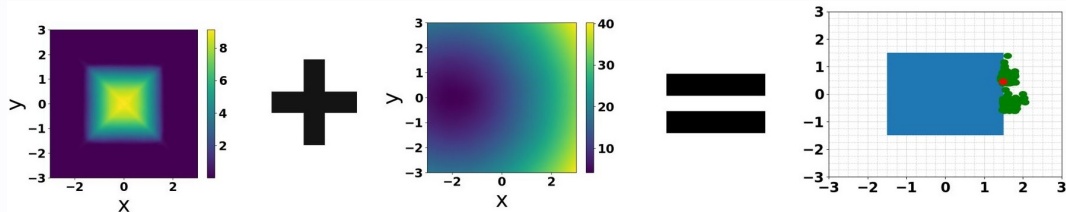$J_2 \rightarrow$ Learn to send one robot to a point

$J_1 \rightarrow$ Learn to avoid square-shaped region

$J_2 \rightarrow$ Learn to go to a point

- The above are heat maps of the two value functions.
- Combining them results in the trajectory on the right.

Definitions of Independence and Orthogonality

$$\min_{u \in \mathcal{U}, \delta \in \mathbb{R}^N} \quad \|u\|^2$$

$$\text{s.t.} \quad L_f J_1(x) + L_g J_1(x) u \leq -\sigma_1(x)$$

$$\vdots$$

$$L_f J_N(x) + L_g J_N(x) u \leq -\sigma_N(x)$$

$J_1, \ldots J_N$ are *independent* at $x \in \mathcal{X} \Leftrightarrow L_g J_1(x)^\top, \ldots, L_g J_N(x)^\top$ are linearly independent

$J_1, \ldots J_N$ are *orthogonal* at $x \in \mathcal{X} \Leftrightarrow \langle L_g J_i(x)^\top, L_g J_j(x)^\top \rangle = 0 \ \forall \ i, j \in \{1, \ldots, N\}$

$$J_{N+1} \approx \min_{u(\cdot)} \int_t^\infty e^{-\beta\tau} \left(q_{N+1}(x) + \|u\|^2 + \sum_{i=1}^{N} (L_g J_i(x)u)^2 \lambda_i \right) d\tau$$

In Proposition 2, we show that by picking large enough values of λ_i and successfully fitting the cost functional, we can make the new task, J_{N+1}, independent to previously trained tasks $J_1, \ldots, J_N$.

Variant of Value Iteration in Continuous Action Space

Can extend previous work in [2] to approximate this new cost functional.

$$J_{N+1} \approx \min_{u(\cdot)} \int_t^\infty e^{-\beta\tau} \left(q_{N+1}(x) + \|u\|^2 + \sum_{i=1}^N (L_g J_i(x)u)^2 \lambda_i \right) d\tau$$

At each iteration, use the following input to to estimate the next iteration of the value function.

$$u^\star = -\frac{1}{2} R(x)^{-1} (L_g J_{N+1}(x))^\top$$

$$R(x) = I + \sum_{i=1}^N (L_g J_i(x))^\top L_g J_i(x)$$

[2] M. Lutter, S. Mannor, J. Peters, D. Fox, and A. Garg, "Value Iteration in Continuous Actions, States and Time," in Proceedings of the 38th International Conference on Machine Learning, Jul. 2021, vol. 139, pp. 7224–7234.

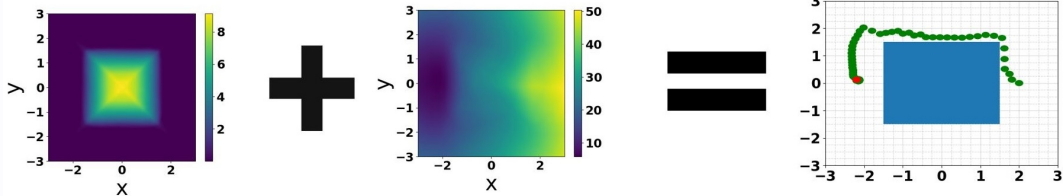## Relating Orthogonality to the Optimal Input

Proposition 3:

Assume that $J_1, \ldots, J_N$ are independent at $x \in \mathcal{X}$.

If we train a new cost-to-go function, J_{N+1} to be independent to $J_1, \ldots, J_N$,

J_{N+1} is orthogonal to each of $J_1, \ldots, J_N$ at $x \in \mathcal{X}$.
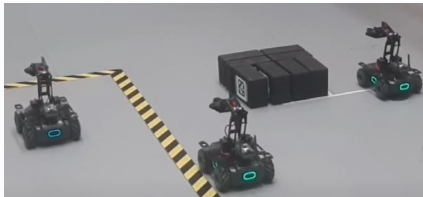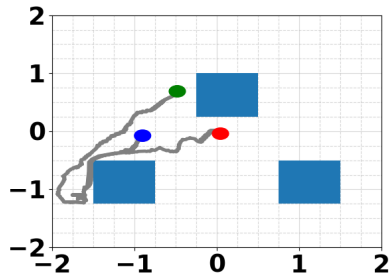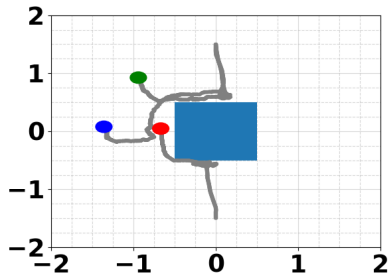
$\Leftrightarrow$

the optimal input is $-\frac{1}{2} L_g J_{N+1}(x)^\top$.

Now they ARE compatible.

- The above are heat maps of the two value functions.
- Combining them results in the trajectory on the right.

We tried our idea on other scenarios.

Summary of Contributions

- Defined notions of independence and orthogonality for learned value functions
- Introduced "interference" for encouraging newly trained value functions to be independent to previous ones
- Introduced variant of value iteration for continuous input/action space that can fit cost functionals with this new "interference" cost
- Tested our idea in a few different scenarios involving controlling a team of mobile robots