# Value Iteration for Learning Concurrently Executable Robotic Control Tasks

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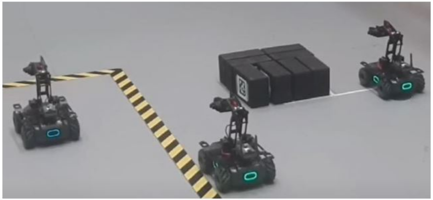
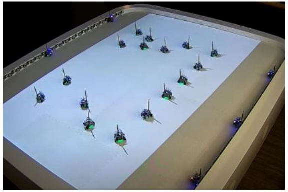
# Motivation





Modern robots are required to do complex tasks and possibly multiple at the same time.

- Let's use RL to learn several control tasks for a robotic system to execute.
    - RL lets us generalize to possibly complex control tasks.
- Let's combine and execute each of these tasks together.
    - Preferably in a way that lets us swap out tasks and/or reorder priorities.
- How do we know that tasks will not interfere with each other?

# Assumptions

- Assume that our robotic system is control-affine:

$$\dot{x} = f(x) + g(x)u, \;\; x \in \mathbb{R}^n, \;\; u \in \mathbb{R}^p$$

- Assume that each RL task we learn is encoded with a "cost-to-go"/value function of the form:

$$J_i(x) \approx \min_{u(\cdot)} \int_t^{\infty} e^{-\beta\tau} \left( q_i(x(\tau)) + \|u(\tau)\|^2 \right) d\tau \qquad\qquad q_i \text{ is P.S.D}$$

# Key Related Works

# Related Work - Combining Learned Tasks Using a Min-Norm Controller

- Treat learned value functions as Control Lyapunov Functions
- Make progress on each task using constrained optimization problem

$$
\begin{aligned}
\min_{u \in \mathcal{U}, \delta \in \mathbb{R}^N} \quad & \|u\|^2 + \kappa \|\delta\|^2 \\
\text{s.t.} \quad & L_f J_1(x) + L_g J_1(x) u \leq -\sigma_1(x) + \delta_1 \\
& \vdots \\
& L_f J_N(x) + L_g J_N(x) u \leq -\sigma_N(x) + \delta_N \\
& K\delta \geq 0
\end{aligned}
$$

Note that: $L_f J_i(x) = \frac{\partial J_i}{\partial x} f(x)$, $L_g J_i(x) = \frac{\partial J_i}{\partial x} g(x)$.

[1] G. Notomista, "A Constrained-Optimization Approach to the Execution of Prioritized Stacks of Learned Multi-robot Tasks," in Distributed Autonomous Robotic Systems, 2024, pp. 479–493

# Related Work - Value Iteration for Continuous Action Spaces

Assume continuous, control-affine dynamics and cost function as mentioned previously.

$$\dot{x} = f(x) + g(x)u, \; x \in \mathbb{R}^n, \; u \in \mathbb{R}^p$$
$$J_i(x) \approx \min_{u(\cdot)} \int_t^{\infty} q_i(x(\tau))) + \|u(\tau)\|^2 d\tau$$

Use expression from solved HJB equation as "optimal input" at each iteration.

$$u^\star = -\frac{1}{2}(L_g J_i(x))^\top = -\frac{1}{2} g(x)^\top \left(\frac{\partial J_i}{\partial x}\right)^\top$$

[2] M. Lutter, S. Mannor, J. Peters, D. Fox, and A. Garg, "Value Iteration in Continuous Actions, States and Time," in Proceedings of the 38th International Conference on Machine Learning, Jul. 2021, vol. 139, pp. 7224–7234.

# Related Work - Independence and Orthogonality for Robotic Control Tasks

- Work in [3] defines notions of independence and orthogonality between Jacobian-based tasks to analyze how multi-joint robot arms can achieve multiple tasks at the same time.
- Work in [4] define these notions for Extended Set-Based Tasks.

[3] Gianluca Antonelli. 2009. Stability Analysis for Prioritized Closed-Loop Inverse Kinematic Algorithms for Redundant Robotic Systems. IEEE Transactions on Robotics 25, 5 (2009), 985–994

[4] Gennaro Notomista, Mario Selvaggio, María Santos, Siddharth Mayya, Francesca Pagano, Vincenzo Lippiello, and Cristian Secchi. 2023. Beyond Jacobian-based tasks: Extended set-based tasks for multi-task execution and prioritization. (2023).

# How do we know that tasks are compatible with each other?

$$
\begin{aligned}
\min_{u\in\mathcal{U},\delta\in\mathbb{R}^N} \quad & \|u\|^2 \\
\text{s.t.} \quad & L_f J_1(x) + L_g J_1(x) u \leq -\sigma_1(x) \\
& \vdots \\
& L_f J_N(x) + L_g J_N(x) u \leq -\sigma_N(x)
\end{aligned}
$$

We do not.

# Sometimes they are compatible

$J_1 \rightarrow$ Learn to avoid circular region

$J_2 \rightarrow$ Learn to go to some point

<ADD “ANIMATION">

# Sometimes they are compatible

$J_1 \rightarrow$ Learn to form a triangle

$J_2 \rightarrow$ Learn to send one robot to a point

<ADD “ANIMATION">

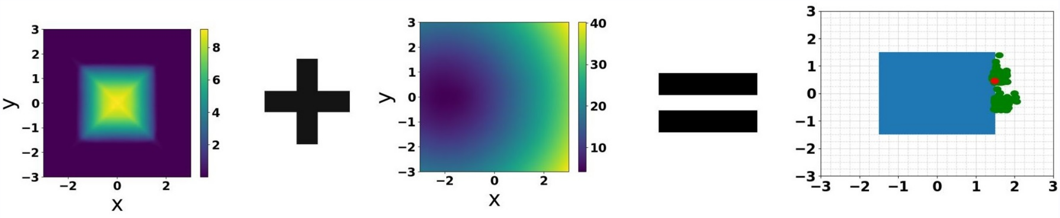
# Sometimes they are NOT compatible

$J_1 \to$ Learn to avoid square-shaped region

$J_2 \to$ Learn to go to a point

<ADD “ANIMATION">

# Sometimes they are NOT compatible




- The above are heat maps of the two value functions.
- Combining them results in the trajectory on the right.

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\min_{u \in \mathcal{U}, \delta \in \mathbb{R}^N} \quad & \|u\|^2 \\
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& L_f J_N(x) + L_g J_N(x) u \leq -\sigma_N(x)
\end{aligned}
$$

# Definitions of *Independence* and *Orthogonality*

$$J_1, \dots J_N \text{ are } \textit{independent} \text{ at } x \in \mathcal{X} \Leftrightarrow L_g J_1(x)^\top, \dots, L_g J_N(x)^\top \text{ are linearly independent}$$

$$J_1, \dots J_N \text{ are } \textit{orthogonal} \text{ at } x \in \mathcal{X} \Leftrightarrow \langle L_g J_i(x)^\top, L_g J_j(x)^\top \rangle = 0 \ \forall\ i, j \in \{1, \dots, N\}$$

# Introducing "Interference" Input Cost

$$J_{N+1} \approx \min_{u(\cdot)} \int_t^{\infty} e^{-\beta\tau} \left( q_{N+1}(x) + \|u\|^2 + \sum_{i=1}^{N} (L_g J_i(x) u)^2 \lambda_i \right) d\tau$$

In Proposition 2, we show that by picking large enough values of $\lambda_i$ and successfully fitting the cost functional, we can make the new task, $J_{N+1}$, independent to previously trained tasks $J_1, \ldots, J_N$.

# Variant of Value Iteration in Continuous Action Space

Can extend previous work in [2] to approximate this new cost functional.

$$J_{N+1} \approx \min_{u(\cdot)} \int_t^\infty e^{-\beta\tau} \left( q_{N+1}(x) + \|u\|^2 + \sum_{i=1}^{N} (L_g J_i(x) u)^2 \lambda_i \right) d\tau$$

At each iteration, use the following input to to estimate the next iteration of the value function.

$$u^\star = -\frac{1}{2} R(x)^{-1} (L_g J_{N+1}(x))^\top$$

$$R(x) = I + \sum_{i=1}^{N} (L_g J_i(x))^\top L_g J_i(x)$$

[2] M. Lutter, S. Mannor, J. Peters, D. Fox, and A. Garg, "Value Iteration in Continuous Actions, States and Time," in Proceedings of the 38th International Conference on Machine Learning, Jul. 2021, vol. 139, pp. 7224–7234.

# Relating Orthogonality to the Optimal Input

Proposition 3:
Assume that $J_1, \ldots, J_N$ are independent at $x \in \mathcal{X}$.
If we train a new cost-to-go function, $J_{N+1}$ to be independent to $J_1, \ldots, J_N$, $J_{N+1}$ is orthogonal to each of $J_1, \ldots, J_N$ at $x \in \mathcal{X}$.
$\Leftrightarrow$
the optimal input is $-\frac{1}{2} L_g J_{N+1}(x)^\top$.

# Now they ARE compatible.

$J_1 \rightarrow$ Learn to avoid square-shaped region

$J_2 \rightarrow$ Learn to go to a point

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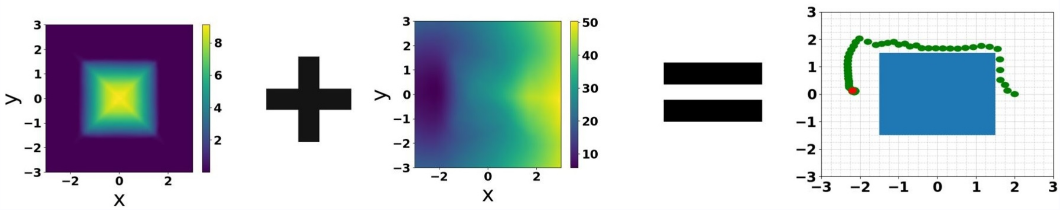


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# We tried our idea on other scenarios.

Add pictures here.