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## ENGINEERING TRIPPOS PART II A

EIETL

MODULE EXPERIMENT 3F3

### RANDOM VARIABLES and RANDOM NUMBER GENERATION Short Report Template

Name:

College:

Lab Group Number:

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This is a template suitable for the short report write-up.  
Simply edit the Latex or Word document to include your  
calculations/ results/ code.

#### 1. Uniform and normal random variables.

Histogram of Gaussian random numbers overlaid on exact Gaussian curve (scaled):

*Include your graphic here*

Histogram of Uniform random numbers overlaid on exact Uniform curve (scaled):

*Include your graphic here*

Kernel density estimate for Gaussian random numbers overlaid on exact Gaussian curve:

*Include your graphic here*

Kernel density estimate for Uniform random numbers overlaid on exact Gaussian curve:

*Include your graphic here*

Comment on the advantages and disadvantages of the kernel density method compared with the histogram method for estimation of a probability density from random samples:

*Text answer here*

Theoretical mean and standard deviation calculation for uniform density as a function of  $N$ :

*Text/maths answer here*

Explain behaviour as  $N$  becomes large:

*Text/maths answer here*

Plot of histograms for  $N = 100$ ,  $N = 1000$  and  $N = 10000$  with theoretical mean and  $\pm 3$  standard deviation lines:

*Include your graphic here*

Are your histogram results consistent with the multinomial distribution theory?

*Text/math answer here*

2. **Functions of random variables** For normally distributed  $\mathcal{N}(x|0, 1)$  random variables, take  $y = f(x) = ax + b$ . Calculate  $p(y)$  using the Jacobian formula:

*Text/math answer here*

Explain how this is linked to the general normal density with non-zero mean and non-unity variance:

*Text/math answer here*

Verify this formula by transforming a large collection of random samples  $x^{(i)}$  to give  $y^{(i)} = f(x^{(i)})$ , histogramming the resulting  $y$  samples, and overlaying a plot of your formula calculated using the Jacobian:

*Include your graphic here*

Now take  $p(x) = \mathcal{N}(x|0, 1)$  and  $f(x) = x^2$ . Calculate  $p(y)$  using the Jacobian formula:

*Text/math answer here*

Verify your result by histogramming of transformed random samples:

*Include your graphic here*

### 3. Inverse CDF method

Calculate the CDF and the inverse CDF for the exponential distribution:

*Text/math answer here*

Matlab/Python code for inverse CDF method for generating samples from the exponential distribution:

*Matlab/Python code here*

Plot histograms/ kernel density estimates and overlay them on the desired exponential density:

*Include your graphic here*

**4. Simulation from a ‘non-standard’ density.**

Matlab/Python code to generate  $N$  random numbers drawn from the distribution of  $X$ :

Plot some histogram density estimates with  $\alpha = 0.5, 1.5$  and several values of  $\beta$ .

Hence comment on the interpretation of the parameters  $\alpha$  and  $\beta$ .