Bottom-Up Parsing

- Attempts to traverse a parse tree bottom up (**post-order traversal**)
- Reduces a sequence of tokens to the start symbol
- At each reduction step, the RHS of a production is replaced with LHS
- A reduction step corresponds to the **reverse of a rightmost derivation**
- Example: given the following grammar

$$E \rightarrow E + T \mid T$$
$$T \rightarrow T * F \mid F$$
$$F \rightarrow (\ E\ ) \mid \mathbf{id}$$

A rightmost derivation for **id + id * id** is shown below:

$$E \Rightarrow_{rm} E + T \Rightarrow_{rm} E + T * F \Rightarrow_{rm} E + T * \mathbf{id}$$
$$\Rightarrow_{rm} E + F * \mathbf{id} \Rightarrow_{rm} E + \mathbf{id} * \mathbf{id} \Rightarrow_{rm} T + \mathbf{id} * \mathbf{id}$$
$$\Rightarrow_{rm} F + \mathbf{id} * \mathbf{id} \Rightarrow_{rm} \mathbf{id} + \mathbf{id} * \mathbf{id}$$

Handles

- If $S \Rightarrow^{+}_{rm} \alpha$ then α is called a **right sentential form**
- A **handle** of a right sentential form is:
  - A substring β that matches the RHS of a production $A \rightarrow \beta$
  - The reduction of β to A is a step along the reverse of a rightmost derivation
- If $S \Rightarrow^{+}_{rm} \gamma A w \Rightarrow_{rm} \gamma\beta w$, where w is a sequence of tokens then
  - The substring β of $\gamma\beta w$ and the production $A \rightarrow \beta$ make the handle
- Consider the reduction of **id + id * id** to the start symbol E

Sentential Form	*Production*	*Sentential Form*	*Production*
$\underline{\mathbf{id}} + \mathbf{id} * \mathbf{id}$	$F \rightarrow \mathbf{id}$	$E + T * \underline{\mathbf{id}}$	$F \rightarrow \mathbf{id}$
$\underline{F} + \mathbf{id} * \mathbf{id}$	$T \rightarrow F$	$E + \underline{T * F}$	$T \rightarrow T * F$
$\underline{T} + \mathbf{id} * \mathbf{id}$	$E \rightarrow T$	$\underline{E + T}$	$E \rightarrow E + T$
$E + \underline{\mathbf{id}} * \mathbf{id}$	$F \rightarrow \mathbf{id}$	E	
$E + \underline{F} * \mathbf{id}$	$T \rightarrow F$		

Stack Implementation of a Bottom-Up Parser

- A bottom-up parser uses an explicit stack in its implementation
- The main actions are **shift** and **reduce**
  - A bottom-up parser is also known as as **shift-reduce** parser
- Four operations are defined: **shift**, **reduce**, **accept**, and **error**
  - **Shift**: parser shifts the next token on the parser stack
  - **Reduce**: parser reduces the RHS of a production to its LHS
    - The handle always appears on top of the stack
  - **Accept**: parser announces a successful completion of parsing
  - **Error**: parser discovers that a syntax error has occurred
- The parser operates by:
  - Shifting tokens onto the stack
  - When a handle β is on top of stack, parser reduces β to LHS of production
  - Parsing continues until an error is detected or input is reduced to start symbol

Example on Bottom-Up Parsing

❖ Consider the parsing of the input string **id + id * id**

Stack	Input	Action
\$	**id + id * id \$**	**shift**
\$<u>**id**</u>	**+ id * id \$**	**reduce** $F \rightarrow$ **id**
\$<u>F</u>	**+ id * id \$**	**reduce** $T \rightarrow F$
\$<u>T</u>	**+ id * id \$**	**reduce** $E \rightarrow T$
\$$E$	**+ id * id \$**	**shift**
\$$E$ +	**id * id \$**	**shift**
\$$E$ + <u>**id**</u>	*** id \$**	**reduce** $F \rightarrow$ **id**
\$$E$ + <u>F</u>	*** id \$**	**reduce** $T \rightarrow F$
\$$E$ + T	*** id \$**	**shift**
\$$E$ + T *	**id \$**	**shift**
\$$E$ + T * <u>**id**</u>	**\$**	**reduce** $F \rightarrow$ **id**
\$$E$ + <u>$T * F$</u>	**\$**	**reduce** $T \rightarrow T * F$
\$<u>$E + T$</u>	**\$**	**reduce** $E \rightarrow E + T$
\$$E$	**\$**	**accept**

$$E \rightarrow E + T \mid T$$
$$T \rightarrow T * F \mid F$$
$$F \rightarrow (\, E \,) \mid \mathbf{id}$$

We use \$ to mark the bottom of the stack as well as the end of input

LR Parsing

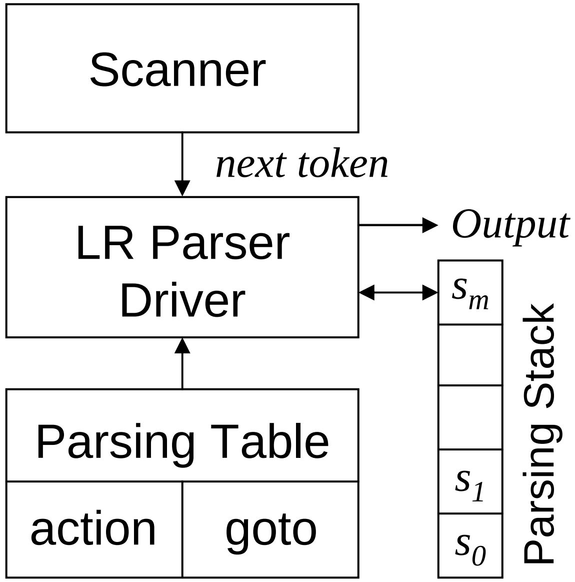
- To have an operational shift-reduce parser, we must determine:
  - Whether a handle appears on top of the stack
  - The reducing production to be used
  - The choice of actions to be made at each parsing step
- LR parsing provides a solution to the above problems
  - Is a general and efficient method of shift-reduce parsing
  - Is used in a number of automatic parser generators
- The LR(*k*) parsing technique was introduced by Knuth in 1965
  - L is for Left-to-right scanning of input
  - R corresponds to a Rightmost derivation done in reverse
  - *k* is the number of lookahead symbols used to make parsing decisions

LR Parsing – cont'd

- LR parsing is attractive for a number of reasons …
  - Is the most general deterministic parsing method known
  - Can recognize virtually all programming language constructs
  - Can be implemented very efficiently
  - The class of LR grammars is a proper superset of the LL grammars
  - Can detect a syntax error as soon as an erroneous token is encountered
  - A LR parser can be generated by a parser generating tool
- Four LR parsing techniques will be considered
  - LR(0) : LR parsing with no lookahead token to make parsing decisions
  - SLR(1) : Simple LR, with one token of lookahead
  - LR(1) : Canonical LR, with one token of lookahead
  - LALR(1) : Lookahead LR, with one token of lookahead
- LALR(1) is the preferable technique used by parser generators

LR Parsers

- An LR parser consists of …
  - Driver program
    - Same driver is used for all LR parsers
  - Parsing stack
    - Contains state information, where s_i is *state i*
    - States are obtained from grammar analysis
  - Parsing table, which has two parts
    - Action section: specifies the parser actions
    - Goto section: specifies the successor states



- The parser driver receives tokens from the scanner one at a time
- Parser uses top state and current token to lookup parsing table
- Different LR analysis techniques produce different tables

LR Parsing Table Example

❖ Consider the following grammar G_1 …

1: $E \rightarrow E + T$ 3: $T \rightarrow$ **ID**

2: $E \rightarrow T$ 4: $T \rightarrow$ **(** E **)**

❖ The following parsing table is obtained after grammar analysis

State	Action					Goto	
	+	**ID**	**(**	**)**	**$**	*E*	*T*
0		S1	S2			G4	G3
1	R3			R3	R3		
2		S1	S2			G6	G3
3	R2			R2	R2		
4	S5				A		
5		S1	S2				G7
6	S5			S8			
7	R1			R1	R1		
8	R4			R4	R4		

Entries are labeled with …

S*n*: Shift token and goto state *n*
(call scanner for next token)

R*n*: Reduce using production *n*

G*n*: Goto state *n* (after reduce)

A: Accept parse
(terminate successfully)

blank : Syntax error

LR Parsing Example

Stack	Symbols	Input	Action
0	$	**id** + **(** **id** + **id** **)** $	S1
0 1	$ **id**	+ **(** **id** + **id** **)** $	R3, G3
0 3	$ T	+ **(** **id** + **id** **)** $	R2, G4
0 4	$ E	+ **(** **id** + **id** **)** $	S5
0 4 5	$ E +	**(** **id** + **id** **)** $	S2
0 4 5 2	$ E + **(**	**id** + **id** **)** $	S1
0 4 5 2 1	$ E + **(** **id**	+ **id** **)** $	R3, G3
0 4 5 2 3	$ E + **(** T	+ **id** **)** $	R2, G6
0 4 5 2 6	$ E + **(** E	+ **id** **)** $	S5
0 4 5 2 6 5	$ E + **(** E +	**id** **)** $	S1
0 4 5 2 6 5 1	$ E + **(** E + **id**	**)** $	R3, G7
0 4 5 2 6 5 7	$ E + **(** $E + T$	**)** $	R1, G6
0 4 5 2 6	$ E + **(** E	**)** $	S8
0 4 5 2 6 8	$ E + **(** E **)**	$	R4, G7
0 4 5 7	$ $E + T$	$	R1, G4
0 4	$ E	$	A

1: $E \rightarrow E + T$
2: $E \rightarrow T$
3: $T \rightarrow$ **id**
4: $T \rightarrow$ **(** E **)**

Grammar symbols do not appear on the parsing stack

They are shown here for clarity

LR Parser Driver

- Let s be the parser stack top state and t be the current input token
- If *action*[s,t] = **shift** n then
  - Push state n on the stack
  - Call scanner to obtain next token
- If *action*[s,t] = **reduce** $A \rightarrow X_1\ X_2\ \ldots\ X_m$ then
  - Pop the top m states off the stack
  - Let s' be the state now on top of the stack
  - Push *goto*[s', A] on the stack (using the goto section of the parsing table)
- If *action*[s,t] = **accept** then return
- If *action*[s,t] = **error** then call error handling routine
- All LR parsers behave the same way
  - The difference depends on how the parsing table is computed from a CFG

LR(0) Parser Generation – Items and States

- ❖ LR(0) grammars can be parsed looking only at the stack
- ❖ Making shift/reduce decisions without any lookahead token
- ❖ Based on the idea of an **item** or a **configuration**
- ❖ **An LR(0) item consists of a production and a dot**

$$A \rightarrow X_1 \, . \, . \, . \, X_i \bullet X_{i+1} \, . \, . \, . \, X_n$$

- ❖ The dot symbol • may appear anywhere on the right-hand side
  - ✸ Marks how much of a production has already been seen
  - ✸ $X_1 \, . \, . \, . \, X_i$ appear on top of the stack
  - ✸ $X_{i+1} \, . \, . \, . \, X_n$ are still expected to appear
- ❖ An LR(0) **state** is a set of LR(0) items
  - ✸ It is the **set of all items that apply** at a given point in parse

LR(0) Parser Generation – Initial State

- Consider the following grammar *G1*:

  1: $E \rightarrow E + T$ 3: $T \rightarrow \mathbf{ID}$

  2: $E \rightarrow T$ 4: $T \rightarrow \mathbf{(}\ E\ \mathbf{)}$

- For LR parsing, grammars are **augmented** with a . . .
  - New start symbol *S*, and a
  - New start production   0: $S \rightarrow E\ \$$
- The input should be reduced to *E* followed by \$
  - We indicate this by the item: $S \rightarrow \bullet\ E\ \$$
- The initial state (numbered 0) will have the item: $S \rightarrow \bullet\ E\ \$$
- An LR parser will start in state 0
- State 0 is initially pushed on top of parser stack

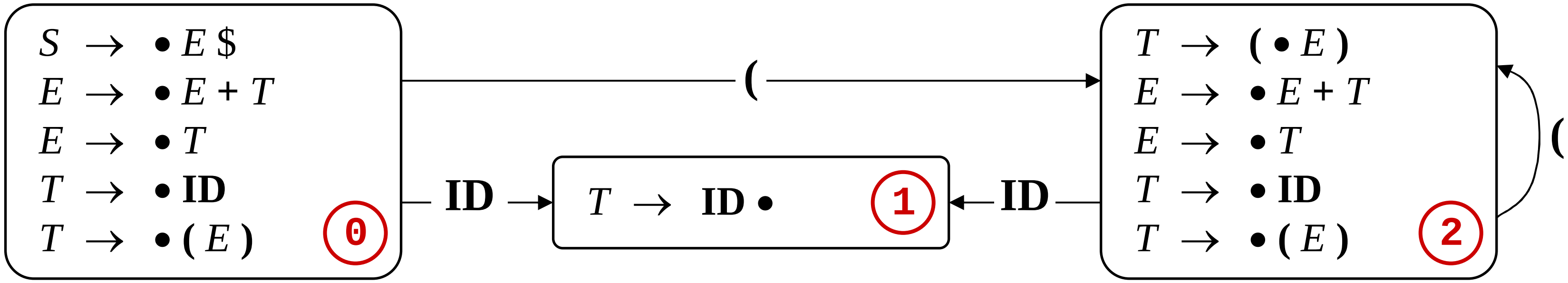
Identifying the Initial State

- Since the dot appears before E, an E is expected
  - There are two productions of E: $E \rightarrow E + T$ and $E \rightarrow T$
  - Either E+T or T is expected
  - The items: $E \rightarrow \bullet\, E + T$ and $E \rightarrow \bullet\, T$ are added to the initial state
- Since T can be expected and there are two productions for T
  - Either **ID** or **(** E **)** can be expected
  - The items: $T \rightarrow \bullet$ **ID** and $T \rightarrow \bullet$ **(** E **)** are added to the initial state
- The initial state (0) is identified by the following set of items

$$\boxed{\begin{array}{lll} S & \rightarrow & \bullet\, E\ \$ \\ E & \rightarrow & \bullet\, E + T \\ E & \rightarrow & \bullet\, T \\ T & \rightarrow & \bullet\, \textbf{ID} \\ T & \rightarrow & \bullet\, \textbf{(}\ E\ \textbf{)} \end{array} \quad 0}$$

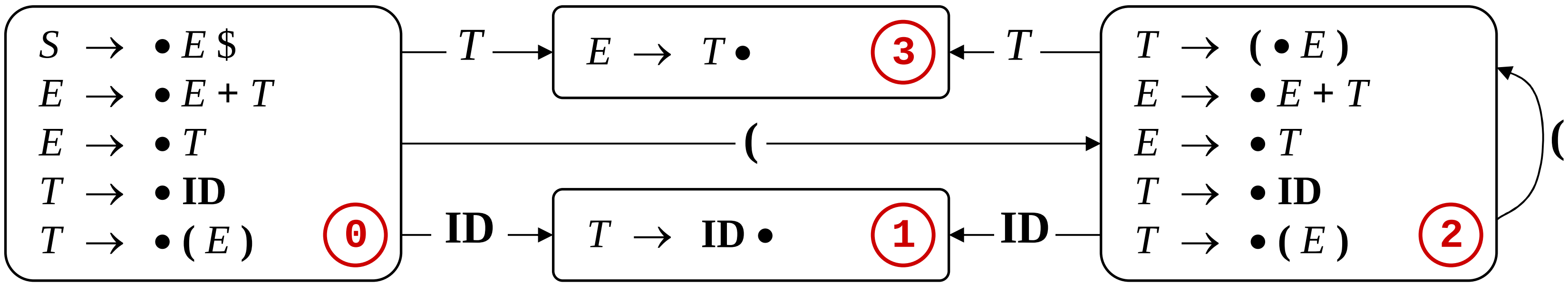
Shift Actions

- In state 0, we can shift either an **ID** or a left parenthesis
  - If we shift an **ID**, we shift the dot past the **ID**
  - We obtain a new item $T \rightarrow \textbf{ID} \bullet$ and a new state (state 1)
  - If we shift a left parenthesis, we obtain $T \rightarrow \textbf{(} \bullet E \textbf{)}$
  - Since the dot appears before E, an E is expected
  - We add the items $E \rightarrow \bullet E + T$ and $E \rightarrow \bullet T$
  - Since the dot appears before T, we add $T \rightarrow \bullet \textbf{ID}$ and $T \rightarrow \bullet \textbf{(} E \textbf{)}$
  - The new set of items forms a new state (state 2)
- In State 2, we can also shift an **ID** or a left parenthesis as shown



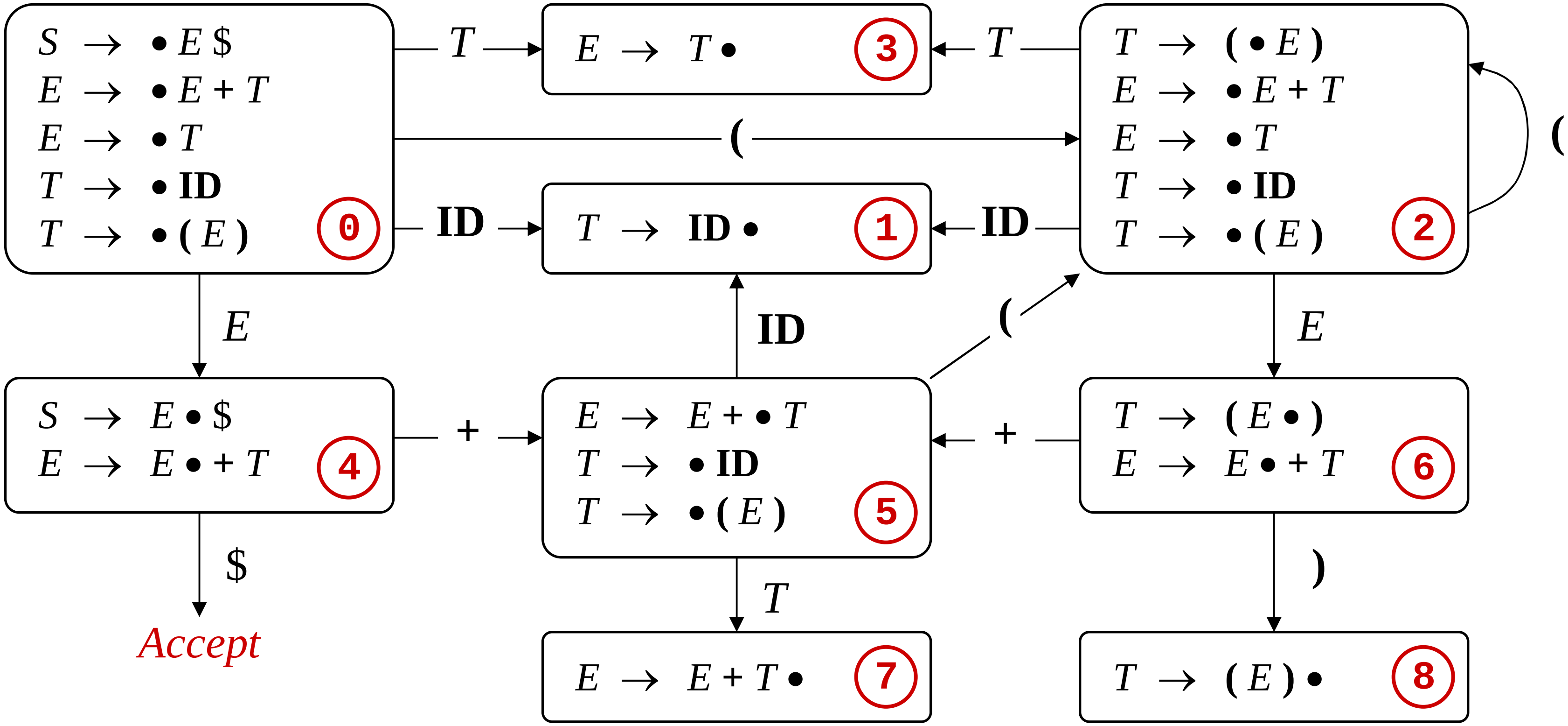
Reduce and Goto Actions

- In state 1, the dot appears at the end of item $T \rightarrow$ **ID** •
  - This means that **ID** appears on top of stack and can be reduced to T
  - When • appears at end of an item, the parser can perform a reduce action
- If **ID** is reduced to T, what is the next state of the parser?
  - **ID** is popped from the stack; Previous state appears on top of stack
  - T is pushed on the stack
  - A new item $E \rightarrow T$ • and a new state (state 3) are obtained
  - If top of stack is state 0 and we push a T, we go to state 3
  - Similarly, if top of stack is state 2 and we push a T, we go also to state 3



DFA of LR(0) States

- We complete the state diagram to obtain the DFA of LR(0) states
- In state 4, if next token is **$**, the parser **accepts** (successful parse)



LR(0) Parsing Table

- The LR(0) parsing table is obtained from the LR(0) state diagram
- The rows of the parsing table correspond to the LR(0) states
- The columns correspond to tokens and non-terminals
- For each state transition $i \rightarrow j$ caused by a token x …
  - Put **Shift** j at position $[i, x]$ of the table
- For each transition $i \rightarrow j$ caused by a nonterminal A …
  - Put **Goto** j at position $[i, A]$ of the table
- For each state containing an item $A \rightarrow \alpha \bullet$ of rule n …
  - Put **Reduce** n at position $[i, y]$ for every token y
- For each transition $i \rightarrow Accept$ …
  - Put **Accept** at position $[i, \$]$ of the table

LR(0) Parsing Table – cont'd

- The LR(0) table of grammar *G1* is shown below
  - For a shift, the token to be shifted determines the next state
  - For a reduce, the state on top of stack specifies the production to be used

State	Action					Goto	
	+	**ID**	**(**	**)**	**$**	*E*	*T*
0		S1	S2			G4	G3
1	R3	R3	R3	R3	R3		
2		S1	S2			G6	G3
3	R2	R2	R2	R2	R2		
4	S5				A		
5		S1	S2				G7
6	S5			S8			
7	R1	R1	R1	R1	R1		
8	R4	R4	R4	R4	R4		

Entries are labeled with …

S*n*: Shift token and goto state *n* (call scanner for next token)

R*n*: Reduce using production *n*

G*n*: Goto state *n* (after reduce)

A: Accept parse (terminate successfully)

blank : Syntax error

LR(0) Closure and Goto Functions

- To construct the LR(0) DFA, we need *closure*(I) and *goto*(I, X)
  - I is a set of items and X is a grammar symbol
- *Closure*(I) returns the complete set of items for an LR(0) state
  - New items are added to a state when dot appears before a non-terminal
- *Goto*(I, X) determines next state for a given state I and symbol X
  - The dot is moved past X and the *closure* of new set of items is obtained

function *closure*(I)
    $J := I$
    forall item $A \to \alpha \bullet B\,\gamma$ in J **do**
        forall production $B \to \beta$ **do**
            if $B \to \bullet\,\beta \notin J$ **then**
                $J := J \cup \{B \to \bullet\,\beta\}$
    return J
end function *closure*

function *goto*(I, X)
    $J := \varnothing$
    forall item $A \to \alpha \bullet X\,\gamma$ in I **do**
        $J := J \cup \{A \to \alpha\, X \bullet \gamma\}$
    return *closure*(J)
end function *goto*

Constructing the DFA of LR(0) States

- The following algorithm builds an LR(0) DFA for a grammar *G*
  - The algorithm determines the set of LR(0) states and the set of transitions
  - The algorithm ends when no additional states or transitions can be added

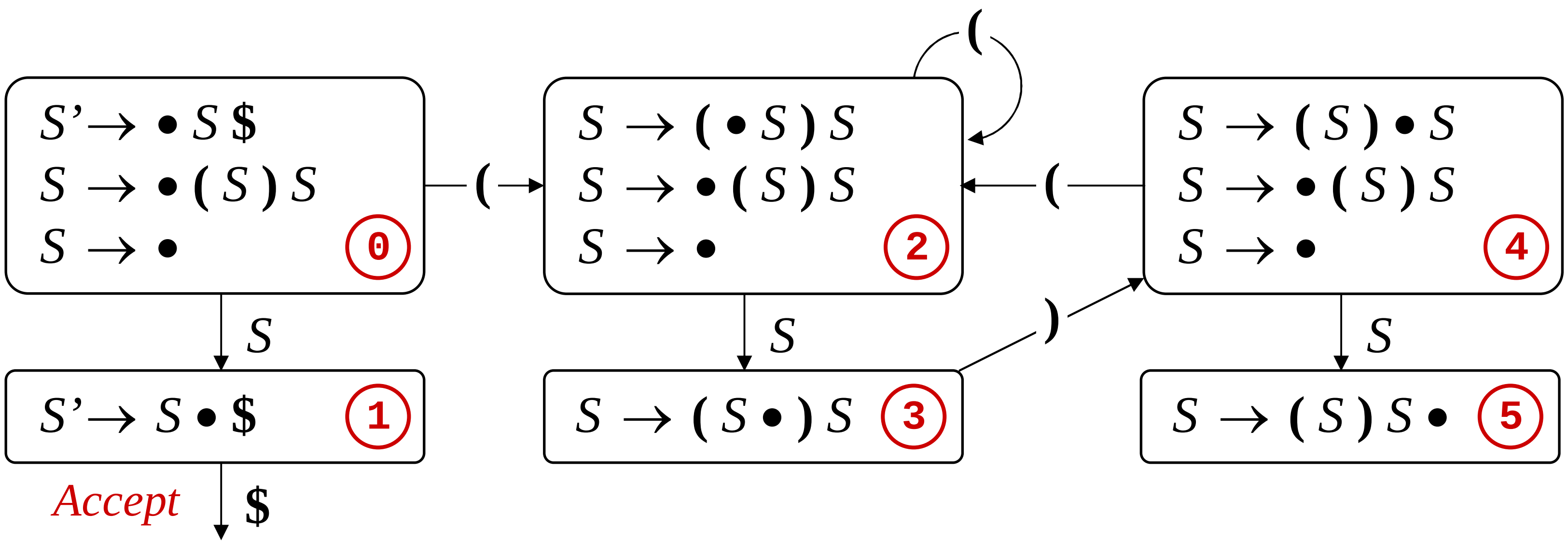
function *LR0_DFA*(*G*)
    $States$:= $\{closure(\{S' \rightarrow \bullet\ S\ \$\})\}$ -- Set of all LR(0) states
    $Edges$:= $\varnothing$ -- Set of all transitions
    forall state I in $States$ **do**
        forall item $A \rightarrow \alpha \bullet X \gamma$ in I **do**
            $J := goto(I, X)$
            if $J \notin States$ **then** $States := States \cup \{J\}$
            $Edges := Edges \cup \{\ I \xrightarrow{X} J\ \}$
    return *States*, *Edges*
end function *LR0_DFA*

Limitations of the LR(0) Parsing Method

- Consider grammar *G2* for matched parentheses

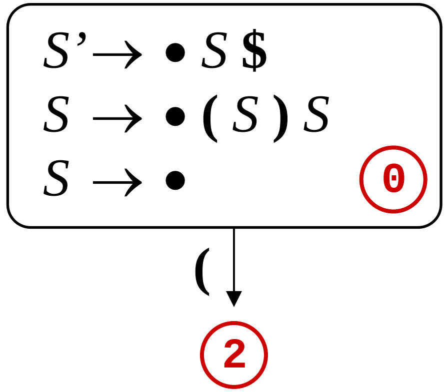
  0: $S' \to S\ \$$ 1: $S \to \mathbf{(}\ S\ \mathbf{)}\ S$ 2: $S \to \varepsilon$

- The LR(0) DFA of grammar *G*2 is shown below
- In states: 0, 2, and 4, parser can shift **(** and reduce ε to *S*



Conflicts

- In state 0 parser encounters a conflict ...
  - It can shift state 2 on stack when next token is **(**
  - It can reduce production 2: $S \rightarrow \varepsilon$
  - This is a called a **shift-reduce** conflict
  - This conflict also appears in states 2 and 4



- Two kinds of conflicts may arise
  - **Shift-reduce** and **reduce-reduce**

> ➢ **Shift-reduce** conflict
>
> Parser can shift and can reduce
>
> ➢ **Reduce-reduce** conflict
>
> Two (or more) productions can be reduced

State	Action			Goto
	(	**)**	**$**	*S*
0	S2,R2	R2	R2	G1
1			A	
2	S2,R2	R2	R2	G3
3		S4		
4	S2,R2	R2	R2	G5
5	R1	R1	R1	

LR(0) Grammars

- The shift-reduce conflict in state 0 indicates that *G2* is not LR(0)
- A grammar is LR(0) if and only if each state is either …
  - A **shift state**, containing only shift items
  - A **reduce state**, containing only a single reduce item
- If a state contains $A \to \alpha \bullet x\, \gamma$ then it cannot contain $B \to \beta \bullet$
  - Otherwise, parser can shift x and reduce $B \to \beta \bullet$ (shift-reduce conflict)
- If a state contains $A \to \alpha \bullet$ then it cannot contain $B \to \beta \bullet$
  - Otherwise, parser can reduce $A \to \alpha \bullet$ and $B \to \beta \bullet$ (reduce-reduce conflict)
- LR(0) lacks the power to parse programming language grammars
  - Because they do not use the lookahead token in making parsing decisions

SLR(1) Parsing

- SLR(1), or simple LR(1), improves LR(0) by …
  - Making use of the lookahead token to eliminate conflicts
- SLR(1) works as follows …
  - It uses the same DFA obtained by the LR(0) parsing method
  - It puts reduce actions only where indicated by the FOLLOW set
- To reduce α to A in $A \rightarrow \alpha \bullet$ we must ensure that …
  - Next token may follow A (belongs to FOLLOW(A))
- We should not reduce $A \rightarrow \alpha \bullet$ when next token $\notin$ FOLLOW(A)
- In grammar $G2$ …
  - 0: $S' \rightarrow S$ \$ 1: $S \rightarrow$ **(** S **)** S 2: $S \rightarrow \varepsilon$
  - FOLLOW(S) = {**\$**, **)**}
  - Productions 1 and 2 are reduced when next token is **\$** or **)** only

SLR(1) Parsing Table

- The SLR(1) parsing table of grammar *G2* is shown below
- The shift-reduce conflicts are now eliminated
  - The R2 action is removed from [0, **(**], [2, **(**], and [4, **(**]
  - Because **(** does not follow *S*
  - S2 remains under [0, **(**], [2, **(**], and [4, **(**]
  - R1 action is also removed from [5, **(**]
- Grammar *G2* is SLR(1)
  - No conflicts in parsing table
  - R1 and R2 for **)** and **$** **only**
  - **Follow set indicates when to reduce**

State	**Action**			**Goto**
	(	**)**	**$**	*S*
0	S2	R2	R2	G1
1			A	
2	S2	R2	R2	G3
3		S4		
4	S2	R2	R2	G5
5		R1	R1	

SLR(1) Grammars

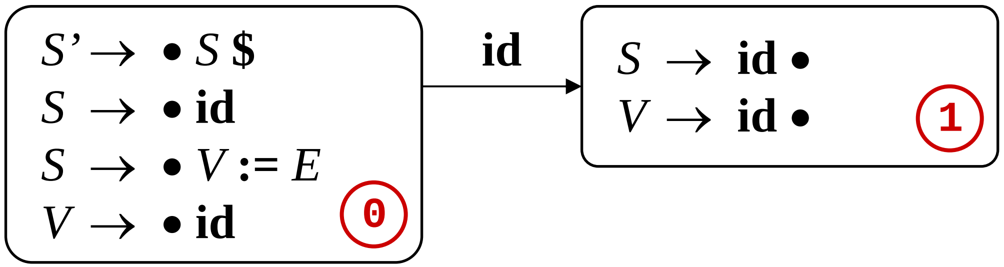
- SLR(1) parsing increases the power of LR(0) significantly
  - Lookahead token is used to make parsing decisions
  - Reduce action is applied more selectively according to FOLLOW set
- A grammar is SLR(1) if two conditions are met in every state …
  - If $A \rightarrow \alpha \bullet x \gamma$ and $B \rightarrow \beta \bullet$ then token $x \notin$ FOLLOW(B)
  - If $A \rightarrow \alpha \bullet$ and $B \rightarrow \beta \bullet$ then FOLLOW(A) $\cap$ FOLLOW(B) $= \varnothing$
- Violation of first condition results in **shift-reduce conflict**
  - $A \rightarrow \alpha \bullet x \gamma$ and $B \rightarrow \beta \bullet$ and $x \in$ FOLLOW(B) then …
  - Parser can shift x and reduce $B \rightarrow \beta$
- Violation of second condition results in **reduce-reduce conflict**
  - $A \rightarrow \alpha \bullet$ and $B \rightarrow \beta \bullet$ and $x \in$ FOLLOW(A) $\cap$ FOLLOW(B)
  - Parser can reduce $A \rightarrow \alpha$ and $B \rightarrow \beta$
- SLR(1) grammars are a superset of LR(0) grammars

Limits of the SLR(1) Parsing Method

- Consider the following grammar *G3* …

  0: $S' \to S$ **$**   1: $S \to$ **id**   2: $S \to V$ **:=** E 3: $V \to$ **id**   4: $E \to V$ 5: $E \to$ **n**

- The initial state consists of 4 items as shown below
  - When **id** is shifted in state 0, we obtain 2 items: $S \to$ **id** • and $V \to$ **id** •
- FOLLOW(S) = {**$**} and FOLLOW($V$) = {**:=** , **$**}
- **Reduce-reduce conflict** in state 1 when lookahead token is **$**
  - Therefore, grammar *G3* is **not** SLR(1)
  - The reduce-reduce conflict is caused by the weakness of SLR(1) method
  - $V \to$ **id** should be reduced only when lookahead token is **:=** (but not **$**)



General LR(1) Parsing – Items and States

- Even more powerful than SLR(1) is the LR(1) parsing method
- LR(1) generalizes LR(0) by including a lookahead token in items
- An LR(1) item consists of …
  - **Grammar production rule**
  - **Right-hand position represented by the dot**, and
  - **Lookahead token**

  $A \rightarrow X_1 \ldots X_i \bullet X_{i+1} \ldots X_n \ , \ l$ where l is a **lookahead token**
- The • represents how much of the right-hand side has been seen
  - $X_1 \ldots X_i$ appear on top of the stack
  - $X_{i+1} \ldots X_n$ are expected to appear
- The lookahead token l is expected **after** $X_1 \ldots X_n$ appear on stack
- **An LR(1) state is a set of LR(1) items**

LR(1) Parser Generation – Initial State

- Consider again grammar *G3* …

  0: $S' \to S$ **$**   1: $S \to$ **id**   2: $S \to V$ **:=** E 3: $V \to$ **id**   4: $E \to V$ 5: $E \to$ **n**

- The initial state contains the LR(1) item: $S' \to \bullet\, S$, **$**
  - $S' \to \bullet\, S$, **$** means that S is expected and to be followed by **$**
- The closure of ($S' \to \bullet\, S$, **$**) produces the initial state items
  - Since the dot appears before *S*, an *S* is expected
  - There are two productions of *S*: $S \to$ **id** and $S \to V$ **:=** E
  - The LR(1) items ($S \to \bullet$ **id** , **$**) and ($S \to \bullet\, V$ **:=** E , **$**) are obtained
    - The lookahead token is **$** (end-of-file token)
  - Since the $\bullet$ appears before *V* in ($S \to \bullet\, V$ **:=** E , **$**), a *V* is expected
  - The LR(1) item ($V \to \bullet$ **id** , **:=**) is obtained
    - The lookahead token is **:=** because it appears after *V* in ($S \to \bullet\, V$ **:=** E , **$**)

Shift Action

- The initial state (state 0) consists of 4 items
- In state 0, we can shift an **id**
  - The token **id** can be shifted in two items
  - When shifting **id**, we shift the dot past the **id**
  - We obtain ($S \rightarrow$ **id** $\bullet$, **\$**) and ($V \rightarrow$ **id** $\bullet$, **:=**)
  - The two LR(1) items form a new state (state 1)
  - The two items are **reduce items**
  - No additional item can be added to state 1

$S' \rightarrow \bullet\, S\,,\ \$$
$S \rightarrow \bullet\, \mathbf{id}\,,\ \$$
$S \rightarrow \bullet\, V \mathbin{:=} E\,,\ \$$
$V \rightarrow \bullet\, \mathbf{id}\,,\ :=$
0

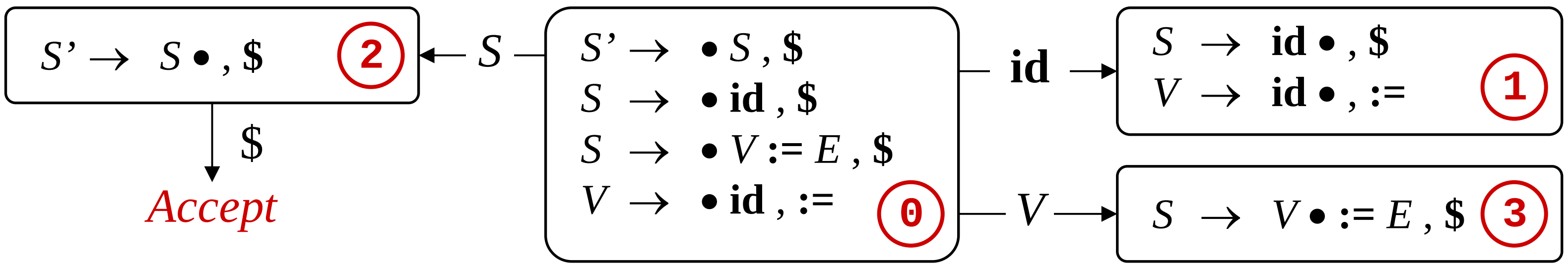
State 0:
$S' \rightarrow \bullet\, S\,,\ \$$
$S \rightarrow \bullet\, \mathbf{id}\,,\ \$$
$S \rightarrow \bullet\, V \mathbin{:=} E\,,\ \$$
$V \rightarrow \bullet\, \mathbf{id}\,,\ :=$
0

$\xrightarrow{\mathbf{id}}$

State 1:
$S \rightarrow \mathbf{id}\, \bullet\,,\ \$$
$V \rightarrow \mathbf{id}\, \bullet\,,\ :=$
1

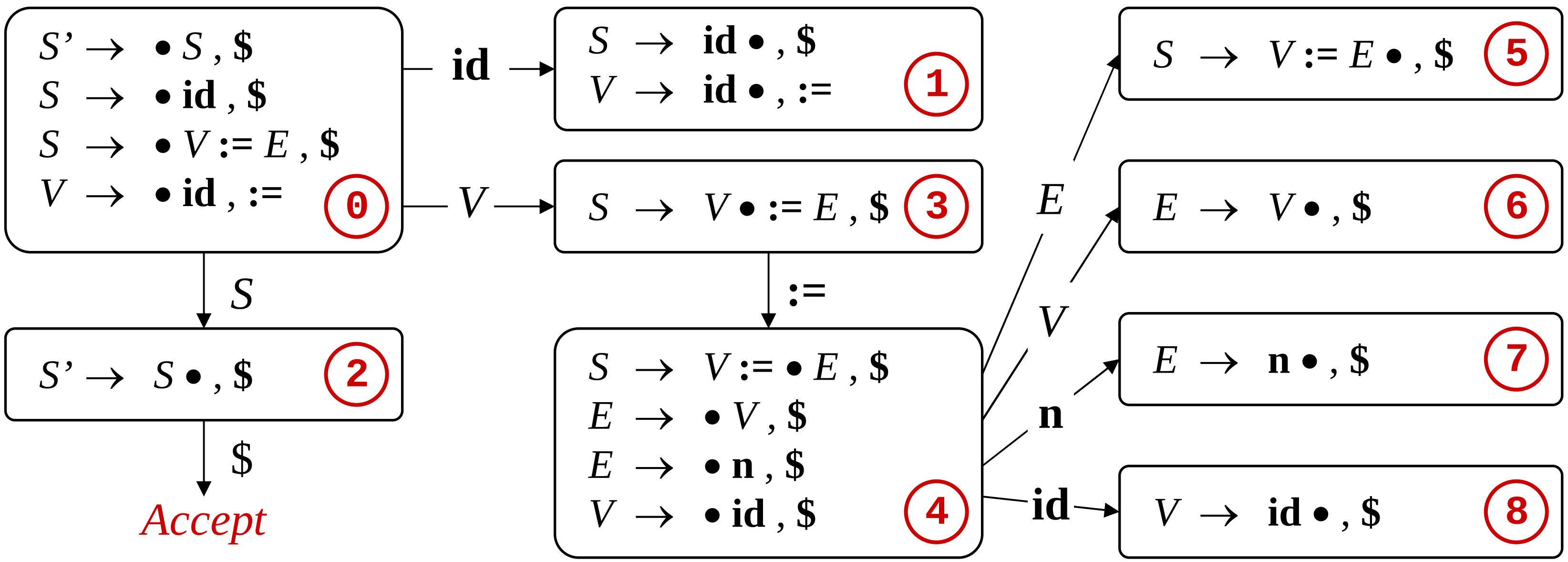
Reduce and Goto Actions

- In state 1, • appears at end of ($S \rightarrow$ **id** • , **$**) and ($V \rightarrow$ **id** • , **:=**)
  - This means that **id** appears on top of stack and can be reduced
  - Two productions can be reduced: $S \rightarrow$ **id** and $V \rightarrow$ **id**
- The lookahead token eliminates the conflict of the reduce items
  - If lookahead token is **$** then **id** is reduced to S
  - If lookahead token is **:=** then **id** is reduced to V
- When in state 0 after a reduce action …
  - If S is pushed, we obtain item ($S' \rightarrow S$ • , **$**) and go to state 2
  - If V is pushed, we obtain item ($S \rightarrow V$ • **:=** E , **$**) and go to state 3



LR(1) State Diagram

- The LR(1) state diagram of grammar *G3* is shown below
- Grammar *G3*, which was not SLR(1), is now LR(1)
- The reduce-reduce conflict that existed in state 1 is now removed
- The lookahead token in LR(1) items eliminated the conflict



LR(1) Closure and Goto Functions

- To construct the LR(1) DFA, we need *closure* and *goto* functions
- *Closure*(I) returns the complete set of LR(1) items for a state
  - For items $(A \to \alpha \bullet B \gamma , l) \in I$, add $(B \to \bullet \beta , x)$ forall $x \in \text{FIRST}(\gamma l)$
  - $x \in \text{FIRST}(\gamma l)$ is what follows B in $(A \to \alpha \bullet B \gamma , l)$
- *Goto*(I, X) determines next state for a given state I and symbol X

function *closure*(I)
    $J := I$
    forall item $(A \to \alpha \bullet B \gamma , l)$ in J **do**
        forall production $B \to \beta$ **do**
            forall $x \in \text{FIRST}(\gamma l)$ **do**
                if $(B \to \bullet \beta , x) \notin J$ **then**
                    $J := J \cup \{(B \to \bullet \beta , x)\}$
    return J
end function *closure*

function *goto*(I, X)
    $J := \varnothing$
    forall item $(A \to \alpha \bullet X \gamma , l)$ in I **do**
        $J := J \cup \{(A \to \alpha X \bullet \gamma , l)\}$
    return *closure*(J)
end function *goto*

LR(1) Grammars

- A grammar is LR(1) if the following two conditions are met …
  - If a state contains ($A \rightarrow \alpha \bullet x \gamma$, a) and ($B \rightarrow \beta \bullet$, b) then $b \neq x$
  - If a state contains ($A \rightarrow \alpha \bullet$, a) and ($B \rightarrow \beta \bullet$, b) then $a \neq b$
- Violation of first condition results in a **shift-reduce conflict**
- If a state contains ($A \rightarrow \alpha \bullet x \gamma$, a) and ($B \rightarrow \beta \bullet$, x) then …
  - It can shift x and can reduce $B \rightarrow \beta$ when lookahead token is x
- Violation of second condition results in **reduce-reduce conflict**
- If a state contains ($A \rightarrow \alpha \bullet$, a) and ($B \rightarrow \beta \bullet$, a) then …
  - It can reduce $A \rightarrow \alpha$ and $B \rightarrow \beta$ when lookahead token is a
- LR(1) grammars are a superset of SLR(1) grammars

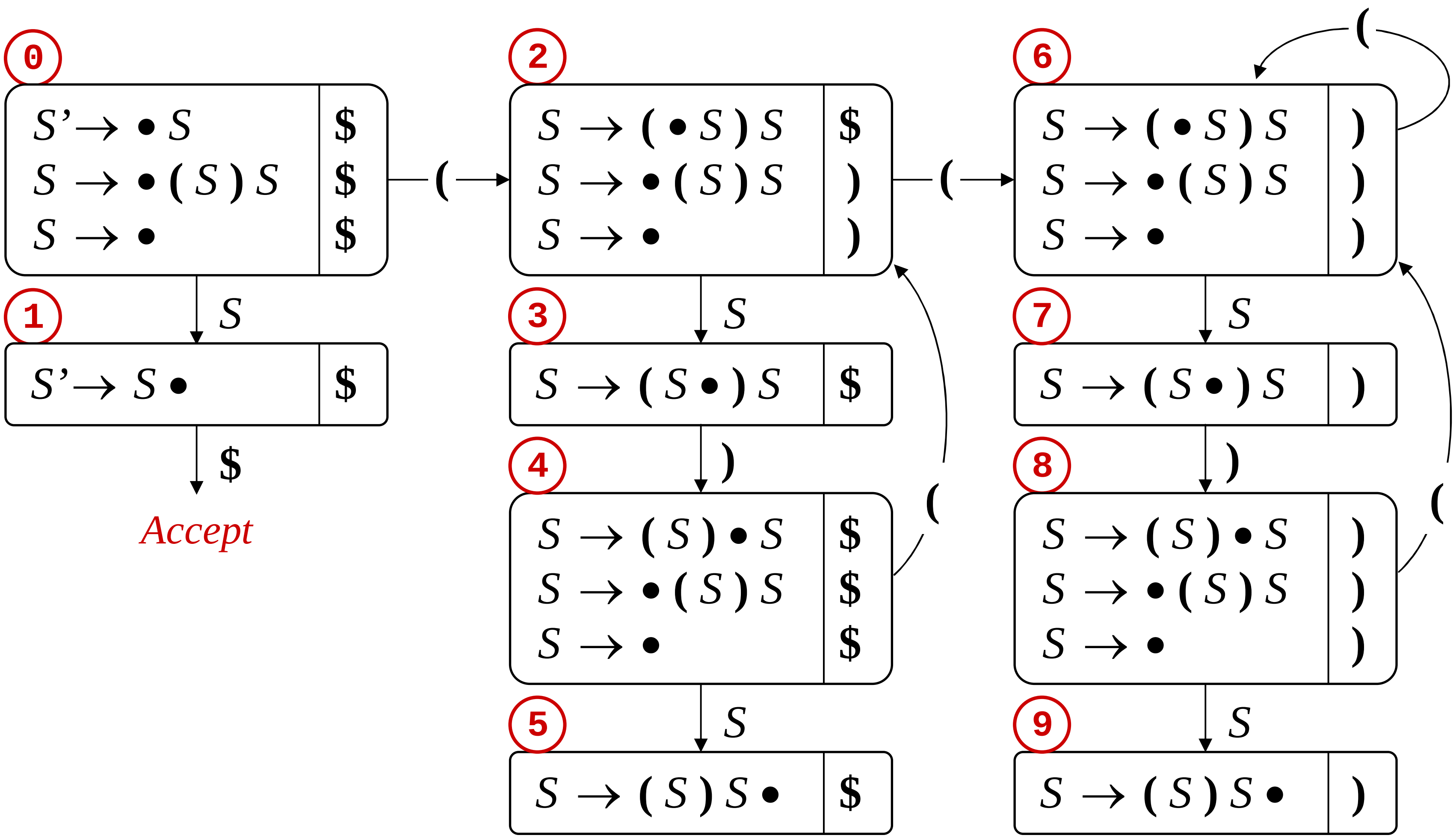
Drawback of LR(1)

- LR(1) can generate very large parsing tables
- For a typical programming language grammar …
  - The number of states is around several hundred for LR(0) and SLR(1)
  - The number of states can be several thousand for LR(1)
- This is why parser generators do not adopt the general LR(1)
- Consider again grammar G2 for matched parentheses

  0: $S' \to S\ \$$ 1: $S \to \mathbf{(}\ S\ \mathbf{)}\ S$ 2: $S \to \varepsilon$

- The LR(1) DFA has 10 states, while the LR(0) DFA has 6

LR(1) DFA of Grammar G2



LALR(1) : Look-Ahead LR(1)

- Preferred parsing technique in many parser generators
- Close in power to LR(1), but with less number of states
- Increased number of states in LR(1) is because
  - Different lookahead tokens are associated with same LR(0) items
- Number of states in LALR(1) = states in LR(0)
- LALR(1) is based on the observation that
  - Some LR(1) states have same LR(0) items
  - Differ only in lookahead tokens
- LALR(1) can be obtained from LR(1) by
  - Merging LR(1) states that have same LR(0) items
  - Obtaining the union of the LR(1) lookahead tokens

LALR(1) DFA of Grammar G2