Lecture 8

Instructor: Amit Kumar Das

Senior Lecturer,

Department of Computer Science & Engineering,

East West University

Dhaka, Bangladesh.

Game theory
(Sections 17.5-17.6)

Game theory

- **Game theory** deals with systems of interacting agents where the outcome for an agent depends on the actions of all the other agents
  - Applied in sociology, politics, economics, biology, and, of course, AI
- **Agent design:** determining the best strategy for a rational agent in a given game
- **Mechanism design:** how to set the rules of the game to ensure a desirable outcome

Modelling behaviour

Game theory in practice

Computing: Software that models human behaviour can make forecasts, outfox rivals and transform negotiations

Sep 3rd 2011 | from the print edition

Christian Montenegro

PROVABLY FAIR SOLUTIONS.

Spliddit offers quick, free solutions to everyday fair division problems, using methods that provide indisputable fairness guarantees and build on decades of research in economics, mathematics, and computer science.

Share Rent

Moving into a new apartment with roommates? Create harmony by fairly assigning rooms and sharing the rent.

Divide Goods

Fairly divide jewelry, artworks, electronics, toys, furniture, financial assets, or even an entire estate.

Assign Credit

Determine the contribution of each individual to a school project, academic paper, or business endeavor.

http://www.spliddit.org

CADE METZ   BUSINESS   09.21.15   7:00 AM

FACEBOOK DOESN'T MAKE AS MUCH MONEY AS IT COULD—ON PURPOSE

YOU CAN THINK of John Hegeman as Facebook's chief economist. He spends his days thinking about the economics of Facebook advertising.

That's an enormous thing. Facebook pulled in $4.04 billion in the second quarter of this year. And the overall economy of Facebook advertising, as Hegeman describes it, is far larger. Advertising, you see, is very much a part of everything else on the world's largest social network. Hegeman doesn't just think about ads. He thinks about how ads fit with the rest of Facebook.

When he joined Facebook in 2007, after getting a master's in economics at Stanford University, Hegeman helped build the online auction that drives the company's advertising system. Auctions are the standard way that online services accept ads from advertisers and place them on web pages and inside smartphone apps. That's what Google uses with AdWords, the system that serves up all those ads when you look for stuff on the company's Internet search engine. Advertisers bid (in dollars) for placement on the results page when you key in a particular word or group of words. But in building Facebook's advertising system, Hegeman and team took online auctions in a new direction.

http://www.wired.com/2015/09/facebook-doesnt-make-much-money-couldon-purpose/

Prisoner's dilemma

DEMO

Prisoner's dilemma

- Two criminals have been arrested and the police visit them separately

- If one player testifies against the other and the other refuses, the one who testified goes free and the one who refused gets a 10-year sentence

- If both players testify against each other, they each get a 5-year sentence

- If both refuse to testify, they each get a 1-year sentence

	Alice: Testify	**Alice: Refuse**
Bob: Testify		
Bob: Refuse		

Prisoner's dilemma

- Alice's reasoning:
  - Suppose Bob testifies. Then I get 5 years if I testify and 10 years if I refuse. So I should testify.
  - Suppose Bob refuses. Then I go free if I testify, and get 1 year if I refuse. So I should testify.
- **Dominant strategy:** A strategy whose outcome is better for the player regardless of the strategy chosen by the other player

	Alice: Testify	**Alice: Refuse**
Bob: Testify	-5,-5	-10,0
Bob: Refuse	0,-10	-1,-1

Nash Equilibrium

"**If there is a set of strategies with the property that no player can benefit by changing her strategy while the other players keep their strategies unchanged, then that set of strategies and the corresponding payoffs constitute the Nash Equilibrium.** "

Prisoner's dilemma

- **Nash equilibrium:** A pair of strategies such that no player can get a bigger payoff by switching strategies, provided the other player sticks with the same strategy
  - (Testify, testify) is a *dominant strategy equilibrium*

- **Pareto optimal outcome:** It is impossible to make one of the players better off without making another one worse off

- In a non-zero-sum game, a Nash equilibrium is not necessarily Pareto optimal!

	Alice: Testify	**Alice: Refuse**
Bob: Testify	-5,-5	-10,0
Bob: Refuse	0,-10	-1,-1

Prisoner's dilemma in real life

- Price war
- Arms race
- Steroid use
- [Diner's dilemma](#)
- Collective action in politics

	Defect	Cooperate
Defect	Lose – lose	Lose big – win big
Cooperate	Win big – lose big	Win – win

http://en.wikipedia.org/wiki/Prisoner's_dilemma

Valentine's Day Problem

Suppose my wife and I have both lost our cell phones and cannot contact each other. Both of us would really like to spend more time at home with each other (utility 3) in the valentine's day. But both of us also have a slight preference in favor of working late and earning some overtime (utility 2). If I go home and my wife's there and I can spend time with her, great. If I stay at work and make some money, that would be pretty okay too. But if I go home and my wife's not there and I have to sit around alone all night, that would be the worst possible outcome (utility 1). Meanwhile, my wife has the same set of preferences: she wants to spend time with me, she'd be okay with working late, but she doesn't want to sit at home alone.

	I go home	I work late
She goes home	($3,3$)	($1,2$)
She works late	($2,1$)	($2,2$)

Is there any way to get a better answer?

- <u>Superrationality</u>
  - Assume that the answer to a symmetric problem will be the same for both players
  - Maximize the payoff to each player while considering only identical strategies
  - Not a conventional model in game theory

- Repeated games
  - If the number of rounds is fixed and known in advance, the equilibrium strategy is still to defect
  - If the number of rounds is unknown, cooperation may become an equilibrium strategy

Review: Game theory

- Agent design and mechanism design
- Dominant strategies
- Nash equilibria
- Pareto optimality
- Examples of games

Game of Chicken

Player 1

Player 2

	S	**C**
S	-10, -10	-1, 1
C	1, -1	0, 0

- Is there a dominant strategy for either player?

- Is there a Nash equilibrium?

  (Straight, chicken) or (chicken, straight)

- *Anti-coordination* game: it is mutually beneficial for the two players to choose different strategies
  - Model of escalated conflict in humans and animals (hawk-dove game)

- How are the players to decide what to do?
  - Pre-commitment or threats
  - Different roles: the "hawk" is the territory owner and the "dove" is the intruder, or vice versa

http://en.wikipedia.org/wiki/Game_of_chicken

Mixed strategy equilibria

Player 1   **Player 2**

Straight

Chicken

Chicken

Straight

	S	C
S	-10, -10	-1, 1
C	1, -1	0, 0

- **Mixed strategy:** a player chooses between the moves according to a probability distribution

- Suppose each player chooses S with probability 1/10. Is that a Nash equilibrium?

- Consider payoffs to P1 while keeping P2's strategy fixed
  - The payoff of P1 choosing S is (1/10)(–10) + (9/10)1 = –1/10
  - The payoff of P1 choosing C is (1/10)(–1) + (9/10)0 = –1/10
  - Can P1 change their strategy to get a better payoff?
  - Same reasoning applies to P2

Finding mixed strategy equilibria

	P1: Choose S with prob. p	**P1:** Choose C with prob. $1-p$
P2: Choose S with prob. q	-10, -10	-1, 1
P2: Choose C with prob. $1-q$	1, -1	0, 0

- Expected payoffs for P1 given P2's strategy:

  P1 chooses S: $q(-10) + (1-q)1 = -11q + 1$

  P1 chooses C: $q(-1) + (1-q)0 = -q$

- In order for P2's strategy to be part of a Nash equilibrium, P1 has to be indifferent between its two actions:

  $-11q + 1 = -q$ or $q = 1/10$

  Similarly, $p = 1/10$

Existence of Nash equilibria

- Any game with a finite set of actions has at least one Nash equilibrium (which may be a mixed-strategy equilibrium)
- If a player has a dominant strategy, there exists a Nash equilibrium in which the player plays that strategy and the other player plays the *best response* to that strategy
- If both players have *strictly dominant* strategies, there exists a Nash equilibrium in which they play those strategies

Computing Nash equilibria

- For a two-player zero-sum game, simple linear programming problem

- For non-zero-sum games, the algorithm has worst-case running time that is exponential in the number of actions

- For more than two players, and for sequential games, things get pretty hairy

Mechanism design (inverse game theory)

- Assuming that agents pick rational strategies, how should we design the game to achieve a socially desirable outcome?

- We have multiple agents and a **center** that collects their choices and determines the outcome

Auctions

- Goals
  - Maximize revenue to the seller
  - Efficiency: make sure the buyer who values the goods the most gets them
  - Minimize transaction costs for buyer and sellers

Ascending-bid auction

- What's the optimal strategy for a buyer?
  - Bid until the current bid value exceeds your *private value*
- Usually revenue-maximizing and efficient, unless the reserve price is set too low or too high
- Disadvantages
  - Collusion
  - Lack of competition
  - Has high communication costs

Sealed-bid auction

- Each buyer makes a single bid and communicates it to the auctioneer, but not to the other bidders
  - Simpler communication
  - More complicated decision-making: the strategy of a buyer depends on what they believe about the other buyers
  - Not necessarily efficient
- **Sealed-bid second-price auction:** the winner pays the price of the second-highest bid
  - Let V be your private value and B be the highest bid by any other buyer
  - If $V > B$, your optimal strategy is to bid above B – in particular, bid V
  - If $V < B$, your optimal strategy is to bid below B – in particular, bid V
  - Therefore, your dominant strategy is to bid V
  - This is a **truth revealing** mechanism

Dollar auction

- A dollar bill is auctioned off to the highest bidder, but the second-highest bidder has to pay the amount of his last bid
    - Player 1 bids 1 cent
    - Player 2 bids 2 cents
    - …
    - Player 2 bids 98 cents
    - Player 1 bids 99 cents
        - If Player 2 passes, he loses 98 cents, if he bids $1, he might still come out even
    - So Player 2 bids $1
        - Now, if Player 1 passes, he loses 99 cents, if he bids $1.01, he only loses 1 cent
    - …
- What went wrong?
    - When figuring out the expected utility of a bid, a rational player should take into account the future course of the game
- What if Player 1 starts by bidding 99 cents?

Dollar auction

- A dollar bill is auctioned off to the highest bidder, but the second-highest bidder has to pay the amount of his last bid

- Dramatization: https://www.youtube.com/watch?v=pA-SNscNADk

Regulatory mechanism design: Tragedy of the commons

- States want to set their policies for controlling emissions
  - Each state can reduce their emissions at a cost of -10 or continue to pollute at a cost of -5
  - If a state decides to pollute, -1 is added to the utility of every other state
- What is the dominant strategy for each state?
  - Continue to pollute
  - Each state incurs cost of -5-49 = -54
  - If they all decided to deal with emissions, they would incur a cost of only -10 each
- Mechanism for fixing the problem:
  - Tax each state by the total amount by which they reduce the global utility (**externality cost**)
  - This way, continuing to pollute would now cost -54

Review: Game theory

- Normal form representation of a game
- Dominant strategies
- Nash equilibria
- Pareto optimal outcomes
- Pure strategies and mixed strategies
- Examples of games
- Mechanism design
  - Auctions: ascending bid, sealed bid, sealed bid second-price, "dollar auction"

Thank You