Lecture 13

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Conditional Independence

- X and Y are independent if

$$\forall x, y \;\; P(x, y) = P(x)P(y) \;\; \dashrightarrow \;\; X \perp\!\!\!\perp Y$$

- X and Y are conditionally independent given Z

$$\forall x, y, z \;\; P(x, y|z) = P(x|z)P(y|z) \;\; \dashrightarrow \;\; X \perp\!\!\!\perp Y|Z$$

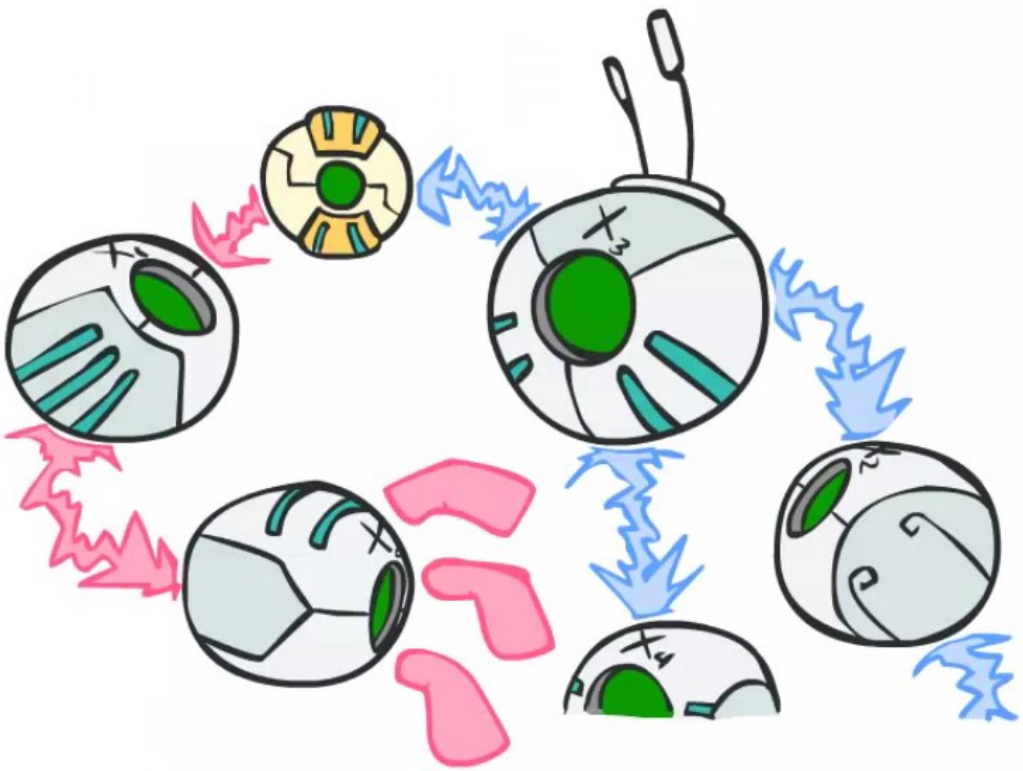
- (Conditional) independence is a property of a distribution

- Example: $Alarm \perp\!\!\!\perp Fire|Smoke$

Conditional Independence

- What about this domain:
  - Traffic
  - Umbrella
  - Raining

D-separation: Outline



Reachability (D-Separation)

- Question: Are X and Y conditionally independent given evidence vars {Z}?
  - Yes, if X and Y "separated" by Z
  - Consider all (undirected) paths from X to Y
  - No active paths = independence!

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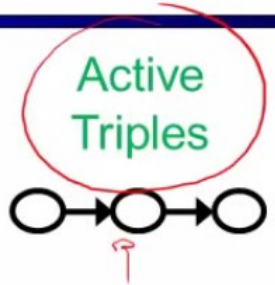
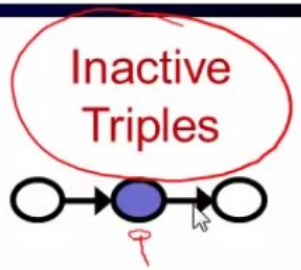
- A path is active if each triple is active:
  - Causal chain $A \rightarrow B \rightarrow C$ where B is unobserved (either direction)





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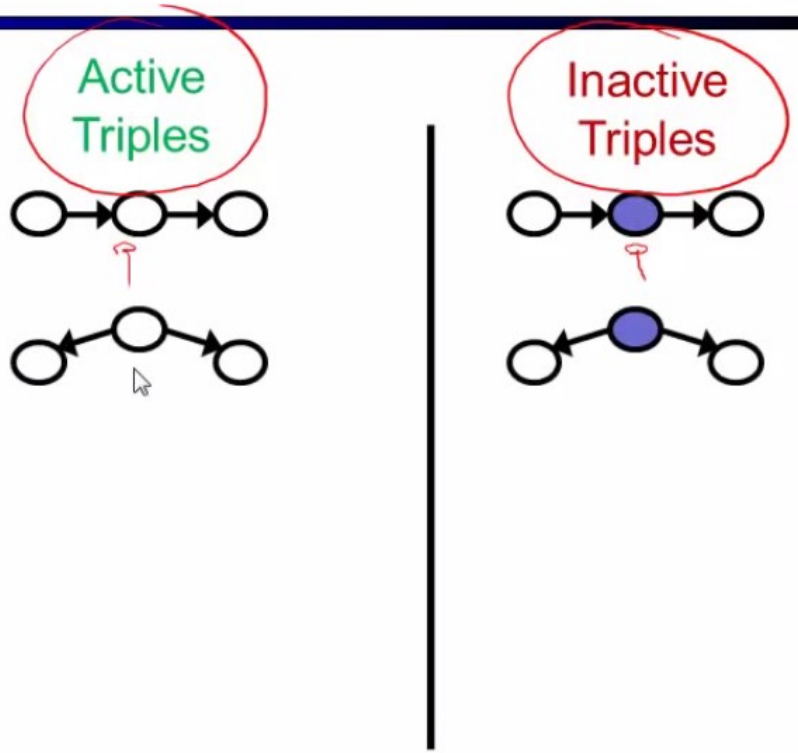

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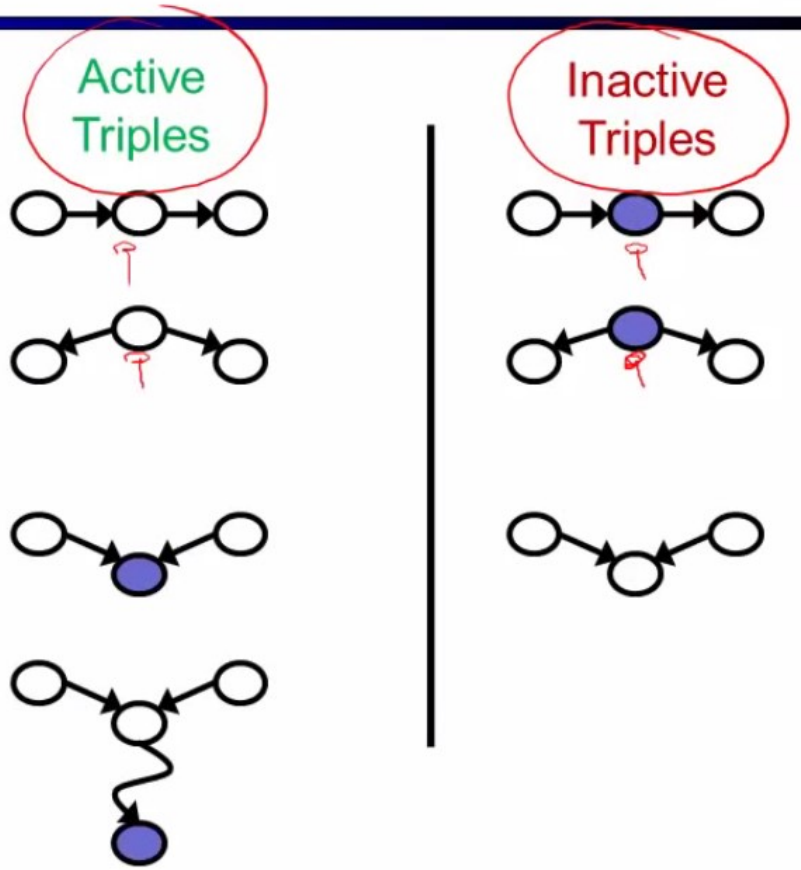
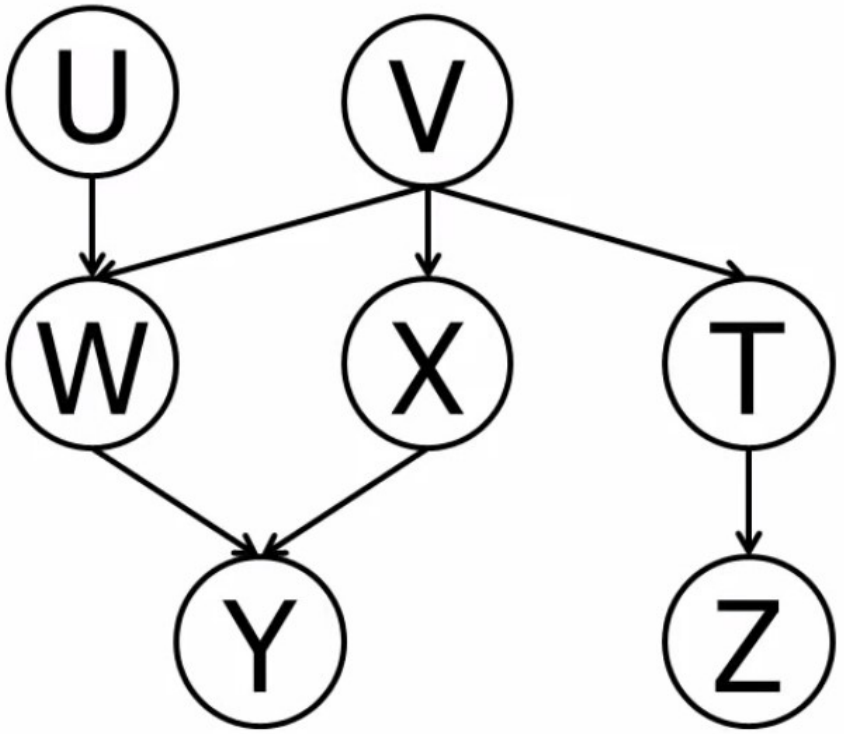
- A path is active if each triple is active:
  - Causal chain $A \rightarrow B \rightarrow C$ where B is unobserved (either direction)
  - Common cause $A \leftarrow B \rightarrow C$ where B is unobserved
  - Common effect (aka v-structure) $A \rightarrow B \leftarrow C$ where B *or one of its descendents* is observed



Example 1

V _||_ Z

- o guaranteed to be true
- o not guaranteed to be true



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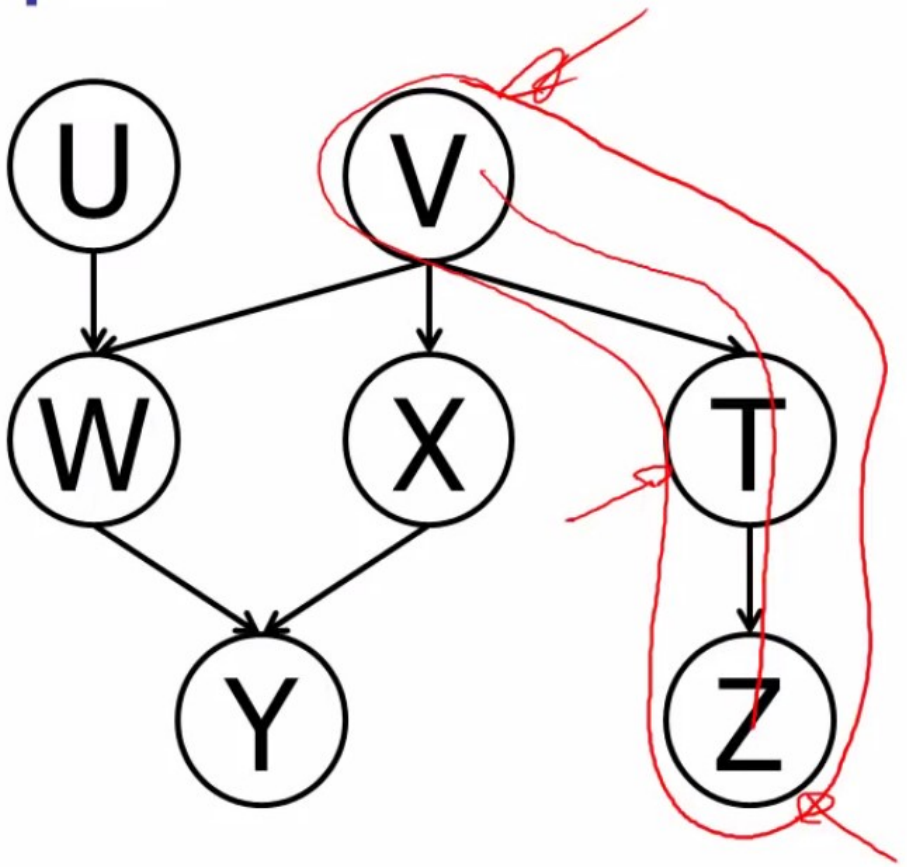
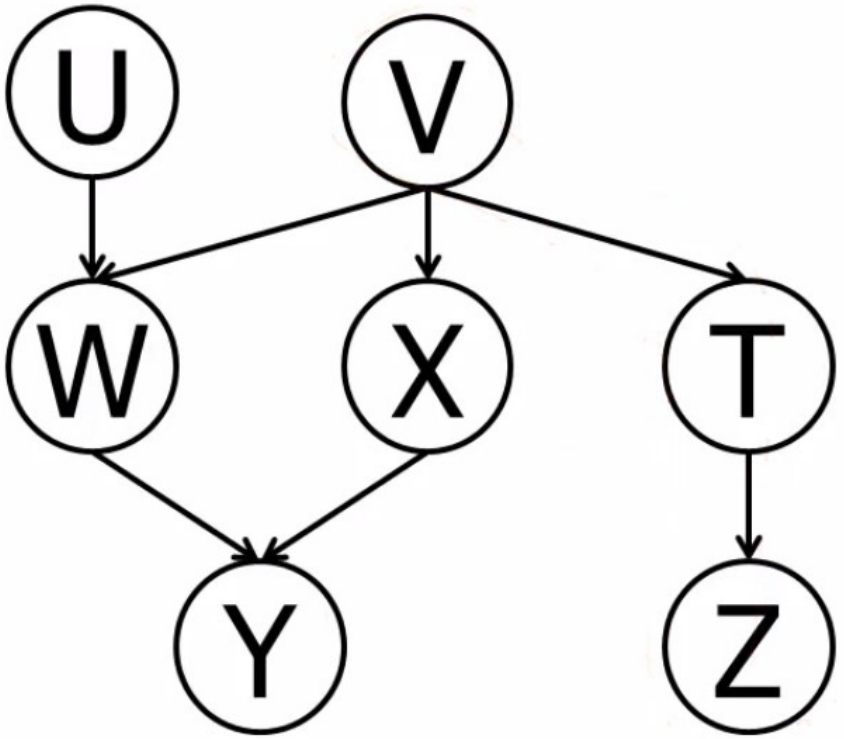


Example 2

V _||_ Z | T

- o guaranteed to be true
- o not guaranteed to be true



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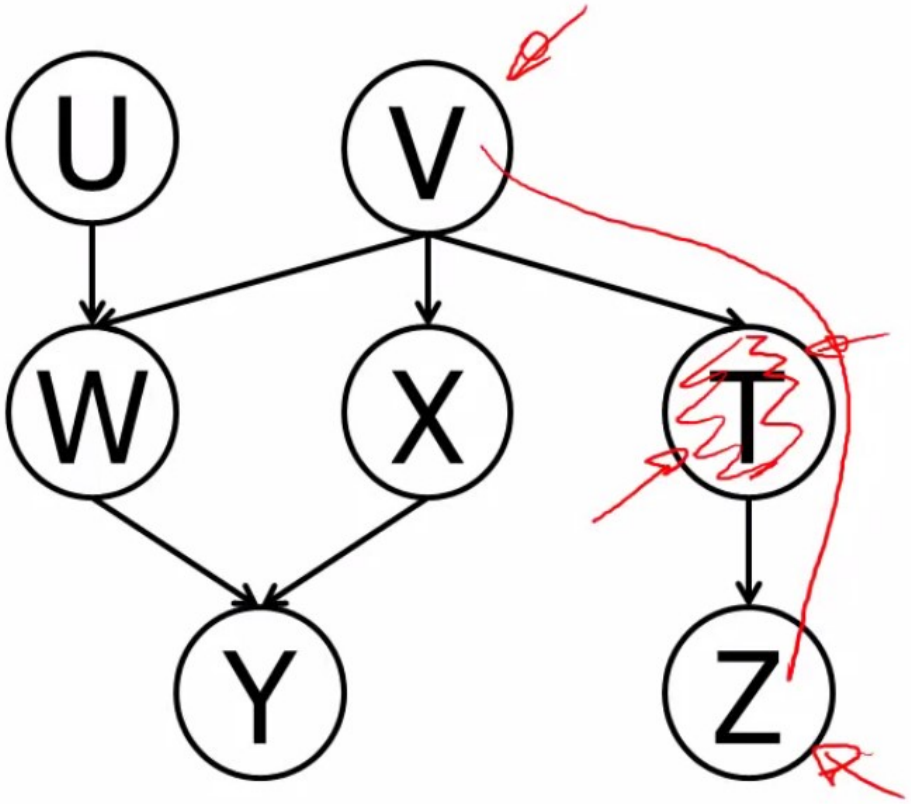
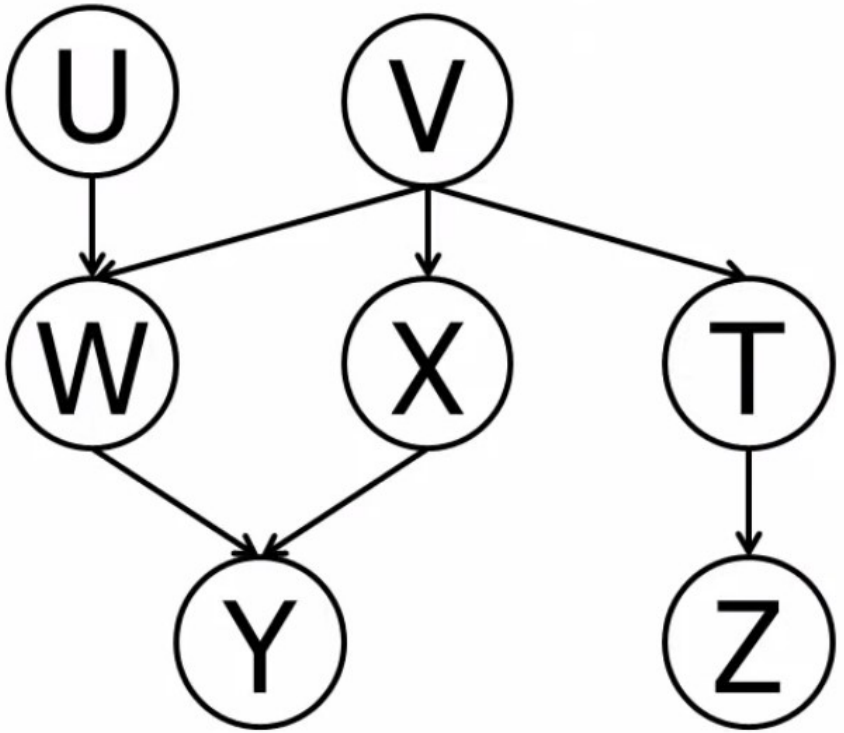


Example 3

U _||_ V

- o guaranteed to be true
- o not guaranteed to be true



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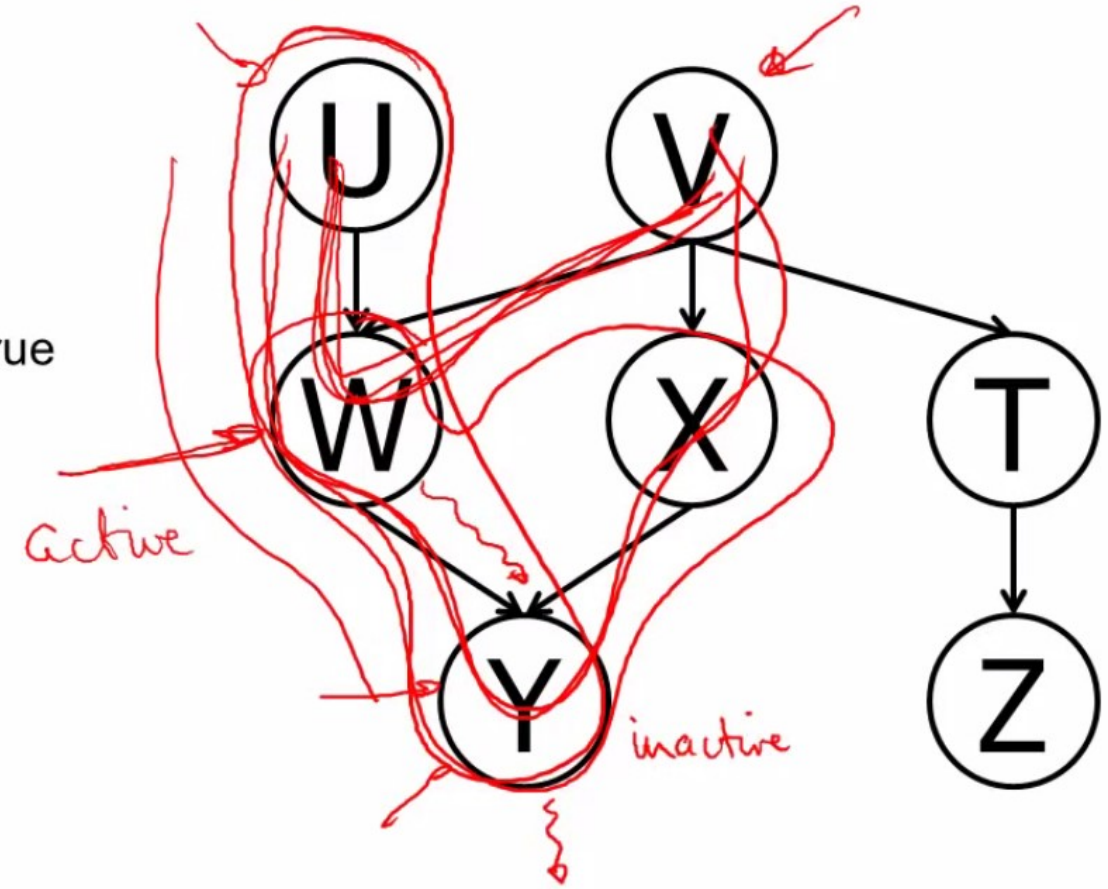
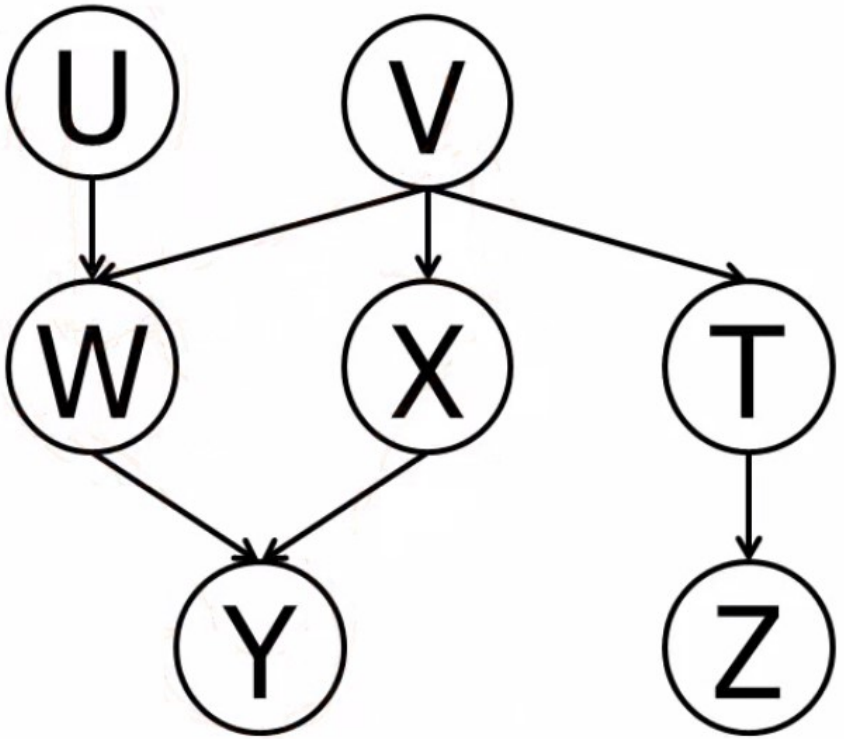


Example 4

U _||_ V | W

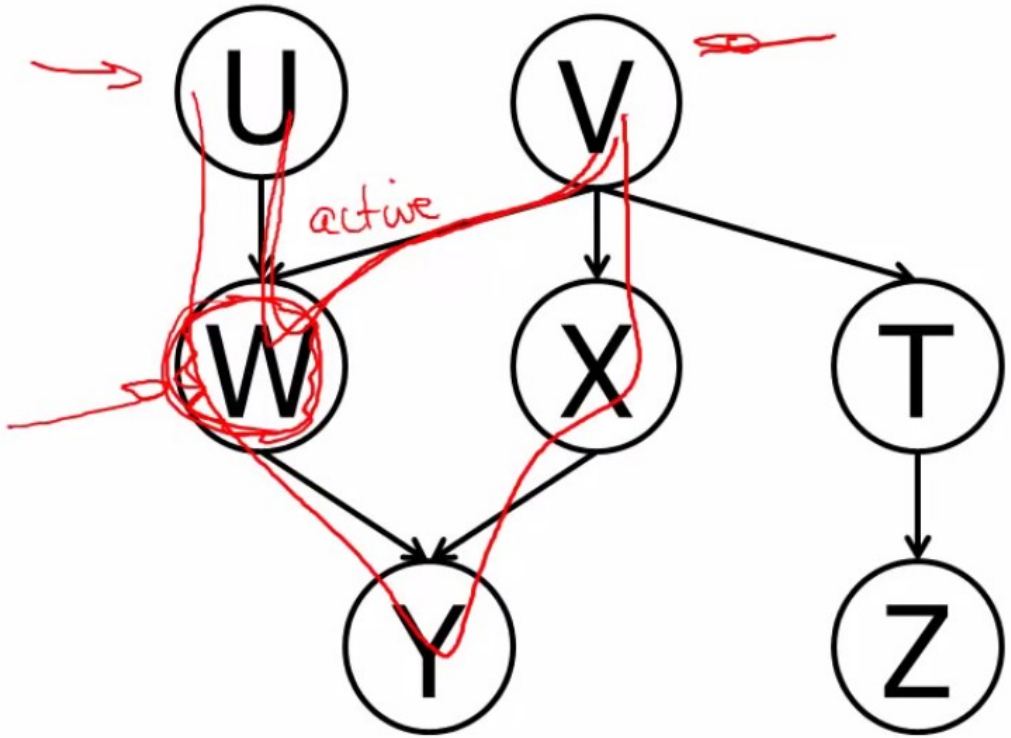
- o guaranteed to be true
- o not guaranteed to be true



Example 4

U _||_ V | W

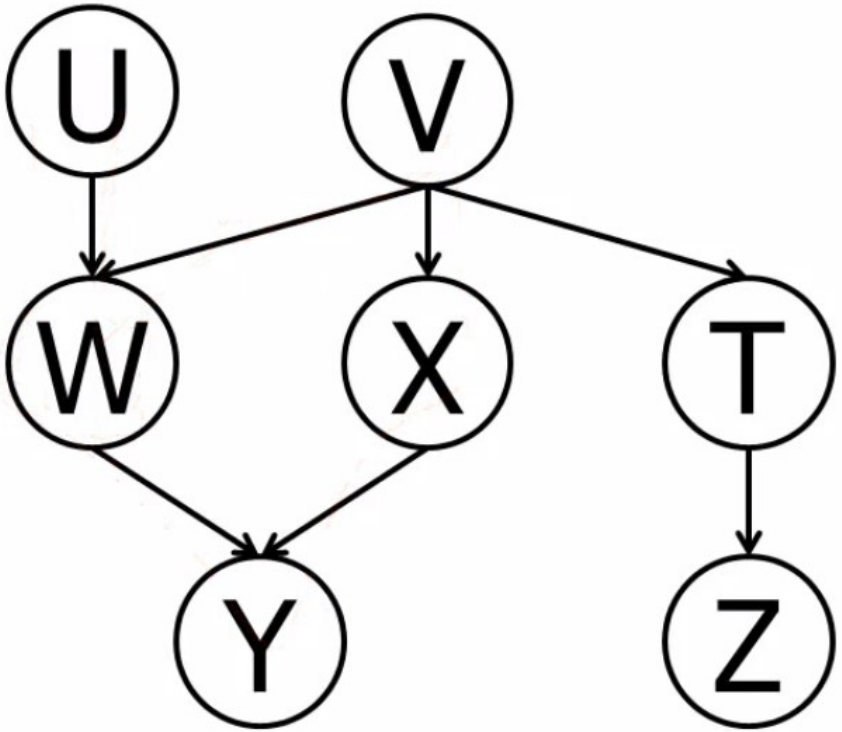
- o guaranteed to be true
- not guaranteed to be true



Example 5

U _||_ V | X

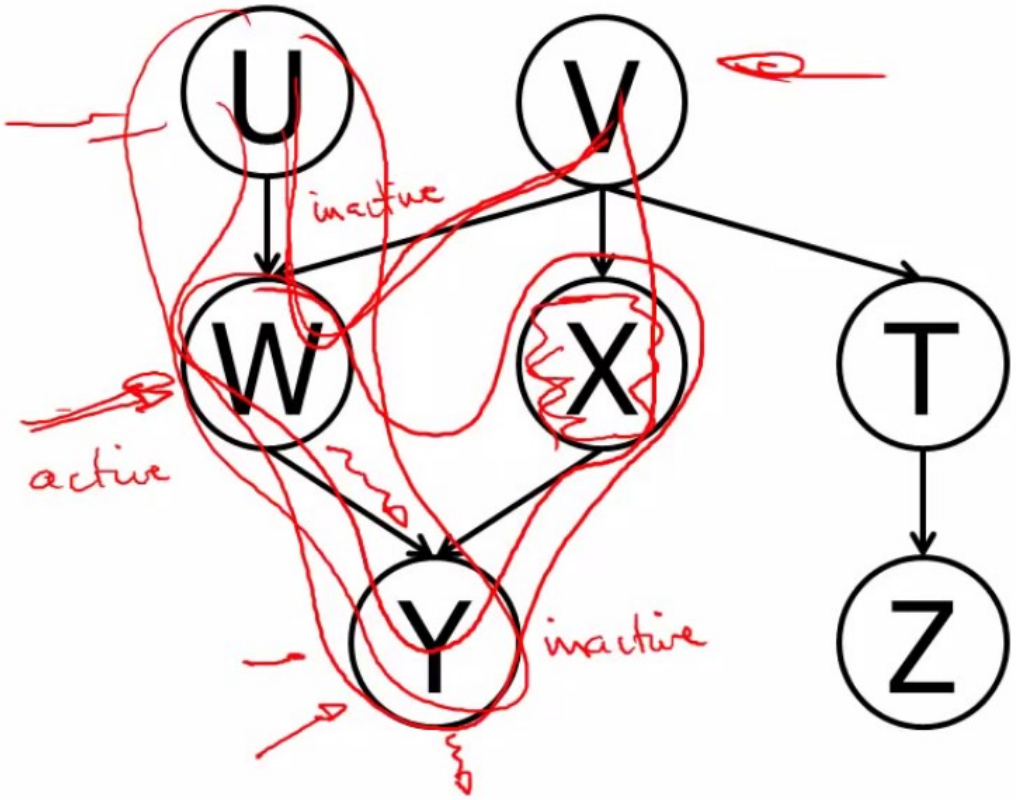
- o guaranteed to be true
- o not guaranteed to be true



Example 5

U _||_ V | X

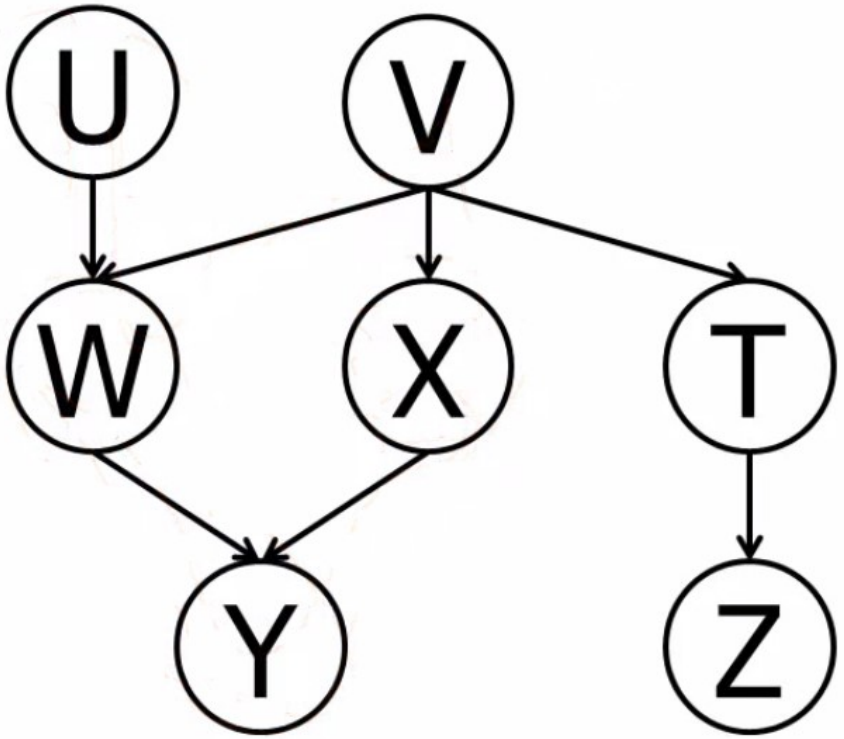
- guaranteed to be true
- not guaranteed to be true



Example 6

U _||_ V | Y

- o guaranteed to be true
- o not guaranteed to be true

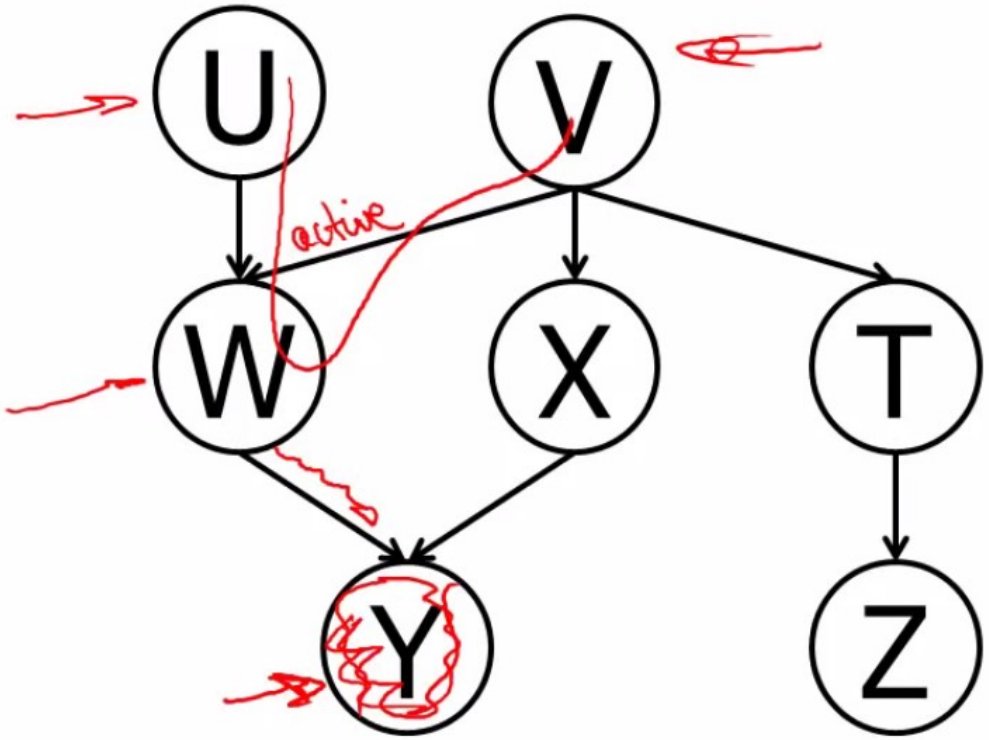


Example 6

U _||_ V | Y

o guaranteed to be true

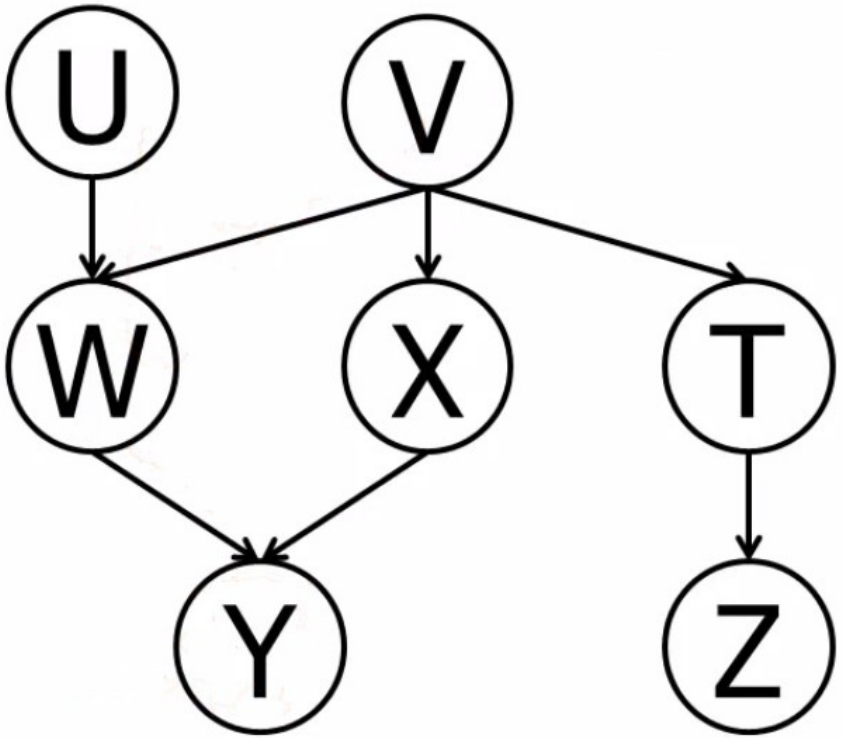
not guaranteed to be true



Example 7

U _||_ V | Z

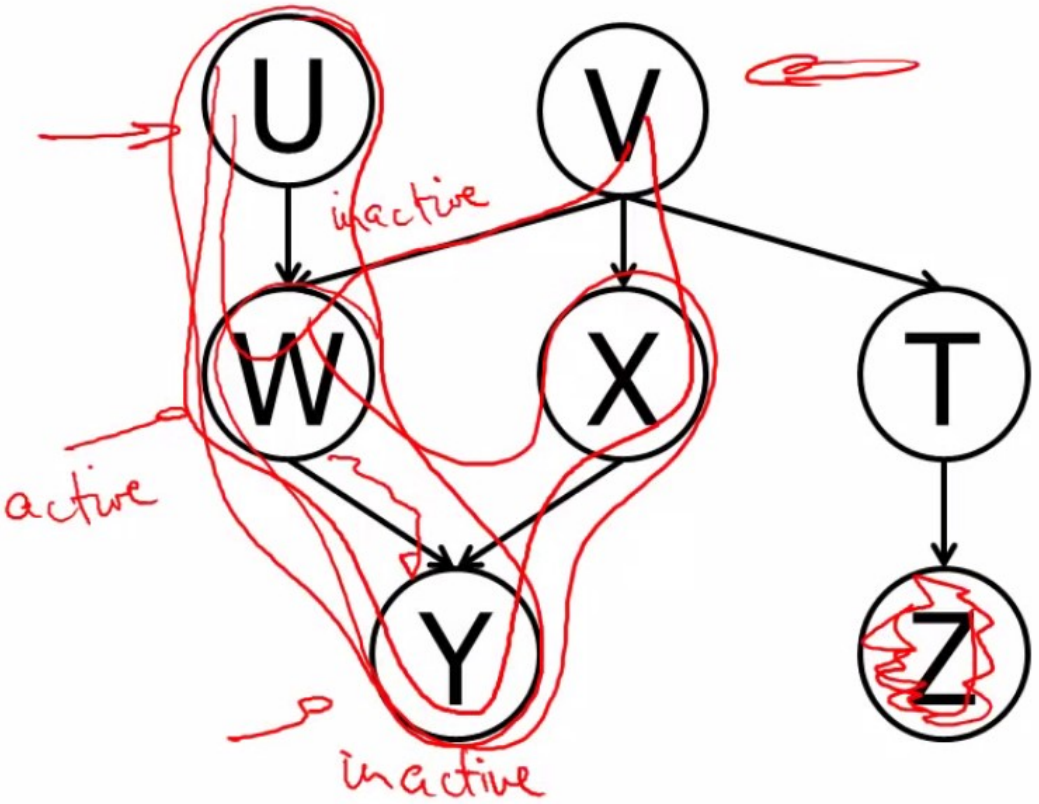
- o guaranteed to be true
- o not guaranteed to be true



Example 7

U _||_ V | Z

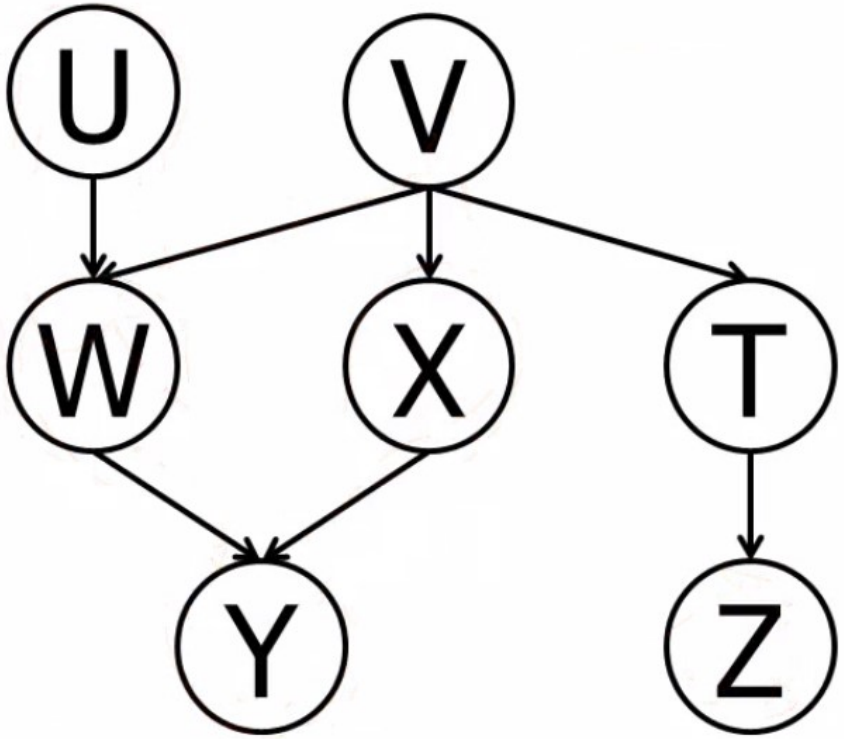
- guaranteed to be true
- not guaranteed to be true



Example 8

W _||_ X

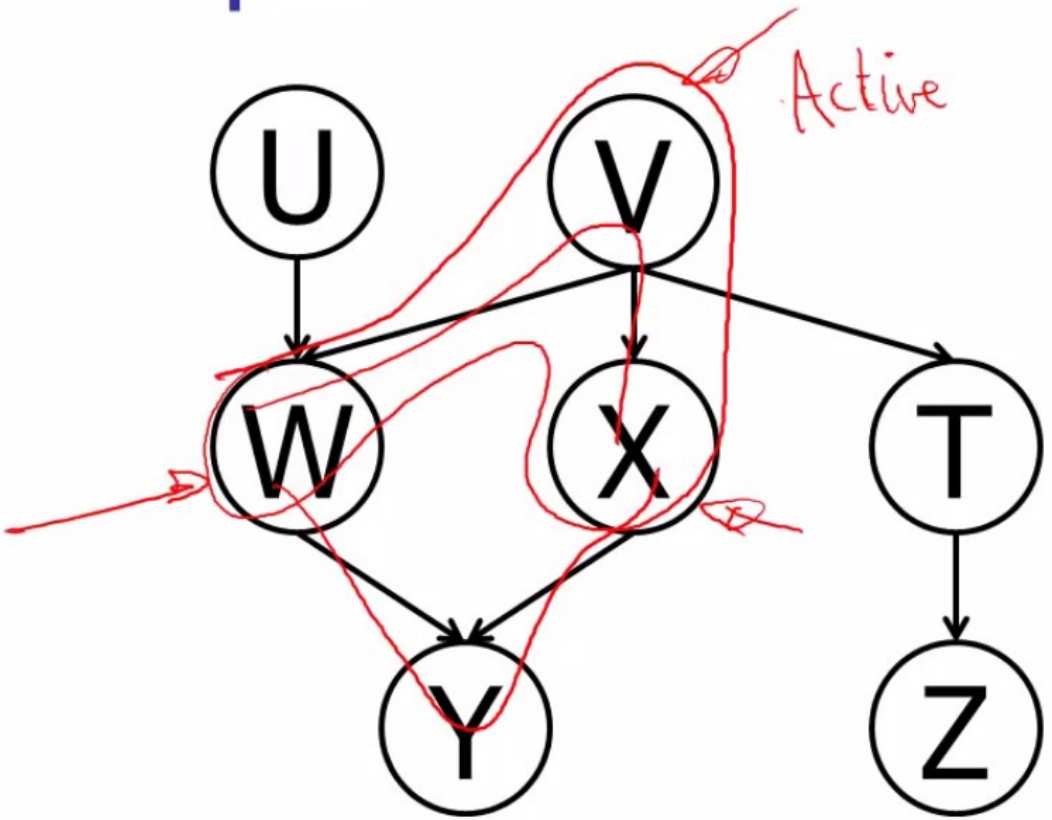
- o guaranteed to be true
- o not guaranteed to be true



Example 8

W _||_ X

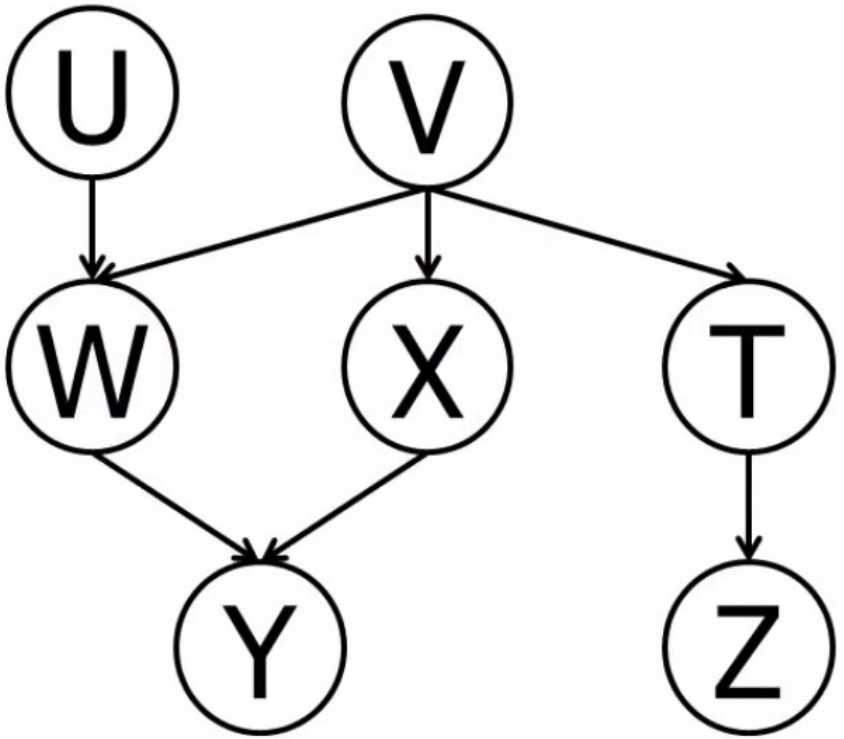
- guaranteed to be true
- not guaranteed to be true



Example 9

X _||_ T | V

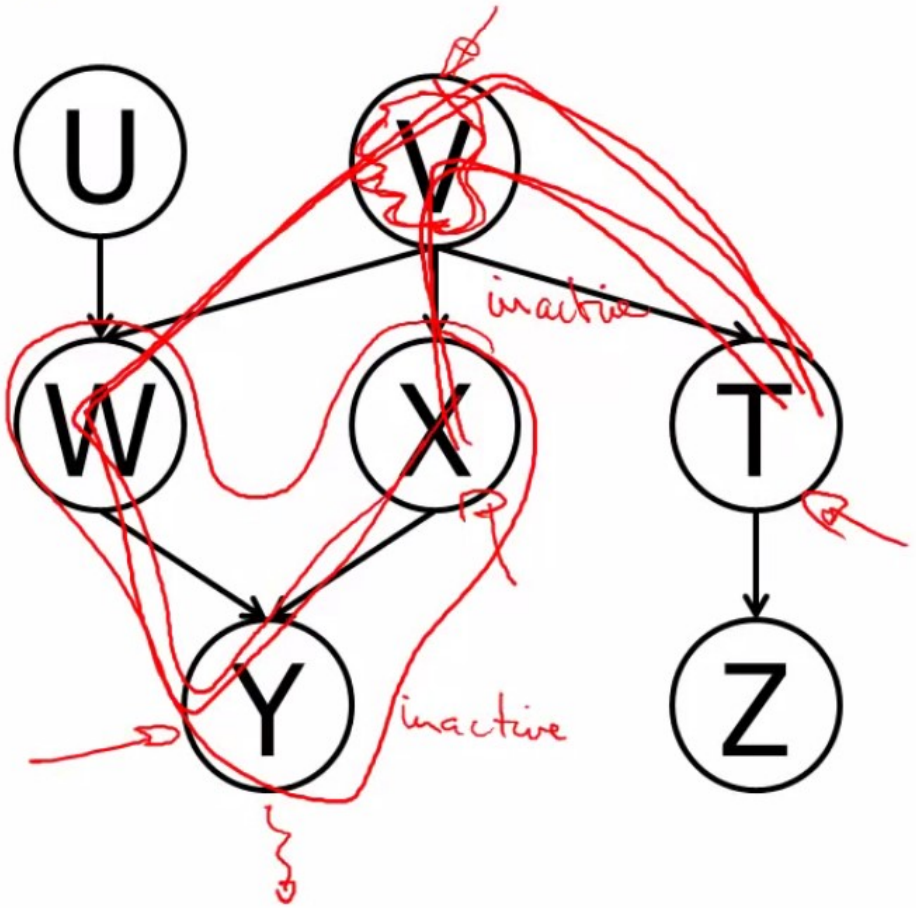
- o guaranteed to be true
- o not guaranteed to be true



Example 9

X _||_ T | V

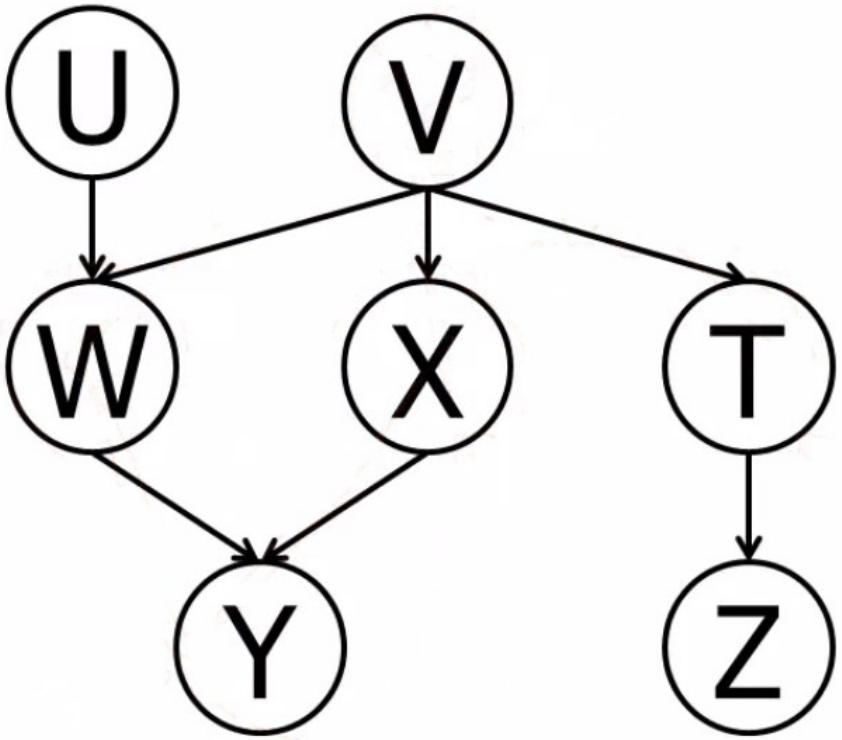
- guaranteed to be true
- not guaranteed to be true



Example 10

X _||_ W | U

- guaranteed to be true
- not guaranteed to be true

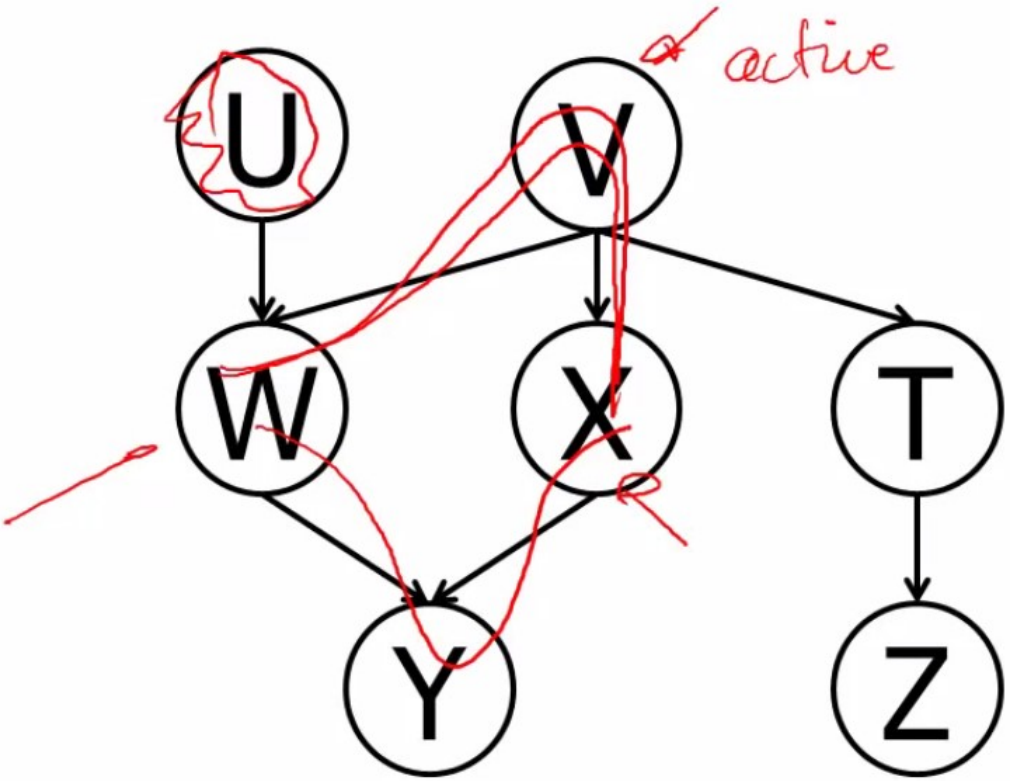


Example 10

X _||_ W | U

o guaranteed to be true

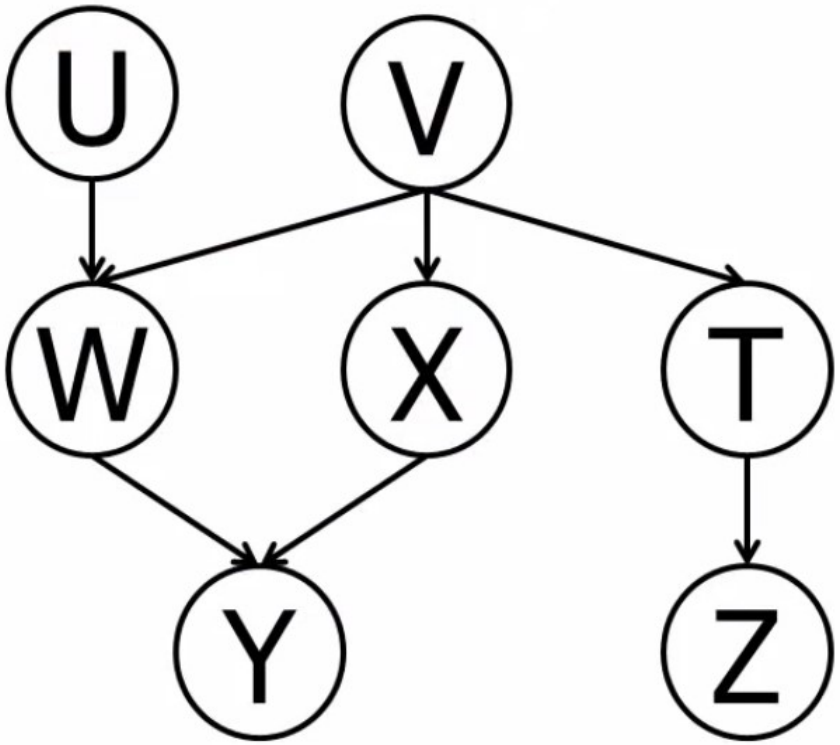
o not guaranteed to be true



Example 11

Y _||_ Z

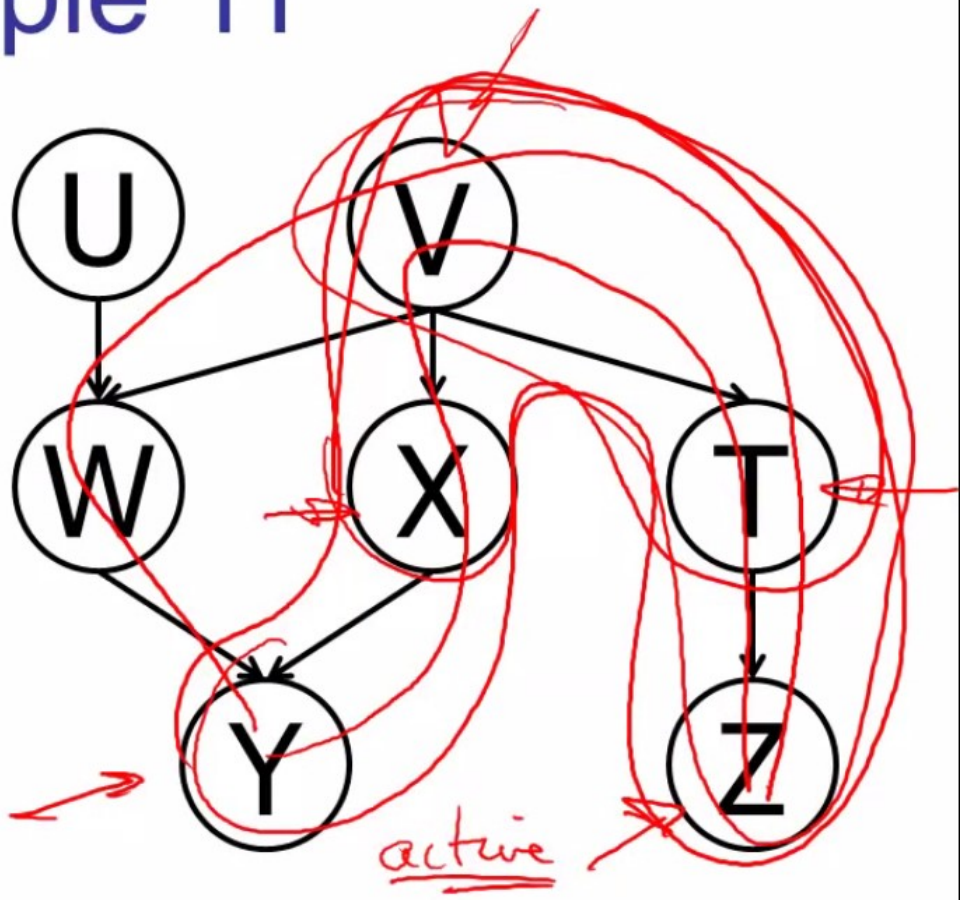
- o guaranteed to be true
- o not guaranteed to be true



Example 11

Y _||_ Z

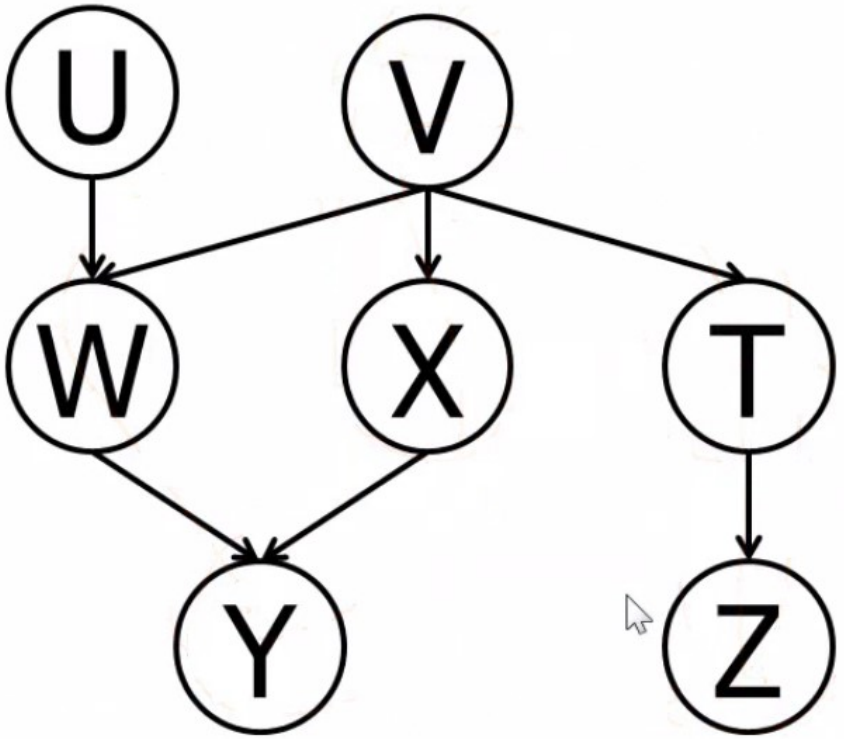
- guaranteed to be true
- not guaranteed to be true



Example 12

Y _||_ Z | T

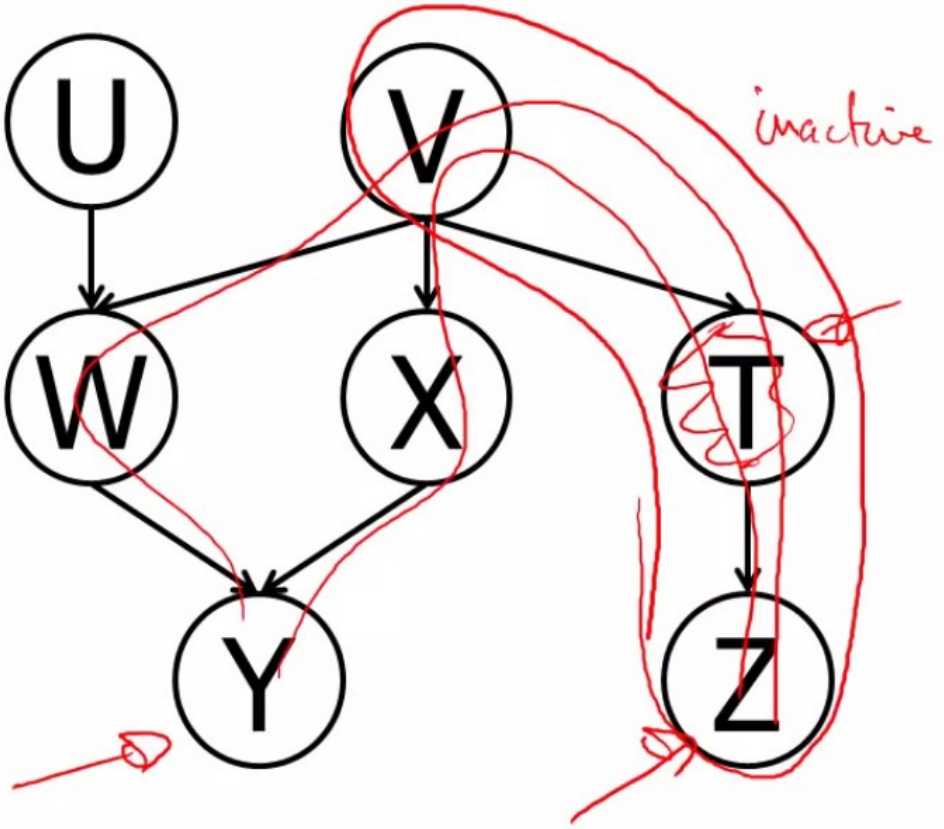
- o guaranteed to be true
- o not guaranteed to be true



Example 12

$Y \perp\!\!\!\perp Z \mid T$

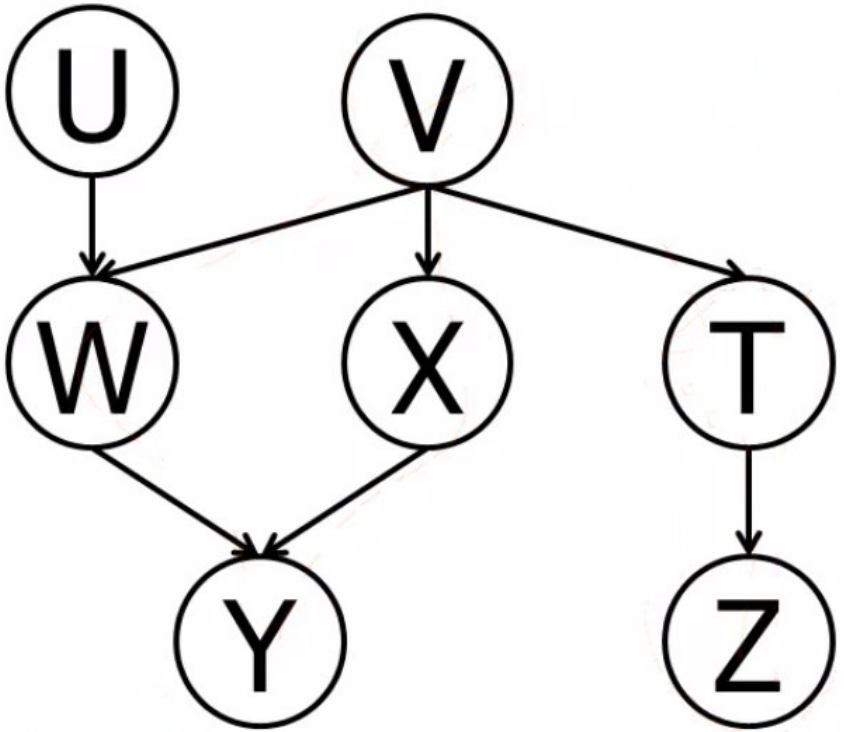
- guaranteed to be true
- not guaranteed to be true



Example 13

$Y \perp\!\!\!\perp Z \mid X$

- guaranteed to be true
- not guaranteed to be true



Example 13

Y _||_ Z | X

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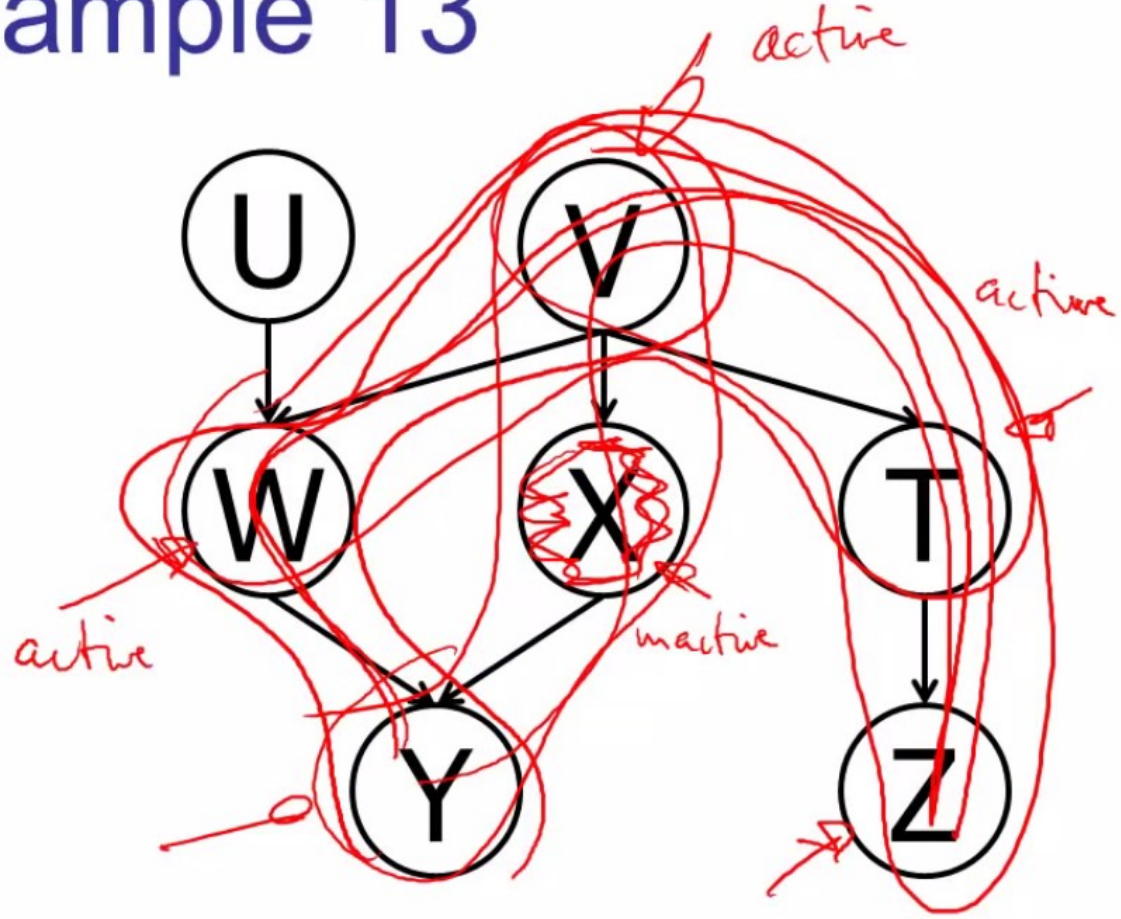
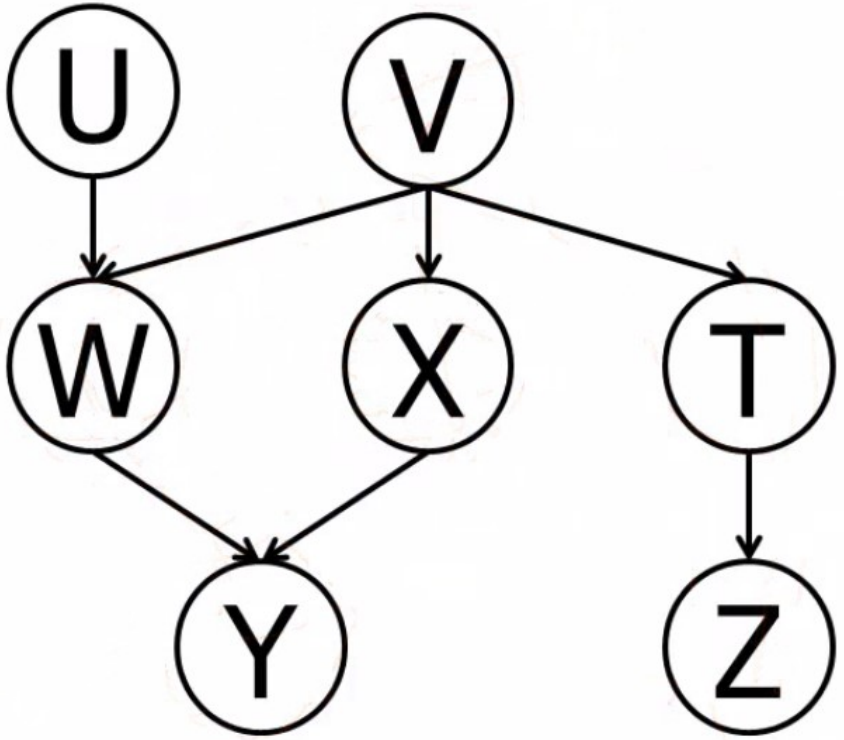


Example 14

Y _||_ Z | V

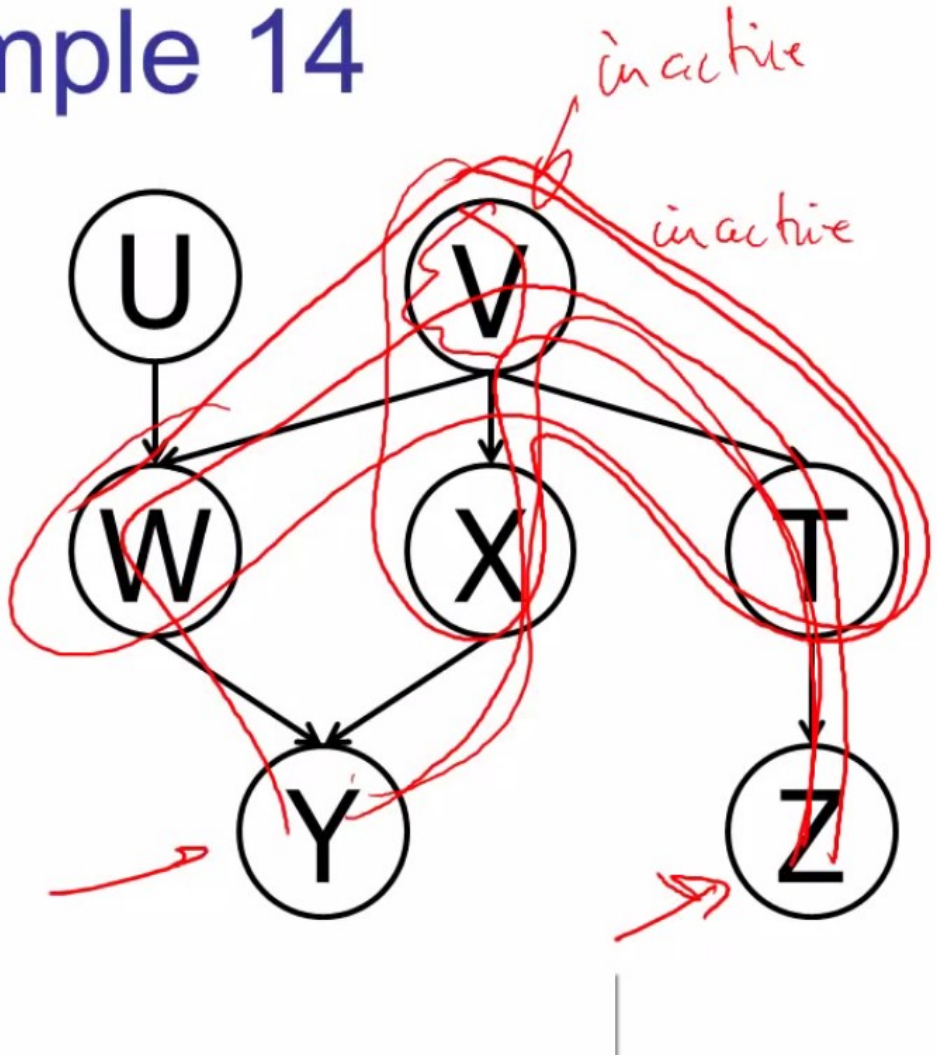
- guaranteed to be true
- not guaranteed to be true



Example 14

Y _||_ Z | V

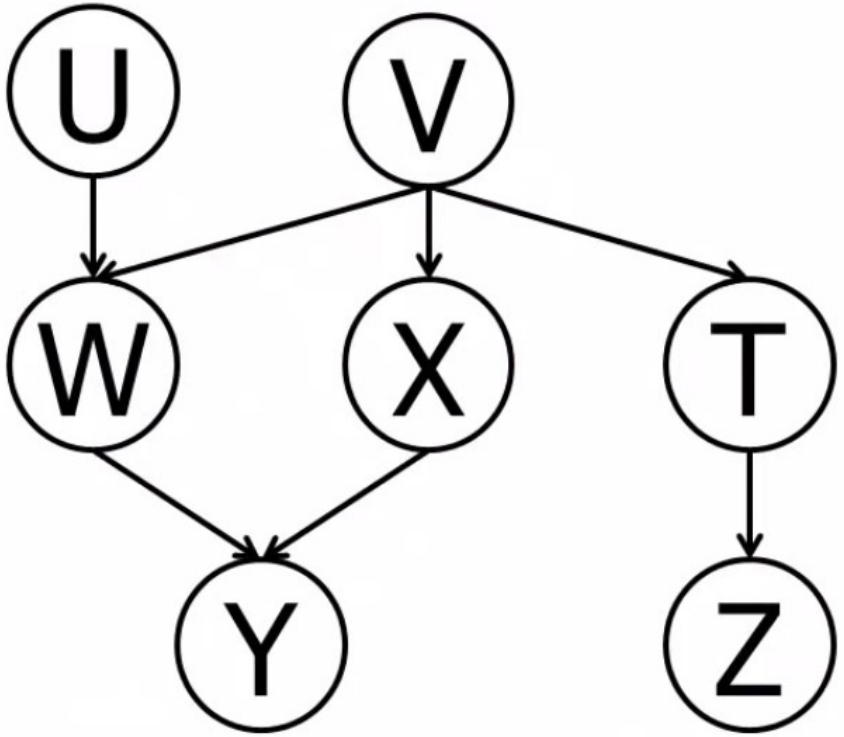
- guaranteed to be true
- not guaranteed to be true



Example 15

W _||_ Z | V

- guaranteed to be true
- not guaranteed to be true



Example 15

W _||_ Z | V

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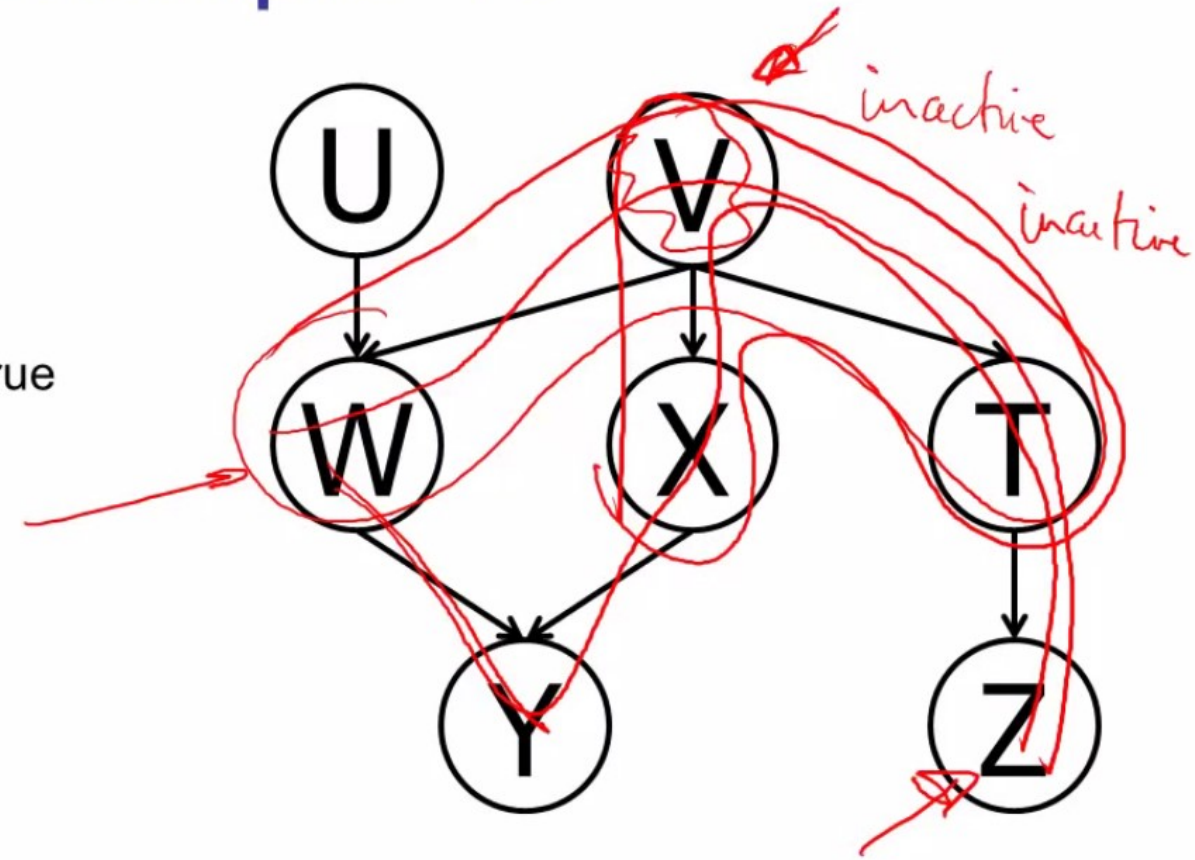
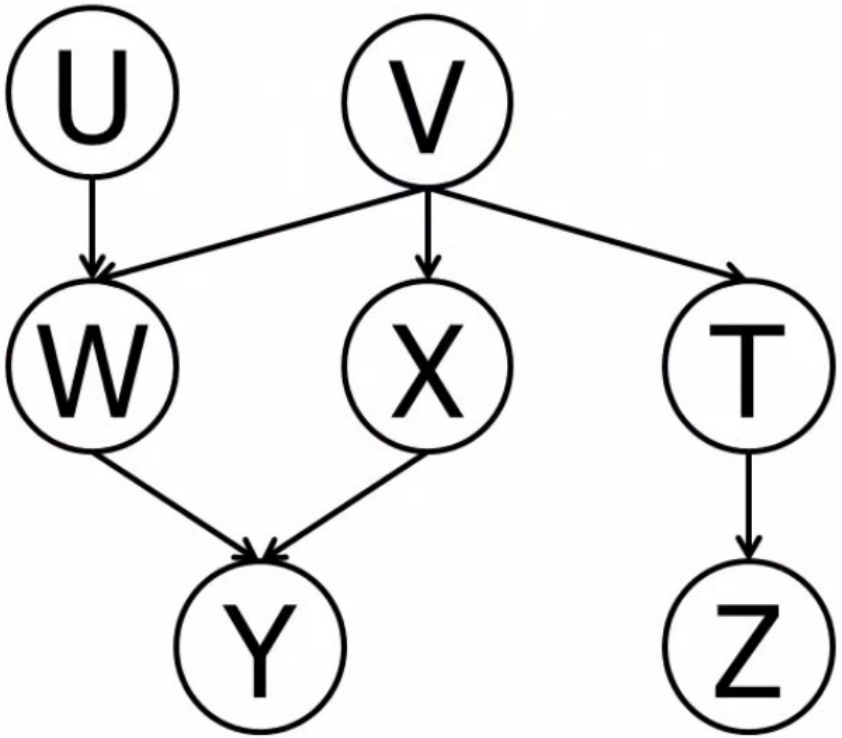


Example 16

U _||_ Z |

- guaranteed to be true
- not guaranteed to be true



Example 16

U _||_ Z |

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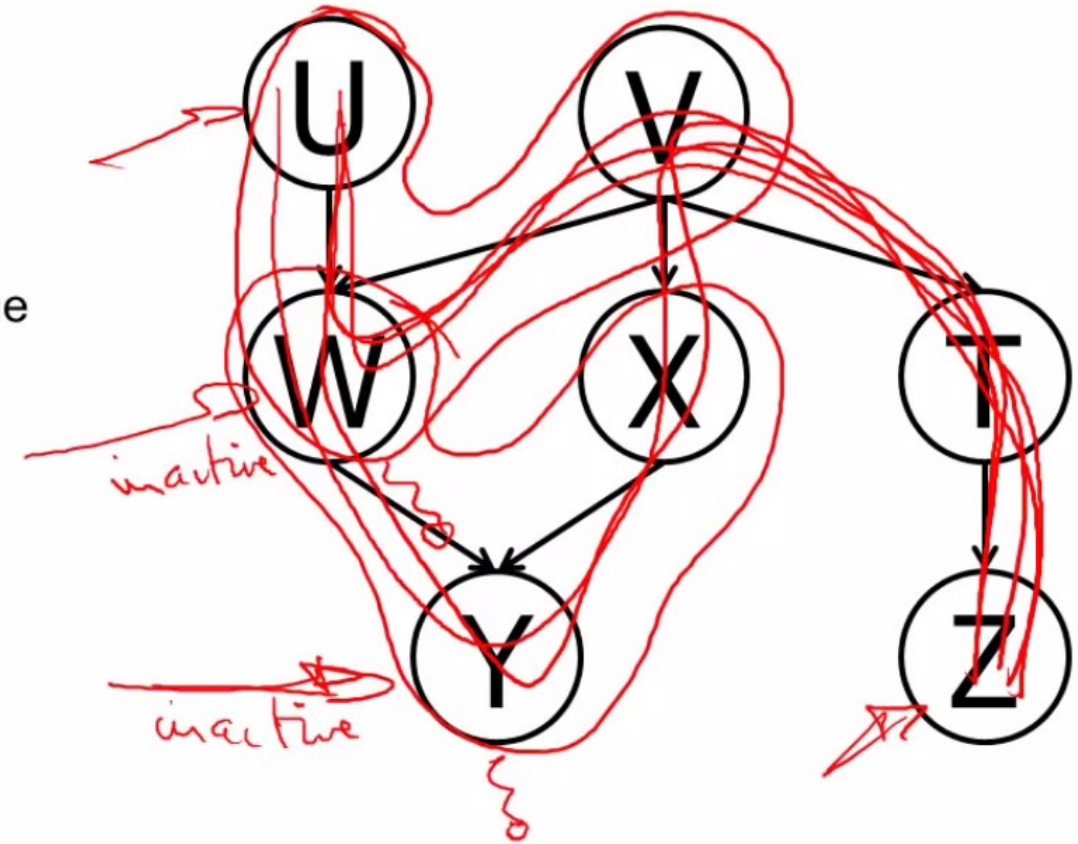
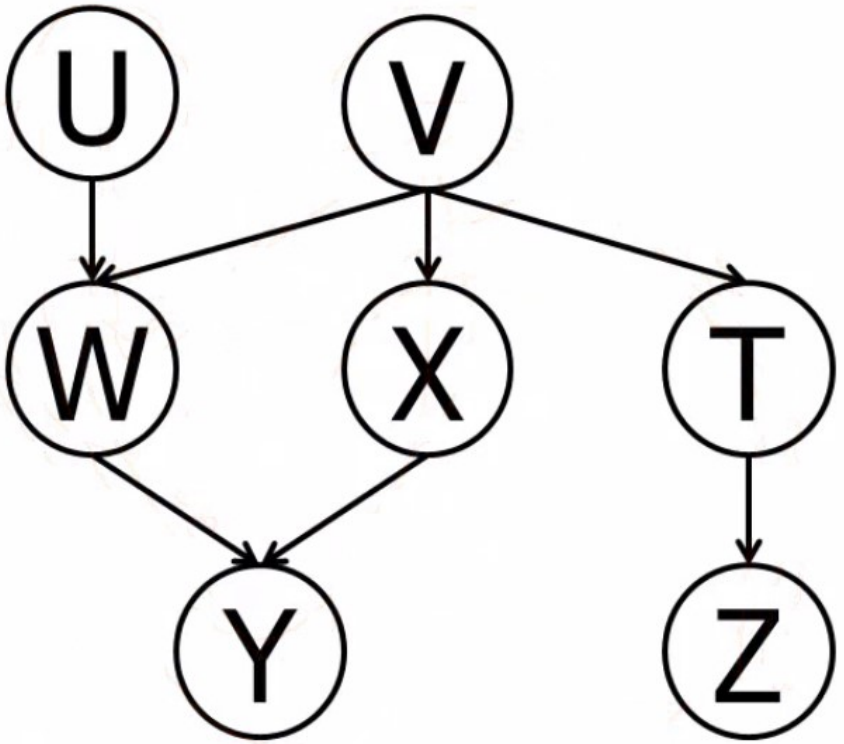


Example 17

U _||_ Z | Y

- guaranteed to be true
- not guaranteed to be true

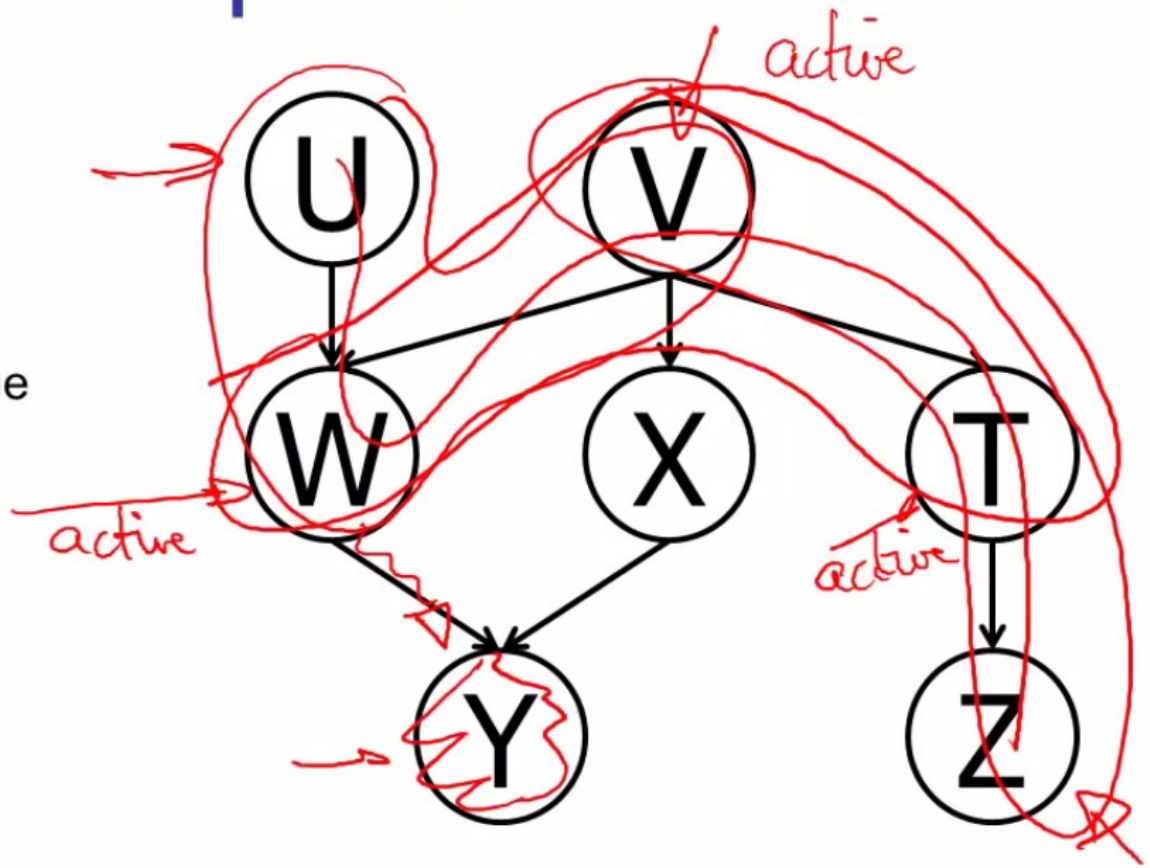


Example 17

U _||_ Z | Y

o guaranteed to be true

not guaranteed to be true



Thank You