17 Drive Systems

CONTENTS

1. HISTORY

In ancient times the Greeks and Romans used slaves to construct their buildings and works of art. In the best case these slaves had simple mechanical tools at there disposal for moving large loads. Slavery disappeared but slave like labour remained since it took many centuries before more powerful mechanical tools became available. The power became available at the end of the eighteenth century with the first steam engine (1775) of James Watt. This steam machine powered the pumps that were needed to drain the flood water from the deep shafts of the coal mines.
The steam machine was responsible for the birth of the industrial revolution. It was the first time in the history of mankind that people had such large mechanical power available to produce rotating motion at the time and place of their choice!
When electricity became the primary form of energy the electric motor replaced the steam machine. The electric motor experienced little change or development during the first half of the twentieth century up until the second world war. At that time the servo mechanisms were developed. These mechanisms enabled large powers to be controlled together with very accurate positioning.
The technical systems that produce an accurate position and (or) speed of a “load” are referred to as a drive system. Theses drive systems can refer to linear or rotational movement (translation and rotation). By means of the servo mechanisms intelligence was added to the drive system.

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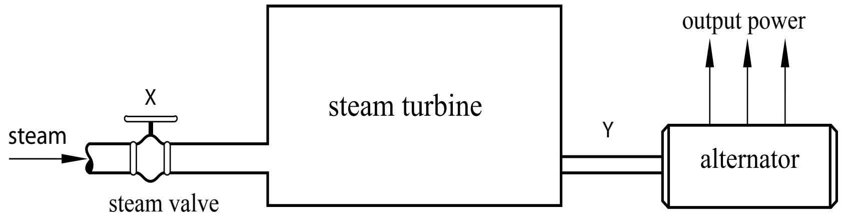
2. CONTROL THEORY

2.1 Open circuits and closed loops

Consider a power plant where a steam turbine drives an alternator. The steam feed and as a result the speed of the turbine can be controlled by a valve in the supply line.



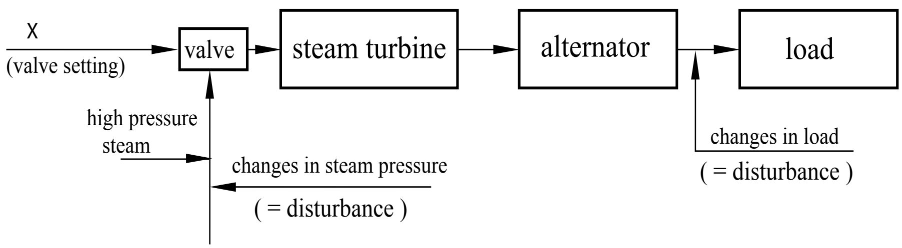


Fig. 17-1: Example of an open loop

If there is no change in the operating conditions of the whole system the position of the valve X could be calibrated as a function of the generator speed. The speed of the turbine is given by Y= k.X, whereby k is a constant that is determined by the operating conditions of the entire system. It is clear that a change in steam pressure or in the load of the alternator will result in a different turbine speed and consequently a different alternator frequency. The system shown in fig. 17-1 is unreliable if we want to maintain a constant frequency of the power grid.
Fig. 17-1 shows a block diagram of an **open loop control system**.
In order to ensure that the speed of the turbine achieves and maintains a predetermined value a configuration as shown in Fig 17-2 is used. The output speed of the turbine is measured using a tachometer. An operator reads the speed compares it with the desired value and subsequently regulates the steam flow with a hand wheel until the output speed is at the desired level. Fig. 17-2 is an example of a **closed loop control system**. The human operator functions not only as an error detector but also as a power amplifier and motor. In a servo system the operator is replaced by a pneumatic, hydraulic or electric system. One theoretical method of operating the steam valve is to use an electric motor as shown in fig. 17-3. Later in this book we will learn about practical solutions.

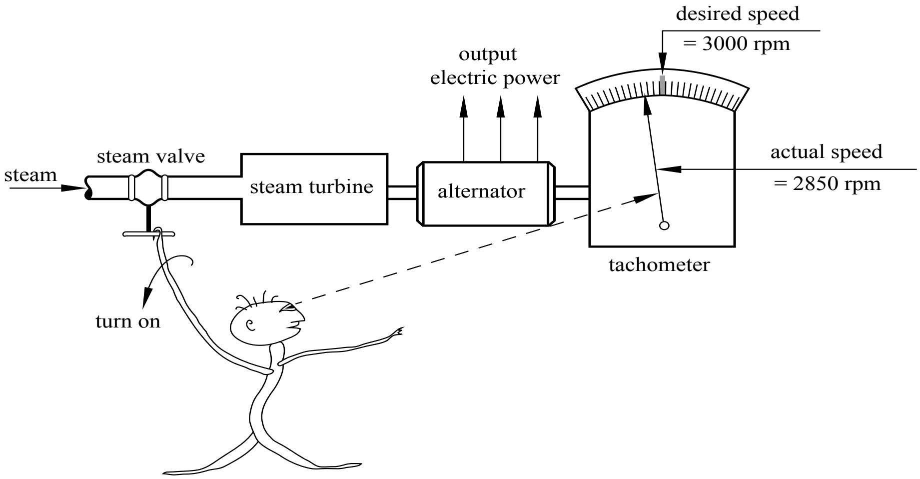


Fig. 17-2: closed loop control (via human operator)

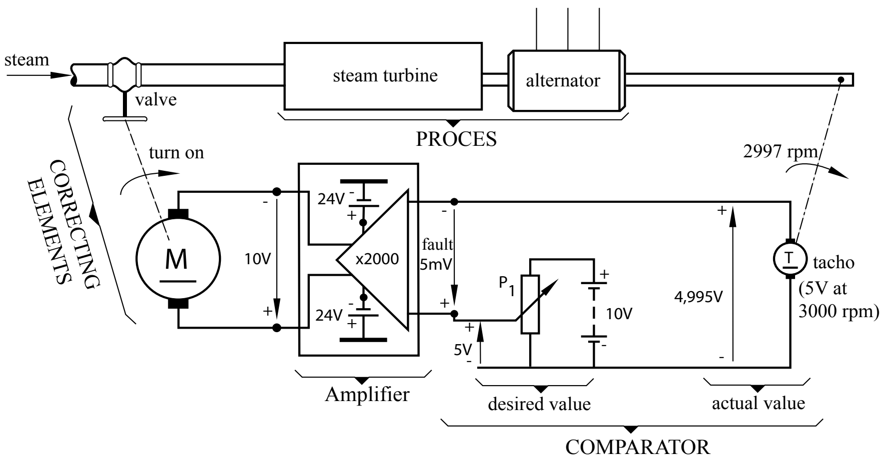


Fig. 17-3: Theoretical speed controller of a steam turbine

The speed is measured via a tacho-generator. Assume that nominal speed of the turbine should be 3000 rpm and the tacho-generator produces 5V at 3000 rpm. The potentiometer P_1 is set to 5V (set point). This set point is then compared with the tacho voltage (measured process value).
If the turbine rotates either too quick or too slow an error voltage results. The polarity of the error voltage is directly related to whether the turbine runs too quick or too slow. This determines the polarity of the armature voltage and thus the direction of rotation of the motor. We need to ensure that when the turbine speed is too high (low) the steam valve is closed (opened).
The block diagram of this closed loop controller is shown in fig. 17-4.

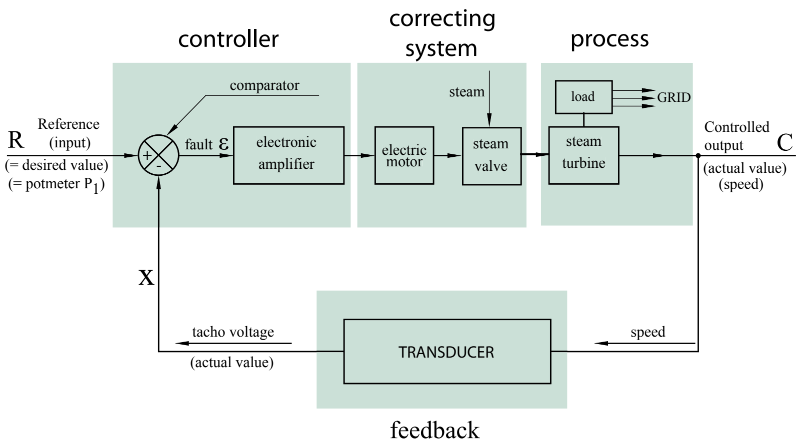
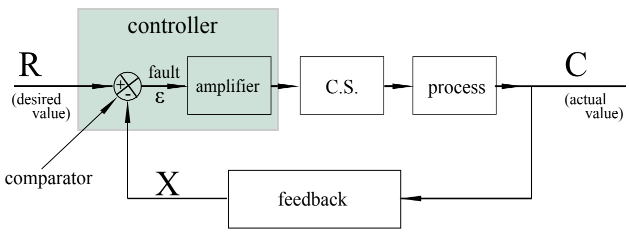
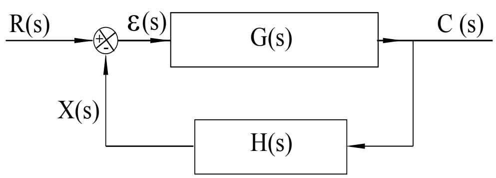


Fig 17-4: Block diagram of configuration of fig. 17-3

2.2 General block diagram of a control loop



(a) Block diagram feedback system



(b) Canonical form feedback system

Fig. 17-5: General block diagrams of a closed loop controller

In fig.17-4 a block diagram of a closed loop controller is illustrated. Based on this diagram a general block diagram for all closed loop controllers has been derived (fig. 17-5). At every instant the actual value (C) of the output value is compared with the set point value (R). Any eventual error (ε) is corrected. The feed back of the output signal to the comparator at the input is referred to as feedback. In the example considered here the output signal is subtracted from the input signal R, this is referred to as negative feedback.

In fig. 17-5b we can write (with Laplace-notations and with ∓ feedback):

$$C(s) = \varepsilon\,(s)\,.\,G(s) \;= G(s)\,.\,[\;R\,(s) \mp X(s)\;] \;=\; G(s)\,.\,[\;R(s) \;\mp\; H(s)\,.\,C(s)]$$

$$\rightarrow\rightarrow\; C(s) \pm H(s)\,.\,G(s)\,.\,C(s) = G(s)\,.\,R(s)$$

So that:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm\; H(s)\,.\,G(s)} \qquad (17\text{-}1)$$

G(s) =	*forward transfer function*	*H(s)* =	*feedback transfer function*
G(s).H(s) =	*loop gain*	*C(s)/R(s)* =	*closed loop transfer function*
ε(s)/R(s) =	*error ratio*		

For a practical application of (17-1): see fig. 19-36 and number 6.1.1 on page 19-35 !

Remark

With positive feedback the comparator produces the sum of the input and output signal. This is undesirable in a control loop. In fig. 17-3 this would mean for example that with a low turbine speed the steam valve would be closed even more. It is clear that this does not result in a well controlled turbine speed.

2.3 History of power electronics

During the second World War a group of engineers and scientists (including the mathematician Norbert Wiener) worked on a project for the US government to develop a fireline control system to combat German aircraft. Firelines are radar controlled cannons. In implementing this project the designers concluded that the heart of the problem lay in comparing what happened and what should have happened. This is the principle of feedback as described above in controlling the speed of a steam turbine. The next step in their research was confirming the similarity between a control process of machines and higher developed organisms. Wiener developed these thoughts in two important books: “Cybernetics” and “The human use of human beings”. Cybernetics was born and in the technical realm the foundation was laid for the theory of servomechanisms. Application of this theory of servomechanisms after the second World War saw the birth of industrial automation. The term cybernetics is derived from an old Greek word for “steersman”.
In solving automation problems the controlling element of the control loop often uses electrical energy. In order to control the electrical energy use was made of electronics. A new technology was born: industrial or power electronics.
From this point in time the three pillars of electrical engineering were **power engineering** for the generation and distribution of electrical energy, **electronic engineering** for processing data and signals and the third was **control or automation** working with closed loop control loops.
An important application of power electronics is the control of motors in electrical drive trains.

2.4 Energy flow in a controlled system

The arrows in the drawing of fig. 17-5 represent the flow of control energy or information and not the supply energy of the system to be controlled (not the steam energy of the turbine example).
In control system block diagrams the energy flow is not included but as an exception this will be included in fig. 17-6 to highlight in which parts of the automated system the drive energy is added.

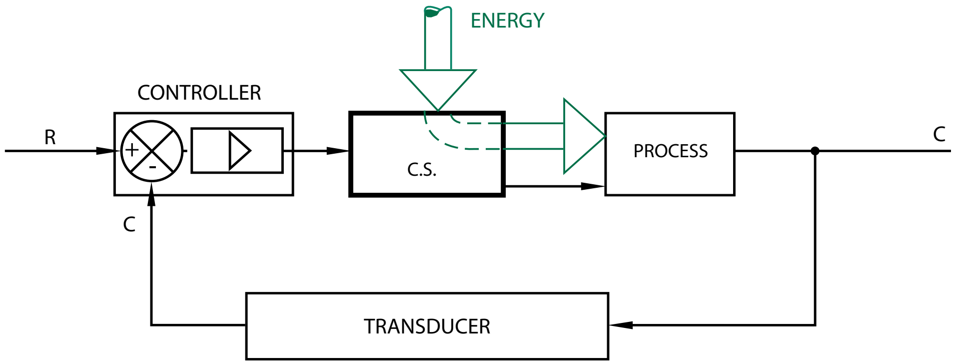


Fig.17-6: Energy flow with automatic control

The energy that flows via the corrective device to the process is not necessarily electrical but can also be hydraulic or pneumatic energy. We then refer to it as a hydraulic or pneumatic drive system respectively rather than an electrical drive system. If the controller and converter are of different physical types to the corrective device then it is referred to as a combined system. In this manner we refer to electro-hydraulic drive systems whereby the control is electrical and the corrective device is hydraulic.

3. TYPES OF DRIVE SYSTEMS

In previous paragraphs we have seen that drives are primarily either hydraulic, pneumatic or electrical. The following table gives an idea of the advantages and disadvantages of the different drive systems.

The advantage of hydraulics is the availability of large force compared to volume and weight of the drive system. Operating canal lock doors is a typical application.

Pneumatics is interesting from the perspective of safety with respect to overload (air is compressible!). The compressibility of air is then a disadvantage when it comes to accurate positioning. Pneumatics also have an advantage in explosive atmospheres. Typical applications are operating doors on trains and department stores, packing machines etc...

Due to the recent applications of micro electronics electrical drives have become more intelligent and this for a very low price. This intelligence makes electrical drives even more attractive. Applications include mechanical machines, pumps, ventilators, bridges, cranes, electrical traction, etc. The intelligent control has as a consequence been responsible for the replacement of hydraulic drives in heavy duty robots with electrical. Arguments for the use of electrical drives include:

- environmentally friendly: electrical energy is “proper”
- energy saving: the energy conversion should have a high efficiency. Of the different drive types electrical drives have the best efficiency.

- optimising the production process: industrial applications require more and more speed, accuracy and reliability. These properties are inherent in electrical systems
- user friendly: thanks to micro electronics drive systems are very user friendly
- easy control and reproduction of speed, torque, acceleration, deceleration …

Table 7-1

SYSTEM PROPERTY	DRIVE SYSTEM		
	Hydraulic	Pneumatic	Electric
• continuous control	• yes	• yes	• yes
• slow linear movement (with large force)	very good	possible	possible
• fast linear movement	possible	very good	good to very good
• rotating movement	easy to realise	easy to realise	very good
• can system operate safely in explosive atmosphere ?	possible	very good	possible
• positional accuracy	good	poor	very good
• cost price energy and installation	high	very high	low
• efficiency	poor	poor	very high
• how is the energy availability ?	very poor (one high pressure supply per installation)	good (central compressor per plant)	very good (electricity is available everywhere)
• energy density	very high	poor	poor
• safety	good	good	good
• intelligent control	moderate	moderate	very good and economical

4. ELECTRONIC DRIVE TECHNOLOGY

4.1 Generalities

In an electric drive system the mechanical energy required to move a load (position and speed) is delivered by an electric motor.
Depending on the type of current we have DC or AC motors. The available energy source is an AC or DC supply.

In order to control the energy flux from electrical power supply to motor with high efficiency, electrical energy converters are used. We distinguish four types of electrical energy converters:

- **Controlled rectifier:** controls the energy flow between AC power grid and DC consumer
- **AC controller:** controls the energy flow between AC power grid and AC consumer
- **Chopper:** allows the energy to be regulated between a DC power source and a DC consumer
- **Inverter:** able to regulate the energy between a DC source and an AC consumer.

Every one of these converters together with an electric motor will play the role of corrective device in the configuration shown in fig. 17-6.

Remarks

1. In the control and regulating of electric motors we distinguish between:
   - speed control (drive system)
   - position control (motion control)
2. According to power level we distinguish between:
   - small power motors: up to 20 kW
   - medium power motors: 20 to 100 kW
   - large power motors: above 100 kW

Motors between 10 and 500 W are extremely low power and motors above 1 MW are extremely large.

3. The following factors influence motor choice:
   1. total cost price of motor + control electronics
   2. maintenance and energy costs
   3. dynamic behaviour.

4.2 Choice of drive train

In the choice of drive train the most important properties of the driven system are:

1. *M-n* curves of the mechanical system
2. Required power on the shaft of the motor
3. Required control properties such as dynamic behaviour, control range, speed.

In addition the properties of the available energy source play a role in the chosen solution.

4.3 *M-n* curves

In drive technology a distinction is made with respect to the torque and speed. One classification can look like this:

1. Torque independent of speed:
   - applications with constant power (winding and unwinding of material, drills,…)
   - applications with constant torque (conveyer belts, lifting equipment with constant load,…)
   - torque varies quadratically with speed (fans, centrifugal pumps,…)
   - torque is proportional to speed (take off roller,…)

2. torque is a function of time: (rolling mill, CNC machines,…); CNC = computer numerical control
3. torque dependent on the angular position (metal cutters, presses,…)

Remarks

1. It is clear that the motor torque at every instant needs to be at least as large as the counter torque of the driven load. In addition some applications require a large starting torque to get moving. Examples of this are plunger pumps, mixers with material that has hardened, equipment whereby at low temperature the lubricant has a higher resistance than during normal service, etc…
   In the choice of drive train account needs to be taken of the required (large) starting torque.
2. We classify the following practical groups of load torque (M_t)
   1. $M_t = K$ = constant (rolling mills, cranes, conveyer belts, mixers, escalators, mills,…)
   2. $M_t = K.\omega$ (paper machines, screw jacks, …)
   3. $M_t = K.\omega^2$ (centrifugal pumps and ventilators)
   4. $M_t = \frac{K}{\omega}$ (wrapping machines, sanders)

   In the groups 1, 2 and 3 an electric motor with constant flux will be used. For group 4, often in part of the operating region, the motor is controlled via flux control.

3. $P = \omega \,.\, M_t$ provides the respective types of load as power:
   1. M_t = constant = K $\rightarrow$ $P = K \,.\, \omega$
   2. $M_t = K \,.\, \omega$ $\rightarrow$ $P = K \,.\, \omega^2$
   3. $M_t = K \,.\, \omega^2$ $\rightarrow$ $P = K \,.\, \omega^3$
   4. $M_t = \frac{K}{\omega}$ $\rightarrow$ P = constant (K)

4.4 Four quadrant service

If the motor speed needs to be controlled in both directions and in addition electrical braking is possible we refer to four quadrant service. Examples are:

- lifting equipment (lifting and lowering with a crane, draw-bridges, elevators,…)
- loads with a large moment of inertia where rundown without braking would take too much time (centrifuges,…)
- rolling mills (also have a large moment of inertia): in addition it can occur that the material to be rolled will have to be rolled several times backwards and forwards resulting in fast braking being required
- metal working machines where time loss (production!) will not allow to switch to slower speed.

4.5 Types of electric motor used with electronic drives

Until 1975 DC motors were used almost exclusively for speed control. Due to the development of the microcontroller, digital signal processor (DSP) and the IGBT, speed control of asynchronous and synchronous motors is now possible for a large range of applications. These days an AC drive is the normal solution. Since the existing DC drives will not be put onto the scrapheap any time soon it is advisable to study speed control of both DC and AC motors. In addition the knowledge of DC drives is useful in understanding vector control of AC motors.

The term DC or AC refers to the type of voltage to which the motor may be directly connected. In addition to the large power synchronous motor (MW!) many small synchronous motors are used. Specifically this includes stepper motors, switched reluctance motors, the brushless DC motor and the AC servo motor, which are discussed in chapters 21 and 22. These motors operate from a DC supply with the inclusion of an electronic converter. We do not call them DC motors since they cannot operate directly from a DC supply. It is more accurate to categorize them with synchronous motors. Fig. 17-7 shows the motors used for speed and position control

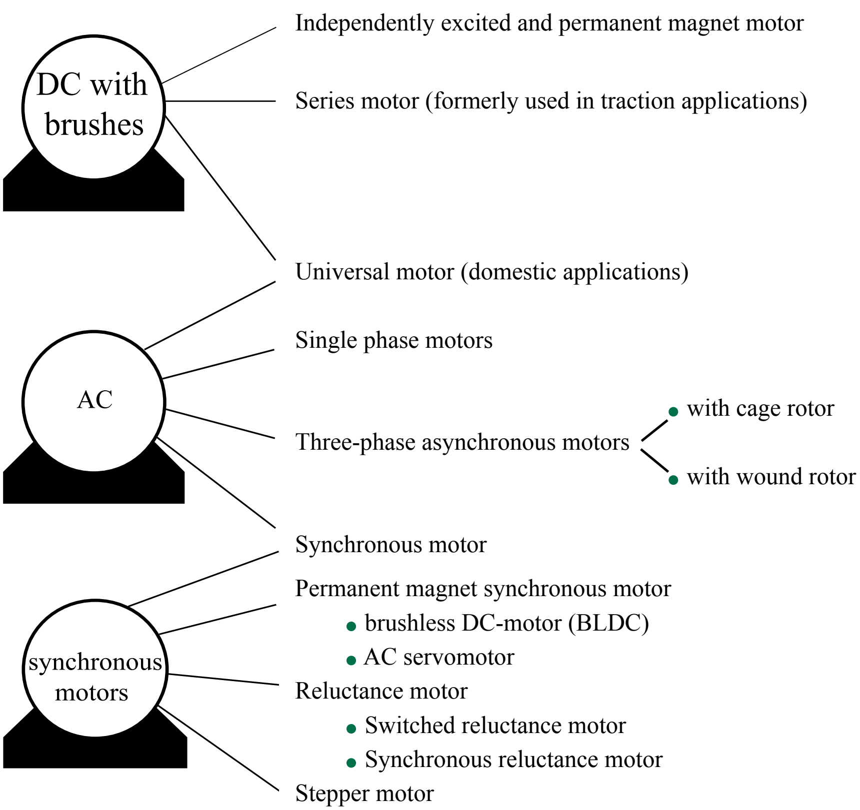


Fig. 17-7: Types of electric motors used in electronic drive systems

Table 17-2 provides an overview of the most commonly used motors and their advantages, disadvantages and typical applications.

Table 17-2

MOTOR TYPE	Advantages	Disadvantages	Typical applications
DC-MOTOR WITH BRUSHES (independently excited type and permanent magnet motor)	- can rotate extremely slowly - small torque ripple - over loadable - simple, inexpensive controllers	- speed limited by counter emf - small power density - brushes and collector (maintenance) - poor thermal performance (losses central in motor!)	- extreme heavy duty compressors and fans - automotive (seats, windows, mirrors,...)
UNIVERSAL MOTOR	- high start torque - small in comparision with single phase motor	- brushes and collector (maintenance)	- household applications (vacuum cleaner, drill,...)
THREE-PHASE ASYNCHRONOUS MOTOR (SQUIRREL CAGE)	- simple construction - good efficiency at full load - quite, smooth operation, low torque ripple - easy to use above "basic" speed	- poor efficiency with small load - poor thermal overload behavior (not suitable for servo applications) - average power density - needs vector control to be competitive	- pumps and ventilators - compressors - general industrial speed control - cheap low power industrial and domestic applications (fridges, air conditioners, washing machines, dish washers, machine tools)
BRUSHLESS DC MOTOR (BLDC)	- controllable across a large speed range - fast acceleration and deceleration - high efficiency at all speeds (if loaded) - best overload properties - quiet, smooth, extremely low torque ripple - large mechanical power density - long lasting reliability	- requires more complex control compared to a classic DC machine - requires vector control to be competitive with a DC machine - poor efficiency at low load - limited choice of application above basic speed - not suitable for AC power, requires a converter	- audio equipment for Cd's (CD-ROM player ,bar-code reader, ticket printer, etc...) - robotics - cooling - electric cars - active auto suspension - electrical servo steering - in standalone industrial drives
SWITCHED RELUCTANCE MOTOR (SRM)	- cheap motor - operates well in rough situations - high starting torque - long lasting reliability - extremely flexible - extremely high speeds	- high torque ripple at low speed - noise caused by torque ripple - the drive design has to be adjusted	- automotive - fans - domestic applications.. - cheap brush-less drive applications with large speed range - hard disk drive
SYNCHRONOUS RELUCTANCE MOTOR (syncRM)	- high efficiency (even at partial load) - efficiency better than IE4 level (IM) - short run-up time (low inertia rotor)	- requires frequency converter (FOC: with special algorithms) - low power factor	- fans and pumps - conveyers - cranes - compressors
STEPPER MOTOR	- quick angular position - accurate operation in open loop - large speed range - reliable operation - no maintenance, long lifespan	- the complete stepper motor with control card is not cheap	- robotics - coordinate tables - recorders and plotters - printers, type writers, cash registers, disc drives

4.6 Comparison table of frequency controlled and DC drive trains

Table 17-3

Property	Frequency control		DC controller
	Scalar	Closed loop vector control	
MOTOR			
. dimensions	. small		. normal
. price	. cheap		. expensive
. protection level	. Ex-protection level		. low
. maintenance	. minimal		. collector and brushes
CONVERTER			
. dimensions	. large		. small
. price	. high		. low
. "electronics"	. complicated		. extremely simple
DRIVE			
. torque control	. no	. extremely good	. extremely good
. torque at stand still	. no	. yes, limited by $I_{inv.}$	. yes, limited by brushes and commutator
. dynamic behaviour		. reasonably good	. extremely good
. constant shaft torque		. extremely good	. extremely good
. "fan" characteristic	. extremely good		. not good
. control of speed:			
. constant torque	. 1-100%		. 0-100%
. constant power	. up to 200%		. up-to 500%
. accuracy	. 2-3%	. 0.5...0.01%	. 0.5...0.01%
maximum speed (50 Hz net)			
. small power	. 6000 rpm		. 5000 rpm
. large power	. 3000 rpm		. 1000 rpm
. efficiency	. high		. extremely high
. power factor	. extremely high		. low to high
. noise	. high		. low
. EMC	. many preventative measures required		. no problems

5. USEFUL MECHANICAL FORMULAS

5.1 Basic equations

5.1.1 Force of inertia

The first basic mechanical formula is : $\Sigma \overrightarrow{F} = \int_0^m \overrightarrow{a} \,.\, dm = m \,.\, \overrightarrow{a}$ = force of inertia.

If a force f_a is applied to a mass *m,* and the opposing force is f_t as shown in fig. 17-8 , then the relationship between force and acceleration *a* is given by:

$$\Sigma \overrightarrow{F} = f_a - f_b = m \,.\, a = m.\frac{dv}{dt} = m \,.\frac{d^2x}{dt^2}$$

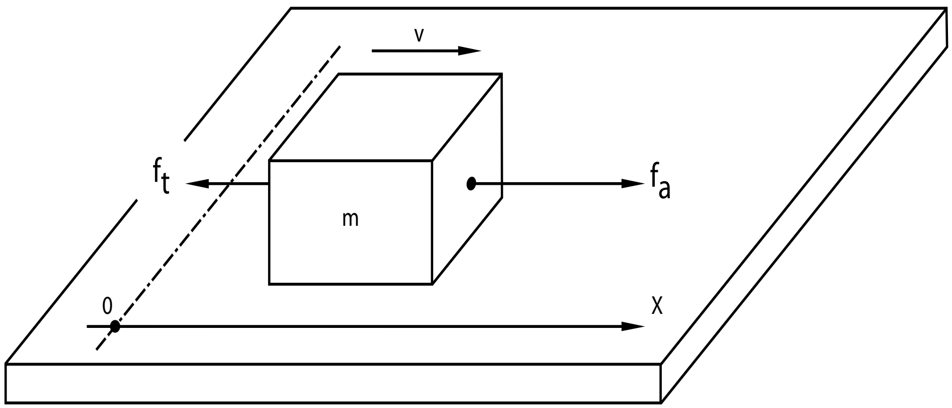


Fig. 17-8: Acceleration by translation

5.1.2 Moment of inertia

A second basic mechanical equation is that of the moment of inertia: $\overrightarrow{M} \,\Sigma F = \overrightarrow{M} \int_0^m a \,.\, dm$

In the case of rotation around the axis of rotation and with M = driving moment and M_t = opposing moment (= load moment) these two basic formulae become:

$$M - M_t = \int a \,.\, dm \,.\, r = \int dv/dt \,.\, r \,.\, dm = \frac{d\omega}{dt} \int r^2 \,.\, dm$$

The term $\int r^2 \,.\, dm$ is in similar fashion to the force of inertia of a translation called the

moment of inertia, so that: $J = \int r^2 .\, dm$

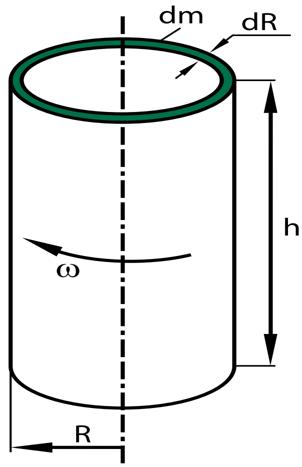
5.1.3 Moment of inertia of cylinders

1. Full cylinder

When the mass is distributed in a homogeneous cylinder (fig. 17-9) with radius R, height h and specific density ρ, then we can determine the moment of inertia J of this cylinder as follows.
We call dm the mass of an infinitely thin tube with thickness dR and height h and we find:
$dm = \rho \,.\, 2 \,.\, \pi \,.\, R \,.\, h \,.\, dR$.
The moment of inertia of the entire cylinder is given by:

$$J = \int_0^R R^2 \,.\, dm = \int_0^R R^2 \,.\, \rho \,.\, 2 \,.\, \pi \,.\, h \,.\, R \,.\, dR = \rho \,.\, 2 \,.\, \pi \,.\, h \,.\, \int_0^R R^3 \,.\, dR$$

$$= \rho \,.\, 2 \,.\, \pi \,.\, h \,.\, \frac{R^4}{4} = \rho \,.\, \pi \,.\, R^2 \,.\, h \,.\, \frac{R^2}{2} = m \cdot \frac{R^2}{2} \quad \rightarrow\rightarrow \quad \boxed{J = m \cdot \frac{R^2}{2}} \qquad (17\text{-}2)$$



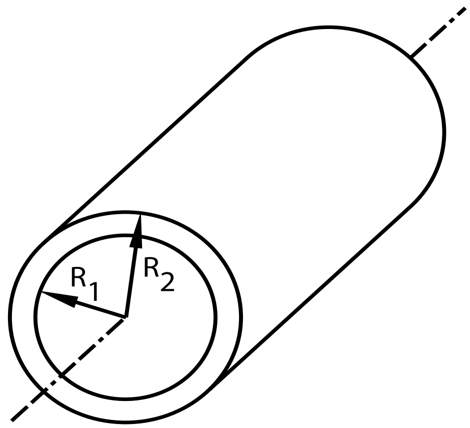


Fig. 17-10: Hollow cylinder

Fig. 17-9: Mass distributed across the homogenous cylinder

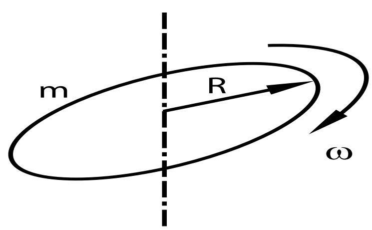


Fig. 17-11: Mass concentrated in thin ring

2. Hollow cylinder

For a hollow cylinder (fig.17-10) we find : $J = \int_{R_1}^{R_2} \rho \,.\, 2 \,.\, \pi \,.\, h \,.\, R^3 \,.\, dR = \frac{\rho \,.\, \pi \,.\, h}{2} \,.\, (R_2^4 - R_1^4)$

with the mass $m = \rho \,.\, \pi \,.\, h \,.\, (R_2^2 - R_1^2)$, gives: $\boxed{J = \frac{m}{2} \cdot (R_2^2 + R_1^2)}$ (17-3)

If the mass is concentrated in a thin ring ($R_1 = R_2 = R$) as in fig. 17-11, then (17-3) becomes: $J = m \,.\, R^2$.

5.2 Moment of inertia of several mechanisms

5.2.1 Equivalent moment of inertia

Assume that the driving force f_a is produced by an electric motor with a belt as shown in fig 17-12.

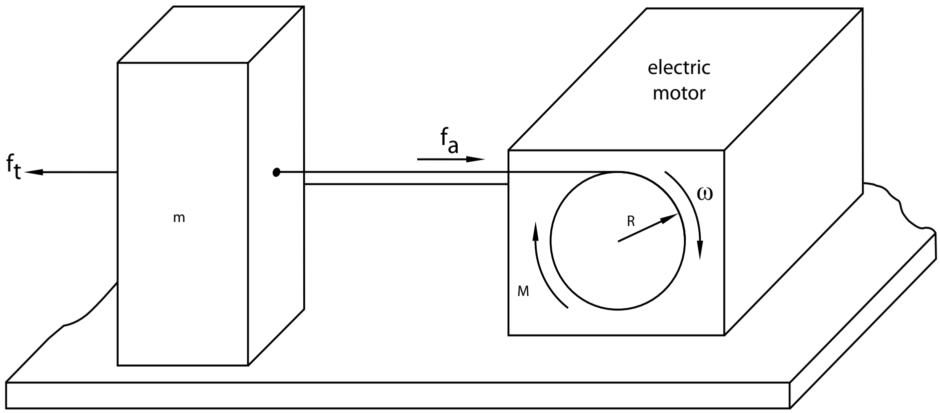


Fig. 17-12: Drive with electric motor

For fig. 17-12 we can write :

$$f_a - f_t = m \,.\, \frac{dv}{dt} \,;\; (f_a - f_t) \,.\, R = m \,.\, R \,.\, \frac{dv}{dt} = m \,.\, R \,.\, \frac{d\omega R}{dt} = m \,.\, R^2 \,.\, \frac{d\omega}{dt}$$

$$\overrightarrow{M}\ \Sigma F = M - M_t = m \,.\, R^2 \,.\, \frac{d\omega}{dt}$$

Here in: M = driving torque (Nm)
M_t = opposing torque (Nm)

From the equation it can be seen that the term $m \cdot R^2$ is present even though the mass m is not rotating. Therefore: $m \,.\, R^2 = J_{eq.}$ = the "equivalent" moment of inertia of the linearly moving mass, reflected to the shaft of the motor (kgm^2).

We find therefore: $\boxed{M - M_t = J_{eq.} \cdot \frac{d\omega}{dt}}$ (17-4)

Remarks

1. $J_{eq.} = m \,.\, R^2 = m \frac{D^2}{4}$ .Sometimes the inertia is quoted as $m \,.\, D^2$, in which case it needs to be divided by 4 to find $J_{eq.}$
2. The kinetic energy $W = \frac{1}{2} \,.\, m \,.\, v^2$ may also be written as:
$W = \frac{1}{2} \,.\, m \,.\, v^2 = \frac{1}{2} \,.\, m \,.\, R^2 \,.\, \omega^2 = \frac{1}{2} \cdot J_{eq.} \cdot \omega^2$
This expression is similar to (electrical) energy:
$W = \frac{1}{2} \cdot L \cdot i^2 \quad and \quad W = \frac{1}{2} \cdot C \cdot V^2$

5.2.2 Pulley

A load with mass m (kg) operates on the circumference of a wheel R, so that $J_{1eq.} = m_1 \,.\, R^2$ (kgm²).

The wheel R with mass m_2 (kg) also needs to be accelerated. We introduce a moment of inertia $J_2 = \frac{m_2 \,.\, R^2}{2}$ so that the equivalent moment of inertia of the pulley is given by

$$J_{eq.} = m_1 \cdot R^2 + \frac{m_2 \cdot R^2}{2} \qquad (17\text{-}5)$$

The moment equation is: $M - M_t = J_{eq.} \,.\, \frac{d\omega}{dt}$.

M is the motor torque and M_t is the load torque:

$$M_t = F \cdot R \qquad (17\text{-}6)$$

Here by: $F = m_1 \,.\, g$.

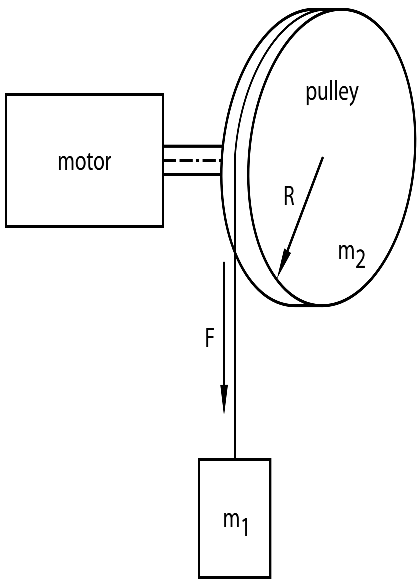


Fig. 17-13: Pulley

The equivalent moment of inertia calculated on the shaft of the machine is also valid for the setup shown in fig. 17-14 and may be expressed by expression (17-5).

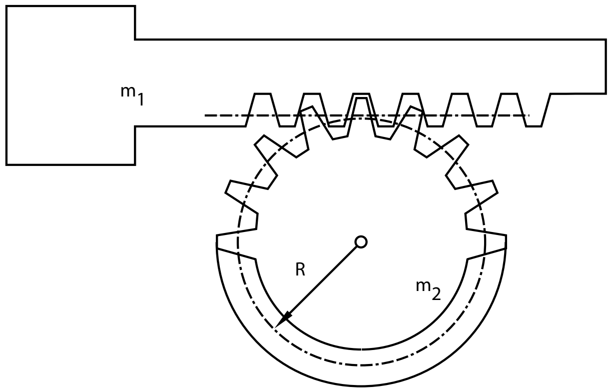


Fig. 17-14: Rack and pinion

5.2.3 Gear wheel transmission

Assume a reduction whereby gearwheel R_2 is driven by gearwheel R_1 with a torque M_a.

The driving power of gearwheel R_1 is F_1. The opposing force $F_2 = F_1 = F$.

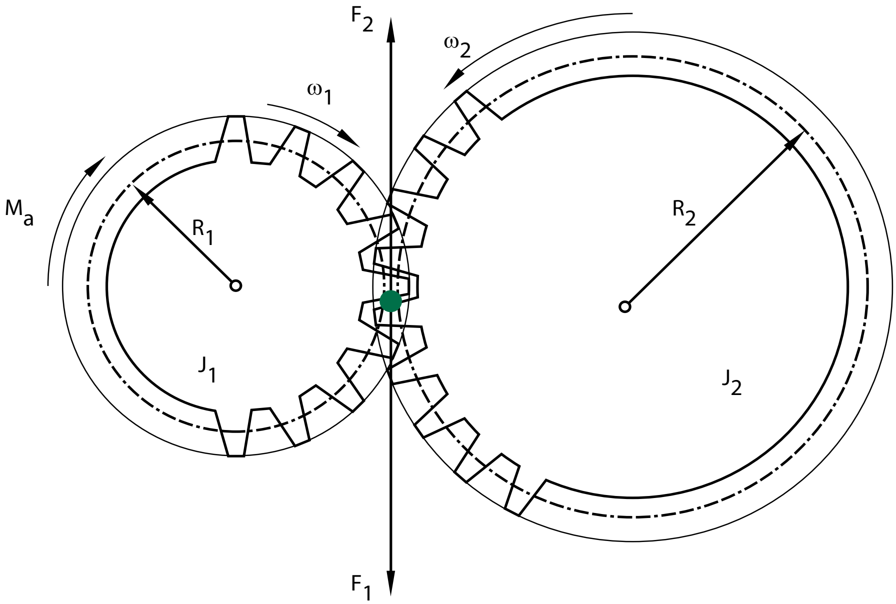


Fig. 17-15: Gearwheel reduction

Problem: Which moment is needed to bring the unloaded gearwheels up to speed? To solve this problem we separate each gearwheel and determine the rotation balance.
By neglecting every form of friction we can write:

- for the left gearwheel: $M_a - R_1 \,.\, F = J_1 \,.\, \dfrac{d\omega_1}{dt}$ or: $M_a = R_1 \,.\, F + J_1 \,.\, \dfrac{d\omega_1}{dt}$
- for the right gearwheel : $R_2 \,.\, F = J_2 \,.\, \dfrac{d\omega_2}{dt}$ then: $R_1 \,.\, F = \dfrac{R_1}{R_2} \,.\, J_2 \,.\, \dfrac{d\omega_2}{dt}$

With $r = \dfrac{R_1}{R_2}$ is $R_1 \,.\, F = r \,.\, J_2 \,.\, \dfrac{d\omega_2}{dt}$ and: $M_a = r \,.\, J_2 \,.\, \dfrac{d\omega_2}{dt} + J_1 \,.\, \dfrac{d\omega_1}{dt}$

From $\omega_2 = \omega_1 \,.\, \dfrac{R_1}{R_2} = \omega_1 \,.\, r$

it follows: $M_a = J_1 \,.\, \dfrac{d\omega_1}{dt} + r \,.\, J_2 \,.\, \dfrac{d(\omega_1 \,.\, r)}{dt} = (J_1 + r^2 \,.\, J_2) \,.\, \dfrac{d\omega_1}{dt}$

The equivalent moment of inertia reflected to the shaft of the gearwheel R_1 is:
$J_{eq.} = J_1 + r^2 \,.\, J_2$

Calculating with revolutions and a transmission ratio N:

$N = \dfrac{\text{input shaft speed}}{\text{output shaft speed}} = \dfrac{1}{r}$, then : $\boxed{J_{eq.} = J_1 + \dfrac{J_2}{N^2}}$ (17-7)

Similarly valid for a reduction (fig. 17-16): $\boxed{J_{eq.} = J_r + \dfrac{J_b}{N^2}}$ (17-8)

J_r = inertia of the reductor (kgm^2)

J_b = inertia of the load connected to the output shaft

N = transmission ratio = $\dfrac{\omega_1}{\omega_2}$

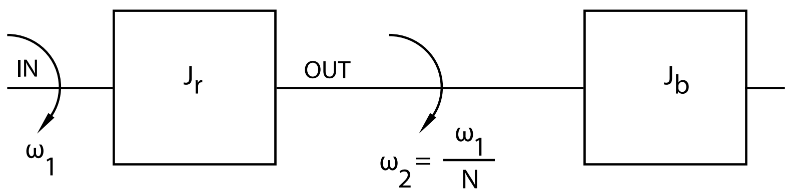
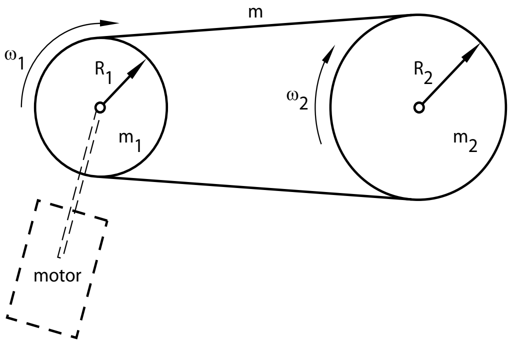


Fig. 17-16: Reduction (N) loaded with moment of inertia

5.2.4 Belt or chain transmission



$$J_{eq.} = \frac{(m_1 + m_2)}{2} \cdot R_1^2 + m \cdot R_1^2 \qquad (17\text{-}9)$$

Fig. 7-17: Belt or chain transmission

$J_{eq.}$ = total equivalent inertia on motor shaft (kgm^2) . If the pulley wheel R_2 is also loaded with inertia J_b then:

$$J_{eq.} = \frac{(m_1 + m_2)}{2} \cdot R_1^2 + m \cdot R_1^2 + \frac{J_b}{N^2} \qquad (17\text{-}10)$$

Here in N = transmission ratio

5.2.5 Worm wheel transmission

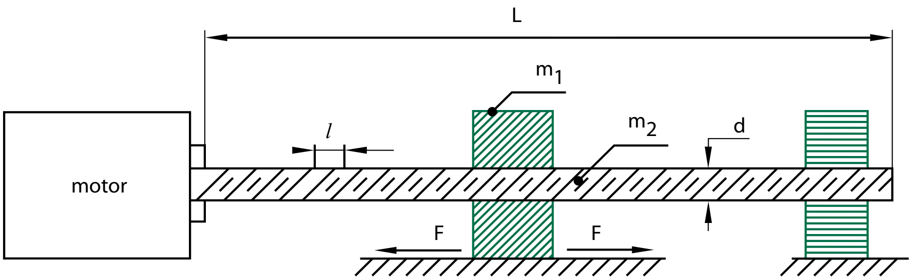


Fig. 17-18: Motor with worm wheel transmission

A rotation of one revolution ($2\,\pi\,r$) of the motor shaft corresponds to a displacement l of the load with mass m_1 , so that $J_{1eq.} = m_1 \cdot r^2 = m_1 \cdot \left[\frac{l}{2\,.\,\pi}\right]^2$. The rotation of the worm wheel with mass m_2 requires a moment of inertia $\frac{m_2 \cdot r^2}{2} = \frac{m_2 \cdot d^2}{8}$. If we express the pitch of the screw and the average diameter of the worm in mm, then the total equivalent inertia moment is:

$$J_{eq.} = \frac{m_1 \cdot l^2}{4 \cdot \pi^2} \cdot 10^{-6} + \frac{m_2 \cdot d^2}{8} \cdot 10^{-6} \quad (\text{kgm}^2) \qquad (17\text{-}11)$$

Remark

The worm and gear produce a friction torque, quantified by:

$$M_f = \frac{F \cdot l \cdot 10^{-3}}{2 \cdot \pi \cdot \eta} \quad \text{(Nm)} \qquad (17\text{-}12)$$

F = force of friction (N)
l = screw pitch (mm)
η = transmission efficiency to the load. This efficiency is between 0.3 and 0.9. A ball screw spindle can achieve an efficiency of 0.9.

5.3 Counter torque on the motor shaft

There are four groups of torques to be overcome:

a. Inertia J_m
Only has affect during a change of motor speed. The total moment of inertia consists of the sum of the moments of inertia of the rotor with the equivalent inertia of the complete load reflected to the shaft of the motor.

b. Counter torque produced by forces
In the example of the pulley (fig. 17-13) the counter torque is $M_t = F\ x\ R$.
If there is a reduction (N) between motor shaft and counter torque then the equivalent torque is :

$$M_{eq.} = \frac{M_{load}}{N} \qquad (17\text{-}13)$$

c. Counter torque as a result of viscous friction
This torque is proportional to speed. It is the result of the action of a gas or liquid on a solid object that moves in this environment. Internally in the motor we find a viscous friction between the rotor and air.

d. Counter torque as a result of dry friction
This torque is the result of movement of one object against another fixed object.
In addition to external friction the internal friction in the motor also need to be included (friction of the motor shaft in its bearings!).

Remark

As already mentioned on p. 17.9 we categorize different load torques (M_t):

1. M_t = constant (rolling mills, cranes, conveyor belts, mixers, escalators, mills,...)
2. M_t = $K.\omega$ (paper machines, screw jacks, …)
3. M_t = $K.\omega^2$ (centrifugal pumps and fans)
4. $M_t = \frac{K}{\omega}$ (wrapping machines, sanders)

6. MOMENT AND POWER OF A MOTOR

The required torque is indicated by:

$$M = M_t + J_m \cdot \frac{d\omega}{dt} \qquad (17\text{-}14)$$

M = required motor torque (Nm)

M_t = sum of all friction torques and other force torques. For example in a pulley transmission M_t = total torque produced by friction + $F\ x\ R$ (pulley)

J_m = sum of all the moments of inertia reflected to the shaft of the motor. In J_m the moment of inertia of the rotor is included. (kgm^2)

$\frac{d\omega}{dt}$ = angular acceleration motor (rad/s^2)

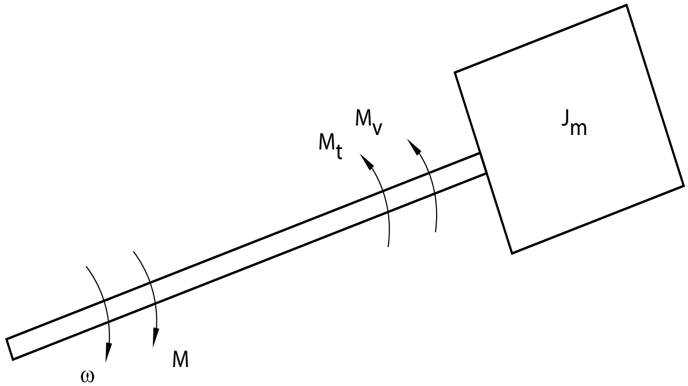


Fig. 17-19: Drive system

For fig. 17-19 we can write: $M = M_t + M_v$ with : $M_v = J_m \cdot \frac{d\omega}{dt}$

Here M_v is the acceleration or deceleration torque, depending on the sign of $\frac{d\omega}{dt}$

Multiplying (17-14) with ω produces: $\omega \cdot M = \omega \cdot M_t + \omega \cdot J_m \cdot \frac{d\omega}{dt}$

$$P_{sh.} = P_t + \omega \cdot J_m \cdot \frac{d\omega}{dt} \qquad (17\text{-}15)$$

$P_{sh.} = \omega \cdot M$ = driving power (on the shaft)

$P_t = \omega \cdot M_t$ = load power

$\omega \cdot J_m \cdot \frac{d\omega}{dt}$ = change of kinetic energy, stored in the rotating masses

7 RUN DOWN TEST TO DETERMINE THE INERTIA OF A DRIVE SYSTEM

From (17-15) follows a method to determine the inertia of a complete drive.
The input power P_i of the motor is measured at different steady state speeds ω. Since these measurements were in steady state (ω = constant) $\omega \,.\, J_m \,.\, \frac{d\omega}{dt} = 0$ and $P_{shaft} = P_t$.

If the copper losses P_{Cu} in the motor are subtracted from P_i , then $P_t = P_i - P_{Cu}$ remains and this for different values of ω. In this way the effective load torque $M_t = \frac{P_t}{\omega}$ for different motor speeds may be determined. Fig. 17-20 shows an example.

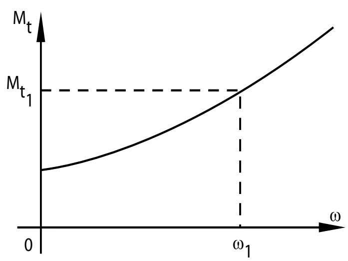


Fig. 17-20: Motor counter torque

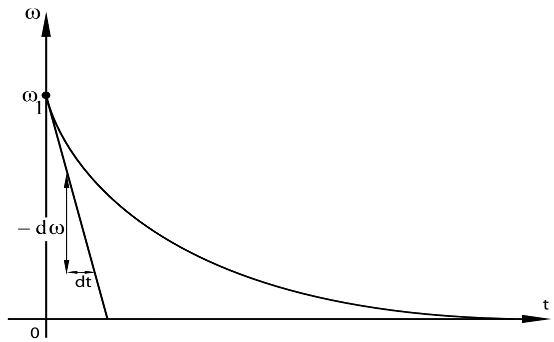


Fig. 17-21: Run down speed of motor

The motor is now accelerated to a value ω_1 with a load torque M_{t1} (fig. 17-20). Then the motor is disconnected from the electrical supply. It will run down to stand still, we note the speed as function of time. (fig. 17-21)

During run down: $0 = M_{t1} + J_m \,.\, \frac{d\omega}{dt}$ so that:

$$J_m = -\frac{M_{t1}}{[d\omega/dt\,]_{\omega_1}} \qquad (17\text{-}16)$$

If we can determine the steepness $\frac{d\omega}{dt}$ of the rundown curve, then from (17-16) the inertia of the drive can be calculated. If the ω-t curve in fig. 17-21 is a straight line, then J_m is easily determined. In the case of a non linear curve we can calculate for tangents at different points in order to determine the average value of J_m .

8. NUMERIC EXAMPLES

1. We call P_{sh} and M_{sh} the nominal values of power and moment of torque on the shaft of the motor.
   Determine the start time of a drive with the following details:
   $P_{sh} = 4$ kW ; $n = 3000$ rpm ; $M_{start} = 1.5 \cdot M_{sh}$; $M_t = 0.75 \cdot M_{sh}$; $J_{rotor} = 0.06$ kgm² ;
   $J_{eq.load} = 0.25$ kgm² .

   SOLUTION:

   Total inertia: : $J = 0.25 + 0.06 = 0.31$ kgm²

   $$\omega = \frac{2 \cdot \pi \cdot n}{60} = 314 \text{ rad/s}$$

   $$P_{sh} = M_{sh} \cdot \omega \;\rightarrow\; M_{sh} = \frac{4500}{314} = 14.32 \text{ Nm}$$

   $M_{sh} = M_t + M_v$ produces at start: $M_{start} = M_t + M_v$, so that the acceleration torque is:
   $M_v = 1.5 \cdot M_{sh} - 0.75 \cdot M_{sh} = 10.74$ Nm

   From $M_v = J \cdot \frac{d\omega}{dt}$ it follows that the start time:

   $$dt = t \;\rightarrow\; t = \frac{J \cdot d\omega}{M_v} = \frac{0.31 \cdot 314}{10.74} = 9 \text{ seconds}$$

2. Determine the nominal power of a motor, if the drive has the following properties:
   $m \,.\, D^2 = 10$ kgm² ; $\Delta n = 0$ to 1500 rpm in 4 seconds ; $M_{start} = 2 \,.\, M_{sh}$;
   $M_t = 0.6 \text{ x } M_{sh}$

   SOLUTION:
   $M_{start} = M_t + M_v = 2 \cdot M_{sh} = 0.6 \cdot M_{sh} + M_v$, so that: $M_v = 1.4 \cdot M_{sh}$

   $$M_v = J \cdot \frac{d\omega}{dt} = \frac{m \cdot D^2}{4} \cdot \frac{d\omega}{dt} = \frac{10}{4} \cdot \frac{2 \cdot \pi \cdot 1500}{60 \cdot 4} = 98.17 \text{ Nm}$$

   $$M_{sh} = \frac{M_v}{1.4} = 70.12 \text{ Nm}$$

   $$P_{sh} = M_{sh} \cdot \omega = 70.12 \cdot \frac{2 \cdot \pi \cdot 1500}{60} = 11 \text{ kW}$$

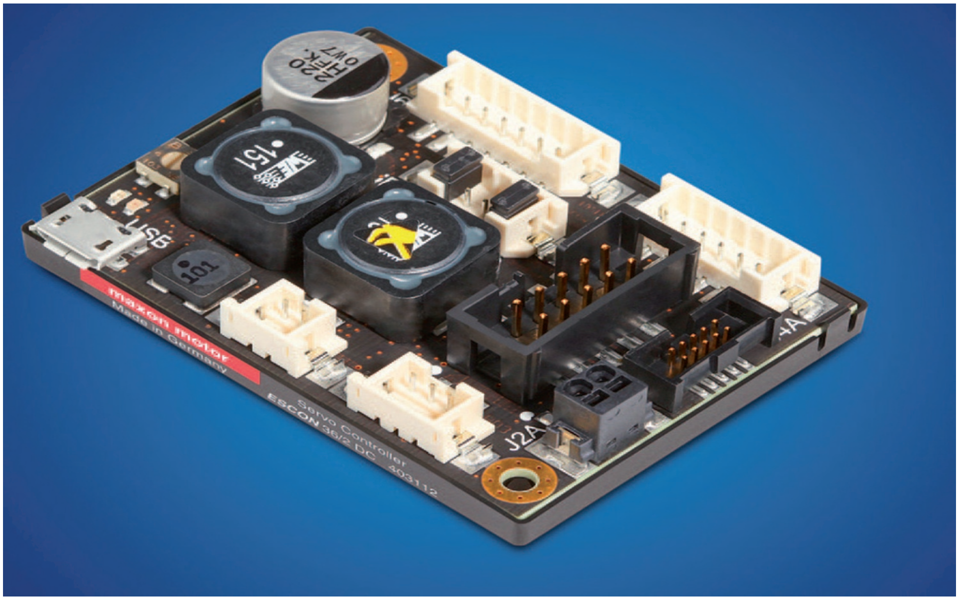


Photo Maxon Motor Benelux:

ESCON 36/2 DC: first product in the new servo controller range by maxon motor. Compact, powerful 4-quadrant PWM servo controller offers efficient control of brushed permanent-magnet DC motors up to 72 W.

Photo LEM: Isolated current and voltage measurement in the industry (this is the cover photo from LEM)