Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.1/BDH_1.1.3.pg`

Consider the differential equation

$$\frac{dP}{dt} = 0.4P\left(1 - \frac{P}{230}\right)$$

where $P(t)$ is the population at time t. (consider only $P > 0$)

(a) For what values of P is the population in equilibrium? $P =$ ___
(enter your answer as a comma-separated list)

(b) For what values of P is the population increasing? ___ $< P <$ ___.

(c) For what values of P is the population decreasing? $P >$ ___.

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.1/BDH_1.1.5.pg`

Consider the differential equation

$$\frac{dy}{dt} = y^3 - y^2 - 12y.$$

(a) For what values of y is $y(t)$ in equilibrium? $y =$ ___
(enter your answer as a comma-separated list)

(b) For what values of y is $y(t)$ increasing? ___ $< y <$ ___ and $y >$ ___.

(c) For what values of y is $y(t)$ decreasing? $y <$ ___ and ___ $< y <$ ___.

Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.1/BDH_1.1.13.pg`

Consider an elementary model of the learning process: Although human learning is an extremely complicated process, it is possible to build models of certain simple types of memorization. For example, consider a person presented with a list to be studied. The subject is given periodic quizzes to determine exactly how much of the list has been memorized. (The lists are usually things like nonsense syllables, randomly generated three-digit numbers, or entries from tables of integrals.) If we let $L(t)$ be the fraction of the list learned at time t, where $L = 0$ corresponds to knowing nothing and $L = 1$ corresponds to knowing the entire list, then we can form a simple model of this type of learning based on the assumption:
The rate $\frac{dL}{dt}$ is proportional to the fraction of the list left to be learned.

Since $L = 1$ corresponds to knowing the entire list, the model is

$$\frac{dL}{dt} = k(1 - L)$$

where k is the constant of proportionality.
For what value of L, $0 \le L \le 1$, does learning occur most rapidly?
$L =$ ___

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.1/BDH_1.1.15.pg`

Consider the following two differential equations that model two students' rates of memorizing a poem. Aly's rate is proportional to the amount to be learned with proportionality constant $k = 2$. Beth's rate is proportional to the square of the amount to be learned with proportionality constant 3. The corresponding differential equations are

$$\frac{dL_A}{dt} = 2(1 - L_A) \qquad \text{and} \qquad \frac{dL_B}{dt} = 3(1 - L_B)^2$$

where $L_A(t)$ and $LB(t)$ are the fractions of the poem learned at time t by Aly and Beth, respectively.

(a) Which student has a faster rate of learning at $t = 0$ if they both start memorizing together having never seen the poem before?
- ?
- Aly
- Beth
- They both have the same rate

(b) Which student has a faster rate of learning at $t = 0$ if they both start memorizing together having already learned one-half of the poem?
- ?
- Aly
- Beth
- They both have the same rate

(c) Which student has a faster rate of learning at $t = 0$ if they both start memorizing together having already learned one-third of the poem?
- ?
- Aly
- Beth
- They both have the same rate

Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.1/BDH_1.1.17.pg`

Suppose a species of fish in a particular lake has a population that is modeled by the logistic population model with growth rate k, carrying capacity N, and time t measured in years:

$$\frac{dP}{dt} = k(1 - P/N)P.$$

Adjust the model to account for each of the following situations.

(a) One hundred fish are harvested each year.

$$\frac{dP}{dt} = \_\_\_\_\_\_\_\_$$

(b) One-third of the fish population is harvested annually.

$$\frac{dP}{dt} = \_\_\_\_\_\_\_\_$$

(c) The number of fish harvested each year is proportional to the square root of the number of fish in the lake.

$$\frac{dP}{dt} = \_\_\_\_\_\_\_\_$$

(use a for the additional constant of proportionality)

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Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.2/BDH_1.2.1.pg`

Bob, Glen, and Paul are once again sitting around enjoying their nice, cold glasses of iced cappucino when one of their students asks them to come up with solutions to the differential equation

$$\frac{dy}{dt} = \frac{y+1}{t+1}.$$

After much discussion, Bob says $y(t) = t$, Glen says $y(t) = 2t+1$, and Paul says $y(t) = t^2 - 2$.

(a) Who is right?
- ?
- Bob
- Glen
- Paul
- Bob and Glen
- Bob and Paul
- Glen and Paul
- All three
- None of them

(b) What solution should they have seen right way? $y(t) =$ ___

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.2/BDH_1.2.3.pg`

Make up a differential equation of the form $\frac{dy}{dt} = f(t,y)$ that has $y(t) = e^{t^3}$ as a solution.
(Try to come up with one whose right-hand side $f(t,y)$ depends explicitly on both t and y.)

$\frac{dy}{dt} =$ ___

Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.2/BDH_1.2.5.pg`

Find the general solution of the differential equation

$$\frac{dy}{dt} = (ty)^2.$$

Use c for the constant appearing in your solution.

$y(t) =$ ___

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.2/BDH_1.2.7.pg`

Find the general solution of the differential equation

$$\frac{dy}{dt} = 2y+1.$$

Use k for the constant appearing in your solution.

$y(t) =$ ___

Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.2/BDH_1.2.9.pg`

Find the general solution of the differential equation

$$\frac{dy}{dt} = e^{-y}.$$

Use c for the constant appearing in your solution.

$y(t) =$ ___

Problem 6. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.2/BDH_1.2.36.pg`

Solve the initial value problem

$$\frac{dy}{dt} = \frac{1}{2y+3}, \quad y(0) = 1.$$

$y(t) =$ ___

Problem 7. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.2/BDH_1.2.17.pg`

Find the general solution of the differential equation

$$\frac{dy}{dt} = y(1-y).$$

Use k for the constant appearing in your solution.

$y(t) =$ ___
(if you need to enter multiple expressions, separate them with a comma)

Problem 8. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.2/BDH_1.2.23.pg`

Find the general solution of the differential equation

$$\frac{dw}{dt} = \frac{w}{t}.$$

Use k for the constant appearing in your solution.

$w(t) =$ ___

Problem 9. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.2/BDH_1.2.35.pg`

Solve the initial value problem

$$\frac{dy}{dt} = (y^2+1)t, \quad y(0) = 1.$$

$y(t) =$ ___

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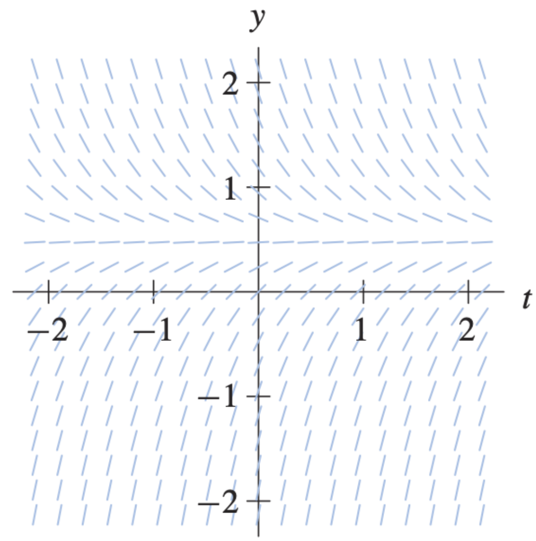
Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.3/BDH_1.3.1.pg`

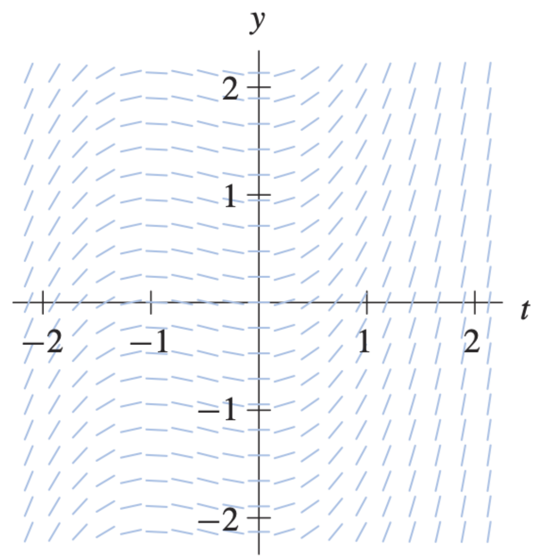
Sketch the slope field for the differential equation

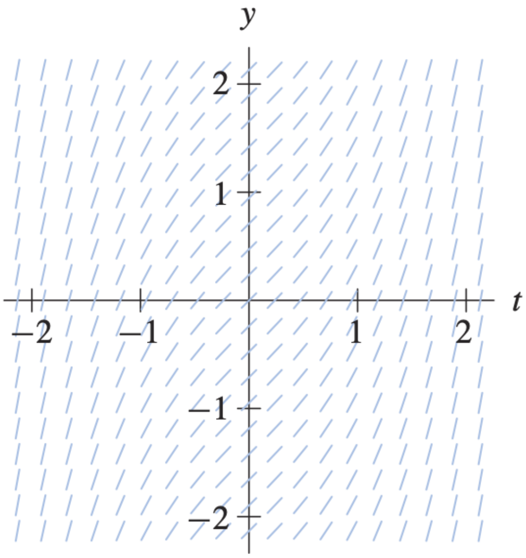
$$\frac{dy}{dt} = t^2 + t$$

as follows:
(a) Pick a few points (t, y) with both $-2 \leq t \leq 2$ and $-2 \leq y \leq 2$ and plot the associated slope marks without the use of technology.
(b) Use HPGSolver to check these individual slope marks.
(c) Make a more detailed drawing of the slope field and then use HPGSolver to confirm your answer.
Which of the following slope fields is the correct one for the differential equation? [?/a/b/c/d]



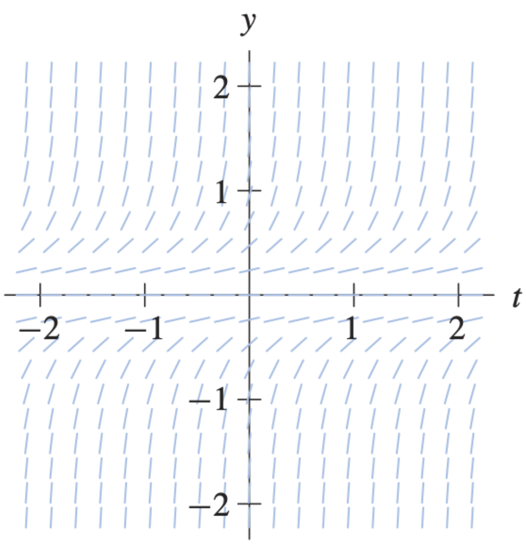
(a) (b)





(c)

(d)

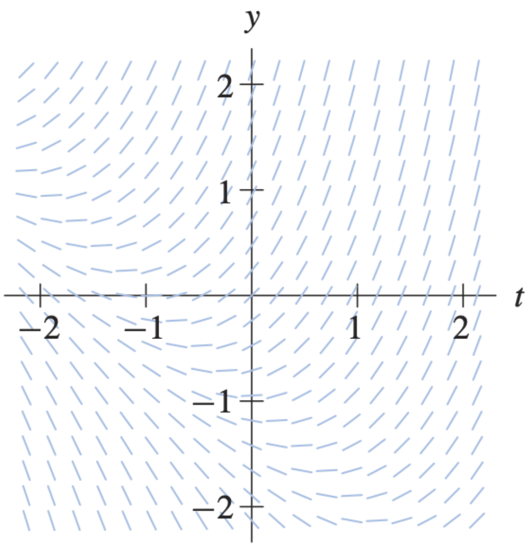


Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.3/BDH_1.3.3.pg`

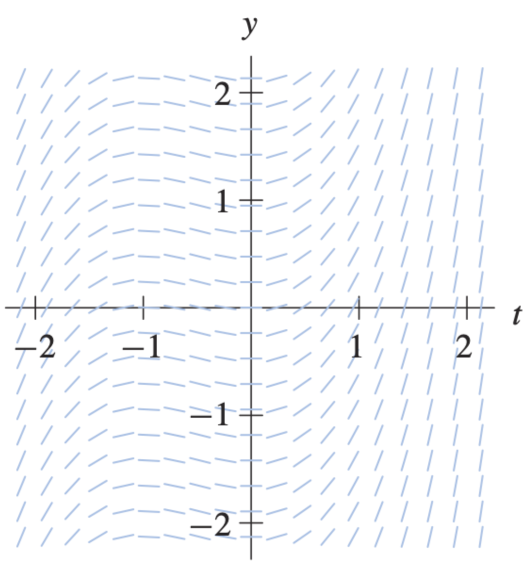
Sketch the slope field for the differential equation

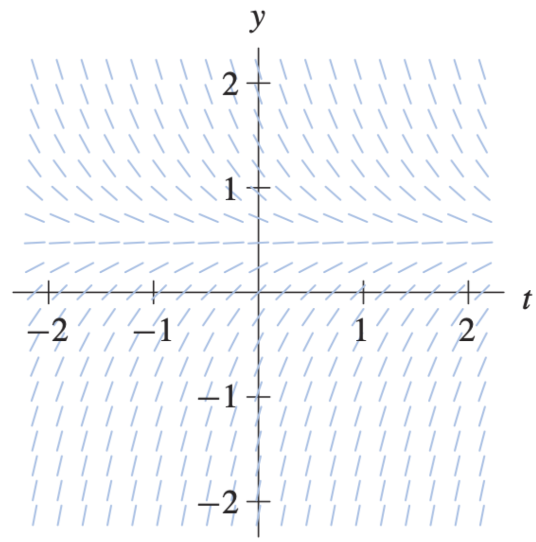
$$\frac{dy}{dt} = 1 - 2y$$

as follows:
(a) Pick a few points (t, y) with both $-2 \le t \le 2$ and $-2 \le y \le 2$ and plot the associated slope marks without the use of technology.
(b) Use HPGSolver to check these individual slope marks.
(c) Make a more detailed drawing of the slope field and then use HPGSolver to confirm your answer.
Which of the following slope fields is the correct one for the differential equation? [?/a/b/c/d]

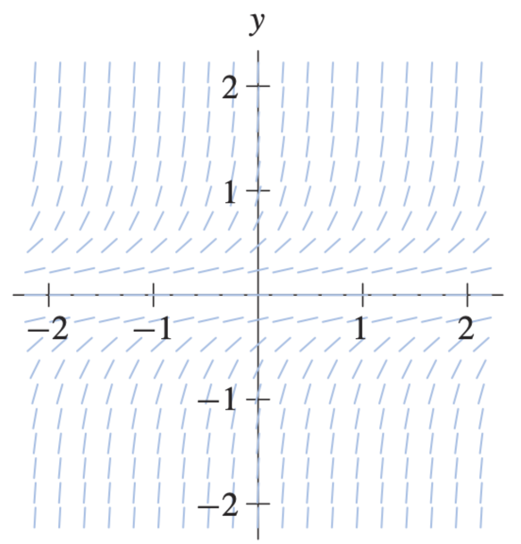


(a) (b)





(c) (d)

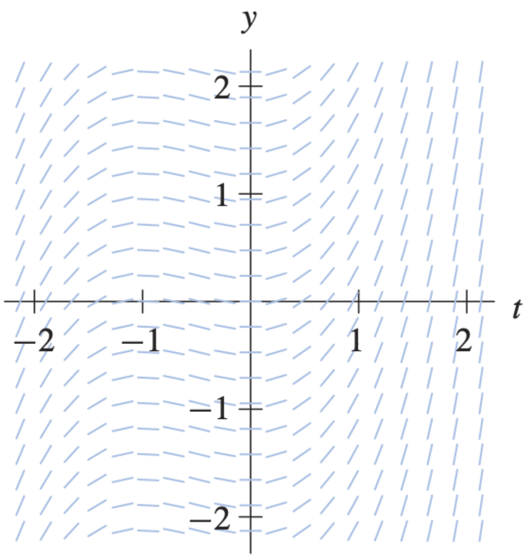


Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.3/BDH_1.3.5.pg`

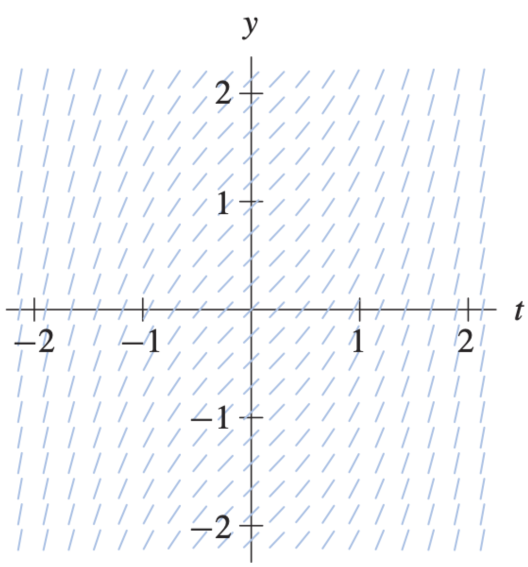
Sketch the slope field for the differential equation

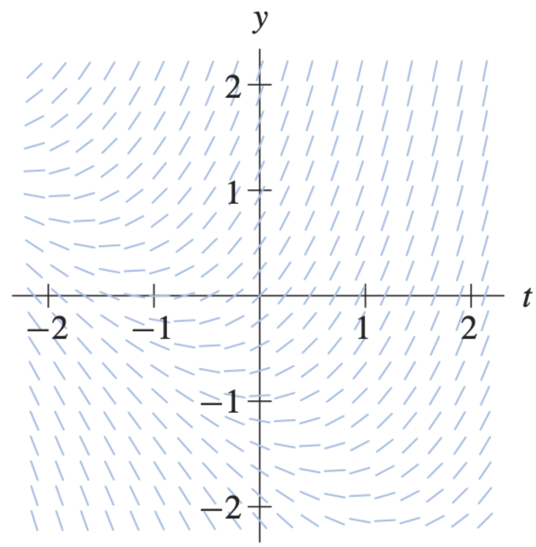
$$\frac{dy}{dt} = 2y(1-y)$$

as follows:
(a) Pick a few points (t, y) with both $-2 \le t \le 2$ and $-2 \le y \le 2$ and plot the associated slope marks without the use of technology.
(b) Use HPGSolver to check these individual slope marks.
(c) Make a more detailed drawing of the slope field and then use HPGSolver to confirm your answer.
Which of the following slope fields is the correct one for the differential equation? [?/a/b/c/d]

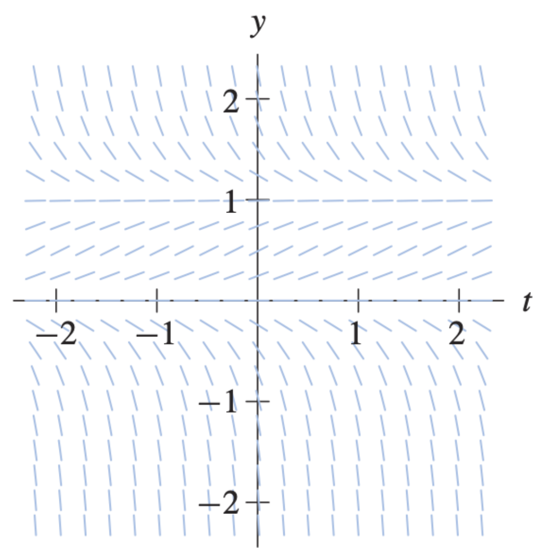


(a) (b)





(c) (d)

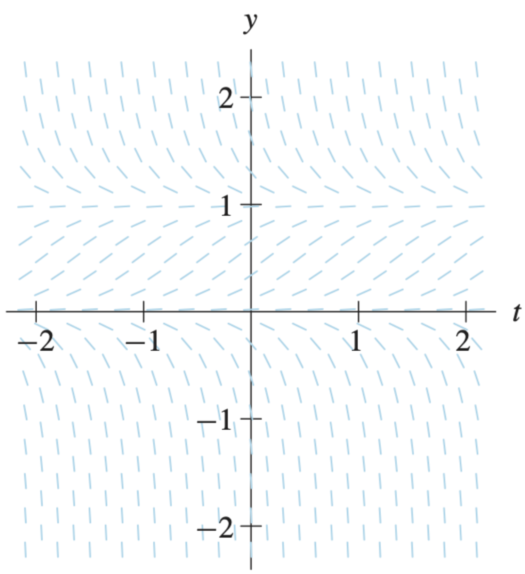


Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.3/BDH_1.3.7.pg`

A differential equation and its associated slope field are given. For each equation,
(a) sketch a number of different solutions on the slope field (do this on paper; nothing to submit online), and
(b) describe briefly the behavior of the solution with $y(0) = 1/2$ as t increases (mark correct behavior below).

You should first answer these exercises without using any technology, and then you should confirm your answer using HPGSolver.

$$\frac{dy}{dt} = 3y(1-y)$$



- **a**. The solution with $y(0) = 1/2$ approaches the equilibrium value $y = 1$ from above as t increases. It increases toward $y = \infty$ as t decreases.

- **b**. The solution with $y(0) = 1/2$ approaches the equilibrium value $y = 1$ from below as t increases. It decreases toward $y = 0$ as t decreases.

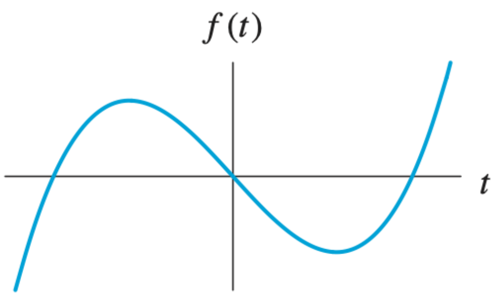
- **c**. The solution with $y(0) = 1/2$ approaches the equilibrium value $y = 0$ from above as t increases. It increases toward $y = 1$ as t decreases.

- **d**. The solution with $y(0) = 1/2$ increases toward $y = \infty$ as t increases. It decreases toward $y = 1$ as t decreases.

Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.3/BDH_1.3.13.pg`

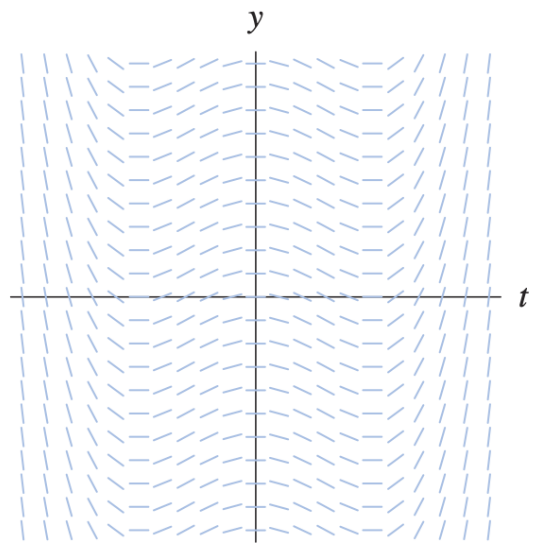
Suppose we know that the graph shown is the graph of the right-hand side $f(t)$ of the differential equation

$$\frac{dy}{dt} = f(t)$$



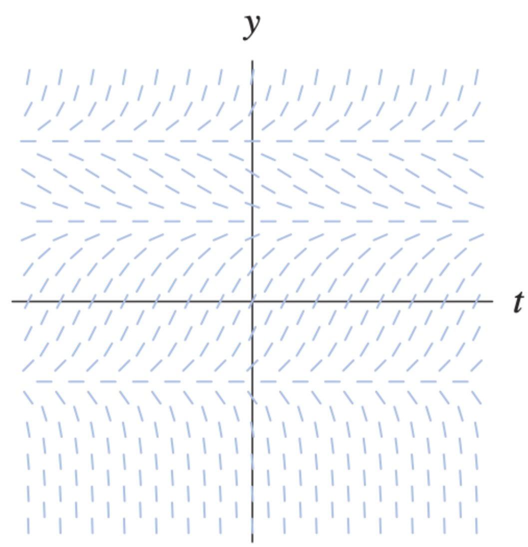
Make a rough sketch of the slope field that corresponds to this differential equation.

Which of the following slope fields is the correct one for this differential equation? [?/a/b]



(a)

(b)



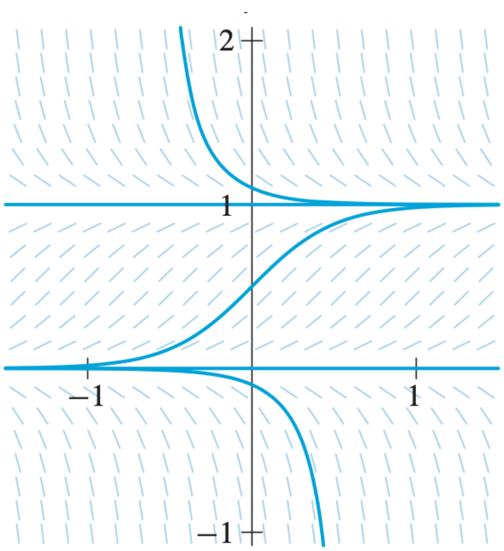
Problem 6. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.3/BDH_1.3.15.pg`

Consider the autonomous differential equation

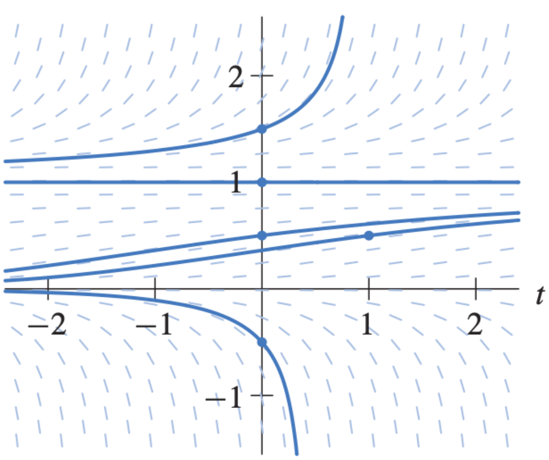
$$\frac{dS}{dt} = S^3 - 2S^2 + S$$

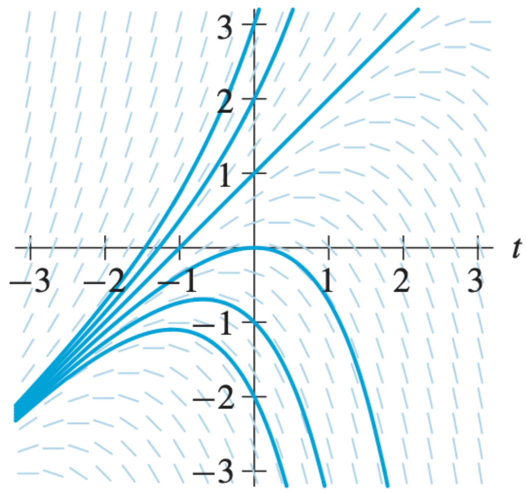
(a) Make a rough sketch of the slope field without using any technology.
(b) Using this drawing, sketch the graphs of the solutions $S(t)$ with the initial conditions $S(0) = 1/2$, $S(1) = 1/2$, $S(0) = 1$, $S(0) = 3/2$, and $S(0) = -1/2$.
(c) Confirm your answer using HPGSolver.

Which of the following slope fields most closely matches your sketch? [?/a/b/c/d]

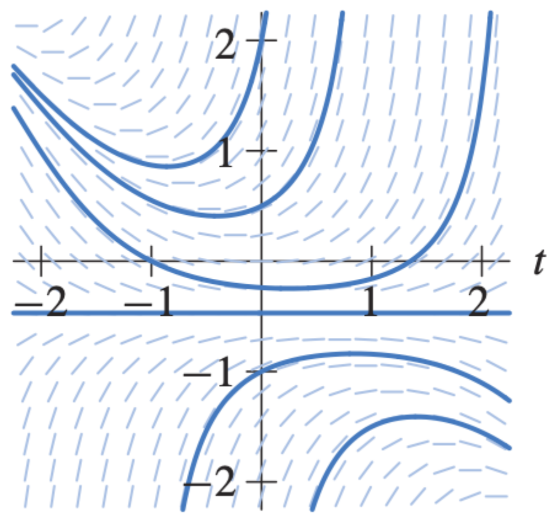


(a) (b)





(c) (d)



Problem 7. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.3/BDH_1.3.16.pg`

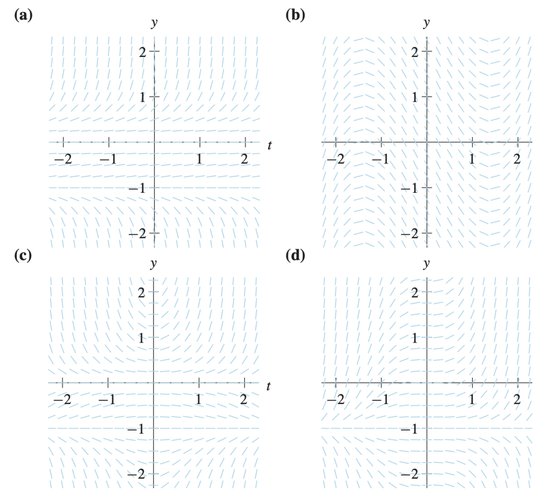
Eight differential equations and four slope fields are given below. Determine the equation that corresponds to each slope field. You should do this exercise without using technology.

(i) $\dfrac{dy}{dt} = y^2 + y$ (ii) $\dfrac{dy}{dt} = y^2 - y$ (iii) $\dfrac{dy}{dt} = y^3 + y^2$ (iv) $\dfrac{dy}{dt} = 2 - t^2$

(v) $\dfrac{dy}{dt} = ty + ty^2$ (vi) $\dfrac{dy}{dt} = t^2 + t^2 y$ (vii) $\dfrac{dy}{dt} = t + ty$ (viii) $\dfrac{dy}{dt} = t^2 - 2$

(a) [?/i/ii/iii/iv/v/vi/vii/viii] (b) [?/i/ii/iii/iv/v/vi/vii/viii]

(c) [?/i/ii/iii/iv/v/vi/vii/viii] (d) [?/i/ii/iii/iv/v/vi/vii/viii]



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Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.4/BDH_1.4.1.pg`

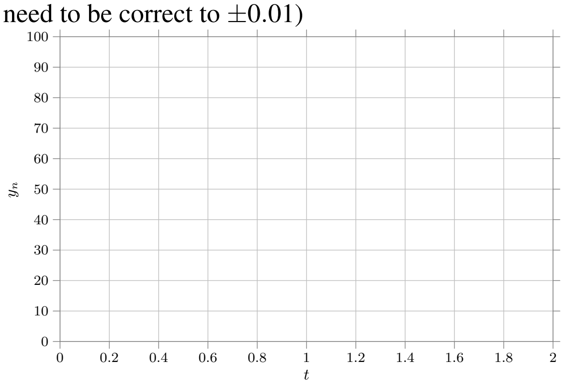
Perform Euler's method **by hand** with the given step size Δt on the initial-value problem over the time interval specified:

$$\frac{dy}{dt} = 2y+1, \quad y(0)=3, \quad 0 \le t \le 2, \quad \Delta t = 0.5.$$

Fill in the table below of the time values and approximate values of the dependent variable. Use decimals throughout, not fractions.

k	t_k	y_k
0	______	______
1	______	______
2	______	______
3	______	______
4	______	______

(keep four decimals in intermediate calculations; your answers need to be correct to ± 0.01)



(click on the graph to embiggen)

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.4/BDH_1.4.5.pg`

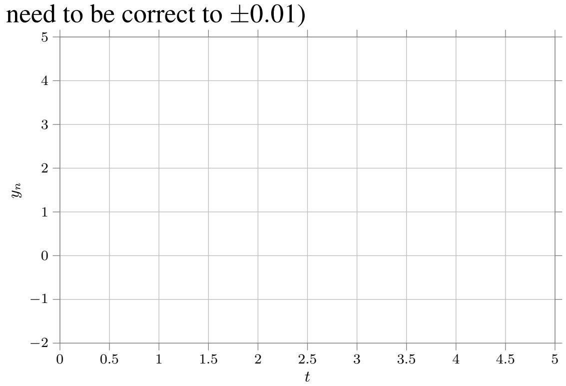
Perform Euler's method **by hand** with the given step size Δt on the initial-value problem over the time interval specified:

$$\frac{dy}{dt} = (3-y)(y+1), \quad y(0)=4, \quad 0 \le t \le 5, \quad \Delta t = 1.0.$$

Fill in the table below of the time values and approximate values of the dependent variable. Use decimals throughout, not fractions.

k	t_k	y_k
0	______	______
1	______	______
2	______	______
3	______	______
4	______	______
5	______	______

(keep four decimals in intermediate calculations; your answers need to be correct to ± 0.01)



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Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.4/BDH_1.4.6_11.pg`

Consider the initial-value problem

$$\frac{dy}{dt} = (3-y)(y+1), \quad y(0) = 0.$$

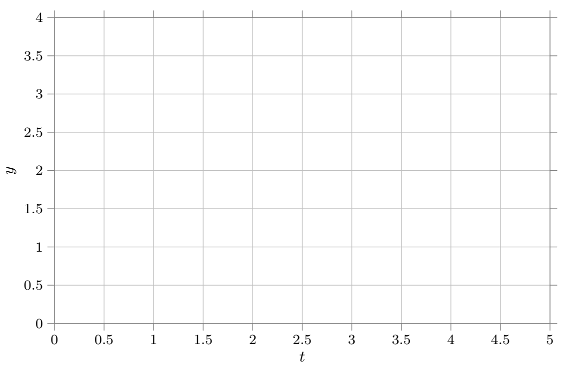
As $t \to \infty$ the solution to this initial-value problem approaches [?/∞/3/0/-1/−∞] from [?/above/below].

Use Euler's method with $n = 8$ steps to approximate the solution to the initial-value problem over the time interval $0 \le t \le 5$. (your answers should to be correct to $\pm 0.0001\%$)

You can use **this online numerical method widget** (opens in new tab).

$y_0, y_1, y_2, \ldots, y_n =$ ________

(enter as a comma-separated list)



(click on the graph to embiggen)

What's wrong with the approximate solution given by Euler's method?

- **a**. The long-term behavior of the numerical approximation does not approach the long-term behavior from the qualitative analysis.
- **b**. As the solution approaches the equilibrium solution corresponding to $y = 3$, its slope decreases. We do not expect the solution to 'jump over' an equilibrium solution, as the numerical approximation does.
- **c**. The numerical solution fails to match the initial condition.

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.4/BDH_1.4.15.pg`

Consider the initial-value problem

$$\frac{dy}{dt} = \sqrt{y}, \quad y(0) = 1.$$

Solve, by hand, the initial value problem:
$y(t) =$ ________

Using Euler's method, compute three different approximate solutions corresponding to $\Delta t = 1.0$, 0.5, and 0.25 over the interval $0 \le t \le 4$.

(your answers should to be correct to $\pm 0.0001\%$)

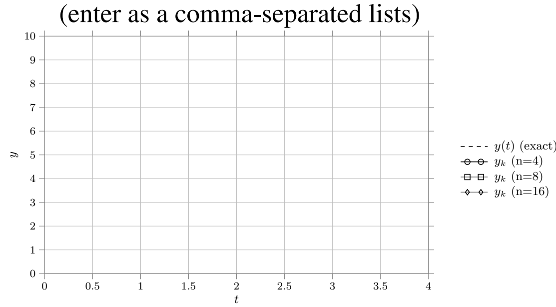
You can use **this online numerical method widget** (opens in new tab).

$\Delta t = 1.0:$ $y_0, y_1, y_2, \ldots, y_4 =$ ________

$\Delta t = 0.5:$ $y_0, y_1, y_2, \ldots, y_8 =$ ________

$\Delta t = 0.25:$ $y_0, y_1, y_2, \ldots, y_{16} =$ ________

(enter as a comma-separated lists)



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What observations can you make about the actual and approximate solutions to the initial-value problem?

- **a**. The error between the numerical solution and the exact solution decreases as the stepsize Δt decreases.
- **b**. The error between the numerical solution and the exact solution increases as the stepsize Δt decreases.
- **c**. The approximate solutions should always be greater than the actual solution.
- **d**. The approximate solutions will always be less than the actual solution.

- **e**. Both (a) and (c).
- **f**. Both (b) and (d).
- **g**. Both (a) and (d).
- **h**. Both (b) and (c).

Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.4/BDH_1.4.2.pg`

Consider the initial-value problem

$$\frac{dy}{dt} = t - y^2, \quad y(0) = 1,$$

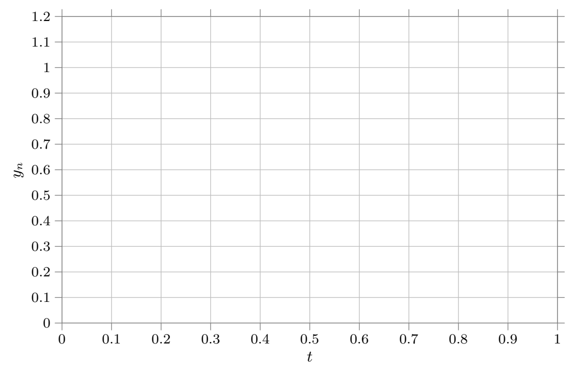
which is a non-linear equation (but not separable) for which we do not know a solution method.

Use Euler's method with $n = 8$ steps to approximate the solution to the initial-value problem over the time interval $0 \leq t \leq 1$. (your answers should to be correct to $\pm 0.0001\%$)

You can use **this online numerical method widget** (opens in new tab).

$y_0, y_1, y_2, \ldots, y_n =$ ____________

(enter as a comma-separated list)



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Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.6/BDH_1.6.1.pg`

Sketch the phase line for the differential equation

$$\frac{dy}{dt} = 3y(y-2)$$

(recall that we usually draw these as a vertical line... maybe rotate your screen?)

1 1 To place a symbol on the line click a symbol button then click a point on the line.
To remove a symbol from the line click the "delete" button then click the symbol on the line.

Classify the equilibria: (Enter "N/A" if there are fewer equilibria than asked for)
The smallest equilibrium point is a [??/sink/source/node].
The next larger equilibrium point is a

- ??
- sink
- source
- node
- `N/A`

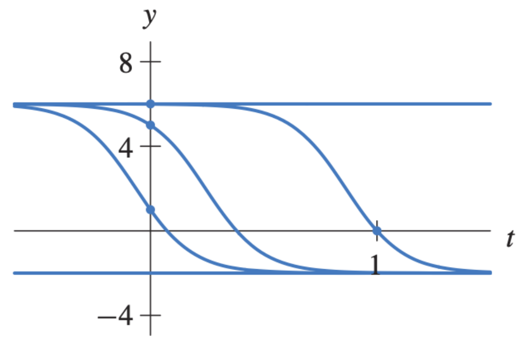
.
The next larger equilibrium point is a

- ??
- sink
- source
- node
- `N/A`

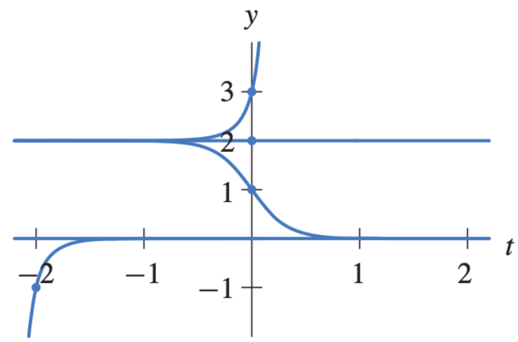
.

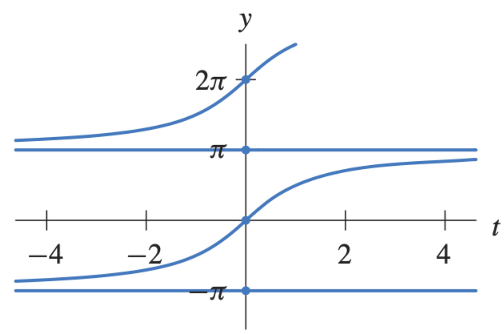
Sketch, on paper and on a single pair of axes, the graphs of the solutions satisfying the initial conditions $y(0) = 1$, $y(-2) = -1$, $y(0) = 3$, and $y(0) = 2$.

Select the graph that most closely matches your sketch: [?/a/b/c/d]

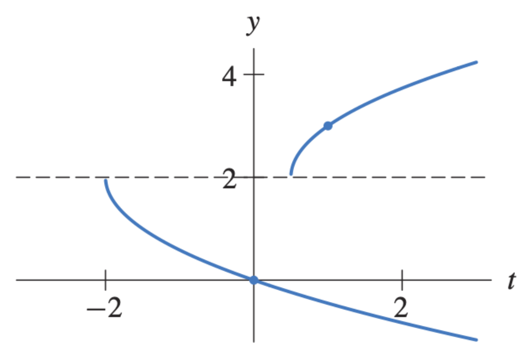


(a) (b)





(c) (d)



Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.6/BDH_1.6.3.pg`

Sketch the phase line for the differential equation

$$\frac{dy}{dt} = \cos y$$

(recall that we usually draw these as a vertical line... maybe rotate your screen?)

1 1 To place a symbol on the line click a symbol button then click a point on the line.
To remove a symbol from the line click the "delete" button then click the symbol on the line.

Classify the equilibria: (Enter "N/A" if there are fewer equilibria than asked for)
The smallest equilibrium point is a [??/sink/source/node].
The next larger equilibrium point is a

- ??
- sink
- source
- node
- `N/A`

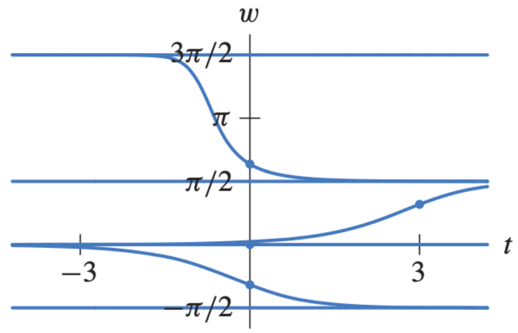
.
The next larger equilibrium point is a

- ??
- sink
- source
- node
- `N/A`

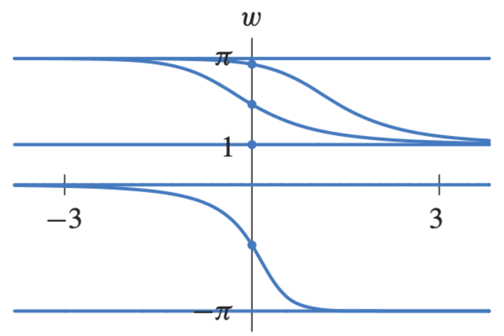
.

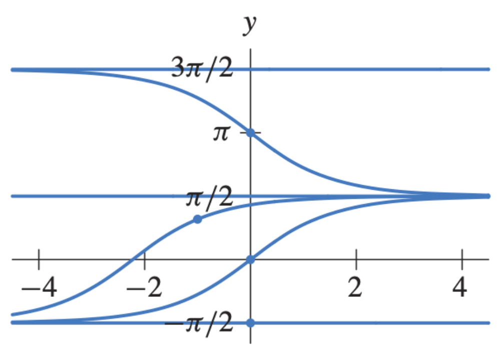
Sketch, on paper and on a single pair of axes, the graphs of the solutions satisfying the initial conditions $y(0) = 0$, $y(-1) = 1$, $y(0) = -\pi/2$, and $y(0) = \pi$.

Select the graph that matches your sketch: [?/a/b/c/d]

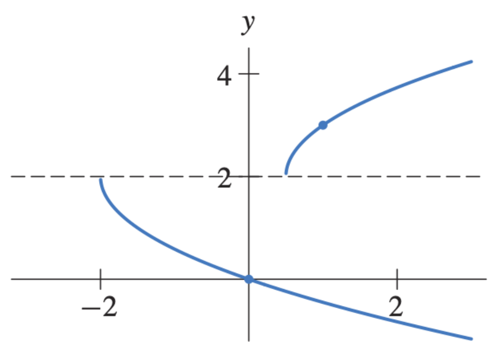


(a) (b)





(c) (d)



Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.6/BDH_1.6.5.pg`

Sketch the phase line for the differential equation

$$\frac{dw}{dt} = (1-w)\sin w$$

(recall that we usually draw these as a vertical line... maybe rotate your screen?)

1 1 To place a symbol on the line click a symbol button then click a point on the line.
To remove a symbol from the line click the ”delete” button then click the symbol on the line.

Classify the equilibria: (Enter ”N/A” if there are fewer equilibria than asked for)
The smallest equilibrium point is a [??/sink/source/node].
The next larger equilibrium point is a

- ??
- sink
- source
- node
- N/A

.
The next larger equilibrium point is a

- ??
- sink
- source
- node
- N/A

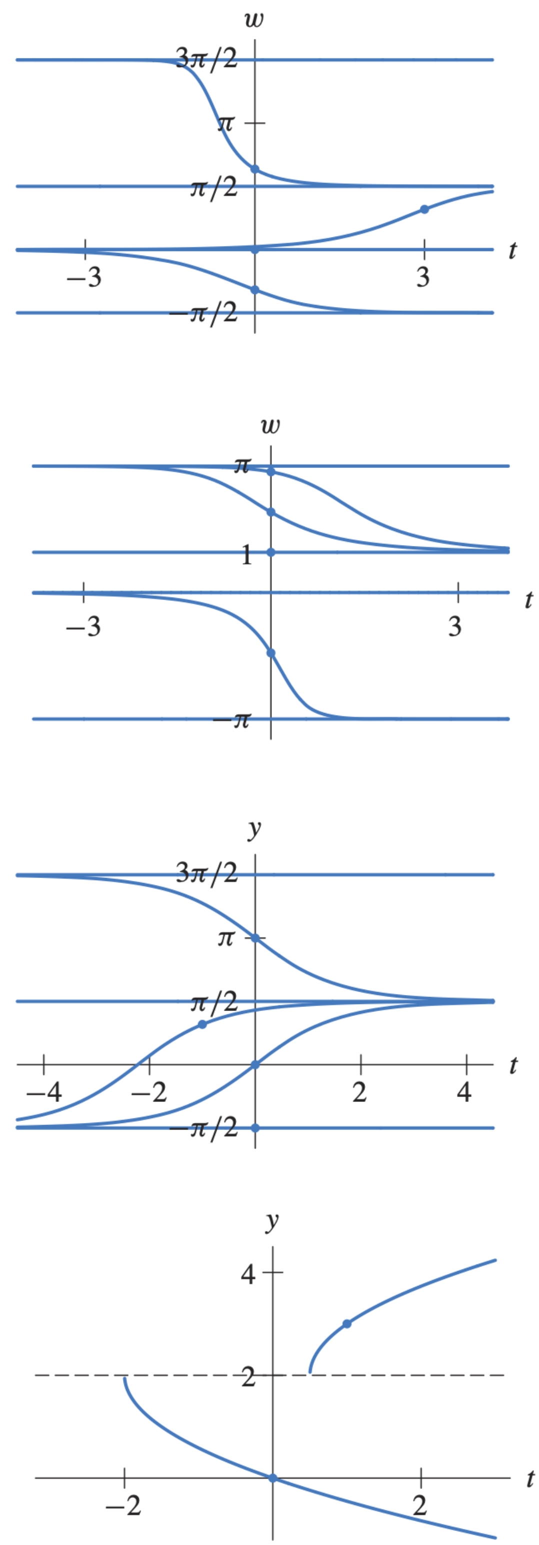
.
The next larger equilibrium point is a

- ??
- sink
- source
- node
- N/A

.

Sketch, on paper and on a single pair of axes, the graphs of the solutions satisfying the initial conditions $w(0) = -3/2$, $w(0) = 1$, $w(0) = 2$, and $w(0) = 3$.

Select the graph that matches your sketch: [?/a/b/c/d]



(a) (b) (c) (d)

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.6/BDH_1.6.2.pg`

Sketch the phase line for the differential equation

$$\frac{dy}{dt} = y^2 - 4y - 12$$

(recall that we usually draw these as a vertical line... maybe rotate your screen?)

1 1 To place a symbol on the line click a symbol button then click a point on the line.
To remove a symbol from the line click the "delete" button then click the symbol on the line.

Classify the equilibria: (Enter "N/A" if there are fewer equilibria than asked for)
The smallest equilibrium point is a [??/sink/source/node].
The next larger equilibrium point is a

- ??
- sink
- source
- node
- N/A

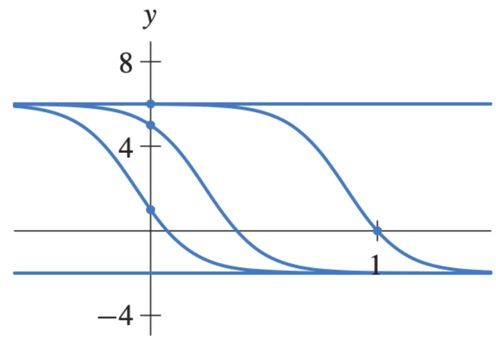
.
The next larger equilibrium point is a

- ??
- sink
- source
- node
- N/A

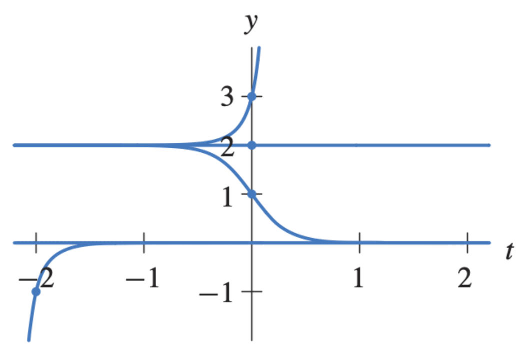
.

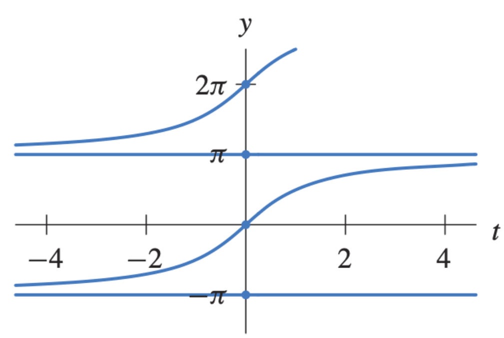
Sketch, on paper and on a single pair of axes, the graphs of the solutions satisfying the initial conditions $y(0) = 1$, $y(1) = 0$, $y(0) = 6$, and $y(0) = 5$.

Select the graph that matches your sketch: [?/a/b/c/d]

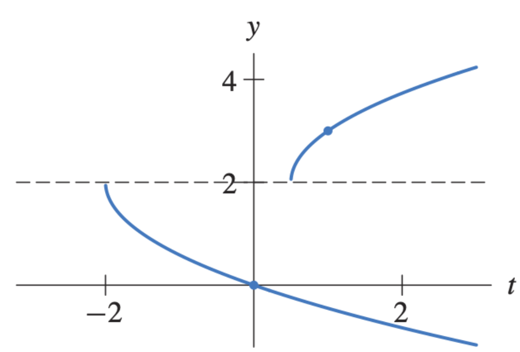


(a) (b)





(c) (d)



Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.6/BDH_1.6.23.pg`

Consider the differential equation

$$\frac{dy}{dt} = y^2 - 4y + 3.$$

Describe the long-term behavior of solutions with the given initial condition.

(*Hint: draw a phase line and use it to answer the questions below.*)

For $y(0) = 2$, the solution is

- ?
- increasing
- decreasing
- an equilibrium

As $t \to \infty$ it tends toward [?/∞/3/0/1/−∞]

As $t \to -\infty$ it tends toward [?/∞/3/0/1/−∞]

For $y(0) = -4$, the solution is

- ?
- increasing
- decreasing
- an equilibrium

As $t \to \infty$ it tends toward [?/∞/3/0/1/−∞]

As $t \to -\infty$ it tends toward [?/∞/3/0/1/−∞]

For $y(3) = 5$, the solution is

- ?
- increasing
- decreasing
- an equilibrium

As $t \to \infty$ it tends toward [?/∞/3/0/1/−∞]

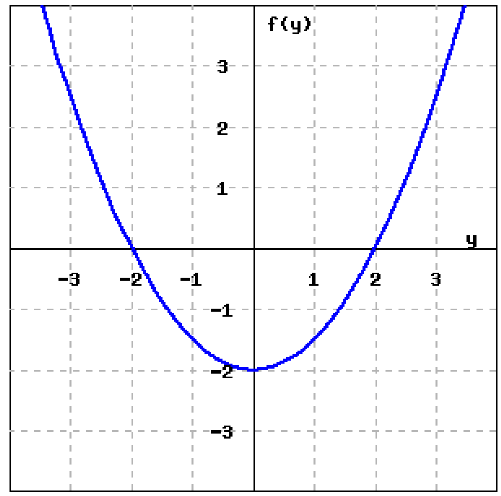
As $t \to -\infty$ it tends toward [?/∞/3/0/1/−∞]

Problem 6. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.6/BDH_1.6.29.pg`

Sketch the phase line for the autonomous differential equation

$$\frac{dy}{dt} = f(y)$$

where the graph of $f(y$ is



(recall that we usually draw these as a vertical line... maybe rotate your screen?)

1 1 To place a symbol on the line click a symbol button then click a point on the line.
To remove a symbol from the line click the "delete" button then click the symbol on the line.

Classify the equilibria: (Enter "N/A" if there are fewer equilibria than asked for)
The smallest equilibrium point is a [??/sink/source/node].
The next larger equilibrium point is a

- ??
- sink
- source
- node
- `N/A`

.

The next larger equilibrium point is a

- ??
- sink
- source
- node
- `N/A`

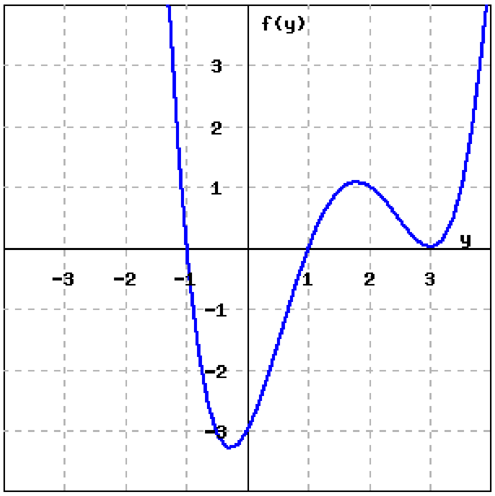
.

Problem 7. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.6/BDH_1.6.31.pg`

Sketch the phase line for the autonomous differential equation

$$\frac{dy}{dt} = f(y)$$

where the graph of $f(y$ is



(recall that we usually draw these as a vertical line... maybe rotate your screen?)

1 1 To place a symbol on the line click a symbol button then click a point on the line.
To remove a symbol from the line click the "delete" button then click the symbol on the line.

Classify the equilibria: (Enter "N/A" if there are fewer equilibria than asked for)

The smallest equilibrium point is a [??/sink/source/node].
The next larger equilibrium point is a

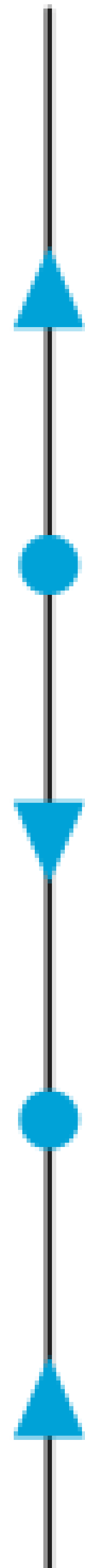
- ??
- sink
- source
- node
- `N/A`

.
The next larger equilibrium point is a

- ??
- sink
- source
- node
- `N/A`

.

Problem 8. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.6/BDH_1.6.33.pg`



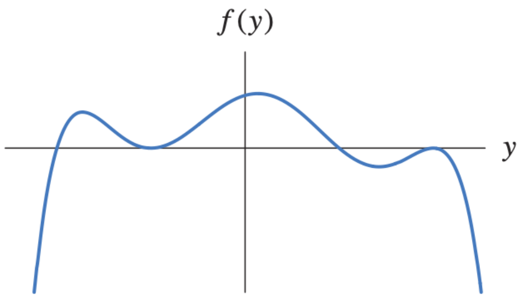
The phase line for an autonomous equation

$$\frac{dy}{dt} = f(y)$$

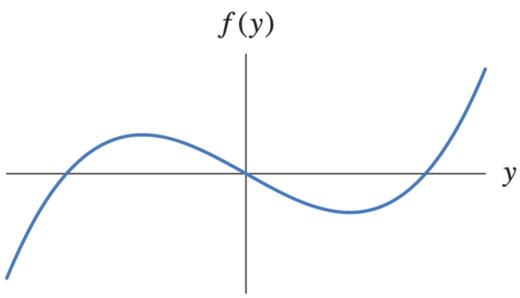
is shown to the right.

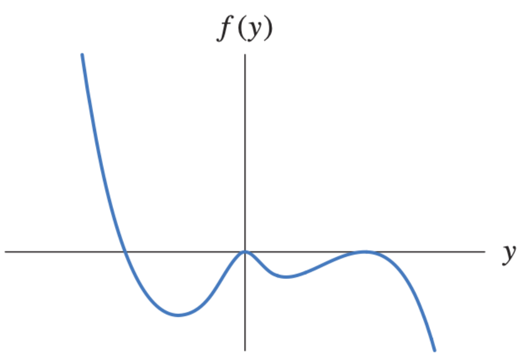
Make, on paper, a rough sketch of the graph of the function $f(y)$.

Which of the following graphs most closely resembles yours? [?/a/b/c/d]

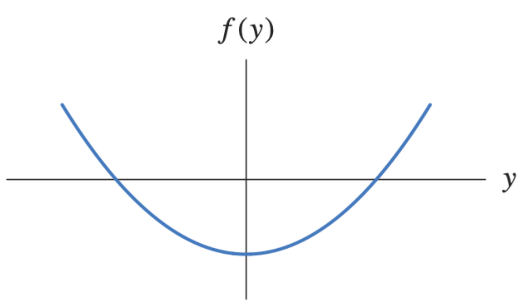


(a) (b)





(c) (d)

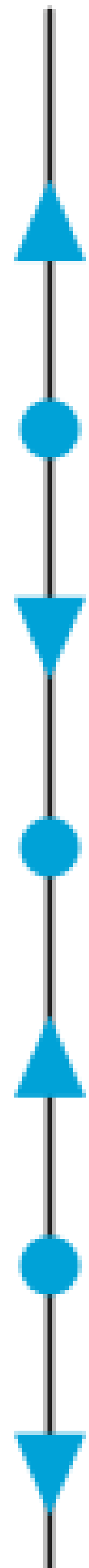


Problem 9. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.6/BDH_1.6.35.pg`

The phase line for an autonomous equation

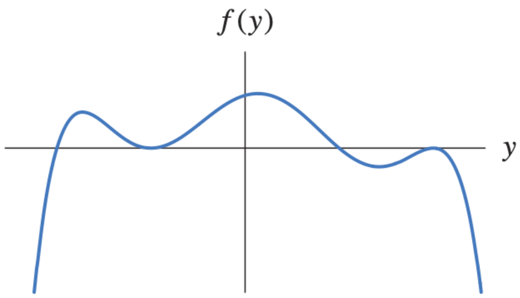
$$\frac{dy}{dt} = f(y)$$

is shown to t he right.



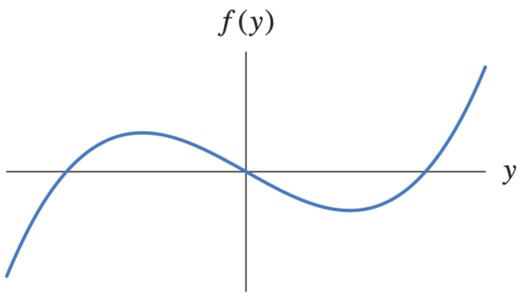
Make, on paper, a rough sketch of the graph of the function $f(y)$.

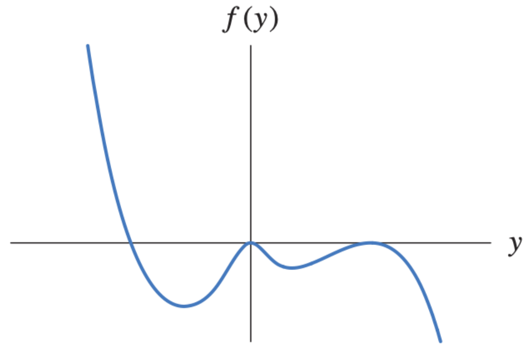
Which of the following graphs most closely resembles yours? [?/a/b/c/d]



(a)

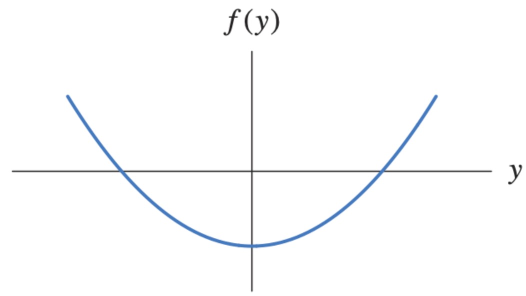
(b)





(c)

(d)



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Thomas Hoft **210S23_Hoft**
Assignment BDH_1.7 due 02/14/2023 at 09:55am CST

Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.7/BDH_1.7.1.pg`

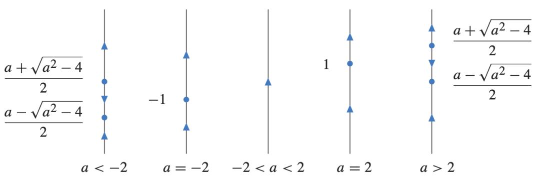
Locate the bifurcation values for the one-parameter family

$$\frac{dy}{dt} = y^2 + a$$

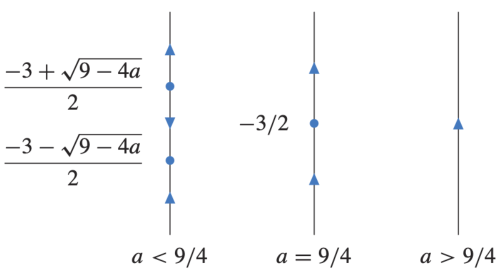
and, on paper, draw the phase lines for values of the parameter slightly smaller than, slightly larger than, and at the bifurcation values.

$a =$ ___ (for multiple values, enter a comma-separated list, i.e. "-7, 42")

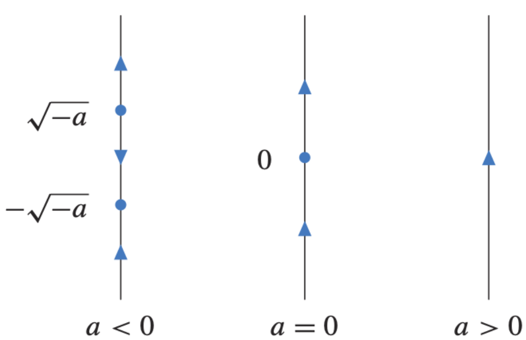
Select the bifurcation diagram that matches your diagram: [?/a/b/c]



(a)



(b)



(c)

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.7/BDH_1.7.3.pg`

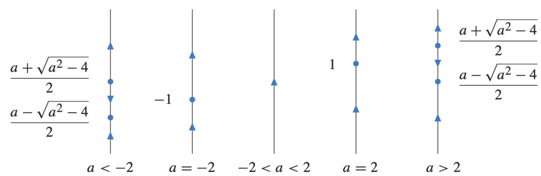
Locate the bifurcation values for the one-parameter family

$$\frac{dy}{dt} = y^2 - ay + 1$$

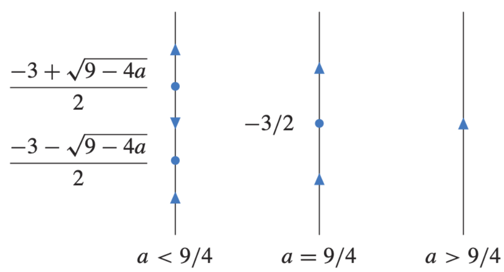
and, on paper, draw the phase lines for values of the parameter slightly smaller than, slightly larger than, and at the bifurcation values.

$a =$ ___ (for multiple values, enter a comma-separated list, i.e. "-7, 42")

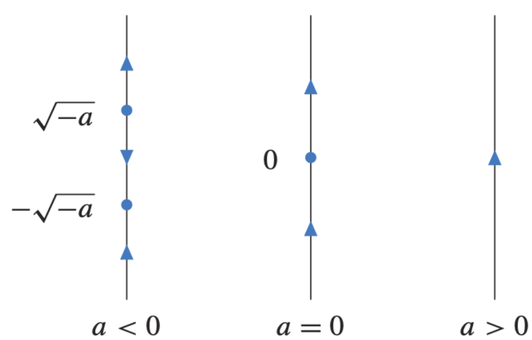
Select the bifurcation diagram corresponding to your diagram: [?/a/b/c]



(a)



(b)



(c)

Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.7/BDH_1.7.9.pg`

Locate the bifurcation values of α for the one-parameter family

$$\frac{dy}{dt} = \sin(y) + \alpha.$$

(You should, on paper, draw the phase lines for values of the parameter slightly smaller than, slightly larger than, and at the bifurcation values.)

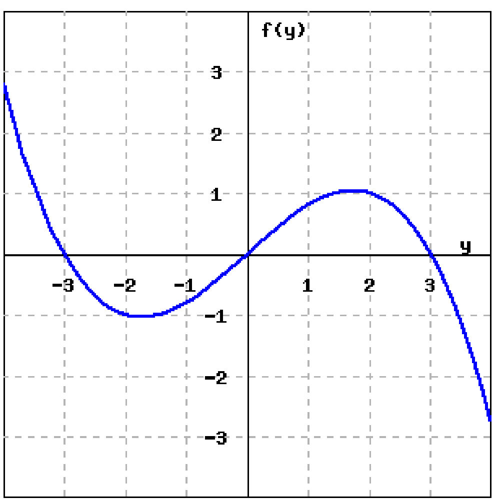
$\alpha =$ ___ (for multiple values, enter a comma-separated list, i.e. "-7, 42")

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.7/BDH_1.7.11.pg`

Locate the bifurcation values that occur in the one-parameter family

$$\frac{dy}{dt} = f(y) + \alpha$$

where a graph of $f(y)$ is



(You should, on paper, draw the phase lines for values of the parameter slightly smaller than, slightly larger than, and at the bifurcation values.)

$\alpha =$ ___ (for multiple values, enter a comma-separated list, i.e. "-7, 42")

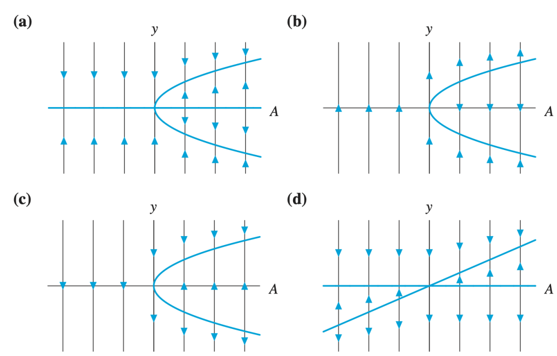
Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.7/BDH_1.7.13.pg`

Six one-parameter families of differential equations depending on the parameter A and four bifurcation diagrams are given below. Determine the one-parameter family that corresponds to each bifurcation diagram; you should be able to state briefly how you know your choice is correct.

(i) $\frac{dy}{dt} = Ay - y^2$ (ii) $\frac{dy}{dt} = Ay + y^2$ (iii) $\frac{dy}{dt} = Ay - y^3$

(iv) $\frac{dy}{dt} = A - y^2$ (v) $\frac{dy}{dt} = y^2 - A$ (vi) $\frac{dy}{dt} = Ay + y^2$

(a) [?/i/ii/iii/iv/v/vi] (b) [?/i/ii/iii/iv/v/vi] (c) [?/i/ii/iii/iv/v/vi] (d) [?/i/ii/iii/iv/v/vi]



Problem 6. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.7/BDH_1.7.19.pg`

Consider the population model

$$\frac{dP}{dt} = 2P - \frac{P^2}{50}$$

for a species of fish in a lake (notice this is a Logistic model). Suppose it is decided that fishing will be allowed, but it is unclear how many fishing licenses should be issued. Suppose the average catch of a fisherman with a license is 3 fish per year (these are hard fish to catch).

(a) What is the largest number of licenses that can be issued if the fish are to have a chance to survive in the lake? ___
(Hint: think about a bifurcation – a qualitative change in the behavior of solutions such as population stability at a non-zero value vs die-off to population of zero.)

(b) Suppose the number of fishing licenses in part (a) is issued. What will happen to the fish population – that is, how does the behavior of the population depend on the initial population?
(Hint: draw a bifurcation diagram with a phase line at the value you found in (a).)

If the initial population is greater than [?/100/90/80/70/60/50/40/30/20/10/0], the population will tend towards the value

- ?
- ∞
- 100
- 90
- 80
- 70
- 60
- 50
- 40
- 30
- 20
- 10
- 0
- $-\infty$

.

If the initial population is smaller than [?/100/90/80/70/60/50/40/30/20/10/0], the population will tend towards the value

- ?
- ∞
- 100
- 90
- 80
- 70
- 60
- 50
- 40
- 30
- 20
- 10
- 0
- $-\infty$

(c) The simple population model above can be thought of as a model of an ideal fish population that is not subject to many of the environmental problems of an actual lake. For the actual fish population, there will be occasional changes in the population that were not considered when this model was constructed. For example, if the water level increases due to a heavy rainstorm, a few extra fish might be able to swim down a usually dry stream bed to reach the lake, or the extra water might wash toxic waste into the lake, killing a few fish. Given the possibility of unexpected perturbations of the population not included in the model, what do you think will happen to the actual fish population if we allow fishing at the level determined in part (b)?

- **a**. The fish will all live happy lives.
- **b**. The fish population will crash to zero if only eleven additional fish die.
- **c**. The fish population will explode, with $P \to \infty$ due to underfishing.
- **d**. There is no circumstance that will result in the population failing to to approach a non-zero equilibrium value.

Problem 7. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.7/BDH_1.7.23.pg`

(a) Use PhaseLines from the DETools software to investigate the bifurcation diagram for the differential equation

$$\frac{dy}{dt} = ay - y^3$$

where a is a parameter.

For $a > 0$ there are [?/0/1/2/3] sinks and [?/0/1/2/3] sources, and and the middle equilibrium is [?/positive/negative/zero]; for $a < 0$ there is one [?/positive/negative/zero] sink.

What are the bifurcation values for this one-parameter family? ____

(b) Use PhaseLines to investigate the bifurcation diagram for the differential equation

$$\frac{dy}{dt} = r + ay - y^3$$

where r is a positive parameter. How does the bifurcation diagram change from the $r = 0$ case (see part (a))?

The bifurcation value of a is now [?/positive/negative/zero]; for $a > 0$ there are [?/0/1/2/3] sinks and [?/0/1/2/3] sources, and the middle equilibrium is [?/positive/negative/zero]; for $a < 0$ there is one [?/positive/negative/zero] sink.

(c) Suppose r is negative in the equation in part (b). How does the bifurcation diagram change?

The bifurcation value of a is now [?/positive/negative/zero]; for $a > 0$ there are [?/0/1/2/3] sinks and [?/0/1/2/3] sources, and the middle equilibrium is [?/positive/negative/zero]; for $a < 0$ there is one [?/positive/negative/zero] sink.

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Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.8/BDH_1.8.1.pg`
Find the general solution of the differential equation
$$\frac{dy}{dt} = -4y + 9e^{-t}.$$
Use k for the constant appearing in your solution.

$y(t) =$ ______

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.8/BDH_1.8.3.pg`
Find the general solution of the differential equation
$$\frac{dy}{dt} = -3y + 4\cos 2t.$$
Use k for the constant appearing in your solution.

$y(t) =$ ______

Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.8/BDH_1.8.5.pg`
Find the general solution of the differential equation
$$\frac{dy}{dt} = 3y - 4e^{3t}.$$
Use k for the constant appearing in your solution.

$y(t) =$ ______

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.8/BDH_1.8.7.pg`
Solve the initial value problem
$$\frac{dy}{dt} + 2y = e^{t/3} \quad y(0) = 1.$$

$y(t) =$ ______

Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.8/BDH_1.8.9.pg`
Solve the initial value problem
$$\frac{dy}{dt} + y = \cos 2t \quad y(0) = 5.$$

$y(t) =$ ______

Problem 6. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.8/BDH_1.8.13.pg`
Consider the nonhomogeneous linear equation
$$\frac{dy}{dt} + 3y = \cos 3t.$$
To find a particular solution, it is pretty clear that our guess must contain a cosine function, but it is not so clear that the guess must also contain a sine function.

(a) Guess $y_p(t) = \alpha\cos 3t$ and substitute this guess into the equation. Is there a value of α such that $y_p(t)$ is a solution?
$\alpha =$ ___ (enter the value of α as a number, or enter **DNE** if there is no such α)

(b) Why is the proper guess for a particular solution $y_p(t) = \alpha\cos 3t + \beta\sin 3t$?

- I plugged it in and it worked.
- The textbook said so.
- Substitution of this guess into the equation leads to two linear algebraic equations in two unknowns, and such systems of equations usually have a unique solution.
- The professor said so.
- All of the above.

Problem 7. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.8/BDH_1.8.17.pg`

Consider the nonhomogeneous linear equation

$$\frac{dy}{dt} = y^2.$$

(a) Show that $y_1(t) = 1/(1-t)$ is a solution.

- I verified that it is a solution.
- I discovered that it is **not** a solution.

(b) Show that $y_2(t) = 2/(1-t)$ is not a solution.

- I verified that it is a solution.
- I discovered that it is **not** a solution.

(c) Why don't these two facts contradict the Linearity Principle?

- The Linearity Principle is wrong.
- The differential equation is not linear.
- The Linearity Principle should hold for this equation.

Problem 8. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.8/BDH_1.8.20.pg`

Consider the nonhomogeneous linear equation

$$\frac{dy}{dt} + 2y = 3t^2 + 2t - 1.$$

In order to find the general solution, we must guess a particular solution $y_p(t)$. Since the right-hand side is a quadratic polynomial, it is reasonable to guess a quadratic for $y_p(t)$, so let

$$y_p(t) = at^2 + bt + c,$$

where a, b, and c are constants. Determine values for these constants so that $y_p(t)$ is a solution.

$a =$ ___

$b =$ ___

$c =$ ___

Problem 9. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.8/BDH_1.8.23.pg`

Find the general solution and the solution that satisfies the initial condition $y(0) = 0$ for the differential equation

$$\frac{dy}{dt} - 3y = 2t - e^{4t}.$$

Use k for the constant appearing in your general solution.

$y(t) =$ __________ (general solution)

$y(t) =$ __________ (specific solution satisfying the inital condition)

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Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.9/BDH_1.9.1.pg`

Find the general solution of the differential equation
$$\frac{dy}{dt} = -\frac{y}{t} + 2.$$
Use c for the constant appearing in your solution.

$y(t) =$ ______

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.9/BDH_1.9.5.pg`

Find the general solution of the differential equation
$$\frac{dy}{dt} - \frac{2t}{1+t^2}\, y = 3.$$
Use c for the constant appearing in your solution.

$y(t) =$ ______

Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.9/BDH_1.9.11.pg`

Solve the initial value problem
$$\frac{dy}{dt} - \frac{2y}{t} = 2t^2, \quad y(-2) = 4.$$

$y(t) =$ ______

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.9/BDH_1.9.21.pg`

Consider the nonhomogeneous equation
$$\frac{dv}{dt} + 0.4\, v = 3\cos 2t.$$
(a) Find the general solution using the method of integrating factors or the method of variation of parameters. Use k for the constant appearing in your solution.
$v(t) =$ ______

(b) Find the general solution using the guessing technique from Section 1.8 (the method of undetermined coefficients). Use k for the constant appearing in your solution.
$v(t) =$ ______

Which method was easier for you?

- Integrating factors / variation of parameters.
- Undetermined coefficients.
- Both are of the same degree of ease / difficulty.
- Neither is easy.

Problem 6. **(1 point)** `ustLibrary/ustDiffEq/setBDH_1.9/BDH_1.R.54.pg`

A 1000-gallon tank initially contains a mixture of 450 gallons of cola and 50 gallons of cherry syrup. Cola is added at the rate of 8 gallons per minute, and cherry syrup is added at the rate of 2 gallons per minute. At the same time, a well mixed solution of cherry cola is withdrawn at the rate of 5 gallons per minute. What percentage of the mixture is cherry syrup when the tank is full?

___%

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Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_2.1/2.1.1.pg`

Consider the following two systems of equations, both predator-prey models:

$$\text{(i)}\quad \frac{dx}{dt} = 10x\left(1-\frac{x}{10}\right) - 20xy \qquad \text{(ii)}\quad \frac{dx}{dt} = 0.3x - \frac{xy}{100}$$

$$\frac{dy}{dt} = -5y + \frac{xy}{20} \qquad \frac{dy}{dt} = 15y\left(1-\frac{y}{15}\right) + 25xy$$

$$x(t) \geq 0,\ y(t) \geq 0 \qquad x(t) \geq 0,\ y(t) \geq 0.$$

In one of these systems, the prey are very large animals and the predators are very small animals, such as elephants and mosquitoes. Thus it takes many predators to eat one prey, but each prey eaten is a tremendous benefit for the predator population. The other system has very large predators and very small prey. Determine which system is which.

System (i) has

- Large prey; small predators.
- Large predators; small prey.

System (ii) has

- Large prey; small predators.
- Large predators; small prey.

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_2.1/2.1.2.pg`

Consider the following two systems of equations, both predator-prey models:

$$\text{(i)}\quad \frac{dx}{dt} = 10x\left(1-\frac{x}{10}\right) - 20xy \qquad \text{(ii)}\quad \frac{dx}{dt} = 0.3x - \frac{xy}{100}$$

$$\frac{dy}{dt} = -5y + \frac{xy}{20} \qquad \frac{dy}{dt} = 15y\left(1-\frac{y}{15}\right) + 25xy$$

$$x(t) \geq 0,\ y(t) \geq 0 \qquad x(t) \geq 0,\ y(t) \geq 0.$$

Find all equilibrium points for the two systems:

(i) $(x,y) =$ ________
(format your answer as a list of points, i.e. (1,2), (3,4), (5,6).) help (points)

(ii) $(x,y) =$ ________
(format your answer as a list of points, i.e. (1,2), (3,4), (5,6).) help (points)

Explain the significance of these points in terms of the predator and prey populations:

The equilibria for system (i) have the propertie(s) that

- **a**. Predators and prey co-exist.
- **b**. Predators die out; prey survive.
- **c**. Prey die out; predators survive.
- **d**. (a) and (b)
- **e**. (a) and (c)
- **f**. (b) and (c)
- **g**. (a) and (b) and (c)

(you can ignore the trivial equilibrium (0,0) if it is present)

The equilibria for system (i) have the propertie(s) that

- **a**. Predators and prey co-exist.

- **b**. Predators die out; prey survive.
- **c**. Prey die out; predators survive.
- **d**. (a) and (b)
- **e**. (a) and (c)
- **f**. (b) and (c)
- **g**. (a) and (b) and (c)

(you can ignore the trivial equilibrium (0,0) if it is present)

Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_2.1/2.1.3.pg`

Consider the following two systems of equations, both predator-prey models:

$$\text{(i)}\quad \frac{dx}{dt} = 10x\left(1-\frac{x}{10}\right) - 20xy \qquad \text{(ii)}\quad \frac{dx}{dt} = 0.3x - \frac{xy}{100}$$
$$\frac{dy}{dt} = -5y + \frac{xy}{20} \qquad \frac{dy}{dt} = 15y\left(1-\frac{y}{15}\right) + 25xy$$

$$x(t) \geq 0,\ y(t) \geq 0 \qquad x(t) \geq 0,\ y(t) \geq 0.$$

Suppose that the predators are extinct at time $t_0 = 0$. For each system, verify that the predators remain extinct for all time.

(i) The equation for dy/dt now reads $\frac{dy}{dt} =$ ________.

The solution to this is $y(t) =$ ________.

(ii) The equation for dy/dt now reads $\frac{dy}{dt} =$ ________.

The solution to this is $y(t) =$ ________.

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_2.1/2.1.4i.pg`

Consider the following system of equations, a predator-prey model:

$$\frac{dx}{dt} = 10x\left(1-\frac{x}{10}\right) - 20xy$$
$$\frac{dy}{dt} = -5y + \frac{xy}{20}$$

$$x(t) \geq 0,\ y(t) \geq 0$$

Describe the behavior of the prey population if the predators are extinct: sketch, on paper, the phase line for the prey population assuming that the predators are extinct, and describe the long-term behavior of the prey population. Then enter the results of your work below.

Note: ignore (do not fill in) the negative population part of the phase line.

1 1

- The prey obey an exponential growth model and grow without bound.
- The prey obey a logistic model and approach a carrying capacity.
- The prey obey an exponential growth model and die out.
- The prey obey a logistic model and die out.

Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_2.1/2.1.4ii.pg`

Consider the following systems of equations, a predator-prey model:

$$\frac{dx}{dt} = 0.3x - \frac{xy}{100}$$
$$\frac{dy}{dt} = 15y\left(1 - \frac{y}{15}\right) + 25xy$$

$$x(t) \geq 0,\ y(t) \geq 0.$$

Describe the behavior of the prey population if the predators are extinct: sketch, on paper, the phase line for the prey population assuming that the predators are extinct, and describe the long-term behavior of the prey population. Then enter the results of your work below.

Note: ignore (do not fill in) the negative population part of the phase line.

1 1

- The prey obey an exponential growth model and grow without bound.
- The prey obey a logistic model and approach a carrying capacity.
- The prey obey an exponential growth model and die out.
- The prey obey a logistic model and die out.

Problem 6. **(1 point)** `ustLibrary/ustDiffEq/setBDH_2.1/2.1.9.pg`

The predator-prey and modified predator-prey systems disussed in the text are:

$$\text{(i)}\ \frac{dR}{dt} = 2R - 1.2RF \qquad \text{(ii)}\ \frac{dR}{dt} = 2R\left(1 - \frac{R}{2}\right) - 1.2RF$$
$$\frac{dF}{dt} = -F + 0.9RF \qquad \frac{dy}{dt} = -F + 0.9RF.$$

How would you modify these systems to include the effect of hunting of the prey at a rate of α units of prey per unit of time? (use "a" in your answer instead of "alpha")

(i) $\frac{dR}{dt} =$ __________

$\frac{dF}{dt} =$ __________

(ii) $\frac{dR}{dt} =$ __________

$\frac{dF}{dt} =$ __________

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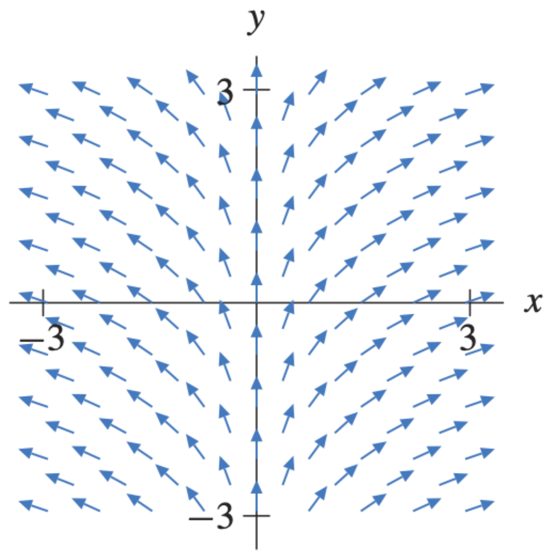
Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_2.2/BDH_2.2.1.pg`

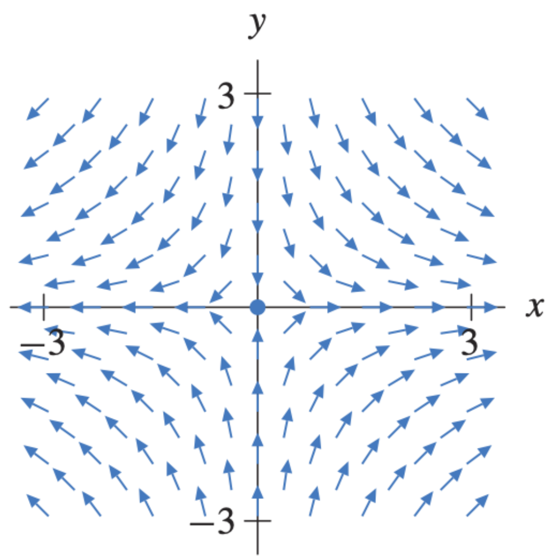
Consider the first-order system of differential equations

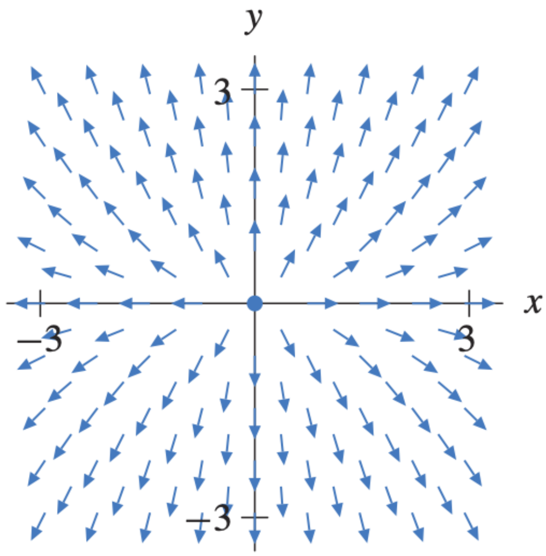
$$\frac{dx}{dt} = 1$$
$$\frac{dy}{dt} = 0$$

(a) Sketch, on paper, enough vectors in the direction field to get a sense of its geometric structure (you should do this part of the exercise without the use of technology) and confirm your work with HPGSystemSolver from the DETools software.
Which of the following direction fields most closely matches your sketch? [?/a/b/c/d]



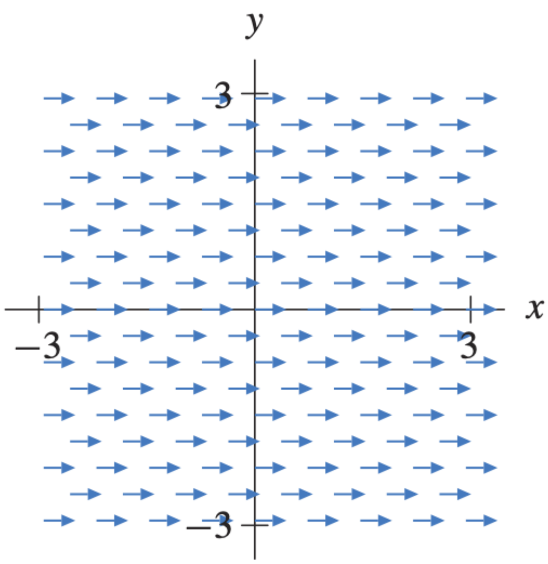
(a) (b)



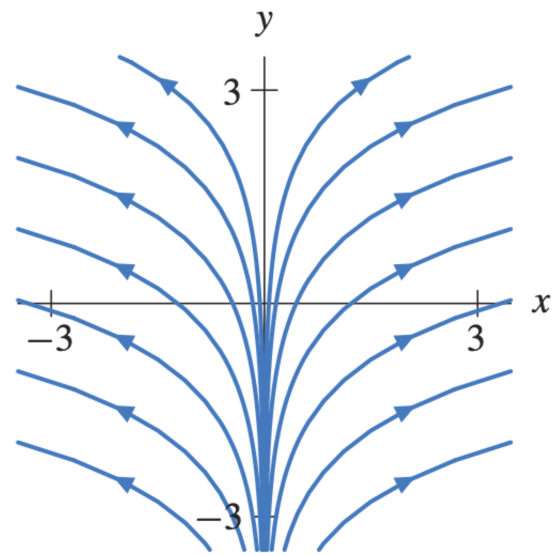


(c)

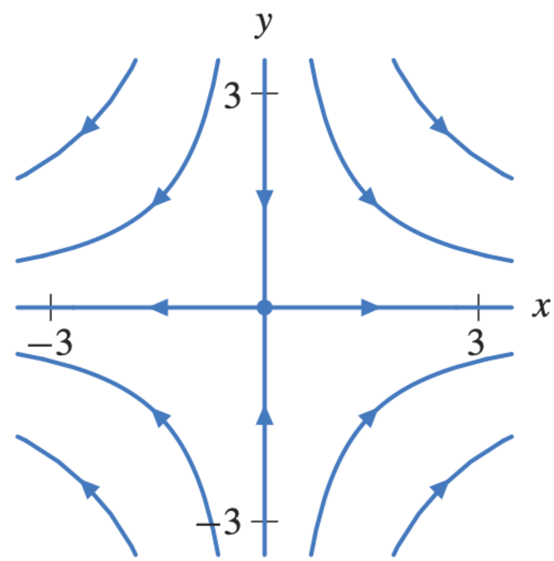
(d)

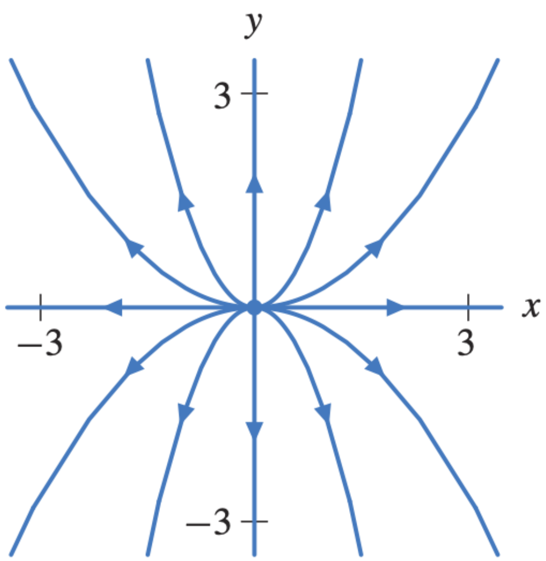


(b) Make a rough sketch, on paper, of the phase portrait of the system (i.e. sketch solution curves on your vector field) and confirm your answer using HPGSystemSolver.
Which of the following phase portraits most closely matches your sketch? [?/a/b/c/d]

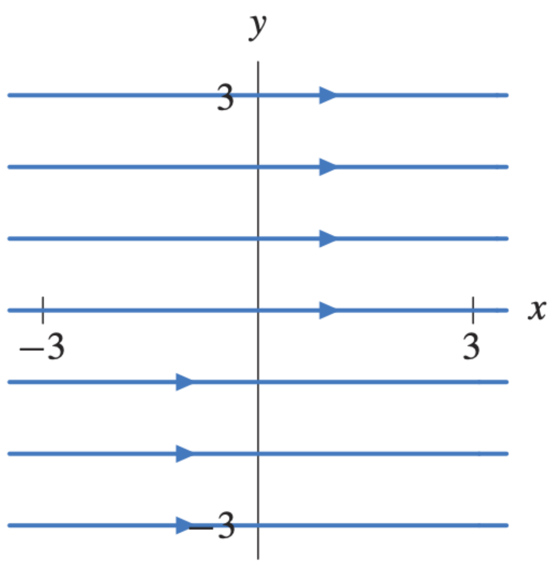


(a)





(c)



(c) Briefly describe the behavior of the solutions.

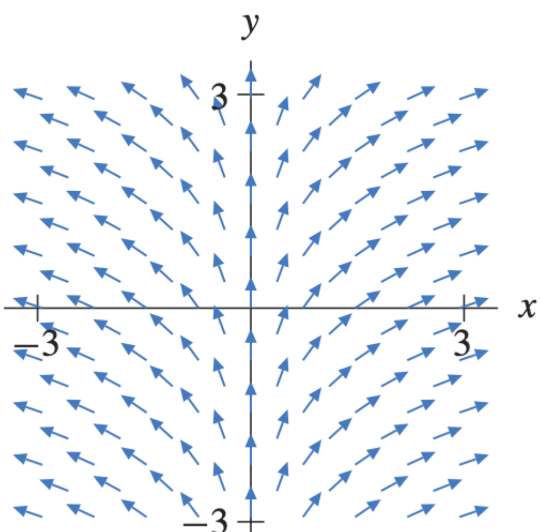
- As t increases, solutions move up and right if $x(0) > 0$, up and left if $x(0) < 0$.
- As t increases, solutions move toward the x-axis in the y-direction and away from the y-axis in the x-direction.
- As t increases, solutions move away from the equilibrium point at the origin.
- As t increases, solutions move along horizontal lines toward the right.

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_2.2/BDH_2.2.5.pg`

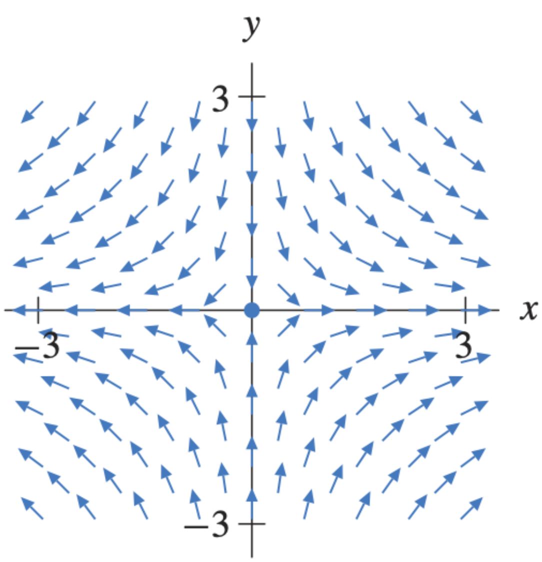
Consider the first-order system of differential equations

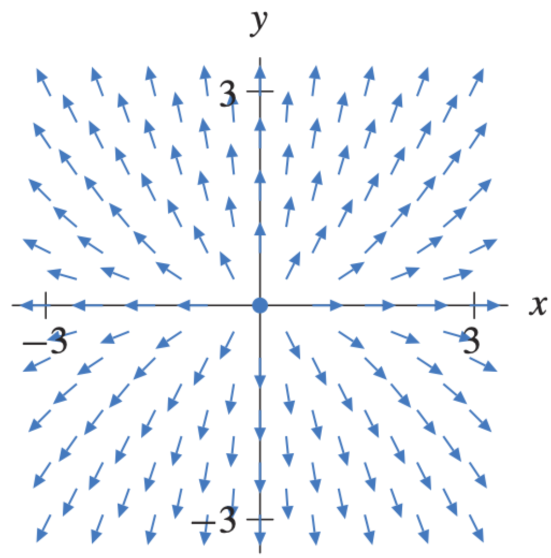
$$\frac{dx}{dt} = x$$
$$\frac{dy}{dt} = -y$$

(a) Sketch, on paper, enough vectors in the direction field to get a sense of its geometric structure (you should do this part of the exercise without the use of technology) and confirm your work with HPGSystemSolver from the DETools software.
Which of the following direction fields most closely matches your sketch? [?/a/b/c/d]

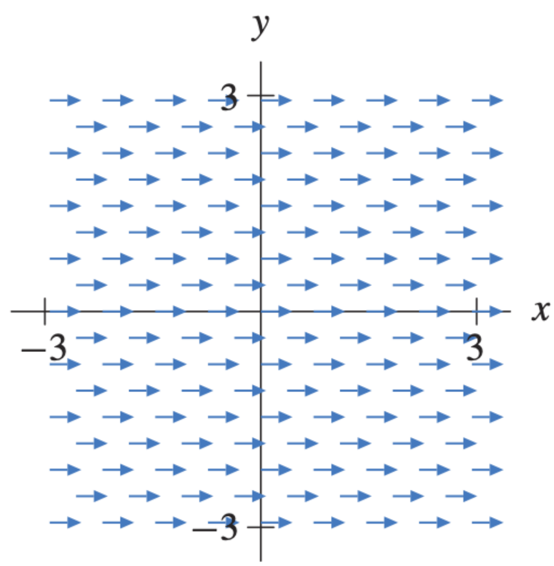


(a) (b)

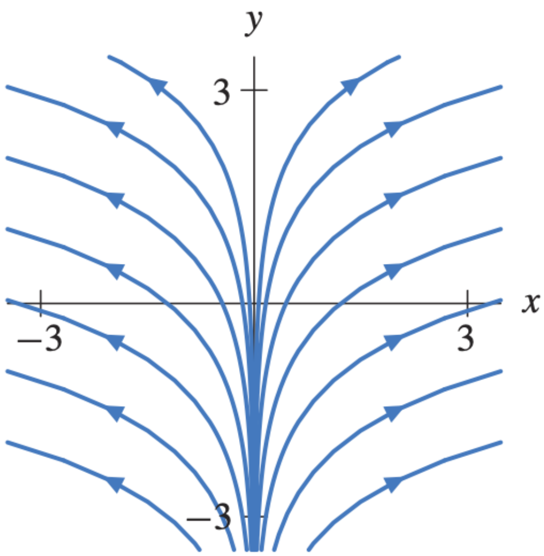




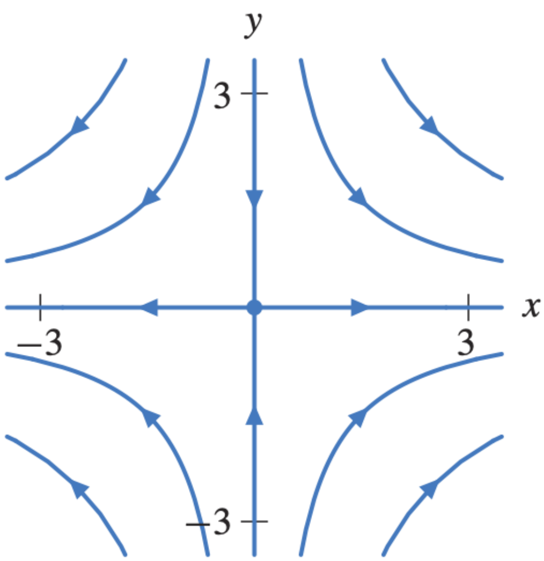
(c) (d)

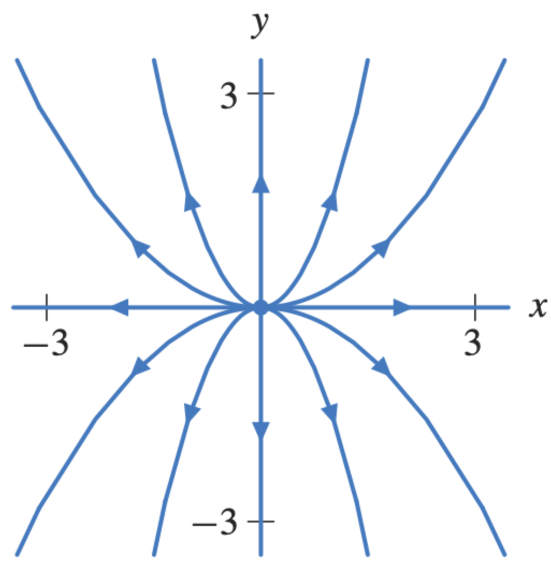


(b) Make a rough sketch, on paper, of the phase portrait of the system (i.e. sketch solution curves on your vector field) and confirm your answer using HPGSystemSolver.
Which of the following phase portraits most closely matches your sketch? [?/a/b/c/d]

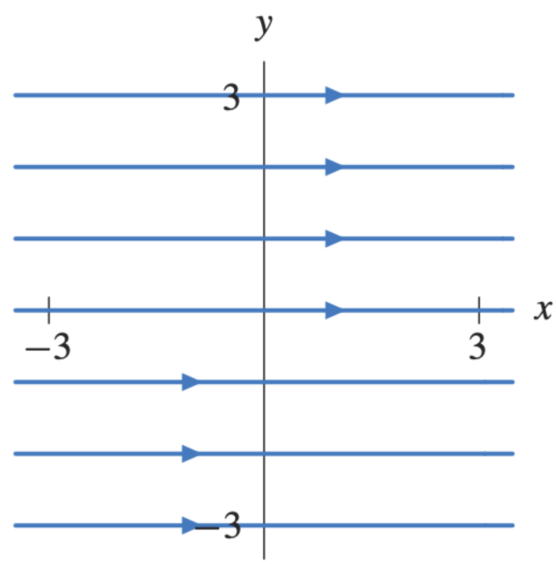


(a) (b)





(c) (d)



(c) Briefly describe the behavior of the solutions.

- As t increases, solutions move up and right if $x(0) > 0$, up and left if $x(0) < 0$.

- As t increases, solutions move toward the x-axis in the y-direction and away from the y-axis in the x-direction.

- As t increases, solutions move away from the equilibrium point at the origin.

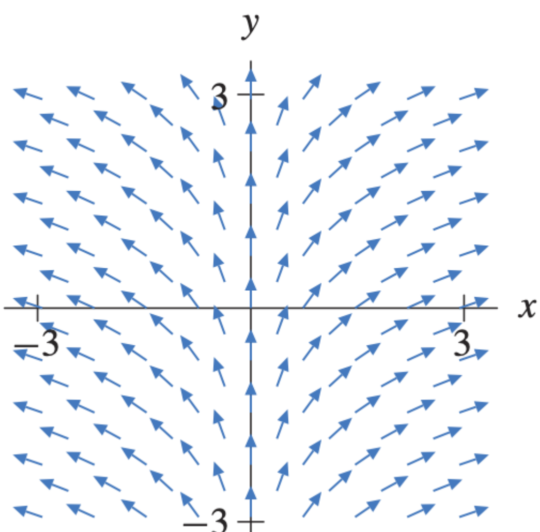
- As t increases, solutions move along horizontal lines toward the right.

Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_2.2/BDH_2.2.6.pg`

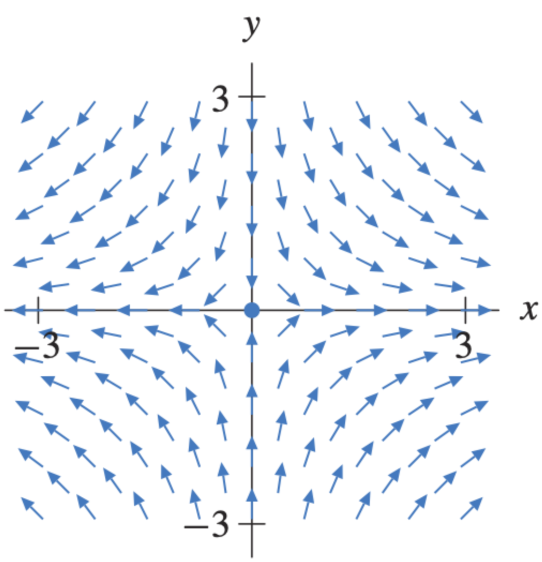
Consider the first-order system of differential equations

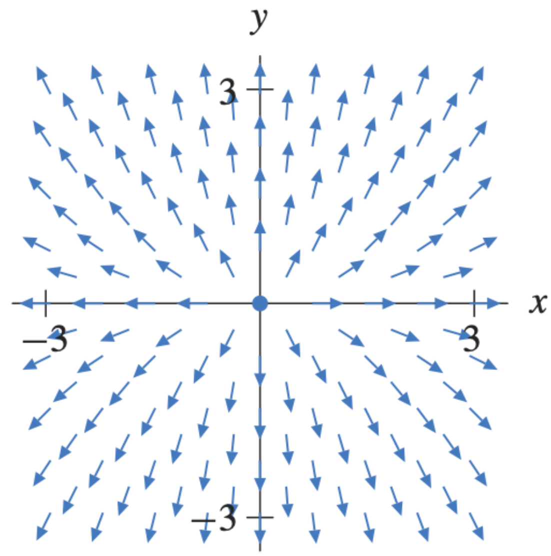
$$\frac{dx}{dt} = x$$
$$\frac{dy}{dt} = 2y$$

(a) Sketch, on paper, enough vectors in the direction field to get a sense of its geometric structure (you should do this part of the exercise without the use of technology) and confirm your work with HPGSystemSolver from the DETools software.
Which of the following direction fields most closely matches your sketch? [?/a/b/c/d]

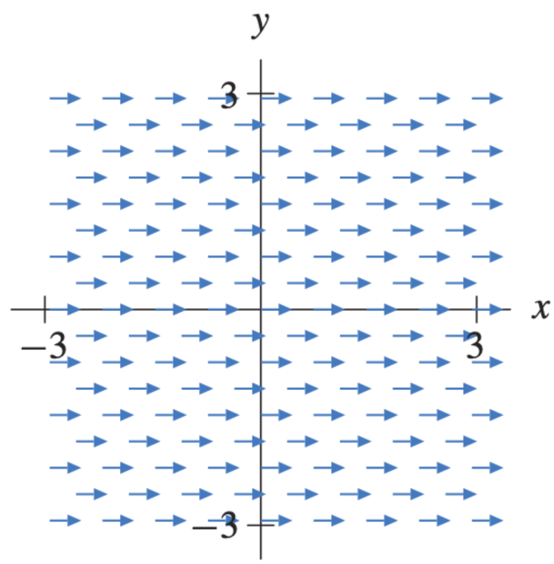


(a) (b)

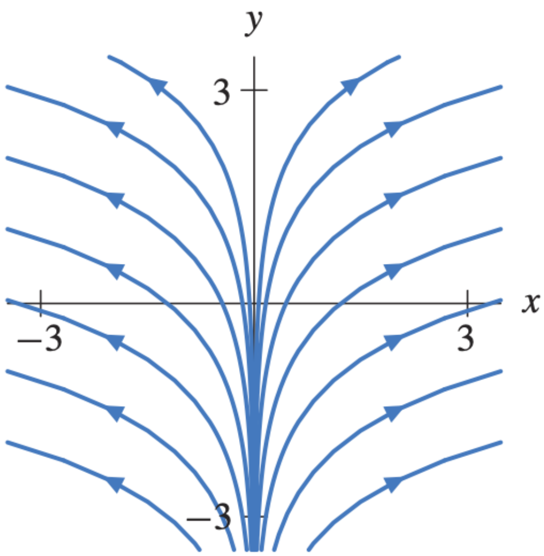




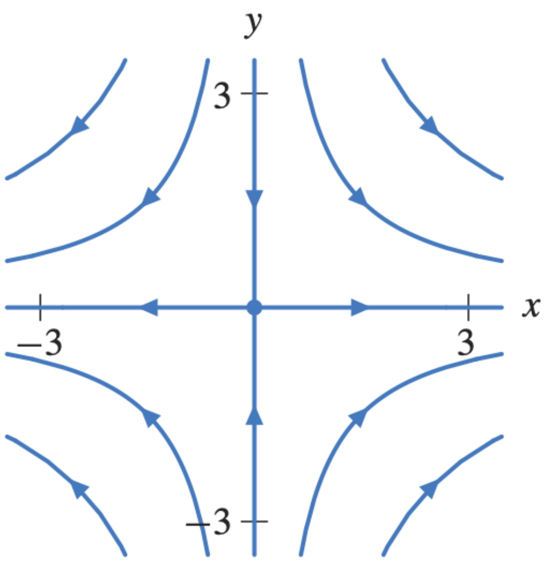
(c) (d)

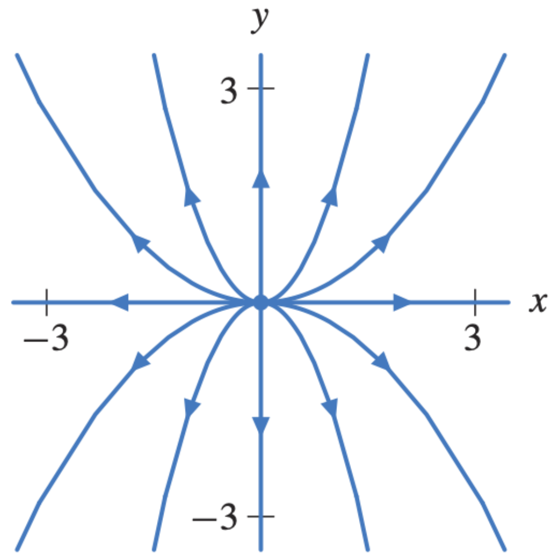


(b) Make a rough sketch, on paper, of the phase portrait of the system (i.e. sketch solution curves on your vector field) and confirm your answer using HPGSystemSolver.
Which of the following phase portraits most closely matches your sketch? [?/a/b/c/d]

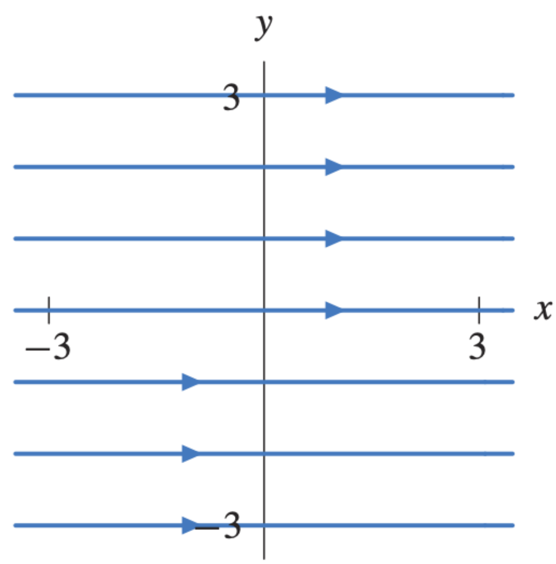


(a) (b)





(c) (d)



(c) Briefly describe the behavior of the solutions.

- As t increases, solutions move up and right if $x(0) > 0$, up and left if $x(0) < 0$.
- As t increases, solutions move toward the x-axis in the y-direction and away from the y-axis in the x-direction.
- As t increases, solutions move away from the equilibrium point at the origin.
- As t increases, solutions move along horizontal lines toward the right.

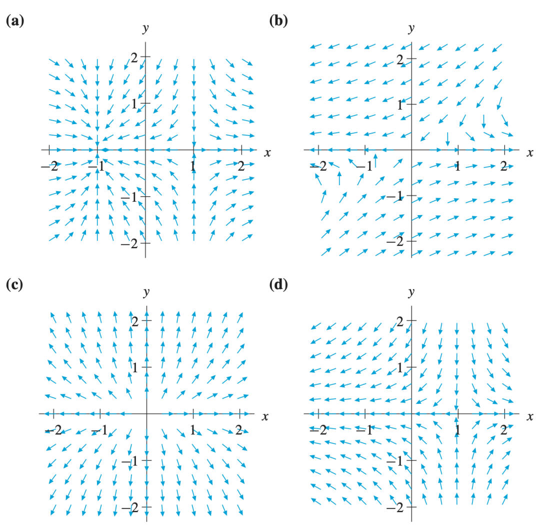
Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_2.2/BDH_2.2.11.pg`

Eight systems of differential equations and four direction fields are given below. Determine the system that corresponds to each direction field. You should do this exercise without using technology.

(i) $\frac{dx}{dt} = -x$, $\frac{dy}{dt} = y - 1$ (ii) $\frac{dx}{dt} = x^2 - 1$, $\frac{dy}{dt} = y$ (iii) $\frac{dx}{dt} = x + 2y$, $\frac{dy}{dt} = -y$ (iv) $\frac{dx}{dt} = 2x$, $\frac{dy}{dt} = y$

(v) $\frac{dx}{dt} = x$, $\frac{dy}{dt} = 2y$ (vi) $\frac{dx}{dt} = x - 1$, $\frac{dy}{dt} = -y$ (vii) $\frac{dx}{dt} = x^2 - 1$, $\frac{dy}{dt} = -y$ (viii) $\frac{dx}{dt} = x - 2y$, $\frac{dy}{dt} = -y$

(a) [?/i/ii/iii/iv/v/vi/vii/viii] (b) [?/i/ii/iii/iv/v/vi/vii/viii] (c) [?/i/ii/iii/iv/v/vi/vii/viii] (d) [?/i/ii/iii/iv/v/vi/vii/viii]



Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_2.2/BDH_2.2.13.pg`

Consider the system of equations

$$\frac{dx}{dt} = 4x - 7y + 2$$
$$\frac{dy}{dt} = 3x + 6y - 1.$$

(a) Find the equilibrium points of the system.
$(x, y) =$ __________
(format your answer as a list of points, i.e. (1,2), (3,4), (5,6).) help (points)

(b) Using HPGSystemSolver from the DETools software, sketch the direction field and phase portrait of the system. (no work to enter here)

(c) Briefly describe the behavior of typical solutions:

- As t increases, typical solutions spiral toward either (1, 0) or (-1, 0) depending on the initial condition.
- As t increases, typical solutions spiral toward one of the equilibria on the x-axis. Which equilibrium point the solution approaches depends on the initial condition.
- As t increases, typical solutions spiral away from the origin in the counter-clockwise direction.
- As t increases, typical solutions move on a circle around the origin, either counter-clockwise inside the unit circle, which consists entirely of equilibrium points, or clockwise outside the unit circle.

Problem 6. **(1 point)** `ustLibrary/ustDiffEq/setBDH_2.2/BDH_2.2.16.pg`

Consider the system of equations

$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = x - x^3 - y.$$

(a) Find the equilibrium points of the system.

$(x,y) =$ ______

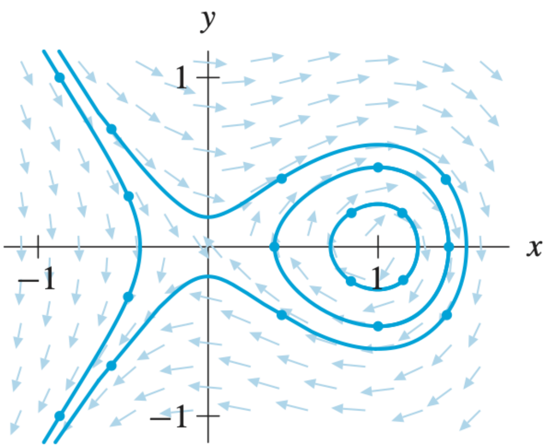
(format your answer as a list of points, i.e. (1,2), (3,4), (5,6).) help (points)

(b) Using HPGSystemSolver from the DETools software, sketch the direction field and phase portrait of the system. (no work to enter here)

(c) Briefly describe the behavior of typical solutions:

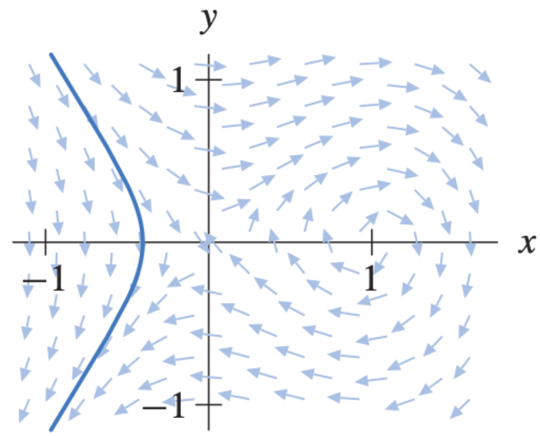
- As t increases, typical solutions spiral toward either (1, 0) or (-1, 0) depending on the initial condition.
- As t increases, typical solutions spiral toward one of the equilibria on the x-axis. Which equilibrium point the solution approaches depends on the initial condition.
- As t increases, typical solutions spiral away from the origin in the counter-clockwise direction.
- As t increases, typical solutions move on a circle around the origin, either counter-clockwise inside the unit circle, which consists entirely of equilibrium points, or clockwise outside the unit circle.

Problem 7. **(1 point)** `ustLibrary/ustDiffEq/setBDH_2.2/BDH_2.2.21.pg`



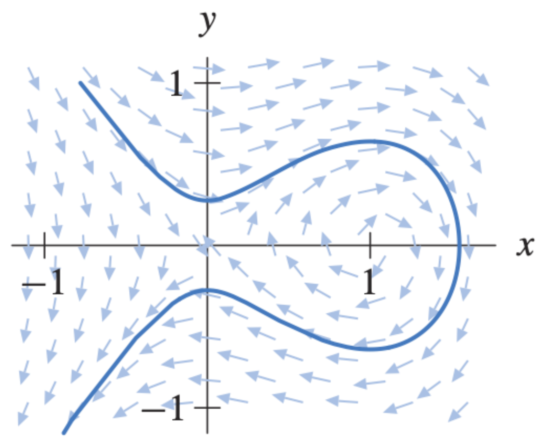
Consider the four solution curves in the phase portrait (i–iv) and the four pairs of $x(t)-$ and $y(t)-$graphs (a–d) shown below.

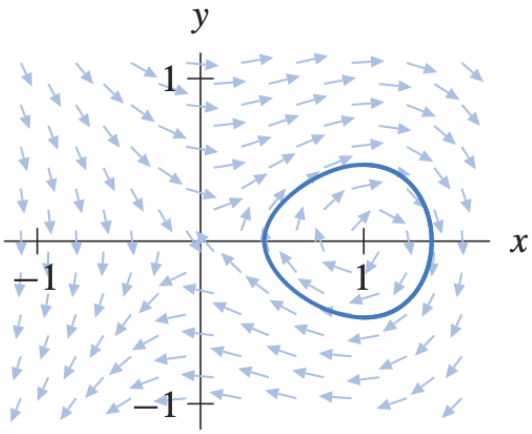
Match each solution curve with its corresponding pair of $x(t)-$ and $y(t)-$graphs.



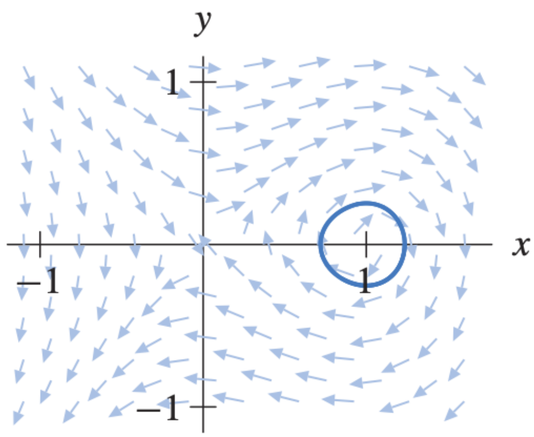
(i)

(ii)



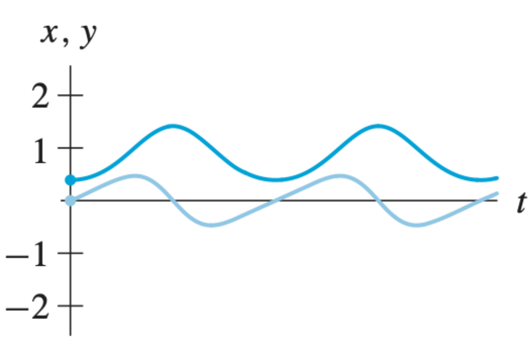


(iii)

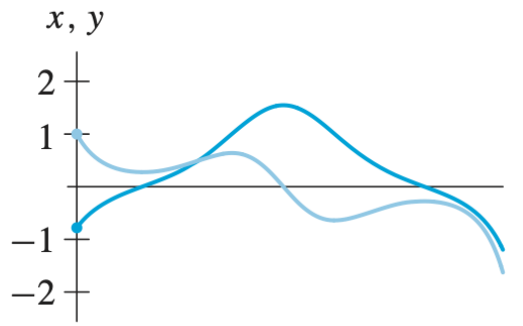


(iv)

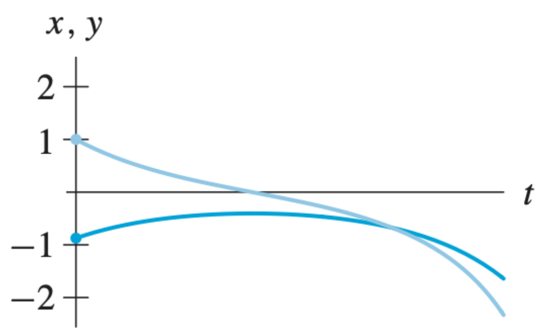
[?/i/ii/iii/iv] (a)



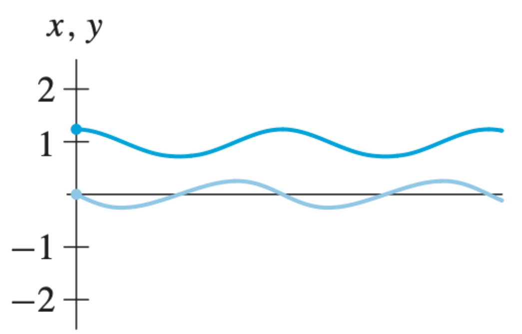
[?/i/ii/iii/iv] (b)



[?/i/ii/iii/iv] (c)



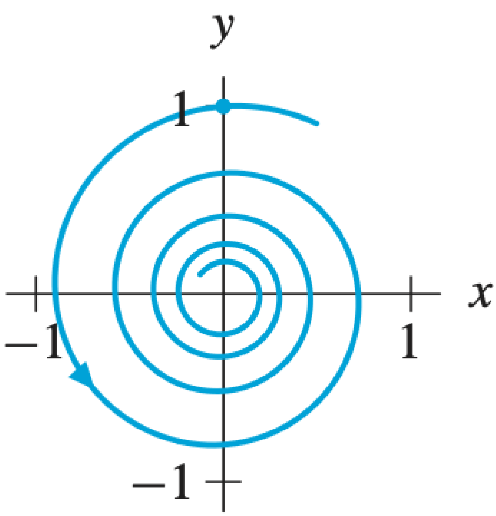
[?/i/ii/iii/iv] (d)



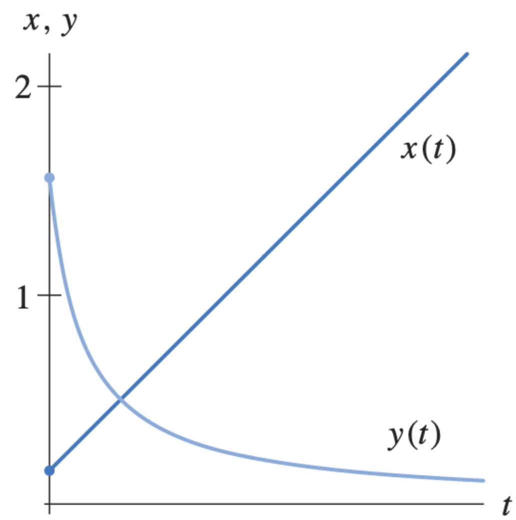
(click on any graph to embiggen)

Problem 8. **(1 point)** `ustLibrary/ustDiffEq/setBDH_2.2/BDH_2.2.23.pg`

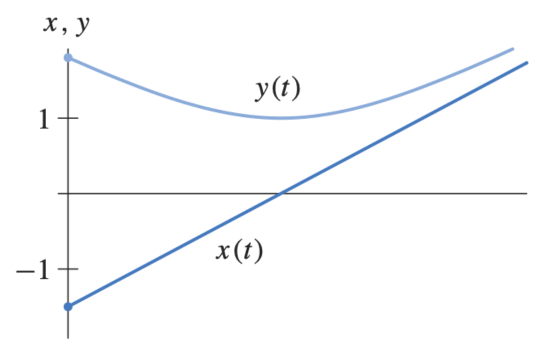
A solution curve in the xy-plane and an initial condition on that curve are specified.

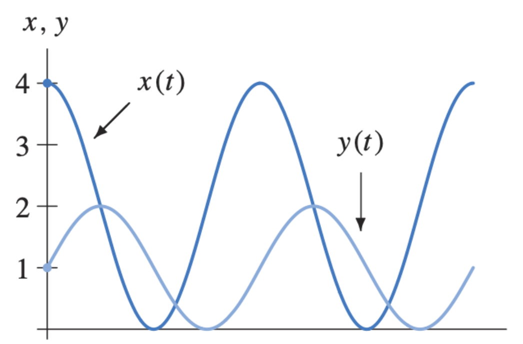


Sketch, on paper, the $x(t)$- and $y(t)$-graphs for the solution. Which of the following direction fields most closely matches your sketch? [?/a/b/c/d]

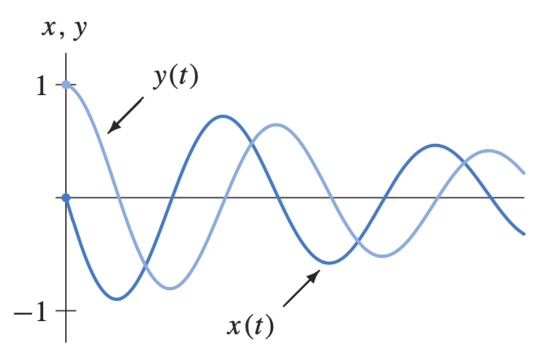


(a) (b)



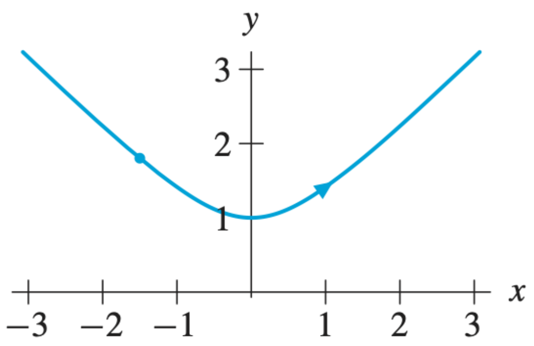


(c) (d)

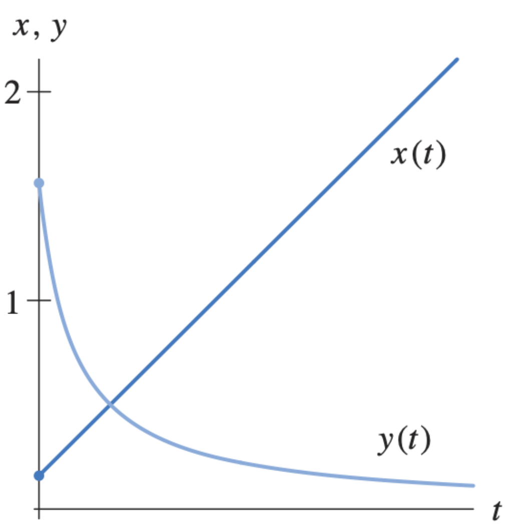


Problem 9. **(1 point)** `ustLibrary/ustDiffEq/setBDH_2.2/BDH_2.2.25.pg`

A solution curve in the xy-plane and an initial condition on that curve are specified.

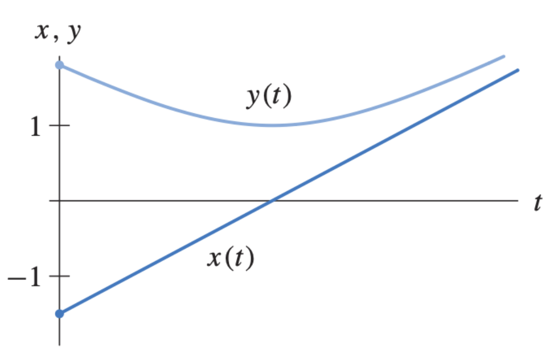


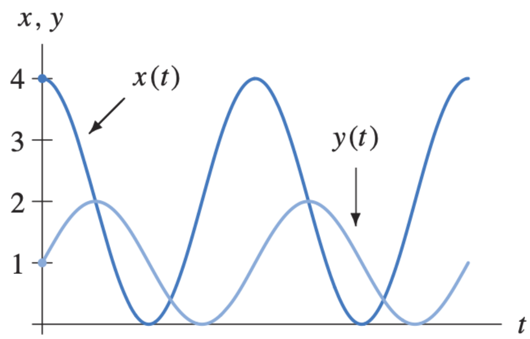
Sketch, on paper, the $x(t)$- and $y(t)$-graphs for the solution. Which of the following direction fields most closely matches your sketch? [?/a/b/c/d]



(a)

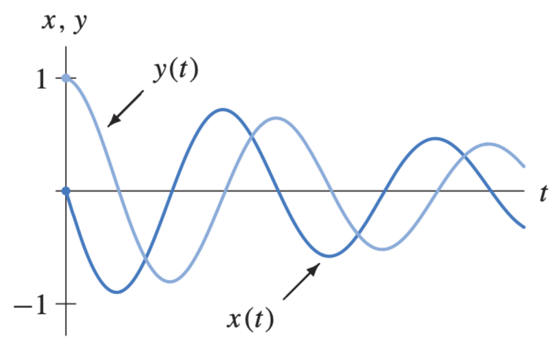
(b)





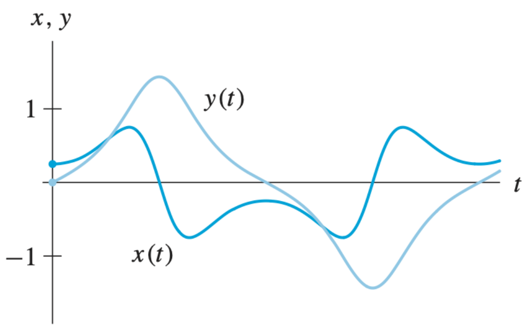
(c)

(d)

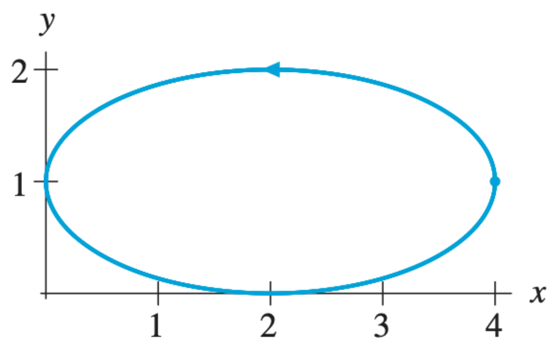


Problem 10. **(1 point)** `ustLibrary/ustDiffEq/setBDH_2.2/BDH_2.2.27.pg`

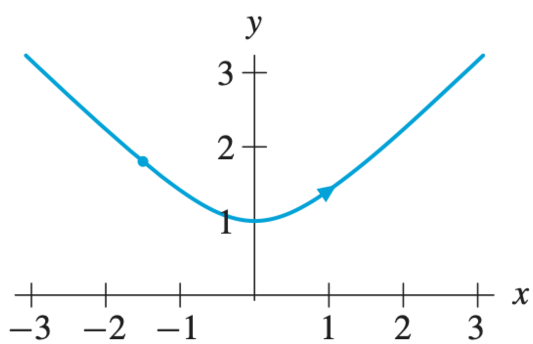
The following graphs are the $x(t)$- and the $y(t)$-graphs for a solution curve in the xy-phase plane.

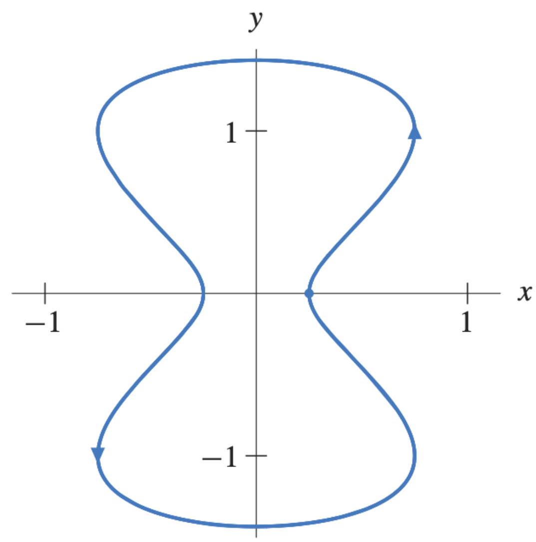


Sketch, on paper, the solution curve, and indicate the direction that the solution travels as time increases.
Which of the following curves most closely matches your sketch? [?/a/b/c/d]

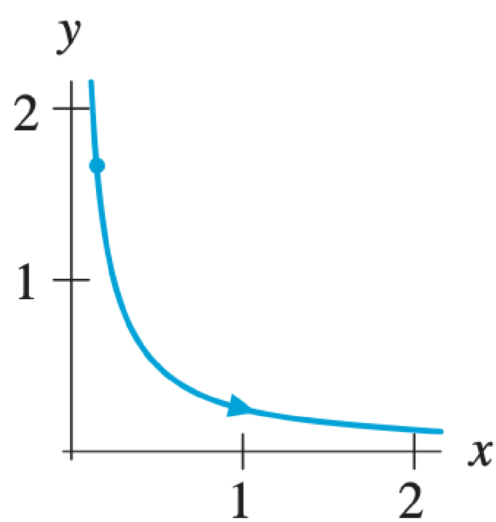


(a) (b)





(c) (d)



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Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_2.3-4/BDH_2.3.1.pg`

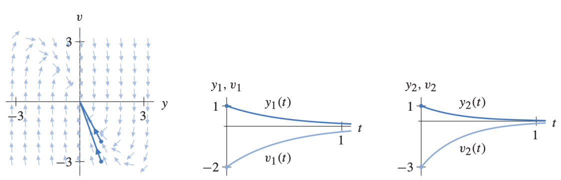
Consider the harmonic oscillator equation for y(t)

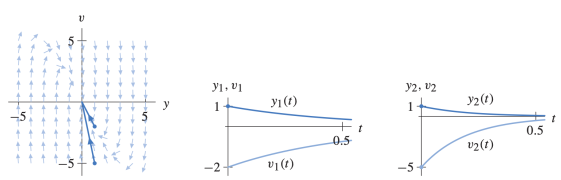
$$\frac{d^2y}{dt^2}+7\frac{dy}{dt}+10y=0.$$

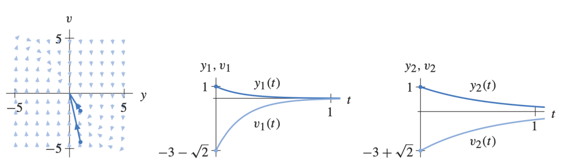
(a) Using HPGSystemSolver from the DETools software, sketch the associated direction field. (you'll need to convert the second-order equation into a first-order system; no work to submit here)

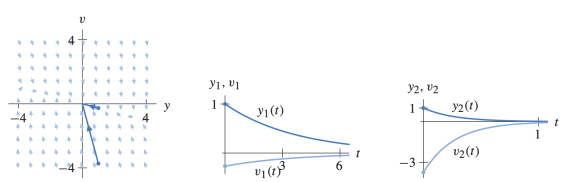
(b) Using the guess-and-test method, find two nonzero solutions that are not multiples of one another.
$y_1(t) =$ __________
$y_2(t) =$ __________

(c) For each solution, plot both its solution curve in the yv-plane and its $y(t)$- and $v(t)$-graphs. Use the direction field from (a), but sketch the solution curves in the yv-plane and the $y(t)$- and $v(t)$-graphs by hand on paper.
Which of the following curves most closely matches your graph? [?/a/b/c/d]

(a) 

(b) 

(c) 

(d) 

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_2.3-4/BDH_2.3.3.pg`

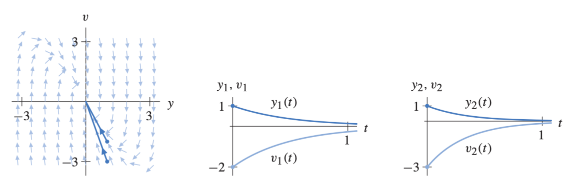
Consider the harmonic oscillator equation for y(t)

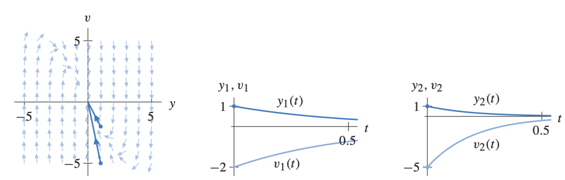
$$\frac{d^2y}{dt^2}+4\frac{dy}{dt}+y=0.$$

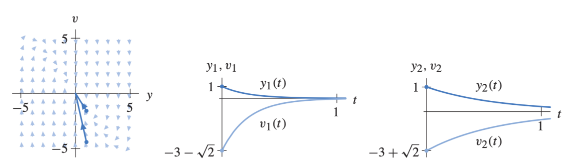
(a) Using HPGSystemSolver from the DETools software, sketch the associated direction field. (you'll need to convert the second-order equation into a first-order system; no work to submit here)

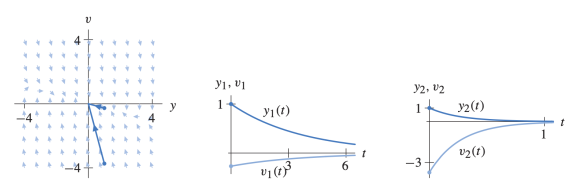
(b) Using the guess-and-test method, find two nonzero solutions that are not multiples of one another.
$y_1(t) =$ __________
$y_2(t) =$ __________

(c) For each solution, plot both its solution curve in the yv-plane and its $y(t)$- and $v(t)$-graphs. Use the direction field from (a), but sketch the solution curves in the yv-plane and the $y(t)$- and $v(t)$-graphs by hand on paper.
Which of the following curves most closely matches your graph? [?/a/b/c/d]

(a) 

(b) 

(c) 

(d) 

Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_2.3-4/BDH_2.3.5.pg`

In the damped harmonic oscillator $m\frac{d^2y}{dt^2}+b\frac{dy}{dt}+ky=0$, we assume that the coefficients m, b, and k are positive. However, the rationale underlying the guess-and-test method made no such assumption, and the same analytic technique can be used if some or all of the coefficients of the equation are negative.

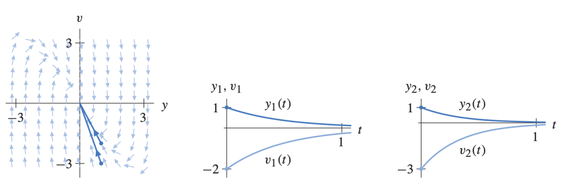
Consider the harmonic oscillator equation for y(t)

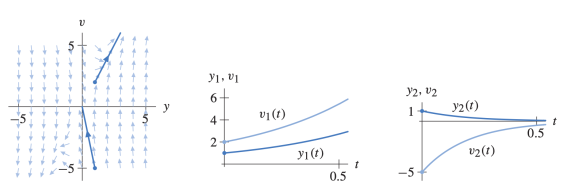
$$\frac{d^2y}{dt^2}+3\frac{dy}{dt}-10y=0.$$

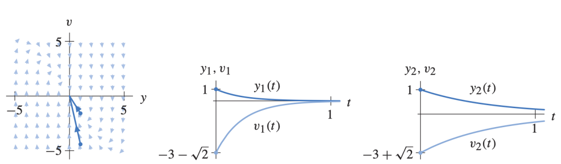
(a) Using HPGSystemSolver from the DETools software, sketch the associated direction field. (you'll need to convert the second-order equation into a first-order system; no work to submit here)

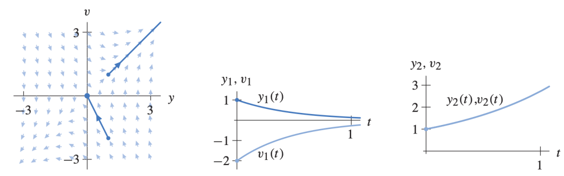
(b) Using the guess-and-test method, find two nonzero solutions that are not multiples of one another.
$y_1(t) =$ __________
$y_2(t) =$ __________

(c) For each solution, plot both its solution curve in the yv-plane and its $y(t)$- and $v(t)$-graphs. Use the direction field from (a), but sketch the solution curves in the yv-plane and the $y(t)$- and $v(t)$-graphs by hand on paper.
Which of the following curves most closely matches your graph? [?/a/b/c/d]
(click on any graph to embiggen)


(a)


(b)


(c)


(d)

What is different in this case, where k is negative?

Problem 4. **(2 points)** `ustLibrary/ustDiffEq/setBDH_2.3-4/BDH_2.4.1.pg`

Consider the system
$$\frac{dx}{dt}=2x+2y$$
$$\frac{dy}{dt}=x+3y.$$
For the given functions $\mathbf{Y}(t)=(x(t),y(t))$, determine if $\mathbf{Y}(t)$ is a solution.

(a) $(x(t),y(t))=(2e^t,-e^t)$

- Yes, a solution
- No, not a solution

(b) $(x(t),y(t))=(2e^t-e^{4t},-e^t+e^{4t})$

- Yes, a solution
- No, not a solution

Problem 5. **(3 points)** `ustLibrary/ustDiffEq/setBDH_2.3-4/BDH_2.4.7.pg`

Consider the partially decoupled system

$$\frac{dx}{dt} = 2x + y$$
$$\frac{dy}{dt} = -y.$$

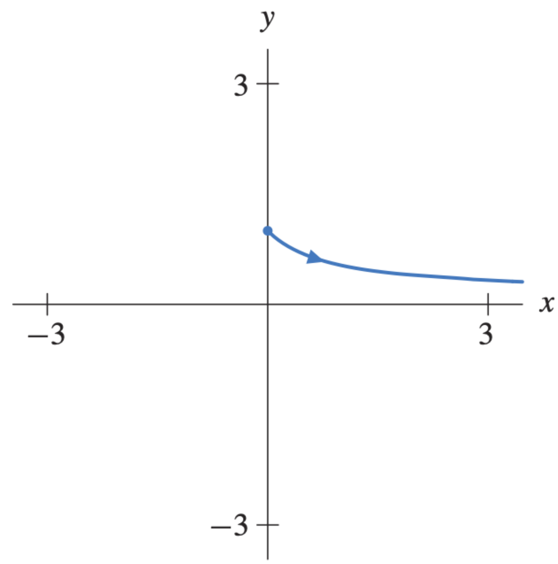
(a) Is $\mathbf{Y}(t) = (x(t), y(t)) = (e^{2t} - e^{-t}, e^{-2t})$ a solution?

- Yes, a solution
- No, not a solution

(b) Derive the general solution to this system.
$x(t) =$ ________ use k1 and k2 for any constants appearing in $x(t)$
$y(t) =$ ________ use k1 for any constant appearing in $y(t)$

(c) Using the result of (b), determine the solution that satisfies the initial condition $\mathbf{Y}(0) = (x(0), y(0)) = (0, 1)$:
$\mathbf{Y}(t) = (x(t), y(t)) = ($ ________ , ________ $)$

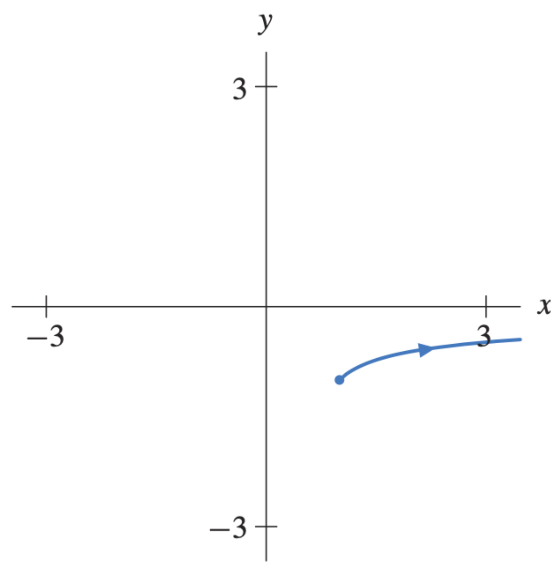
(d) Using HPGSystemSolver, generate the direction field for the system. Plot, on paper, the solution curve from (c) in the xy- phase plane.
Which of the following most closely matches your sketch? [?/i/ii]



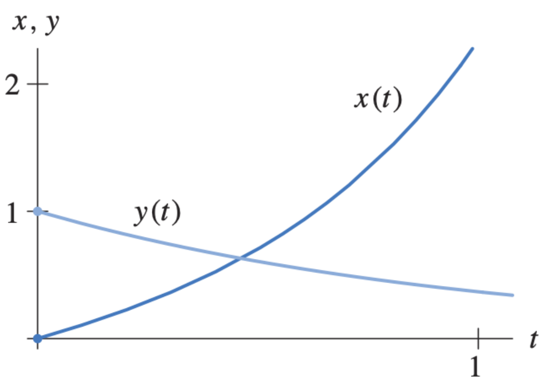
(i)

,

(ii)

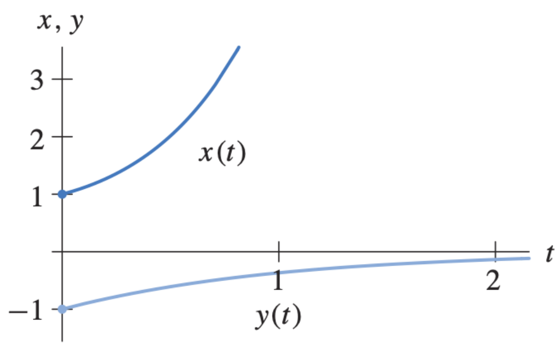


(e) Using only your solution curve from (d), sketch, on paper, the $x(t)$- and $y(t)$-graphs.
Which of the following most closely matches your sketch? [?/i/ii]



(i)

(ii)



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Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.1/BDH_3.1.5.pg`

Rewrite the linear system

$$\frac{dx}{dt} = 2x + y$$
$$\frac{dy}{dt} = x + y$$

in matrix form.

$$\frac{\mathbf{dY}}{dt} = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix} \mathbf{Y}, \text{ where } \mathbf{Y} = \begin{pmatrix} x \\ y \end{pmatrix}$$

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.1/BDH_3.1.9.pg`

Rewrite the linear system

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \begin{pmatrix} -3 & 2\pi \\ 4 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

in component form.

$\frac{dx}{dt} =$ ______________________________

$\frac{dy}{dt} =$ ______________________________

Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.1/BDH_3.1.25.pg`

Consider the linear system

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

(a) Show that the function

$$\mathbf{Y}(t) = \begin{pmatrix} te^{2t} \\ -(t+1)e^{2t} \end{pmatrix}$$

is a solution to the differential equation.

- I verified that it is a solution.
- I discovered that it is **not** a solution.

(b) Solve the initial value problem

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}, \quad \mathbf{Y}(0) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}.$$

$$\mathbf{Y}(t) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$$

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.1/BDH_3.1.27.pg`

Consider the linear system

$$\frac{d\mathbf{Y}}{dt} = \mathbf{AY}, \text{ where } \mathbf{Y}(t) = \begin{pmatrix} x \\ y \end{pmatrix}$$

with coefficient matrix

$$\mathbf{A} = \begin{pmatrix} -2 & -1 \\ 2 & -5 \end{pmatrix}.$$

Consider the two functions

$$\mathbf{Y}_1(t) = \left(e^{-3t} - 2e^{-4t},\, e^{-3t} - 4e^{-4t}\right)$$

and

$$\mathbf{Y}_2(t) = \left(2e^{-3t} + e^{-4t},\, 2e^{-3t} + 2e^{-4t}\right)$$

and the initial value

$$\mathbf{Y}(0) = (2, 3).$$

(a) Solutions?

(a) Check that the two functions are solutions of the system; if they are not solutions, then stop.

- I verified they are both solutions.
- $\mathbf{Y}_1(t)$ is **not** a solution.
- $\mathbf{Y}_2(t)$ is **not** a solution.
- Neither function is a solution.

(b) Linearly independent?

(b) Check that the two solutions are linearly independent; if they are not linearly independent, then stop.

- I verified they are linearly independent.
- I discovered that they are **not** linearly independent.

(c) Initial value.

(c) Find the solution to the linear system with the given initial value.

$$\mathbf{Y}(t) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$$

Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.1/BDH_3.1.29.pg`

Consider the linear system

$$\frac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y}, \text{ where } \mathbf{Y}(t) = \begin{pmatrix} x \\ y \end{pmatrix}$$

with coefficient matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}.$$

Consider the two functions

$$\mathbf{Y}_1(t) = \left(-e^{-t} + 12e^{3t},\ e^{-t} + 4e^{3t}\right)$$
$$\mathbf{Y}_2(t) = \left(e^{-t},\ 2e^{-t}\right)$$

and the initial value

$$\mathbf{Y}(0) = (2, 3).$$

(a) Solutions?

(a) Check that the two functions are solutions of the system; if they are not solutions, then stop.

- I verified they are both solutions.
- $\mathbf{Y}_1(t)$ is **not** a solution.
- $\mathbf{Y}_2(t)$ is **not** a solution.
- Neither function is a solution.

(b) Linearly independent?

- I said stop, but can't figure out how to make WeBWorK do that.

(c) Initial value.

- I said stop, but can't figure out how to make WeBWorK do that.

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Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.2I/BDH_3.2.1abc.pg`

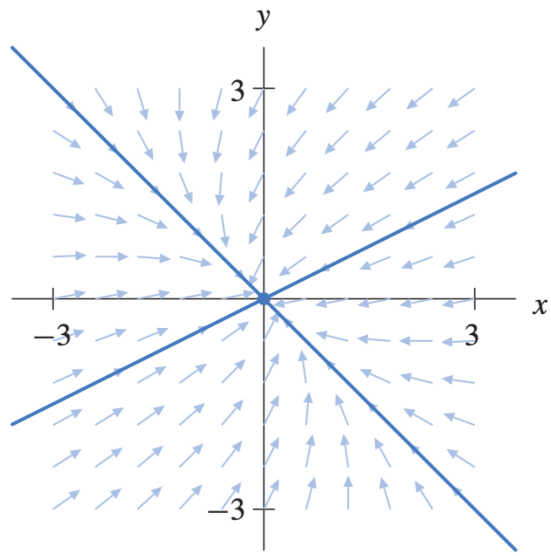
Consider the system

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 3 & 2 \\ 0 & -2 \end{pmatrix} \mathbf{Y}.$$

(a) Compute the eigenvalues.
$\lambda =$ _____ (enter your answer as a comma-separated list of numbers)

(b) For each eigenvalue, compute an associated eigenvector.
For the smaller eigenvalue, $v =$ _____
For the larger eigenvalue, $v =$ _____
(enter your answers using angle brackets "¡1,2¿", or enter "DNE")

(c) Using HPGSystemSolver, sketch the direction field for the system, and plot the straight-line solutions. Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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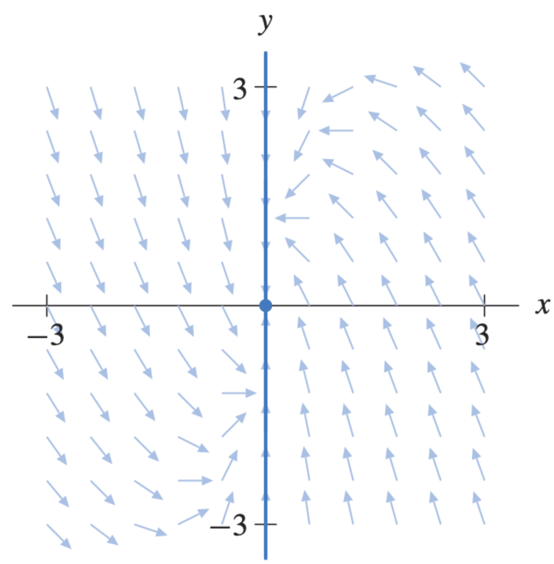

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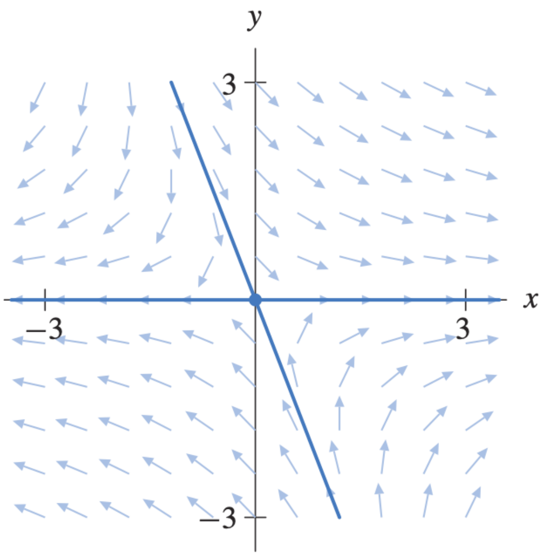

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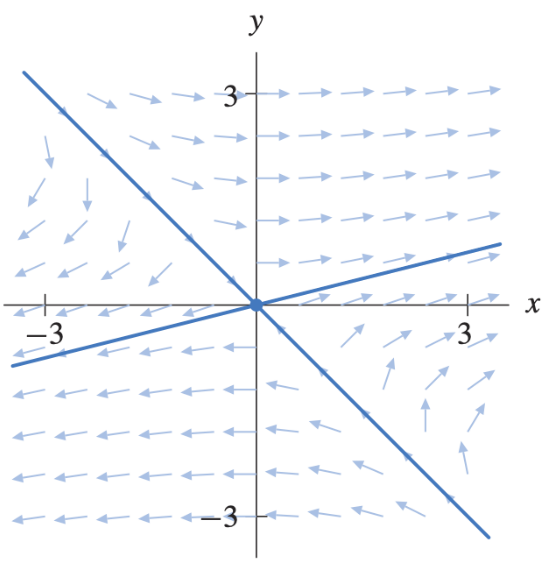


iv)

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.2I/BDH_3.2.3abc.pg`

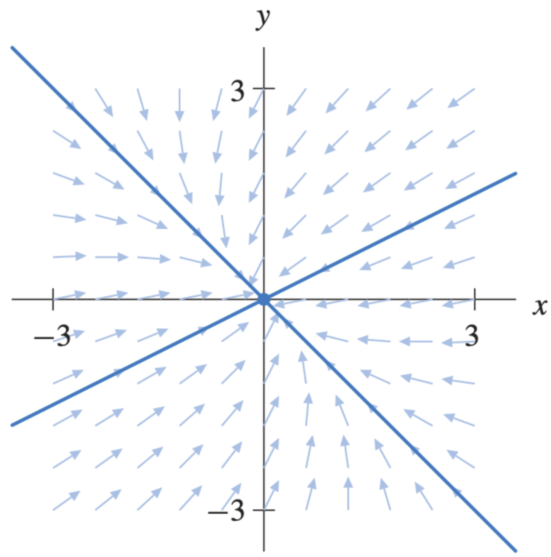
Consider the system

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \begin{pmatrix} -5 & -2 \\ -1 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

(a) Compute the eigenvalues.
$\lambda =$ _____ (enter your answer as a comma-separated list of numbers)

(b) For each eigenvalue, compute an associated eigenvector.
For the smaller eigenvalue, $v =$ _____
For the larger eigenvalue, $v =$ _____
(enter your answers using angle brackets "¡1,2¿", or enter "DNE")

(c) Using HPGSystemSolver, sketch the direction field for the system, and plot the straight-line solutions. Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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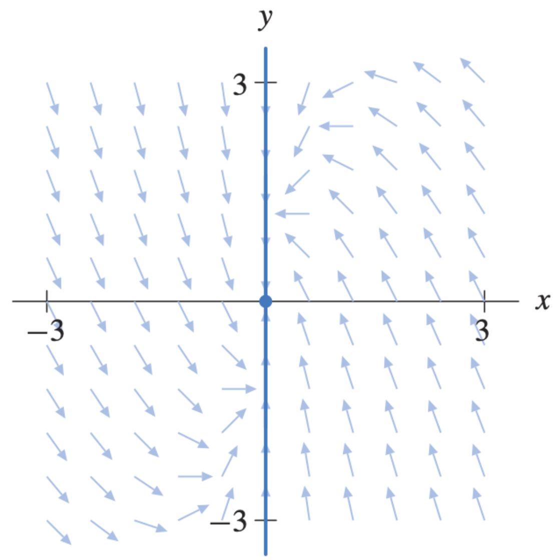

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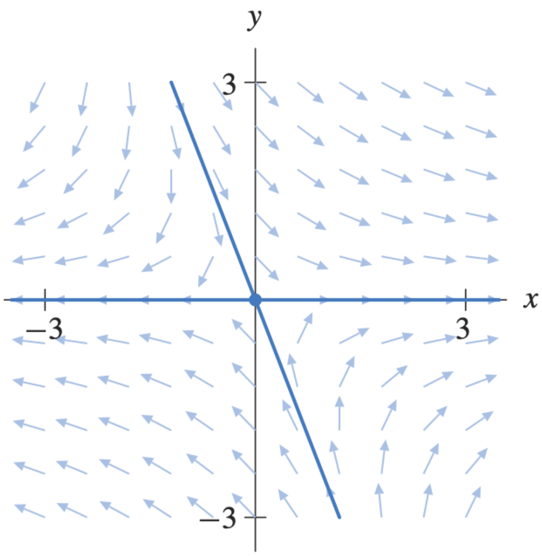

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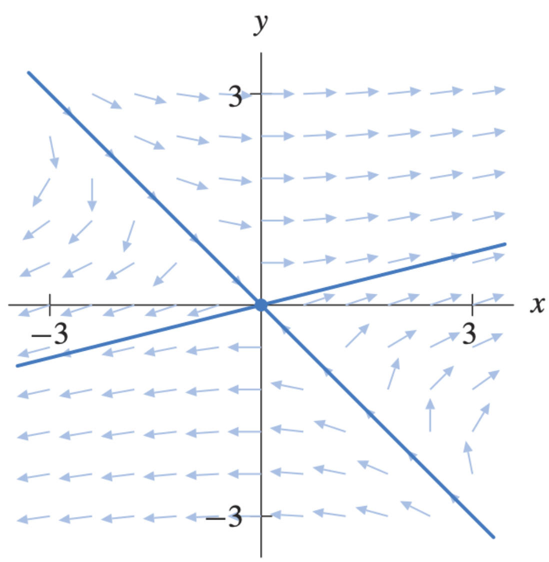

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Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.2I/BDH_3.2.5abc.pg`

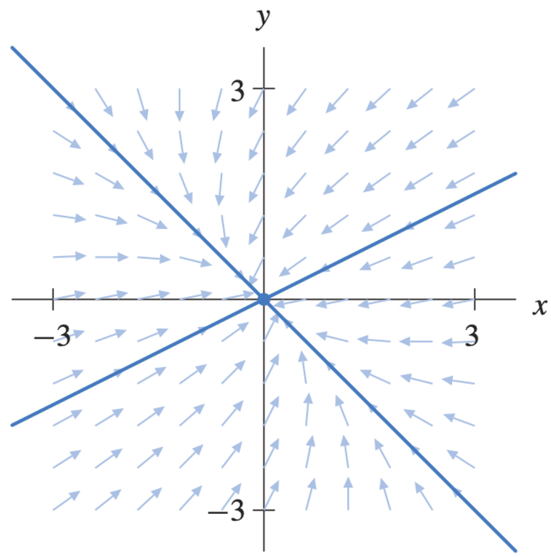
Consider the system

$$\frac{dx}{dt} = -\frac{x}{2}$$
$$\frac{dy}{dt} = x - \frac{y}{2}.$$

(a) Compute the eigenvalues.
$\lambda =$ ____ (enter your answer as a comma-separated list of numbers)

(b) For each eigenvalue, compute an associated eigenvector.
For the smaller eigenvalue, $v =$ ____
For the larger eigenvalue, $v =$ ____
(enter your answers using angle brackets "¡1,2¿", or enter "DNE")

(c) Using HPGSystemSolver, sketch the direction field for the system, and plot the straight-line solutions. Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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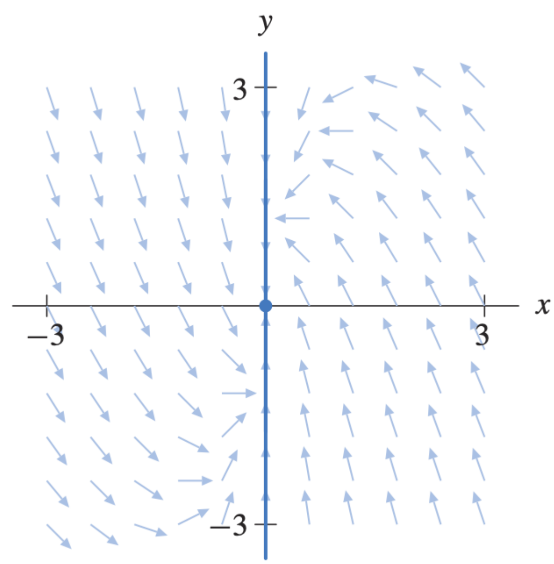

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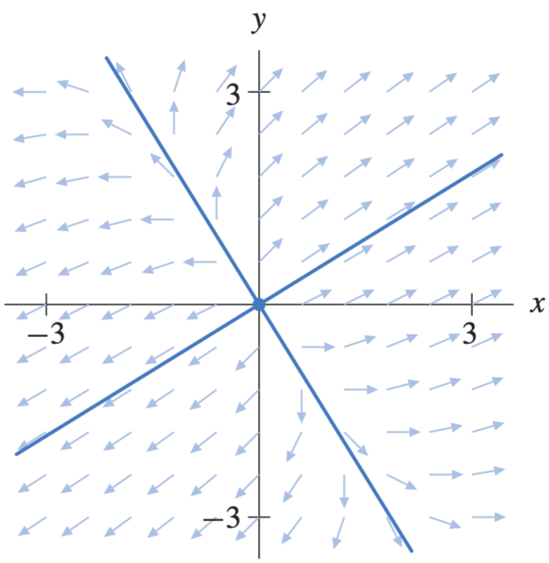

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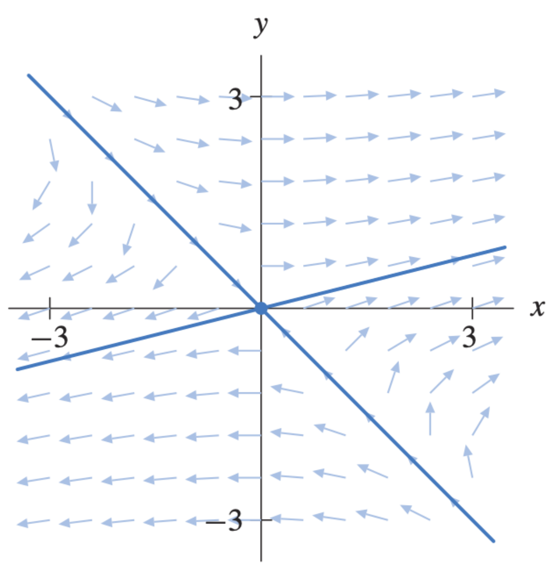


Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.2I/BDH_3.2.7abc.pg`

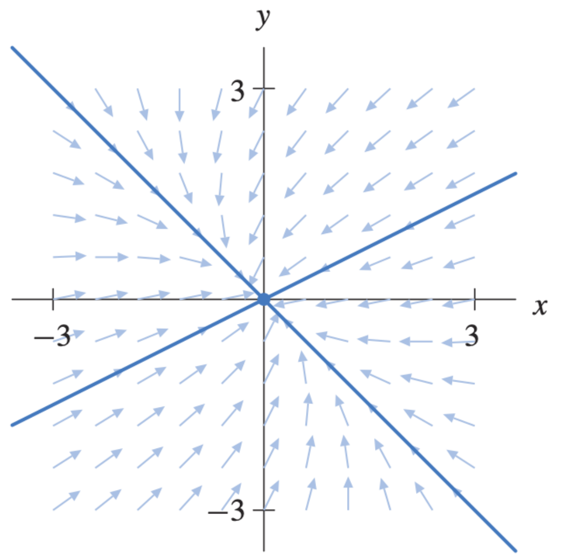
Consider the system

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

(a) Compute the eigenvalues.
$\lambda =$ _____ (enter your answer as a comma-separated list of numbers)

(b) For each eigenvalue, compute an associated eigenvector.
For the smaller eigenvalue, $v =$ _____
For the larger eigenvalue, $v =$ _____
(enter your answers using angle brackets "¡1,2¿", or enter "DNE")

(c) Using HPGSystemSolver, sketch the direction field for the system, and plot the straight-line solutions. Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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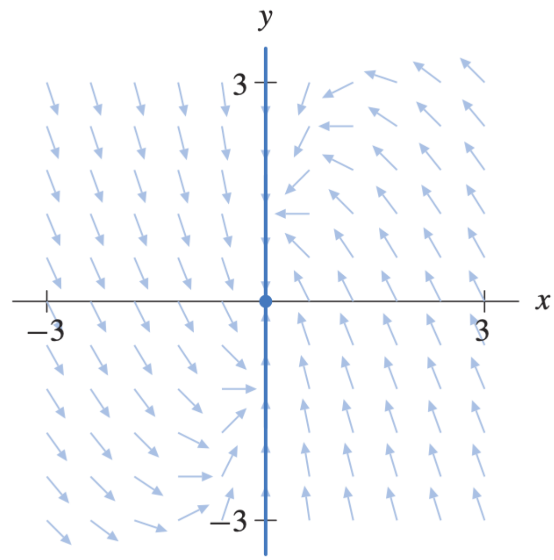

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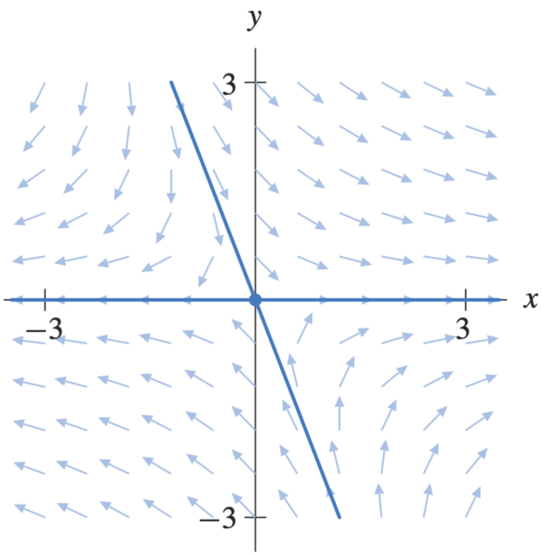

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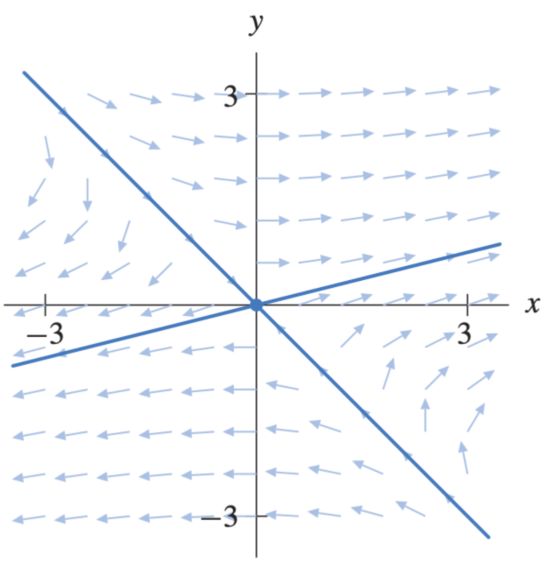


iv)

Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.2I/BDH_3.2.9abc.pg`

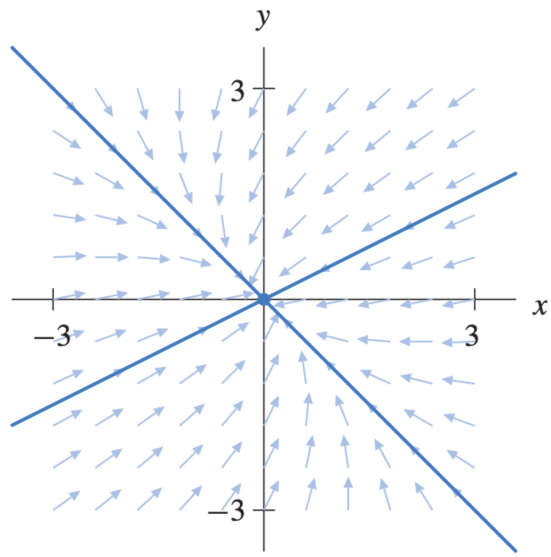
Consider the system

$$\frac{dx}{dt} = 2x + y$$
$$\frac{dy}{dt} = x + y.$$

(a) Compute the eigenvalues.
$\lambda =$ ________ (enter your answer as a comma-separated list of numbers)

(b) For each eigenvalue, compute an associated eigenvector.
For the smaller eigenvalue, $v =$ ________
For the larger eigenvalue, $v =$ ________
(enter your answers using angle brackets "¡1,2¿", or enter "DNE")

(c) Using HPGSystemSolver, sketch the direction field for the system, and plot the straight-line solutions. Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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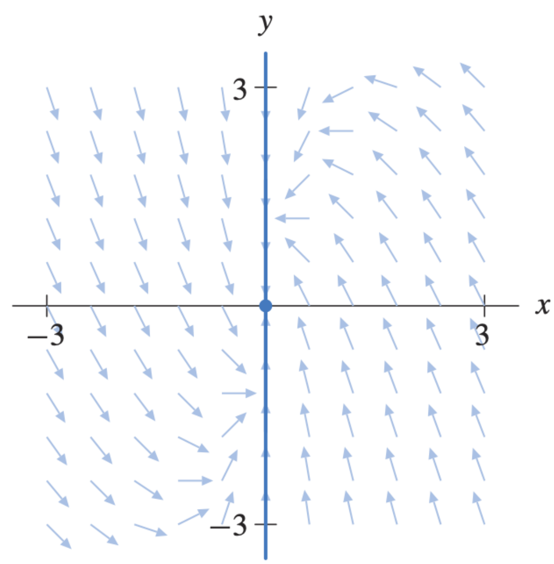

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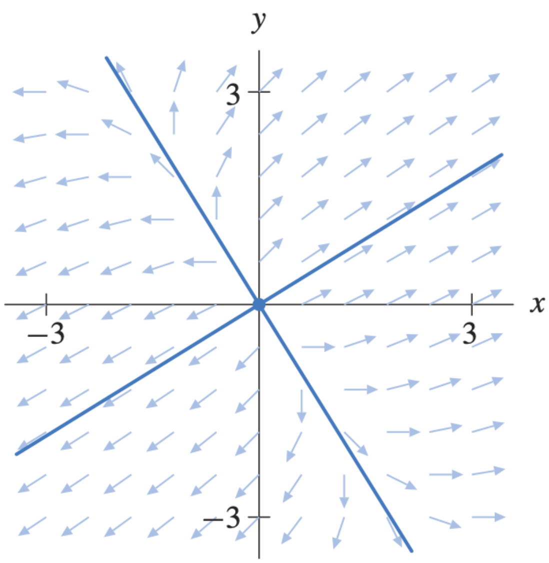
(iii) 

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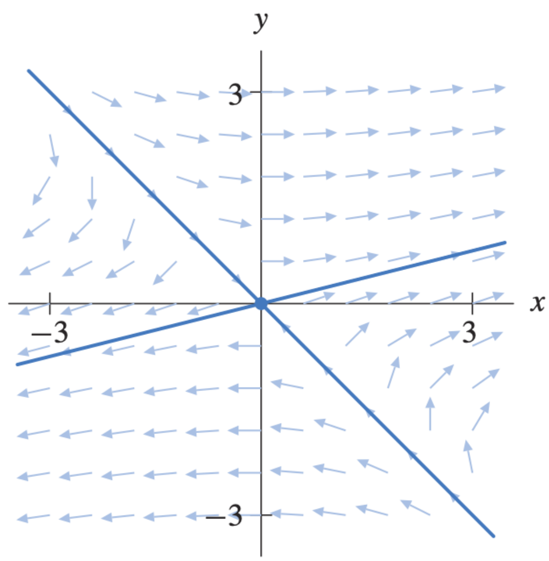
iv) 

Problem 6. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.2I/BDH_3.2.15.pg`

A matrix of the form

$$\mathbf{A} = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$$

is called *diagonal* .

(a) Find the eigenvalues of **A**.
λ_1 = ___
λ_2 = ___

(b) Which of the following best describes the eigenvalues of **A**?

- There are no eigenvectors.
- The only eigenvector is (7,-8).
- Every vector (x_0, y_0) is an eigenvector.
- The only eigenvectors are (1,0) and (0,1).

Problem 7. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.2I/BDH_3.2.16.pg`

A matrix of the form

$$\mathbf{A} = \begin{pmatrix} a & b \\ 0 & d \end{pmatrix}$$

is called *upper triangular* . Suppose that $b \neq 0$ and $a \neq d$.

Find the eigenvalues of **A**.
λ = ___ (enter your answer as a comma-separated list)

Problem 8. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.2I/BDH_3.2.17.pg`

A matrix of the form

$$\mathbf{A} = \begin{pmatrix} a & b \\ b & d \end{pmatrix}$$

is called *symmetric* .

(a) Find the eigenvalues of **A**.
λ = ______________________________
(enter your answer as a comma-separated list)

(b) Which of the following best describes the eigenvalues of **A**?

- One is positive; the other is negative.
- They are complex (imaginary numbers).
- They are real.
- They could be real or complex.

(c) If $b \neq 0$, the eigenvalues of **A** are [?/the same/distinct].

Problem 9. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.2I/BDH_3.2.18.pg`

A matrix of the form

$$\mathbf{A} = \begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$$

has no special name.

(a) Find the eigenvalues of $\mathbf{A}$.

$\lambda =$ ________________________

(enter your answer as a comma-separated list)

(b) Which of the following best describes the eigenvalues of $\mathbf{A}$?

- One is positive; the other is negative.
- They are complex (imaginary numbers).
- They are real.
- They could be real or complex.

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Thomas Hoft — 210S23_Hoft
Assignment BDH_3.2II due 03/16/2023 at 09:55am CDT

Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.2II/BDH_3.2.1de.pg`

Consider the system

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 3 & 2 \\ 0 & -2 \end{pmatrix} \mathbf{Y}.$$

Earlier, you computed the eigenvalues and eigenvectors of the coefficient matrix for this system, and sketched the direction field with straight-line solutions.

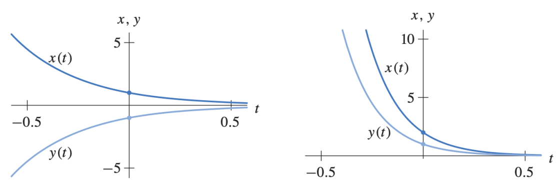
$$\lambda_1 = -2,\ \mathbf{v}_1 = \begin{pmatrix} -2 \\ 5 \end{pmatrix}; \quad \lambda_2 = 3,\ \mathbf{v}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

(d) for each eigenvalue, specify a corresponding straight-line solution and plot its $x(t)$- and $y(t)$-graphs.

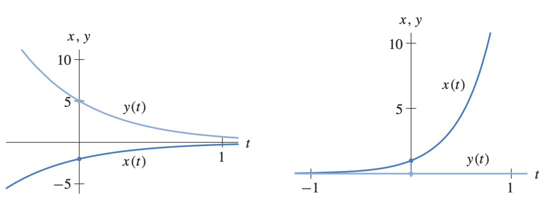
$\mathbf{Y_1}(t) =$ ______ (corresponding to λ_1 and $\mathbf{v_1}$)

$\mathbf{Y_2}(t) =$ ______ (corresponding to λ_2 and $\mathbf{v_2}$)
(enter your answers using angle brackets "3 e^(-t) $< 2,1 >$"; if the system does not have two distinct eigenvalues, enter "DNE" for $\mathbf{Y_2}(t)$)

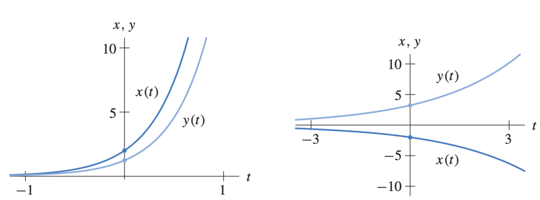
Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



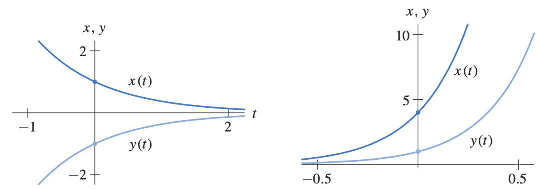
(i) The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$. the $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.



(ii) The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$. The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.



(iii) The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$. The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.



(iv) The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$. The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.

(e) if the system has two distinct eigenvalues, compute the general solution.

$\mathbf{Y}(t) =$ ______
(Use "c1" and "c2" as the constants and angle brackets as above; enter "DNE" if the system does not have two distinct eigenvalues.)

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.2II/BDH_3.2.3de.pg`

Consider the system

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \begin{pmatrix} -5 & -2 \\ -1 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

Earlier, you computed the eigenvalues and eigenvectors of the coefficient matrix for this system, and sketched the direction field with straight-line solutions.

$$\lambda_1 = -3,\ \mathbf{v}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}; \quad \lambda_2 = -6,\ \mathbf{v}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

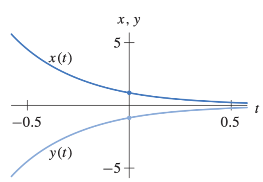
(d) for each eigenvalue, specify a corresponding straight-line solution and plot its $x(t)$- and $y(t)$-graphs.

$\mathbf{Y_1}(t) =$ __________ (corresponding to λ_1 and $\mathbf{v_1}$)

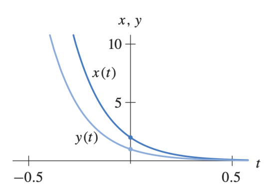
$\mathbf{Y_2}(t) =$ __________ (corresponding to λ_2 and $\mathbf{v_2}$)
(enter your answers using angle brackets "3 e^(-t) $< 2, 1 >$"; if the system does not have two distinct eigenvalues, enter "DNE" for $\mathbf{Y_2}(t)$)

Select the graph corresponding to your sketch. [?/i/ii/iii/iv]

(i)

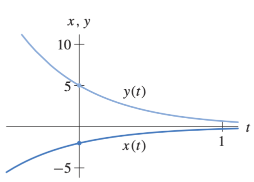


The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$.

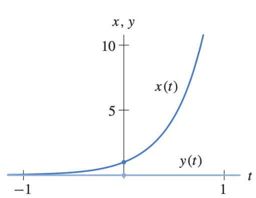


the $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.

(ii)

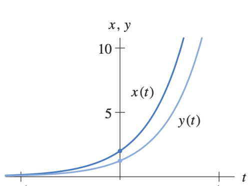


The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$.

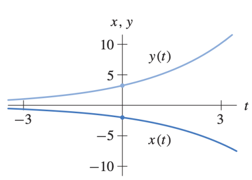


The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.

(iii)

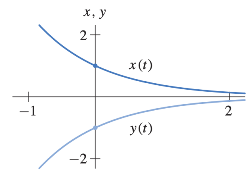


The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$.

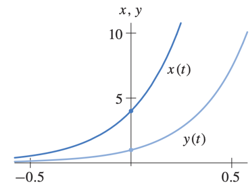


The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.

(iv)



The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$.



The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.

(e) if the system has two distinct eigenvalues, compute the general solution.

$\mathbf{Y}(t) =$ ______________
(Use "c1" and "c2" as the constants and angle brackets as above; enter "DNE" if the system does not have two distinct eigenvalues.)

Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.2II/BDH_3.2.5de.pg`

Consider the system

$$\frac{dx}{dt} = -\frac{x}{2}$$
$$\frac{dy}{dt} = x - \frac{y}{2}.$$

Earlier, you computed the eigenvalues and eigenvectors of the coefficient matrix for this system, and sketched the direction field with straight-line solutions.

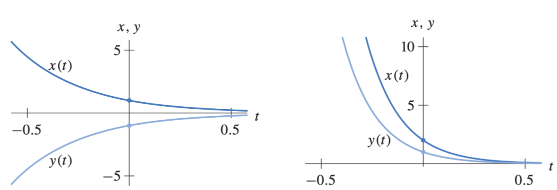
$$\lambda_1 = -0.5, \ \mathbf{v}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

(d) for each eigenvalue, specify a corresponding straight-line solution and plot its $x(t)$- and $y(t)$-graphs.

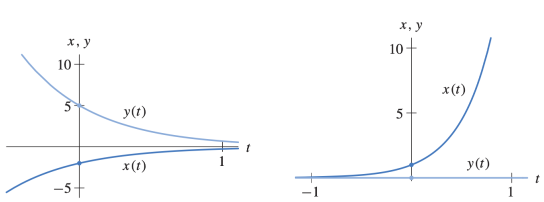
$\mathbf{Y_1}(t) =$ ________ (corresponding to λ_1 and $\mathbf{v_1}$)

$\mathbf{Y_2}(t) =$ ________ (corresponding to λ_2 and $\mathbf{v_2}$)
(enter your answers using angle brackets "3 e^(-t) < 2,1 >"; if the system does not have two distinct eigenvalues, enter "DNE" for $\mathbf{Y_2}(t)$)

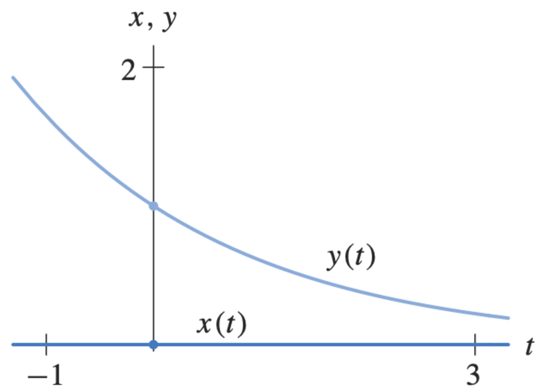
Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



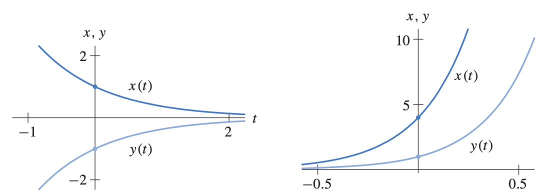
(i) The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$. the $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.



(ii) The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$. The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.



(iii) The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}(t)$.



(iv) The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$. The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.

(e) if the system has two distinct eigenvalues, compute the general solution.

$\mathbf{Y}(t) =$ ____________
(Use "c1" and "c2" as the constants and angle brackets as above; enter "DNE" if the system does not have two distinct eigenvalues.)

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.2II/BDH_3.2.7de.pg`

Consider the system

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

Earlier, you computed the eigenvalues and eigenvectors of the coefficient matrix for this system, and sketched the direction field with straight-line solutions.

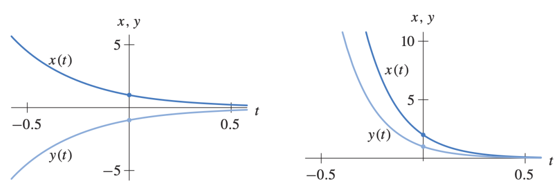
$$\lambda_1 = -1,\ \mathbf{v}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}; \quad \lambda_2 = 4,\ \mathbf{v}_2 = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

(d) for each eigenvalue, specify a corresponding straight-line solution and plot its $x(t)$- and $y(t)$-graphs.

$\mathbf{Y_1}(t) =$ __________ (corresponding to λ_1 and $\mathbf{v_1}$)

$\mathbf{Y_2}(t) =$ __________ (corresponding to λ_2 and $\mathbf{v_2}$)
(enter your answers using angle brackets "3 e^(-t) $< 2,1 >$"; if the system does not have two distinct eigenvalues, enter "DNE" for $\mathbf{Y_2}(t)$)

Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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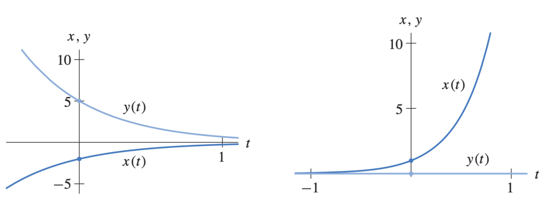
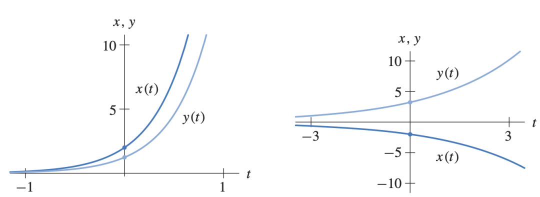
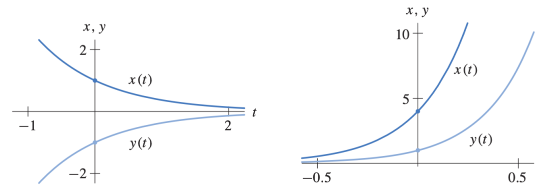


(ii) The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$. The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.



(iii) The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$. The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.



(iv) The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$. The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.

(e) if the system has two distinct eigenvalues, compute the general solution.

$\mathbf{Y}(t) =$ ______________
(Use "c1" and "c2" as the constants and angle brackets as above; enter "DNE" if the system does not have two distinct eigenvalues.)

Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.2II/BDH_3.2.9de.pg`

Consider the system

$$\frac{dx}{dt} = 2x + y$$
$$\frac{dy}{dt} = x + y.$$

Earlier, you computed the eigenvalues and eigenvectors of the coefficient matrix for this system, and sketched the direction field with straight-line solutions.

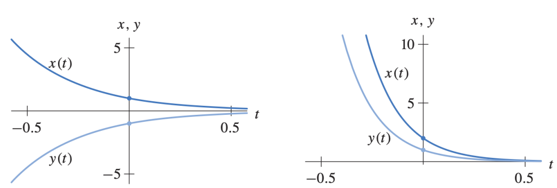
$$\lambda_1 = \frac{3+\sqrt{5}}{2},\ \mathbf{v}_1 = \begin{pmatrix} 2 \\ -1+\sqrt{5} \end{pmatrix}; \quad \lambda_2 = \frac{3-\sqrt{5}}{2},\ \mathbf{v}_2 = \begin{pmatrix} -2 \\ 1+\sqrt{5} \end{pmatrix}$$

(d) for each eigenvalue, specify a corresponding straight-line solution and plot its $x(t)$- and $y(t)$-graphs.

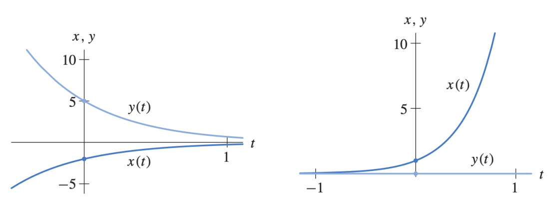
$\mathbf{Y_1}(t) =$ ________ (corresponding to λ_1 and $\mathbf{v_1}$)

$\mathbf{Y_2}(t) =$ ________ (corresponding to λ_2 and $\mathbf{v_2}$)
(enter your answers using angle brackets "3 e^(-t) < 2, 1 >"; if the system does not have two distinct eigenvalues, enter "DNE" for $\mathbf{Y_2}(t)$)

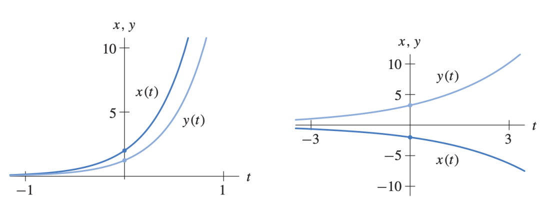
Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



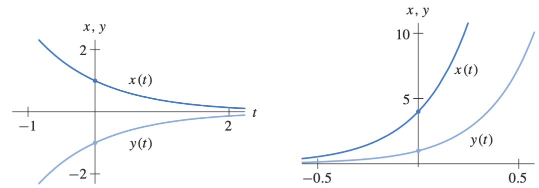
(i) The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$. the $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.



(ii) The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$. The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.



(iii) The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$. The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.



(iv) The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$. The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.

(e) if the system has two distinct eigenvalues, compute the general solution.

$\mathbf{Y}(t) =$ ____________
(Use "c1" and "c2" as the constants and angle brackets as above; enter "DNE" if the system does not have two distinct eigenvalues.)

Problem 6. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.2II/BDH_3.2.11.pg`

Solve the initial-value problem

$$\frac{dx}{dt} = -2x - 2y$$
$$\frac{dy}{dt} = -2x + y,$$

where the initial condition $(x(0), y(0))$ is:

(a) $(1, 0)$ $\mathbf{Y}(t) =$ ____________

(b) $(0, 1)$ $\mathbf{Y}(t) =$ ____________

(c) $(1, -2)$ $\mathbf{Y}(t) =$ ____________

(enter your answers using angle brackets "2 e^(t) < 1, 2 > - 3 e^(-t) < 2, 1 >")

Problem 7. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.2II/BDH_3.2.13.pg`

Solve the initial-value problem

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -4 & 1 \\ 2 & -3 \end{pmatrix} \mathbf{Y}, \quad \mathbf{Y}(0) = \mathbf{Y}_0.$$

where the initial condition $\mathbf{Y}_0$ is:

(a) $\mathbf{Y}_0 = (1, 0)$ $\mathbf{Y}(t) =$ ____________

(b) $\mathbf{Y}_0 = (2, 1)$ $\mathbf{Y}(t) =$ ____________

(c) $\mathbf{Y}_0 = (-1, -2)$ $\mathbf{Y}(t) =$ ____________

(enter your answers using angle brackets "2 e^(t) < 1, 2 > - 3 e^(-t) < 2, 1 >")

Problem 8. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.2II/BDH_3.2.21.pg`

Consider the harmonic oscillator equation for $y(t)$

$$\frac{d^2y}{dt^2}+7\frac{dy}{dt}+10y=0.$$

(a) convert the equation to a first-order, linear system;

$\frac{dy}{dt}=$ __________

$\frac{dv}{dt}=$ __________

(b) compute the eigenvalues and eigenvectors of the system;

$\lambda_1 =$ __________ (the smaller eigenvalue)
$\mathbf{v}_1 =$ __________ (the corresponding eigenvector, using angle brackets "$<1,2>$")
$\lambda_2 =$ __________ (the larger eigenvalue)
$\mathbf{v}_2 =$ __________ (the corresponding eigenvector, using angle brackets "$<1,2>$")

(c) for each eigenvalue, pick an associated eigenvector, and determine the solution $\mathbf{Y}(t)$ to the system;

$\mathbf{Y_1}(t) =$ __________ (corresponding to λ_1 and $\mathbf{v_1}$)

$\mathbf{Y_2}(t) =$ __________ (corresponding to λ_2 and $\mathbf{v_2}$)

(enter your answers using angle brackets "3 e^(-t) $<2,1>$")

(d) and compare the results of your calculations in part (c) with the results that you obtained when you used the guess-and-test method of Section 2.3.

Problem 9. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.2II/BDH_3.2.23.pg`

Consider the harmonic oscillator equation for $y(t)$

$$\frac{d^2y}{dt^2}+4\frac{dy}{dt}+y=0.$$

(a) convert the equation to a first-order, linear system;

$\frac{dy}{dt}=$ __________

$\frac{dv}{dt}=$ __________

(b) compute the eigenvalues and eigenvectors of the system;

$\lambda_1 =$ __________ (the smaller eigenvalue)
$\mathbf{v}_1 =$ __________ (the corresponding eigenvector, using angle brackets "$<1,2>$")
$\lambda_2 =$ __________ (the larger eigenvalue)
$\mathbf{v}_2 =$ __________ (the corresponding eigenvector, using angle brackets "$<1,2>$")

(c) for each eigenvalue, pick an associated eigenvector, and determine the solution $\mathbf{Y}(t)$ to the system;

$\mathbf{Y_1}(t) =$ _______________ (corresponding to λ_1 and $\mathbf{v_1}$)

$\mathbf{Y_2}(t) =$ _______________ (corresponding to λ_2 and $\mathbf{v_2}$)

(enter your answers using angle brackets "3 e^(-t) $<2,1>$")

(d) and compare the results of your calculations in part (c) with the results that you obtained when you used the guess-and-test method of Section 2.3.

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Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.3/BDH_3.3.1.pg`

Consider the system

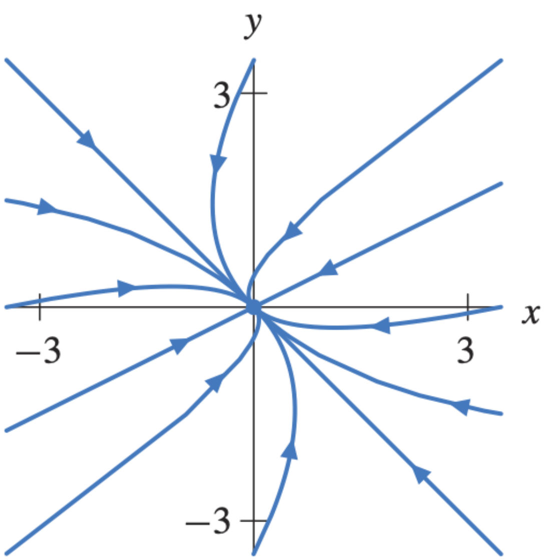
$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 3 & 2 \\ 0 & -2 \end{pmatrix} \mathbf{Y}.$$

Earlier, you computed the eigenvalues and eigenvectors of the coefficient matrix for this system:

$$\lambda_1 = -2,\ \mathbf{v}_1 = \begin{pmatrix} -2 \\ 5 \end{pmatrix}; \quad \lambda_2 = 3,\ \mathbf{v}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Sketch, on paper, the phase portrait for this system.

Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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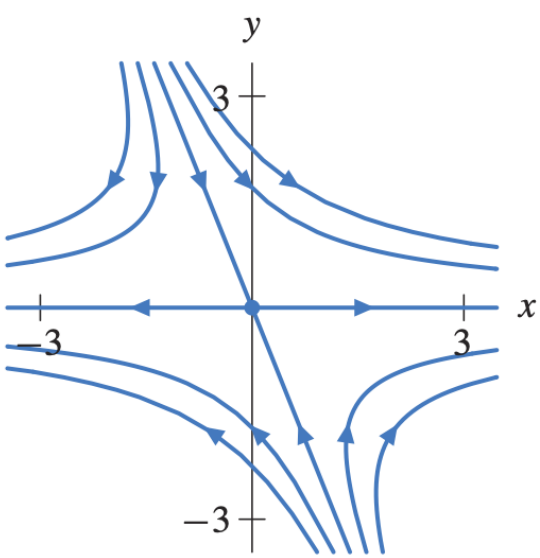

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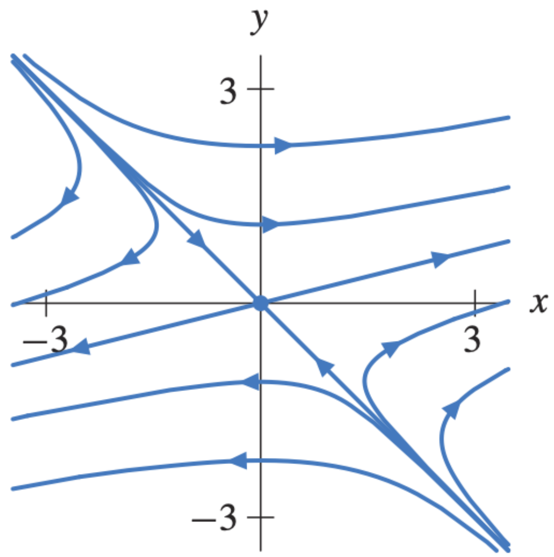

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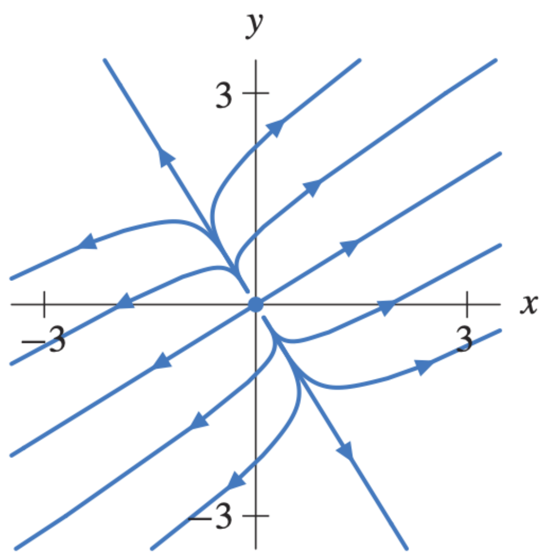

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Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.3/BDH_3.3.3.pg`

Consider the system

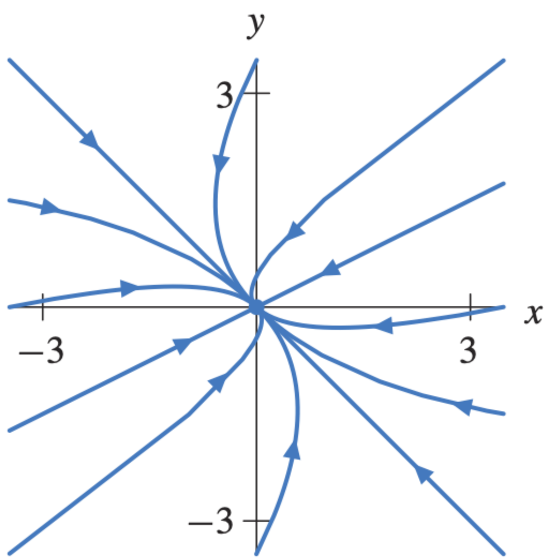
$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \begin{pmatrix} -5 & -2 \\ -1 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

Earlier, you computed the eigenvalues and eigenvectors of the coefficient matrix for this system:

$$\lambda_1 = -3,\ \mathbf{v}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}; \quad \lambda_2 = -6,\ \mathbf{v}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Sketch, on paper, the phase portrait for this system.

Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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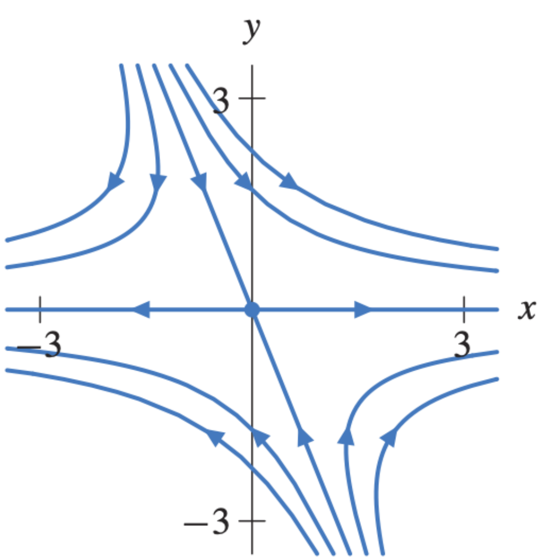

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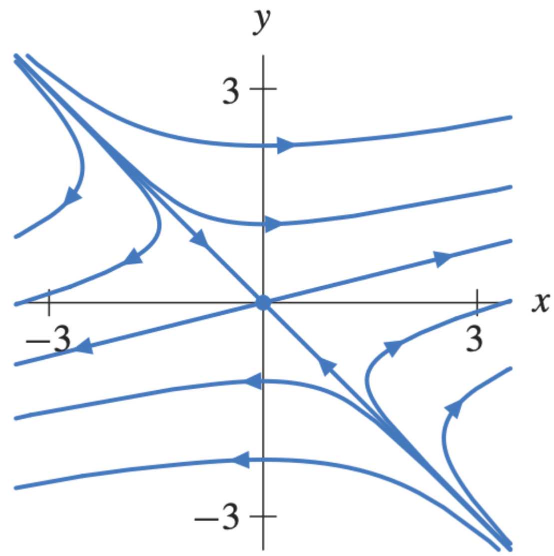

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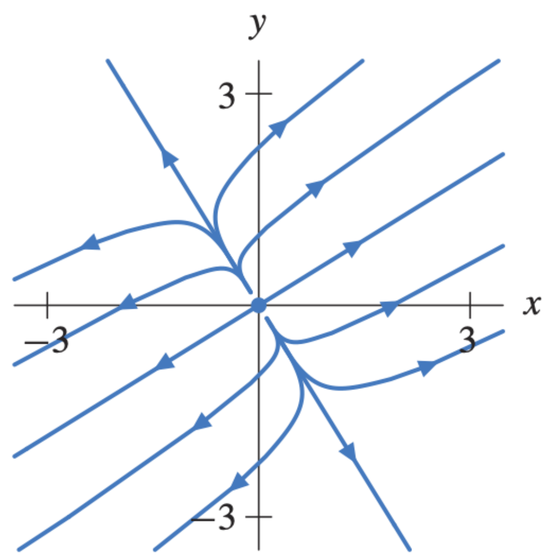

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Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.3/BDH_3.3.5.pg`

Consider the system

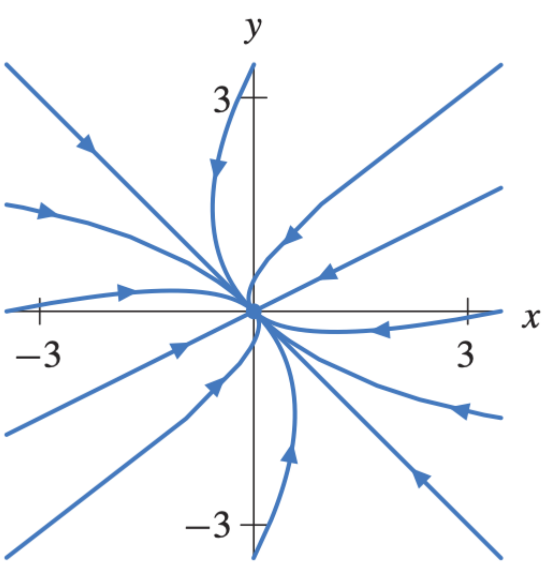
$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

Earlier, you computed the eigenvalues and eigenvectors of the coefficient matrix for this system:

$$\lambda_1 = -1,\ \mathbf{v}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}; \quad \lambda_2 = 4,\ \mathbf{v}_2 = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

Sketch, on paper, the phase portrait for this system.

Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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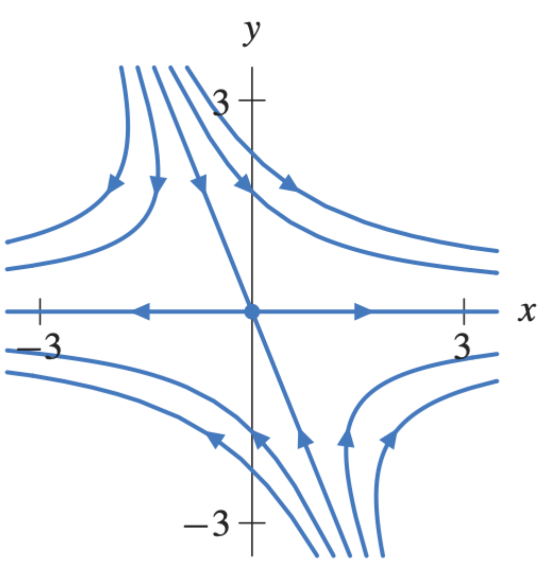

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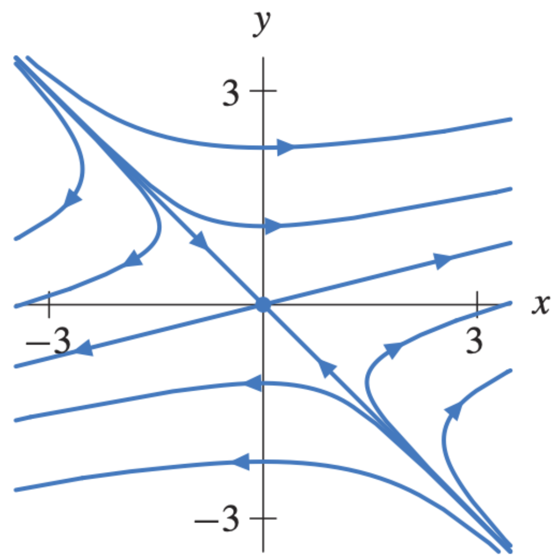

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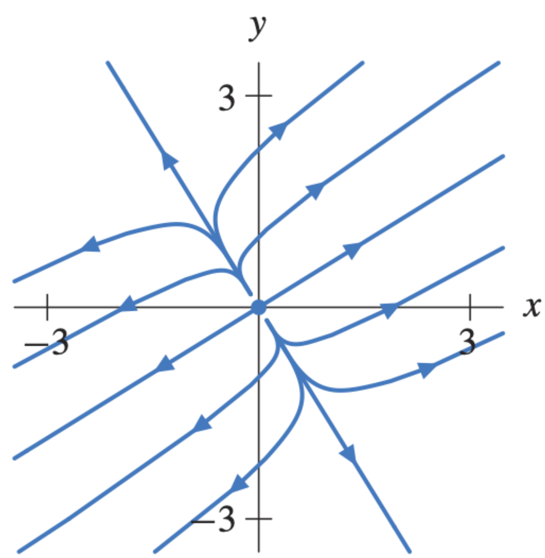

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Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.3/BDH_3.3.7.pg`

Consider the system

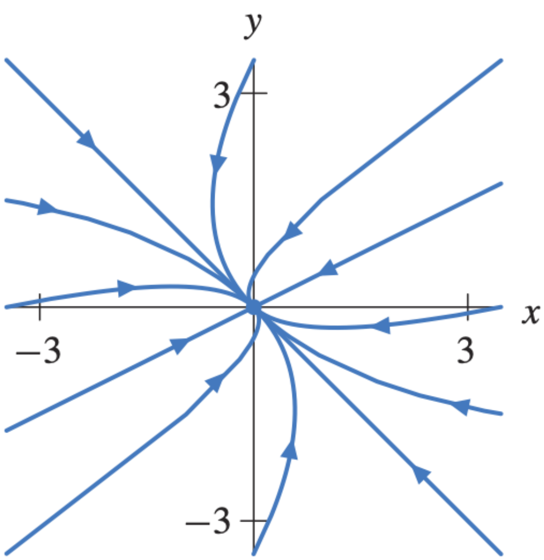
$$\frac{dx}{dt} = 2x + y$$
$$\frac{dy}{dt} = x + y.$$

Earlier, you computed the eigenvalues and eigenvectors of the coefficient matrix for this system:

$$\lambda_1 = \frac{3+\sqrt{5}}{2},\ \mathbf{v}_1 = \begin{pmatrix} 2 \\ -1+\sqrt{5} \end{pmatrix}; \quad \lambda_2 = \frac{3-\sqrt{5}}{2},\ \mathbf{v}_2 = \begin{pmatrix} -2 \\ 1+\sqrt{5} \end{pmatrix}$$

Sketch, on paper, the phase portrait for this system.

Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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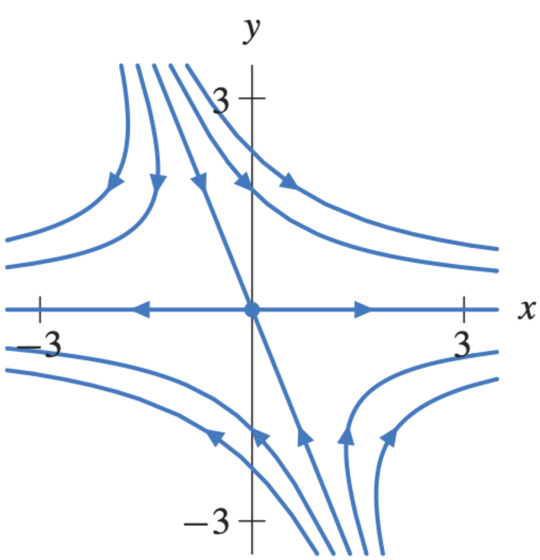

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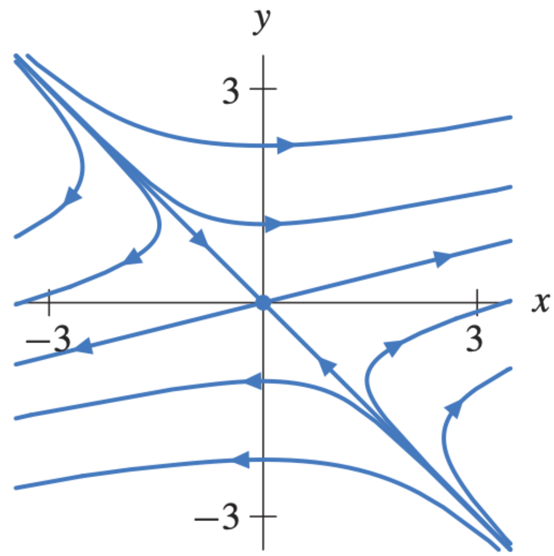

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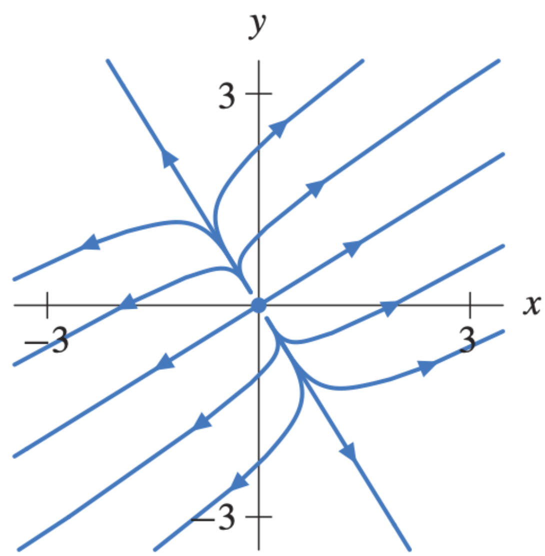

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Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.3/BDH_3.3.9.pg`

Consider the initial-value problem

$$\frac{dx}{dt} = -2x - 2y$$
$$\frac{dy}{dt} = -2x + y,$$

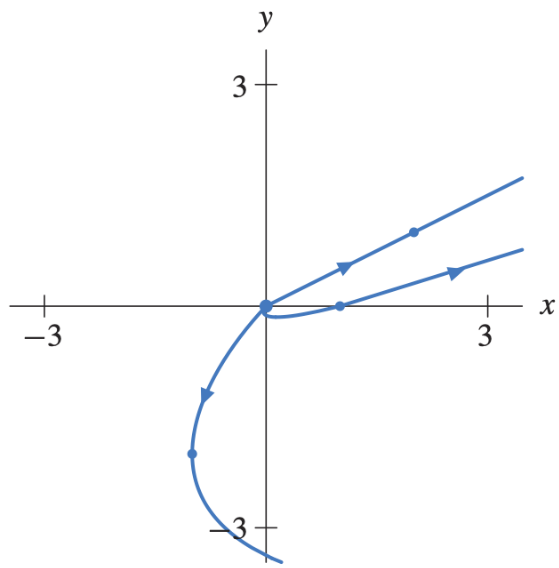
where the initial condition $\mathbf{Y}(0) = (x(0), y(0))$ is specified below.

Earlier, you computed the eigenvalues and eigenvectors of the coefficient matrix for this system:

$$\lambda_1 = 2,\ \mathbf{v}_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix};\quad \lambda_2 = -3,\ \mathbf{v}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Sketch, on paper, the phase portrait for this system.

Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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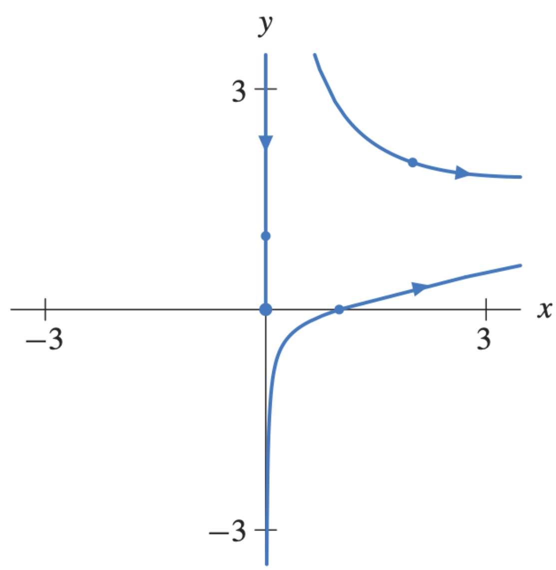

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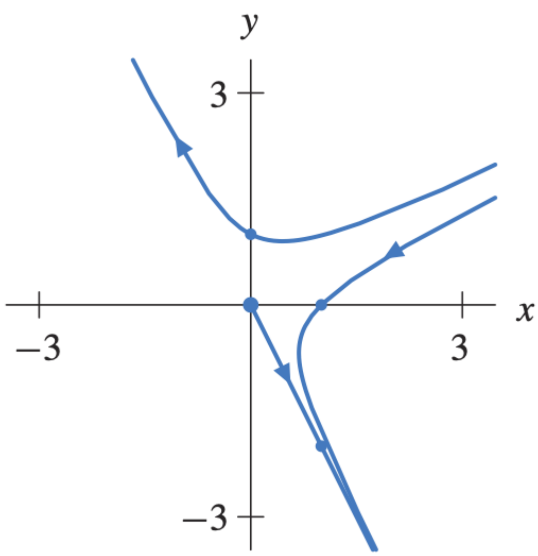

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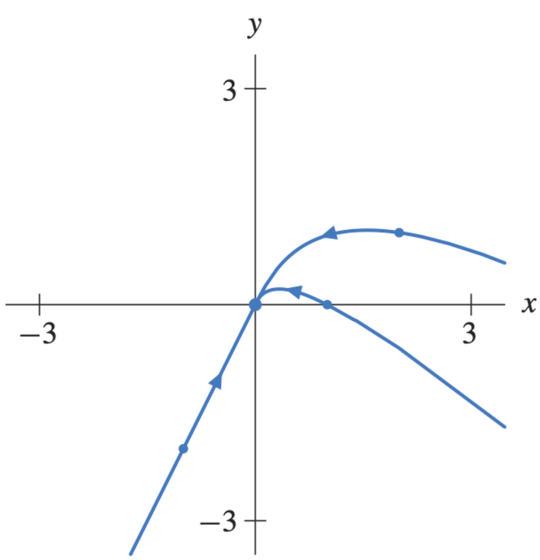

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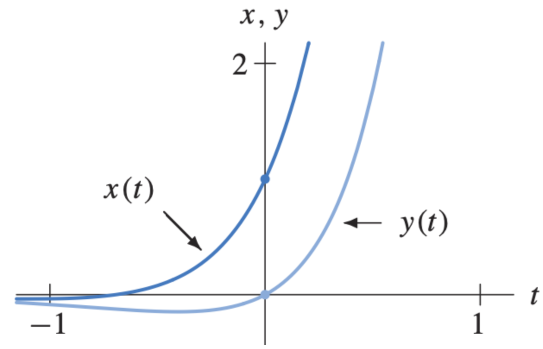
Sketch, on paper, the $x(t)$- and $y(t)$-graphs for the solutions corresponding to the initial conditions specified below

(a) $\mathbf{Y}(0) = (1, 0)$

Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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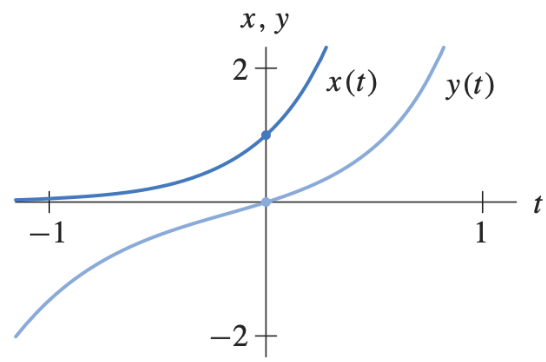

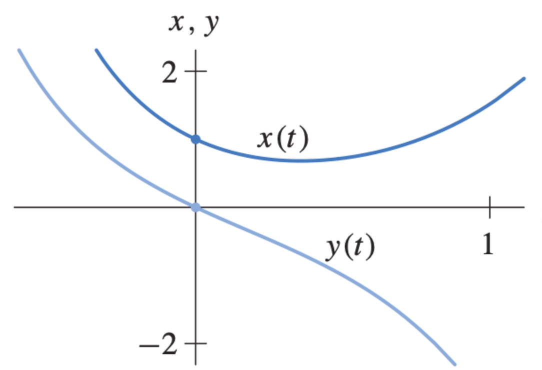

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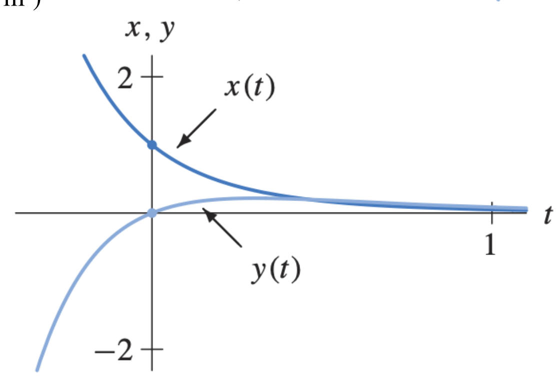

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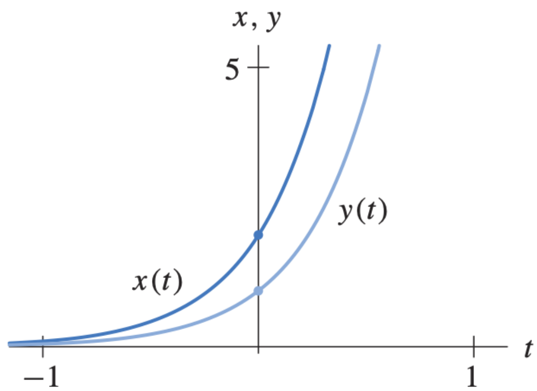
(b) $\mathbf{Y}(0) = (0, 1)$

Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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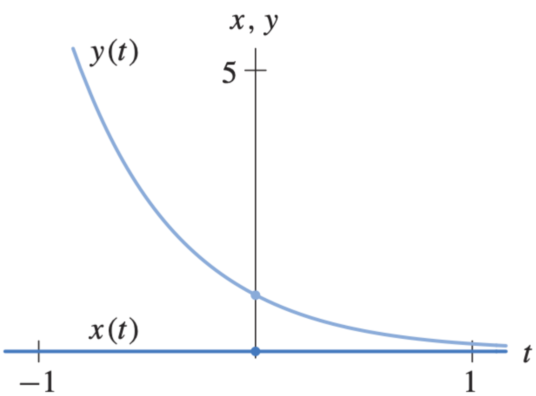

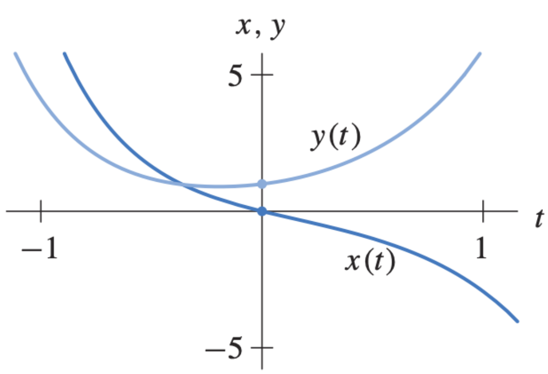

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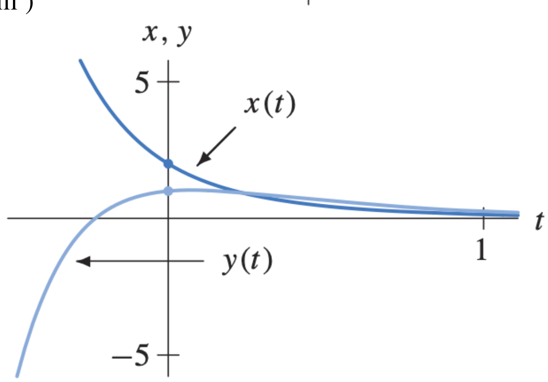

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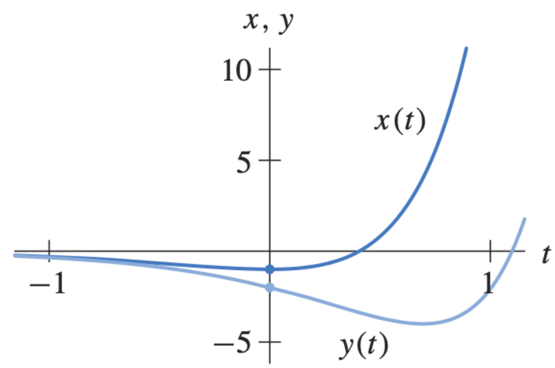
(c) $\mathbf{Y}(0) = (1, -2)$

Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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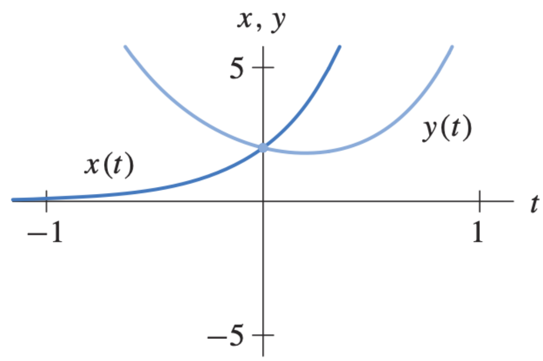

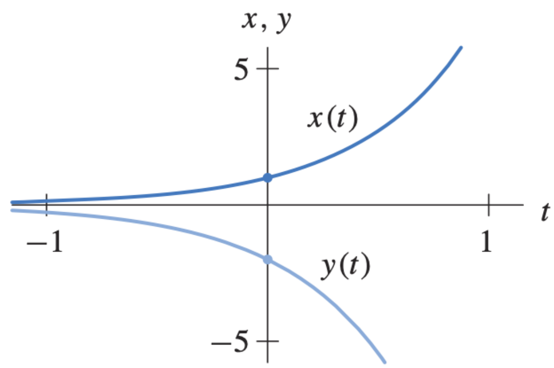

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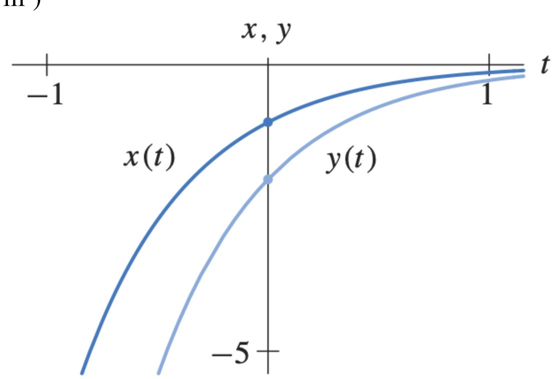

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Problem 6. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.3/BDH_3.3.11.pg`

Consider the initial-value problem

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -4 & 1 \\ 2 & -3 \end{pmatrix} \mathbf{Y}, \quad \mathbf{Y}(0) = \mathbf{Y}_0,$$

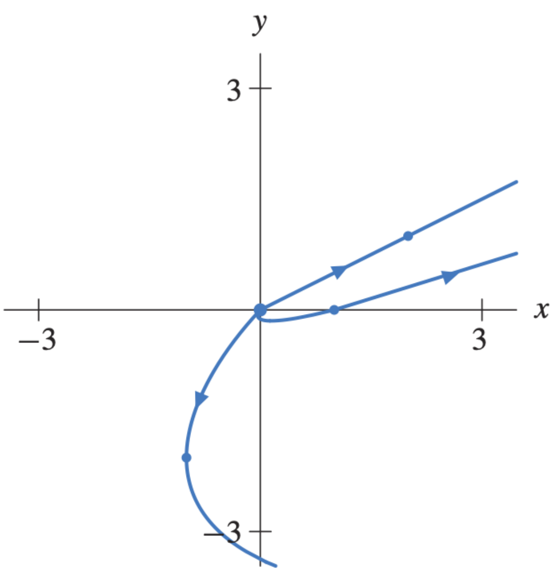
where the initial condition $\mathbf{Y}(0) = (x(0), y(0))$ is specified below.

Earlier, you computed the eigenvalues and eigenvectors of the coefficient matrix for this system:

$$\lambda_1 = -5,\ \mathbf{v}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}; \quad \lambda_2 = -2,\ \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Sketch, on paper, the phase portrait for this system.

Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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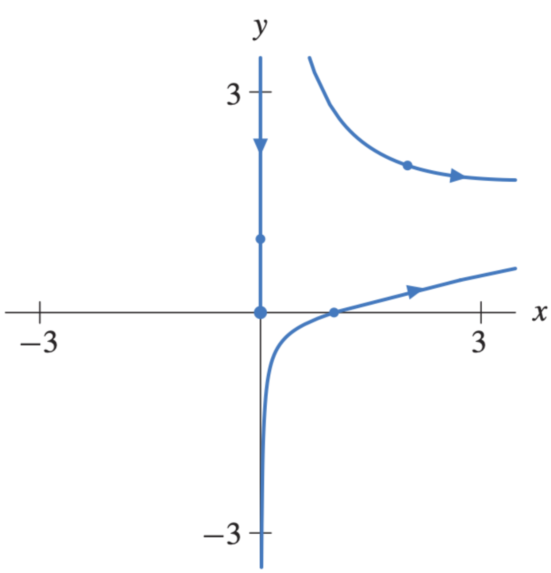

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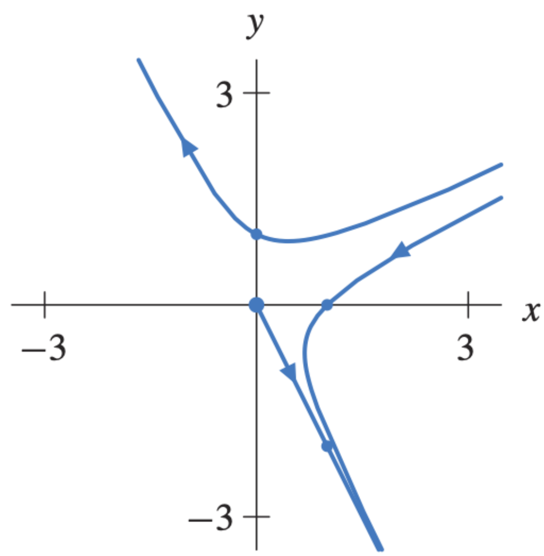

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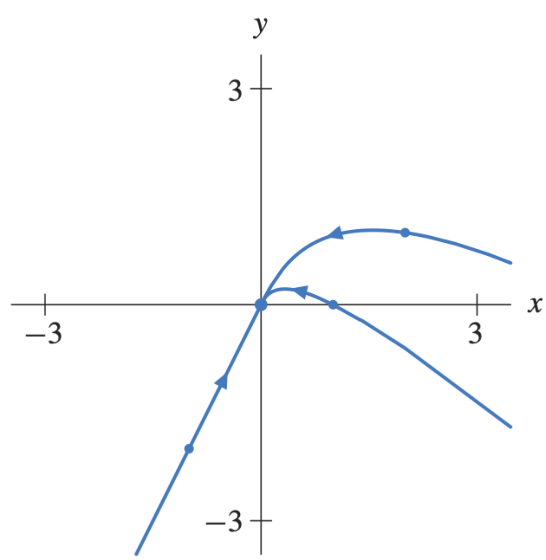

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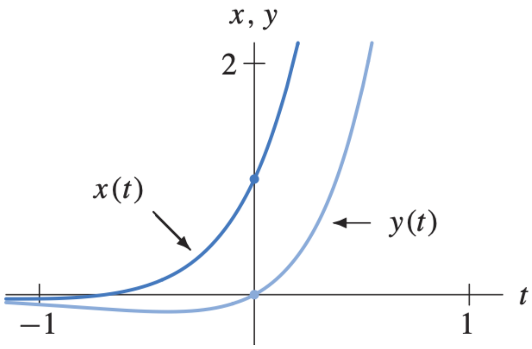
Sketch, on paper, the $x(t)$- and $y(t)$-graphs for the solutions corresponding to the initial conditions specified below

(a) $\mathbf{Y}(0) = (1, 0)$

Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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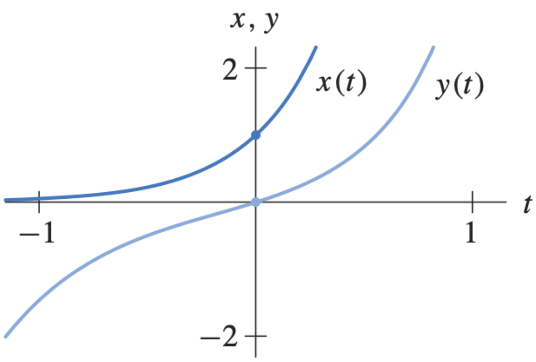

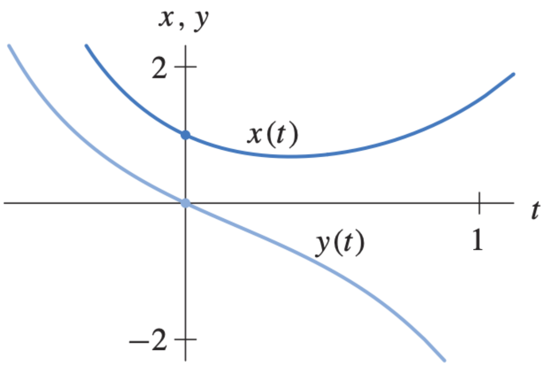

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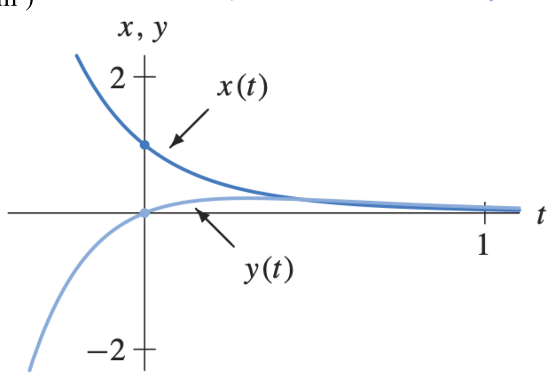

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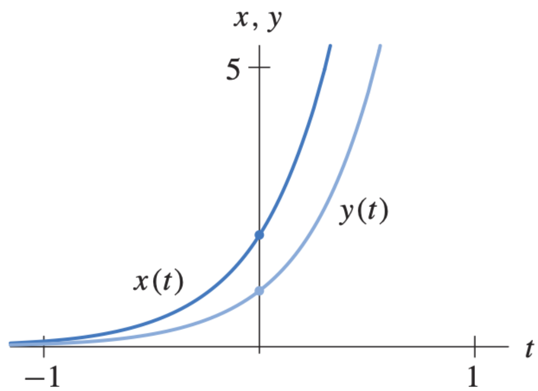
(b) $\mathbf{Y}(0) = (2, 1)$

Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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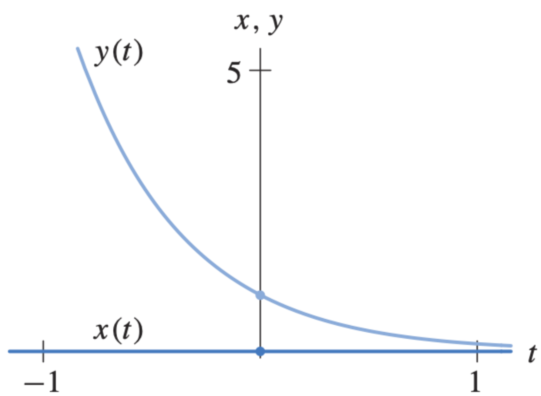

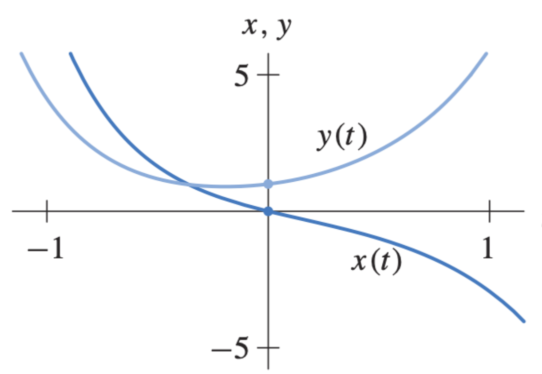

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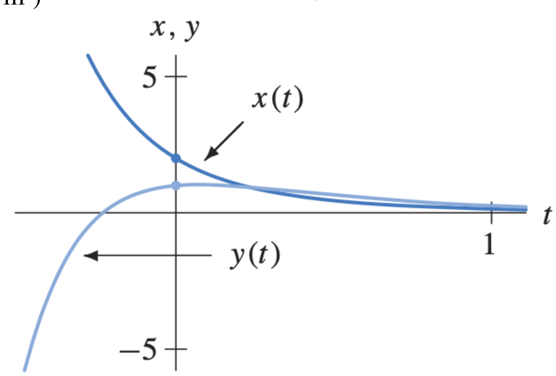

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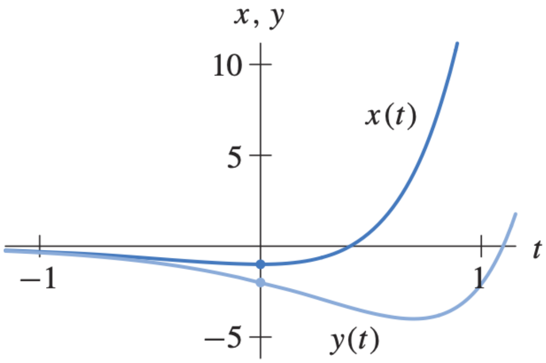
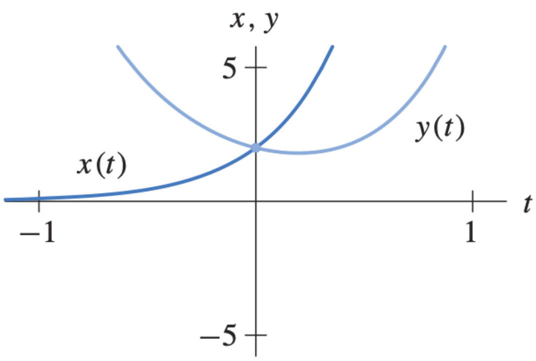
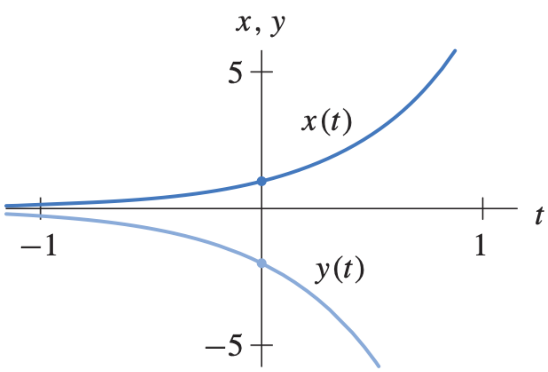
(c) $\mathbf{Y}(0) = (-1, -2)$

Select the graph corresponding to your sketch. [?/i/ii/iii/iv]

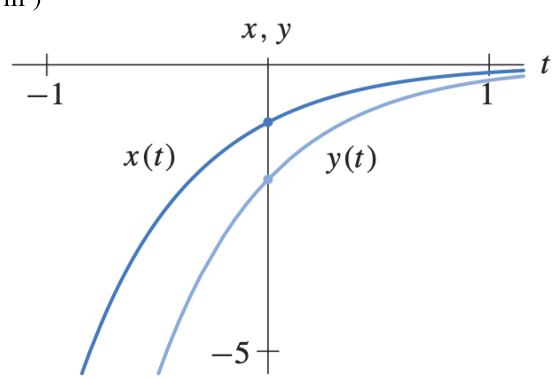


(i) (ii)





(iii) (iv



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Problem 7. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.3/BDH_3.3.15.pg`

Consider the harmonic oscillator equation for $y(t)$

$$\frac{d^2y}{dt^2}+4\frac{dy}{dt}+y=0.$$

Earlier, you converted this equation to a first-order system

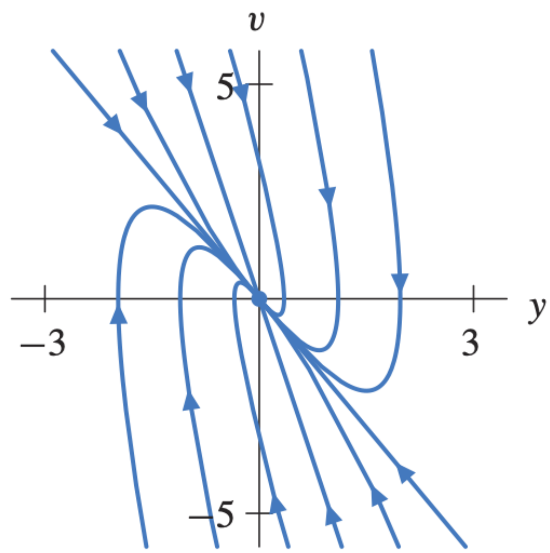
$$\begin{pmatrix}\frac{dy}{dt}\\ \frac{dv}{dt}\end{pmatrix}=\begin{pmatrix}0 & 1\\ -1 & -4\end{pmatrix}\begin{pmatrix}y\\ v\end{pmatrix}.$$

and computed the eigenvalues and eigenvectors of the coefficient matrix:

$$\lambda_1=-2-\sqrt{3},\ \mathbf{v}_1=\begin{pmatrix}1\\ -2-\sqrt{3}\end{pmatrix};\quad \lambda_2=-2+\sqrt{3},\ \mathbf{v}_2=\begin{pmatrix}1\\ -2+\sqrt{3}\end{pmatrix}$$

Sketch, on paper, the phase portrait for this system.

Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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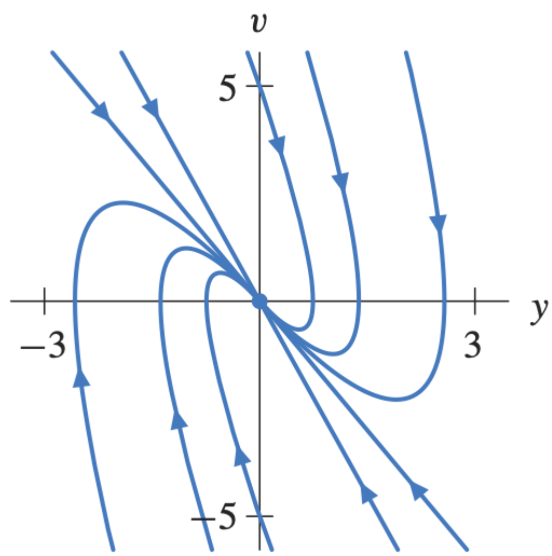

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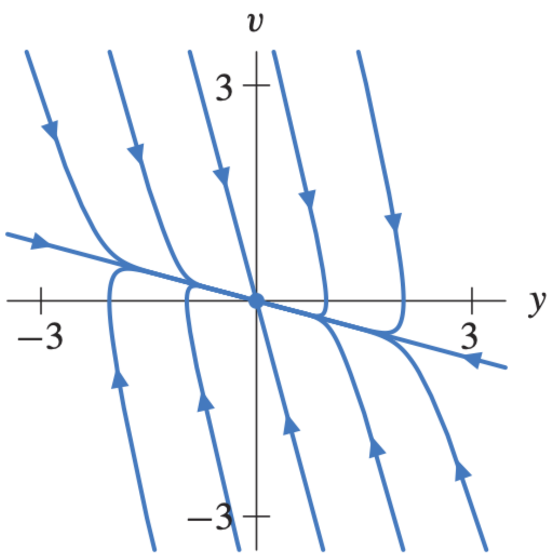

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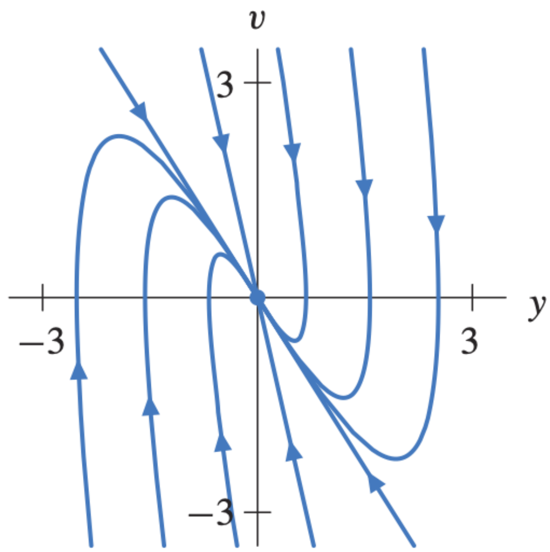

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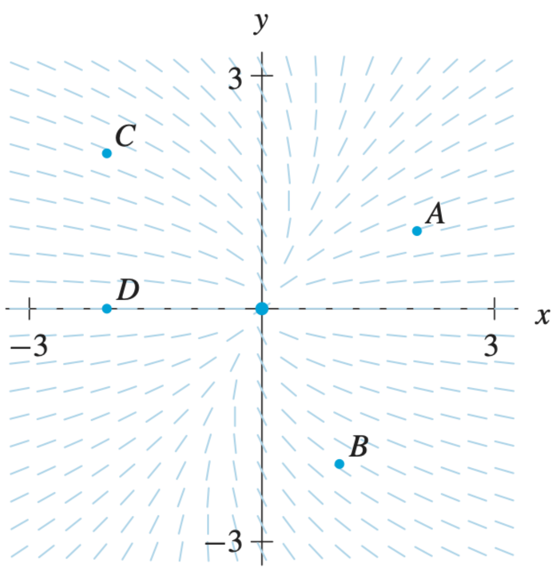
Problem 8. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.3/BDH_3.3.19.pg`

The slope field for the system

$$\frac{dx}{dt} = -2x + \frac{1}{2}y$$
$$\frac{dy}{dt} = -y$$

is shown to the right.



(a) Determine the type of the equilibrium point at the origin. [?/sink/source/saddle]
Hint: you cannot do this from the slope field without arrows – find the eigenvalues of the coefficient matrix of the system.

(b) Calculate all straight-line solutions.

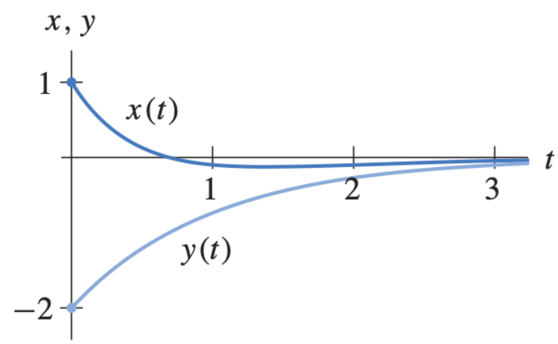
$\mathbf{Y_1}(t) =$ ______________

$\mathbf{Y_2}(t) =$ ______________
Do not include constants c1, c2 or k1, k2 in your solutions.
Enter your answers using angle brackets ”e^(-t) < 2, 1 >”.

(c) Plot, on paper, the $x(t)$- and $y(t)$-graphs, $(t \geq 0)$, for the initial conditions A = (2,1), B = (1,-2), C = (-2,2), and D = (-2, 0).

Select the graphs corresponding to your sketches.

A: [?/i/ii/iii/iv]; B: [?/i/ii/iii/iv]; C: [?/i/ii/iii/iv]; D: [?/i/ii/iii/iv]



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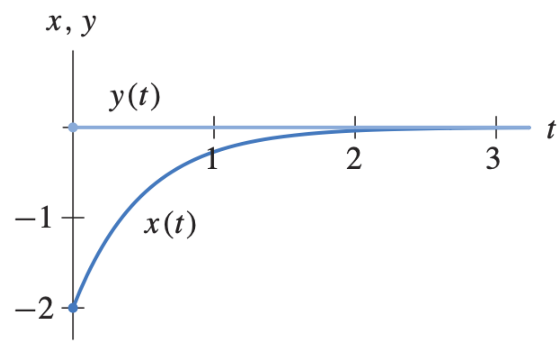

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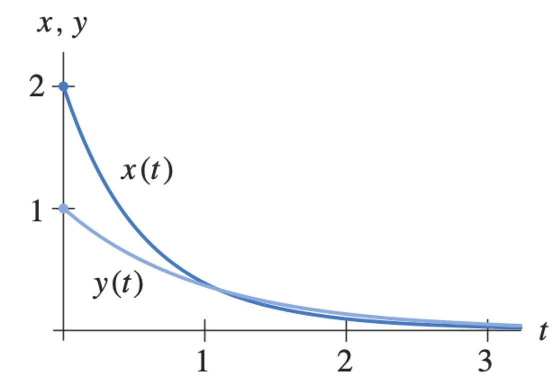

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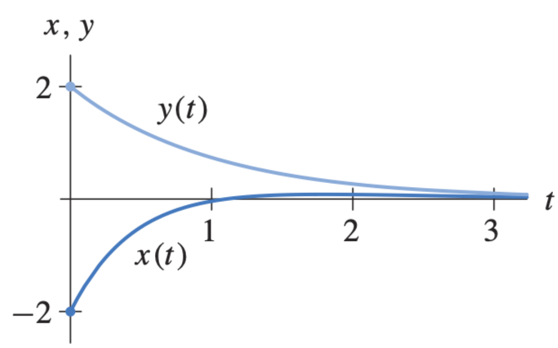

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Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.4/BDH_3.4.1.pg`

Suppose the 2×2 matrix $\mathbf{A}$ has $\lambda = 1 + 3i$ as an eigenvalue with eigenvector

$$\mathbf{Y}_0 = \begin{pmatrix} 2+i \\ 1 \end{pmatrix}.$$

Compute the general solution to $d\mathbf{Y}/dt = \mathbf{A}\mathbf{Y}$.

$\mathbf{Y}(t) =$ ______________________________

Use "k1" and "k2" as the constants and enter your answer using angle brackets "3 e^(-t) <cos(t),sin(t)>".

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.4/BDH_3.4.3_9.pg`

Consider the linear system $\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} \mathbf{Y}$, with initial condition $\mathbf{Y}_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

(a) find the eigenvalues of the coefficient matrix;
$\lambda_1, \lambda_2 =$ ________ (comma-separated list; use "i" for the imaginary unit)

(b) determine if the origin is a spiral sink, a spiral source, or a center;
- ?
- spiral sink
- spiral source
- center

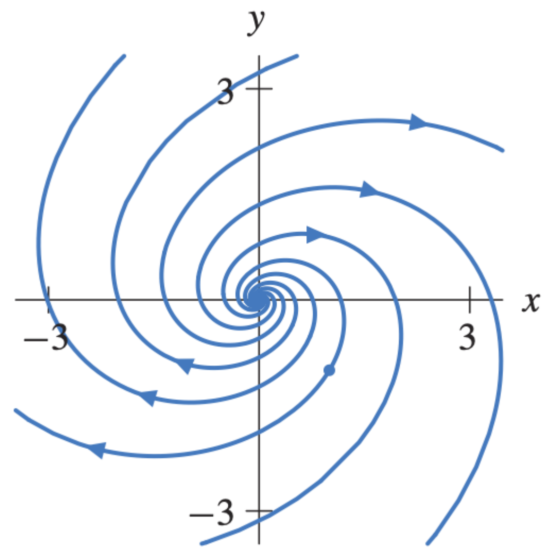
(c) determine the natural period and natural frequency of the oscillations;
$T_0 =$ _____, $\omega_0 =$ _____ (use "pi" for π)

(d) determine the direction of the oscillations in the phase plane (do the solutions go clockwise or counterclockwise around the origin?);
- ?
- clockwise
- counterclockwise

(e) find the general solution;
$\mathbf{Y}(t) =$ ________________________
(Use "k1" and "k2" as the constants and enter your answer using angle brackets "3 e^(-t) <cos(t),sin(t)>")

(f) find the particular solution with the given initial value;
$\mathbf{Y}(t) =$ ________________________

(g) sketch, on paper, the xy-phase portrait and the $x(t)$- and $y(t)$-graphs of the particular solution;
Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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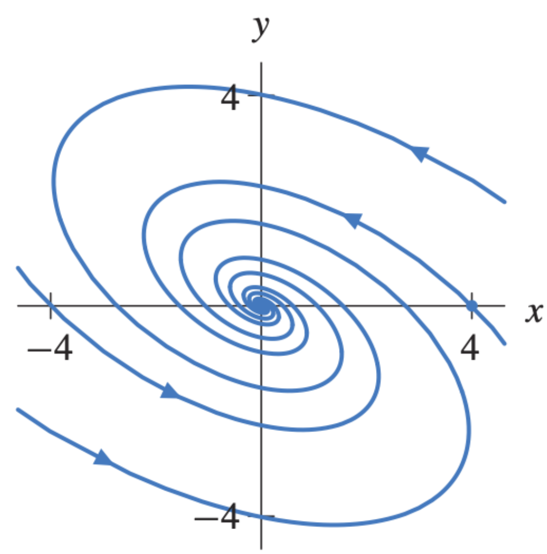

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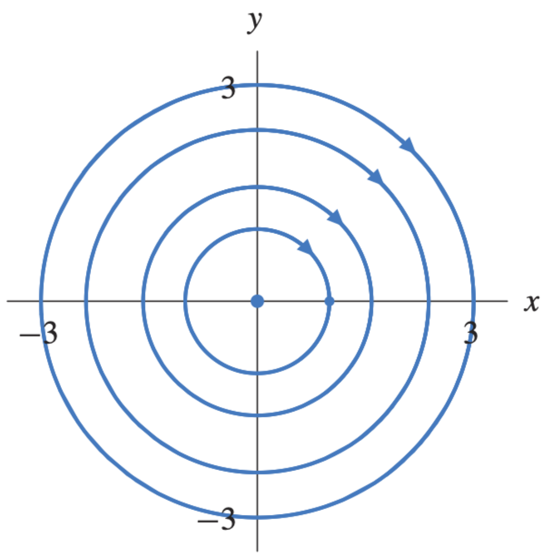

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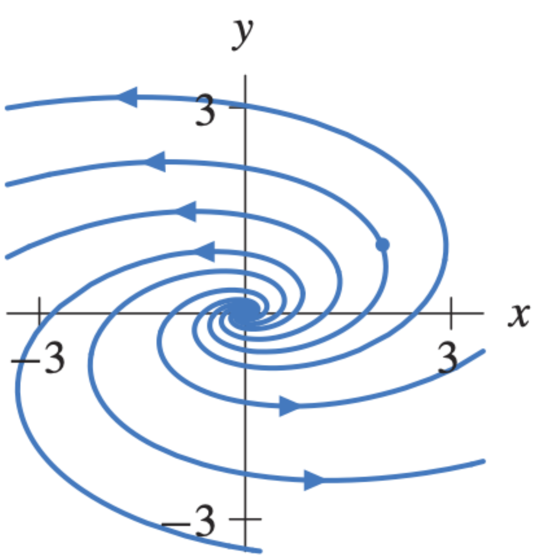

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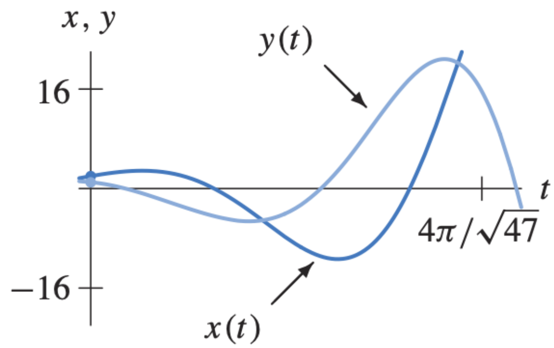
(h) sketch, on paper, the $x(t)$- and $y(t)$-graphs of the particular solution.
Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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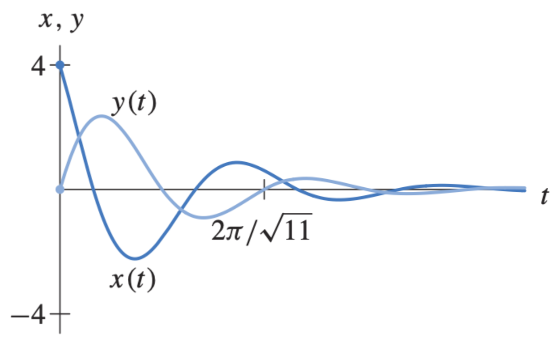

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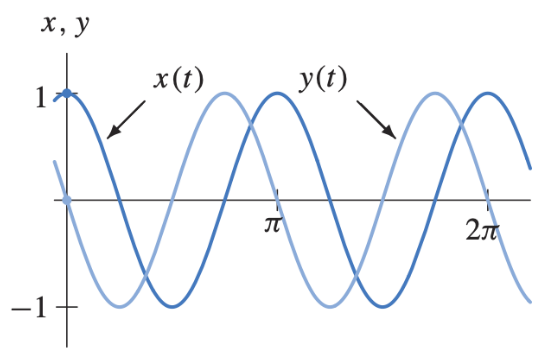

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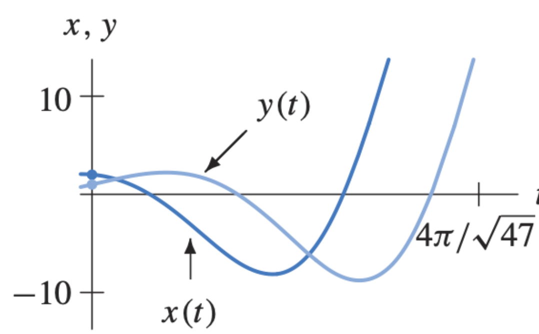

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Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.4/BDH_3.4.5_11.pg`

Consider the linear system $\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -3 & -5 \\ 3 & 1 \end{pmatrix} \mathbf{Y}$, with initial condition $\mathbf{Y}_0 = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$.

(a) find the eigenvalues of the coefficient matrix;
$\lambda_1, \lambda_2 =$ ________ (comma-separated list; use ”i” for the imaginary unit)

(b) determine if the origin is a spiral sink, a spiral source, or a center;
- ?
- spiral sink
- spiral source
- center

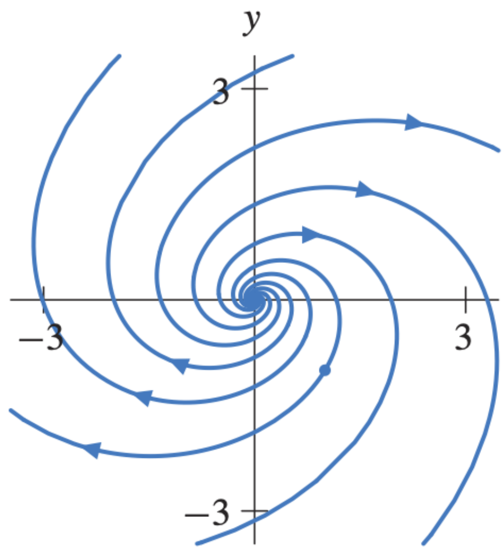
(c) determine the natural period and natural frequency of the oscillations;
$T_0 =$ ____, $\omega_0 =$ ____ (use ”pi” for π)

(d) determine the direction of the oscillations in the phase plane (do the solutions go clockwise or counterclockwise around the origin?);
- ?
- clockwise
- counterclockwise

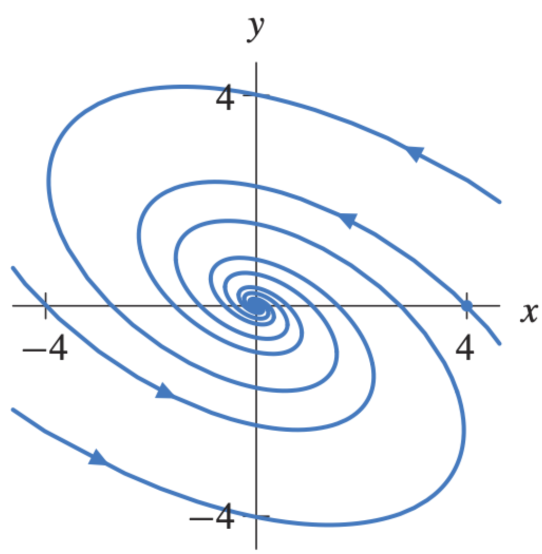
(e) find the general solution;
$\mathbf{Y}(t) =$ ______________________________
(Use ”k1” and ”k2” as the constants and enter your answer using angle brackets ”3 e^(-t) <cos(t),sin(t)>”)

(f) find the particular solution with the given initial value;
$\mathbf{Y}(t) =$ ______________________________

(g) sketch, on paper, the xy-phase portrait and the $x(t)$- and $y(t)$-graphs of the particular solution;
Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



(i) (ii



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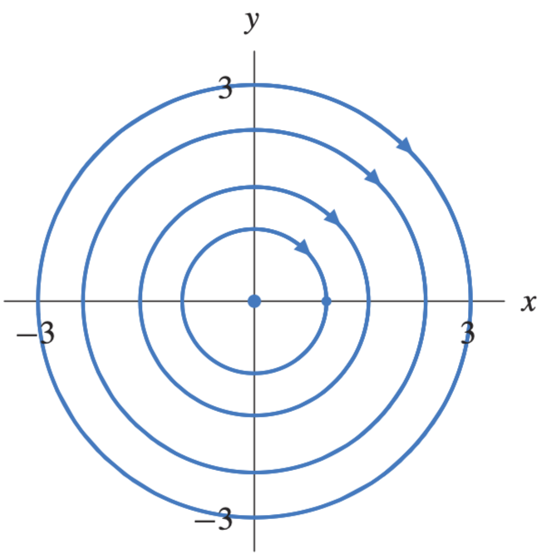

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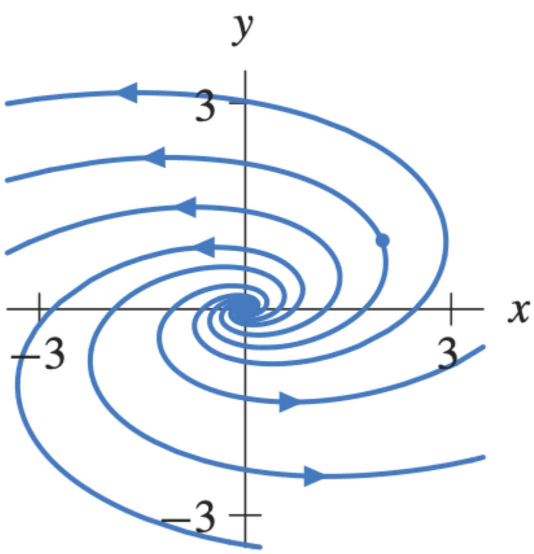

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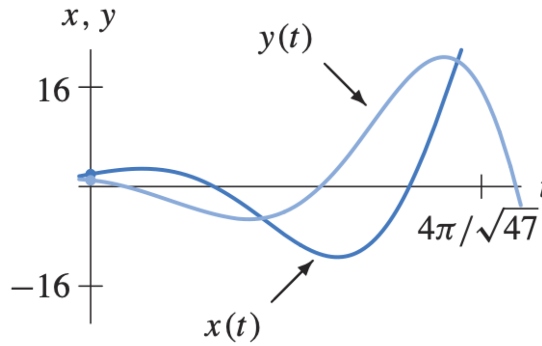
(h) sketch, on paper, the $x(t)$- and $y(t)$-graphs of the particular solution.
Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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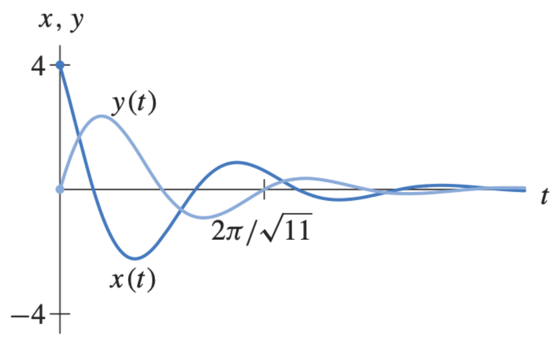

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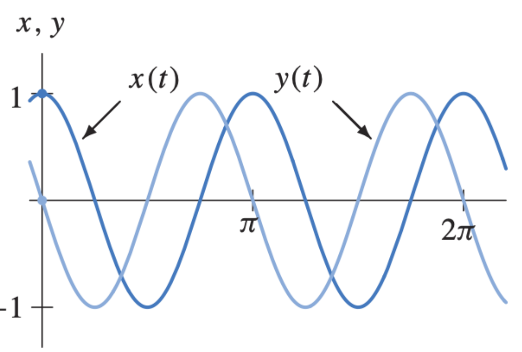
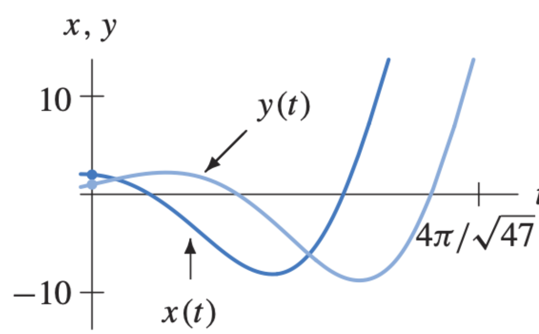


(iii) (iv



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Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.4/BDH_3.4.7_13.pg`

Consider the linear system $\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 2 & -6 \\ 2 & 1 \end{pmatrix} \mathbf{Y}$, with initial condition $\mathbf{Y}_0 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

(a) find the eigenvalues of the coefficient matrix;
$\lambda_1, \lambda_2 =$ ______ (comma-separated list; use "i" for the imaginary unit)

(b) determine if the origin is a spiral sink, a spiral source, or a center;
- ?
- spiral sink
- spiral source
- center

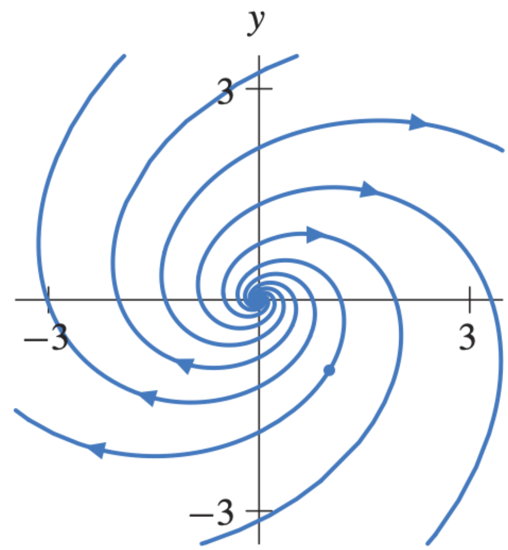
(c) determine the natural period and natural frequency of the oscillations;
$T_0 =$ ____, $\omega_0 =$ ____ (use "pi" for π)

(d) determine the direction of the oscillations in the phase plane (do the solutions go clockwise or counterclockwise around the origin?);
- ?
- clockwise
- counterclockwise

(e) find the general solution;
$\mathbf{Y}(t) =$ ______
(Use "k1" and "k2" as the constants and enter your answer using angle brackets "3 e^(-t) <cos(t),sin(t)>")

(f) find the particular solution with the given initial value;
$\mathbf{Y}(t) =$ ______

(g) sketch, on paper, the xy-phase portrait and the $x(t)$- and $y(t)$-graphs of the particular solution;
Select the graph corresponding to your sketch. [?/i/ii/iii/iv]



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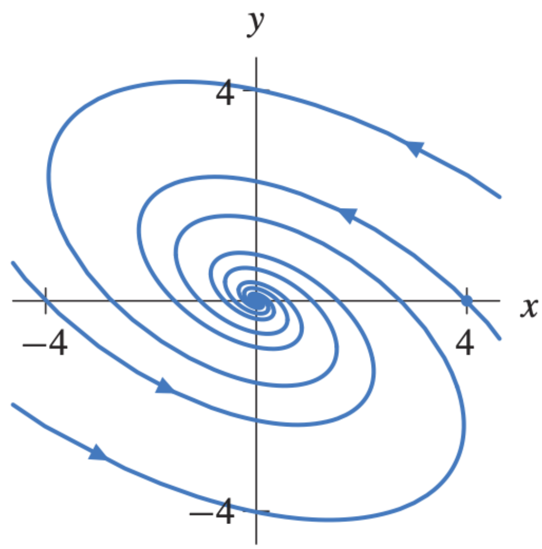

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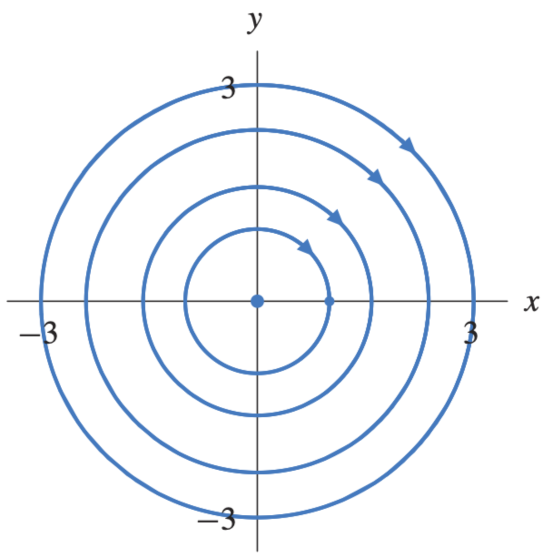

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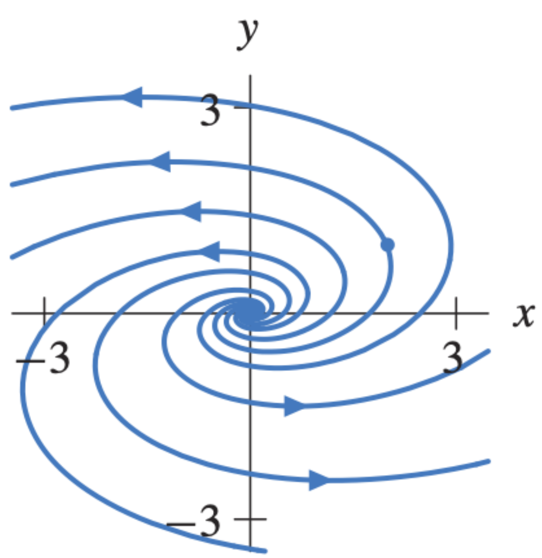

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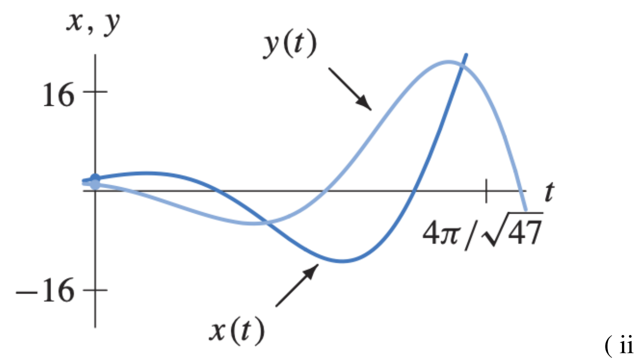

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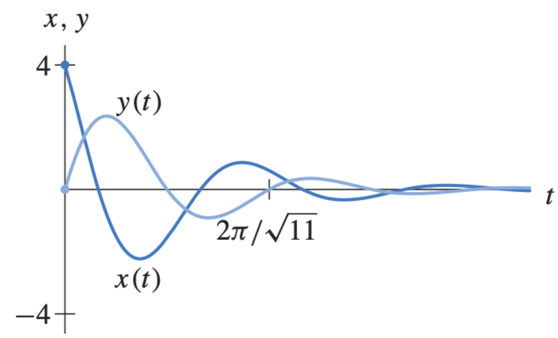

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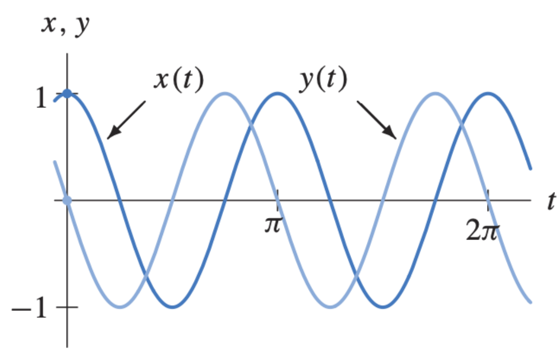

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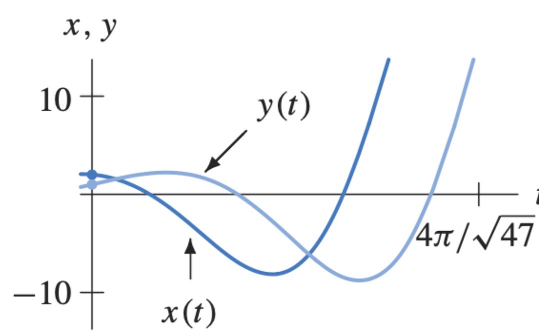

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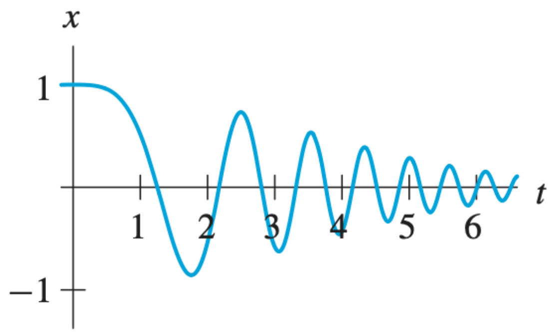
(h) sketch, on paper, the $x(t)$- and $y(t)$-graphs of the particular solution.
Select the graph corresponding to your sketch. [?/i/ii/iii/iv]

Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.4/BDH_3.4.15.pg`

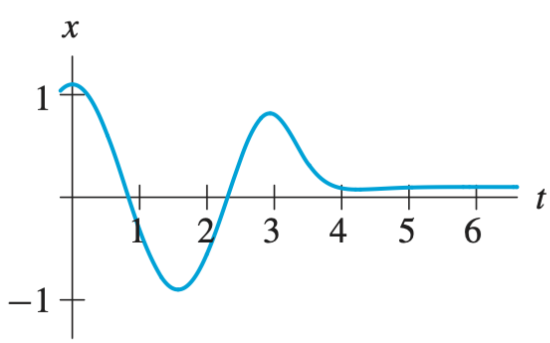
(i) Consider the following graph of a function $x(t)$.



(a)
Can this be an $x(t)$-graph for a solution of a linear system with complex eigenvalues? [?/yes/no]
(b)
Why is it not an $x(t)$-graph for a solution of a linear system with complex eigenvalues?

- The amplitude is not monotonically decreasing or increasing.
- Oscillation stops at some t.
- Oscillation starts at some t. There was no prior oscillation.
- The time between successive zeros is not constant.

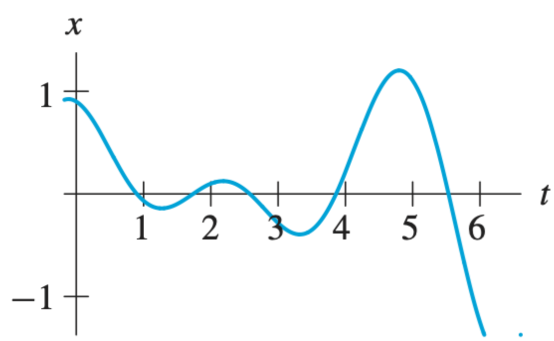
(ii) Consider the following graph of a function $x(t)$.



(a)
Can this be an $x(t)$-graph for a solution of a linear system with complex eigenvalues? [?/yes/no]
(b)
Why is it not an $x(t)$-graph for a solution of a linear system with complex eigenvalues?

- The amplitude is not monotonically decreasing or increasing.
- Oscillation stops at some t.
- Oscillation starts at some t. There was no prior oscillation.
- The time between successive zeros is not constant.

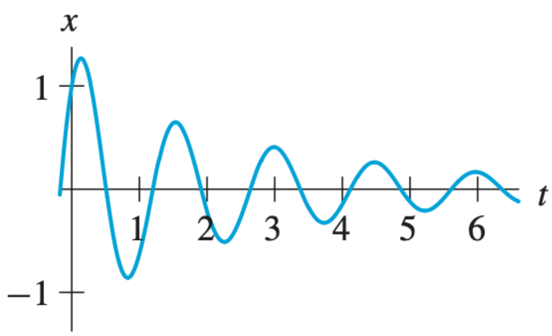
(iii) Consider the following graph of a function $x(t)$.



(a)
Can this be an $x(t)$-graph for a solution of a linear system with complex eigenvalues? [?/yes/no]
(b)
Why is it not an $x(t)$-graph for a solution of a linear system with complex eigenvalues?

- The amplitude is not monotonically decreasing or increasing.
- Oscillation stops at some t.
- Oscillation starts at some t. There was no prior oscillation.
- The time between successive zeros is not constant.

(iv) Consider the following graph of a function $x(t)$.

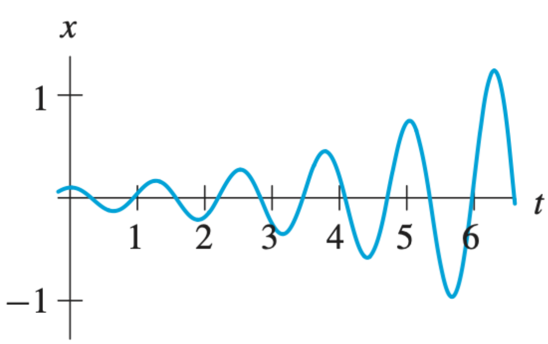


(a)
Can this be an $x(t)$-graph for a solution of a linear system with complex eigenvalues? [?/yes/no]
(b)
Classify the origin as a spiral sink, a spiral source, or a center:

- ?
- spiral sink
- spiral source
- center

Determine the natural period of the system: $T_0 =$ _____

(v) Consider the following graph of a function $x(t)$.



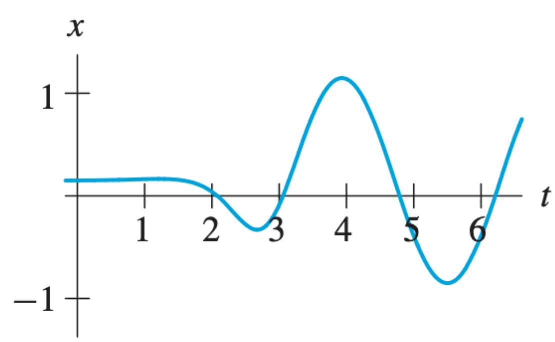
(a)
Can this be an $x(t)$-graph for a solution of a linear system with complex eigenvalues? [?/yes/no]

(b)
Classify the origin as a spiral sink, a spiral source, or a center:

- ?
- spiral sink
- spiral source
- center

Determine the natural period of the system: $T_0 =$ _____

(vi) Consider the following graph of a function $x(t)$.



(a)
Can this be an $x(t)$-graph for a solution of a linear system with complex eigenvalues? [?/yes/no]
(b)
Why is it not an $x(t)$-graph for a solution of a linear system with complex eigenvalues?

- The amplitude is not monotonically decreasing or increasing.
- Oscillation stops at some t.
- Oscillation starts at some t. There was no prior oscillation.
- The time between successive zeros is not constant.

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Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.6I/BDH_3.6.1.pg`

Find the general solution (in scalar form) of the second-order equation

$$\frac{d^2y}{dt^2}+7\frac{dy}{dt}+10y=0.$$

$y(t) =$ ______________

Use "k1" and "k2" for the constants in your solution.

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.6I/BDH_3.6.3.pg`

Find the general solution (in scalar form) of the second-order equation

$$\frac{d^2y}{dt^2}+6\frac{dy}{dt}+9y=0.$$

$y(t) =$ ______________

Use "k1" and "k2" for the constants in your solution.

Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.6I/BDH_3.6.7.pg`

Find the solution of the initial-value problem

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}-3y=0$$
$$y(0)=6,\ y'(0)=-2$$

$y(t) =$ ______________

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.6I/BDH_3.6.9.pg`

Find the solution of the initial-value problem

$$\frac{d^2y}{dt^2}-4\frac{dy}{dt}+13y=0$$
$$y(0)=1,\ y'(0)=-4$$

$y(t) =$ ______________

Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.6I/BDH_3.6.31.pg`

Suppose $y(t)$ is a complex-valued solution of

$$\frac{d^2y}{dt}+p\frac{dy}{dt}+qy=0,$$

where p and q are real numbers. Show that if $y(t)=y_{\mathrm{re}}(t)+i\,y_{\mathrm{im}}(t)$, where $y_{\mathrm{re}}(t)$ and $y_{\mathrm{im}}(t)$ are real valued, then both $y_{\mathrm{re}}(t)$ and $y_{\mathrm{im}}(t)$ are solutions of the second-order equation.

Online version in development – do this on paper.

- I showed it!

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Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.6II/BDH_3.6.13_21.pg`

Consider a harmonic oscillator with mass $m = 1$, spring constant $k = 7$, and damping coefficient $b = 8$, with initial conditions $y(0) = -1$, $v(0) = 5$.

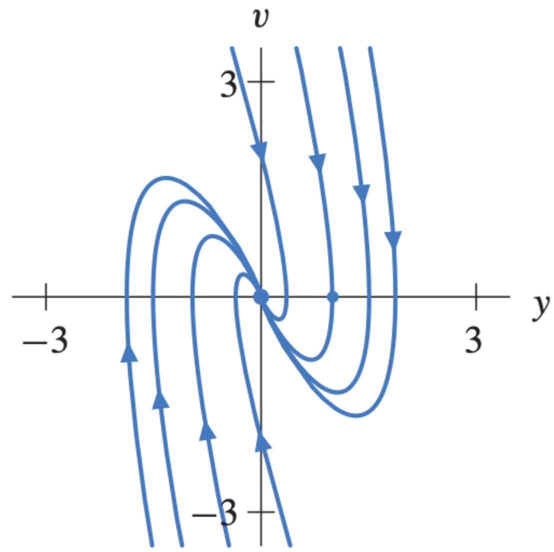
(a) Write the second-order differential equation (on paper; I haven't figured out how to get WeBWorK to do this in a reasonable way),
and write the corresponding first-order system.

$$\frac{d\mathbf{Y}}{dt} = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix} \mathbf{Y}$$

(b) Find the eigenvalues and eigenvectors of the linear system.
$\lambda_1 =$ ____, $v_1 =$ ____
$\lambda_2 =$ ____, $v_2 =$ ____
Enter vectors using angle brackets: "$< 2, 1 >$".

(c) Classify the oscillator:
- ?
- underdamped
- overdamped
- critically damped
- undamped

(d) Sketch, on paper, the phase portrait of the linear system.
Select the graph corresponding to your sketch: [?/i/ii/iii/iv]



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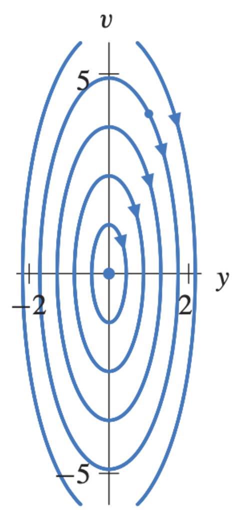

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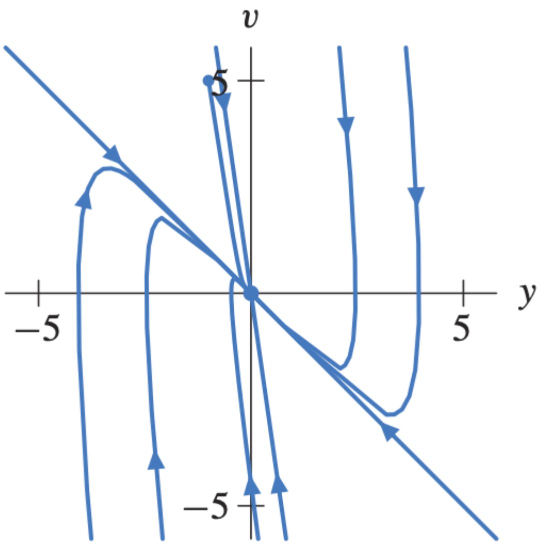

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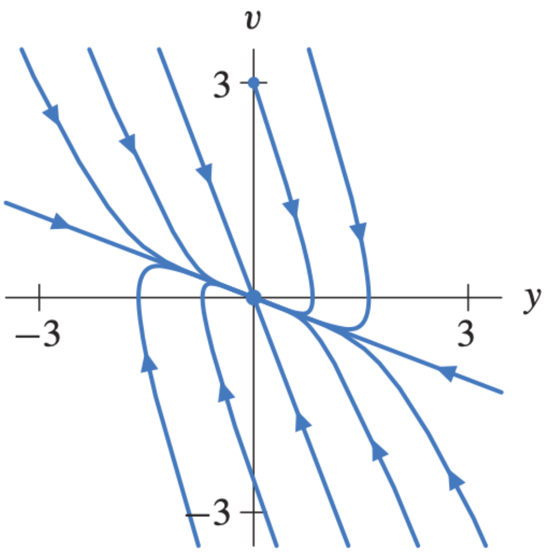
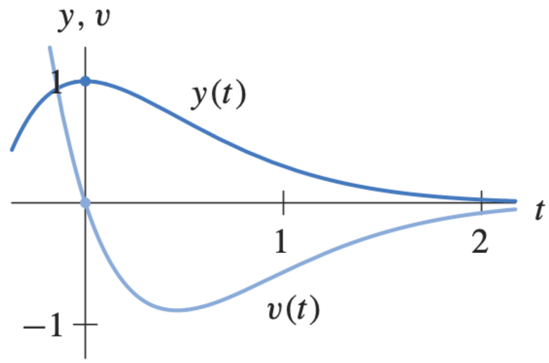


iv)

(e) Sketch, on paper, the $y(t)$- and $v(t)$-graphs of the solution with the given initial condition.
Select the graph corresponding to your sketch: [?/i/ii/iii/iv]



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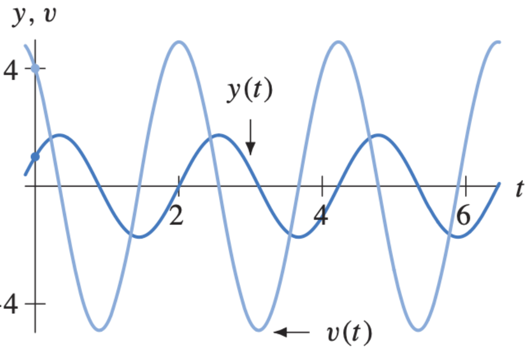

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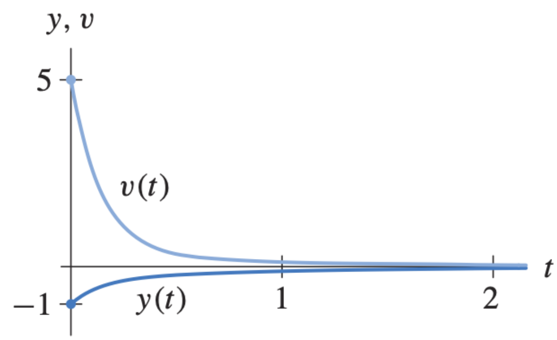

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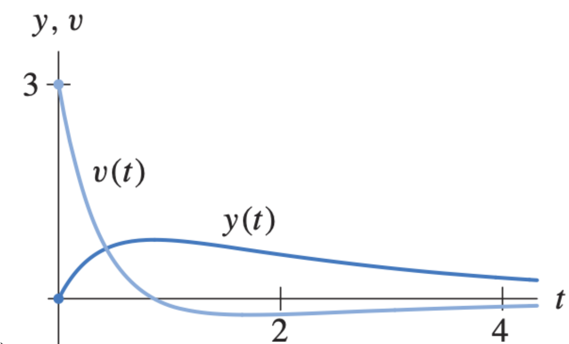


iv)

(f) Find the general solution of the second-order equation that models the motion of the oscillator.

$y(t) =$ ____________________

Use ”k1” and ”k2” for the constants in your solution.

(g) Find the particular solution for the given initial condition.

$y(t) =$ ____________________

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.6II/BDH_3.6.15_23.pg`

Consider a harmonic oscillator with mass $m = 1$, spring constant $k = 5$, and damping coefficient $b = 4$, with initial conditions $y(0) = 1$, $v(0) = 0$.

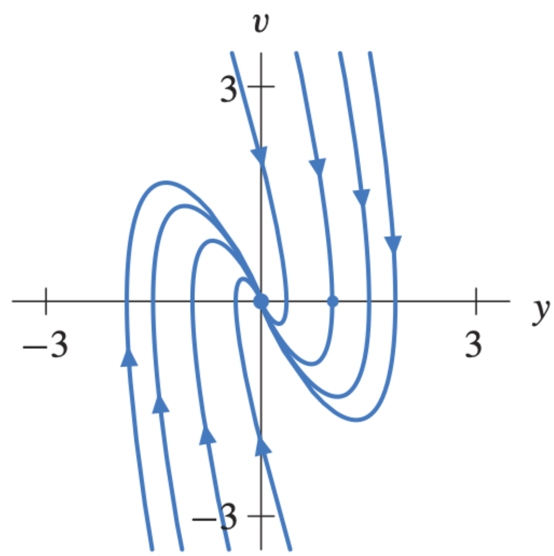
(a) Write the second-order differential equation (on paper; I haven't figured out how to get WeBWorK to do this in a reasonable way),
and write the corresponding first-order system.

$$\frac{d\mathbf{Y}}{dt} = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix} \mathbf{Y}$$

(b) Find the eigenvalues of the linear system.
$\lambda =$ __________ (comma-separated list; use "i" for the imaginary unit)

(c) Classify the oscillator:
- ?
- underdamped
- overdamped
- critically damped
- undamped

(d) Sketch, on paper, the phase portrait of the linear system.
Select the graph corresponding to your sketch: [?/i/ii/iii/iv]



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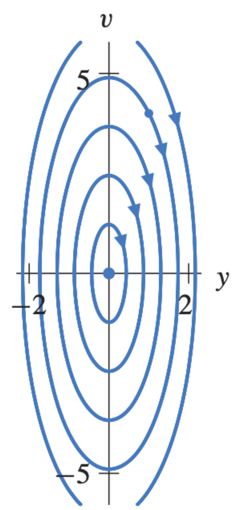

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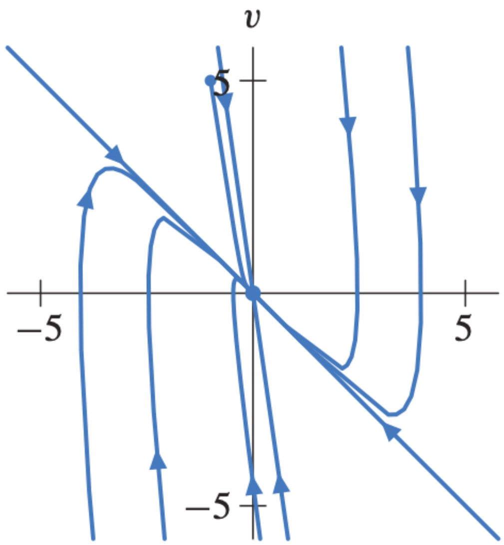

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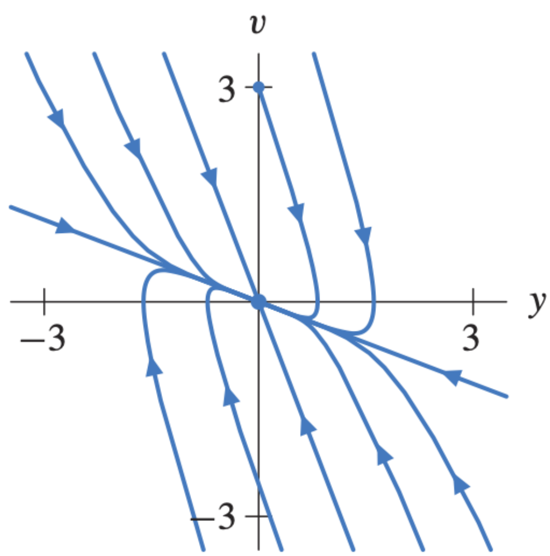
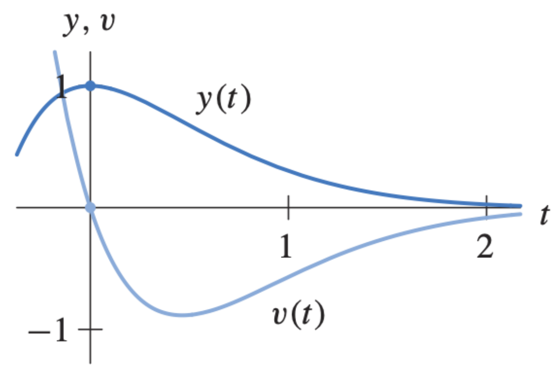


iv)

(e) Sketch, on paper, the $y(t)$- and $v(t)$-graphs of the solution with the given initial condition.
Select the graph corresponding to your sketch: [?/i/ii/iii/iv]



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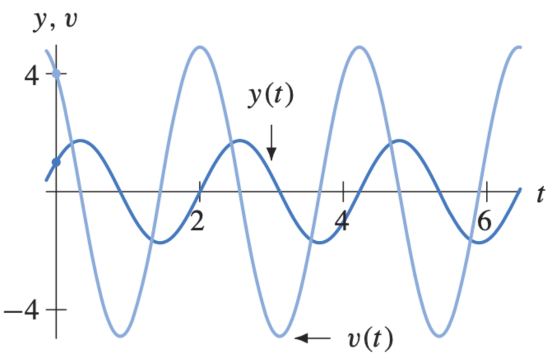

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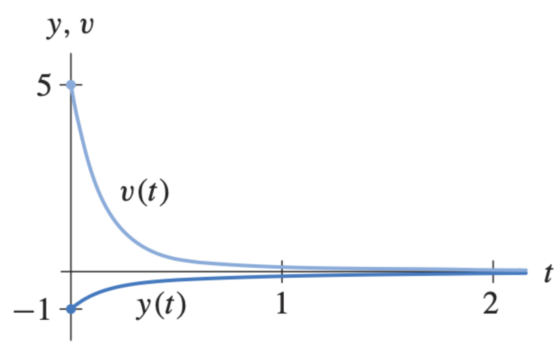

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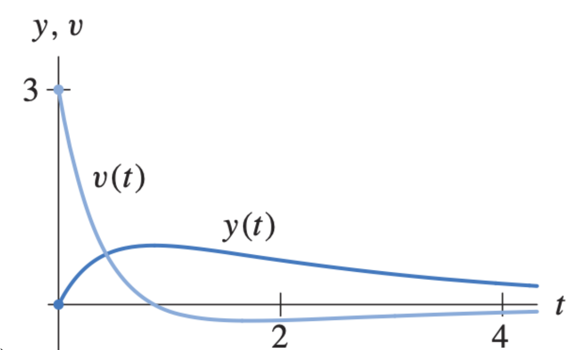


iv)

(f) Find the general solution of the second-order equation that models the motion of the oscillator.

$y(t) =$ ____________________

Use ”k1” and ”k2” for the constants in your solution.

(g) Find the particular solution for the given initial condition.

$y(t) =$ ____________________

Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.6II/BDH_3.6.16_24.pg`

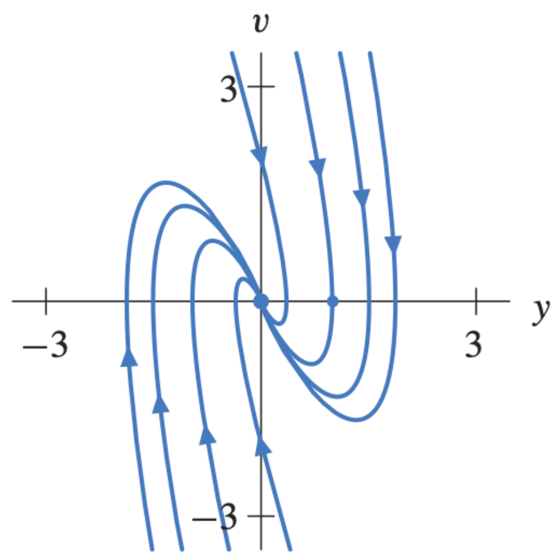
Consider a harmonic oscillator with mass $m = 1$, spring constant $k = 8$, and damping coefficient $b = 0$, with initial conditions $y(0) = 1$, $v(0) = 4$.

(a) Write the second-order differential equation (on paper; I haven't figured out how to get WeBWorK to do this in a reasonable way),
and write the corresponding first-order system.
$\dfrac{d\mathbf{Y}}{dt} = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix} \mathbf{Y}$

(b) Find the eigenvalues of the linear system.
$\lambda =$ __________ (comma-separated list; use "i" for the imaginary unit)

(c) Classify the oscillator:
- ?
- underdamped
- overdamped
- critically damped
- undamped

(d) Sketch, on paper, the phase portrait of the linear system.
Select the graph corresponding to your sketch: [?/i/ii/iii/iv]



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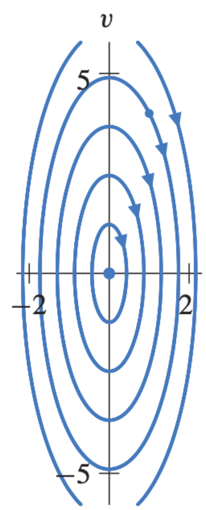

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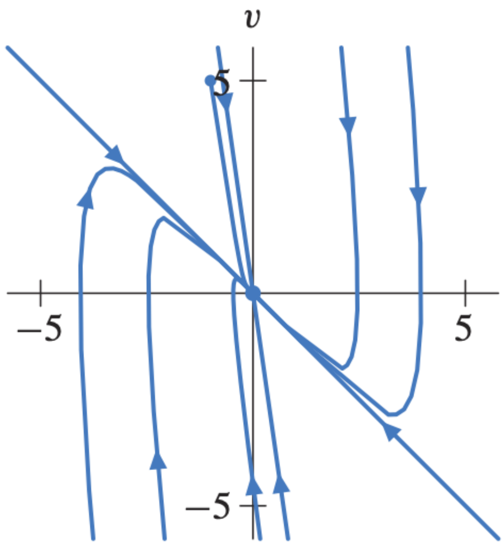

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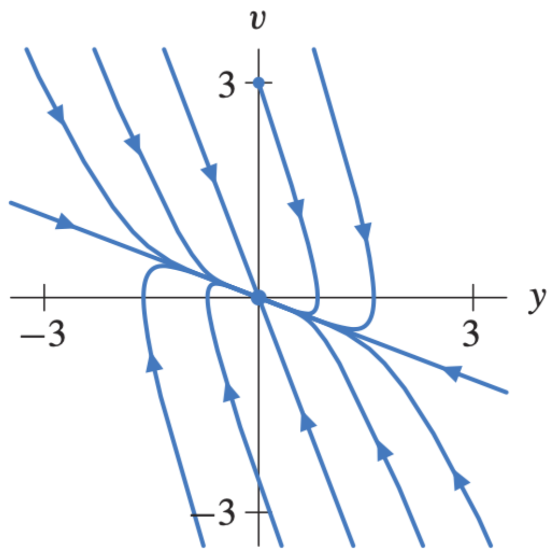
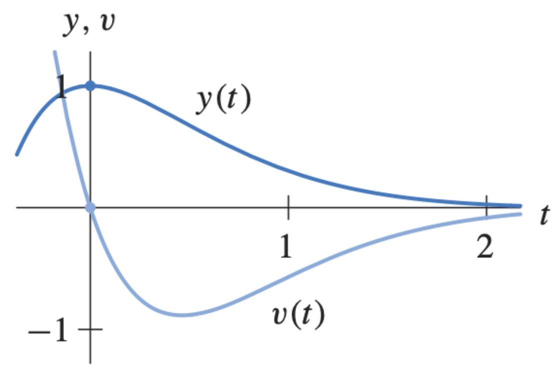


iv)

(e) Sketch, on paper, the $y(t)$- and $v(t)$-graphs of the solution with the given initial condition.
Select the graph corresponding to your sketch: [?/i/ii/iii/iv]



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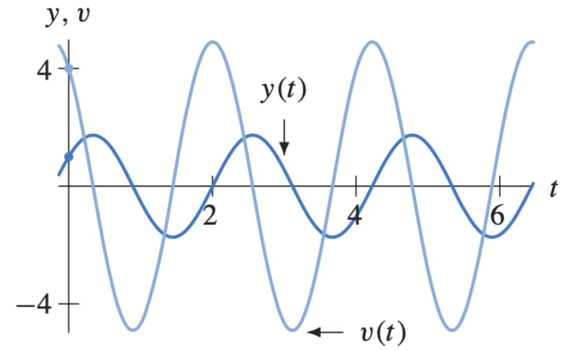

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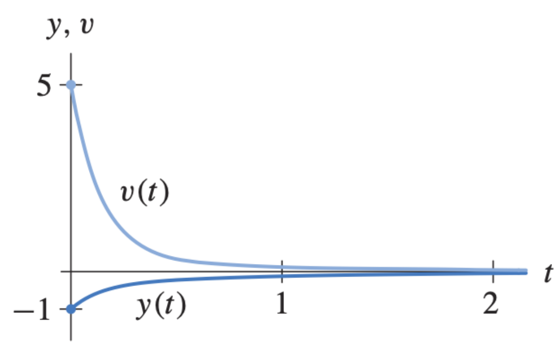

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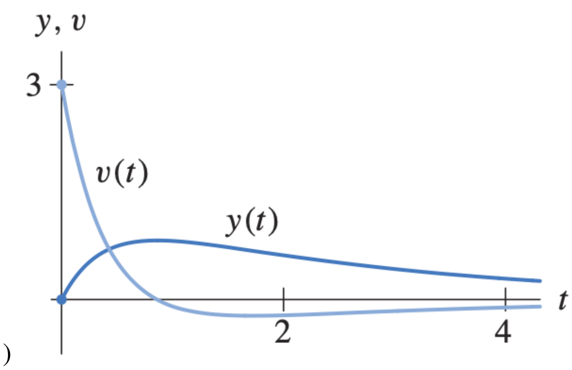


iv)

(f) Find the general solution of the second-order equation that models the motion of the oscillator.

$y(t) =$ ____________________

Use ”k1” and ”k2” for the constants in your solution.

(g) Find the particular solution for the given initial condition.

$y(t) =$ ____________

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.6II/BDH_3.6.17_25.pg`

Consider a harmonic oscillator with mass $m = 2$, spring constant $k = 1$, and damping coefficient $b = 3$, with initial conditions $y(0) = 0$, $v(0) = 3$.

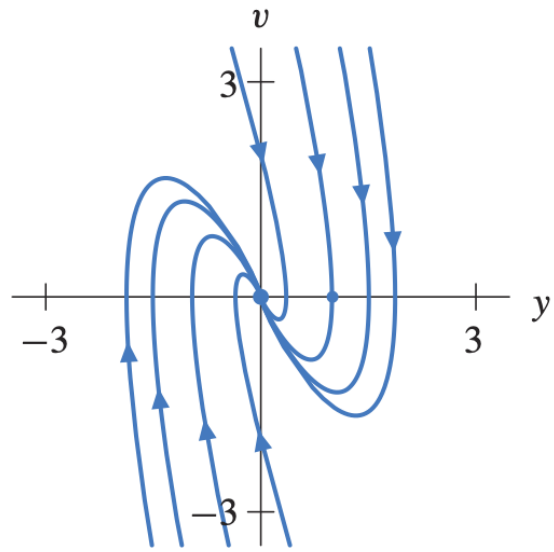
(a) Write the second-order differential equation (on paper; I haven't figured out how to get WeBWorK to do this in a reasonable way),
and write the corresponding first-order system.

$$\frac{d\mathbf{Y}}{dt} = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix} \mathbf{Y}$$

(b) Find the eigenvalues and eigenvectors of the linear system.
$\lambda_1 =$ _____, $v_1 =$ _____
$\lambda_2 =$ _____, $v_2 =$ _____
Enter vectors using angle brackets: "$< 2, 1 >$".

(c) Classify the oscillator:
- ?
- underdamped
- overdamped
- critically damped
- undamped

(d) Sketch, on paper, the phase portrait of the linear system.
Select the graph corresponding to your sketch: [?/i/ii/iii/iv]



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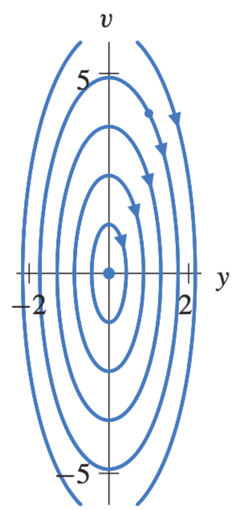

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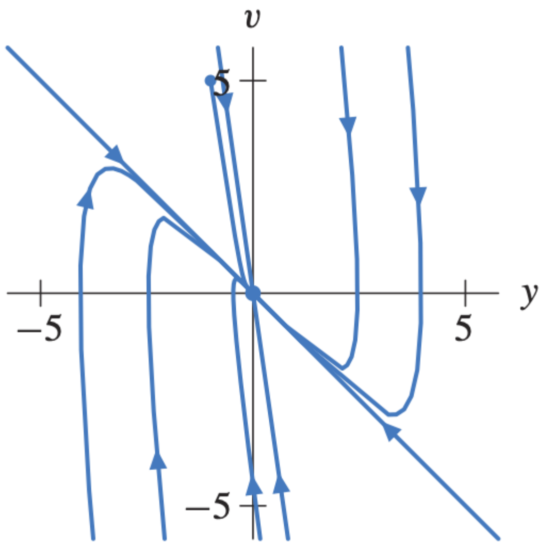

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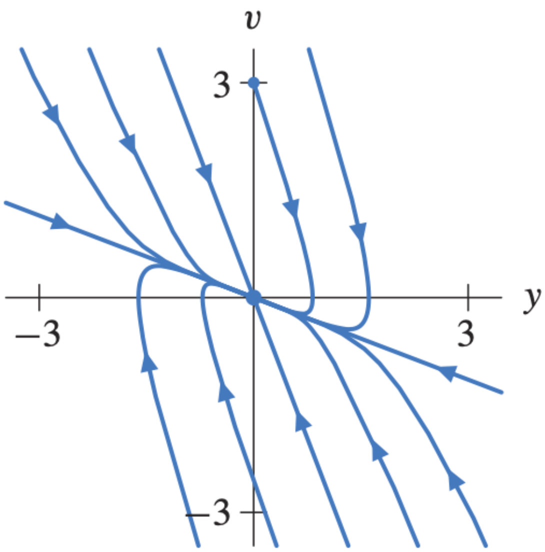
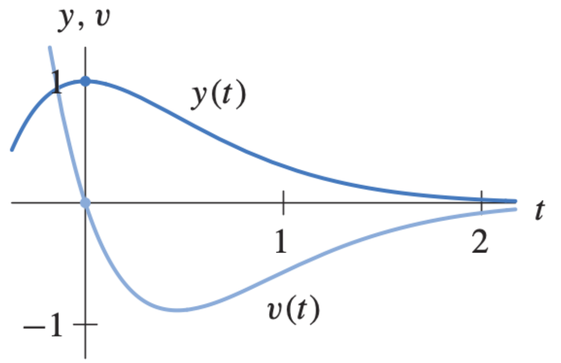


iv)

(e) Sketch, on paper, the $y(t)$- and $v(t)$-graphs of the solution with the given initial condition.
Select the graph corresponding to your sketch: [?/i/ii/iii/iv]



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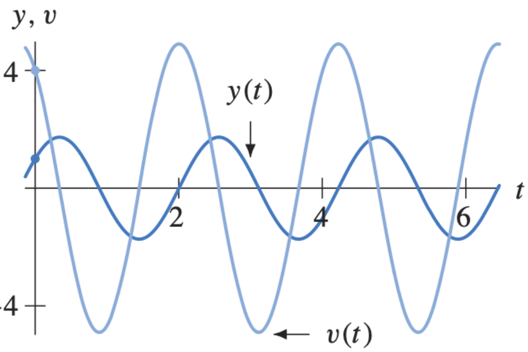

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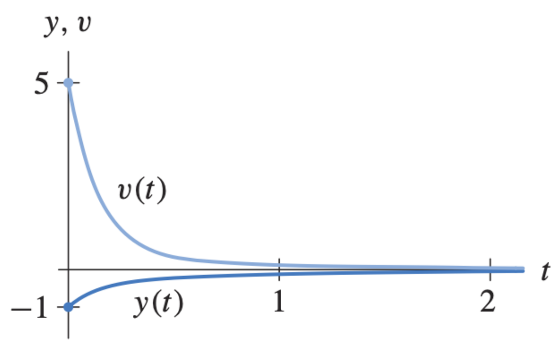

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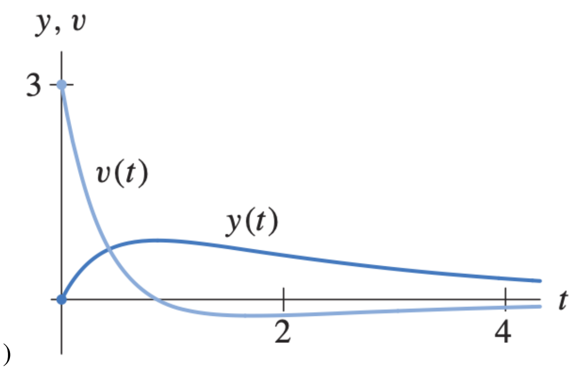

iv)

(f) Find the general solution of the second-order equation that models the motion of the oscillator.

$y(t) =$ ________________

Use "k1" and "k2" for the constants in your solution.

(g) Find the particular solution for the given initial condition.

$y(t) =$ ______

Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.6II/BDH_3.6.36.pg`

An automobile's suspension system consists essentially of large springs with damping. When the car hits a bump, the springs are compressed. It is reasonable to use a harmonic oscillator to model the up-and-down motion, where $y(t)$ measures the amount the springs are stretched or compressed and $v(t)$ is the vertical velocity of the bouncing car.

Suppose that you are working for a company that designs suspension systems for cars. One day your boss comes to you with the results of a market research survey indicating that most people want shock absorbers that "bounce twice" when compressed, then gradually return to their equilibrium position from above. That is, when the car hits a bump, the springs are compressed. Ideally they should expand, compress, and expand, then settle back to the rest position. After the initial bump, the spring would pass through its rest position three times and approach the rest position from the expanded state.

(a) Sketch a graph of the position of the spring after hitting a bump, where $y(t)$ denotes the state of the spring at time t, $y > 0$ corresponds to the spring being stretched, and $y < 0$ corresponds to the spring being compressed.

(b) Explain (politely) why the behavior pictured in the figure is impossible with standard suspension systems that are accurately modeled by the harmonic oscillator system.

(c) What is your suggestion for a choice of a harmonic oscillator system that most closely approximates the desired behavior? Justify your answer with an essay.

No online answer-entry – think about this and jot notes on paper.

- I did it!

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Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.8/BDH_3.8.1.pg`

Consider the linear system

$$\frac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y} = \begin{pmatrix} 0 & 0.1 & 0 \\ 0 & 0 & 0.2 \\ 0.4 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

Check that the functions

$$\mathbf{Y}_2(t) = e^{-0.1t} \begin{pmatrix} -\cos\left(\sqrt{0.03}\,t\right) - \sqrt{3}\sin\left(\sqrt{0.03}\,t\right) \\ -2\cos\left(\sqrt{0.03}\,t\right) + 2\sqrt{3}\sin\left(\sqrt{0.03}\,t\right) \\ 4\cos\left(\sqrt{0.03}\,t\right) \end{pmatrix}$$

and

$$\mathbf{Y}_3(t) = e^{-0.1t} \begin{pmatrix} -\cos\left(\sqrt{0.03}\,t\right) + \sqrt{3}\sin\left(\sqrt{0.03}\,t\right) \\ -2\cos\left(\sqrt{0.03}\,t\right) - 2\sqrt{3}\sin\left(\sqrt{0.03}\,t\right) \\ 4\cos\left(\sqrt{0.03}\,t\right) \end{pmatrix}$$

are solutions to the system.

- I plugged $\mathbf{Y}_2$ into the LHS and the RHS and verified that they are the same.
- I plugged $\mathbf{Y}_3$ into the LHS and the RHS and verified that they are the same.

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.8/BDH_3.8.5.pg`

Consider the linear system

$$\frac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y}$$

with the coefficient matrix

$$\mathbf{A} = \begin{pmatrix} -2 & 3 & 0 \\ 3 & -2 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

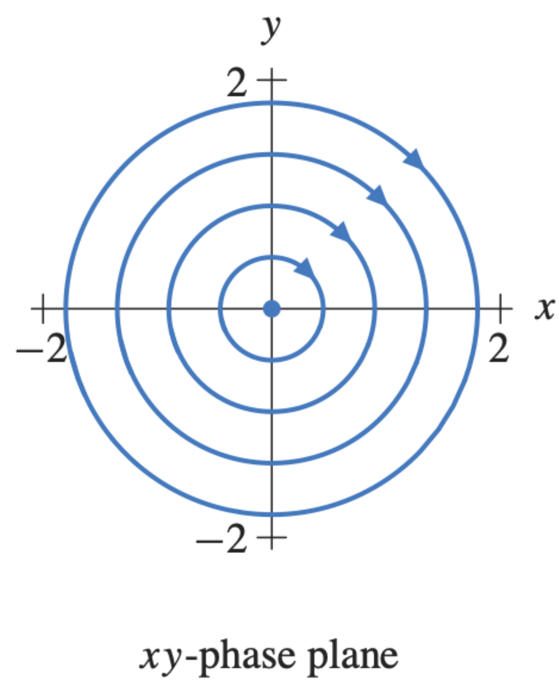
This system decouples into a two-dimensional system and a one-dimensional system.

(a) Compute the eigenvalues: _____ (enter as a comma-separated list)

(b) The system decouples into a

- two-dimensional system in the xy-plane and a one-dimensional system on the z-axis.
- two-dimensional system in the xz-plane and a one-dimensional system on the y-axis.
- two-dimensional system in the yz-plane and a one-dimensional system on the x-axis.

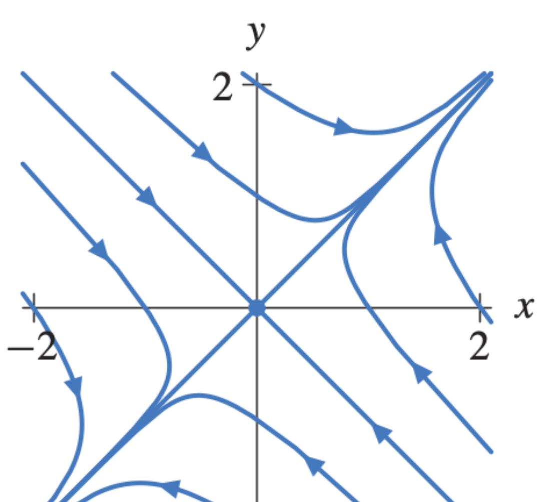
(c) Sketch, on paper, the two-dimensional phase plane and one-dimensional phase line for the decoupled systems.
Select the graph corresponding to your sketch: [?/i/ii/iii/iv]



(i)

z-phase line

(ii)



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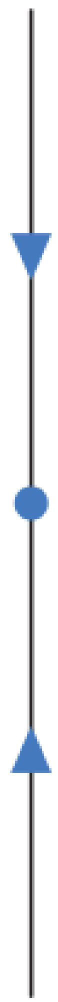
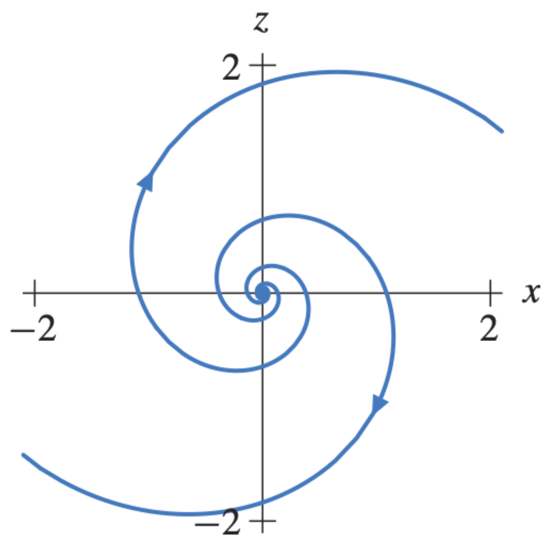


z-phase line

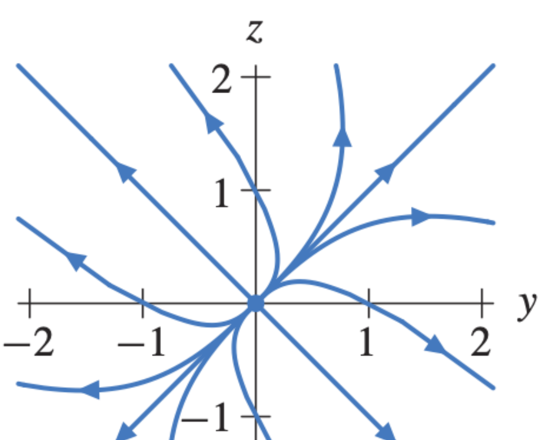


(iii)

xz-phase plane

z-phase line

(iv)



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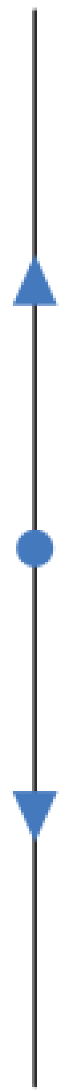


x-phase line

Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.8/BDH_3.8.7.pg`

Consider the linear system

$$\frac{d\mathbf{Y}}{dt} = \mathbf{AY}$$

with the coefficient matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}.$$

This system decouples into a two-dimensional system and a one-dimensional system.

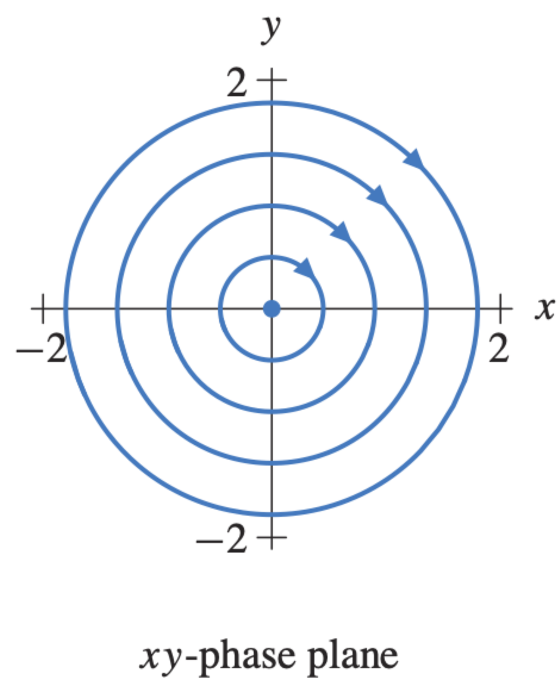
(a) Compute the eigenvalues: ____ (enter as a comma-separated list)

(b) The system decouples into a

- two-dimensional system in the xy-plane and a one-dimensional system on the z-axis.
- two-dimensional system in the xz-plane and a one-dimensional system on the y-axis.
- two-dimensional system in the yz-plane and a one-dimensional system on the x-axis.

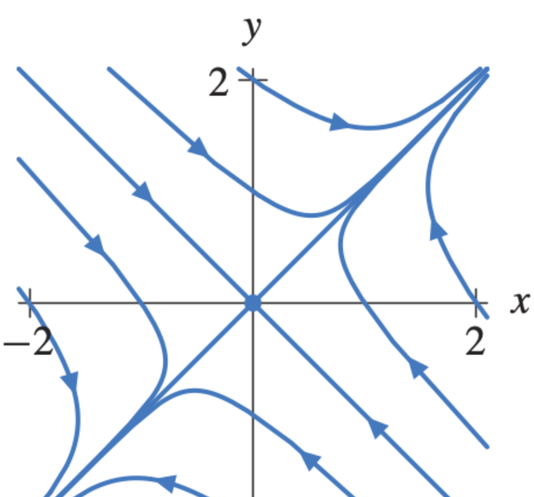
(c) Sketch, on paper, the two-dimensional phase plane and one-dimensional phase line for the decoupled systems.
Select the graph corresponding to your sketch: [?/i/ii/iii/iv]

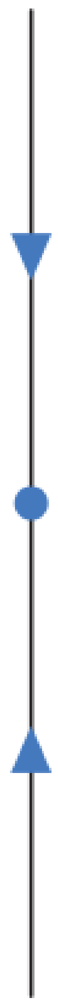
(i)



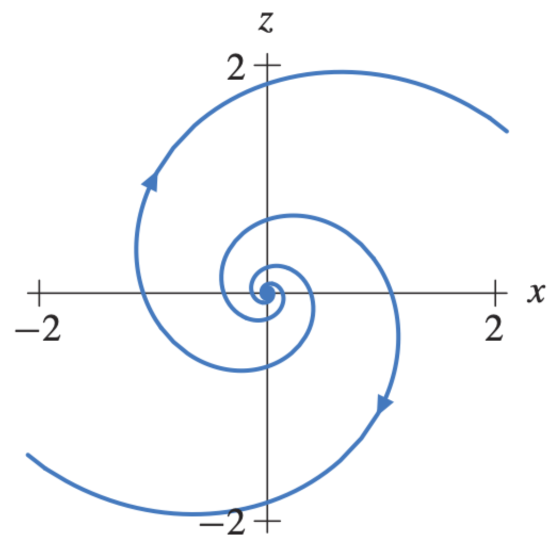
z-phase line

(ii)

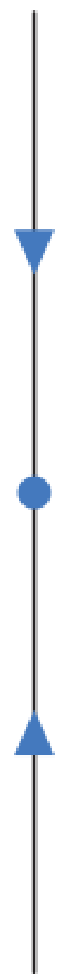




z-phase line

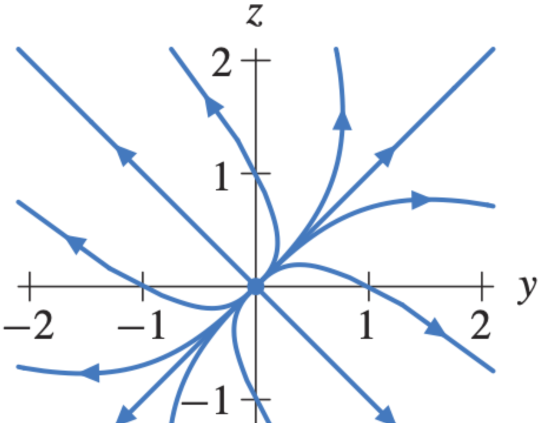


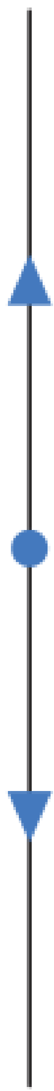
(iii) *xz*-phase plane



z-phase line

(iv)





x-phase line

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.8/BDH_3.8.16.pg`

For the linear system

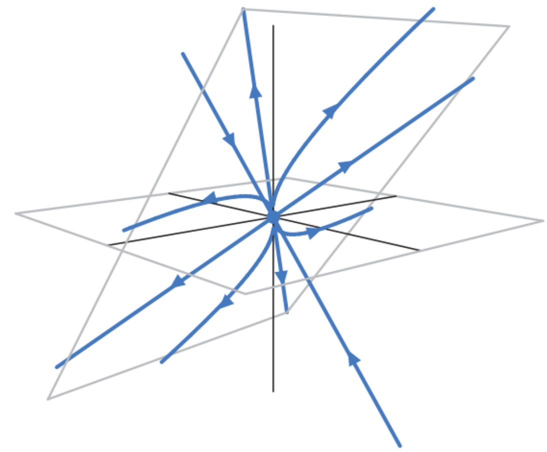
$$\frac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y} = \begin{pmatrix} 2 & -1 & 0 \\ 0 & -2 & 3 \\ -1 & 3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} :$$

(a) Show that $\mathbf{v_1} = (1,1,1)$ is an eigenvector of the coefficient matrix by computing $\mathbf{Av_1}$.
What is the eigenvalue for this eigenvector? $\lambda_1 =$ ___

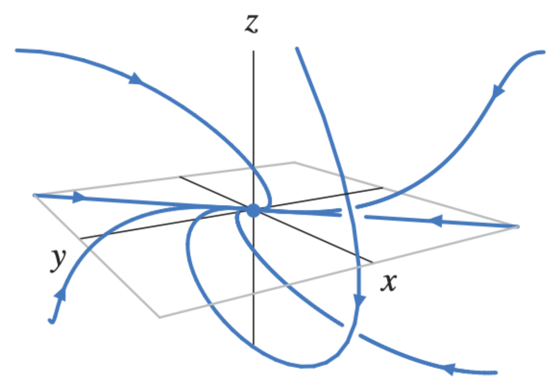
(b) Find the other two eigenvalues for the matrix:
$\lambda_2, \lambda_3 =$ _________ (comma-separated list)
Hint: use (a) to factor the characteristic polynomial.

(c) Classify the system: The equilibrium point at the origin is a
- ?
- sink
- source
- saddle
- spiral sink
- spiral source
- center

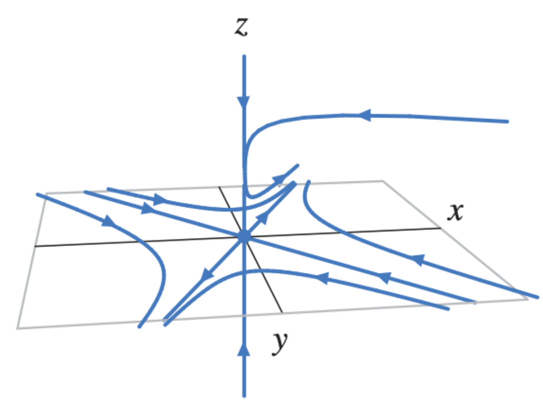
(d) Attempt to sketch, on paper, the three-dimensional phase portrait.
Select the graph corresponding to your sketch: [?/i/ii/iii]



(i) (



ii)



(iii)

Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_3.8/BDH_3.8.17.pg`

For the linear system

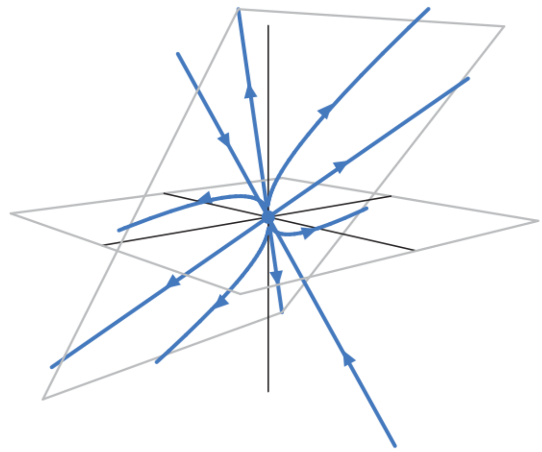
$$\frac{d\mathbf{Y}}{dt} = \mathbf{AY} = \begin{pmatrix} -4 & 3 & 0 \\ 0 & -1 & 1 \\ 5 & -5 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} :$$

(a) Show that $\mathbf{v_1} = (1,1,0)$ is an eigenvector of the coefficient matrix by computing $\mathbf{Av_1}$.
What is the eigenvalue for this eigenvector? $\lambda_1 =$ ___

(b) Find the other two eigenvalues for the matrix:
$\lambda_2, \lambda_3 =$ ________ (comma-separated list)
Hint: use (a) to factor the characteristic polynomial.

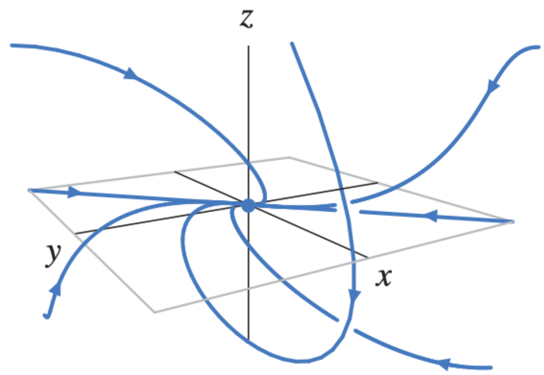
(c) Classify the system: The equilibrium point at the origin is a
- ?
- sink
- source
- saddle
- spiral sink
- spiral source
- center

(d) Attempt to sketch, on paper, the three-dimensional phase portrait.
Select the graph corresponding to your sketch: [?/i/ii/iii]

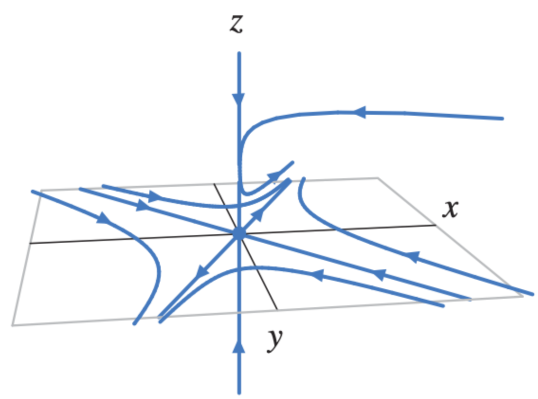


(i)

(



ii)



(iii)

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Thomas Hoft **210S23_Hoft**

Assignment BDH_4.1 due 04/06/2023 at 09:55am CDT

Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_4.1/BDH_4.1.1.pg`

Find the general solution of the differential equation

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 6y = e^{4t}$$

Use "k1" and "k2" for the constants in your solution.

$y(t) =$ ________________

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_4.1/BDH_4.1.7.pg`

Find the general solution of the differential equation

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 4y = e^{4t}$$

Use "k1" and "k2" for the constants in your solution.

$y(t) =$ ________________

Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_4.1/BDH_4.1.11.pg`

Find the solution of the initial value problem

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 13y = -3e^{-2t}, \quad y(0) = y'(0) = 0.$$

$y(t) =$ ________________

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_4.1/BDH_4.1.13.pg`

Consider the differential equation

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = e^{-t/2}.$$

(a) Compute the general solution.

$y(t) =$ ________________

Use "k1" and "k2" for the constants in your solution.

(b) Compute the solution with $y(0) = y'(0) = 0$.

$y(t) =$ ________________

Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_4.1/BDH_4.1.21.pg`

One of the most common forcing functions is constant forcing. Consider the following equation for a harmonic oscillator with constant forcing:

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 4y = 5.$$

(a) Compute the general solution.

$y(t) =$ ________________

Use "k1" and "k2" for the constants in your solution.

(b) Compute the solution with $y(0) = y'(0) = 0$.

$y(t) =$ ________________

Problem 6. **(1 point)** `ustLibrary/ustDiffEq/setBDH_4.1/BDH_4.1.25.pg`

The discussion of the qualitative behavior of solutions of equations for damped forced harmonic oscillators relies on the fact that solutions of the unforced equation (natural response) tend to zero as t increases. If damping is not present, then the natural response does not tend to zero and the qualitative behavior of solutions is more complicated (see Section 4.3). However, the Extended Linearity Principle and the Method of Undetermined Coefficients apply to all nonhomogeneous, linear equations.

Consider the undamped forced harmonic oscillator

$$\frac{d^2y}{dt^2} + 9y = e^{-t}.$$

(a) Compute the general solution.

$y(t) =$ ____________________

Use "k1" and "k2" for the constants in your solution.

(b) Compute the solution with $y(0) = y'(0) = 0$.

$y(t) =$ ____________________

(c) Use your favorite graphing tool to plot the solution with $y(0) = y'(0) = 0$, and describe its long-term behavior.

- The solution oscillates about a constant value of y with constant amplitude oscillations.
- The solution quickly approaches a solution of the unforced oscillator.
- The solution tends to infinity, but it oscillates about an exponentially-increasing value as it does so.

Problem 7. **(1 point)** `ustLibrary/ustDiffEq/setBDH_4.1/BDH_4.1.31.pg`

In order to use the Method of Undetermined Coefficients to find a particular solution, we must be able to make a reasonable guess (up to multiplicative constants – the undetermined coefficients) of a particular solution. In this section we discussed exponential forcing. If the forcing is some other type of function, then we must adjust our guess accordingly. For example, for an equation of the form

$$\frac{d^2y}{dt^2} + 4y = -3t^2 + 2t + 3,$$

the forcing function is the quadratic polynomial $g(t) = -3t^2 + 2t + 3$. It is reasonable to guess that a particular solution in this case is also a quadratic polynomial. Hence we guess $y_p(t) = at^2 + bt + c$. The constants a, b, and c are determined by substituting $y_p(t)$ into the equation.

(a) Compute the general solution of the equation above.

$y(t) =$ ____________________

Use "k1" and "k2" for the constants in your solution.

(b) Compute the solution with $y(0) = 2$, $y'(0) = 0$.

$y(t) =$ ____________________

Problem 8. **(1 point)** `ustLibrary/ustDiffEq/setBDH_4.1/BDH_4.1.39.pg`

Consider the damped forced harmonic oscillator

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = 2t + e^{-t}.$$

(a) Compute the general solution.

$y(t) =$ ____________________

Use "k1" and "k2" for the constants in your solution.

(b) Compute the solution with $y(0) = y'(0) = 0$.

$y(t) =$ ____________________

(c) Use your favorite graphing tool to plot the solution from (b), and describe its long-term behavior.

- All of the exponential terms tend to 0. Hence, the solution tends to a constant.
- The solution tends to infinity.
- All exponential terms in the solution tend to zero. Hence, the solution tends to infinity linearly in t.

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Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_4.2/BDH_4.2.1.pg`

Find the general solution of the differential equation

$$\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y=\cos t$$

$y(t) =$ ______________

Use "k1" and "k2" for the constants in your solution.

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_4.2/BDH_4.2.5_11.pg`

(a) Find the general solution of the differential equation

$$\frac{d^2y}{dt^2}+6\frac{dy}{dt}+8y=\cos t.$$

$y(t) =$ ______________

Use "k1" and "k2" for the constants in your solution.

(b) Find the solution of the initial-value problem

$$\frac{d^2y}{dt^2}+6\frac{dy}{dt}+8y=\cos t, \quad y(0)=y'(0)=0.$$

$y(t) =$ ______________

Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_4.2/BDH_4.2.9_13.pg`

(a) Find the general solution of the differential equation

$$\frac{d^2y}{dt^2}+4\frac{dy}{dt}+20y=-3\sin 2t.$$

$y(t) =$ ______________

Use "k1" and "k2" for the constants in your solution.

(b) Find the solution of the initial-value problem

$$\frac{d^2y}{dt^2}+4\frac{dy}{dt}+20y=-3\sin 2t, \quad y(0)=y'(0)=0.$$

$y(t) =$ ______________

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_4.2/BDH_4.2.15.pg`

To find a particular solution of a forced equation with sine or cosine forcing, we solved the corresponding complexified equation. This method is particularly efficient, but there are other approaches. Find a particular solution via the Method of Undetermined Coefficients for the equation

$$\frac{d^2y}{dt^2}+3\frac{dy}{dt}+y=\cos 3t$$

(a) by using the guess $y_p(t)=a\cos 3t+b\sin 3t$ (with a and b as undetermined coefficients)

$y(t) =$ ______________

(b) by using the guess $y_p(t)=A\cos(3t+\phi)$ (with A and ϕ as undetermined coefficients)

$y(t) =$ ______________

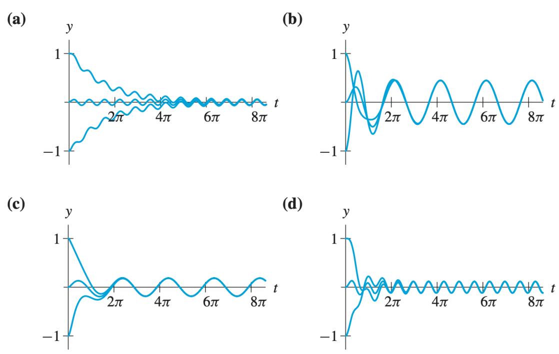
Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_4.2/BDH_4.2.17.pg`

Six second-order equations and four $y(t)$-graphs are given below. The equations are all of the form

$$\frac{d^y}{dt^2} + p\frac{dy}{dt} + qy = \cos \omega t$$

for various values of the parameters p, q, and ω. For each $y(t)$-graph, determine the second-order equation for which $y(t)$ is a solution. You should do this exercise without using technology and be able to state briefly how you know your choice is correct.

(i) $p = 5, q = 3, \omega = 1$ (ii) $p = 1, q = 3, \omega = 1$ (iii) $p = 5, q = 2, \omega = 2$
(iv) $p = 1, q = 3, \omega = 2$ (v) $p = 5, q = 1, \omega = 3$ (vi) $p = 1, q = 1, \omega = 3$



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(a) [?/i/ii/iii/iv/v/vi] (b) [?/i/ii/iii/iv/v/vi] (c) [?/i/ii/iii/iv/v/vi]
(d) [?/i/ii/iii/iv/v/vi]

Problem 6. **(1 point)** `ustLibrary/ustDiffEq/setBDH_4.2/BDH_4.2.19.pg`

(a) Find the general solution of

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 20y = e^{-t}\cos t.$$

Hint : Consider the forcing function to be the real part of a complex exponential.

$y(t) =$ ______________________________

Use "k1" and "k2" for the constants in your solution.

(b) Determine the long-term behavior of solutions of this equation.

- All of the exponential terms tend to 0. Hence, the solution tends to a constant.
- The solution tends to infinity.
- All solutions oscillate with decaying amplitude and the period of the forcing function since the natural oscillations decay faster than the forcing oscillations.
- All exponential terms in the solution tend to zero. Hence, the solution tends to infinity linearly in t.

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Thomas Hoft **210S23_Hoft**
Assignment BDH_4.3 due 04/11/2023 at 09:55am CDT

Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_4.3/BDH_4.3.1_9.pg`

(a) Compute the general solution of the differential equation

$$\frac{d^2y}{dt^2} + 9y = \cos t.$$

$y(t) =$ ______________________

Use "k1" and "k2" for the constants in your solution.

(b) Compute the solution of the initial-value problem

$$\frac{d^2y}{dt^2} + 9y = \cos t, \quad y(0) = y'(0) = 0.$$

$y(t) =$ ______________________

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_4.3/BDH_4.3.5_13.pg`

(a) Compute the general solution of the differential equation

$$\frac{d^2y}{dt^2} + 9y = 2\cos 3t.$$

$y(t) =$ ______________________

Use "k1" and "k2" for the constants in your solution.

(b) Compute the solution of the initial-value problem

$$\frac{d^2y}{dt^2} + 9y = 2\cos 3t, \quad y(0) = 2,\ y'(0) = -9.$$

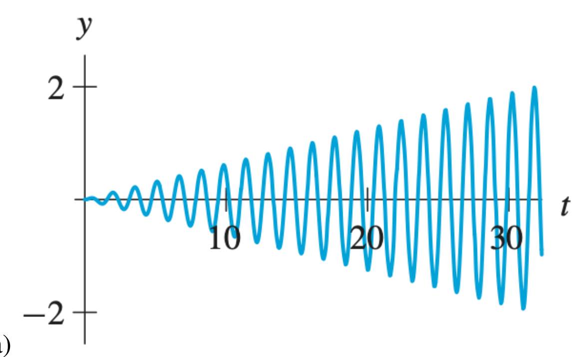
$y(t) =$ ______________________

Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_4.3/BDH_4.3.21ac.pg`

Four second-order equations and two $y(t)$-graphs are given below. For each $y(t)$-graph, determine the second-order equation for which $y(t)$ is a solution. You should do this exercise without using technology and be able to state briefly how you know your choice is correct.

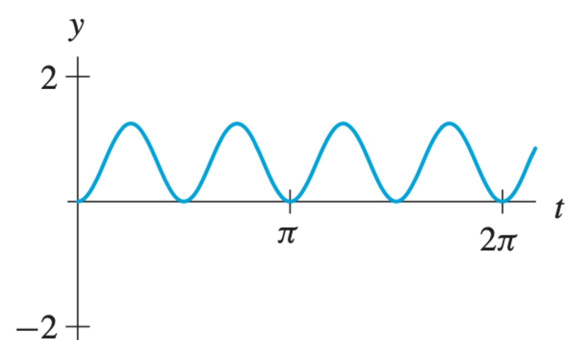
(i) $\dfrac{d^2y}{dt^2} + 16y = 10$ (ii) $\dfrac{d^2y}{dt^2} + 16y = -10$

(iii) $\dfrac{d^2y}{dt^2} + 16y = \dfrac{1}{2}\cos 4t$ (iv) $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} + 16y = \cos 4t$



(a)

[?/i/ii/iii/iv]



(c)

[?/i/ii/iii/iv]

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_4.3/BDH_4.3.24.pg`

Suppose the suspension system of the average car can be fairly well modeled by an underdamped harmonic oscillator with a natural period of 2 seconds. How far apart should speed bumps be placed so that a car traveling at 10 miles per hour over several bumps will bounce more and more violently with each bump?

_____ miles

___ feet and ___ inches

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Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_5.1/5.1.3.pg`

Consider the system

$$\frac{dx}{dt} = -2x + y$$
$$\frac{dy}{dt} = -y + x^2$$

which depends on the parameter a.

(a) Find the linearized system for the equilibrium point (0,0).

The Jacobian is $J(0,0) = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

$\frac{dx}{dt} =$ _________

$\frac{dy}{dt} =$ _________

(b) The equilibrium at (0,0) is a
- ?
- sink
- source
- saddle
- spiral sink
- spiral source
- center

(c) Sketch, on paper, the phase portrait for the linearized system near (0,0). (nothing to enter online)

(d) Find the linearized system for the equilibrium point (2,4).

The Jacobian is $J(2,4) = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

$\frac{dx}{dt} =$ _________

$\frac{dy}{dt} =$ _________

(e) The equilibrium at (2,4) is a
- ?
- sink
- source
- saddle
- spiral sink
- spiral source
- center

(f) Sketch, on paper, the phase portrait for the linearized system near (2,4). (nothing to enter online)

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_5.1/5.1.6.pg`

For the competing species population model

$$\frac{dx}{dt} = 2x\left(1 - \frac{x}{2}\right) - xy$$
$$\frac{dy}{dt} = 3y\left(1 - \frac{y}{3}\right) - 2xy$$

the equilibrium point (1,1) is a saddle.

The Jacobian is $J(x,y) = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

(a) Find the linearized system for the equilibrium point (0,0).
$\frac{dx}{dt} =$ __________
$\frac{dy}{dt} =$ __________

(b) The equilibrium at (0,3) is a

- ?
- sink
- source
- saddle
- spiral sink
- spiral source
- center

(c) Sketch, on paper, the phase portrait for the linearized system near (0,0). (nothing to enter online)

(d) Find the linearized system for the equilibrium point (0,3).
$\frac{dx}{dt} =$ __________
$\frac{dy}{dt} =$ __________

(e) The equilibrium at (0,3) is a

- ?
- sink
- source
- saddle
- spiral sink
- spiral source
- center

(f) Sketch, on paper, the phase portrait for the linearized system near (0,3). (nothing to enter online)

(h) Find the linearized system for the equilibrium point (2,0).
$\frac{dx}{dt} =$ __________
$\frac{dy}{dt} =$ __________

(i) The equilibrium at (2,0) is a

- ?
- sink
- source
- saddle
- spiral sink
- spiral source
- center

(j) Sketch, on paper, the phase portrait for the linearized system near (2,0). (nothing to enter online)

Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_5.1/5.1.7.pg`

Consider the system

$$\frac{dx}{dt} = x(-x-3y+150)$$
$$\frac{dy}{dt} = y(-2x-y+100)$$

where we restrict our attention to $x, y \geq 0$.

(a)

The equilibria are ______.

(enter a comma-separated list of points (x_0, y_0))

(b)

Classify all equilibria.

The equilibrium at (0,0) is a

- ?
- sink
- source
- saddle
- spiral sink
- spiral source
- center

The equilibrium at (0,100) is a

- ?
- sink
- source
- saddle
- spiral sink
- spiral source
- center

The equilibrium at (150,0) is a

- ?
- sink
- source
- saddle
- spiral sink
- spiral source
- center

The equilibrium at (30,40) is a

- ?
- sink
- source
- saddle
- spiral sink
- spiral source
- center

(c)

Sketch, on paper, the phase portrait for the linearized system near each equilibrium. Use HPGSystemSolver to compare the actual phase portrait to the phase portraits of the linearizations. (You may need to set $\Delta t = 0.0001$.) (nothing to enter online)

- I did it!

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_5.1/5.1.9.pg`

Consider the system

$$\frac{dx}{dt} = x(100 - x - 2y)$$
$$\frac{dy}{dt} = y(150 - x - 6y)$$

where we restrict our attention to $x, y \geq 0$.

(a)

The equilibria are ____________________.

(enter a comma-separated list of points (x_0, y_0))

(b)

Classify all equilibria.

The equilibrium at (0,0) is a

- ?
- sink
- source
- saddle
- spiral sink
- spiral source
- center

The equilibrium at (0,25) is a

- ?
- sink
- source
- saddle
- spiral sink
- spiral source
- center

The equilibrium at (100,0) is a

- ?
- sink
- source
- saddle
- spiral sink
- spiral source
- center

The equilibrium at (75,12.5) is a

- ?
- sink
- source
- saddle
- spiral sink
- spiral source
- center

(c)

Sketch, on paper, the phase portrait for the linearized system near each equilibrium. Use HPGSystemSolver to compare the actual phase portrait to the phase portraits of the linearizations. (You may need to set $\Delta t = 0.0001$.) (nothing to enter online)

- I did it!

Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_5.1/5.1.11.pg`

Consider the system

$$\frac{dx}{dt} = x(-x - y + 40)$$
$$\frac{dy}{dt} = y(-x^2 - y^2 + 2500)$$

where we restrict our attention to $x, y \geq 0$.

(a)

The equilibria are ____________________.

(enter a comma-separated list of points (x_0, y_0))

(b)

Classify all equilibria.

The equilibrium at (0,0) is a

- ?
- sink
- source
- saddle
- spiral sink
- spiral source
- center

The equilibrium at (0,50) is a

- ?
- sink
- source
- saddle
- spiral sink
- spiral source
- center

The equilibrium at (40,0) is a

- ?
- sink
- source
- saddle
- spiral sink
- spiral source
- center

(c)

Sketch, on paper, the phase portrait for the linearized system near each equilibrium. Use HPGSystemSolver to compare the actual phase portrait to the phase portraits of the linearizations. (You may need to set $\Delta t = 0.0001$.) (nothing to enter online)

- I did it!

Problem 6. **(1 point)** `ustLibrary/ustDiffEq/setBDH_5.1/5.1.23.pg`

The system

$$\frac{dx}{dt} = y - ax^3$$
$$\frac{dy}{dt} = y - x$$

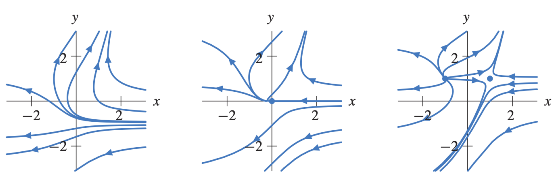
depends on the parameter a.

(a) Find all equilibrium points: ____________
(enter as a comma-separated list of points (x_i, y_i))

(b) Determine all values of a at which a bifurcation occurs: ____
(enter as a comma-separated list of values)

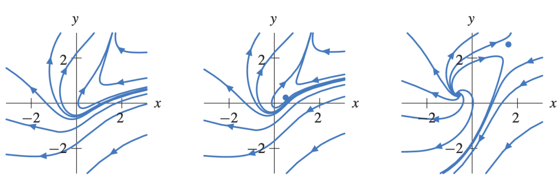
(c) Sketch, on paper, the phase portrait before, at, and after each bifurcation value.
Select the graph corresponding to your sketch: [?/i/ii/iii/iv]

(i)



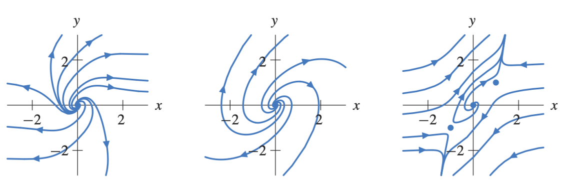
$a < 0$ $a =$
0 $a > 0$

(ii)



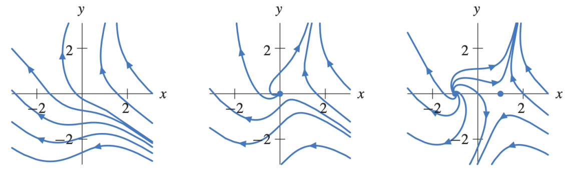
$a < 0$ $a =$
0 $a > 0$

(iii)



$a < 0$ $a =$
0 $a > 0$

(iv)



$a < 0$ $a =$
0 $a > 0$

Problem 7. **(1 point)** `ustLibrary/ustDiffEq/setBDH_5.1/5.1.25.pg`

The system

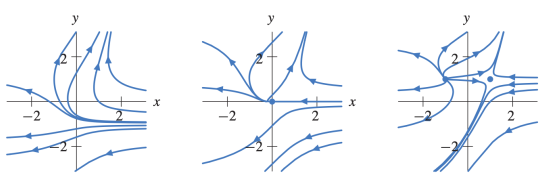
$$\frac{dx}{dt} = y - x^2 + a$$
$$\frac{dy}{dt} = y + x^2$$

depends on the parameter a.

(a) Find all equilibrium points: ________________
(enter as a comma-separated list of points (x_i, y_i))

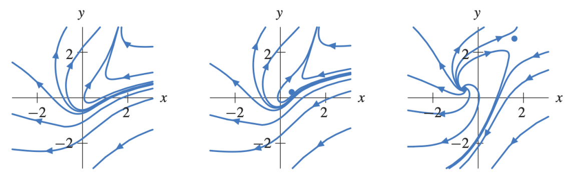
(b) Determine all values of a at which a bifurcation occurs: ____
(enter as a comma-separated list of values)

(c) Sketch, on paper, the phase portrait before, at, and after each bifurcation value.
Select the graph corresponding to your sketch: [?/i/ii/iii/iv]



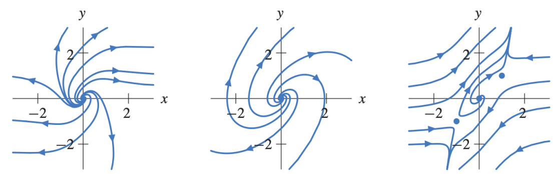
(i)

$a < 0$ $a =$
0 $a > 0$



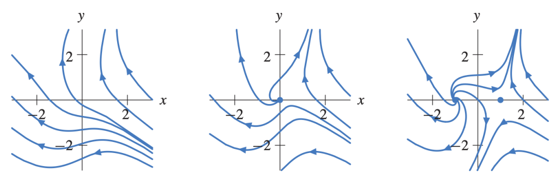
(ii)

$a < 0$ $a =$
0 $a > 0$



(iii)

$a < 0$ $a =$
0 $a > 0$



(iv)

$a < 0$ $a =$
0 $a > 0$

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Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_5.2/BDH_5.2.1.pg`

Consider the system

$$\frac{dx}{dt} = 2 - x - y$$
$$\frac{dy}{dt} = y - x^2$$

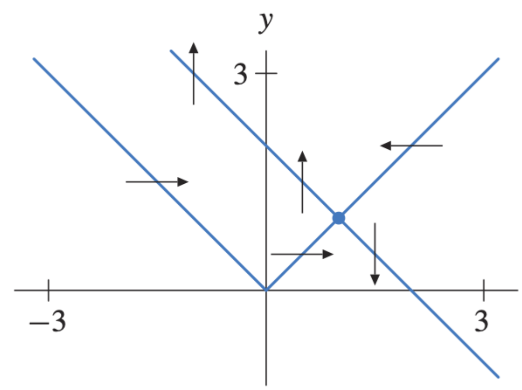
and the initial conditions

(a) $x_0 = 2,\ y_0 = 1$
(b) $x_0 = 0,\ y_0 = -1$
(c) $x_0 = 0,\ y_0 = 1$

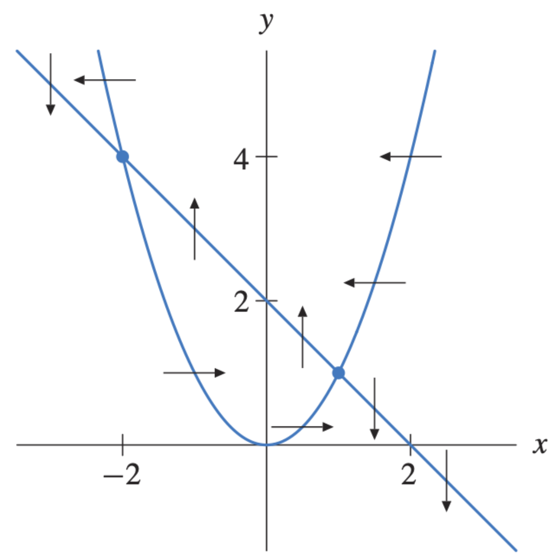
Sketch, on paper, the x- and y-nullclines of the system. Then find all equilibrium points. Using the direction of the vector field between the nullclines, describe the possible fate of the solution curves corresponding to the initial conditions (a), (b), and (c).

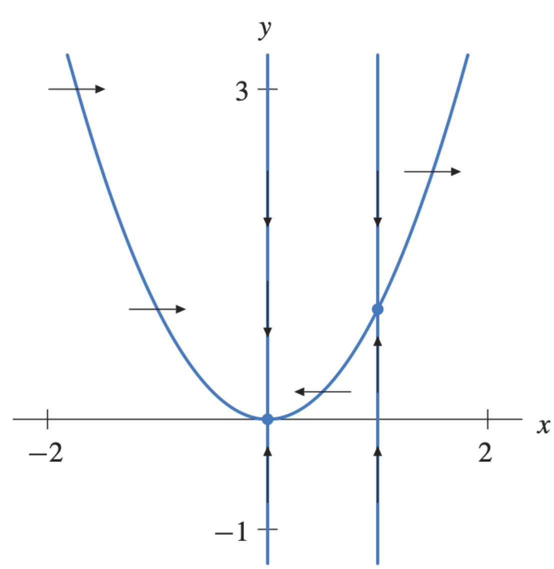
The equilibrium points are: ______________________
Enter as a comma-separated list of points (x_i, y_i).

Select the graph corresponding to your sketch of the nullclines: [?/i/ii/iii]



(i) (ii)





(iii)

Describe the long-term behviour of the solutions with initial conditions (a-c):

- **a**. All three solutions will go down and to the right without bound.
- **b**. The solutions for (a) and (c) tend to infinity in the left-up region; the solution for (b) tends to infinity in the right-down region.
- **c**. The solutions for (a) and (b) tend toward the equilibrium point at (0, 0); the solutions for (c) tends toward the equilibrium point at (1, 1).

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_5.2/BDH_5.2.3.pg`

Consider the system

$$\frac{dx}{dt} = x(x-1)$$
$$\frac{dy}{dt} = x^2 - y$$

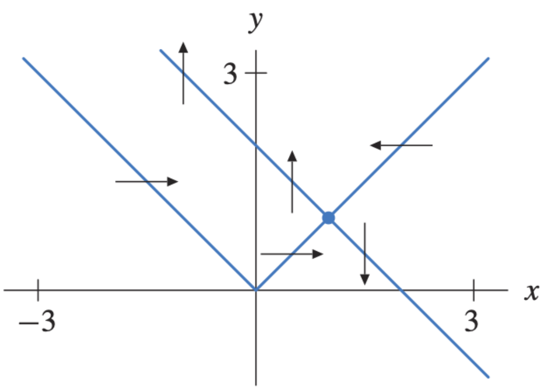
and the initial conditions

$$\text{(a) } x_0 = -1,\ y_0 = 0$$
$$\text{(b) } x_0 = 0.8,\ y_0 = 0$$
$$\text{(c) } x_0 = 1,\ y_0 = 3$$

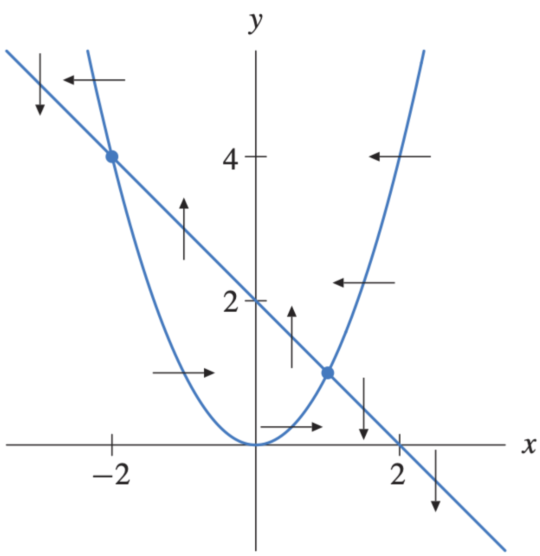
Sketch, on paper, the x- and y-nullclines of the system. Then find all equilibrium points. Using the direction of the vector field between the nullclines, describe the possible fate of the solution curves corresponding to the initial conditions (a), (b), and (c).

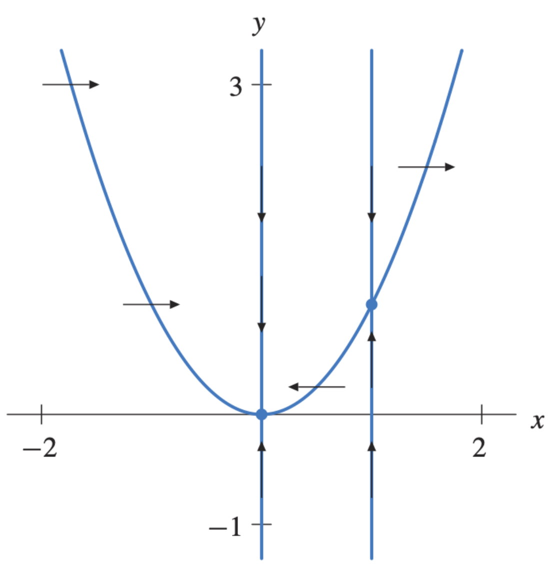
The equilibrium points are: ______________
Enter as a comma-separated list of points (x_i, y_i).

Select the graph corresponding to your sketch of the nullclines: [?/i/ii/iii]



(i) (ii)





(iii)

Describe the long-term behviour of the solutions with initial conditions (a-c):

- **a**. All three solutions will go down and to the right without bound.
- **b**. The solutions for (a) and (c) tend to infinity in the left-up region; the solution for (b) tends to infinity in the right-down region.
- **c**. The solutions for (a) and (b) tend toward the equilibrium point at (0, 0); the solutions for (c) tends toward the equilibrium point at (1, 1).

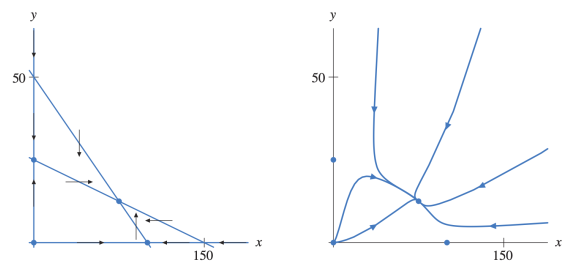
Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_5.2/BDH_5.2.5.pg`

Consider the system

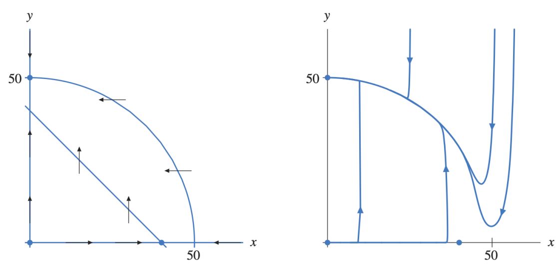
$$\frac{dx}{dt} = x(-x-3y+150)$$
$$\frac{dy}{dt} = y(-2x-y+100)$$

with $x, y \geq 0$.
Sketch, on paper, the x- and y-nullclines and the phase portrait of the system. Describe the long-term behavior of solutions to the system.

Select the graph corresponding to your sketch: [?/i/ii/iii/iv/v]



(i)



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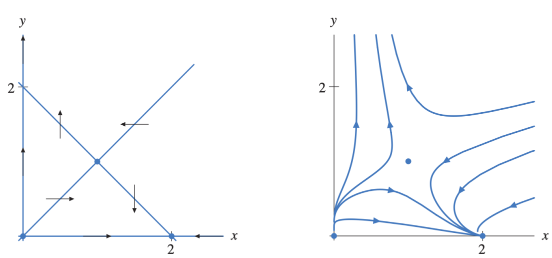

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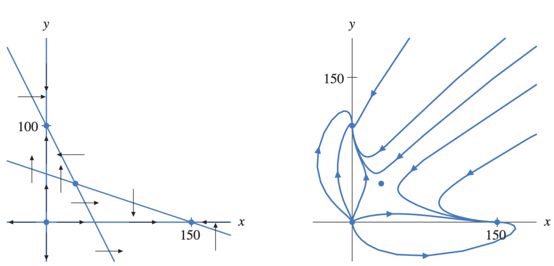

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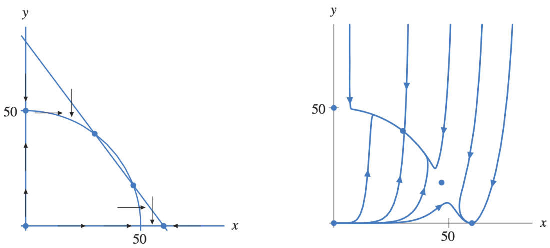


(v)

Describe the long-term behviour of solutions:

- **a**. Most solutions tend towards one of the equilibria on the co-ordinate axes.
- **b**. All solutions off the axes tend towards the solution in the first quadrant.
- **c**. Solutions off the x-axis tend toward the sink on the y-axis.
- **d**. Most solutions tend towards the sink in the first quadrant or the sink on the x-axis.
- **e**. Most solutions tend toward either the sink on the y-axis or toward infinity in the y-direction.

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_5.2/BDH_5.2.7.pg`

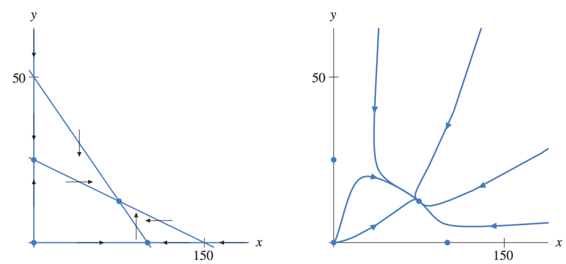
Consider the system

$$\frac{dx}{dt} = x(100 - x - 2y)$$
$$\frac{dy}{dt} = y(150 - x - 6y)$$

with $x, y \geq 0$.

Sketch, on paper, the x- and y-nullclines and the phase portrait of the system. Describe the long-term behavior of solutions to the system.

Select the graph corresponding to your sketch: [?/i/ii/iii/iv/v]



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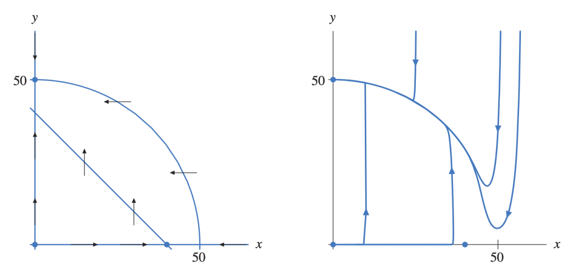

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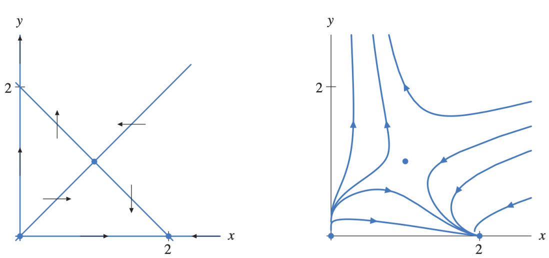

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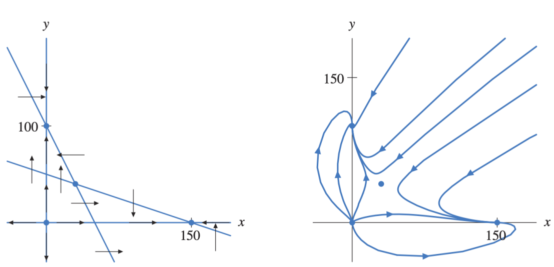

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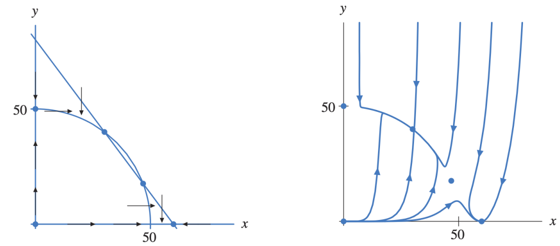


(v)

Describe the long-term behviour of solutions:

- **a**. Most solutions tend towards one of the equilibria on the co-ordinate axes.
- **b**. All solutions off the axes tend towards the solution in the first quadrant.
- **c**. Solutions off the x-axis tend toward the sink on the y-axis.
- **d**. Most solutions tend towards the sink in the first quadrant or the sink on the x-axis.
- **e**. Most solutions tend toward either the sink on the y-axis or toward infinity in the y-direction.

Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_5.2/BDH_5.2.9.pg`

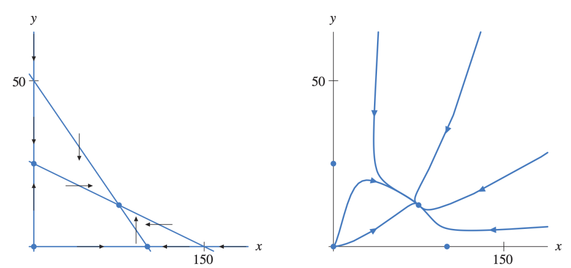
Consider the system

$$\frac{dx}{dt} = x(-x-y+40)$$
$$\frac{dy}{dt} = y(-x^2-y^2+2500)$$

with $x, y \geq 0$.

Sketch, on paper, the x- and y-nullclines and the phase portrait of the system. Describe the long-term behavior of solutions to the system.

Select the graph corresponding to your sketch: [?/i/ii/iii/iv/v]



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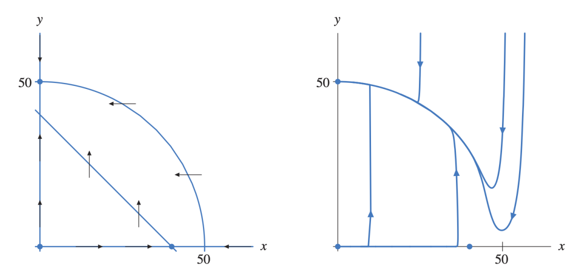

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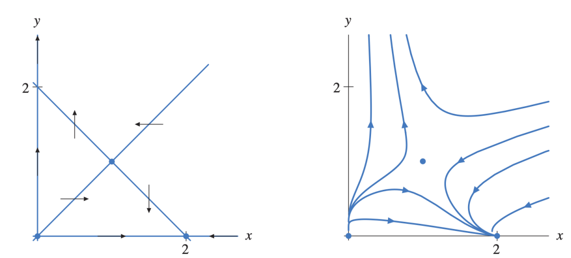

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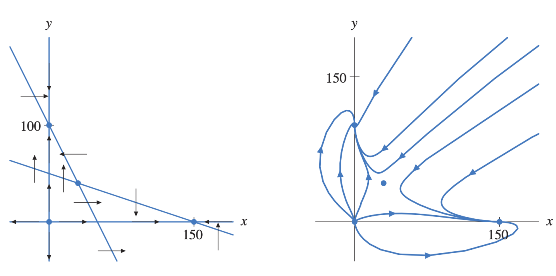

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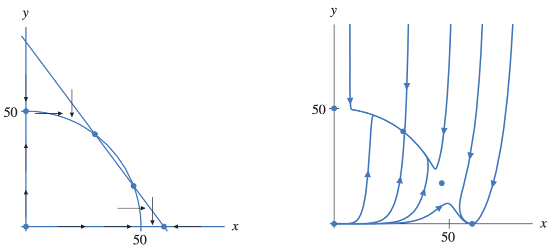


(v)

Describe the long-term behviour of solutions:

- **a**. Most solutions tend towards one of the equilibria on the co-ordinate axes.
- **b**. All solutions off the axes tend towards the solution in the first quadrant.
- **c**. Solutions off the x-axis tend toward the sink on the y-axis.
- **d**. Most solutions tend towards the sink in the first quadrant or the sink on the x-axis.
- **e**. Most solutions tend toward either the sink on the y-axis or toward infinity in the y-direction.

Problem 6. **(1 point)** `ustLibrary/ustDiffEq/setBDH_5.2/BDH_5.2.11.pg`

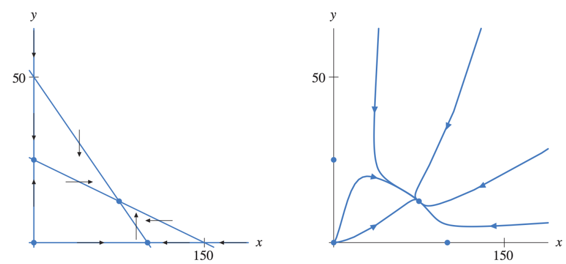
Consider the system

$$\frac{dx}{dt} = x(-8x - 6y + 480)$$
$$\frac{dy}{dt} = y(-x^2 - y^2 + 2500)$$

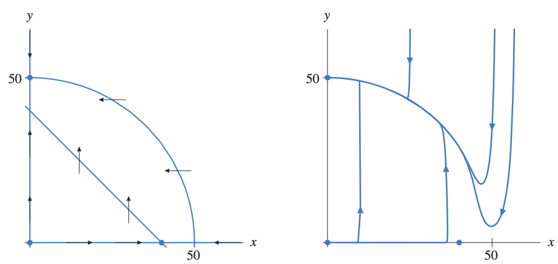
with $x, y \geq 0$.

Sketch, on paper, the x- and y-nullclines and the phase portrait of the system. Describe the long-term behavior of solutions to the system.

Select the graph corresponding to your sketch: [?/i/ii/iii/iv/v]



(i)



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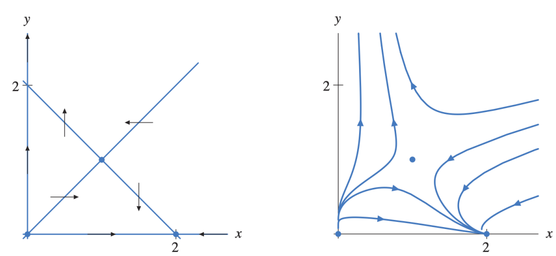

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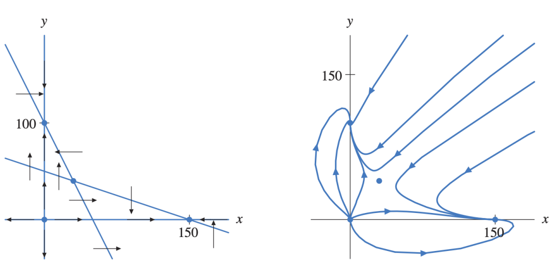

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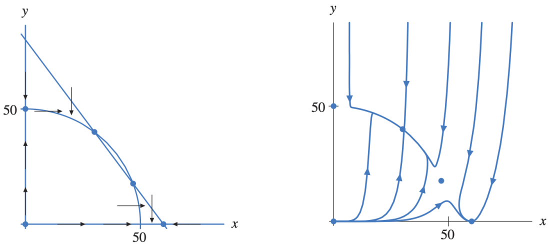


(v)

Describe the long-term behviour of solutions:

- **a**. Most solutions tend towards one of the equilibria on the co-ordinate axes.
- **b**. All solutions off the axes tend towards the solution in the first quadrant.
- **c**. Solutions off the x-axis tend toward the sink on the y-axis.
- **d**. Most solutions tend towards the sink in the first quadrant or the sink on the x-axis.
- **e**. Most solutions tend toward either the sink on the y-axis or toward infinity in the y-direction.

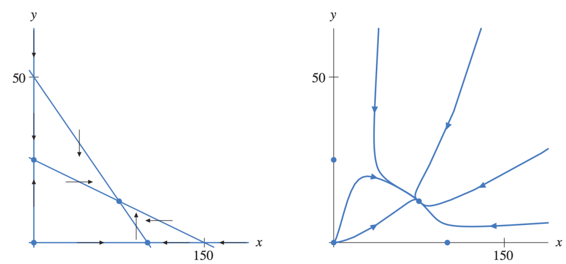
Problem 7. **(1 point)** `ustLibrary/ustDiffEq/setBDH_5.2/BDH_5.2.13.pg`

Consider the system

$$\frac{dx}{dt} = x(2-x-y)$$
$$\frac{dy}{dt} = y(y-x)$$

with $x, y \geq 0$.
Sketch, on paper, the x- and y-nullclines and the phase portrait of the system. Describe the long-term behavior of solutions to the system.

Select the graph corresponding to your sketch: [?/i/ii/iii/iv/v]



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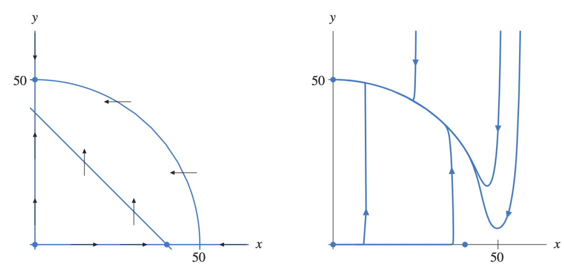

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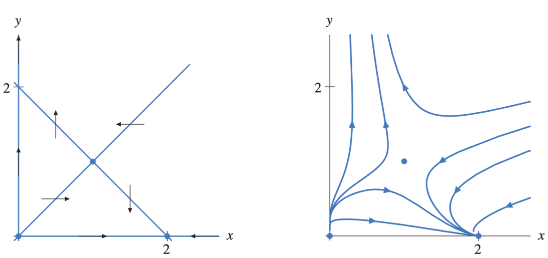

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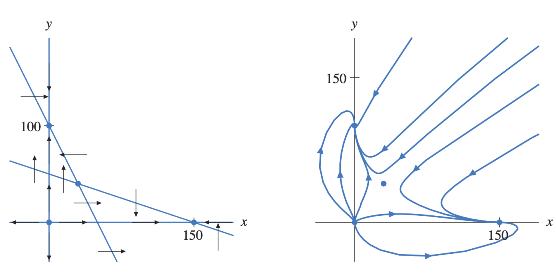

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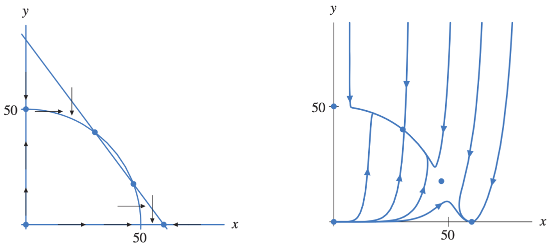


(v)

Describe the long-term behviour of solutions:

- **a**. Most solutions tend towards one of the equilibria on the co-ordinate axes.
- **b**. All solutions off the axes tend towards the solution in the first quadrant.
- **c**. Solutions off the x-axis tend toward the sink on the y-axis.
- **d**. Most solutions tend towards the sink in the first quadrant or the sink on the x-axis.
- **e**. Most solutions tend toward either the sink on the y-axis or toward infinity in the y-direction.

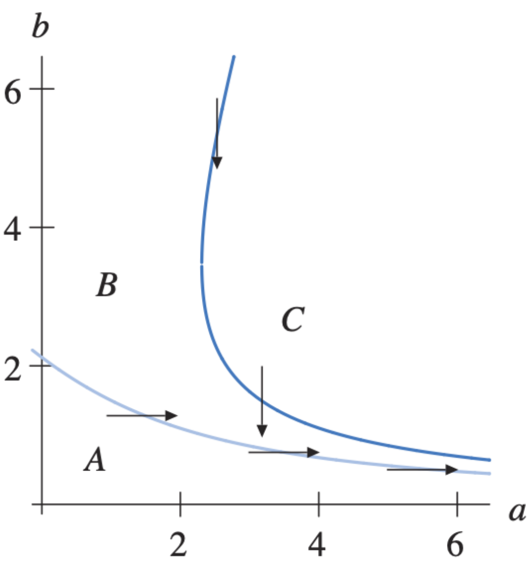
Problem 8. **(1 point)** `ustLibrary/ustDiffEq/setBDH_5.2/BDH_5.2.17.pg`

The system below models a chemical reaction:

$$\frac{da}{dt} = 2 - \frac{ab}{2}$$
$$\frac{db}{dt} = \frac{3}{2} - \frac{ab}{2}$$

Here $a(t)$ is the amount of substance A in a solution, and $b(t)$ is the amount of substance B in a solution at time t. We only need to consider nonnegative $a(t)$ and $b(t)$.

(a) Sketch, on paper, the nullclines and indicate the direction of the vector field along the nullcline, Select the graph corresponding to your sketch: [?/i/ii/iii/iv]



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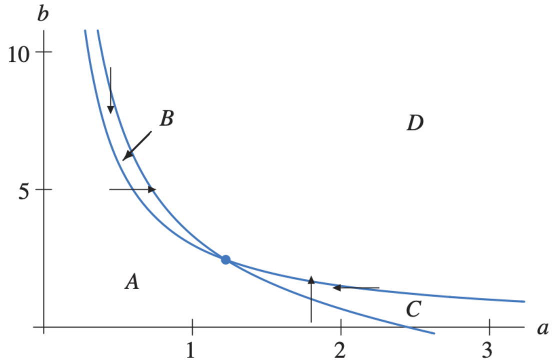

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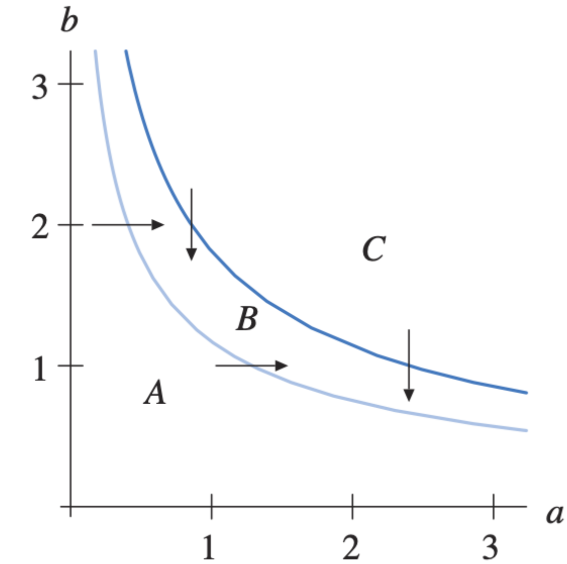

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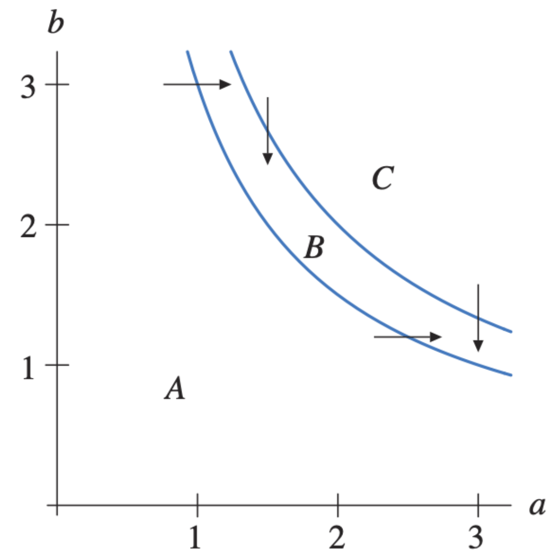


iv)

(b) identify the regions that solutions cannot leave and determine the fate of solutions in these regions as time increases.

- **a**. Solutions that start in region A eventually enter region B. Solutions that start in region C head down and to the left until they enter region B. Solutions that start in region B stay in region B. All solutions head off to infinity in the a-direction.
- **b**. All solutions in the first quadrant tend to the equilibrium point in the first quadrant.
- **c**. Solutions that start in region A eventually enter region B. Solutions that start in region C head down and to the

left until they enter region B. Solutions that start in region B either stay in B or go to C and back to B. All solutions head off to infinity in the a-direction.

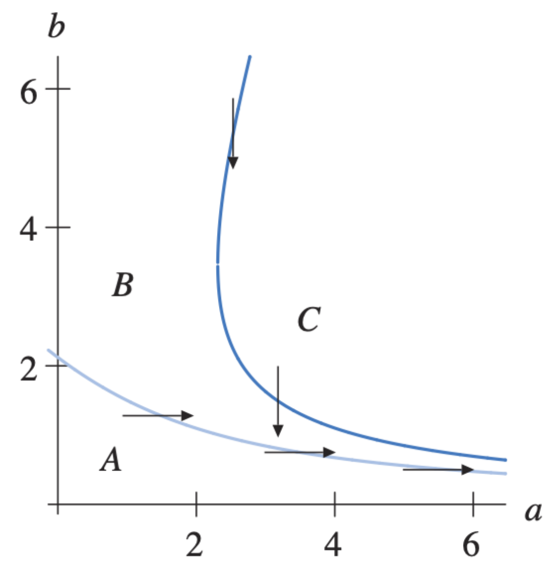
Problem 9. **(1 point)** `ustLibrary/ustDiffEq/setBDH_5.2/BDH_5.2.19.pg`

The system below models a chemical reaction:

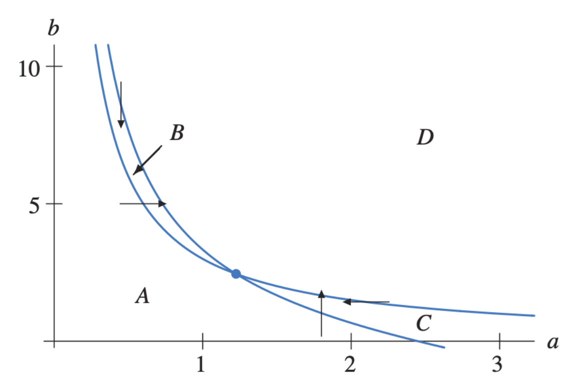
$$\frac{da}{dt} = 2 - \frac{ab}{2}$$
$$\frac{db}{dt} = \frac{3}{2} - \frac{ab}{2}$$

Here $a(t)$ is the amount of substance A in a solution, and $b(t)$ is the amount of substance B in a solution at time t. We only need to consider nonnegative $a(t)$ and $b(t)$.

(a) Sketch, on paper, the nullclines and indicate the direction of the vector field along the nullcline, Select the graph corresponding to your sketch: [?/i/ii/iii/iv]



(i) (ii



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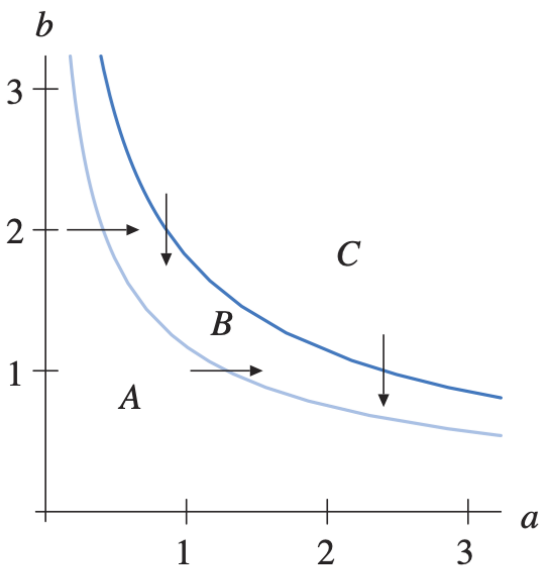
(iii)



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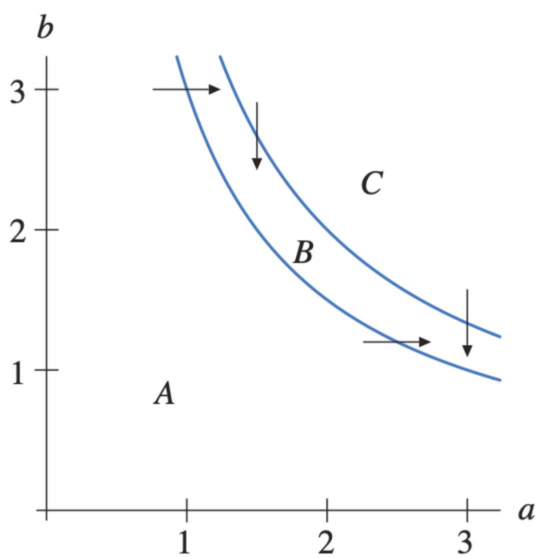

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(

(b) identify the regions that solutions cannot leave and determine the fate of solutions in these regions as time increases.

- **a**. Solutions that start in region A eventually enter region B. Solutions that start in region C head down and to the left until they enter region B. Solutions that start in region B stay in region B. All solutions head off to infinity in the a-direction.

- **b**. All solutions in the first quadrant tend to the equilibrium point in the first quadrant.

- **c**. Solutions that start in region A eventually enter region B. Solutions that start in region C head down and to the left until they enter region B. Solutions that start in region B either stay in B or go to C and back to B. All solutions head off to infinity in the a-direction.

Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.1/BDH_6.1.1.pg`

Compute the Laplace transform of $f(t) = 3$, from the definition.

$\mathcal{L}[f(t)](s) =$ __________

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.1/BDH_6.1.3.pg`

Compute the Laplace transform of $f(t) = -5t^2$, from the definition.

$\mathcal{L}[f(t)](s) =$ __________

Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.1/BDH_6.1.7.pg`

Find the inverse Laplace transform of $F(s) = \dfrac{1}{s-3}$.

$\mathcal{L}^{-1}[F(s)](t) =$ __________

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.1/BDH_6.1.9.pg`

Find the inverse Laplace transform of $F(s) = \dfrac{2}{3s+5}$.

$\mathcal{L}^{-1}[F(s)](t) =$ __________

Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.1/BDH_6.1.11.pg`

Find the inverse Laplace transform of $F(s) = \dfrac{4}{s(s+3)}$.

$\mathcal{L}^{-1}[F(s)](t) =$ __________

Problem 6. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.1/BDH_6.1.13.pg`

Find the inverse Laplace transform of $F(s) = \dfrac{2s+1}{(s-1)(s-2)}$.

$\mathcal{L}^{-1}[F(s)](t) =$ __________

Problem 7. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.1/BDH_6.1.15.pg`

Consider the initial value problem

$$\frac{dy}{dt} = -y + e^{-2t}, \quad y(0) = 2.$$

(a) Compute the Laplace transform of both sides of the equation; substitute the initial conditions and solve for the Laplace transform of the solution.
$\mathcal{L}[y] =$ ______________

(b) Find a function whose Laplace transform is the same as the solution.
$y(t) =$ ______________

(c) Check that you have found the solution of the initial-value problem.

- I did it!

Problem 8. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.1/BDH_6.1.17.pg`

Consider the initial value problem

$$\frac{dy}{dt} + 7y = 1, \quad y(0) = 3.$$

(a) Compute the Laplace transform of both sides of the equation; substitute the initial conditions and solve for the Laplace transform of the solution.
$\mathcal{L}[y] =$ ______________

(b) Find a function whose Laplace transform is the same as the solution.
$y(t) =$ ______________

(c) Check that you have found the solution of the initial-value problem.

- I did it!

Problem 9. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.1/BDH_6.1.23.pg`

Consider the initial value problem

$$\frac{dy}{dt} = -y + t^2, \quad y(0) = 1.$$

(a) Compute the Laplace transform of both sides of the equation; substitute the initial conditions and solve for the Laplace transform of the solution.
$\mathcal{L}[y] =$ ______________

(b) Find a function whose Laplace transform is the same as the solution.
$y(t) =$ ______________

(c) Check that you have found the solution of the initial-value problem.

- I did it!

Problem 10. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.1/BDH_6.1.25.pg`

Find the general solution of the equation

$$\frac{dy}{dt} = 2y + 2e^{-3t}$$

using the Laplace transform. Use c for the constant in your solution.
(This equation is linear, but please use the method of Laplace transforms.)
$y(t) =$ ______________

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Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.2/BDH_6.2.1.pg`

For $a \geq 0$, let $g_a(t)$ denote the function

$$g_a(t) = \begin{cases} 1, & \text{if } t < a; \\ 0, & \text{if } t \geq a. \end{cases}$$

(a) Give an expression for $g_a(t)$ using the Heaviside function $u_a(t)$.

$g_a(t) =$ ________

Note: in WeBWorK, enter $u_a(t)$ as $u(t-a)$.

(b) Compute $\mathcal{L}[g_a] =$ ________

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.2/BDH_6.2.3.pg`

Suppose $a \geq 0$. Compute the Laplace transform of the function

$$g_a(t) = \begin{cases} t/a, & \text{if } t < a; \\ 1, & \text{if } t \geq a. \end{cases}$$

Hint: your solution should contain a as a parameter.

$\mathcal{L}[g_a] =$ ________

Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.2/BDH_6.2.5.pg`

Compute the inverse Laplace transform of the function $\dfrac{e^{-3s}}{(s-1)(s-2)}$.

____

Note: in WeBWorK, enter $u_a(t)$ as $u(t-a)$.

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.2/BDH_6.2.7.pg`

Compute the inverse Laplace transform of the function $\dfrac{14e^{-s}}{(3s+2)(s-4)}$.

____

Note: in WeBWorK, enter $u_a(t)$ as $u(t-a)$.

Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.2/BDH_6.2.9.pg`

Solve the initial value problem

$$\frac{dy}{dt} + 9y = u_5(t), \quad y(0) = -2.$$

$y(t) =$ ________

Note: in WeBWorK, enter $u_a(t)$ as $u(t-a)$.

Problem 6. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.2/BDH_6.2.13.pg`

Solve the initial value problem

$$\frac{dy}{dt} = -y + u_1(t)(t-1), \quad y(0) = 2.$$

$y(t) =$ ________

Note: in WeBWorK, enter $u_a(t)$ as $u(t-a)$.

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Thomas Hoft **210S23_Hoft**

Assignment BDH_6.3 due 05/02/2023 at 09:55am CDT

Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.3/BDH_6.3.1.pg`

Compute, from the definition, the Laplace transform of the function $\sin \omega t$.

______

Note: enter 'w' for ω.

(This exercise is mainly a review of integration by parts.)

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.3/BDH_6.3.11_15.pg`

(a) Write the quadratic $s^2+2s+10$ in the form $(s+\alpha)^2+\beta^2$ (that is, complete the square).

______

(b) Compute the inverse Laplace transform of the function $\dfrac{1}{s^2+2s+10}$ using the result of (a).

______

Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.3/BDH_6.3.13_17.pg`

(a) Write the quadratic s^2+s+1 in the form $(s+\alpha)^2+\beta^2$ (that is, complete the square).

______

(b) Compute the inverse Laplace transform of the function $\dfrac{2s+3}{s^2+s+1}$ using the result of (a).

______

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.3/BDH_6.3.27.pg`

Solve the initial value problem

$$\frac{d^2y}{dt^2}+4y=8, \quad y(0)=11,\ y'(0)=5.$$

(a) Compute the Laplace transform of both sides of the differential equation, substitute in the initial conditions and simplify to obtain the Laplace transform of the solution:

$\mathcal{L}[y]=$ ______

(b) find the solution by taking the inverse Laplace transform:

$y(t)=$ ______

Problem 5. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.3/BDH_6.3.29.pg`

Solve the initial value problem

$$\frac{d^2y}{dt^2}-4\frac{dy}{dt}+5y=2e^t, \quad y(0)=3,\ y'(0)=1.$$

(a) Compute the Laplace transform of both sides of the differential equation, substitute in the initial conditions and simplify to obtain the Laplace transform of the solution:

$\mathcal{L}[y]=$ ______

(b) find the solution by taking the inverse Laplace transform:

$y(t)=$ ______

Problem 6. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.3/BDH_6.3.31.pg`

Solve the initial value problem

$$\frac{d^2y}{dt^2}+4y=\cos 2t, \quad y(0)=-2,\ y'(0)=0.$$

(a) Compute the Laplace transform of both sides of the differential equation, substitute in the initial conditions and simplify to obtain the Laplace transform of the solution:

$\mathcal{L}[y]=$ ______

(b) find the solution by taking the inverse Laplace transform:

$y(t)=$ ______

Problem 7. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.3/BDH_6.3.33.pg`

Solve the initial value problem

$$\frac{d^2y}{dt^2}+4\frac{dy}{dt}+9y=20u_2(t)\sin(t-2), \quad y(0)=1,\ y'(0)=2.$$

(a) Compute the Laplace transform of both sides of the differential equation, substitute in the initial conditions and simplify to obtain the Laplace transform of the solution:

$\mathcal{L}[y]=$ ______

(b) find the solution by taking the inverse Laplace transform:

$y(t)=$ ______

Note: in WeBWorK, enter $u_a(t)$ as $u(t-a)$.

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Thomas Hoft **210S23_Hoft**
Assignment BDH_6.4 due 05/04/2023 at 09:55am CDT

Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.4/BDH_6.4.1.pg`

Compute the limit $\lim_{\Delta t \to 0} \left(\frac{e^{s\Delta t} - e^{-s\Delta t}}{2\Delta t} \right) =$ ___.

Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.4/BDH_6.4.3.pg`

Solve the initial value problem

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = \delta_3(t), \quad y(0) = 1,\ y'(0) = 1.$$

(a) Compute the Laplace transform of both sides of the differential equation, substitute in the initial conditions and simplify to obtain the Laplace transform of the solution:

$\mathcal{L}[y] =$ ______________________________

(b) find the solution by taking the inverse Laplace transform:

$y(t) =$ ______________________________

Note: in WeBWorK, enter $u_a(t)$ as $u(t-a)$.

Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.4/BDH_6.4.5.pg`

Solve the initial value problem

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 3y = \delta_1(t) - 3\delta_4(t), \quad y(0) = 0,\ y'(0) = 0.$$

(a) Compute the Laplace transform of both sides of the differential equation, substitute in the initial conditions and simplify to obtain the Laplace transform of the solution:

$\mathcal{L}[y] =$ ______________________________

(b) find the solution by taking the inverse Laplace transform:

$y(t) =$ ______________________________

Note: in WeBWorK, enter $u_a(t)$ as $u(t-a)$.

Problem 4. **(1 point)** `ustLibrary/ustDiffEq/setBDH_6.4/BDH_6.4.6.pg`

Consider the damped harmonic oscillator with impulse forcing

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 3y = \delta_4(t), \quad y(0) = 1,\ y'(0) = 0.$$

(a) Determine the qualitative behavior of the solution of the initial-value problem:
The natural (angular) frequency is $\omega_0 =$ _____.
The natural period is $T_0 =$ _____.
For $0 \le t < 4$ the system is
• ?
• oscillating with increasing amplitude
• oscillating with decreasing amplitude
• in equilibrium
and approaching [?/∞/1/0/−∞].
At $t = 4$ the system is given a jolt, and the solution increases for a short time.
For $t > 4$ the system is
• ?
• oscillating with increasing amplitude
• oscillating with decreasing amplitude
• in equilibrium
.
As $t \to \infty$ the solution approaches [?/∞/1/0/−∞].

(b) Compute the Laplace transform of both sides of the differential equation, substitute in the initial conditions and simplify to obtain the Laplace transform of the solution:

$\mathcal{L}[y] =$ ______________________________

(c) find the solution by taking the inverse Laplace transform:

$y(t) =$ ______________________________

Note: in WeBWorK, enter $u_a(t)$ as $u(t-a)$.

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Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_7.1/BDH_7.1.a.pg`

Consider the initial-value problem

$$\frac{dy}{dt} = \cos^2\left(\frac{y}{4}\right), \quad y(0) = 0.$$

Solve, by hand, the initial value problem:

$y(t) =$ ________

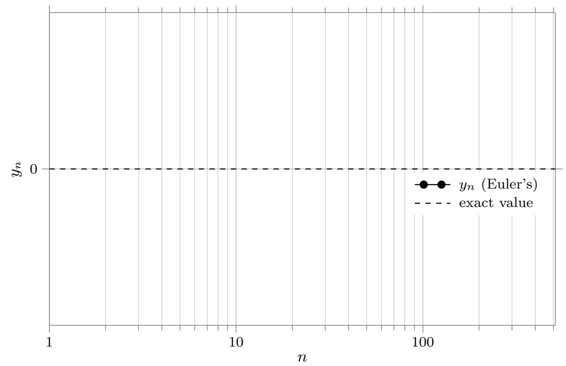
Compute the exact value:

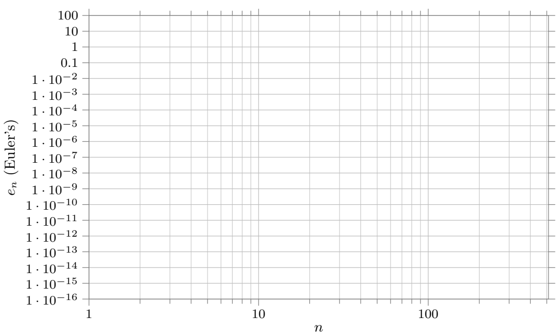
$y(4) =$ ________

Calculate the Euler's method approximation y_n to $y(4)$ using $n = 2, 4, 8, 16, \ldots, 512$, and the corresponding errors e_n. (your answers should to be correct to $\pm 0.0001\%$)

You can use **this online numerical method widget** (opens in new tab).

n	y_n	e_n
2	________	________
4	________	________
8	________	________
16	________	________
32	________	________
64	________	________
128	________	________
256	________	________
512	________	________





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Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_7.2/BDH_7.2.a.pg`

Consider the initial-value problem

$$\frac{dy}{dt} = \cos^2\left(\frac{y}{4}\right), \quad y(0) = 0.$$

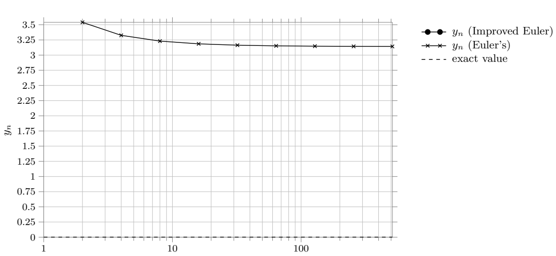
Solve, by hand, the initial value problem:
$y(t) =$ ________

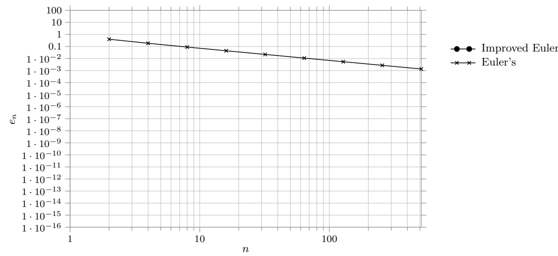
Compute the exact value:
$y(4) =$ ________

Calculate the Improved Euler's method approximation y_n to $y(4)$ using $n = 2, 4, 8, 16, \ldots, 512$, and the corresponding errors e_n. (your answers should to be correct to $\pm 0.0001\%$)

You can use **this online numerical method widget** (opens in new tab).

n	y_n	e_n
2	________	________
4	________	________
8	________	________
16	________	________
32	________	________
64	________	________
128	________	________
256	________	________
512	________	________





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Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_7.2/BDH_7.2.5.pg`

Consider the initial-value problem

$$\frac{dy}{dt} = (3 - y)(y + 1), \quad y(0) = 0.$$

As $t \to \infty$ the solution to this initial-value problem approaches [?/∞/3/0/-1/−∞] from [?/above/below].

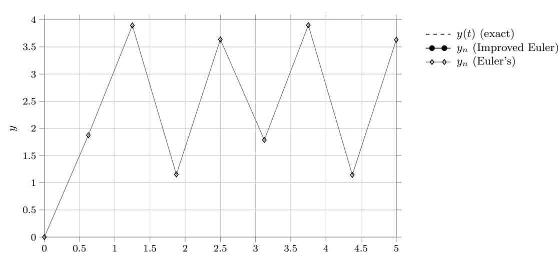
Solve, by hand, the initial value problem:
$y(t) =$ ________________

Use Improved Euler's method with $n = 8$ steps to approximate the solution to the initial-value problem over the time interval $0 \le t \le 5$. (your answers should to be correct to $\pm 0.0001\%$)

You can use **this online numerical method widget** (opens in new tab).

$y_0, y_1, y_2, \ldots, y_n =$ ________________

(enter as a comma-separated list)



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Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_7.2/BDH_7.2.2.pg`
Consider the initial-value problem

$$\frac{dy}{dt} = t - y^2, \quad y(0) = 1,$$

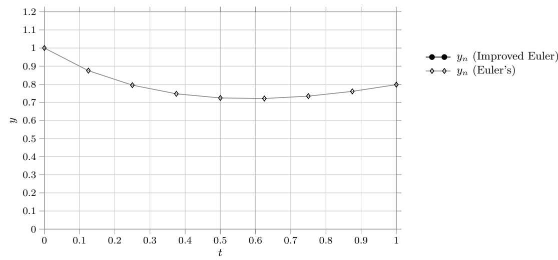
which is a non-linear equation (but not separable) for which we do not know a solution method.

Use Improved Euler's method with $n = 8$ steps to approximate the solution to the initial-value problem over the time interval $0 \leq t \leq 1$. (your answers should to be correct to $\pm 0.0001\%$)

You can use **this online numerical method widget** (opens in new tab).

$y_0, y_1, y_2, \ldots, y_n =$ ____________

(enter as a comma-separated list)



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Problem 1. **(1 point)** `ustLibrary/ustDiffEq/setBDH_7.3/BDH_7.3.a.pg`

Consider the initial-value problem

$$\frac{dy}{dt} = \cos^2\left(\frac{y}{4}\right), \quad y(0) = 0.$$

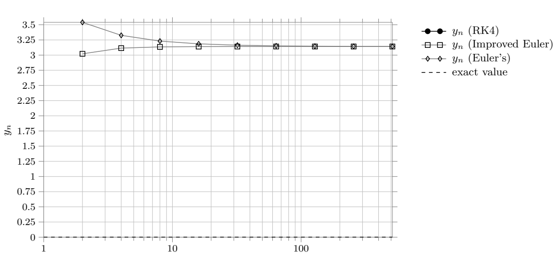
Solve, by hand, the initial value problem:
$y(t) =$ ________

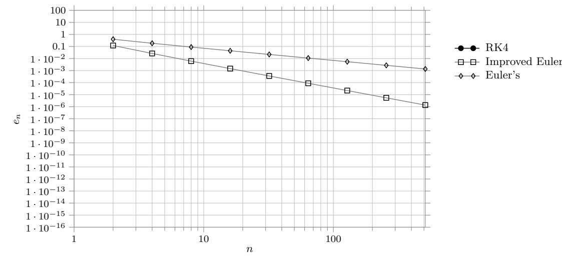
Compute the exact value:
$y(4) =$ ________

Calculate the RK4 method approximation y_n to $y(4)$ using $n = 2, 4, 8, 16, \ldots, 512$, and the corresponding errors e_n. (your answers should to be correct to $\pm 0.0001\%$)

You can use **this online numerical method widget** (opens in new tab).

n	y_n	e_n
2	________	________
4	________	________
8	________	________
16	________	________
32	________	________
64	________	________
128	________	________
256	________	________
512	________	________





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Problem 2. **(1 point)** `ustLibrary/ustDiffEq/setBDH_7.3/BDH_7.3.3.pg`

Consider the initial-value problem

$$\frac{dy}{dt} = (3 - y)(y + 1), \quad y(0) = 0.$$

As $t \to \infty$ the solution to this initial-value problem approaches [?/∞/3/0/-1/−∞] from [?/above/below].

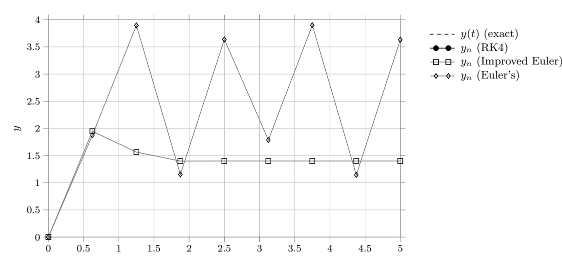
Solve, by hand, the initial value problem:
$y(t) =$ ________________

Use RK4 with $n = 8$ steps to approximate the solution to the initial-value problem over the time interval $0 \le t \le 5$. (your answers should to be correct to $\pm 0.0001\%$)

You can use **this online numerical method widget** (opens in new tab).

$y_0, y_1, y_2, \ldots, y_n =$ ________________

(enter as a comma-separated list)



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Problem 3. **(1 point)** `ustLibrary/ustDiffEq/setBDH_7.3/BDH_7.3.2.pg`

Consider the initial-value problem

$$\frac{dy}{dt} = t - y^2, \quad y(0) = 1,$$

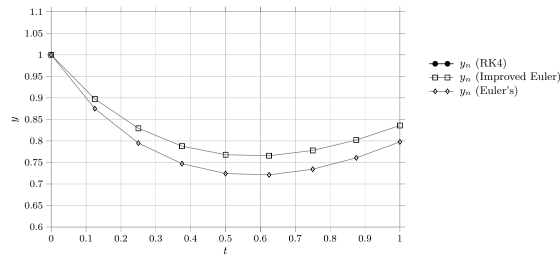
which is a non-linear equation (but not separable) for which we do not know a solution method.

Use RK4 with $n = 8$ steps to approximate the solution to the initial-value problem over the time interval $0 \leq t \leq 1$. (your answers should to be correct to $\pm 0.0001\%$)

You can use **this online numerical method widget** (opens in new tab).

$y_0, y_1, y_2, \ldots, y_n =$ ________________

(enter as a comma-separated list)



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