jee2020-paper1

1. If $g(x) = x^2 + x - 1$ and $(g \circ f)(x) = 4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to:

   (a) $\frac{3}{2}$
   (b) $-\frac{1}{2}$
   (c) $-\frac{3}{2}$
   (d) $\frac{1}{2}$

2. If $\text{Re}\left(\frac{z-1}{2z+i}\right) = 1$, where $z = x + iy$, then the point (x, y) lies on a:

   (a) Circle whose centre is at $\left(-\frac{1}{2}, -\frac{3}{2}\right)$.
   (b) Circle whose diameter is $\frac{\sqrt{5}}{2}$.
   (c) Straight line whose slope is $\frac{3}{2}$.
   (d) Straight line whose slope is $-\frac{2}{3}$.

3. Five numbers are in arithmetic progression A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$, then the greatest number among them is:

   (a) $\frac{21}{2}$
   (b) 27
   (c) 16
   (d) 7

4. If

$$y(\alpha) = \sqrt{2\left(\frac{\tan\alpha + \cot\alpha}{1 + \tan^2\alpha}\right) + \frac{1}{\sin^2\alpha}}, \quad \alpha \in \left(\frac{3\pi}{4}, \pi\right),$$

   then

$$\frac{dy}{d\alpha} \text{ at } \alpha = \frac{5\pi}{6} \text{ is:}$$

   (a) 4
   (b) $-\frac{1}{4}$
   (c) $\frac{4}{3}$

(d) -4

5. Let α be a root of the equation $x^2 + x + 1 = 0$, and let the matrix $A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$, then the matrix A^{31} is equal to:

   (a) A^3
   (b) A
   (c) A^2
   (d) $I3$

6. If $y = mx + 4$ is a tangent to both the parabolas $y^2 = 4x$ and $x^2 = 2by$, then b is equal to:

   (a) 128
   (b) 264
   (c) 128
   (d) -32

7. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is:

   (a) $\sqrt{3}$
   (b) $2\sqrt{3}$
   (c) $3\sqrt{2}$
   (d) $\frac{3}{\sqrt{2}}$

8. An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for $k = 3, 4, 5$, otherwise X takes the value 1. Then the expected value of X is:

   (a) $\frac{3}{16}$
   (b) $-\frac{3}{16}$
   (c) $\frac{1}{8}$
   (d) $-\frac{1}{8}$

9. The area of the region enclosed by the circle $x^2 + y^2 = 2$ which is not common to the region bounded by the parabola $y^2 = x$ and the straight line $y = x$, is:

   (a) $\frac{1}{3}(12\pi - 1)$
   (b) $\frac{1}{6}(12\pi - 1)$
   (c) $\frac{1}{6}(24\pi - 1)$

(d) $\frac{1}{3}(l6\pi - 1)$

10. Let $x^k + y^k = a^k$, $(a, k > 0)$ and $\frac{dy}{dx} + (\frac{y}{x})^{\frac{1}{3}} = 0$, then k is:

    (a) $\frac{1}{2}$
    (b) $\frac{1}{3}$
    (c) $\frac{2}{3}$
    (d) $\frac{4}{3}$

11. If $y = y(x)$ is the solution of the differential equation $e^y \left\{ \frac{dy}{dx} - 1 \right\} = e^x$ such that $y(0) = 0$, then $y(1)$ is equal to:

    (a) $2 + \log_e 2$
    (b) $2e$
    (c) $log_e 2$
    (d) $1 + \log_e 2$

12. Total number of 6 digit numbers in which only and all the five digits 1,3,5,7 and 9 appers is:

    (a) $\frac{5}{2}(6!)$
    (b) 5^6
    (c) $\frac{1}{2}(6!)$
    (d) $6!$

13. Let P be a plane passing through the points $(2, 1, 0)$, $(4, 1, 1)$, and $(5, 0, 1)$, and let R be the point $(2, 1, 6)$. Then the image of R in the plane P is:

    (a) $(6, 5, -2)$
    (b) $(4, 3, 2)$
    (c) $(3, 4, -2)$
    (d) $(6, 5, 2)$

14. A vector $\mathbf{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ $(\alpha, \beta \in \mathbb{R})$ lies in the plane of the vectors $\mathbf{b} = \hat{i} + \hat{j}$ and $\mathbf{c} = \hat{i} - \hat{j} + 4\hat{k}$. If $\mathbf{a}$ bisects the angle between $\mathbf{b}$ and $\mathbf{c}$, then:

    (a) $\mathbf{a} \cdot \hat{i} + 1 = 0$
    (b) $\mathbf{a} \cdot \hat{i} + 3 = 0$
    (c) $\mathbf{a} \cdot \hat{k} + 4 = 0$
    (d) $\mathbf{a} \cdot \hat{k} + 2 = 0$

15. If $f(a+b+1-x) = f(x)$ for all x, where a and b are fixed positive real numbers, then

$$\frac{1}{a+b}\int_a^b x\big(f(x)+f(x+1)\big)\,dx$$

is equal to:

(a) $\int_{a+1}^{b+1} f(x)\,dx$

(b) $\int_{a+1}^{b+1} f(x+1)\,dx$

(c) $\int_{a-1}^{b-1} f(x+1)\,dx$

(d) $\int_{a-1}^{b-1} f(x)\,dx$

16. Let the function $f : [-7, 0] \to \mathbb{R}$ be continuous on $[-7, 0]$ and differentiable on $(-7, 0)$. If $f(-7) = -3$ and $f'(x) \leq 2$ for all $x \in (-7, 0)$, then for all such functions f, $f(-1) + f(0)$ lies in the interval:

(a) $[-6, 20]$

(b) $(-\infty, 20]$

(c) $(-\infty, 11]$

(d) $[-3, 11]$

17. If the system of linear equations

$$\begin{aligned} 2x + 2ay + az &= 0, \\ 2x + 3by + bz &= 0, \\ 2x + 4cy + cz &= 0, \end{aligned}$$

where a, b, c are non-zero and distinct, has a non-zero solution, then:

(a) a, b, c are in A.P.

(b) $a + b + c = 0$

(c) a, b, c are in G.P.

(d) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

18. Let α and β be two real roots of the equation

$$(k+1)\tan^2 x - \sqrt{2}\cdot\lambda\tan x = (1-k),$$

where $k(\neq -1)$ and λ are real numbers. If

$$\tan^{-1}(\alpha+\beta) = 50,$$

then the value of λ is:

(a) 5

(b) 10

(c) $5\sqrt{2}$

(d) $10\sqrt{2}$

19. The logical statement $(p \Rightarrow q) \wedge (q \Rightarrow \neg p)$

(a) p

(b) q

(c) p

(d) q

20. The greatest positive integer k, for which $49^k + 1$ is a factor of the sum $19^{125} + 49^{124} + \cdots + 49^2 + 49 + 1$, is:

(a) 32

(b) 60

(c) 63

(d) 65

21. $$\lim_{x \to 2} \frac{3^3 + 3^{3-x} - 12}{3^{-x/2} - 3^{1-x}}$$

is equal to _____.

22. If the variance of the first n natural numbers is 10 and the variance of the first m even natural numbers is 16, then $m + n$ is equal to _____.

23. If the sum of the coefficients of all even powers of x in the product(1+x+ $x^2 + \ldots + x^{2n}$) (1-x+ x^2 - $x^3 + \ldots + x^{2n}$)

24. Let $A(1, 0)$, $B(6, 2)$, and $C(\frac{3}{2}, 6)$ be the vertices of a triangle ABC. If P is a point inside the triangle ABC such that the triangles APC, APB, and BPC have equal areas, then the length of the line segment PQ, where Q is the point $(-\frac{7}{6}, -\frac{1}{3})$, is _____

25. Let S be the set of points where the function $f(x) = |2 - |x - 3||, \quad x \in R$ is not differentiable. Then,$\sum_{x \in S} f(f(x))$ is equal to _____.