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A Simple PID Control Design for Systems with Time Delay

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ABSTRACT

In this article a new simple method for calculating PID parameters for a wide class of SISO process models is presented. The method utilizes one self oscillation experiment and one open loop step response experiment for determining the parameters of a suitable PID controller. A specialization of the method to first order models with time delay (FOTD) introduces a simple approximation of the ultimate gain which eliminates the need for any self oscillation experiment, thus giving rise to a method based solely on a step response of the model. Combining this method with a simple method for estimation of the three parameters in a FOTD model finally results in a design method based purely on an open loop step response. The methods presented are evaluated by simulation on a number of process models and the functionality of the step response based method is tested on a real process (a soldering iron).

Keywords: PID, controller design, Ziegler-Nichols method, time delay, rational approximation

1 Introduction

Ever since the days of the emergence of the Ziegler-Nichols famous methods ([1]) for choice of parameters in a PI or PID controller for a certain process model, there has been attempts to find improved methods for tuning the PID parameters. In this article we propose an assumingly new method for determining suitable parameters of a PID controller. The method uses three measurements of the process model, one from a self oscillation experiment and two from an open loop step response.

In [2] there is a survey of the most common parameter tuning rules for PID controllers available today. The article [3] specifically treats the so called AMIGO methods.

2 PID controller design

Given a process model transfer function

$$G_p(s) = \frac{B(s)}{A(s)}\, e^{-Ls}$$

the parameters K_u, k_p and L_p are estimated, where K_u is the ultimate feedback gain for which the closed loop system oscillates with constant amplitude, k_p is the process model static gain and T_p is the total (equivalent first order) time constant defined by

$$G_p(s) = \frac{k_p}{1 + T_p s + O(s^2)} = k_p(1 - T_p s + O(s^2))$$

which defines the first order low frequency properties of the process model. The parameter T_p is also referred to as *average residence time* ([3]). For a FOTD (First Order plus Time Delay) model

$$G(s) = \frac{k_p}{1 + Ts}\, e^{-Ls}$$

it is immediately clear that the total time constant is given by $T_p = T + L$. In this case T_p is directly obtained from the open loop step response by noting the time for which the output reaches the level $(1 - e^{-1})k_p \approx 0.63k_p$. It turns out that, although this method for estimating T_p is only approximate in the general case, the method is still useful for a broad class of process models.
The controller structure consists of the classical PID controller on parallell form (with filtered derivative part):

$$G_c(s) = K\left(1 + \frac{1}{T_i}\frac{1}{s} + \frac{T_d s}{1 + T_f s}\right)$$

where $T_f = \frac{T_d}{N}$ for some filter factor N (typically $N = 10$). From the three estimated process parameters we make use of the following choice of PID parameters:

$$K = c_K K_u \quad , \quad T_i = \frac{T_p K k_p}{K k_p + \sigma} \quad \text{and} \quad T_d = c_d T_i \tag{1}$$

where c_K, c_d and σ are design parameters. The parameter σ is most often chosen to be 0.5, which gives a vertical LF-asymptote $\mathrm{Re}\, s = -\sigma = -0.5$ of the loop transfer nyquist curve. This follows from the fact that the loop transfer function can be written as

$$\begin{aligned}
&G_c(s)G_p(s) = \\
&= K\left(1 + \frac{1}{T_i s} + O(s)\right) k_p(1 - T_p s + O(s^2)) \\
&= \frac{K k_p}{T_i s}(1 + T_i s + O(s^2))(1 - T_p s + O(s^2)) \\
&= \frac{K k_p}{T_i s}(1 + (T_i - T_p)s + O(s^2))
\end{aligned}$$

The formula for T_i then follows from

$$\lim_{\omega \to 0} \operatorname{Re} G_c(i\omega)G_p(i\omega) = Kk_p \left(1 - \frac{T_p}{T_i}\right) = -\sigma$$

The coefficient c_K, which is ususally chosen to be less than 0.5, can essentially be regarded as a “bandwidth parameter” for the closed loop system behaviour. The parameter c_d (the quotient of T_d and T_i) is a relative measure of the amount of derivative action ($c_d = 0$ corresponds to pure PI control). The parameter σ indirectly influences the phase margin and the value 0.5 suggested above approximately gives a phase margin of 60°. A larger value of σ decreases the phase margin resulting in less damped responses to steps in the reference signal. This can, however, be utilized to obtain a faster response to load disturbances, especially in cases with high ultimate gain K_u.

3 Non-resonant systems

A common type of process model used in the process industry is

$$G(s) = G_0(s)e^{-Ls},$$

where $G_0(s)$ typically is rational with well-damped poles and of low order. Some examples will be given of PID control of such systems for different choices of the parameters c_K and c_d.

Example 1. Consider the system

$$G(s) = \frac{1 - 2s}{(1 + 4s)(1 + s)^3} e^{-3s}$$

Fig. 1 shows the closed loop responses for PID control of the system with four sets of parameters. Notice how the response speed is directly correlated to the gain factor c_K. The choice of the derivative factor c_d is done with regard to keeping the same damping in all cases. In all cases the parameter σ is chosen as 0.5. The only process data needed for this design was $k_p = 1$, $K_u \approx 1.51$, and $T_p = 2 + 4 + 3 \cdot 1 + 3 = 12$ s. □

The method is not dependent on the considered process models to have any time delay, since the process data required does not depend on the time delay explicitly. High order models without time delay can often exhibit a behaviour similar to low order models with time delay. One problem with models of low order with small relative time delay is illustrated by the following example.

Example 2. For the system

$$G(s) = \frac{1}{1 + 4s} e^{-0.4s}$$

problems appear for the larger values of c_K. This is shown in Fig. 2. Stability problems start to appear when the gain factor is increased to $c_K = 0.3$. This is due to the fact that the derivative factor c_d is much too high for such small relative time delays when the gain is increased. Since the ultimate feedback gain K_u is large for small values of the relative time delay L/T, this results in a large

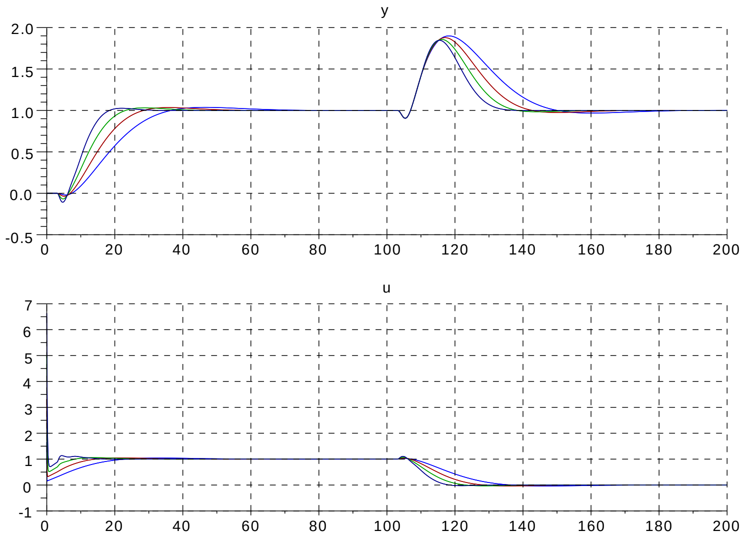

Figure 1: closed loop response for PID control of a fourth order system with time delay. Parameters chosen are $(c_K, c_d) = (0.1, 0.05)$, $(0.2, 0.1)$, $(0.3, 0.2)$, and $(0.4, 0.25)$. As expected the response speed increases with increasing value of the gain factor c_K.

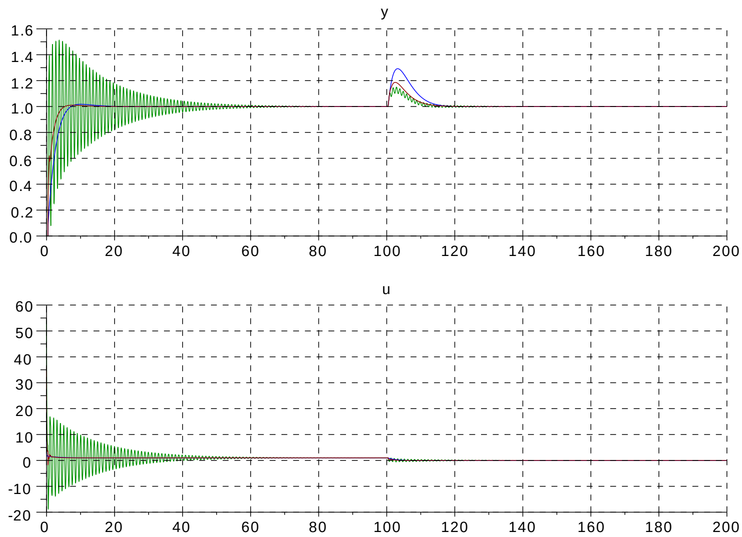

Figure 2: Closed loop responses for PID control of a first order system with time delay. The parameter values are in this case $(c_K, c_d) = (0.1, 0.05)$, $(0.2, 0.1)$, and $(0.3, 0.2)$.

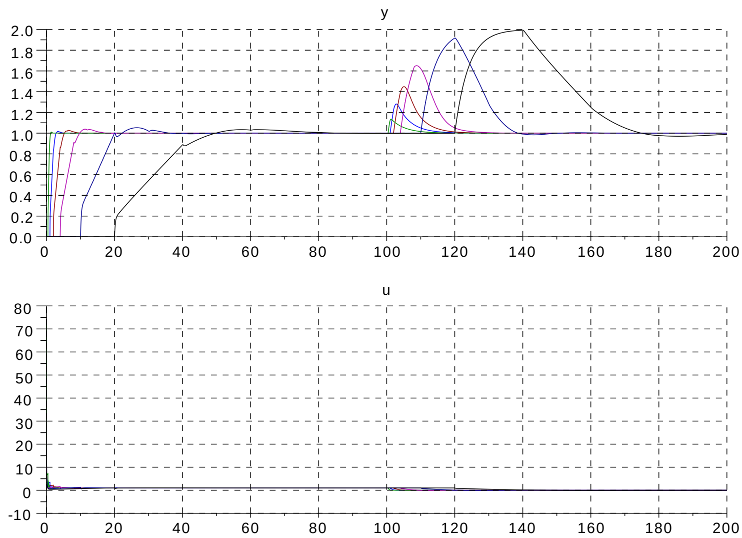

Figure 3: Closed loop responses for PID control of a first order system with time delay. The gain factor is $c_K = 0.4$ and the values of the time delay are $L = 0.4$, 1, 2, 4, 10, and 20 s. The time constant is given by $T = 4$ s. A unit step load disturbance is introduced at time $t = 100$ s.

gain $K = c_K K_u$. As the derivative action further amplifies the nyquist curve at high frequencies, the spiral arms are magnified so much that they approach the critical point -1, thereby deteriorating the stability. □

This may seem as a clear disadvantage of the proposed method, but the problem is remedied in the following example where greater care is taken when choosing the derivative factor $c_d = T_d/T_i$.

Example 3. The first order system with time delay is revisited. This time, however, the system is studied for several different values of the time delay L.

$$G(s) = \frac{1}{1+4s}e^{-Ls}$$

As L approaches 0, the derivative action must be eliminated, since, as noted in previous example, the ultimate gain K_u then gets large. By manually tuning c_d for each L so that the closed loop response has an overshoot of 5%, an approximate formula for the derivative factor c_d is given by

$$c_d = 0.3\left(1 - e^{-0.7\frac{L}{T}}\right) \tag{2}$$

for $c_K = 0.4$. For $c_K = 0.3$ the following formula gives better curve fitting:

$$c_d = 0.2 - 0.25e^{-0.8\frac{L}{T}} + 0.05e^{-2.3\frac{L}{T}}$$

The result is depicted in Fig. 3, where the closed loop responses for several different time delays L are shown and where the gain factor is given by $c_K = 0.4$. □

The accuracy achieved for the formulas for c_d decreases with increasing values of c_K. For values such large as $c_K = 0.4$ the behaviour of the system worsens for values of L/T larger than 4 and the formula is then no longer useful. For lower values of c_K the closed loop behaviour becomes gradually more independent of the value of c_d (but slower). The artefacts in the closed loop responses, due to the pure time delay combined with feedback, also become less noticeable for decreasing values of c_K.

4 Resonant systems

For systems with poorly damped dynamics, the choice of derivative factor c_d must be modified. Systems with small relative damping combined with small relative timedelay L/T are not feasible to design PID controllers for, using the methods described in this article, so only mildly oscillative systems are considered. This is due to the fact that poorly damped systems demands some phase retardation in the PID controller, which in turn forces negative values on the PID parameters, since the controller then must have zeros in the right half plane. For a second order system with time delay

$$G_p(s) = \frac{k_p}{1 + Ts + a_2 s^2} e^{-Ls}$$

the parameter c_d should be chosen near a_2/T^2 to give a near cancellation of the weakly damped poles by the controller zeros. This works best for smaller values of L/T, whereas for larger values of L/T the value of c_d can be chosen more independently of the quotient a_2/T^2. For $\sigma = 0.5$ and a specified overshoot of 5%, an approximate formula for c_d, when $c_K = 0.3$, is given by

$$c_d = 0.2 - (0.30 - 1.01x - 1.82x^2)e^{-0.8\tau} + (0.076 - 0.056x - 2.07x^2)e^{-2.3\tau} \quad (3)$$

where $x = a_2/T^2$ and $\tau = L/T$. This formula (Eq. (3)) is valid for $0.2 < L/T < 5$ and for $0 < a_2/T^2 < 1$ (i.e. $\zeta > 1/2$, where ζ is the relative damping). For $L/T > 5$ higher frequency properties of the time delayed system starts to degrade the behaviour of the closed loop response.

Example 4. A moderately resonant system with relative damping $\zeta = 0.5$ and with time delay L is given by

$$G(s) = \frac{1}{1 + 4s + 16s^2} e^{-Ls}$$

Using the PID parameter formula above, the closed loop reference value step responses and load disturbance response is shown in Fig. 4 for four values of the time delay, namely $L = 1$, 4, 16 and 30 s, respectively. □

Just as in the case $a_2 = 0$ in previous section, this effect for large time delays can be explained by the derivative high frequency blow-up of the spiral arms of the loop transfer nyquist curve, which introduces high frequency resonances. To some extent this can be counteracted by decreasing the gain factor c_K, thereby sacrificing closed loop bandwidth. The formula in Eq. (3) for c_d is the result of least squares curve fitting, first with respect to L/T and then with respect to a_2/T^2. The coefficients 0.8 and 2.3 in the exponents of the "base functions" are,

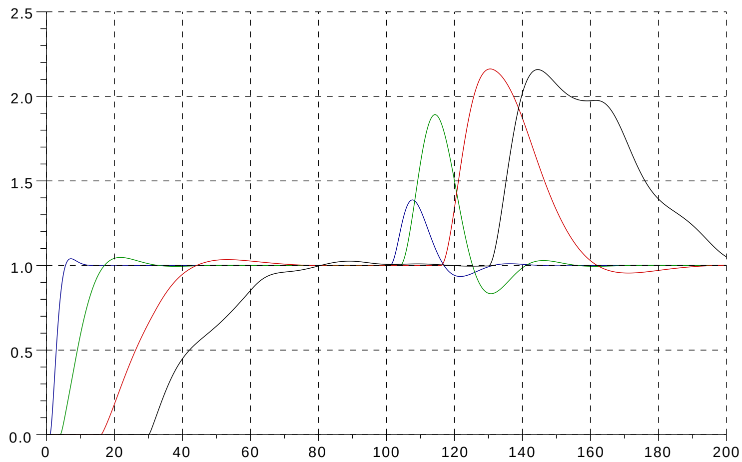

Figure 4: PID control of systems with relative damping $\zeta = 0.5$ and a time delay L. The four curves correspond to $L = 1$, 4, 16 and 30 s respectively ($T = 4$ s and $a_2 = 16$ s^2). The formula in Eq. (3) has been used for the choice of c_d in the PID design.

however, the result of some manually tuning. The overshoot in the closed loop step response varies around 5% (3-6%) within the specified parameter range given above. Extrapolating to systems with parameters $a_2/T^2 > 1$ using the formula for c_d gives overshoots gradually more deviating from the nominal 5% but it should be noted that in that case a 5% overshoot is often not even achievable for any value of c_d whatsoever. For $L/T = 0.25$ and $a_2/T^2 = 4$ ($\zeta = \frac{T}{\sqrt{a_2}}/2 = 1/4$) the minimum attainable overshoot lies around 24%, for example. So even if the valid operation range for the c_d formula can be extended (by least squares or otherwise) there are limitations. However, by decreasing c_K to values less than 0.3 it is possible to find values of c_d for which the overshoot can be decreased to 5%, but then all PID parameters will be negative (including the derivative filter factor N) and the system must be slowed down to a bandwidth well below the resonance frecuency. The negativity of the PID parameters can be seen as a consequence of the phase lag needed to achieve the lowering of the bandwidth in order to rotate the resonance frequency region of the nyquist curve of the loop transfer function clock-wise from the left half plane to the right half plane. This implies the appearance of zeros in the right half plane, which in turn manifests itself as negative coefficients in the numerator of the loop transfer function, thus resulting in negative values in one or several of the PID parameters.

5 Approximations of the ultimate gain for FOTD systems

Determining the ultimate gain of a process model is traditionally done by some sort of self oscillation based test method. Even if the classical self oscillation experiment often is replaced by more practical but approximate relay feedback oscillation methods it is tempting to find a pure step response based design method. This would be possible to achieve by finding an approximation of the ultimate gain K_u from step response data. Another approach is to find such an approximate formula for K_u based on a simple process model. In process industry the most efficient process model seems to be a first order system with time delay, which only contains three parameters, namely the static gain k_p, the time constant T, and the time delay L:

$$G_p(s) = \frac{k_p}{1+Ts}e^{-Ls}$$

The ultimate gain for this system is given by

$$K_u = \frac{1}{k_p}\sqrt{1+x^2}$$

where x is defined by the trancendental equation

$$\arctan x = \pi - \frac{L}{T}x$$

An approximate solution to this equation is obtained by simply using $\arctan x \approx \pi/2$ resulting in $x \approx \frac{\pi T}{2L}$ which gives

$$K_u \approx \frac{1}{k_p}\sqrt{1+\left(\frac{\pi T}{2L}\right)^2} \tag{4}$$

This approximation is relevant for small values of the relative time delay L/T. A more accurate solution is obtained by making use of the approximation

$$\arctan x \approx \frac{\pi}{2}\frac{x}{x+1}$$

The maximal absolute error for this approximation is

$$\max_{x>0}\left|\arctan x - \frac{\pi}{2}\frac{x}{x+1}\right| \approx 0.071$$

Remark. This rational approximation is actually the best rational uniform approximation $r(x)$ of degree $(1,1)$ to $\arctan x$ under the interpolation constraints $r(0)=0$ and $r(\infty)=\pi/2$. Since the constraints directly yields

$$r(x) = \frac{\pi}{2}\frac{ax}{ax+1}$$

for $a>0$, it is sufficient to show that the error function $\epsilon(x) = \arctan x - r(x)$ has one local maximum and one local minimum for $0 < x < \pi/2$ and that these have equal magnitude precisely when $a = 1$, which guarantees the minimax property of the approximation.

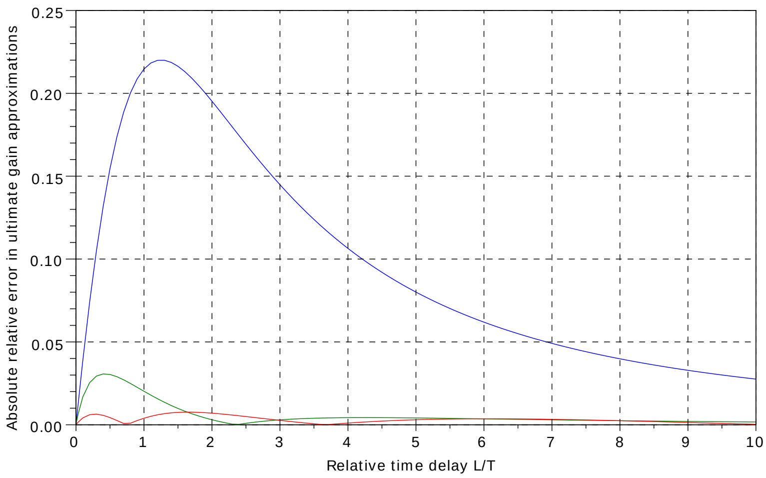

Figure 5: The absolute relative error curves for the three approximation methods Eq. (4), Eq. (5), and Eq. (6), of the ultimate gain as a function of the relative time delay L/T giving maximum absolute relative errors of 22%, 3%, and 1% respectively.

This results in the following sharpened formula for the ultimate gain:

$$K_u \approx \frac{1}{k_p}\sqrt{1+\frac{1}{4}\left(x-1+\sqrt{x^2+6x+1}\right)^2} \tag{5}$$

where $x = \frac{\pi T}{2L}$. Note, as a peculiarity, that by just switching the 6 to a 2 in this formula we arrive at the previous cruder approximation. Of course, higher order rational approximations of $\arctan x$ will give better result but this will not be addressed here. As an example of another type of approximation of K_u the following can be mentioned:

$$K_u \approx \frac{1}{k_p}\sqrt{1+\left(\frac{\pi T}{2L}\right)^2}\frac{1}{1+0.2e^{-\frac{2L}{T}}-0.2e^{-\frac{L}{5T}}-0.1\frac{L}{T}e^{-\frac{L}{T}}} \tag{6}$$

This is an ad hoc parametrized and manually tuned correction of the crude approximation in Eq. (4), where the correction factor consists of the inverse of a sum of exponentials.

The absolute relative error curves for the three approximation methods are shown in Fig. 5. From this figure, the maximum absolute relative errors for the approximation methods in Eq. (4), Eq. (5), and Eq. (6), can be estimated to 22%, 3%, and 1% respectively.

6 Step based method for FOTD systems

Aiming at a purely step response based PID parameter formula for first order systems with time delay (FOTD systems)

$$G(s) = \frac{k_p e^{-Ls}}{1 + Ts}$$

the crude approximation in Eq. (4) is used, which gives an absolute relative error of the ultimate gain K_u of at most around 20% (as can be seen in Fig. (5)). Actually, the approximation is *underestimating* the true ultimate gain by about 20% in the worst case. The parameter c_K in Eq. (1) is chosen to $c_K = 0.4$. Together with the underestimation of the ultimate gain this implies that $c_K = 0.4$ in the worst case corresponds to an actual value of $c_K \approx 0.3$. The only side effect of this is that the design in worst case gives a slightly slower response, without any other form of performance degradation. For the parameter $c_d = T_d/T_i$ the formula in Eq. (2) is utilized. These choices gives the following simple explicit formula for the PID parameters:

$$\begin{cases} K = \dfrac{0.4}{k_p}\sqrt{1 + \left(\dfrac{\pi T}{2L}\right)^2} \\ T_i = \dfrac{L + T}{1 + \dfrac{1}{2Kk_p}} \\ T_d = 0.3\left(1 - e^{-0.7\frac{L}{T}}\right) T_i \end{cases} \tag{7}$$

For low values of the relative time delay L/T this PID parameter formula results in slow responses to load disturbances. A modified version of the formula for T_i, which constitutes a trade-off between on the one hand (a) slower response to load disturbances but smaller overshoot in the reference value step response, and, on the other hand (b) faster response to load disturbances but larger overshoot in the reference value step response is given by

$$T_i = \frac{L + T}{1 + \dfrac{1 + \alpha (Kk_p)^2}{2Kk_p}} \tag{8}$$

where the parameter α interpolates between the two cases ($\alpha = 0$ for case (a) and $\alpha = 1$ for case (b)). There is a clear advantage to choose values of α closer to 1 since the larger overshoot in the reference value step response can easily be reduced by making use of reference value weighting.
Case (b) ($\alpha = 1$) is derived from the observation that for $L = 0$ (case without time delay) and pure PI control ($T_d = 0$) the characteristic equation for the closed loop system becomes

$$s^2 + \frac{1 + Kk_p}{T}s + \frac{Kk_p}{TT_i} = 0 \tag{9}$$

By substituting $T_p = T + L = T$ (since $L = 0$) into the formula (picked from Eq. (1)):

$$T_i = \frac{T_p}{1 + \dfrac{\sigma}{Kk_p}}$$

and then substituting this into Eq. (9) the following characteristic equation is obtained:

$$s^2 + \frac{1 + Kk_p}{T} s + \frac{Kk_p + \sigma}{T^2} = 0 \tag{10}$$

Comparing coefficients between Eq. (10) and the characteristic equation of a second order system with standard parametrization

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

where ζ is the relative damping, gives the following relation between σ and ζ:

$$\sigma = \frac{(1 + Kk_p)^2}{4\zeta^2} - Kk_p$$

Choosing the relative damping $\zeta = 1/\sqrt{2}$ yields

$$\sigma = \frac{(1 + Kk_p)^2}{2} - Kk_p = \frac{1 + (Kk_p)^2}{2}$$

which corresponds to the case $\alpha = 1$ above.

Example 5. The system

$$G(s) = \frac{1}{1 + 20s}\, e^{-2s}$$

has a relative time delay of $L/T = 0.1$, which is fairly small. In Fig. 6 three cases of PID control of the system are shown, $\alpha = 0$, $\alpha = 1$ without reference value weighting and $\alpha = 1$ with reference value weighting respectively. □

When choosing values of c_K other than 0.4 it is empirically found that the parameter c_d should be scaled according to $c_d = (c_K/0.4)0.3(1 - e^{-0.7L/T})$. The formula is then summarized as

$$\begin{cases} K = \dfrac{c_K}{k_p}\sqrt{1 + \left(\dfrac{\pi T}{2L}\right)^2} \\[2ex] T_i = \dfrac{L + T}{1 + \dfrac{1 + \alpha(Kk_p)^2}{2Kk_p}} \\[2ex] T_d = 0.75\, c_K \left(1 - e^{-0.7\frac{L}{T}}\right) T_i \end{cases} \tag{11}$$

A useful formula for the parameter α is given by

$$\alpha = \frac{f_\alpha}{f_\alpha + L/T}$$

where $f_\alpha = 0.1$ which empirically gives a good response to load disturbances. By using a simple reference value filter

$$F_r(s) = \frac{1 + T_{\mathrm{num}} s}{1 + T_{\mathrm{ref}} s}$$

the increased overshoot in the reference value step response can be decreased by proper choice of time constants T_{ref} and T_{num}.

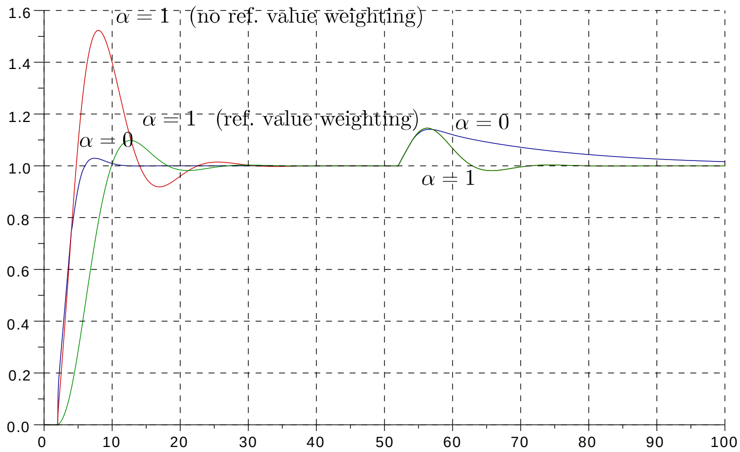

Figure 6: PID control using Eq. (7) with modified integral time according to Eq. (8) for system with $L = 2$ s and $T = 20$ s in three cases, $\alpha = 0$, $\alpha = 1$ with ref. value weighting and $\alpha = 1$ without ref. value weighting.

7 Estimation of L and T from a step response

The classic ("school book") estimation of three parameter (FOTD) model is to draw the tangent line of the measured step response at the inflection point. Assuming the initial value to be 0 the intersection of the tangent line with the time axis gives the dead time L. By drawing the vertical line through the point of intersection between the tangent line and the horizontal line through the final value, the time constant T is then obtained as the width of the base of the triangle thus defined. As an illustration, this method is applied to the step response of the system

$$G(s) = \frac{2}{(1+2s)^3}\, e^{-2s}$$

The result is shown in Fig. 7 together with the step response of the FOTD model obtained (Approximation 1). A simple and straightforward method to estimate L and T from a step respone $y(t)$ is to find the FOTD model which interpolates (coincides with) the measured step response at two points. An easily computable formula for this procedure is given by

$$\begin{cases} T = \dfrac{t_{y(t)=\lambda_2 k_p} - t_{y(t)=\lambda_1 k_p}}{\ln(1-\lambda_1) - \ln(1-\lambda_2)} \\ L = t_{y(t)=\lambda_1 k_p} + T \ln(1-\lambda_1) \end{cases} \tag{12}$$

where k_p is the static gain and $0 < \lambda_1 < \lambda_2 < 1$, where the step response is interpolated at the values $\lambda_1 k_p$ and $\lambda_2 k_p$. Empirically useful values have been found to be $\lambda_1 = 0.3$ and $\lambda_2 = 0.8$ (interpolation of the step response at 30%

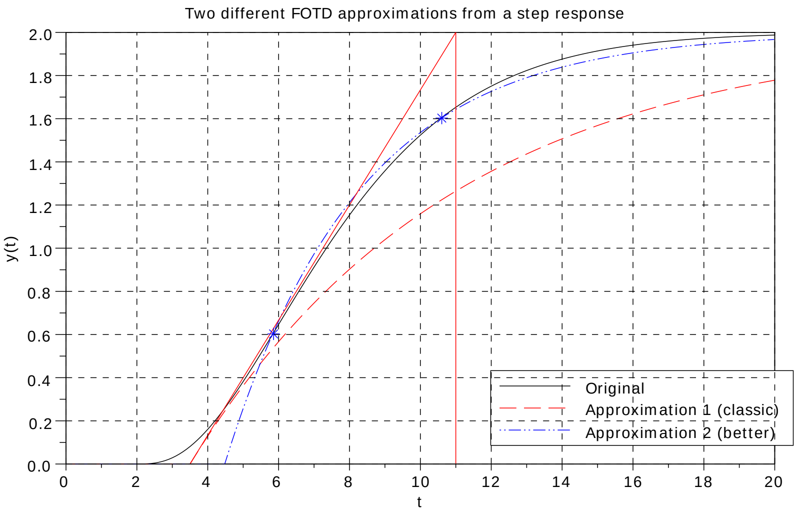

Figure 7: Step response of a third order system with a time delay, together with two different approximations based on FOTD models

and 80% of the static gain). This gives the approximate formulas

$$\begin{cases} T = 0.80\,(t_{y(t)=0.8k_p} - t_{y(t)=0.3k_p}) \\ L = t_{y(t)=0.3k_p} - 0.357\,T \end{cases} \tag{13}$$

The resulting FOTD model is also shown in Fig. 7 (Approximation 2), from which the approximate values $t_{y(t)=0.3k_p} \approx 5.9$ s and $t_{y(t)=0.8k_p} \approx 10.6$ s are read. From Eq. (13) the values $L \approx 4.56$ s and $T \approx 3.76$ s then are obtained. The choice of λ_1 and λ_2 affects the frequency weighting. For example, by decreasing these values to $\lambda_1 = 0.1$ and $\lambda_2 = 0.7$ the high frequency properties get higher priority. A straightforward generalization is to use a least-squares approximation using a larger number of data points.

The Skogestad "half rule"

The so called "half rule" from [4] means that the largest time constant to be neglected is distributed fifty-fifty on the smallest non-neglected time constant and the time delay in the approximation. For a system with two time time constants T_1 and T_2, where $T_2 \ll T_1$ (or at least $T_2 < T_1$), this means that the time constant and the time delay in the FOTD approximation are given by $T = T_1 + T_2/2$ and $L = T_2/2$ respectively. In this case it is easily shown that $T = \sqrt{T_1^2 + T_2^2}$ and $L = T_1 + T_2 - T$ will give an asymptotically correct rule of second order as $s \to 0$. This is seen by matching $(1 + T_1 s)(1 + T_2 s)$ and $(1 + Ts)e^{Ls}$ up to second order terms, which gives a unique solution for T and L. The Skogestad "half rule" gives a non-unique first order approximation. The "half rule" tends to give approximations, where the step responses of the system

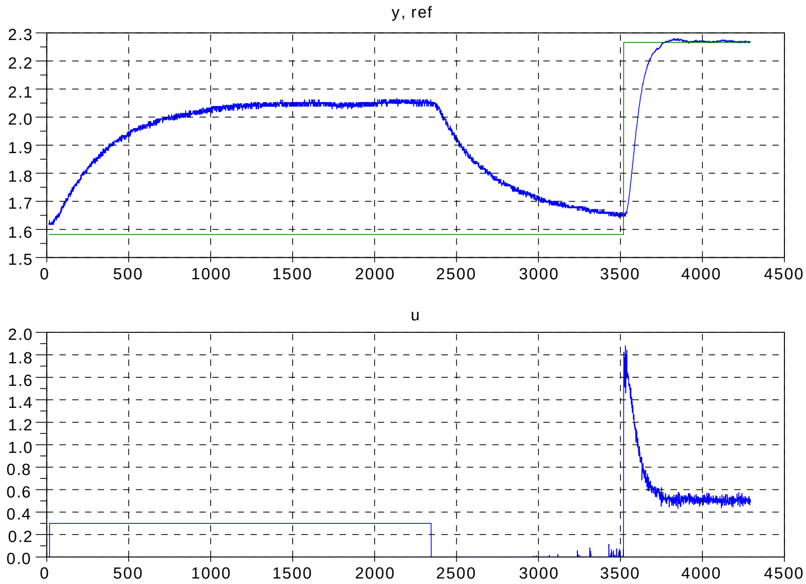

Figure 8: Soldering iron open loop step response experiment and a consecutive cooling followed by a step response of the system when controlled by a PID controller with parameters computed from the open loop step response.

and its FOTD approximation coincide at two points corresponding to a low level of 2-10% and a high level of 70-80%. From a frequency domain perspective this can be viewed as employing heavier weighting at high frequencies, since larger priority is given to matching the step response at small times (i.e. at low levels of the step response).

8 Applications

In order to assess the step response based method it is appropriate to test it on some physical process.

Example 6. The method was applied to a process consisting of a soldering iron with a Pt100 sensor installed on it. The control signal was generated from a solid state relay in the form of a PWM signal with a switch frequency of 0.5 Hz. Fig. 8 shows an open loop step response experiment, where the times for passage of the 30% and 80% levels ($\lambda_1 = 0.3$ and $\lambda_2 = 0.8$) were utilized for calculation of k_0, L and T in a FOTD model using Eq. (12) (in this case specialized to Eq. (13)). The FOTD parameters thus obtained were $k_p = 1.32$, $L = 46.3$ s, and $T = 255$ s. Using the formulas in Eq. (7) gave the result $K = 2.68$, $T_i = 264$ s, and $T_d = 9.46$ s. Note that only the analog control signal u is shown and not the actual PWM signal. After the open loop experiment, the soldering iron was cooled off, whereafter a step was introduced in the reference signal. This resulted in a well-damped closed loop step response with a small overshoot.

□

9 Conclusion

The various methods proposed are demonstrated to yield satisfactory results for a variety of process models. The “base” method depends on a self oscillation experiment (to estimate the ultimate gain) and a step response (to estimate the total time constant T_p, also known as “average residence time”). If the transfer function is known (such as in the given examples) the value of T_p is easily calculated. A decent approximation of T_p for many process models is given by $T_{63\%}$. By specializing to first order systems with time delay (FOTD) and combining this with a simple method for estimating the three FOTD parameters from a step response, a PID design method based only on a step response is obtained. Different estimates of the ultimate gain for a FOTD system is considered and the simplest of these is incorporated into the step response based method in order to eliminate the need for a self oscillation experiment or a relay feedback experiment.

In order to enhance the response to load disturbances, especially for low values of the relative time delay L/T, a parameter $0 \leq \alpha \leq 1$ is introduced in the formula for the integration time T_i, thereby selling of some of the performance of the step response from the reference value.

Some attempts are also made to find some adjustment rules for a second order system with a time delay based on the “base method” in Eq. (1). A special formula for the parameter c_d (i.e. T_d/T_i) is devised which involves the second order properties of the process model. This gives acceptable performance in the reference value responses for process models with relative damping $\zeta > 0.5$.

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