Flight Mechanics: Homework 3

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Policy

You should write your own answer/code by yourself. Cheating is highly discouraged for it could mean a zero or negative grade from the homework. If a question is not clear, please let us know via email.

Submission Instructions

Please submit your homework through the Ninova website. Please zip and upload all your files using filename *studentID.rar*. You must provide all functions you wrote with your zipped file. Functions you do not submit will cause you to lose a portion of your grade. Please make sure that you comment on your code. Make also sure that the plots you produce are readable and they have labels and legends.
You **MUST** include the *report.pdf* file with your homework. Include there:

1. answers to the questions,
2. outputs for each question,
3. how to call your functions for each question.

Problems

In this homework, you are expected to implement full aircraft motion including aerodynamic, propulsive and fuel consumption models. The aircraft dynamic model is given by the set of equations below:

$$\dot{x}_1 = x_4 \cos(x_5)\cos(u_3) \tag{1}$$
$$\dot{x}_2 = x_4 \sin(x_5)\cos(u_3) \tag{2}$$
$$\dot{x}_3 = x_4 \sin(u_3) \tag{3}$$
$$\dot{x}_4 = -\frac{C_D S \rho x_4^2}{2x_6} - g\sin(u_3) + \frac{u_1}{x_6} \tag{4}$$
$$\dot{x}_5 = -\frac{C_L S \rho x_4}{2x_6}\sin(u_2) \tag{5}$$
$$\dot{x}_6 = -f \tag{6}$$

The model is a dynamical system with three control inputs and six state variables. The state variables of the aircraft are the horizontal position (x_1 and x_2), altitude (x_3), the true airspeed (x_4), the heading angle (x_5) and the mass of the aircraft (x_6). The control inputs of the aircraft are the engine thrust (u_1), the bank angle (u_2) and the flight path angle (u_3). In the equation set above, aerodynamic lift and drag coefficients are represented by C_L and C_D, total wing surface area is S, air density is represented as ρ and the fuel consumption is represented as f. These coefficients can be obtained from BADA as a function of state variables.

By implementing a numerical integration method (i.e., Euler method, Runge-Kutta 4^{th} Order Method) yourself from scratch and using the performance parameters (e.g. lift coefficient, fuel consumption) for the aircraft presented in "aircraft.OPF", simulate the motion of aircraft defined via the set of equations above. Perform the following missions via numerical integration:

(a) With the aircraft fully loaded and fully fueled, perform a best-range cruise flight at a starting altitude of $4500m$ to cover the maximum range using Method 1. Find the maximum range.

(b) With the aircraft fully loaded and fully fueled, perform a best-range cruise flight at a starting altitude of $4500m$ to cover the maximum range using Method 3. Find the maximum range.

(c) After defining appropriate values for $h_1, h_2, h_3, \gamma_1, \gamma_2, \gamma_3, \phi, \eta,\ t$ and explaining the logic behind your selection strategy, perform the following flight segments consecutively:

- Climb from h_1 $[m]$ to h_2 $[m]$ with γ_1° flight path angle.
- Perform a cruise flight for t minutes.
- Turn with ϕ° bank angle until heading is changed by η°.
- Descend from h_2 $[m]$ to h_1 $[m]$ with γ_2° flight path angle.
- Perform a cruise flight for t minutes.
- Descend to h_3 $[m]$ with γ_3° flight path angle and also turn with ϕ° bank angle during the descent.

For each of the missions (i.e., part (a), (b), and (c)), present simulation results as listed below.

- 3D position graph
- $V_{TAS} - Time$ graph
- $V_{vertical} - Time$ graph
- $Heading - Time$ graph
- $Mass - Time$ graph
- $Thrust - Time$ graph
- $BankAngle - Time$ graph
- $FlightPathAngle - Time$ graph
- And specify the total fuel consumption during mission

Also, compare the results obtained in part (a) and (b), and discuss your findings.

Note: Initial conditions must be specified at the beginning of the simulation. Nominal speed schedule in Airline procedure model must be used as the initial condition for V_{TAS}.