DSA/ISE 5113 Advanced Analytics and Metaheuristics

Homework 4

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Problem 1(a)

i) .mod file

prob1a_HW4.mod | prob1aii_HW4.mod | problem1aiii_HW4.mod | *prob1aiv_HW4.mod

```
reset;
option solver cplex;


#decision variables
var XA>=0;   #Project A Investment
var XB>=0;   #Project B Investment
var XC>=0;   #Project C Investment
var XD>=0;   #Project D Investment
var XE>=0;   #Project E Investment
var Xb1>=0;  #annual investment allocations to the bank for year 1
var Xb2>=0;  #annual investment allocations to the bank for year 2
var Xb3>=0;  #annual investment allocations to the bank for year 3

#objective

maximize Return: XB + 1.4*XE + 1.75*XD + 1.06*Xb3;#Maximizing the Return

#constraints
subject to A1 : XA + XC + XD + Xb1 = 1000000; #Initial Investment
subject to A2 : 0.3*XA + 1.1*XC + 1.06*Xb1 = XB + Xb2;#Year1 balance
subject to A3 : XA + 0.3*XB + 1.06*Xb2 = XE + Xb3;#Year2 balance
subject to A4 : XA <= 500000;#Limit on Project A
subject to A5 : XB <= 500000;#Limit on Project B
subject to A6 : XE <= 750000;#Limit on Project E

#Display the Results
solve;
display XA,XB,XC,XD,XE;
display Xb1,Xb2,Xb3;
display Return;
```

Output:

```
AMPL
ampl: model 'C:\Users\shrey\prob1a_HW4.mod';
CPLEX 20.1.0.0: optimal solution; objective 1797600
2 dual simplex iterations (1 in phase I)
XA = 5e+05
XB = 0
XC = 0
XD = 5e+05
XE = 659000

Xb1 = 0
Xb2 = 150000
Xb3 = 0

Return = 1797600
```

From the above output we can infer that the investment of $500,000 in Project A, $500,000 in Project D, and $659,000 in Project E, but not investing in Projects B and C. No funds are allocated to the bank in Year 1 or Year 3, but $150,000 is invested in the bank in Year 2. This yields the maximum return of $1,797,600, achieved by prioritizing high-return projects (A, D, E).

ii) .mod file

prob1a_HW4.mod | prob1aii_HW4.mod | problem1aiii_HW4.mod | *prob1aiv_HW4.mod

```
reset;
option solver cplex;


#decision variables
var XA>=0;   #Project A Investment
var XB>=0;   #Project B Investment
var XC>=0;   #Project C Investment
var XD>=0;   #Project D Investment
var XE>=0;   #Project E Investment
var Xb1>=0;  #annual investment allocations to the bank for year 1
var Xb2>=0;  #annual investment allocations to the bank for year 2
var Xb3>=0;  #annual investment allocations to the bank for year 3

#objective
minimize TotalRisk: 0.10*XA + 0.12*XB + 0.05*XC + 0.20*XD + 0.05*XE;#Minimizing Risk


#constraints
subject to A1 : XA + XC + XD + Xb1 = 1000000; #Initial Investment
subject to A2 : 0.3*XA + 1.1*XC + 1.06*Xb1 = XB + Xb2;#Year1 balance
subject to A3 : XA + 0.3*XB + 1.06*Xb2 = XE + Xb3;#Year2 balance
subject to A4 : XA <= 500000;#Limit on Project A
subject to A5 : XB <= 500000;#Limit on Project B
subject to A6 : XE <= 750000;#Limit on Project E


#Display the Results
solve;
display XA,XB,XC,XD,XE;
display Xb1,Xb2,Xb3;
display TotalRisk;
```

Output:

Console

AMPL

```
ampl: model 'C:\Users\shrey\prob1aii_HW4.mod';
CPLEX 20.1.0.0: optimal solution; objective 0
0 dual simplex iterations (0 in phase I)
XA = 0
XB = 0
XC = 0
XD = 0
XE = 0

Xb1 = 1e+06
Xb2 = 1060000
Xb3 = 1123600

TotalRisk = 0
```

The solution minimizes risk by allocating all funds to the bank over the three years (Xb1=1,000,000, Xb2=1,060,000, Xb3=1,123,600) and not investing in any of the projects (A, B, C, D, E). By focusing solely on the bank, which has a risk coefficient of 0, the solution eliminates all exposure to potential losses, aligning perfectly with the goal of minimizing risk.

iii) .mod file

prob1a_HW4.mod | prob1aii_HW4.mod | *problem1aiii_HW4.mod | *prob1aiv_HW4.mod

```
var Xb1>=0;  #annual investment allocations to the bank for year 1
var Xb2>=0;  #annual investment allocations to the bank for year 2
var Xb3>=0;  #annual investment allocations to the bank for year 3


#objectives
maximize Return: XB + 1.4*XE + 1.75*XD + 1.06*Xb3;#Maximizing Return
minimize TotalRisk: 0.10*XA + 0.12*XB + 0.05*XC + 0.20*XD + 0.05*XE;#Minimizing Risk


#constraints
subject to A1 : XA + XC + XD + Xb1 = 1000000; #Initial Investment
subject to A2 : 0.3*XA + 1.1*XC + 1.06*Xb1 = XB + Xb2;#Year1 balance
subject to A3 : XA + 0.3*XB + 1.06*Xb2 = XE + Xb3;#Year2 balance
subject to A4 : XA <= 500000;#Limit on Project A
subject to A5 : XB <= 500000;#Limit on Project B
subject to A6 : XE <= 750000;#Limit on Project E


#SCALARIZATION-----------------------------------------------------------------------------------------------------------

param lambda1;
param lambda2;


maximize objWeightedSum : ((lambda1*( XB + 1.4*XE + 1.75*XD + 1.06*Xb3))-(lambda2*(0.10*XA + 0.12*XB + 0.05*XC + 0.20*XD + 0.05*XE)));

problem maxScalarized: objWeightedSum,XA,XB,XC,XD,XE,Xb1,Xb2,Xb3,A1,A2,A3,A4,A5,A6;

for {k in 0..4} {
let lambda1 := k/4;
let lambda2 := 1 - lambda1;
solve maxScalarized;

printf "\n\nlambda1 = %6.2f; lambda2 = %6.2f \n", lambda1,lambda2;
printf "Profit generated: %6.2f\n", Return;
printf "Total Risk: %6.2f \n\n", TotalRisk ;

printf "%d, %3.2f, %3.2f, %7.4f, %7.4f, %7.4f, %7.4f,%7.4f, %7.4f, %7.4f\n",
k,lambda1,lambda2,XA,XB,XC,XD,XE,Return,TotalRisk > "C:\Users\shrey\Downloads\lambda_output.txt";
}
```

Output:

```
AMPL
ampl: model 'C:\Users\shrey\problem1aiii_HW4.mod';
CPLEX 20.1.0.0: optimal solution; objective 0
0 dual simplex iterations (0 in phase I)


lambda1 =   0.00; lambda2 =   1.00
Profit generated: 1191016.00
Total Risk:   0.00

CPLEX 20.1.0.0: optimal solution; objective 333379
1 simplex iterations (0 in phase I)


lambda1 =   0.25; lambda2 =   0.75
Profit generated: 1446016.00
Total Risk: 37500.00

CPLEX 20.1.0.0: optimal solution; objective 807325
3 simplex iterations (0 in phase I)


lambda1 =   0.50; lambda2 =   0.50
Profit generated: 1797600.00
Total Risk: 182950.00

CPLEX 20.1.0.0: optimal solution; objective 1302462.5
0 simplex iterations (0 in phase I)


lambda1 =   0.75; lambda2 =   0.25
Profit generated: 1797600.00
Total Risk: 182950.00

CPLEX 20.1.0.0: optimal solution; objective 1797600
0 simplex iterations (0 in phase I)


lambda1 =   1.00; lambda2 =   0.00
Profit generated: 1797600.00
Total Risk: 182950.00
```

The objective is to have a balance between maximizing profit and minimizing risk. By using the scalarization method, we explore different combinations of lambda 1 and lambda 2 values which are assigned to these two objectives. When risk is prioritized, all funds are kept in the bank, resulting in zero risk but lower profits. As the focus shifts toward profit, more investments are made in high-return projects, increasing both potential returns and risk.

iv) .mod file

prob1a_HW4.mod | prob1aii_HW4.mod | problem1aiii_HW4.mod | *prob1aiv_HW4.mod

```
display Return, XA, XB, XC, XD, XE, Xb1, Xb2, Xb3;


printf "\nMinimize Risk\n";
solve minRisk;
display TotalRisk, XA, XB, XC, XD, XE, Xb1, Xb2, Xb3;


#Epsilon Constraint-------------------------------------------------------------------

printf "\nEpsilon Constraint\n";

param upperRisk;
param lowerRisk;

solve minRisk;
let lowerRisk:=TotalRisk;

solve maxProfit;
let upperRisk:=TotalRisk;


param epsilon;
let epsilon := lowerRisk;


subject to epsilonRisk: 0.10*XA + 0.12*XB + 0.05*XC + 0.20*XD + 0.05*XE<= epsilon;

problem epsConst:Return,XA,XB,XC,XD,XE,Xb1,Xb2,Xb3,A1,A2,A3,A4,A5,A6,epsilonRisk;

param steps = 20;


printf "\nVarious values of Epsilon\n";
for {eps in 0..steps} {
let epsilon := lowerRisk + eps*(upperRisk - lowerRisk)/(steps);
solve epsConst;
display Return, epsilon, TotalRisk,XA,XB,XC,XD,XE,Xb1,Xb2,Xb3;
printf "%d, %7.4f, %7.4f, %7.4f\n",
eps, epsilon, Return,TotalRisk > "C:\Users\shrey\Downloads\epsilon_output.txt";
}
```

Output:

AMPL

```
ampl: model 'C:\Users\shrey\prob1aiv_HW4.mod';

Maximum Returns
CPLEX 20.1.0.0: optimal solution; objective 1797600
2 dual simplex iterations (1 in phase I)
Return = 1797600
XA = 5e+05
XB = 0
XC = 0
XD = 5e+05
XE = 659000
Xb1 = 0
Xb2 = 150000
Xb3 = 0


Minimize Risk
CPLEX 20.1.0.0: optimal solution; objective 0
4 simplex iterations (0 in phase I)
TotalRisk = 0
XA = 0
XB = 0
XC = 0
XD = 0
XE = 0
Xb1 = 1e+06
Xb2 = 1060000
Xb3 = 1123600
```

```
Epsilon Constraint
CPLEX 20.1.0.0: optimal solution; objective 0
0 simplex iterations (0 in phase I)
CPLEX 20.1.0.0: optimal solution; objective 1797600
4 simplex iterations (0 in phase I)

Various values of Epsilon
Solution determined by presolve;
objective Return = 1191016.
Return = 1191020
epsilon = 0
TotalRisk = 0
XA = 0
XB = 0
XC = 0
XD = 0
XE = 0
Xb1 = 1e+06
Xb2 = 1060000
Xb3 = 1123600

CPLEX 20.1.0.0: optimal solution; objective 1253219
3 dual simplex iterations (2 in phase I)
Return = 1253220
epsilon = 9147.5
TotalRisk = 9147.5
XA = 0
XB = 0
XC = 0
XD = 0
XE = 182950
Xb1 = 1e+06
Xb2 = 1060000
Xb3 = 940650
```

```
CPLEX 20.1.0.0: optimal solution; objective 1315422
0 simplex iterations (0 in phase I)
Return = 1315420
epsilon = 18295
TotalRisk = 18295
XA = 0
XB = 0
XC = 0
XD = 0
XE = 365900
Xb1 = 1e+06
Xb2 = 1060000
Xb3 = 757700

CPLEX 20.1.0.0: optimal solution; objective 1377625
0 simplex iterations (0 in phase I)
Return = 1377620
epsilon = 27442.5
TotalRisk = 27442.5
XA = 0
XB = 0
XC = 0
XD = 0
XE = 548850
Xb1 = 1e+06
Xb2 = 1060000
Xb3 = 574750
```

```
CPLEX 20.1.0.0: optimal solution; objective 1439828
0 simplex iterations (0 in phase I)
Return = 1439830
epsilon = 36590
TotalRisk = 36590
XA = 0
XB = 0
XC = 0
XD = 0
XE = 731800
Xb1 = 1e+06
Xb2 = 1060000
Xb3 = 391800

CPLEX 20.1.0.0: optimal solution; objective 1469039.154
1 dual simplex iterations (0 in phase I)
Return = 1469040
epsilon = 45737.5
TotalRisk = 45737.5
XA = 0
XB = 0
XC = 0
XD = 41187.5
XE = 750000
Xb1 = 958812
Xb2 = 1016340
Xb3 = 327322
```

```
CPLEX 20.1.0.0: optimal solution; objective 1494605.684
0 simplex iterations (0 in phase I)
Return = 1494610
epsilon = 54885
TotalRisk = 54885
XA = 0
XB = 0
XC = 0
XD = 86925
XE = 750000
Xb1 = 913075
Xb2 = 967860
Xb3 = 275931

CPLEX 20.1.0.0: optimal solution; objective 1520172.215
0 simplex iterations (0 in phase I)
Return = 1520170
epsilon = 64032.5
TotalRisk = 64032.5
XA = 0
XB = 0
XC = 0
XD = 132662
XE = 750000
Xb1 = 867338
Xb2 = 919378
Xb3 = 224540
```

```
CPLEX 20.1.0.0: optimal solution; objective 1545738.746
0 simplex iterations (0 in phase I)
Return = 1545740
epsilon = 73180
TotalRisk = 73180
XA = 0
XB = 0
XC = 0
XD = 178400
XE = 750000
Xb1 = 821600
Xb2 = 870896
Xb3 = 173150

CPLEX 20.1.0.0: optimal solution; objective 1571305.276
0 simplex iterations (0 in phase I)
Return = 1571310
epsilon = 82327.5
TotalRisk = 82327.5
XA = 0
XB = 0
XC = 0
XD = 224138
XE = 750000
Xb1 = 775862
Xb2 = 822414
Xb3 = 121759
```

```
CPLEX 20.1.0.0: optimal solution; objective 1596871.807
0 simplex iterations (0 in phase I)
Return = 1596870
epsilon = 91475
TotalRisk = 91475
XA = 0
XB = 0
XC = 0
XD = 269875
XE = 750000
Xb1 = 730125
Xb2 = 773932
Xb3 = 70368.5

CPLEX 20.1.0.0: optimal solution; objective 1622438.338
0 simplex iterations (0 in phase I)
Return = 1622440
epsilon = 100622
TotalRisk = 100622
XA = 0
XB = 0
XC = 0
XD = 315612
XE = 750000
Xb1 = 684388
Xb2 = 725451
Xb3 = 18977.8
```

```
CPLEX 20.1.0.0: optimal solution; objective 1644857.538
1 dual simplex iterations (0 in phase I)
Return = 1644860
epsilon = 109770
TotalRisk = 109770
XA = 42862.8
XB = 0
XC = 0
XD = 339919
XE = 750000
Xb1 = 617219
Xb2 = 667111
Xb3 = 0

CPLEX 20.1.0.0: optimal solution; objective 1665433.969
0 simplex iterations (0 in phase I)
Return = 1665430
epsilon = 118918
TotalRisk = 118918
XA = 110822
XB = 0
XC = 0
XD = 351677
XE = 750000
Xb1 = 537502
Xb2 = 602998
Xb3 = 0
```

```
CPLEX 20.1.0.0: optimal solution; objective 1686010.401
0 simplex iterations (0 in phase I)
Return = 1686010
epsilon = 128065
TotalRisk = 128065
XA = 178781
XB = 0
XC = 0
XD = 363435
XE = 750000
Xb1 = 457785
Xb2 = 538886
Xb3 = 0

CPLEX 20.1.0.0: optimal solution; objective 1706586.832
0 simplex iterations (0 in phase I)
Return = 1706590
epsilon = 137212
TotalRisk = 137212
XA = 246740
XB = 0
XC = 0
XD = 375192
XE = 750000
Xb1 = 378067
Xb2 = 474774
Xb3 = 0
```

```
CPLEX 20.1.0.0: optimal solution; objective 1727163.264
0 simplex iterations (0 in phase I)
Return = 1727160
epsilon = 146360
TotalRisk = 146360
XA = 314699
XB = 0
XC = 0
XD = 386950
XE = 750000
Xb1 = 298350
Xb2 = 410661
Xb3 = 0

CPLEX 20.1.0.0: optimal solution; objective 1747739.695
0 simplex iterations (0 in phase I)
Return = 1747740
epsilon = 155508
TotalRisk = 155508
XA = 382658
XB = 0
XC = 0
XD = 398708
XE = 750000
Xb1 = 218633
Xb2 = 346549
Xb3 = 0
```

```
CPLEX 20.1.0.0: optimal solution; objective 1768316.127
0 simplex iterations (0 in phase I)
Return = 1768320
epsilon = 164655
TotalRisk = 164655
XA = 450617
XB = 0
XC = 0
XD = 410466
XE = 750000
Xb1 = 138916
Xb2 = 282437
Xb3 = 0

CPLEX 20.1.0.0: optimal solution; objective 1786344.67
1 dual simplex iterations (0 in phase I)
Return = 1786340
epsilon = 173802
TotalRisk = 173802
XA = 5e+05
XB = 0
XC = 0
XD = 436396
XE = 730465
Xb1 = 63603.8
Xb2 = 217420
Xb3 = 0
```

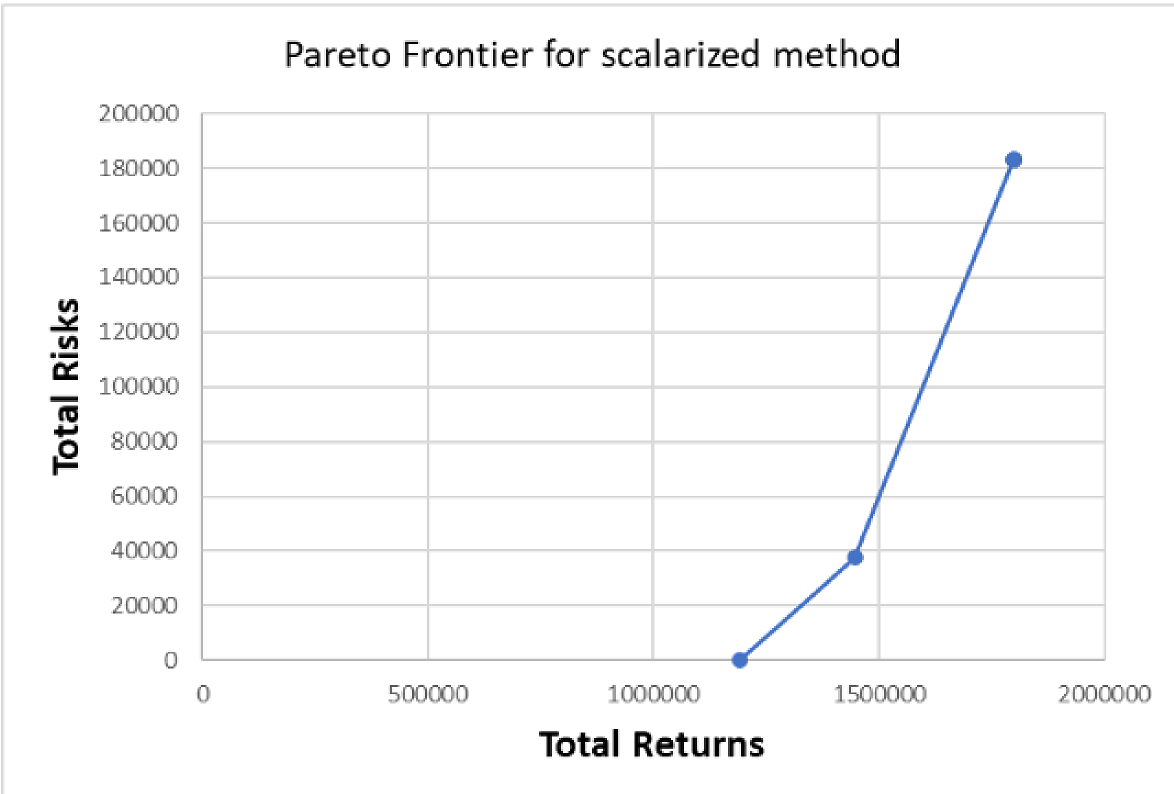
```
CPLEX 20.1.0.0: optimal solution; objective 1797600
0 simplex iterations (0 in phase I)
Return = 1797600
epsilon = 182950
TotalRisk = 182950
XA = 5e+05
XB = 0
XC = 0
XD = 5e+05
XE = 659000
Xb1 = 5.82077e-11
Xb2 = 150000
Xb3 = 0

ampl:
```

The epsilon constraint method explores the balance between maximizing profit and minimizing risk by adjusting the allowable risk across a range of values. At the lowest risk level, the solution allocates all funds to the bank, achieving zero risk but yielding a lower return. As epsilon increases, more funds are gradually shifted toward higher-return projects like E and D, increasing both returns and risk. At the highest risk level, the solution achieves the maximum return of 1,797,600 by investing heavily in projects A, D, and E, but this comes with the highest risk exposure.

Problem 1(b)

k	lambda1	lambda1	total_profit	total_risk
0	0	1	1191016	0
1	0.25	0.75	1446016	37500
2	0.5	0.5	1797600	182950
3	0.75	0.25	1797600	182950
4	1	0	1797600	182950



Pareto Frontier: The curved line in the graph is the Pareto frontier, which represents a set of optimal investment strategies. Each point on this curve represents a balance between risk and return, where any attempt to improve one objective (e.g., return) will necessarily worsen the other objective (e.g., risk).

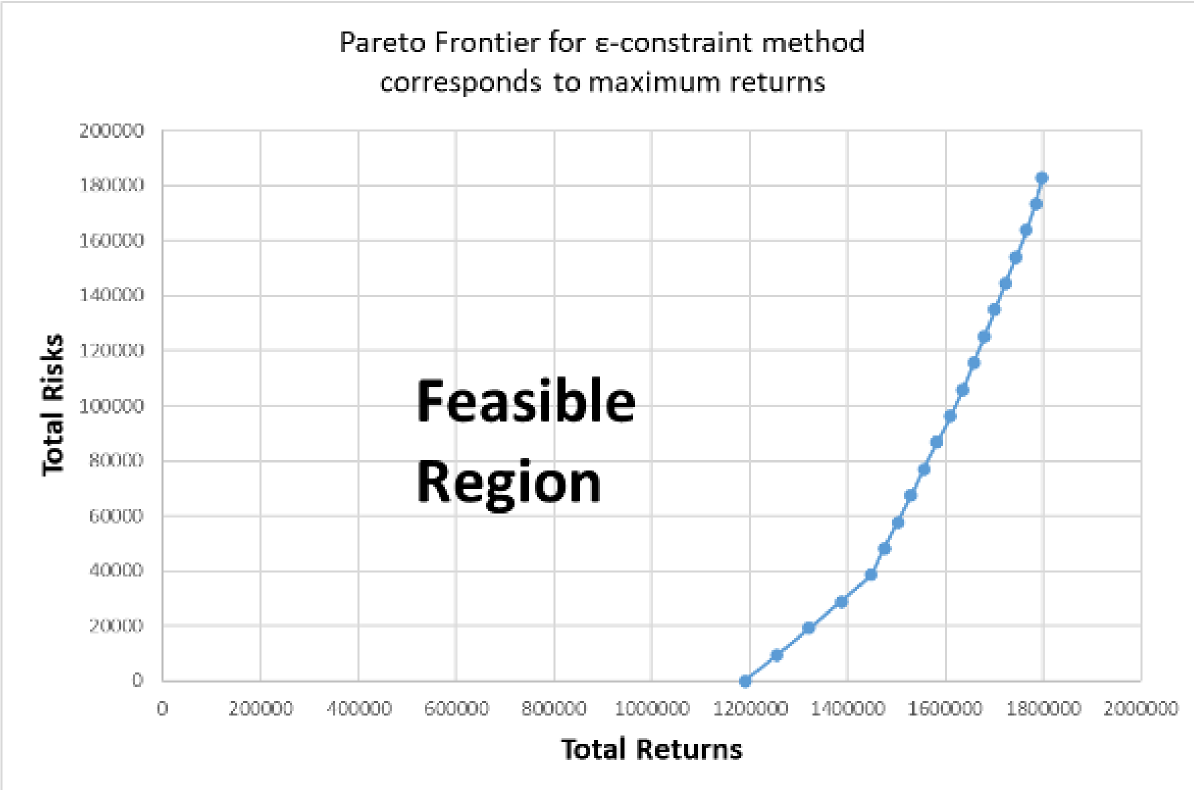
INTERPRETATION

- **Points on the frontier:** These represent portfolios that offer the best possible return for a given level of risk, or conversely, the least amount of risk for a given level of return. There's no single "best" solution on the frontier, as the preferred option depends on the risk tolerance of Titan Enterprises.

- **Higher return comes at a higher risk:** As you move from left to right along the frontier, the total return increases, but so does the total risk. Investors seeking a higher return on their investment must be willing to accept a higher level of risk.
- **Choosing an investment strategy:** The committee can use this graph to visualize the risk-return trade-off for different investment portfolios. By understanding their risk tolerance, they can select a point on the Pareto frontier that aligns with their investment goals. For instance, if the committee is very risk-averse, they might choose a portfolio closer to the left side of the frontier, where the return is lower, but the risk is minimized. Conversely, if the committee has a higher tolerance for risk, they might opt for a portfolio on the right side of the frontier to achieve a higher potential return.

Problem 1 (c)
Maximum returns achieved

eps	epsilon	total_profit	total_risk
0	0	1191016	0
1	9147.5	1253219	9147.5
2	18295	1315422	18295
3	27442.5	1377625	27442.5
4	36590	1439828	36590
5	45737.5	1469039.15	45737.5
6	54885	1494605.68	54885
7	64032.5	1520172.21	64032.5
8	73180	1545738.75	73180
9	82327.5	1571305.28	82327.5
10	91475	1596871.81	91475
11	100622.5	1622438.34	100622.5
12	109770	1644857.54	109770
13	118917.5	1665433.97	118917.5
14	128065	1686010.4	128065
15	137212.5	1706586.83	137212.5
16	146360	1727163.26	146360
17	155507.5	1747739.7	155507.5
18	164655	1768316.13	164655
19	173802.5	1786344.67	173802.5
20	182950	1797600	182950



INTERPRETATIONS

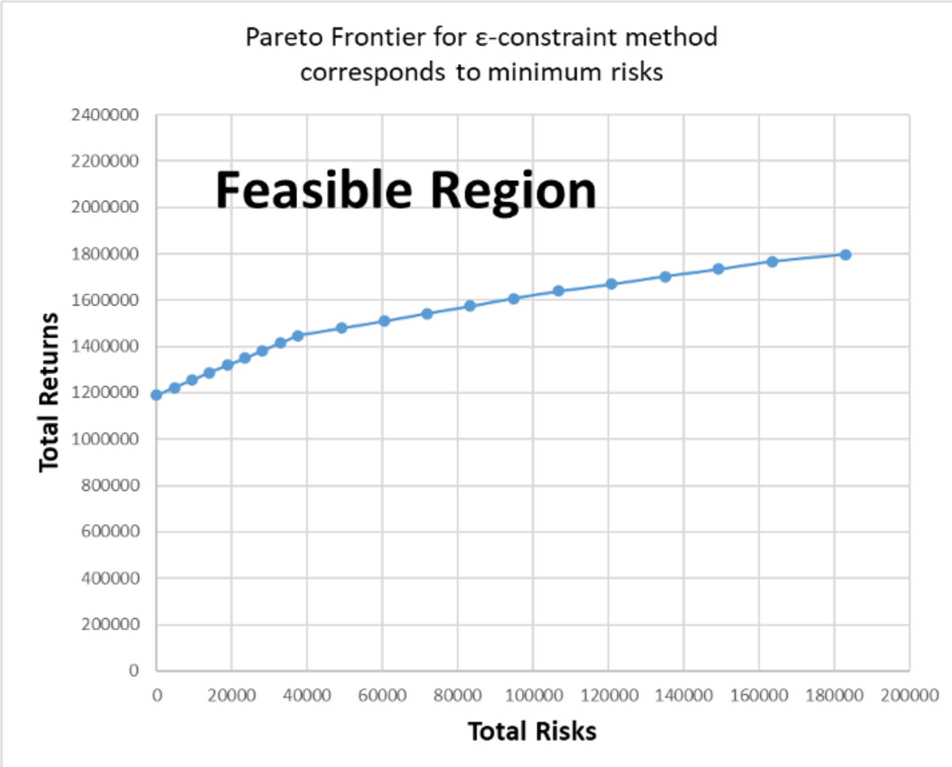
Decision Making for the Committee:

The Investment Review Committee can utilize this graph to make better informed decisions based on their risk tolerance:

- If the committee is very risk-averse, they should choose a portfolio closer to the bottom left of the frontier, where the priority is on minimizing risk.
- If they have a higher tolerance for risk and are seeking higher potential returns, they can select a portfolio located closer to the top right of the frontier.
- The graph helps them visualize the range of potential returns achievable for a given level of acceptable risk.

Minimum risk achieved

eps	epsilon	total_risk	total_profit
0	1191016	0	1191016
1	1222941.474	4694.9226	1222941.474
2	1254866.947	9389.8452	1254866.947
3	1286792.421	14084.7678	1286792.421
4	1318717.895	18779.6904	1318717.895
5	1350643.368	23474.613	1350643.368
6	1382568.842	28169.5356	1382568.842
7	1414494.316	32864.4582	1414494.316
8	1446419.79	37644.4726	1446419.79
9	1478345.263	49067.1515	1478345.263
10	1510270.737	60489.8304	1510270.737
11	1542196.211	71912.5093	1542196.211
12	1574121.684	83335.1882	1574121.684
13	1606047.158	94757.8671	1606047.158
14	1637972.632	106709.2322	1637972.632
15	1669898.105	120902.0854	1669898.105
16	1701823.579	135094.9385	1701823.579
17	1733749.053	149287.7917	1733749.053
18	1765674.526	163480.6448	1765674.526
19	1797600	182950	1797600



INTERPRETATIONS

- **Low Risk Tolerance:** If the committee has a low risk tolerance, they should prioritize minimizing risk. They would likely choose a portfolio closer to the bottom left side of the frontier within the feasible region.
- **High Risk Tolerance:** Conversely, a committee with a high-risk tolerance might be more willing to accept risk in pursuit of potentially higher returns. They would likely select a portfolio located closer to the top right side of the frontier within the feasible region. This portfolio might allocate more funds to potentially high-risk, high-reward projects.
- **Intermediate Risk Tolerance:** Most committees likely fall somewhere between these two extremes. They would choose a portfolio from the middle range of the frontier within the feasible region, balancing risk and return based on their specific risk tolerance level.