DSFP: Image Processing

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I'm only going to talk about an easier problem today.

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- How should I measure a star's brightness?

Aperture fluxes

One solution would be to add up all the intensity within some region centered on a star.

Image

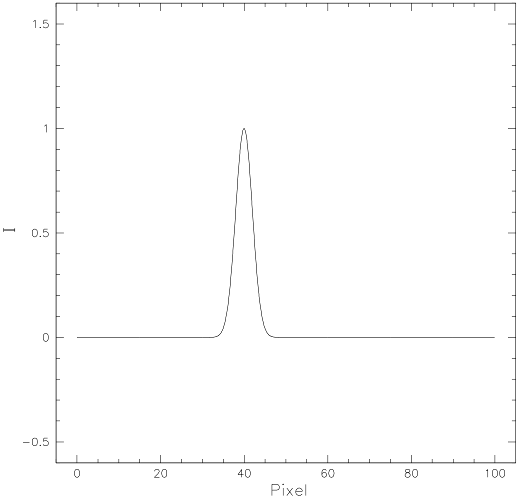
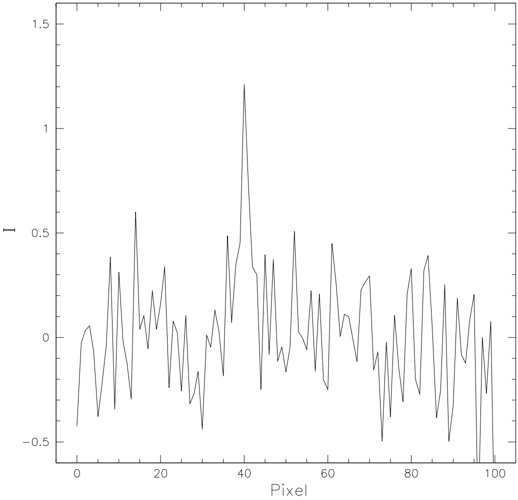
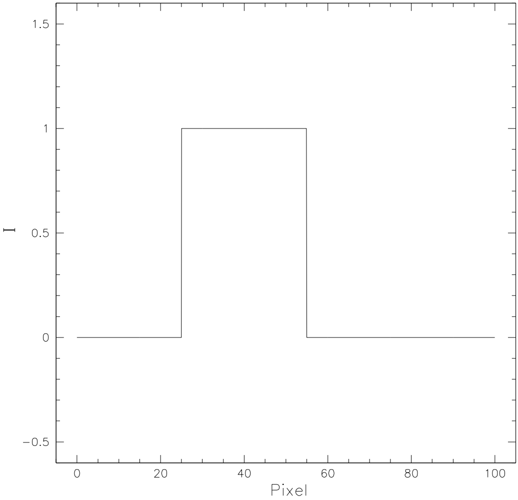


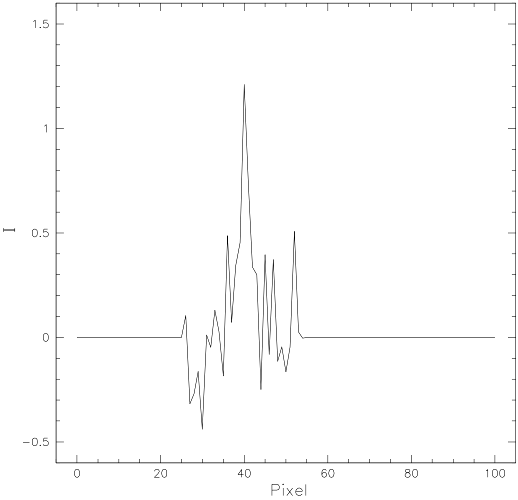
Image with noise



My Aperture



Aperture Flux with noise



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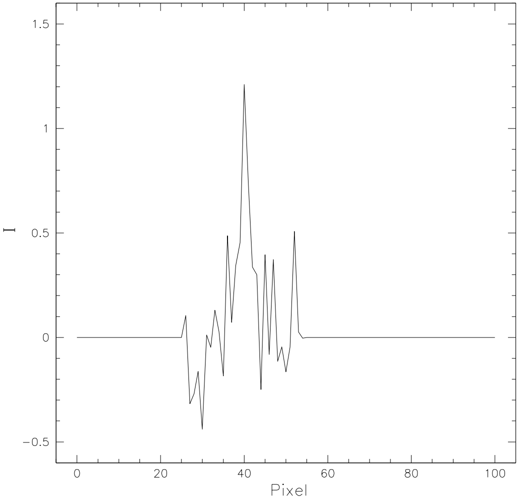

This is obviously a noisy measurement

Isolated Stars

We have a PSF ϕ, so a part of the sky containing a single star with flux F_0 at x_0 may be modelled as

$$S + F_0\,\delta(x - x_0)$$

where S is the background, taken to be constant.

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I.e.

$$I(x) = S + F_0\,\phi(x - x_0) + n(x)$$

Noise

In the optical, to a very good approximation, n is almost entirely shot noise due to the finite number of photons detected, *i.e.* it is a Poisson process. This may be approximated by a Gaussian with mean and variance equal to the signal, I.

Background Estimation

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For today let's assume that S is known, and accordingly set it to 0.

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Our model is that

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Rearranging, we find that

$$n(x) = F_0\,\phi(x - x_0) - I(x)$$

and this is a Gaussian, $N(0, \sigma(x)^2)$; *i.e.*

$$P(n(x)) = \frac{1}{\sqrt{2\pi\sigma(x)^2}} e^{-n(x)^2/2\sigma(x)^2}$$

Likelihoods

If we have pixellated data it's convenient to write $n_i \equiv n(x_i)$, so we can write the total likelihood of our data as

$$\mathcal{L}(I|F_0) = \prod_i \frac{1}{\sqrt{2\pi\sigma_i^2}} \mathrm{e}^{-n_i^2/2\sigma_i^2}$$

but it's easier to work with the logarithm:

$$\ln \mathcal{L}(I|F_0) = -\frac{1}{2}\left(\sum_i \ln 2\pi\sigma_i^2 + \sum_i \frac{n_i^2}{\sigma_i^2}\right)$$

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I'm going to ignore the priors for now. Unfortunately this isn't always possible.

Likelihoods

Substituting our formula for n this becomes

$$\ln \mathcal{L}(I|F_0, x_0) \sim -\sum_i \frac{(I_i - F_0\,\phi_i)^2}{\sigma_i^2}$$

where $\sigma^2 = S + I$.

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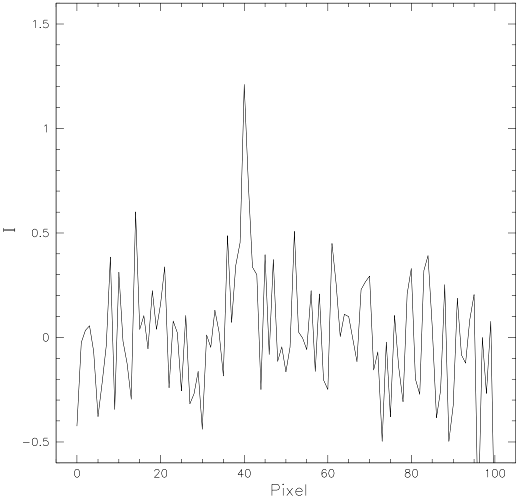
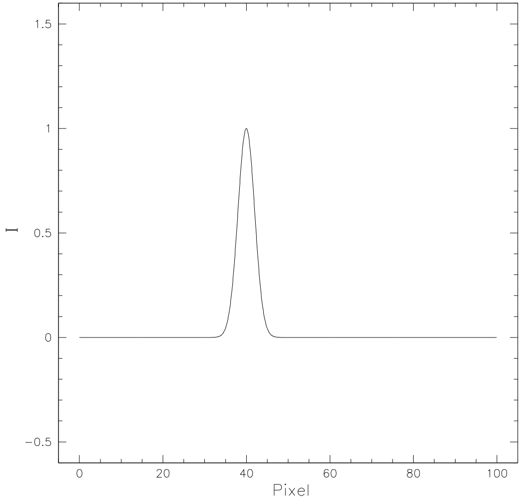
where $\sigma^2 = S + I$. The MLE results from minimising that sum of squares. We've recovered a simple χ^2 estimator -- but now we know it's because we assumed that the noise was Gaussian and chose an asymptotically optimal approach. Differentiation with respect to F_0 we find that this is maximised at

$$\hat{F_0} = \frac{\sum_i I_i\phi_i/\sigma_i^2}{\sum_i \phi_i^2/\sigma_i^2}$$

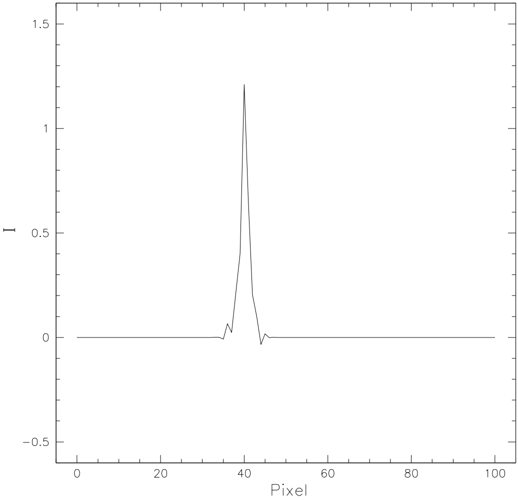
Image with noise



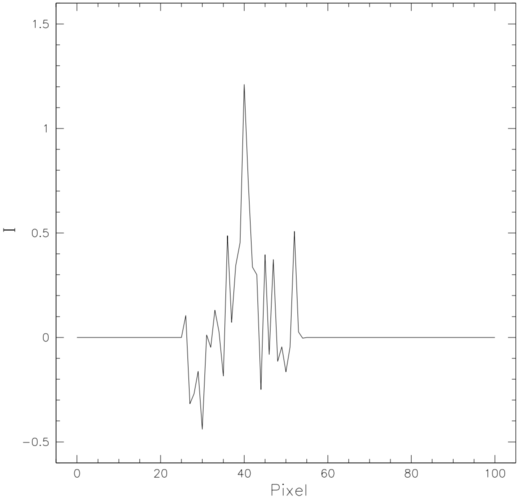
PSF



PSF flux



Aperture flux



Source Detection

I'm primarily interested in faint sources, so the noise is dominated by S which is the same in all pixels. We then have

$$\ln \mathcal{L}(I|F_0, x_0) \propto -\sum_i \left(I_i - F_0\, \phi_i(x_0)\right)^2$$

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The maximum likelihood estimate of the position of our object is thus given by the maximum of the initial data, convolved with the PSF.

Measuring fluxes using the Psf

For faint sources (so all pixels have the same variance, $\sigma^2 \equiv S$), the flux is given by

$$\hat{F_0} = \frac{\sum_i I_i \phi_i}{\sum_i \phi_i^2}$$

So each photon is weighted by the PSF's profile, *i.e.* the probability that it belongs to the source.

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In this limit, the noise in the measurement is

$$\frac{\left(\sum_i \phi_i\right)^2}{\sum_i \phi_i^2}\,\sigma^2 \equiv n_{\mathrm{eff}}\,\sigma^2$$

If the PSF is Gaussian $N(0, \alpha^2)$, $n_{\mathrm{eff}} = 4\pi\alpha^2$

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If you like algebra, you can take the expression

$$\hat{F_0} = \frac{\sum_i I_i \phi_i / \sigma_i^2}{\sum_i \phi_i^2 / \sigma_i^2}$$

and substitute $\sigma_i^2 = F_0 \phi_i$ to find that

$$\hat{F_0} = \sum_i I_i$$

i.e. an aperture flux.