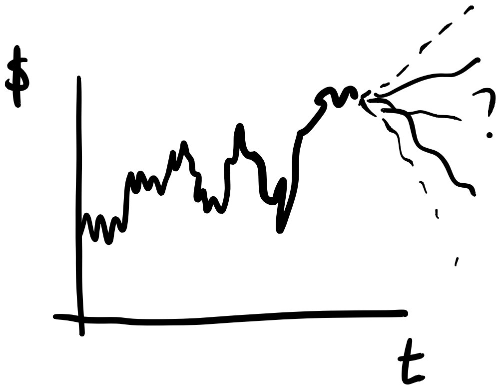
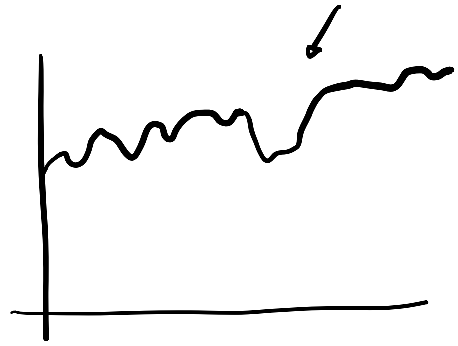
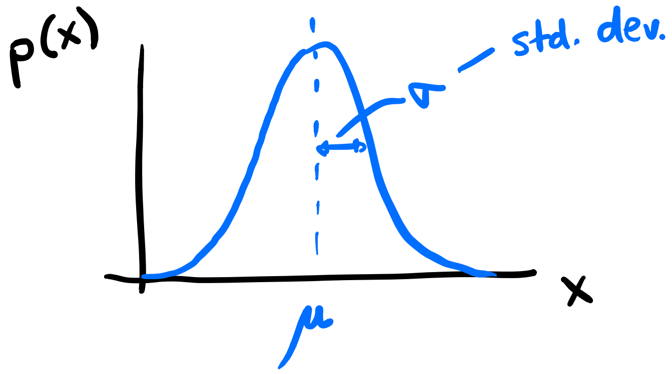
rodluger@gmail.com



PREDICTION

## GAUSSIAN DISTRIBUTION (1D)

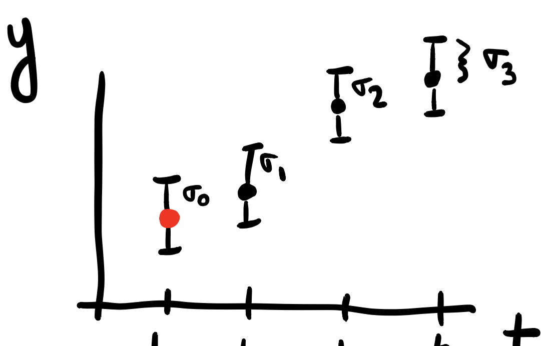



$\sigma^2$ — variance

$$\rightarrow \quad p(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$x \sim \mathcal{N}(\mu, \sigma^2)$$

## GAUSSIAN DISTRIBUTION (many D)




$$y_0 \sim \mathcal{N}(\mu_0, \sigma_0^2)$$

$$y_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$$

$t_0 \quad t_1 \quad t_2 \quad t_3 \quad t$

covariance matrix

$$\underline{y} \sim \mathcal{N}(\underline{\mu}, \underline{\underline{\Sigma}})$$

$$\underline{\underline{\Sigma}} = \begin{bmatrix} \sigma_0 & & & 0 \\ & \sigma_1 & & \\ & & \sigma_2 & \\ 0 & & & \sigma_3 \end{bmatrix}$$

sometimes

$$\underline{\underline{\Sigma}} = \sigma^2 \underline{\underline{I}}$$

$$p(\underline{y} \mid \underline{\mu}, \underline{\underline{\Sigma}}) = \frac{1}{\sqrt{2\pi\sigma_0^2}} e^{-\frac{1}{2}\left(\frac{y_0-\mu_0}{\sigma_0}\right)^2}$$

$$\times \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{1}{2}\left(\frac{y_1-\mu_1}{\sigma_1}\right)^2}$$

$$\times \ldots$$

EXERCISE 1

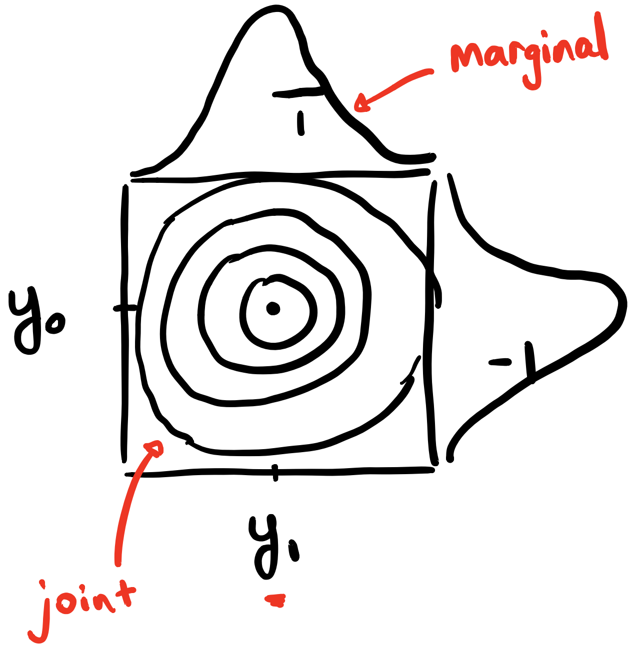
$$= \prod_{n=0}^{N-1} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{1}{2}\sum_{n=0}^{N-1}\left(\frac{y_n-\mu_n}{\sigma_n}\right)^2}$$

$$= \boxed{\frac{1}{\sqrt{(2\pi)^N |\underline{\underline{\Sigma}}|}} e^{-\frac{1}{2}(\underline{y}-\underline{\mu})^T \underline{\underline{\Sigma}}^{-1} (\underline{y}-\underline{\mu})}}$$

GENERAL

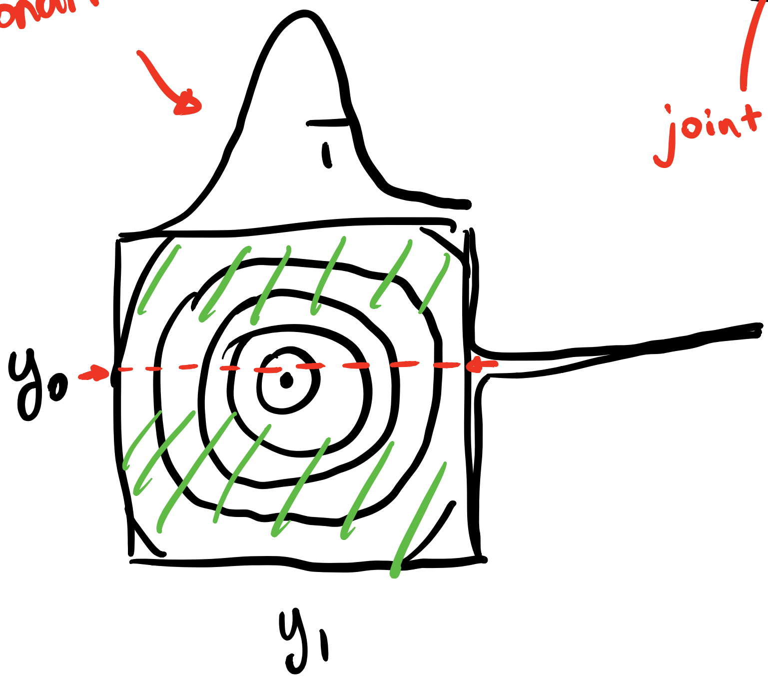
## COVARIANCE

$$\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

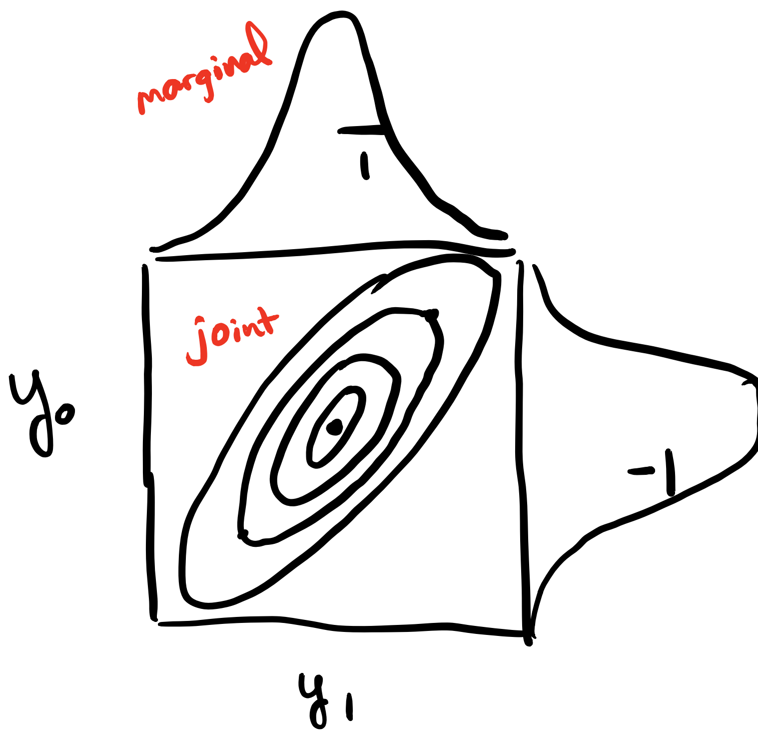


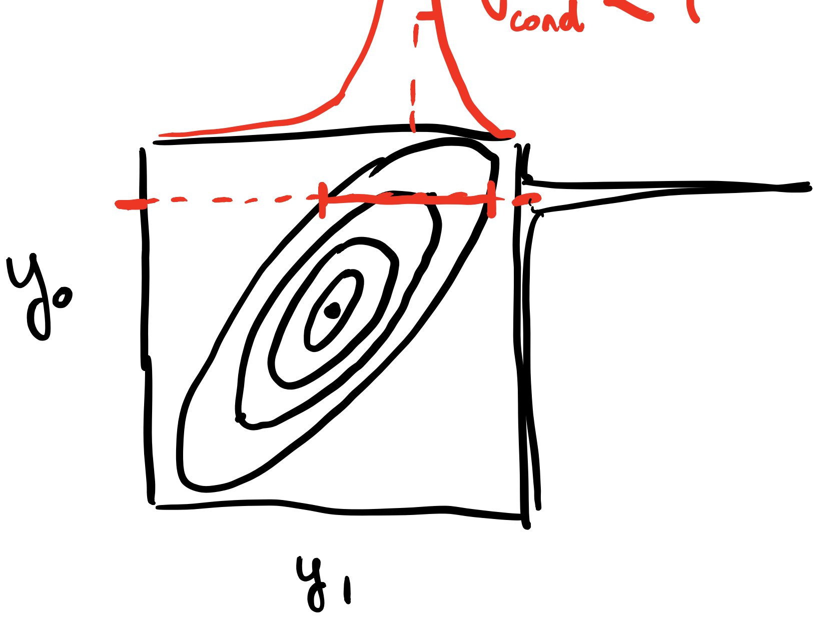

conditional

$p(y_1 | y_0)$



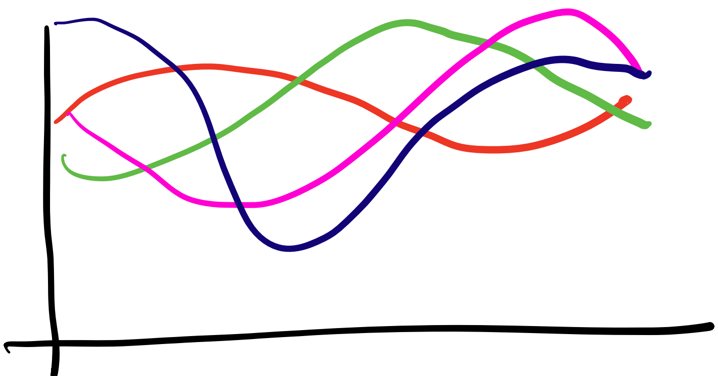

$$\Sigma = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$




**EXERCISE 2**

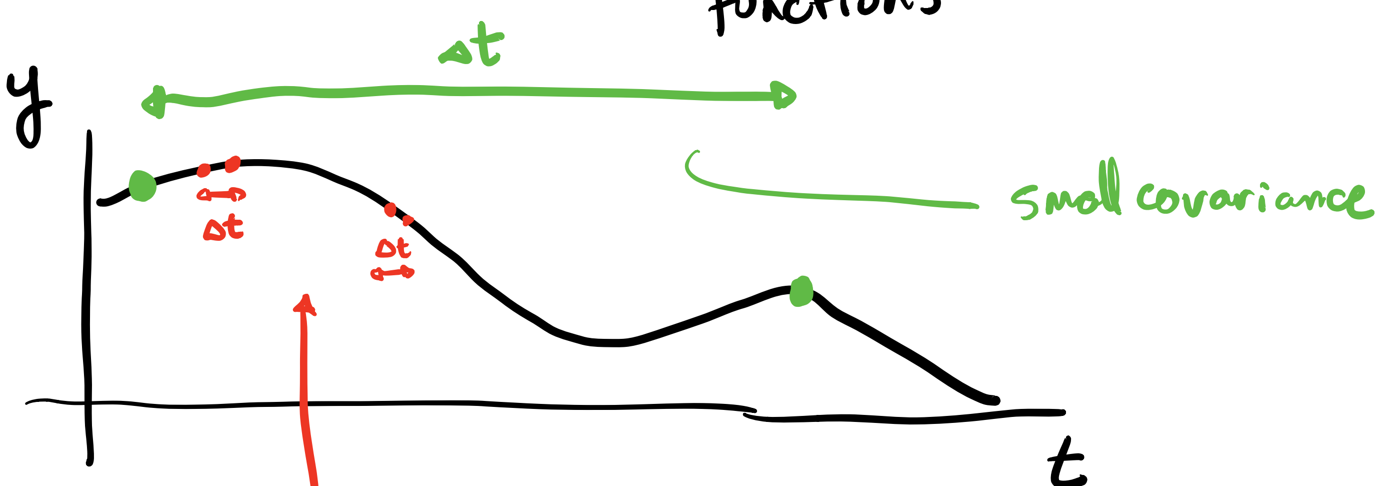
## BACK TO GPs



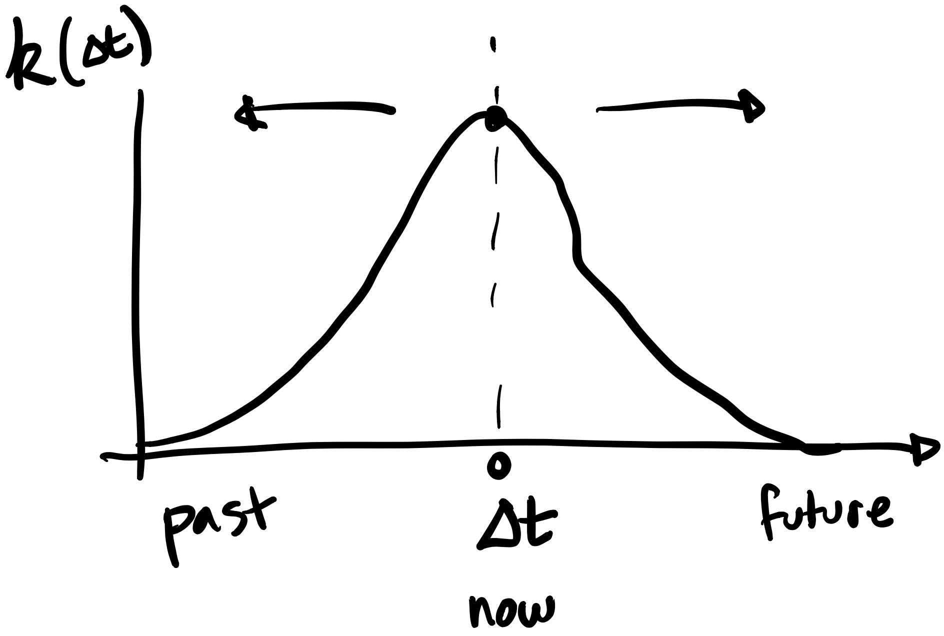
GP = GAUSS DISTR OVER FUNCTIONS

$$\underline{y} \sim GP(m, \boxed{k}(\cdot,\cdot))$$

functions



large covariance




$$k(t_i, t_j) = A^2 e^{-\frac{1}{2}\left(\frac{t_i - t_j}{\ell}\right)^2}$$

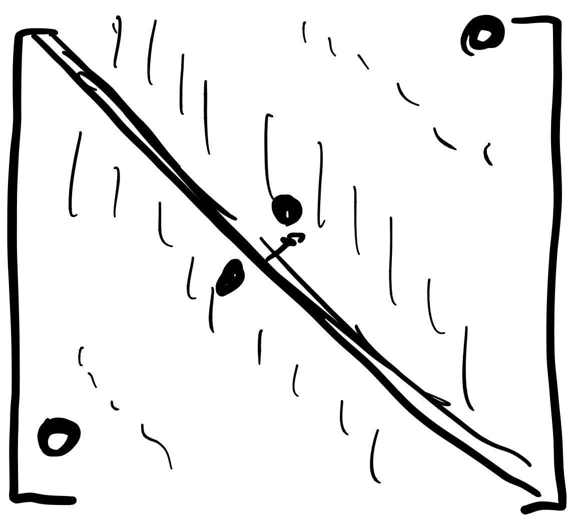
amplitude

lengthscale

SQUARED EXPONENTIAL KERNELS

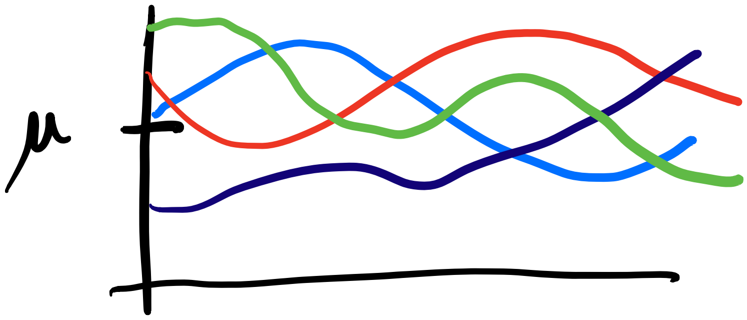
$\boxed{\text{EXERCISES } 3, 4}$

$\Sigma =$

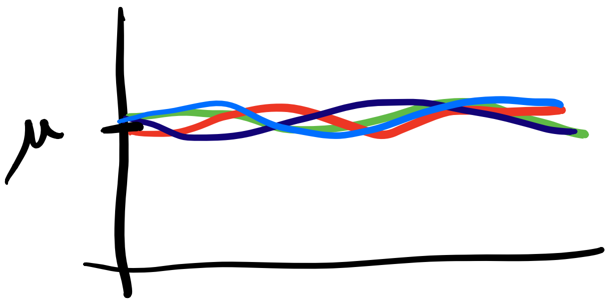


## EXAMPLE

$$\underline{y} \sim \mathcal{N}\left(\underline{\mu},\ \underline{\underline{\Sigma}}(A, \ell)\right)$$

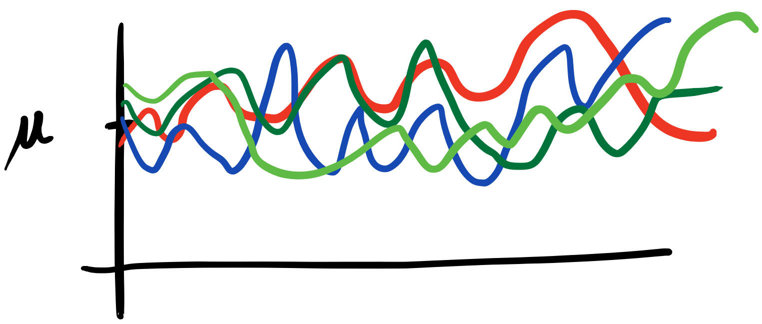



$A_0,\ \ell_0$



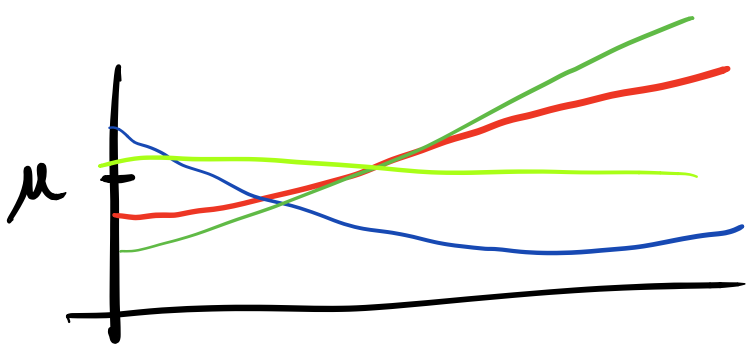

$A \ll A_0$

$\ell = \ell_0$




$A = A_0$

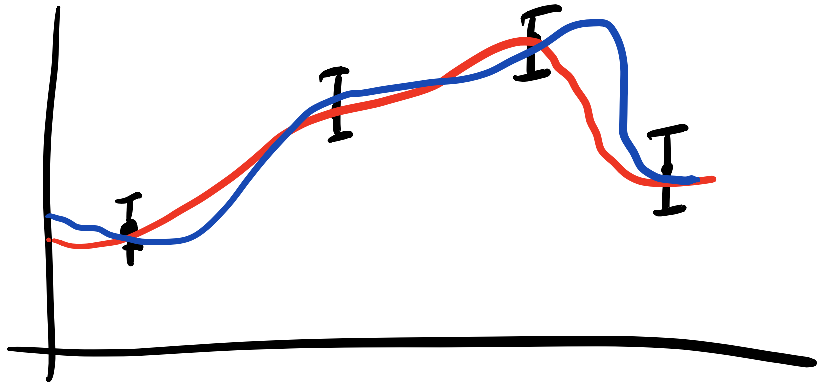
$\ell \ll \ell_0$



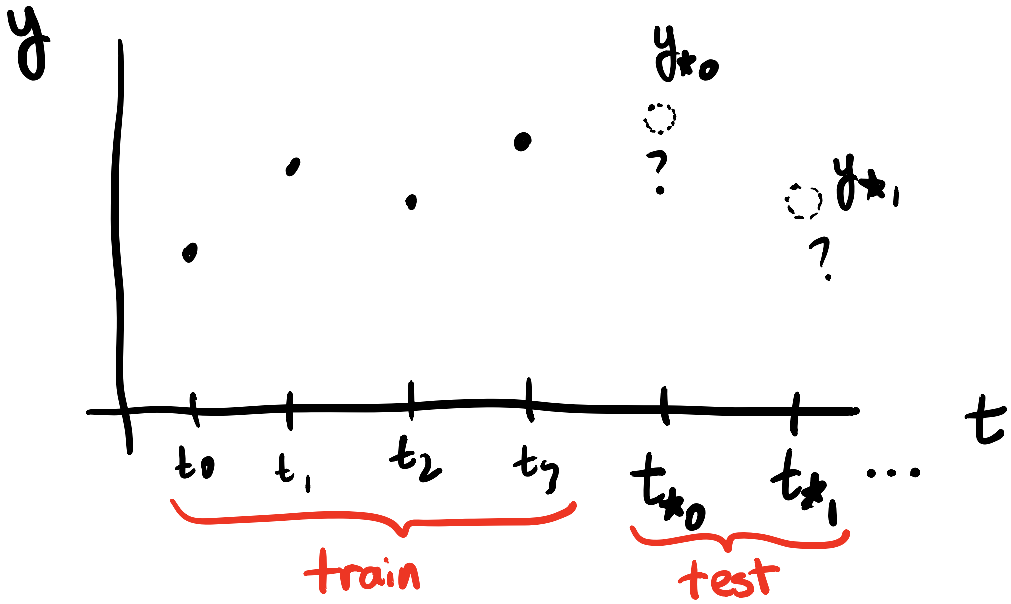

$A = A_0$

$\ell \gg \ell_0$

## EXERCISE 5



REJECTION SAMPLING




EXERCISES 6,7,8

$$y_* \sim \mathcal{N}(\mu', \Sigma')$$

$$\mu' = \Sigma(t_*, t)\,\Sigma(t, t)^{-1} y$$

$$\Sigma' = \Sigma(t_*, t_*) - \Sigma(t_*, t)\,\Sigma(t, t)^{-1}\,\Sigma(t, t_*)$$

# TUNING THE GP

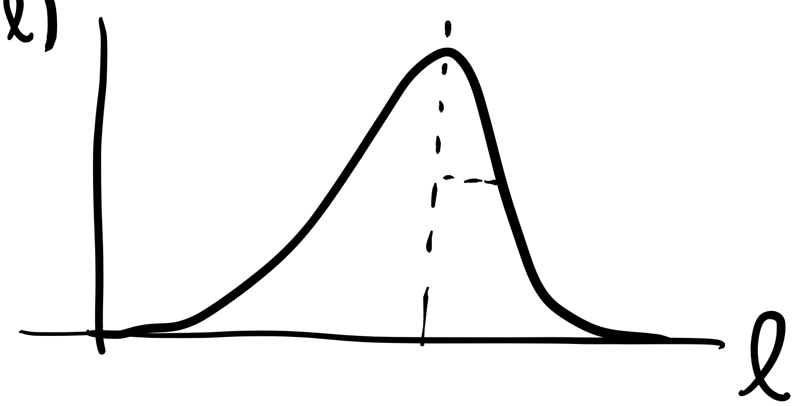
EXERCISE 9

$$p(y|\mu,\Sigma) = \frac{1}{\sqrt{(2\pi)^N |\underline{\underline{\Sigma}}|}} e^{-\frac{1}{2}(\underline{y}-\underline{\mu})^T \underline{\underline{\Sigma}}^{-1} (\underline{y}-\underline{\mu})}$$

$\Sigma = \Sigma(A, \ell)$

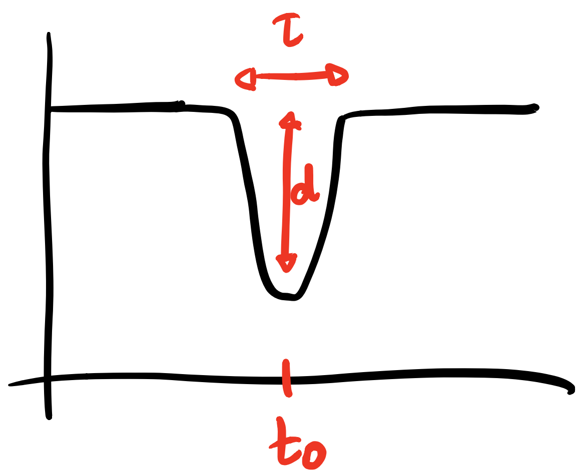
$p(y \,|\, \mu, A, \ell)$

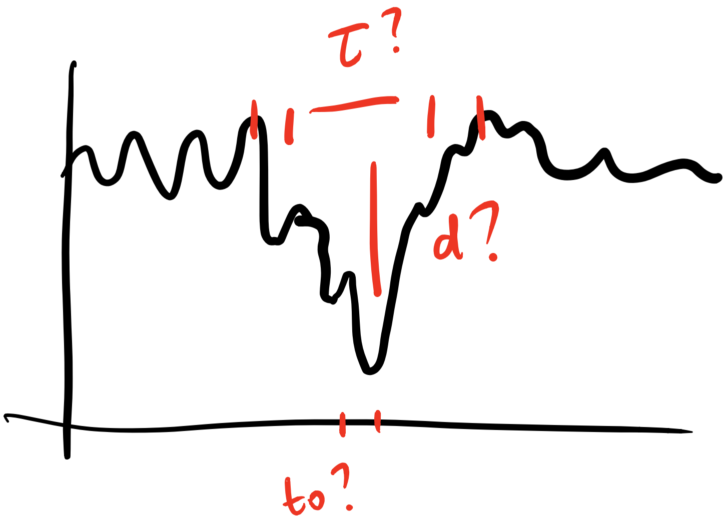
$p(y|\ell)$



# Inference

**Exercise 10**




$$p(y \mid \underbrace{d, \tau, t_0}_{\text{model parameters}}, \underbrace{A, \ell}_{\text{GP parameters}})$$

$$= \frac{1}{\sqrt{(2\pi)^N |\Sigma|}} e^{-\frac{1}{2}(y-\mu)^T \Sigma^{-1} (y-\mu)}$$

$$\Sigma = \Sigma(A, \ell) \leftarrow k$$

$$\mu = \mu(d, \tau, t_0)$$

$\boxed{\text{MCMC}}$

$\boxed{\text{emcee}}$

$p(d \mid y)$

$p(\tau \mid y)$

...

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# BAYES' THEOREM

$$\underbrace{p(\text{model} \mid \text{data})}_{\text{posterior}} \propto \underbrace{p(\text{data} \mid \text{model})}_{\substack{\text{likelihood} \\ \text{(GP)}}} \; \underbrace{p(\text{model})}_{\text{prior}}$$