Conversions Between Bases

From Base A to Base B

Conversion Between Bases

- Types of Conversions
  - Base-B to Decimal
  - Decimal to Base-B
  - Base-B to Base-C

Base-B to Decimal

There are two algorithms we will use to convert between bases:

1. Sum Expanded Representation
2. Horner’s Rule

Base-B to Decimal

SUM EXPANDED REPRESENTATION

Each number is represented as a sequence of digits from left to right. Starting with the largest positional value on the left and the smallest value on the right.

Each digit in the string has an associated *weight* that increases by value as you move to the left.

Base-B to Decimal

SUM EXPANDED REPRESENTATION

Example expansion:

- $7829_{10} = 7000 + 800 + 20 + 9$
- $= (7*1000) + (8*100) + (2*10) + (9*1)$
- $= (7*10^3) + (8*10^2) + (2*10^1) + (9*10^0)$

Sum Expansion Representation

- Numbers represented as a sequence of digits
  - Most significant to the left, least significant to the right
- Each digit in the sequence has a weight
  - Weight increases by multiple of B as you move left, starting with B^0
  - Weight of rightmost digit is one (1)

Sum Expansion Representation

- Example Expansion:
  - $7829_{10} = 7000_{10} + 800_{10} + 20_{10} + 9_{10}$
  - $= (7*1000_{10}) + (8*100_{10}) + (2*10_{10}) + (9*1_{10})$
  - $= (7*10^3) + (8*10^2) + (2*10^1) + (9*10^0)$
- Example Expansion 2:
  - $1010_2 = 1000_2 + 0_2 + 10_2 + 0_2$
  - $= (1*1000_2) + (0*100_2) + (1*10_2) + (0*1_2)$
  - $= (1*2^3) + (0*2^2) + (1*2^1) + (0*2^0)$

Sum Expansion Representation

- Value of the number equals the sum of all the values
- Expanding a number in any base in this way and working out the calculation will result in a base-10 number
- Any number in any base B can be expressed in this method

Sum Expansion Representation

- A number $d_n d_{n-1} \ldots d_1 d_0$ in base B can be expressed using sum expanded notation
  - Each d_n will have a value such that $0 <= d_n <= (B-1)$
  - The exponent to expand with is B^n
  - Expansion is $d_n * B^n + d_{n-1} * B^{n-1} + \ldots + d_1 * B^1 + d_0 * B^0$

Base-B to Decimal

Example: Lets convert **11011_2** to **Base-10** using the sum expanded method

Base: 2 (Binary)

POSITION	4	3	2	1	0
DIGIT	1	1	0	1	1

$$11011_2 = (1*2^4) + (1*2^3) + (0*2^2) + (1*2^1) + (1*2^0)$$
$$= (1*16) + (1*8) + (0*4) + (1*2) + (1*1)$$
$$= 16 + 8 + 0 + 2 + 1$$
$$= 27$$

Answer: 27_{10}

Base-B to Decimal

Example: Lets convert **$A5_{16}$** to **Base-10** using the sum expanded method

Base: 16 (Hex)

POSITION	1	0
DIGIT	A	5

$$
\begin{aligned}
A5_{16} &= (A*16^1) + (5*16^0) \\
&= (10*16) + (5*1) \\
&= 160 + 5 \\
&= 165
\end{aligned}
$$

Answer: 165_{10}

Base-B to Decimal

Your turn to do a few conversions to decimal using sum expanded representation

1. 1001_2
2. 423_5
3. $A2J_{20}$

Base-B to Decimal

Example: Lets convert 1001_2 to **Base-10** using the sum expanded method

Base: 2 (Binary)

POSITION	3	2	1	0
DIGIT	1	0	0	1

$$1001_2 = (1*2^3) + (0*2^2) + 0*2^1 + 1*2^0$$
$$= (1*8) + (1*1)$$
$$= 8 + 1$$
$$= 9$$

Answer: 9_{10}

Base-B to Decimal

Example: Lets convert 423_5 to **Base-10** using the sum expanded method

Base: 5

POSITION	2	1	0
DIGIT	4	2	3

$$423_5 = (4*5^2) + (2*5^1) + 3*5^0$$
$$= (4*25) + (2*5) + (3*1)$$
$$= 100 + 10 + 3$$
$$= 113$$

Answer: 113_{10}

Base-B to Decimal

Example: Lets convert $A2J_{20}$ to **Base-10** using the sum expanded method

Base: 20

POSITION	2	1	0
DIGIT	A	2	J

$$
\begin{aligned}
A2J_{20} &= (A*20^2) + (2*20^1) + J*20^0 \\
&= (10*400) + (2*20) + (19*1) \\
&= 4000 + 40 + 19 \\
&= 4059
\end{aligned}
$$

Answer: 4059_{10}

Base-B to Decimal - Horner's Rule

- In math: A polynomial
  - $A(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + ... +a_nx^n$
- Can be written as
  - $A(x) = a_0 + x (a_1 + x (a_2 + x (a_3 + ... + x(a_{n-1} + x a_n) ...)))$
- See http://everything2.com/title/Horner%2527s+rule for more information

Base-B to Decimal - Horner's Rule

Example: $A(x) = a_0 + a_1x + a_2x^2 + a_3x^3$

$= a_0 + x(a_1 + a_2x + a_3x^2)$

$= a_0 + x(a_1 + x(a_2 + a_3x))$

$= a_0 + x(a_1 + x(a_2 + x(a_3)))$

Example: $A(x) = 5 + 7x - 2x^2s$

$= 5 + x(7 - 2x)$

$= 5 + x(7 - x(2))$

Base-B to Decimal - Horner's Rule

Simplified Algorithm

Set sum = 0_{10}

For all digits in the number starting from the left and moving to the right do the following:

Sum = (Sum * B) + digit value

Where B is the base of the number

Sum at the end will be your base-10 converted number

Base-B to Decimal - Horner's Rule

Example: Convert $AF452_{16}$ to Base-10

Base = 16 (hexadecimal)

Sum = 0

Steps: *Sum = Sum * Base + digit value*

1. $AF452_{16}$ => Sum = Sum * Base + A = 0 * 16 + 10 = 10
2. $AF452_{16}$ => Sum = Sum * Base + F = 10 * 16 + 15 = 175
3. $AF452_{16}$ => Sum = Sum * Base + 4 = 175 * 16 + 4 = 2804
4. $AF452_{16}$ => Sum = Sum * Base + 5 = 2804 * 16 + 5 = 44869
5. $AF452_{16}$ => Sum = Sum * Base + 2 = 44869 * 16 + 2 = 717906

Answer = 717906_{10}

Base-B to Decimal - Horner's Rule

Your turn to do a few conversions to Decimal

1011_2

433_5

$A2B_{16}$

Simplified Algorithm
Set sum = 0_{10}
For all digits in the number starting from the left and moving to the right do the following
    Sum = (Sum * Base) + digit value.

Sum at the end will be your base-10 converted number

Base-B to Decimal - Horner's Rule

Example: Convert 1011_2 to Base-10

Base = 2 (binary)

Sum = 0

Steps: *Sum = Sum * Base + digit value*

1. 1011_2 => Sum = Sum * Base + 1 = 0 * 2 + 1 = 1
2. 1011_2 => Sum = Sum * Base + 0 = 1 * 2 + 0 = 2
3. 1011_2 => Sum = Sum * Base + 1 = 2 * 2 + 1 = 5
4. 1011_2 => Sum = Sum * Base + 1 = 5 * 2 + 1 = 11

Answer = 11_{10}

Base-B to Decimal - Horner's Rule

Example: Convert 433_5 to Base-10

Base = 5

Sum = 0

Steps: ***Sum = Sum * Base + digit value***

1. 433_5 => Sum = Sum * Base + 4 = 0 * 5 + 4 = 4

2. 433_5 => Sum = Sum * Base + 3 = 4 * 5 + 3 = 23

3. 433_5 => Sum = Sum * Base + 3 = 23 * 5 + 3 = 118

Answer = 118_{10}

Base-B to Decimal - Horner's Rule

Example: Convert $A2B_{16}$ to Base-10

Base = 16 (hexadecimal)

Sum = 0

Steps: *Sum = Sum * Base + digit value*

1. $A2B_{16}$ => Sum = Sum * Base + A = 0 * 16 + 10 = 10
2. $A2B_{16}$ => Sum = Sum * Base + 2 = 10 * 16 + 2 = 162
3. $A2B_{16}$ => Sum = Sum * Base + B = 162 * 16 + 11 = 2603

Answer = 2603_{10}

Decimal to Base-B

There are two algorithms we will use to convert between bases:

1. Decomposition (Quick) method
2. Repeated division by B

Decimal to Base-B

The Decomposition (Quick) Method - N_{10} to base-B

1) List all powers of base B until you find an exponent x where $B^x > N$, first power used will be $y = x-1$

2) Determine the factor A such that $A*B^y < N$,
where $0 <= A < B$

3) Set $N = N - A*B^y$
Set $y = y-1$

4) Repeat steps 2) and 3) while $N > 0$

Decimal to Base-B

The quick method when doing work on paper

Example: Convert 14_{10} to base-2 (Step 1)

2^0
1

$2^0 = 1 < 14_{10}$ Keep Going

Decimal to Base-B

The quick method when doing work on paper

Example: Convert 14_{10} to base-2 (Step 2)

	2^1	2^0
	2	1

$2^1 = 2 < 14_{10}$ Keep Going

Decimal to Base-B

The quick method when doing work on paper

Example: Convert 14_{10} to base-2 (Step 3)

	2^2	2^1	2^0
	4	2	1

$2^2 = 4 < 14_{10}$ Keep Going

Decimal to Base-B

The quick method when doing work on paper

Example: Convert 14_{10} to base-2 (Step 4)

	2^3	2^2	2^1	2^0
	8	4	2	1

$2^3 = 8 < 14_{10}$ Keep Going

Decimal to Base-B

The quick method when doing work on paper

Example: Convert 14_{10} to base-2 (Step 5)

2^4	2^3	2^2	2^1	2^0
16	8	4	2	1

$2^4 = 16 > 14_{10}$ Stop

Decimal to Base-B

The quick method when doing work on paper

Example: Convert 14_{10} to base-2 (Step 6)

2^4	2^3	2^2	2^1	2^0
16	8	4	2	1
	1			

N: 14_{10}

8 goes into 14 once

N = 14 – (1*8)

N = 14 – 8 = 6

Decimal to Base-B

The quick method when doing work on paper

Example: Convert 14_{10} to base-2 (Step 7)

2^4	2^3	2^2	2^1	2^0
16	8	4	2	1
	1	1		

N: 6_{10}

4 goes into 6 once

N = 6 – (1*4)

N = 6 – 4 = 2

Decimal to Base-B

The quick method when doing work on paper

Example: Convert 14_{10} to base-2 (Step 8)

2^4	2^3	2^2	2^1	2^0
16	8	4	2	1
	1	1	1	

N: 2_{10}

2 goes into 2 once

N = 2 – (1*2)

N = 2 – 2 = 0

Decimal to Base-B

The quick method when doing work on paper

Example: Convert 14_{10} to base-2 (Step 9)

N: 0_{10}

All remaining digits are now set to zero

2^4	2^3	2^2	2^1	2^0
16	8	4	2	1
	1	1	1	0

Decimal to Base-B

The quick method when doing work on paper

Example: Convert 592_{10} to base-2

2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1024	512	256	128	64	32	16	8	4	2	1
Binary number	1	0	0	1	0	1	0	0	0	0
Reduced # to Convert	80	80	80	16	16	0	0	0	0	0

Decimal to Base-B

The quick method when doing work on paper

Example: Convert 45_{10} to base-4

4^3	4^2	4^1	4^0
64	16	4	1
Binary Number	2	3	1
Reduced # to Convert	13	1	0

Decimal to Base-B

Your turn to do a few conversations

61_{10} to base-2

14_{10} to base-3

Algorithm:

1) **List all powers of base B until you find an exponent*where B^x > N, first power used will be y = x-1**
2) **Determine the factor A such that $A*B^y$ < N, where 0 <= A < B**
3) **Set N = N - $A*B^y$**
   Set y = y-1
4) **Repeat steps 2) and 3) while N > 0**

Decimal to Base-B

The quick method when doing work on paper

Example: Convert 61_{10} to base-2

2^6	2^5	2^4	2^3	2^2	2^1	2^0
64	32	16	8	4	2	1
Binary number	1	1	1	1	0	1
Reduced # to Convert	29	13	5	1	1	0

Decimal to Base-B

The quick method when doing work on paper

Example: Convert 14_{10} to base-3

3^3	3^2	3^1	3^0
27	9	3	1
Base-3 Number	1	1	2
Reduced # to Convert	5	2	0

Decimal to Base-B

Repeated Division Method

- Repeatedly divide the base-10 number by the base you want to convert to (B) until the quotient is 0
  - Using integer division
- Remainders are kept after each division step
- Reversed sequence of remainders is the new number in Base-B

Repeated Division Algorithm

N = the base-10 number

B = the base we want to convert to

R = the final result, starts empty

while(N > 0)

Add (N % B) to the beginning of R, where % is the modulus operator that gives back the remainder of dividing N by B

N = N/B, using integer division (drop the decimal places)

R contains the result in base B

NOTE: R is the N%B read from last calculated to first calculated

Decimal to Base-B

Repeated Division Method (Step 0)

Example: Convert 57_{10} to Binary (Base-2)

Step	N	(N % B)	(N / B)	R
0	57			

Decimal to Base-B

Repeated Division Method (Step 1)

Example: Convert 57_{10} to Binary (Base-2)

Step	N	(N % B)	(N / B)	R
0	57			
1	57	(57%2) = 1	(57/2)=28.5=28	1

Decimal to Base-B

Repeated Division Method (Step 2)

Example: Convert 57_{10} to Binary (Base-2)

Step	N	(N % B)	(N / B)	R
0	57			
1	57	(57%2) = 1	(57/2)=28.5=28	1
2	28	(28%2) = 0	(28/2)=14	01

Decimal to Base-B

Repeated Division Method (Step 3)

Example: Convert 57_{10} to Binary (Base-2)

Step	N	(N % B)	(N / B)	R
0	57			
1	57	(57%2) = 1	(57/2)=28.5=28	1
2	28	(28%2) = 0	(28/2)=14	01
3	14	(14%2) = 0	(14/2)=7	001

Decimal to Base-B

Repeated Division Method (Step 4)

Example: Convert 57_{10} to Binary (Base-2)

Step	N	(N % B)	(N / B)	R
0	57			
1	57	(57%2) = 1	(57/2)=28.5=28	1
2	28	(28%2) = 0	(28/2)=14	01
3	14	(14%2) = 0	(14/2)=7	001
4	7	(7%2) = 1	(7/2) = 3.5 = 3	1001

Decimal to Base-B

Repeated Division Method (Step 5)

Example: Convert 57_{10} to Binary (Base-2)

Step	N	(N % B)	(N / B)	R
0	57			
1	57	(57%2) = 1	(57/2)=28.5=28	1
2	28	(28%2) = 0	(28/2)=14	01
3	14	(14%2) = 0	(14/2)=7	001
4	7	(7%2) = 1	(7/2) = 3.5 = 3	1001
5	3	(3%2) = 1	(3/2) = 1.5 = 1	11001

Decimal to Base-B

Repeated Division Method (Step 6)

Example: Convert 57_{10} to Binary (Base-2)

Step	N	(N % B)	(N / B)	R
0	57			
1	57	(57%2) = 1	(57/2)=28.5=28	1
2	28	(28%2) = 0	(28/2)=14	01
3	14	(14%2) = 0	(14/2)=7	001
4	7	(7%2) = 1	(7/2) = 3.5 = 3	1001
5	3	(3%2) = 1	(3/2) = 1.5 = 1	11001
6	1	(1%2) = 1	(1/2) = 0.5 = 0	111001

Decimal to Base-B

Repeated Division Method (Step 7)

Example: Convert 57_{10} to Binary (Base-2)

Step	N	(N % B)	(N / B)	R
0	57			
1	57	(57%2) = 1	(57/2)=28.5=28	1
2	28	(28%2) = 0	(28/2)=14	01
3	14	(14%2) = 0	(14/2)=7	001
4	7	(7%2) = 1	(7/2) = 3.5 = 3	1001
5	3	(3%2) = 1	(3/2) = 1.5 = 1	11001
6	1	(1%2) = 1	(1/2) = 0.5 = 0	111001
7	0			

Note: that the value in R is the values in the remainder column (N%B) written from the bottom towards the top.

Decimal to Base-B

Repeated Division Method (Step 0)

Example: Convert 124_{10} to Base-7

Step	N	(N % B)	(N / B)	R
0	124			

Decimal to Base-B

Repeated Division Method (Step 1)

Example: Convert 124_{10} to Base-7

Step	N	(N % B)	(N / B)	R
0	124			
1	124	(124%7) = 5	(124/7)=17.7=17	5

Decimal to Base-B

Repeated Division Method (Step 2)

Example: Convert 124_{10} to Base-7

Step	N	(N % B)	(N / B)	R
0	124			
1	124	(124%7) = 5	(124/7)=17.7=17	5
2	17	(17%7) = 3	(17/7)=2.4=2	35

Decimal to Base-B

Repeated Division Method (Step 3)

Example: Convert 124_{10} to Base-7

Step	N	(N % B)	(N / B)	R
0	124			
1	124	(124%7) = 5	(124/7)=17.7=17	5
2	17	(17%7) = 3	(17/7)=2.4=2	35
3	2	(2%7) = 2	(2/7)=0	235

Decimal to Base-B

Repeated Division Method (Step 4)

Example: Convert 124_{10} to Base-7

Step	N	(N % B)	(N / B)	R
0	124			
1	124	(124%7) = 5	(124/7)=17.7=17	5
2	17	(17%7) = 3	(17/7)=2.4=2	35
3	2	(2%7) = 2	(2/7)=0	235
4	0			

Decimal to Base-B

Your turn to do a few conversions

13_{10} to base-2

23_{10} to base-3

Algorithm:

- N = the base-10 number
- B = the base we want to convert to
- R = the final result, starts empty
- While(N > 0)
  - Add (N % B) to the beginning of R, where % is the modulus operator that gives back the remainder of dividing N by B
  - N = N/B, using integer division
- R contains the result in base B

Base-B to Base-C

There are three algorithms we will use to convert between bases:

1. Base-B to Decimal then Decimal to Base-C
   a. Combines the two previous conversions
2. Related base conversions
   a. Can only be done under specific criteria
3. Convert directly from Base-B to Base-C
   a. requires doing math in Base-C or Base-B, tends to be difficult

Base-B to Decimal to Base-C

1. **Convert base-B to base-10**
    - Here you can use the expanded sums method or the method developed using Horner’s rule
2. **Now convert the base-10 number to base-C**
    - Here now you can use the repeated division method covered in the last section

Base-B to Base-C

Example:

Let's convert 111001_2 to a base 5 number.

(step 1) Use sum expanded representation

$$111001_2 = (1*2^5)+(1*2^4)+(1*2^3)+(0*2^2)+(0*2^1)+ (1*2^0)$$

$$= (1*32)+(1*16)+(1*8)+(0*4)+(0*2)+ (1*1)$$

$$= 32 + 16 + 8 + 0 + 0 + 1$$

$$= 32 + 16 + 8 + 1$$

$$= 57_{10}$$

Base-B to Base-C

Example: Let's convert 111001_2 to a base 5 number.

(step 2) Now convert 57_{10} to a base-5 number

(use the repeated division by B algorithm)

N = 57 , B = 5, R =

Step	N	(N % B)	(N / B)	R
0	57			
1	57	(57%5) = 2	(57/5) = 11	2
2	11	(11%5) = 1	(11/5) = 2	12
3	2	(2%5) = 2	(2/5) = 0	212
4	0			

Base-B to Base-C

Your turn to do a few conversions

13_5 to base-3

11001_2 to base-5

5^0	5^1	5^2	5^3
1	5	25	125

3^0	3^1	3^2	3^3
1	3	9	27

2^0	2^1	2^2	2^3	2^4	2^5	2^6
1	2	4	8	16	32	64

Base-B to Base-C

Your turn to do a few conversions

13_5 to base-3 $= 22_3$

11001_2 to base-5 $= 100_5$

5^0	5^1	5^2	5^3
1	5	25	125

3^0	3^1	3^2	3^3
1	3	9	27

2^0	2^1	2^2	2^3	2^4	2^5	2^6
1	2	4	8	16	32	64

Base-B to Base-C

Related bases: a larger base that is a power of a smaller base.

Ex. **$16 = 2^4$**

Note: Each hex digit corresponds to a 4 digit binary number, and vice versa.

Therefore, <u>hex can be used as a binary shorthand</u>. A byte (8-bit number) can be represented by a 2-digit hex number.

Base-B to Base-C

E.g. convert 10110011_2 to hex.

$\underline{1011}\ \underline{0011}$ ← break into 4-bit chunks starting from the right end

convert each 4-bit chunk to a base-16 digit

B 3

i.e. $10110011_2 = B3_{16}$

Base-B to Base-C

Note: Each octal digit corresponds to a 3 digit binary number.

$$\mathbf{8 = 2^3}$$

E.g. convert 10110011_2 to octal.

$\underline{010}$ $\underline{110}$ $\underline{011}$ ← break into 3-bit chunks, left padding the number with zeros (0) as needed, starting from right convert each 3-bit chunk to a base-8 digit

2 6 3

i.e. $10110011_2 = 263_8$

Base-B to Base-C

E.g. convert 17_{16} (hex) to octal.

$16 = 2^4$ and $8 = 2^3$

Both are powers of 2 (binary), convert hex into binary and organize into groups of 4 bits, then separate them into groups of 3 binary bits to convert to octal

1 7

0001 0111

000 010 111

0 2 7

Therefore $17_{16} = 27_8$

Base-B to Base-C

Your turn to do a few conversations

52_8 to base-16 (Hex)

$A2F_{16}$ to base-8 (Octal)

2310_4 to base-16 (Hex)

2110_3 to base-9

Base-B to Base-C

Your turn to do a few conversations

52_8	to base-16	$= 2A_{16}$
$A2F_{16}$	to base-8	$= 5057_8$
2310_4	to base-16	$= B4_{16}$
2110_3	to base-9	$= 73_9$