Generalized Compliments

N’s & N-1’s Comp Base-B

Binary Digits are Complimentary

- The digits in binary are complimentary
- Add the two digits together and you get largest digit as the sum

+ve digits	-ve digits	sum
0	1	1

Compliments in Other Bases

- Hex is shorthand for binary so
  - 2’s complement numbers can be expressed in hex
- Example:

$00000010_2 = 02_{16} = 2_{10}$ ***note: 00-7F$_{16}$ are +ve***

$00000001_2 = 01_{16} = 1_{10}$

$00000000_2 = 00_{16} = 0_{10}$

$11111111_2 = FF_{16} = -1_{10}$

$11111110_2 = FE_{16} = -2_{10}$ ***note: 80-FF$_{16}$ are –ve***

Compliments in Other Bases

Realize that the digits are complements

NOTE: In hex, there are 16 digits, and that for the leading digit value, half will indicate a +ve, and half will indicate a -ve number

+ve digits	-ve digits	sum
0	F	F
1	E	F
2	D	F
3	C	F
4	B	F
5	A	F
6	9	F
7	8	F

Compliments in Other Bases

Are the following 2's Comp binary numbers, expressed in hex, positive or negative?

- $801F_{16}$
  - Negative because MS digit is 8_{16} (1000_2), which is >= 8
- $6FFF_{16}$
  - Positive because MS digit is 6_{16} (0110_2), which is < 8

Compliments in Other Bases

Convert $801F_{16}$ 2's complement to decimal

1. Leading digit of 8 indicates -ve value ← must justify
   therefore apply $NEG_{2's\ Comp}$ → flip the digits and add 1

```
801F    using table from 2 slides previously
||||
7FE0 + 1 = 7FE1
```

2. 7FE1 converted to decimal = ((7 * 16 + 15) * 16 + 14) * 16 + 1 = 32737

3. 801F is negative therefore

```
result = -1 * 32737 = -32737
```

Devpac does not do signed conversion, so this skill **will be** required

Generalized Complements

- In fixed length binary there exists:
  - 1’s complement
  - 2’s complement
- In fixed length base n there exists:
  - n-1’s complement flip digits
  - n’s complement flip digits, then add 1
- Example: In hex we have:
  - 15’s complement hex **and** 16’s complement hex

Generalized Complements

- Even bases have same number of positive/negative indicators
  - base 2 2 digits / 2 = 1 (1 digit positive, 1 digit negative)
  - base 16 16 digits / 2= 8 (8 digits positive, 8 digits negative)
- Odd bases indicators do not split evenly
  - base 5 5 digits / 2 = 2.5
  - one leading symbol is both positive and negative
  - in base 5 that symbol is 2

+ve digits	-ve digits	sum
0	4	4
1	3	4
2	2	4

Generalized Complements - Example

Convert 3211 in 4-digit 5's comp base 5 number to 4-digit 7's comp base 8

+ve or -ve?

-ve since MS digit is a 3 → 4, 3 and part of 2 are -ve

since n's comp base n → flip the digits then add 1

then convert to base 8 through decimal

since original is -ve need to make result -ve, but represented in n-1's comp base n, so flip the digits (draw digit table if necessary)

Generalized Complements - Example

Since -ve 5's comp flip digits and add 1

```
3211
||||
1233 + 1 = 1234
```

Convert to decimal:

$$1*125 + 2*25 + 3*5 + 4*1$$
$$= 125 + 50 + 15 + 4 = 194_{10}$$

+ve digits	-ve digits	sum
0	4	4
1	3	4
2	2	4

Generalized Complements - Example

Convert 194_{10} to base 8

```
8   |   194     R
          24    2
           3    0
           0    3
```

$194_{10} = 302_8 \rightarrow 0302_8$ (it is 4-digit)

+ve digits	-ve digits	sum
0	7	7
1	6	7
2	5	7
3	4	7

Original was -ve so to make 7's comp base 8, flip the digits:

`0302 → 7475`

$\therefore 3211_5$ 5's comp = 7475_8 7's comp

Find Last Positive in Odd Bases

- In odd numbered bases one digit is both positive and negative
- If a number starts with that digit is it +ve or -ve
- One method is to do this via range
- For 3-digit base 5 5's comp (remember fixed length)
  - Number of values = $5^3 = 125$
    but one zero → 125 - 1 = 124 total non-zero values
  - 124 / 2 = 62 since half +ve & half -ve

Find Last Positive in Odd Bases

- Convert 62_{10} to base 5

N	N/5	N%5
62	12	2
12	2	2
2	0	2
0		

$= 222_5$

∴ the values 000 to 222 are +ve and 223 to 444 are -ve