One's Complement Notation

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Odometer Numbers

- Completely unrelated to signed magnitude notation
- Look at a 3 digit odometer
 - smallest digit is 000 and largest digit is 999
 - 1000 different values represented
 - moving forward from 000 gives 001, 002, 003
 - moving backward from 000 gives 999, 998, 997
 - can use these as negatives

- Use the odometer idea, and split the unsigned range
 - In half by convention
- Advancing from 0 gives positive
- Reversing from 0 gives negative
- Same overall number of values able to represented
- Same range as Signed Magnitude
 - Assignment of bit patterns to negative numbers is different
- Next slide shows bit pattern assignment for 4-bit One's Complement Binary

Assume a 4-bit fixed length

Decimal	One's Complement	Hex
Number	Representation	
7	0111	7
6	0110	6
5	0101	5
4	0100	4
3	0011	3
2	0010	2
1	0001	1
0	0000	0
-0	1111	F
-1	1110	Е
-2	1101	D
-3	1100	С
-4	1011	В
-5	1010	Α
-6	1001	9
-7	1000	8

Notice:

- All positives start with a zero (0), all negatives start with a (1)
- Positives same as regular binary, but only uses n-1 bits
 - Bit n-1 (MSBit) must be zero for the value to be valid
 - \circ ex. 13₁₀ = 1101₂ bit 3 is a 1 therefore 13 not a valid 4-bit number

- One's Complement no longer sets the MSBit to indicate the sign
- Use the NEG_{rep} of the positive number to represent the negative number
- This changes the MSBit from an explicit sign bit, to an implicit sign bit
- An MSBit of 0 is still positive
- An MSBit of 1 is still negative
- While the MSBit indicates a sign it is not a sign bit
 - It is part of the number

Add any positive and negative 4-bit value (e.g. 5 and -5)

```
0101
+ <u>1010</u>
1111 (maximum 4-bit value)
```

- positive n-bit + negative n-bit = maximum n-bit value
 - \circ NEG_{1's comp}(N) = max value N or
 - \circ NEG_{1's comp}(N) = flip each bit

Decimal to One's Complement Conversion

Convert Decimal to N-bit One's Complement Binary

- 1. result = abs(num) converted to binary
- IF MSBit (result) != 0 OR length(result) > N bits THEN
 - "num cannot be represented in N bits"
- 3. **ELSE**
 - IF num < 0 THENresult = NEG_{1's Comp}(result)

NEG_{rep} is the negation operation **NOT** make negative Thus, the NEG operation can take **ANY** value

Convert 115₁₀ to 8-bit one's complement binary, expressed in hex

- 1. abs(115) = 115
- 2. 115₁₀ converted to 8-bit binary is 01110011₂
- 3. Since the bit 7 is 0 (and no bit 8 or more was required) 115_{10} is valid
- 4. Since the number is positive nothing further is done
- 5. In hex => 73_{16}

Convert -67₁₀ to 8-bit one's complement binary, expressed in hex

- 1. abs(-67) = 67
- 2. 67₁₀ converted to 8-bit binary is 01000011₂
- 3. Since bit 7 is 0 (and no bit 8 or more was required) -67₁₀ is valid
- 4. Since the number is negative apply $NEG_{1's comp}(01000011_2) = 101111100_2$
- 5. In hex => BC_{16}

One's Complement to Decimal Conversion

Convert One's Complement Binary to Decimal

```
    IF MSBit(binary) == 1 THEN
        temp = NEG<sub>1's comp</sub>(binary)
    ELSE
        temp = binary
```

- 2. result = temp converted to decimal
- 3. **IF** MSBit(binary) == 1 **THEN** result = -1 * result

Convert 00011011₂ 1's comp to base 10

- 1. The MSBit is 0 so this is a positive number No changes made to the number
- 2. 00011011_2 converted to base-10: result = $16 + 8 + 2 + 1 = 27_{10}$
- 3. Since the MSBit = 0 nothing further needs to be done result = 27_{10}

Convert 10100110₂ 1's comp to base 10

- 1. The MSBit is 1 so this is a negative number $NEG_{1,s comp}$ (10100110₂) = 01011001₂
- 2. 01011001_2 converted to base-10 result = $64 + 16 + 8 + 1 = 89_{10}$
- 3. The MSBit is 1

 result = $-1 * result = -89_{10}$

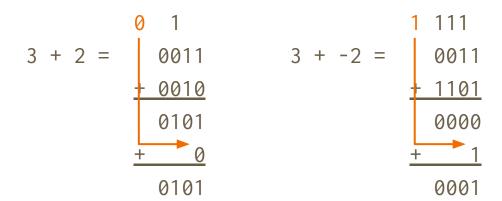
Try the following:

$$\begin{array}{r}
 1 \\
 3 + 2 = 0011 \\
 + 0010 \\
 \hline
 0101
 \end{array}$$

WHAT?

One's Complement - End Around Carry

To make addition work, the carry out bit **must** be added to the result of the addition



Therefore 1's complement addition requires two binary adds

One's Complement - Eliminating Subtraction

Unlike for Signed Magnitude subtraction can be eliminated

$$3 - 2 = 3 + (-2)$$
 $x - y = x + (-y)$ (generalized form)

 $0011 \iff 0011$
 $-0010 \implies 1101$
 $0000 \implies 1$
 0000

- A fixed-length number using one's complement representation has range:
 - \circ -(2^(N-1) 1) to 2^(N-1) 1 (same as SM)
- Half the values are positive and half the values are negative
- Zero can be represented in two ways (same as SM)
 - 00000000 (+0)
 - 11111111 (-0) (negative 0 differs between SM and 1's complement)

Pros:

- ✓ Easy (one step) negation
- ✓ Subtraction is just addition of negation (no extra circuit required)
- ✓ All numbers have inverses
- ✓ MSB indicates sign (negative circuit test is simple)

Cons:

- × two zeros, so extra logic circuit
- different circuit required for signed/unsigned addition (end around carry for signed addition)
- × negative numbers not directly identifiable