Conversions Between Bases

From Base A to Base B

Conversion Between Bases

- Types of Conversions
 - Base-B to Decimal
 - Decimal to Base-B
 - Base-B to Base-C

There are two algorithms we will use to convert between bases:

- 1. Sum Expanded Representation
- 2. Horner's Rule

SUM EXPANDED REPRESENTATION

Each number is represented as a sequence of digits from left to right. Starting with the largest positional value on the left and the smallest value on the right.

Each digit in the string has an associated weight that increases by value as you move to the left.

SUM EXPANDED REPRESENTATION

Example expansion:

- Numbers represented as a sequence of digits
 - Most significant to the left, least significant to the right
- Each digit in the sequence has a weight
 - Weight increases by multiple of B as you move left, starting with B⁰
 - Weight of rightmost digit is one (1)

Example Expansion:

Example Expansion 2:

```
0 	 1010_2 = 1000_2 + 0_2 + 10_2 + 0_2
0 	 = (1*1000_2) + (0*100_2) + (1*10_2) + (0*1_2)
0 	 = (1*2^3) + (0*2^2) + (1*2^1) + (0*2^0)
```

- Value of the number equals the sum of all the values
- Expanding a number in any base in this way and working out the calculation will result in a base-10 number
- Any number in any base B can be expressed in this method

- A number d_nd_{n-1}...d₁d₀ in base B can be expressed using sum expanded notation
 - Each d_n will have a value such that $0 \le d_n \le (B-1)$
 - The exponent to expand with is Bⁿ
 - Expansion is $d_n^*B^n + d_{n-1}^*B^{n-1} + ... + d_1^*B^1 + d_0^*B^0$

Example: Lets convert **11011**₂ to **Base-10** using the sum expanded method

Base: 2 (Binary)

	POSITION	4	3	2	1	0		
	DIGIT	1	1	0	1	1		
	K		1	V	1	1		
1101	$1_2 = (1*2^4)$) + (1*)	2^3) + ($0*2^2$) +	$(1*2^1)$) + (1*	20)	
	= (1*16) + (1*8) + (0*4) + (1*2) + (1*1)							
	= 16 + 8 + 0 + 2 + 1							
	= 27							

Answer: 27₁₀

Example: Lets convert **A5**₁₆ to **Base-10** using the sum expanded method **Base:** 16 (Hex)

POSITION	1	0
DIGIT	А	5
	-0)	
$A5_{16} = (A*16^{1}) + (5$	*16°)	
= (10*16) + (5	*1)	
= 160 + 5		
= 165		

Answer: 165₁₀

Your turn to do a few conversions to decimal using sum expanded representation

- **1.**1001₂
- **2.423**₅
- 3.A2J₂₀

Example: Lets convert 1001₂ to **Base-10** using the sum expanded method

Base: 2 (Binary)

	POSITION	3	2	1	0
	DIGIT	1	0	0	1
		- 2)	1		
1001 ₂ =	$(1*2^3) + (0*1)^3$	*2 ²) +	0*2' +	1*2°	
=	(1*8) + (1*	1)			
=	8 + 1				
=	9				

Answer: 9₁₀

Example: Lets convert 423₅ to **Base-10** using the sum expanded method

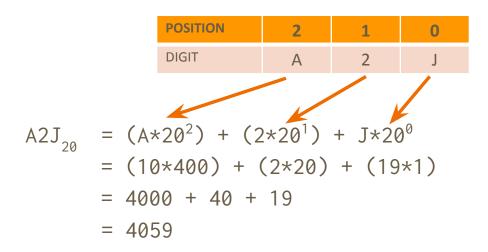
Base: 5

			POSITIO	V	2	1	0
			DIGIT		4	2	3
			2)				
423 ₅	=	(4*	5 ²) +	(2*!	5') +	3*5°	
	=	(4*)	25) +	(2*	5) +	(3*1)	
	=	100	+ 10	+ 3			
	=	113					

Answer: 113₁₀

Example: Lets convert A2J₂₀ to **Base-10** using the sum expanded method

Base: 20



Answer: 4059₁₀

• In math: A polynomial

$$\circ$$
 A(x) = $a_0 + a_1x + a_2x^2 + a_3x^3 + + a_nx^n$

Can be written as

$$A(x) = a_0 + x (a_1 + x (a_2 + x (a_3 + ... + x(a_{n-1} + x a_n) ...)))$$

• See http://everything2.com/title/Horner%2527s+rule for more information

Example:
$$A(x) = a_0 + a_1x + a_2x^2 + a_3x^3$$

 $= a_0 + x(a_1 + a_2x + a_3x^2)$
 $= a_0 + x(a_{1+}x(a_2 + a_3x))$
 $= a_0 + x(a_1 + x(a_2 + x(a_3)))$
Example: $A(x) = 5 + 7x - 2x^2s$
 $= 5 + x(7 - 2x)$
 $= 5 + x(7 - x(2))$

Simplified Algorithm

Set sum =
$$0_{10}$$

For all digits in the number starting from the left and moving to the right do the following:

Sum = (Sum * B) + digit value

Where B is the base of the number

Sum at the end will be your base-10 converted number

```
Example: Convert AF452<sub>16</sub>
                                            to Base-10
Base = 16 (hexadecimal)
Sum = 0
Steps: Sum = Sum * Base + digit value
1.AF452_{16} => Sum = Sum * Base + A = 0 * 16 + 10
                                                          = 10
2.AF452<sub>16</sub> => Sum = Sum * Base + F = 10 * 16 + 15
                                                          = 175
3.AF452<sub>16</sub> => Sum = Sum * Base + 4 = 175 * 16 + 4
                                                          = 2804
4.AF452<sub>16</sub> => Sum = Sum * Base + 5 = 2804 * 16 + 5
                                                               = 44869
5.AF452<sub>16</sub> => Sum = Sum * Base + 2 = 44869 * 16 + 2
                                                        = 717906
Answer = 717906_{10}
```

Your turn to do a few conversions to Decimal

1011₂

433₅

A2B₁₆

Simplified Algorithm

Set sum = 0_{10}

For all digits in the number starting from the left and moving to the right do the following

Sum = (Sum * Base) + digit value.

Sum at the end will be your base-10 converted number

```
Example: Convert
                           1011,
                                          to Base-10
Base = 2 (binary)
Sum = 0
Steps: Sum = Sum * Base + digit value
1.1011<sub>2</sub> => Sum = Sum * Base + 1 = 0 * 2 + 1
                                                      = 1
2.1011_2 => Sum = Sum * Base + 0 = 1 * 2 + 0
                                                      = 2
3.1011_2 => Sum = Sum * Base + 1 = 2 * 2 + 1
                                                      = 5
4.1011<sub>2</sub> => Sum = Sum * Base + 1 = 5 * 2 + 1
                                                      = 11
Answer = 11_{10}
```

```
Example: Convert 433_5 to Base-10
Base = 5
Sum = 0
Steps: Sum = Sum * Base + digit value
1.433_5 => Sum = Sum * Base + 4 = 0 * 5 + 4 = 4
2.433_5 => Sum = Sum * Base + 3 = 4 * 5 + 3 = 23
3.433_5 => Sum = Sum * Base + 3 = 23 * 5 + 3 = 118
Answer = 118_{10}
```

```
Example: Convert A2B<sub>16</sub> to Base-10

Base = 16 (hexadecimal)

Sum = 0

Steps: Sum = Sum * Base + digit value

1.A2B<sub>16</sub> => Sum = Sum * Base + A = 0 * 16 + 10 = 10

2.A2B<sub>16</sub> => Sum = Sum * Base + 2 = 10 * 16 + 2 = 162

3.A2B<sub>16</sub> => Sum = Sum * Base + B = 162 * 16 + 11 = 2603

Answer = 2603<sub>10</sub>
```

There are two algorithms we will use to convert between bases:

- **1.** Decomposition (Quick) method
- 2. Repeated division by B

The Decomposition (Quick) Method - N₁₀ to base-B

- 1) List all powers of base B until you find an exponent x where $B^x > N$, first power used will be y = x-1
- 2) Determine the factor A such that $A*B^y < N$, where $0 \le A \le B$
- 3) Set $N = N A*B^y$ Set y = y-1
- 4) Repeat steps 2) and 3) while N > 0

The quick method when doing work on paper

Example: Convert 14₁₀ to base-2 (Step 1)

$$2^0 = 1 < 14_{10}$$
 Keep Going

The quick method when doing work on paper

Example: Convert 14₁₀ to base-2 (Step 2)

2 ¹	2 ⁰
2	1

$$2^1 = 2 < 14_{10}$$
 Keep Going

The quick method when doing work on paper

Example: Convert 14₁₀ to base-2 (Step 3)

2 ²	2 ¹	2 ⁰
4	2	1

$$2^2 = 4 < 14_{10}$$
 Keep Going

The quick method when doing work on paper

Example: Convert 14₁₀ to base-2 (Step 4)

2 ³	2 ²	2 ¹	2 ⁰
8	4	2	1

$$2^3 = 8 < 14_{10}$$
 Keep Going

The quick method when doing work on paper

Example: Convert 14₁₀ to base-2 (Step 5)

2 ⁴	2 ³	2 ²	2 ¹	2 ⁰
16	8	4	2	1

$$2^4 = 16 > 14_{10}$$
 Stop

The quick method when doing work on paper

Example: Convert 14₁₀ to base-2 (Step 6)

2 ⁴	2 ³	2 ²	2 ¹	2 ⁰
16	8	4	2	1
	1			



8 goes into 14 once

$$N = 14 - (1*8)$$

$$N = 14 - 8 = 6$$



The quick method when doing work on paper

Example: Convert 14₁₀ to base-2 (Step 7)

2 ⁴	2 ³	2 ²	2 ¹	2 ⁰
16	8	4	2	1
	1	1		

N: 6₁₀

4 goes into 6 once

$$N = 6 - (1*4)$$

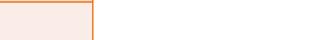
$$N = 6 - 4 = 2$$



The quick method when doing work on paper

Example: Convert 14₁₀ to base-2 (Step 8)

2 ⁴	2 ³	2 ²	2 ¹	2 ⁰
16	8	4	2	1
	1	1	1	_





2 goes into 2 once

$$N = 2 - (1*2)$$

$$N = 2 - 2 = 0$$

The quick method when doing work on paper

Example: Convert 14₁₀ to base-2 (Step 9)

2 ⁴	2 ³	2 ²	2 ¹	2 ⁰
16	8	4	2	1
	1	1	1	0

N: 0₁₀

All remaining digits are

now set to zero



The quick method when doing work on paper

Example: Convert 592₁₀ to base-2

2 ¹⁰	2 ⁹	2 ⁸	2 ⁷	2 ⁶	2 ⁵	2 ⁴	2 ³	2 ²	2 ¹	2 ⁰
1024	512	256	128	64	32	16	8	4	2	1
Binary number	1	0	0	1	0	1	0	0	0	0
Reduced # to	80	80	80	16	16	0	0	0	0	0
Convert										

The quick method when doing work on paper

Example: Convert 45₁₀ to base-4

4 ³	4 ²	4 ¹	4 ⁰
64	16	4	1
Binary Number	2	3	1
Reduced # to	13	1	0
Convert			

Your turn to do a few conversations

```
61<sub>10</sub> to base-2
```

14₁₀ to base-3

Algorithm:

- List all powers of base B until you find an exponent*where B^x > N, first power used will be y = x-1
- Determine the factor A such that A*B^y < N, where 0 <= A < B</p>
- 3) Set N = N A*B^y Set y = y-1
- 4) Repeat steps 2) and 3) while N > 0

The quick method when doing work on paper

2 ⁶	2 ⁵	2 ⁴	2 ³	2 ²	2 ¹	2 ⁰
64	32	16	8	4	2	1
Binary number	1	1	1	1	0	1
Reduced # to Convert	29	13	5	1	1	0

The quick method when doing work on paper

3 ³	3 ²	3 ¹	3 ⁰
27	9	3	1
Base-3 Number	1	1	2
Reduced # to	5	2	0
Convert			

Repeated Division Method

- Repeatedly divide the base-10 number by the base you want to convert to
 (B) until the quotient is 0
 - Using integer division
- Remainders are kept after each division step
- Reversed sequence of remainders is the new number in Base-B

Repeated Division Algorithm

N = the base-10 number

B = the base we want to convert to

R = the final result, starts empty

while (N > 0)

Add (N % B) to the beginning of R, where % is the modulus operator that gives back the remainder of dividing N by B

N = N/B, using integer division (drop the decimal places)

R contains the result in base B

NOTE: R is the N%B read from last calculated to first calculated

Repeated Division Method (Step 0)

Step	N	(N % B)	(N / B)	R
0	57			

Repeated Division Method (Step 1)

Step	N	(N % B)	(N / B)	R
0	57			
1	57	(57%2) = 1	(57/2)=28.5=28	1

Repeated Division Method (Step 2)

Step	N	(N % B)	(N / B)	R
0	57			
1	57	(57%2) = 1	(57/2)=28.5=28	1
2	28	(28%2) = 0	(28/2)=14	01

Repeated Division Method (Step 3)

Step	N	(N % B)	(N / B)	R
0	57			
1	57	(57%2) = 1	(57/2)=28.5=28	1
2	28	(28%2) = 0	(28/2)=14	01
3	14	(14%2) = 0	(14/2)=7	001

Repeated Division Method (Step 4)

Step	N	(N % B)	(N / B)	R
0	57			
1	57	(57%2) = 1	(57/2)=28.5=28	1
2	28	(28%2) = 0	(28/2)=14	01
3	14	(14%2) = 0	(14/2)=7	001
4	7	(7%2) = 1	(7/2) = 3.5 = 3	1001

Repeated Division Method (Step 5)

Step	N	(N % B)	(N / B)	R
0	57			
1	57	(57%2) = 1	(57/2)=28.5=28	1
2	28	(28%2) = 0	(28/2)=14	01
3	14	(14%2) = 0	(14/2)=7	001
4	7	(7%2) = 1	(7/2) = 3.5 = 3	1001
5	3	(3%2) = 1	(3/2) = 1.5 = 1	11001

Repeated Division Method (Step 6)

Step	N	(N % B)	(N / B)	R
0	57			
1	57	(57%2) = 1	(57/2)=28.5=28	1
2	28	(28%2) = 0	(28/2)=14	01
3	14	(14%2) = 0	(14/2)=7	001
4	7	(7%2) = 1	(7/2) = 3.5 = 3	1001
5	3	(3%2) = 1	(3/2) = 1.5 = 1	11001
6	1	(1%2) = 1	(1/2) = 0.5 = 0	111001

Repeated Division Method (Step 7)

Example: Convert 57₁₀ to Binary (Base-2)

Step	N	(N % B)	(N / B)	R
0	57			
1	57	(57%2) = 1	(57/2)=28.5=28	1
2	28	(28%2) = 0	(28/2)=14	01
3	14	(14%2) = 0	(14/2)=7	001
4	7	(7%2) = 1	(7/2) = 3.5 = 3	1001
5	3	(3%2) = 1	(3/2) = 1.5 = 1	11001
6	1	(1%2) = 1	(1/2) = 0.5 = 0	111001
7	0			

Note: that the value in R is the values in the remainder column (N%B) written from the bottom towards the top.

Repeated Division Method (Step 0)

Step	N	(N % B)	(N / B)	R
0	124			

Repeated Division Method (Step 1)

Step	N	(N % B)	(N / B)	R
0	124			
1	124	(124%7) = 5	(124/7)=17.7=17	5

Repeated Division Method (Step 2)

Step	N	(N % B)	(N / B)	R
0	124			
1	124	(124%7) = 5	(124/7)=17.7=17	5
2	17	(17%7) = 3	(17/7)=2.4=2	35

Repeated Division Method (Step 3)

Step	N	(N % B)	(N / B)	R
0	124			
1	124	(124%7) = 5	(124/7)=17.7=17	5
2	17	(17%7) = 3	(17/7)=2.4=2	35
3	2	(2%7) = 2	(2/7)=0	235

Repeated Division Method (Step 4)

Step	N	(N % B)	(N / B)	R
0	124			
1	124	(124%7) = 5	(124/7)=17.7=17	5
2	17	(17%7) = 3	(17/7)=2.4=2	35
3	2	(2%7) = 2	(2/7)=0	235
4	0			

Your turn to do a few conversions

13₁₀ to base-2

23₁₀ to base-3

Algorithm:

- N = the base-10 number
- B = the base we want to convert to
- R = the final result, starts empty
- While(N > 0)
 - Add (N % B) to the beginning of R, where % is the modulus operator that gives back the remainder of dividing N by B
 - N = N/B, using integer division
- R contains the result in base B

There are three algorithms we will use to convert between bases:

- 1. Base-B to Decimal then Decimal to Base-C
 - a. Combines the two previous conversions
- 2. Related base conversions
 - a. Can only be done under specific criteria
- 3. Convert directly from Base-B to Base-C
 - a. requires doing math in Base-C or Base-B, tends to be difficult

Base-B to Decimal to Base-C

1. Convert base-B to base-10

 Here you can use the expanded sums method or the method developed using Horner's rule

2. Now convert the base-10 number to base-C

 Here now you can use the repeated division method covered in the last section

Example:

Let's convert 111001, to a base 5 number.

(step 1) Use sum expanded representation

$$111001_{2} = (1*2^{5})+(1*2^{4})+(1*2^{3})+(0*2^{2})+(0*2^{1})+ (1*2^{0})$$

$$= (1*32)+(1*16)+(1*8)+(0*4)+(0*2)+ (1*1)$$

$$= 32 + 16 + 8 + 0 + 0 + 1$$

$$= 32 + 16 + 8 + 1$$

$$= 57_{10}$$

Example: Let's convert 111001₂ to a base 5 number.

(step 2) Now convert 57₁₀ to a base-5 number (use the repeated division by B algorithm)

$$N = 57$$
, $B = 5$, $R =$

Step	N	(N % B)	(N / B)	R
0	57			
1	57	(57%5) = 2	(57/5) = 11	2
2	11	(11%5) = 1	(11/5) = 2	12
3	2	(2%5) = 2	(2/5) = 0	212
4	0			

Your turn to do a few conversions

13₅ to base-3

11001₂ to base-5

5 ⁰	5 ¹	5 ²	5 ³
1	5	25	125

3 ⁰	3 ¹	3 ²	3 ³
1	3	9	27

2 ⁰	2 ¹	2 ²	2 ³	2 ⁴	2 ⁵	2 ⁶
1	2	4	8	16	32	64

Your turn to do a few conversions

$$13_5$$
 to base-3 = 22_3
 11001_2 to base-5 = 100_5

5 ⁰	5 ¹	5 ²	5 ³
1	5	25	125

3 ⁰	3 ¹	3 ²	3 ³
1	3	9	27

2 ⁰	2 ¹	2 ²	2 ³	2 ⁴	2 ⁵	2 ⁶
1	2	4	8	16	32	64

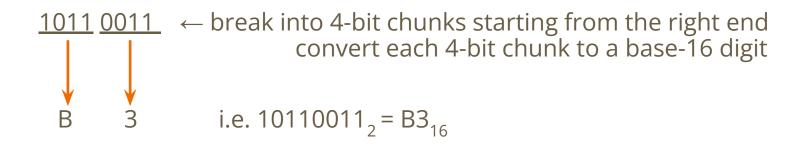
Related bases: a larger base that is a power of a smaller base.

Ex. $16 = 2^4$

Note: Each hex digit corresponds to a 4 digit binary number, and vice versa.

Therefore, <u>hex can be used as a binary shorthand</u>. A byte (8-bit number) can be represented by a 2-digit hex number.

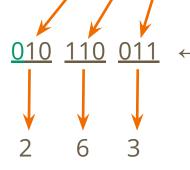
E.g. convert 10110011₂ to hex.



Note: Each octal digit corresponds to a 3 digit binary number.

$$8 = 2^3$$

E.g. convert 10110011, to octal.



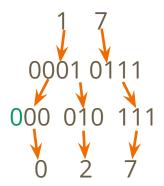
break into 3-bit chunks, left padding the number with zeros (0) as needed, starting from right convert each 3-bit chunk to a base-8 digit

i.e.
$$10110011_2 = 263_8$$

E.g. convert 17_{16} (hex) to octal.

$$16 = 2^4$$
 and $8 = 2^3$

Both are powers of 2 (binary), convert hex into binary and organize into groups of 4 bits, then separate them into groups of 3 binary bits to convert to octal



Therefore $17_{16} = 27_{8}$

Your turn to do a few conversations

```
52_{8} to base-16 (Hex)

A2F_{16} to base-8 (Octal)

2310_{4} to base-16 (Hex)

2110_{3} to base-9
```

Your turn to do a few conversations

```
52_8 to base-16 = 2A_{16}
A2F_{16} to base-8 = 5057_8
2310_4 to base-16 = B4_{16}
2110_3 to base-9 = 73_9
```