1 Preliminaries

This is a supplement to the section 7 notes. Please let me know ASAP if you find or suspect a mistake (email atai1@berkeley.edu).

2 One-shot deviation principle

One-shot deviation principle. In infinitely repeated games (with some discount factor), a strategy profile is SPNE if and only if there are no profitable "one-shot deviations" for each subgame for every player. A one shot deviation is a change in the player's strategy in a single round, reverting to the original strategy afterwards.

We can use this to check SPNE in infinite horizon bargaining games.

Checking SPNE in infinite horizon bargaining games:

1. Have a candidate SPNE strategy profile. Usually this will be obvious or given to you in the problem. *Note that in SPNE in bargaining games, the players agree to a deal at the first proposal.*
2. Consider deviations in the **first round**. Each round the game that is repeated over and over; usually it's (offer, response, counteroffer, response).
3. In later rounds, there are no deviations. So if both players reject the offers in the first round, they revert to the candidate SPNE profile in the second round. This means they agree to the same deal as in (1), and now the payoffs are discounted (or cost is subtracted).

3 Example

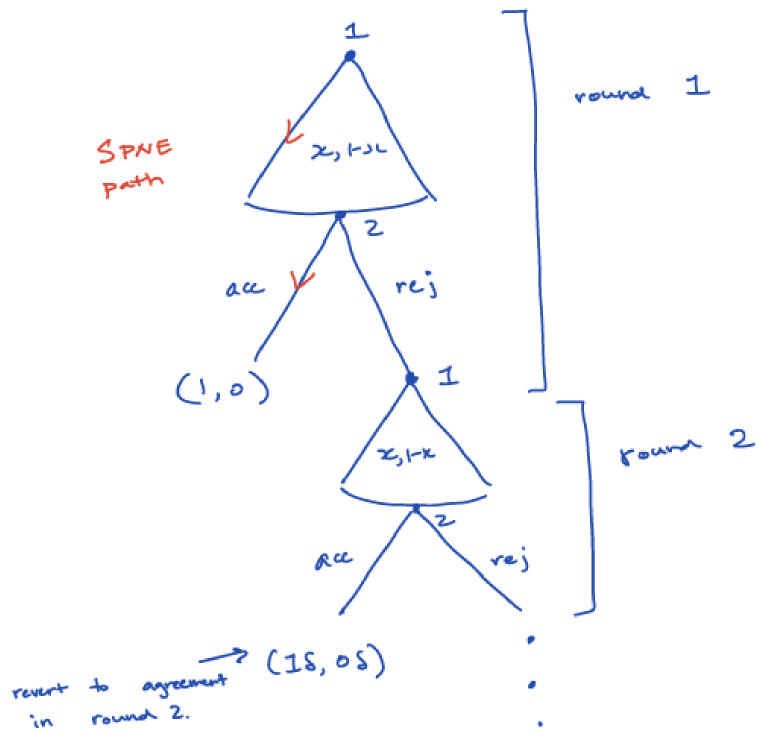
This is exercise 2 from the section notes.

One-sided offers. Two players are splitting something worth 1 utility. Every round, player 1 makes an offer $(x, 1-x)$. Then player 2 either accepts or rejects the offer; if he rejects, then player 1 makes another offer, etc. After every round (player 1 offers, and player 2 responds) the payoff is discounted by $0 < \delta < 1$.

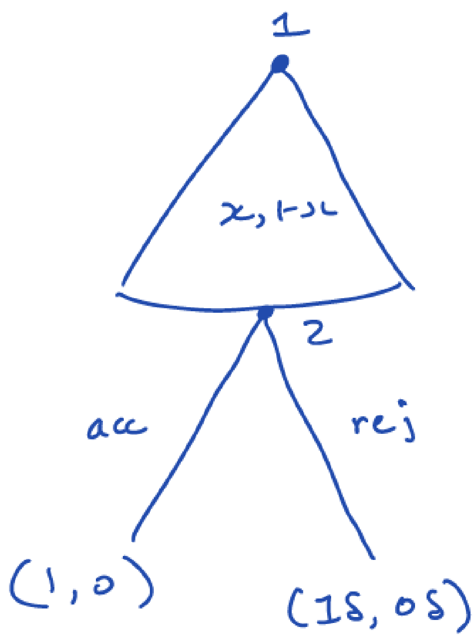
Check that player 1 offering $(1, 0)$ in every round is part of an SPNE strategy profile. (Note that the SPNE strategy profile also has to specify player 2's strategy).

1. Consider the first round of this game (p1 offers, p2 responds). If the proposed strategy is indeed part of an SPNE, then in equilibrium, p1 offers $(1, 0)$ and p2 will accept the offer.

2. If either player deviates in the first round, they might reach the second round. In the second round, assume both players revert to the SPNE strategy. This means that p1 offers $(1, 0)$, and p2 will accept. But now the payoffs are $(1\delta, 0\delta)$. This is shown below:



3. This means we can consider the game shown below. We want to prove that "p1 offers $(1, 0)$" is consistent with an SPNE.



4. Consider the subgame starting at p2's response. The SPNE strategy is: "accept if $x \geq 0\delta = 0$, and reject otherwise". Of course, this just means "accept any offer".

5. Now consider the subgame starting at p1's offer. Given p2's strategy, the SPNE strategy is "offer $(1, 0)$", which indeed matches our proposed SPNE. We're done! The SPNE is:

   (a) p1 offers $(1, 0)$ in every round.

   (b) p2 accepts any offer in every round.