Section 9: Consumer Theory

Econ 104, Spring 2021

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1 Objectives

- Define various consumer demand concepts and solve for them
- Understand the correspondence between utility functions and preferences
- Prove simple results about utility functions

2 Philosophy

Have you ever seen a utility function? I haven't. I've also never seen a preference. We only observe choices. We can use those choices to infer **revealed preferences** $\succsim^*$, we model these with preferences $\succsim$, and we can represent preferences with utility functions.

Definition 1. A utility function $u(\cdot)$ represents a preference $\succsim$ if for all bundles of goods $x, y \in X$:

$$u(x) \geq u(y) \Longleftrightarrow x \succsim y$$

Utility functions are purely models – they aren't "real". No one actually "has" a utility function.

Note. A preference can have many utility function representations.

Exercise 1. Prove that if $u(\cdot)$ represents $\succsim$, then so does $ln(u(\cdot))$.

3 Consumer demand

Definition 2. Let an agent be represented by a utility function $u(\mathbf{x})$, with total income w and prices $\mathbf{p}$ for goods $\mathbf{x}$. Note: $\mathbf{x}$ and $\mathbf{p}$ are vectors.

- The demand function is $\mathbf{x}(\mathbf{p}, w)$, solving $\max_{\mathbf{x} \in X} u(x)\ s.t.\ w \geq \mathbf{p} \cdot \mathbf{x}$. Note this is a vector, so is actually a list of functions.
- The indirect utility function is $v(\mathbf{p}, w) = u(\mathbf{x}(\mathbf{p}, w))$. I.e., if you give me income w and prices $\mathbf{p}$, this is the utility I can get from this.
- The Hicksian demand function is $\mathbf{x}^h(\mathbf{p}, \underline{\mathrm{u}})$, solving $\min \mathbf{p} \cdot \mathbf{x}\ s.t.\ u(\mathbf{x}) \geq \underline{\mathrm{u}}$. I.e., if you tell me the minimum amount of utility to reach, this is the consumption bundle that achieves it.
- The expenditure function is $e(\mathbf{p}, \underline{\mathrm{u}}) = p \cdot \mathbf{x}^h(\mathbf{p}, \underline{\mathrm{u}})$. I.e., the spending required to do the above.

Proposition 1. Kuhn-Tucker conditions. Consider the utility maximization problem. Let $(\mathbf{p}, w) >> 0$. Let $u(\mathbf{x})$ be continuously differentiable. If $\mathbf{x}^*$ is a solution to $\max_{\mathbf{x} \in X} u(x)\ s.t.\ m \geq \mathbf{p} \cdot \mathbf{x}$, then there exists λ such that

1. $\lambda \geq 0$
2. $\frac{\partial u(x)}{\partial x_\ell} \leq \lambda p_\ell$ for all ℓ (ℓ is the dimension of $\mathbf{x}$), and $\frac{\partial u(x)}{\partial x_\ell} = \lambda p_\ell$ if $x^*_\ell > 0$ (i.e., the solution is "interior")
3. $\lambda(p \cdot x^* - w) = 0$

Notes:

- Note the logic here: any solution must satisfy 1-3. Just because a point satisfies 1-3 doesn't mean it's a solution. I.e., you can't just solve 1-3 and say the point you find is the optimal consumption.
- If $u(\mathbf{x})$ is convex, then there is a unique maximizer. In this case, a point that satisfies 1-3 must be a solution (why?), so you can just solve 1-3 and find the solution.
- This is a generalization of the Lagrange multiplier. If the constraint is $w = \mathbf{p} \cdot \mathbf{x}$, then this is the Lagrange multiplier. It is not hard to re-derive conditions 2 and 3 from the Lagrange multiplier in this case. If $u(\mathbf{x})$ is monotone, then the constraint might as well be $w = \mathbf{p} \cdot \mathbf{x}$ (why?).
- It's usually easy to see whether a utility function $u(\mathbf{x})$ will have an interior solution by studying the function. E.g. consider $u(x_1, x_2) = x_1 + 2x_2$ with prices $p_1 = p_2 = 1$.

Exercise 2. Let $u(x_1, x_2) = ln(x_1 x_2)$, and the budget set be $w \geq x_1 p_1 + x_2 p_2$. Is the utility function convex and/or monotone?

Find the demand function, indirect utility, Hicksian demand, and expenditure.