Section 5: Subgame perfect Nash equilibrium

Econ C110 / PoliSci C135

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In this section, we will solve for subgame perfect Nash equilibria (SPNE or SPE).

1 Subgame perfect Nash equilibrium

Definition 1. A **subgame** is a collection of nodes and branches that satisfies three properties:

1. It starts at a single decision node.
2. It contains every alternative at this node and all nodes after this node.
3. It doesn't split up any information sets. (For now, we won't worry about information sets)

Note that the entire game is a subgame.

Definition 2. A **subgame perfect Nash equilibrium (SPNE or SPE)** is an equilibrium in an extensive form game. The strategy profile is a Nash equilibrium in every subgame, even ones that aren't reached. Subgame perfect Nash equilibria are a subset of Nash equilibria.

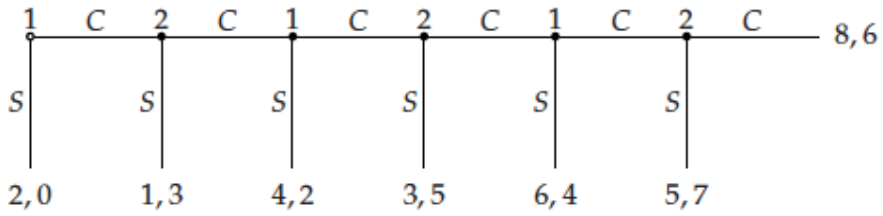
1.1 Finding SPNE

There are two ways of finding SPNE.

1. In a finite game of perfect information, solve via backward induction. Start at the last node; what would the player do at this node? Given this, what would happen at the second to last node(s)? Etc.
2. Check all the Nash equilibria.
   (a) Find all the NE. If necessary, write the game in strategic form (as a matrix) and find all NE this way.
   (b) Find all subgames of the extensive form game.
   (c) Find the NE of each subgame.
   (d) Check which NE of the whole game are also NE in every subgame. I.e., check that each players' strategies are consistent with the subgame NE.

2 Exercises

Exercise 1. (Selten's Horse) Find the SPNE of the following game.



Exercise 2. (Hungry lions)[1] There are n hungry lions in a hierarchical pack who encounter a prey. Lion 1 can choose either to eat the prey or not eat it. If he doesn't eat it, the prey escapes and the game ends. If he eats, he becomes fat and slow, and Lion 2 can eat Lion 1. If Lion 2 doesn't eat Lion 1, the game ends. If he eats Lion 1, then Lion 3 can eat Lion 2, and so on. Each lion prefers being full over being hungry over being eaten. Find the SPNE for a generic n number of lions. *Hint: Start with* $n = 1$ *and* $n = 2$.

[1] Osborne, 2004. Ex 202.1

Exercise 2. (Ultimatum game)[2] Two people are dividing a dollar. Player 1 makes an offer $x \in [0, 1]$. Then player 2 either accepts or rejects the offer. If she accepts, then the players get $(1 - x, x)$. If she rejects, then both players get nothing.

- What are the values of x that can be offered in a Nash equilibrium?
- Suppose x can only be in cents, so $x \in [0.00, 0.01, 0.02, ...0.99, 1.00]$. What is (are?) the SPNE?

[2] Osborne, 2004. Ex 183