Section 2: Dominance and Nash Equilibrium

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1 Objectives

- Find dominated strategies
- Use iterated elimination of dominated strategies to solve for games
- Find best responses
- Define Nash equilibrium
- Understand how the alternative definitions of NE are the same
- Use best responses to solve for NE in formal games
- Apply the definition of NE to solve for games in new settings

2 Dominated strategies

Definition 1. A strategy (pure or mixed) σ_i **strictly dominates** pure strategy s_i if

$$u_i(\sigma_i, s_{-i}) > u_i(s_i, s_{-i})$$

for any strategy profile s_{-i} of other players. I.e., σ_i gives player i a higher (expected) utility than s_i, given any strategy profile of the other players.

Definition 2. Iterated (strict) dominance. Eliminate any strictly dominated strategies for either player. Disregarding the eliminated strategies, eliminate any strategies that are subsequently strictly dominated. Repeat until no more strategies are strictly dominated. For 2-player games, the remaining set of strategy profiles are the **rationalizable** strategy profiles.

Exercise 1. (Prisoner's dilemma). For each player, what strategies are dominated? What are the rationalizable strategy profiles?

	C	D
C	2,2	0,4
D	4,0	1,1

Exercise 2. What strategies survive iterated strict dominance, allowing for mixed strategies?

	L	R
A	2,1	0,0
B	0,1	2,0
C	0,0	0,2

Proposition 1. If iterated strict dominance leads to a single strategy profile, it is a Nash equilibrium. (Note that the converse isn't true – Nash equilibria may exist even when strict dominance doesn't solve for them).

Exercise 3. What is/are the NE of exercise 1?

3 Nash equilibrium

Definition 3. Player i's **belief** $\theta_{-i} \in \Delta S_{-i}$ is a probability distribution over other players' possible strategies. Formally, the **best response** $BR_i(\theta_{-i})$ is the set of best strategies for player i, given his belief of the other players' strategies θ_{-i}. *Note:* Player 1 has a BR to each one of player 2's possible strategies, and each BR may be multiple strategies.

Exercise 4. Consider the game below from player 1's perspective. $\Delta S_{-1} = \Delta S_2$ is the set of probability distributions over $\{L, R\}$. Then let $\theta_{-1} = \theta_2 \in [0, 1]$ represent the probability that player 2 plays L. What is $BR_1(\theta_2 = 1)$? How about $BR_1(\theta_2 = 0)$ and $BR_1(\theta_2 = 0.5)$?

	L	R
T	12,5	1,1
B	3,0	6,15

Definition 4. A **Nash equilibrium (NE)** is a strategy profile in which each player is playing a best response to the other players' strategies (which are also best responses!). I.e., all players are best responding to each other. Here is an alternative definition:

- At the NE strategy profile, no single player can profitably deviate. I.e., assuming the other players stick to the same strategies, player 1 can't play a different strategy and get a *strictly* higher payoff.

(Why are these the same?) Note that a game may have multiple or no NE.

Exercise 5. Find all pure strategy NE in the following game, if any exist.

	L	M	R
A	1,1	0,1	2,0
B	2,1	2,1	1,1
C	2,1	2,0	0,0

Exercise 6. Find all NE in the following game, if any exist.

	L	M	R
A	0,0	0,0	0,0
B	0,0	5,5	0,0
C	0,0	0,0	0,0

Exercise 7. Create a 2x2 or 3x3 game with no Nash equilibria.

Exercise 8. Cournot duopoly.

Suppose there are two firms $i = 1, 2$. Each firm's cost function is given by $C_i(q_i) = c_i q_i$. The inverse demand function is linear where it is positive, given by

$$P(Q) = \begin{cases} A - Q & \text{if} \quad Q \leq A \\ 0 & \text{if} \quad Q > A \end{cases}$$

where $Q = q_1 + q_2$ and $A > c_1 > c_2$. Each firm's objective is to maximize profit.

- Find each firm's best response function.
- Use these to solve for the Nash equilibrium. (Note that there are two cases, depending on the sizes of c_1 and c_2).

Exercise 9. Hotelling election.

Suppose voters are distributed uniformly on an ideological continuum on $[0, 1]$. Two candidates are picking positions $x_i \in [0, 1]$. Voters pick the candidate closer to themselves, and the candidate with the most votes wins. If an equal density of voters picks both candidates, they tie; candidates prefer $win \succ tie \succ lose$. What is/are the Nash equilibria?

Exercise 10. Hotelling election with 3 candidates.

Show that there are no Nash equilibria when there are 3 candidates.