Section 2: Nash Equilibria
Econ C110 / PoliSci C135
Fall 2020
GSI: Andrew Tai

This section will cover the the definition of Nash equilibria (NE) and solution strategies.

1 IEWDS

Recall. We can use **iterated elimination of strictly dominated strategies (IESDS)** to solve games. Remove strategies if they are strictly dominated. If a strategy becomes strictly dominated after earlier deletions, then this can be removed as well.

Definition 1. Iterated elimination of weakly dominated strategies (IEWDS) is another solution method. At each step, remove any *strictly* dominated strategies first. When there are no strictly dominated strategies left, remove *weakly* dominated strategies.

Note. Order of deletion can lead to different solutions!

Exercise 1. Solve this game using IEWDS in different ways. *Hint 1:* Player 2 eliminates M first. *Hint 2:* Player 2 eliminates R first.

	L	M	R
A	$1,1$	$0,0$	$2,0$
B	$2,1$	$2,1$	$1,1$
C	$2,1$	$2,0$	$0,0$

	L	M	R
A	$1,1$	$0,0$	$2,0$
B	$2,1$	$2,1$	$1,1$
C	$2,1$	$2,0$	$0,0$

2 Nash equilibrium

IESDS and IEWDS are solution *methods.* They attempt to solve for a *Nash equilibrium*, which is our first type of equilibrium.

Definition 2. P1's **best response (BR)** to one of P2's strategies is the strategy(ies) that yields the highest payoff given P2's strategy. *Note:* Player 1 has a BR to each one of player 2's possible strategies, and each BR may be multiple strategies.

Exercise 2. What is player 1's BR to L? to R?

	L	R
T	$12, 5$	$1, 1$
B	$3, 0$	$6, 15$

Definition 3. A **Nash equilibrium (NE)** is a strategy profile in which each player is playing a best response to the other players' strategies (which are also best responses!). Here are some alternative definitions:

- At the NE strategy profile, no single player can profitably deviate. I.e., assuming the other players stick to the same strategies, player 1 can't play a different strategy and get a *strictly* higher payoff.

- Each player *conjectures* (guesses) what the other players will do, then plays a best response to the conjectured strategies. All players' conjectures turn out to match the other players' strategies.

(On your own, you should think about why these 3 are equivalent ways to define the same thing.) Note that a game may have multiple or no NE.

Proposition 1. *If* IESDS or IEWDS lead to solutions, they are Nash equilibria. However, they may not find *all* NE, and they may not lead to a solution, even when there are NE.

Exercise 3. Find all NE of exercise 2, if any exist.

Exercise 4. Find all NE of exercise 1. (Reproduced here)

	L	M	R
A	$1,1$	$0,0$	$2,0$
B	$2,1$	$2,1$	$1,1$
C	$2,1$	$2,0$	$0,0$

Exercise 5. Find all NE, if any exist.

	L	M	R
A	$0,0$	$0,0$	$0,0$
B	$0,0$	$5,5$	$0,0$
C	$0,0$	$0,0$	$0,0$

Exercise 6. Determine whether these statements are true or false:

1. A strategy that is strictly dominated is never part of a Nash equilibrium.

2. A strategy that is weakly dominated is never part of a Nash equilibrium.

Exercise 7. Create a 2x2 or 3x3 game with no Nash equilibria.