Section 1

Econ 152

Spring 2020

GSI: Andrew Tai

Introduction

In this section, we will review consumer theory basics. This includes utility functions, optimization, and the income & substitution effects.

Utility functions

Definition. A **utility function** is a function that represents an individual's preferences over bundles of goods. It assigns higher numbers to more preferred bundles.

We often graphically represent utility functions by using indifference curves.

Definition. An **indifference curve** is the set of points that yields the same level of utility.

Indifference curves typically have the following properties:

1. Indifference curves are downward sloping
2. Higher indifference curves indicate higher utility
3. Indifference curves do not intersect (why?)
4. Indifference curves are convex to the origin

Drawing indifference curves (for two goods):

1. Set the utility function equal to a constant.
2. Solve for one good in terms of the other.
3. Draw the curve for this; the axes are the two goods.
4. Draw the concentric indifference curves if desired.

Example 1. Suppose there are two goods, x_1 and x_2, and an individual has preferences represented by the utility function $U(x_1, x_2) = x_1^{2/3} x_2^{1/3}$. (This is a *Cobb-Douglas* utility function.)

Exercise 1. Draw an indifference curve for this utility function. Make sure to label the axes.

Marginal utility

The following concepts will be important for analyzing consumers' choices. I'll present these definitions according to specific goods, though of course they can be any goods in the utility function.

Definition. The **marginal utility** of x_1 (sometimes denoted MU_1) is the utility gained by consuming one more of this good. It is the partial derivative of the utility function with respect to x_1,

$$MU_1 = \frac{\partial U}{\partial x_1}$$

When marginal utility is declining as x_1 increases, we say that the utility function has **decreasing/diminishing marginal utility** in x_1. Analogously for increasing and constant marginal utility.

Definition. The **marginal rate of substitution** between x_1 and x_2 is the ratio of marginal utilities for x_1 and x_2. It is the amount of x_2 the individual would give up in exchange for one additional x_1.

$$MRS_{12} = \frac{MU_1}{MU_2}$$

Note that $MRS_{12} \neq MRS_{21}$. Don't mix up the order!

Exercise 2. Find the marginal utilities of x_1 and x_2 for the utility function in Example 1. Are these increasing, decreasing, or constant marginal utilities? Find MRS_{12}.

Optimization

We're often going to be interested in solving for consumers' optimal (utility maximizing) choices given some budget constraint. We usually do this using Lagrange multipliers.

1. Write the budget constraint equation – often something like $Y = p_1x_1 + p_2x_2$, where Y is income, and p_1 and p_2 are the prices of respective goods.

2. Write the Lagrangian equation:

$$\mathcal{L} = \underbrace{U(x_1, x_2)}_{\text{objective}} - \underbrace{\lambda}_{\text{multiplier}} \underbrace{(Y - p_1x_1 - p_2x_2)}_{\text{budget constraint}}$$

3. Calculate partial derivatives with respect to each good (here, x_1 and x_2) and λ, and set these equal to 0.

$$\frac{\partial \mathcal{L}}{\partial x_1} = 0$$
$$\frac{\partial \mathcal{L}}{\partial x_2} = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0$$

4. Solve this system of equations. The solution will give an "interior" solution *if there is one.* Interior means that the solution isn't "corner" (see the next step).

5. Check the corner cases. The corner cases are consuming only x_1 or only x_2 (or nothing at all!).

Note. The solution point to the two good maximization problem is tangent to an indifference curve. I.e., a point where $MRS_{12} = \frac{p_1}{p_2}$. (Why is this?) This gives an alternative way of quickly solving many problems. We sometimes call this the **tangency condition**.

Exercise 3. Suppose $p_1 = 2$, $p_2 = 1$, and income $= 12$. Use the utility function from Example 1.

1. Write down the budget constraint. Graph it.
2. Solve for the consumer's optimal choice. Try doing it two ways: using the Lagrangean and the tangency condition.
3. Draw an indifference curve representing the maximum feasible utility.

Substitution and income effects

Definition. What happens if the price of one good increases? Let's say the price of x_1 increases. There are two effects:

- The **income effect**. The consumer is poorer, since real income is lower now. This decreases consumption of "regular" goods.
- The **substitution effect**. x_2 became *relatively* cheaper. The consumer substitute away from x_1 in favor of x_2. This effect is as if the consumer were compensated for the decreased real income – she would still consume less of x_1 and more of x_2!

Note. There are separate income and substitution effects *for each good*!

Calculating the income and substitution effects. (for x_1)

1. Note the total change in consumption of x_1.
2. Find the substitution effect. Graphically, this is represented by keeping the *old* indifference curve, and finding the point tangent to the *new* budget slope.
   (a) Set $U(x_1, x_2) =$ old utility. Use the *new* tangency condition $MRS_{12} = \frac{p_{1,new}}{p_{2,new}}$. Solve these two equations.
   (b) This gives the amount the consumer would consume if compensated for the change. The difference between this amount and the old consumption of x_1 is the substitution effect.
3. Find the income effect. The difference between the total effect and the substitution effect is the income effect. In other words, this is the rest of the change.

Exercise 4. Continue Exercise 3. Suppose the price of x_1 increases to $p_1 = 4$.

1. Find and graphically represent the new consumption bundle.
2. What are the substitution and income effects for x_1? Represent these graphically.
3. (At home) Repeat 2 for x_2.