Section 1: Strategic Games and Strategies

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1 Objectives

- List the components of formal games
- Identify these components from a matrix, and vice versa
- Understand the interpretation of dynamic game trees
- List strategies and strategy profiles from

2 Components of games

Definition 1. A game has four components:

1. The **players** (set N)
2. The players' **actions/strategies** for each player (set A_i with elements a_i)
3. The **payoffs** – the utilities that each player receives for each outcome of the game (payoff/utility functions $u_i(s_i, s_{-i})$; s_{-i} denotes everyone else's strategies)
4. The **timing** – who plays, and when. (We will deal with simultaneous games first)

The typical way to represent a game is in **strategic form**, also called **matrix form**.

Note: in a two-player matrix, we list payoffs as (player 1, player 2). Player 1's strategies are rows; and player 2's strategies are columns. I.e.,

$Player 2$

$Player 1$

	a	b
a	$1, 1$	$1, 0$
b	$0, 1$	$0, 0$

Exercise 1. Represent the following game in strategic (matrix) form. Cities A and B are wooing Amazon to build new offices and bring jobs. Each city can charge high taxes ("h") or low taxes ("l"). Amazon will build offices in the city with the lower taxes, but if the two cities charge an equal tax rate, Amazon will build half in each city. The benefit to a city of having all of the offices is 4, half of them is 2, and none of them is 0. Additionally, the gain of charging high taxes is 2 and low taxes is 1 (assuming there is an Amazon to charge a tax to). List the players, strategies, and payoff functions.

3 Dynamic games with complete information

We also have dynamic games, in which players take their actions sequentially.

Definition 2. A player's **strategy** specifies an action *at every node* she is active . This includes nodes that she would never reach. A strategy has as many components as there are nodes for a player!

Definition 3. A game's **histories** is the set (usually called H) of possible paths/events of the game. The **terminal histories** (usually called $Z \subseteq H$) is the set of histories that lead to the end of the game.

Exercise 2. (Entry Game, Osborne). A monopoly firm is facing the possibility of entry by a new challenger. First, the challenger has three choices: stay out of the market, enter the market prepared, or enter the market unprepared. Preparation by the challenger is costly but reduces the loss from a fight. If the challenger enters the market, the monopoly can either fight or acquiesce. The monopoly observes the challenger's choice. The preferences are as follows:

- Monopoly: $out \succ (unprepared, fight) \succ (unprepared, acquiesce) \sim (prepared, acquiesce) \succ (prepared, fight)$
- Challenger: $(unprepared, acquiesce) \succ (prepared, acquiesce) \succ out \succ (prepared, fight) \succ (unprepared, fight)$

Represent this dynamic game as a game tree with appropriate payoffs. List the set of histories and each players' strategies.

4 Dynamic games with incomplete information

Definition 4. An **information set** connects nodes that a player cannot distinguish. On an extensive form game, this is drawn as a dotted line or circle.

Definition 5. A **strategy** specifies an action at every information set at which a player is active.

Exercise 3. Now suppose that the monopoly observes whether the challenger enters or stays out, but not the challenger's level of preparation. Represent this game as a game tree. List each player's information sets and each player's strategies.