Section 4

Econ 152

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Introduction

Today we'll consider labor demand concepts through an extended example problem.

Definitions

Let a firm have production function $q = f(E, K)$, where q is quantity produced, E is employment, and K is capital (from German).

Assume that product is sold at price p. E can be hired at wage w, and K can be rented at price r.

- Marginal product of labor (MPE – E stands for employment). $MP_E = \frac{\partial f(E,K)}{\partial E}$
- Average product of labor. $AP_E = \frac{f(E,K)}{E}$
- Value of marginal product of labor. $VMP_E = p \cdot MP_E$
- Value of average product of labor. $VAP_E = p \cdot AP_E$
- Marginal rate of technical substitution. $MRTS_{EK} = \frac{MP_E}{MP_K}$

Optimization

- Short run. In the short run, we assume K is fixed. So our only choice is to hire the right amount of labor. The optimal point is $VMP_E = w$ (why?), as long as $w = VMP_E < VAP_E$ (why?). This gives the short run labor demand.
- Long run. We choose both the optimal K and optimal E. The tangency condition is given by $MRTS_{EK} = \frac{w}{r}$. This is analogous to the tangency condition in labor supply.

Elasticity

The **elasticity of substitution** measures the percentage change in the capital/labor ratio caused by a 1% change in the price of labor relative to capital, holding output constant. Elasticity of substitution is given by

$$\sigma_{sub} = \left.\frac{\%\Delta(K/E)}{\%\Delta(w/r)}\right|_q$$

The $|_q$ denotes a specific level of q: σ_{sub} can be different at different levels of production.

The **elasticity of labor demand** measures the response of labor demand to a wage change. It is given by

$$\delta = \frac{\%\Delta E}{\%\Delta w}$$

Note. There are separate short run and long run elasticities of labor demand.

Note. Elasticity can be expressed by natural log:

$$\begin{aligned}\sigma_{sub} &= \left.\frac{\%\Delta(K/E)}{\%\Delta(w/r)}\right|_q \\ &= \frac{\partial}{\partial \log(w/r)} \log(K/E)\end{aligned}$$

This means the partial derivative of $\log(K/E)$ with respect to $\log(w/r)$. In other words, solve for $\log(K/E)$, **and treat** $\log(w/r)$ **- the whole thing - as a variable**. You can do a similar thing for δ_{LR} .

Example

Let production be given by $q = AE^{\alpha}K^{\beta}$, where $\alpha, \beta > 0$ and $\alpha + \beta < 1$. Let wage be w and capital price be r; product can be sold at p.

Short run

Exercise 1. Find the MP_E and AP_E.

Exercise 2. Find the VMP_E and VAP_E.

Exercise 3. Let $K = K_0$. Find the short run labor demand.

Exercise 4. Find the short run elasticity of labor demand.

Long run

Exercise 5. Find the $MRTS$.

Exercise 6. Find the elasticity of substitution, σ_{sub}.

Exercise 7. Find the long run labor demand (in terms of w, r, p, and α, β).