

## Section 3

Econ 152

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## Introduction

Today we'll review ideas difference-in-differences and do an example of labor supply with home production.

## Difference-in-Differences (D in D)

### Motivation

Remember (?) from last section omitted variable bias. Let our model be

$$Y_i = \alpha + \beta X_i + \varepsilon_i$$

This is true by definition, since  $\varepsilon_i$  is everything else that affects  $Y_i$ . If we estimate an OLS regression

$$Y_i = \hat{\alpha} + \hat{\beta} X_i + e_i$$

we might have omitted variable bias given by

$$\hat{\beta} = \beta + \frac{Cov(X_i, \varepsilon_i)}{Var(X_i)}$$

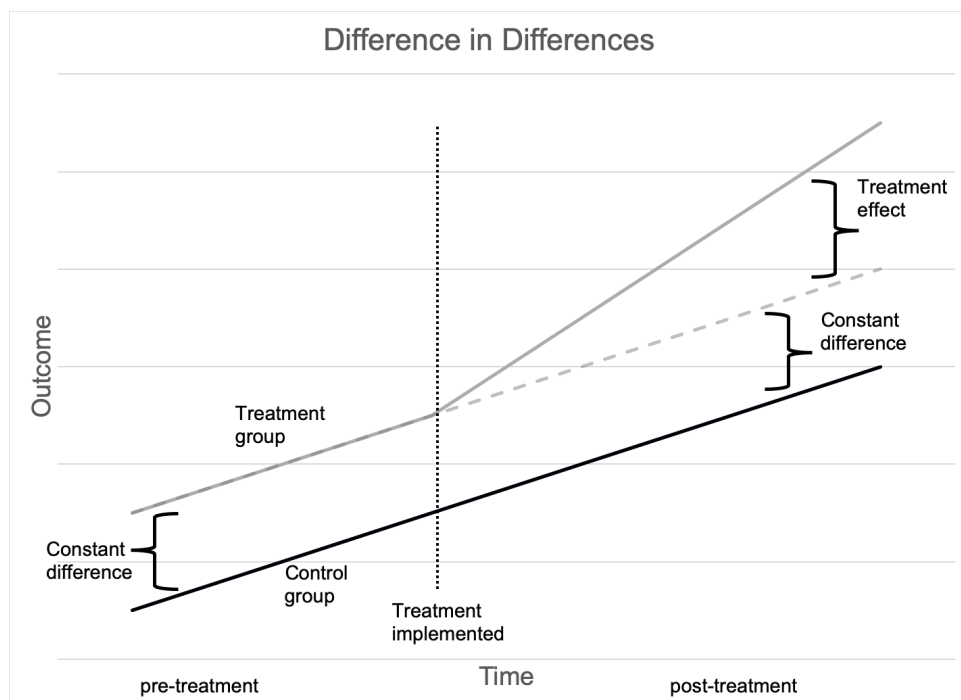
We hope that  $\hat{\beta} = \beta$ , but this isn't always true.

The ideal thing to do is to run an experiment, since this ensures that  $Cov(X_i, \varepsilon_i) = 0$  (why?). But this isn't usually possible in economics. **Difference-in-differences** (also, D-in-D and diff-in-diff) is a simple technique to try to approximate an experiment.

**Basic idea.** We basically apply an experimental analysis to a “quasi-experiment” – an even that isn't actually a random experiment, but looks close to one. There's a “control group” and a “treatment group”. Before the treatment, they look similar. We measure the treatment effect after controlling for the change in the “control group”.

## Estimator

This figure summarizes the situation we're hoping for.



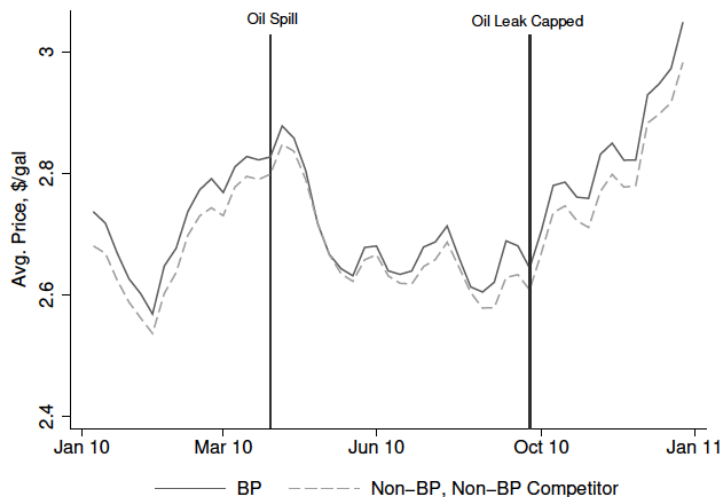
Pre-treatment, we have **two groups that are not the same, but look like they're changing in the same way**. This is called the **parallel pre-trends assumption**. After treatment, the treatment group now changes in a different way.

The estimator formula comes straight from this picture:

$$\text{effect} = \underbrace{(\bar{Y}_{post}^{treatment} - \bar{Y}_{post}^{control})}_{\text{constant difference} + \text{treatment effect}} - \underbrace{(\bar{Y}_{pre}^{treatment} - \bar{Y}_{pre}^{control})}_{\text{constant difference}}$$

Here's an example of diff-in-diff in the wild:

Figure 1: Average Weekly Retail Price for BP and Comparison Group Stations



# Home production

Suppose we have an individual with utility function  $U(x, l)$  and a market wage  $w$ . For simplicity, we'll ignore non-labor income, so  $V = 0$ . Now we introduce “non-market” home production  $f(h_N)$ , where  $h_N$  is hours devoted to non-market work.

- Home production can be interpreted as monetizable informal self-employment, like knitting and selling sweaters. It could also be interpreted as household chores. For simplicity, we'll just assign a consumption-equivalent value to this.

Drawing & writing the budget constraint:

1. Solve  $f'(h_N) = w$  for  $h_N$ . This gives the value of  $h_N^*$  where the consumer switches from home production to market labor. (Why?)
  - What happens if  $f'(h_N) < w$  always, or  $f'(h_N) > w$  always?
2. For the first  $h_N^*$  hours of work, draw the home production function (adding  $V$  if necessary)
3. For the last  $T - h_N^*$  hours of work, draw the wage.
4. The budget constraint function is defined piecewise:

$$C \leq \begin{cases} f(T - L) + V & L > T - h_N^* \\ w \cdot (T - L - h_N^*) + f(h_N^*) + V & L \leq T - h_N^* \end{cases}$$

- What's the interpretation of each part?

Optimizing:

1. Check the tangency condition along the wage slope:  $MRS_{LC} = w$  and plug into the market work portion of the BC. What happens if this gives a solution  $L > T - h_N^*$ ?
2. If you got the solution  $L < h_N^*$  above, check the tangency condition along the home production portion:  $MRS_{LC} = f'(T - L)$  and plug into the home work portion of the BC.

**Exercise 1.** Let utility be  $U(C, L) = C^{1/2}L^{1/2}$ . Suppose  $w = 3$  and home production is  $f(h_N) = 12 \cdot \sqrt{h_n}$ . For simplicity, let  $V = 0$  and  $T = 24$ .

1. Draw the budget constraint.
2. Express the budget constraint as a function.
3. Solve for the optimal bundle.
4. (At home) Suppose home production is given by  $f(h_N) = 24 \cdot \sqrt{h_n}$ . What's the optimal bundle?