Section 7: Perfect Bayesian equilibrium

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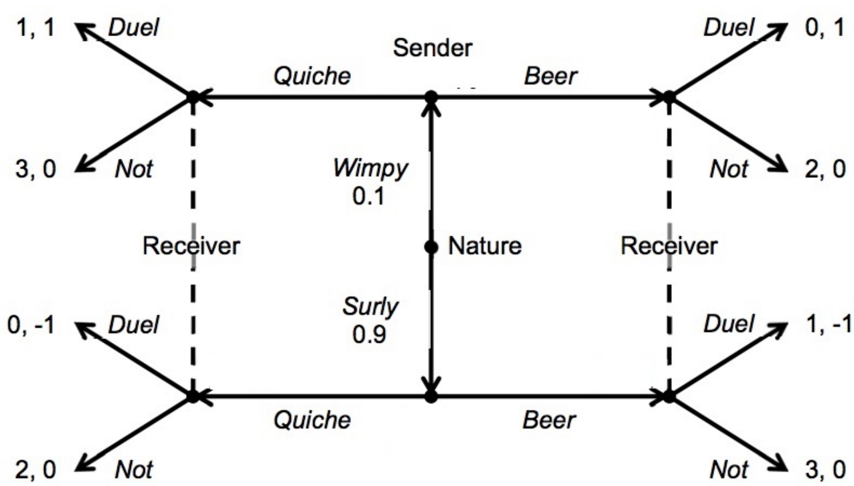
1 Objectives

- Define perfect Bayesian equilibrium
- Find pooling and separating PBE in signaling games, both in formal extensive form games and otherwise

2 Signaling games

Signaling games are games of asymmetric information. There are **senders**, whose types are hidden and send a signal. The **receiver** interprets the signal, forms a belief about the sender's type (called an **assessment**), then chooses an action to play.

Example 1. Beer-quiche signaling game. This game models a dumb interaction in a bar. The **sender** is one of two types: t_1 = "Wimpy" with probability 0.1, and t_2 = "Surly" with probability 0.9. The **receiver** decides whether to duel the sender; he wants to fight, but will only win against the Wimpy type. The sender's signal is the choice of whether to have beer or quiche for breakfast. The Wimpy type prefers quiche, and the Surly type prefers beer; neither type wants to duel. Note that the Wimpy type might be persuaded to drink beer if it deters the receiver from dueling...



3 Perfect Bayesian equilibrium

Definition 1. A perfect Bayesian equilibrium has three elements, which all have to be consistent with each other:

1. Strategy of the senders (for each type!). The strategies have to be best responses given the receiver's strategy.

2. Beliefs (assessment) of the receiver given the signal. The assessment has to be consistent: this means that *if a signal is actually sent in equilibrium*, the belief has to match the senders' actions. E.g., if Wimpy sends Quiche, and Surly sends Beer, then we require $P(Wimpy|Quiche) = 1$ and $P(Surly|Beer) = 1$. Make sure to specify beliefs for **every** possible signal, including ones that aren't played in equilibrium. E.g., if both types eat quiche, then you still need to specify a belief over Beer.

3. Strategy of the receiver in response to the signal. Note: the receiver cannot directly respond to the sender's type – only the signal. This strategy has to be a best response given the receiver's beliefs.

There are three types of **perfect Bayesian equilibria (PBE)** (we'll focus on the first two):

1. **Separating equilibria.** Each type of sender chooses a different signal. The receiver therefore knows what type the sender is based on the signal and can act accordingly.

2. **Pooling equilibria.** All types of senders choose the same signal.

3. **Semi-separating/semi-pooling equilibria.** This can occur with more than 2 senders. Some types pool on the same signal, while other types give unique signals.

4 Finding PBE

Pooling equilibria:

1. Set all sender types' signals to be the same. Let's call it s_1. (You can also try a common mixed strategy).
2. Set the receiver's strategy in order to make the senders' strategy optimal. If this isn't possible, then the signal in step 1 can't be a PBE.
3. Set the assessment. The belief given s_1 has to match the true proportion given by Nature. The beliefs over the other signals can be anything *that makes the above strategy optimal*. If this isn't possible, this PBE isn't possible. Watch out if there's more than 2 types!
4. Check that the receiver's strategy is optimal given the assessment. If not, then this isn't a PBE!

Exercise 1. Find a pooling PBE for Beer-Quiche in which both types signal Quiche. Determine whether a pooling PBE in which both types signal Beer is possible.

Separating equilibria:

1. Set all sender types' signals to be unique. E.g., type 1 sends s_1, type 2 sends s_2.
2. (The same) Set the receiver's strategy in order to make the senders' strategy optimal. If this isn't possible, then the signal in step 1 can't be a PBE.
3. Set the assessment. The beliefs over the signals have to actually match the senders' types. E.g., $P(type1|s_1) = 1$ and $P(type2|s_2) = 1$.
4. (The same) Check that the receiver's strategy is optimal given the assessment. If not, then this isn't a PBE!

Exercise 2. Are there any separating PBE of the Beer-Quiche game?

The Intuitive criterion (IC)

Def. The IC eliminates some pooling equilibria. Consider a signal s_0 that is never sent in equilibrium. If there is any type that would always do worse by sending that signal than the equilibrium, then the IC enforces that the receiver should put 0 probability of that type upon seeing s_0. A PBE either passes or doesn't pass the IC (it's still a PBE though).

Exercise 4. Do any pooling PBE of the Beer-Quiche game pass the IC?