Section 5: Bargaining games

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1 Objectives

- Recognize finite bargaining games and apply backward induction to solve for SPNE
- Apply the one-shot deviation principle to solve infinite bargaining games

2 Finite bargaining games

Note. Recall the following:

- A player's strategy specifies an action *at every node* in which he is called to play. In a bargaining game with continuous offers, the strategy has to specify a response to every possible offer!
- SPNE are equilibria in which strategies are NE in every subgame.
- For finite horizon games, we can use backwards induction to find SPNE.

Exercise 1. (Bargaining with discounting) Consider the bargaining game with a pie of size 1 and two periods: If player 2 rejects player 1's initial proposal, player 2 may make a counterproposal. If this player 1 rejects this proposal, the pie is destroyed. Any payoffs in the second period are discounted by $\delta \in (0, 1)$ for both players. I.e., if player 1 accepts player 2's counterproposal $(y, 1-x)$, the payoffs are $(y\delta, (1-y)\delta)$. Represent this game as a tree and find the SPNE.

Exercise 2. (Bargaining with fixed cost of delay) Consider the same game as above. Instead of discounting, let the period 2 cost of delay be fixed c to each player. E.g., if player 1 accepts player 2's counterproposal $(y, 1-y)$, the payoffs are $(y-c, 1-y-c)$. If player 1 rejects player 2's counterproposal, the payoffs are $(-c, -c)$. Represent this game as a tree and find the SPNE.

3 Infinite bargaining games

One-shot deviation principle. In infinitely repeated games (with some discount factor), a strategy profile is SPNE if and only if there are no profitable "one-shot deviations" for each subgame for every player. A one shot deviation is a change in the player's strategy in a single round, reverting to the original strategy afterwards.

1. Have a candidate SPNE strategy profile. Usually this will be obvious or given to you in the problem. *Note that in SPNE in bargaining games, the players agree to a deal at the first proposal.*
2. Consider deviations in the **first round**. Each round the game that is repeated over and over; usually it's (offer, response, counteroffer, response).
3. In later rounds, there are no deviations. So if both players reject the offers in the first round, they revert to the candidate SPNE profile in the second round. This means they agree to the same deal as in (1), and now the payoffs are discounted (or cost is subtracted).

This means you can apply backward induction in the first round to show SPNE (why?).

Exercise 2. (One-sided offers). Two players are splitting something worth 1 utility. Every round, player 1 makes an offer $(x, 1-x)$. Then player 2 either accepts or rejects the offer; if he rejects, then player 1 makes another offer, etc. After every round (player 1 offers, and player 2 responds) the payoff is discounted by $0 < \delta < 1$. Show that the following is an SPNE: player 1 offers $(1, 0)$ every round; player 2 accepts any offer.