Section 8: Preferences

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1 Objectives

- Define binary relations, preference relations, choice structures, and their properties.
- Apply mathematical notation and logic to these definitions.
- Write proofs involving the above, including for unfamiliar results and properties.

2 Binary relations

Definition 1. A **binary relation** R over S is $R \subseteq S \times S$ (S-cross-S or S-times-S). I.e., R is a set of ordered pairs (x, y) where $x, y \in S$.[1] A binary relation is...

- **reflexive** if for all $a \in S$, $(a, a) \in R$
- **complete** if for all $x, y \in S$, either $(x, y) \in R$ or $(y, x) \in R$ (or both)
- **symmetric** if $(x, y) \in R$ implies $(y, x) \in R$
- **transitive** if $(x, y) \in R$ and $(y, z) \in R$ implies $(x, z) \in R$

We can either write $(x, y) \in R$ or xRy.

[1] An ordered pair means (x, y) is not the same as (y, x).

Exercise 1. Let $S = \{1, 2, 3, 4, 5\}$ and $R \subseteq S \times S$ be the binary relation $\geq$. Note that $S \times S = \{(1, 1), ..(1, 5), (2, 1), ..., (2, 5), ..., (5, 5)\}$.

1. Write $\geq = R \subseteq S \times S$ in set notation.
2. Is $\geq = R$ reflexive, complete, symmetric, transitive? What if R is $>$ instead?

Exercise 2. Let S be the set of all living people. Decide if the following relations are reflexive, symmetric, transitive, and/or complete:

1. "is married to"
2. "is the son or daughter of"
3. "is an ancestor or descendant of"
4. "is weakly taller than"

3 Preferences and choices

Definition 2. A **preference relation** or **rational preference relation** is a binary relation that is complete and transitive. This is usually written $\succsim$. We usually write the strict relation as $\succ$; note that $\succ \subseteq \succsim$.

Definition 3. Let X be a set of consumption bundles, and $x \in X$ be a specific bundle. $WP(x)$ is the set of bundles weakly preferred to x, i.e. $\{y \in X : y \succsim x\}$. $SP(x) = \{y \in X : y \succ x\}$.

Exercise 3. Let $x \succsim y$. Prove that $WP(x) \subseteq WP(y)$.

Definition 4. A **choice structure** is a pair of sets and a set-function $(\mathcal{B}, C(\cdot))$ such that

- $\mathcal{B}$ are sets $B \subseteq X$ (sometimes called *menus*). A note: B is a set/menu, $\mathcal{B}$ is the set of all menus (i.e. a set of sets)
- $C(\cdot)$ is a function from a menu to a set of choices from the menus. We require $C(B) \neq \emptyset$ (you have to choose something) and $C(B) \subseteq B$ (you have to choose from your choices). E.g. $C(\{x, y\}) = \{x\}$ means x is chosen from a menu of x and y.

Note that a choice may have multiple elements (implying they are indifferent to each other).

Definition 5. Weak Axiom of Revealed Preference (WARP). For all $x, y \in X$ and menus B, B' such that $x, y \in B, B'$, if $x \in C(B)$ and $y \in C(B')$, then it must be that $x \in C(B')$.

Exercise 4. A choice rule satisfies Sen's α (also called Independence of Irrelevant Alternatives) if the following holds: If $x \in B \subseteq A$ and $x \in C(A)$, then $x \in C(B)$.

- What is the intuition of Sen's α?
- Show that if a choice structure $(\mathcal{B}, C(\cdot))$ satisfies WARP, then it satisfies Sen's α.