

Section 1: Strategic Games

Econ C110 / PoliSci C135

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This section will cover the basics of representing games in strategic forms. We will also begin to solve games using iterated elimination of strictly dominated strategies (IESDS).

1 Components of games

A game has four components:

1. The **players**
2. The players' **actions/strategies**
3. The **payoffs** – the utilities that each player receives for each outcome of the game
4. The **timing** – who plays, and when. (In the first part of the course, we'll only consider simultaneous games – everyone plays at once).

The typical way to represent a game is in **strategic form**, also called **matrix form**. **Note:** in a two-player matrix, we typically list payoffs as (player 1, player 2). Player 1's strategies are rows; and player 2's strategies are columns. I.e.,

Player2

	<i>a</i>	<i>b</i>
<i>a</i>	1, 1	1, 0
<i>b</i>	0, 1	0, 0

Exercise 1. Represent the following game in strategic (matrix) form. Cities A and B are wooing Amazon to build new offices and bring jobs. Each city can charge high taxes (“h”) or low taxes (“l”). Amazon will build offices in the city with the lower taxes, but if the two cities charge an equal tax rate, Amazon will build half in each city. The benefit to a city of having all of the offices is 4, half of them is 2, and none of them is 0. Additionally, the gain of charging high taxes is 2 and low taxes is 1.

2 Solving games using IESDS

Classical game theory assumes the following of the players:

1. All players are **rational**. That is, they act to maximize their own utility and make no errors.
2. All players have **common knowledge**. They know all players' possible strategies and payoffs. They also know other players are also rational.

IESDS will be our first (and easiest) way of “solving” games. (“Solving” games means finding Nash equilibria or other related concepts, which we’ll cover later.)

Definition 1. A strategy a **strictly dominates** another strategy b if a yields a strictly higher ($>$) payoff no matter what all other players do.

Definition 2. A strategy a **weakly dominates** another strategy b if a yields a weakly higher (\geq) payoff no matter what all other players do.

Definition 3. IESDS is a solution method. Sequentially eliminate strictly dominated strategies. After you eliminate a strategy, it’s no longer considered for the sake of other strategies’ strict dominance. I.e., if you eliminate a strictly dominated strategy, and another strategy afterward becomes strictly dominated, you can eliminate this one too. (Why?)

Exercise 2. Solve the game in exercise 1 using IESDS.

Exercise 3. Prosecutors are interviewing the President’s campaign manager C and personal lawyer L . C and L must each decide whether to stay quiet (q) or flip (f). If they both stay quiet, they decrease their prison sentences compared to both flipping. Yet if one flips while the other stays quiet, the former will go free while the latter will receive a harsh sentence.

		L	
		q	f
C	q	$-1, -1$	$-20, 0$
	f	$0, -20$	$-5, -5$

Solve this game using IESDS.

Exercise 4. A) Are there any strictly dominated strategies in the following game? B) What are the action profiles that survive IESDS? At each step of the elimination what are the rationality and knowledge assumptions?

	C_1	C_2	C_3	C_4
R_1	1, 0	0, 2	2, 2	4, 5
R_2	1, 4	0, 0	3, 2	1, 3
R_3	2, 1	1, 2	2, 0	2, 1