Section 4: Extensive Form Games

Econ C110 / PoliSci C135

Fall 2020

GSI: Andrew Tai

In this section, we will start dynamic games. We will represent dynamic games in extensive form. We will also represent them in matrix form to solve for NE.

1 Dynamic games

In dynamic games, players execute their actions sequentially rather than simultaneously.

Definition 1. The **normal form** of a game specifies the following:

1. The **players**
2. Each player's possible **strategies**
3. The **timing** of each action
4. Each player's **payoffs** for each strategy profile

We can usually represent these things concisely by drawing the extensive form.

Definition 2. For now, we only consider games of **perfect information**. Every player knows the actions that were taken earlier in the game.

Later, we'll consider imperfect information, where a player might not know what another player did earlier.

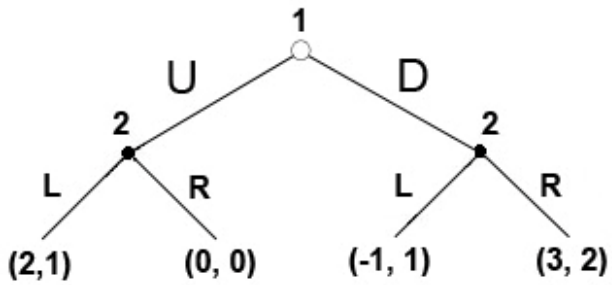
2 Extensive form and strategies

Definition 2. The extensive form is a graphical representation of a dynamic game. When a player takes an action, it shows the possible plays as branches from a **node**.

Definition 3. A player's **strategy** specifies an action *at every node* she is active. It includes nodes that she would never reach. A strategy has as many components as there are nodes for a player!

Definition 4. A game's **histories** is the set (usually called H) of possible paths/events of the game. The **terminal histories** (usually called $Z \subseteq H$) is the set of histories that lead to the end of the game.

Exercise 1. Consider the following extensive form game.



1. How many players are there? Who moves first?
2. How many strategies does player 1 have? What are they? Player 2?
3. What are the histories H? The terminal histories Z?

3 Nash equilibrium of extensive form games

Definition 4. Nash equilibria have the same definition in dynamic games as in static games. Every player is playing a BR to other players' strategies. Equivalently, no single player has a profitable deviation in strategy.

You can also represent a dynamic game in matrix form. Do this as you would in a static game – one row/column for each strategy. Remember that the strategies specify an action at each node! Of course, you lose the timing information by doing this. But this doesn't matter for finding NE.

Proposition 1. The Nash equilibria of an extensive form game are the same as the NE of its static representation. (This includes the mixed strategy NE.)

Exercise 2. What are the NE of exercise 1?

Exercise 3. Two players are dividing two identical items between them. Player 1 makes the offer: he can keep both, give 1 and keep 1, or give both. Player 2 can either accept the offer or reject; if she rejects the offer, both items self-destruct and no one gets anything.

1. Represent this game in extensive form.
2. Represent this game in matrix form.
3. What are the pure strategy NE?