Section 4
Econ 152
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### Introduction

Today we'll consider labor demand concepts through an extended example problem.

#### **Definitions**

Let a firm have production function q = f(E, K), where q is quantity produced, E is employment, and K is capital (from German).

Assume that product is sold at price p. E can be hired at wage w, and K can be rented at price r.

- Marginal product of labor (MPE E stands for employment).  $MP_E = \frac{\partial f(E,K)}{\partial E}$
- Average product of labor.  $AP_E = \frac{f(E,K)}{E}$
- Value of marginal product of labor.  $VMP_E = p \cdot MP_E$
- Value of average product of labor.  $VAP_E = p \cdot AP_E$
- Marginal rate of technical substitution.  $MRTS_{EK} = \frac{MP_E}{MP_K}$

#### Optimization

- Short run. In the short run, we assume K is fixed. So our only choice is to hire the right amount of labor. The optimal point is  $VMP_E = w$  (why?), as long as  $w = VMP_E < VAP_E$  (why?). This gives the short run labor demand.
- Long run. We choose both the optimal K and optimal E. The tangency condition is given by  $MRTS_{EK} = \frac{w}{r}$ . This is analogous to the tangency condition in labor supply.

#### Elasticity

The **elasticity of substitution** measures the percentage change in the capital/labor ratio caused by a 1% change in the price of labor relative to capital, holding output constant. Elasticity of substitution is given by

$$\sigma_{sub} = \left. \frac{\% \Delta(K/E)}{\% \Delta(w/r)} \right|_{q}$$

The  $|_q$  denotes a specific level of q:  $\sigma_{sub}$  can be different at different levels of production.

The **elasticity of labor demand** measures the response of labor demand to a wage change. It is given by

$$\delta = \frac{\% \Delta E}{\% \Delta w}$$

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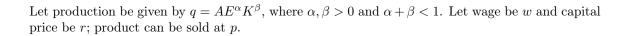
 $\bf Note.$  There are separate short run and long run elasticities of labor demand.

**Note.** Elasticity can be expressed by natural log:

$$\sigma_{sub} = \frac{\%\Delta(K/E)}{\%\Delta(w/r)} \Big|_{q}$$
$$= \frac{\partial}{\partial \log(w/r)} \log(K/E)$$

This means the partial derivative of  $\log(K/E)$  with respect to  $\log(w/r)$ . In other words, solve for  $\log(K/E)$ , and treat  $\log(w/r)$  - the whole thing - as a variable. You can do a similar thing for  $\delta_{LR}$ .

# Example



### Short run

**Exercise 1.** Find the  $MP_E$  and  $AP_E$ .

**Exercise 2.** Find the  $VMP_E$  and  $VAP_E$ .

**Exercise 3.** Let  $K = K_0$ . Find the short run labor demand.

Exercise 4. Find the short run elasticity of labor demand.

## Long run

Exercise 5. Find the MRTS.

**Exercise 6.** Find the elasticity of substitution,  $\sigma_{sub}$ .

**Exercise 7.** Find the long run labor demand (in terms of w, r, p, and  $\alpha, \beta$ ).