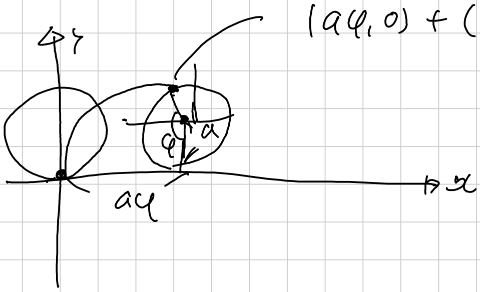
[1]



$(a\varphi, 0) + (-a\cos(\varphi - \frac{\pi}{2}), a + a\sin(\varphi - \frac{\pi}{2}))$

$\cos(\varphi - \frac{\pi}{2}) = \sin\varphi$
$\sin(\varphi - \frac{\pi}{2}) = -\cos\varphi$

(1) $= (a\varphi - a\sin\varphi,\ a - a\cos\varphi)$ //

(2) $P(\varphi) = a[\varphi - \sin\varphi,\ 1 - \cos\varphi]$
$P(\Delta\varphi) = a[(\varphi + \Delta\varphi) - \sin(\varphi + \Delta\varphi),\ 1 - \cos(\varphi + \Delta\varphi)]$

$P(\Delta\varphi + \varphi) - P(\varphi) = \int\sqrt{\left(\frac{\Delta x}{\Delta\varphi}\right)^2 + \left(\frac{\Delta y}{\Delta\varphi}\right)^2}\,d\varphi = \int\sqrt{\left(\frac{dx}{d\varphi}\right)^2 + \left(\frac{dy}{d\varphi}\right)^2}\,d\varphi$ (φ a 2:k$\ge$2)

$\frac{dx}{d\varphi} = a(1 - \cos\varphi)$. $\frac{dy}{d\varphi} = a(\sin\varphi)$

$\therefore (\ )^2 + (\ )^2 = \underbrace{a^2(2 - 2\cos\varphi)}_{2ay} = 4a^2\sin^2\left(\frac{\varphi}{2}\right)d\ell$

$\therefore P(\varphi + \Delta\varphi) - P(\varphi) = 2a\sin\left(\frac{\varphi}{2}\right) = d\ell$

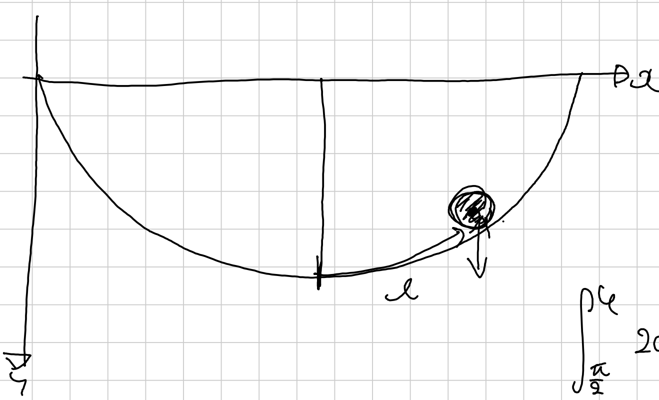
$\therefore \ell = \int_0^{\varphi} 2a\sin\left(\frac{\varphi}{2}\right)d\varphi = 2a\left[-2\cos\frac{\varphi}{2}\right]_0^{\varphi} = 4a\left[1 - \cos\frac{\varphi}{2}\right]$ //

$k = \frac{m\ddot{x}}{x} = \frac{[MLT^{-2}]}{[L]}$

$\rightarrow T$ は 時間 $\quad \overset{[M]}{k}x = m\ddot{x}$

[B]

(4)



$k \cdot m^p \cdot a^q \cdot g^r$
$[M]^p \cdot [L]^q \cdot [LT^{-2}]^r = [T]$

$p = 0,\ r = -\frac{1}{2},\ q = \frac{1}{2}$ //

$\int_{\frac{\pi}{2}}^{\varphi} 2a\sin\left(\frac{\varphi}{2}\right)d\varphi = 2a\left[-2\cos\frac{\varphi}{2}\right]_{\frac{\pi}{2}}^{\varphi}$
$= 4a\left[\frac{\sqrt{2}}{2} - \cos\frac{\varphi}{2}\right]$

(5) $K = \frac{m}{2}(\dot{x}^2 + \dot{y}^2)$ $\qquad$ $(x = a(\varphi - \sin\varphi),\ y = a(1-\cos\varphi))$

$\dot{x} = -a\cos\varphi \cdot \dot{\varphi}, \quad \dot{y} = a\sin\varphi \cdot \dot{\varphi}$

$\therefore K = \frac{ma^2}{2}(\cos^2\varphi \cdot (\dot{\varphi})^2 + \sin^2\varphi \cdot (\dot{\varphi})^2) = \frac{ma^2}{2}(\dot{\varphi})^2$ //

$\varphi = \theta + \pi$

(6) $l = \int_{\pi}^{\theta+\pi} 2a\sin(\frac{\varphi}{2})\,d\varphi = 4a\left[-\cos(\frac{\varphi}{2})\right]_{\pi}^{\theta+\pi} = -4a\cos(\frac{\theta}{2} + \frac{\pi}{2})$

$= \boxed{4a\sin(\frac{\theta}{2})}$ //

$\boxed{y = a(1 - \cos(\theta + \pi)) = a(1 + \cos\theta)}$

$\frac{y}{a} - 1 = \cos\theta$

$\sin^2(\frac{\theta}{2}) = \frac{1 - \cos\theta}{2}$

$(\frac{l}{4a})^2 = \frac{1 - \frac{y}{a} + 1}{2} = \boxed{1 - \frac{y}{2a}}$ $\qquad$ $y = 2a(1 - (\frac{l}{4a})^2)$ //

$V = -mgy = -mg \cdot 2a(1 - (\frac{l}{4a})^2)$ //

$L = \frac{ma^2}{2}\dot{\varphi}^2 + mg \cdot 2a(1 - (\frac{l}{4a})^2)$

$\dot{l} = \frac{4a}{2}\cos(\frac{\theta}{2}) \cdot \dot{\theta}, =$ $\qquad$ $(\dot{\theta})^2 = \left(\frac{\dot{l}}{2a\cos(\frac{\theta}{2})}\right)^2 = (\dot{l})^2 \cdot \frac{1}{4a^2} \cdot \frac{1}{\cos^2(\frac{\theta}{2})}$

$\cos^2(\frac{\theta}{2}) = \frac{y}{2a} = 1 - (\frac{l}{4a})^2$ $\qquad$ $\boxed{= (\dot{l})^2 \cdot \frac{1}{4a^2} \cdot \frac{1}{1 - (\frac{l}{4a})^2}}$

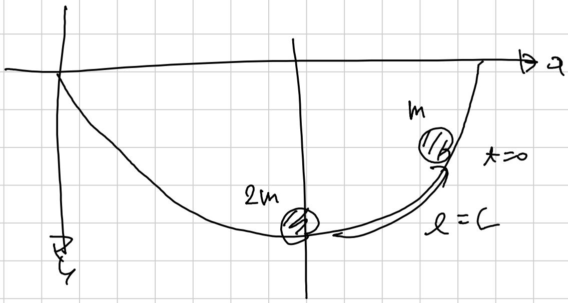
$L = \overbrace{\frac{ma^2}{2}\left[(\dot{l})^2 \cdot \frac{1}{4a^2}\right.}^{\frac{m}{8}} \left.\cdot \frac{1}{1 - (\frac{l}{4a})^2}\right] + mg\,2a \cdot (1 - (\frac{l}{4a})^2)$

$\mathcal{L}[l] = \frac{m}{8} \cdot \underbrace{\frac{d}{dt}\left(2\dot{l} \cdot \frac{1}{1 - (\frac{l}{4a})^2}\right)}_{\shortparallel} + \left(\frac{m}{8} \cdot (\dot{l})^2 \frac{1}{(1 - (\frac{l}{4a})^2)^2} \cdot \frac{2l}{16a^2}\right.$

$2\ddot{l} \cdot \frac{1}{(1 - (\frac{l}{4a})^2)}$ $\qquad$ $\left. + mg\,2a \cdot \frac{2l}{16a^2}\right) = 0$

$-2\dot{l} \cdot \frac{1}{(1 - (\frac{l}{4a})^2)} \cdot (-\frac{l}{2a}) \cdot \dot{l}$

[C]



$$\ddot{l} = -\frac{2gr}{a^2} l$$ であらわすことができる。

⇒完全弾性衝突する場合の解析力学

(ex)

⇒周期はmに依存しない?? →S.

$m_1 = m$, $m_2 = 2m$

① : $v_{10} = v_1$, $v_{20} = 0$ ⇒ $v_{11} = \frac{1}{3} v_1$, $v_{21} = \frac{4}{3} v_1$

$\left(\frac{1}{3}\right)^N v_1$ $\quad \left(\frac{4}{3}\right)^N$

エネルギー保存から解ける.

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(1) $$\mathbb{E} = \begin{cases} 0 & (r<a) \\ \dfrac{q}{4\pi a^2} \dfrac{e_r}{r^2} & (r>a) \end{cases}$$

(2) $$\phi = -\int_{\infty}^{r} \mathbb{E}\, dr = \begin{cases} \dfrac{q}{4\pi a} \\ \end{cases}$$

$$U = \int_{\infty}^{a} \int_{0}^{q} -\frac{q'}{4\pi\varepsilon_0} \cdot \frac{1}{r^2} \cdot dq' dr = \underline{\frac{q^2}{8\pi\varepsilon_0 a^2}}\ /\!/$$

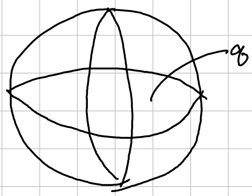
同じになる.

or $$\int_V \frac{1}{2}\varepsilon_0 E^2 dV = \int_a^{\infty} \int_0^{2\pi} \int_0^{\pi} \frac{1}{2}\varepsilon_0 \cdot \frac{q^2}{16\pi^2 a^4} \cdot \frac{1}{r^4} \cdot r^2 \sin\theta\, d\theta\, d\varphi\, dr$$

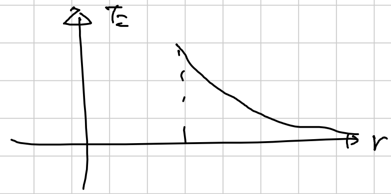
(3) $\underline{4\pi\varepsilon_0 a}$ //

[2]

[A].

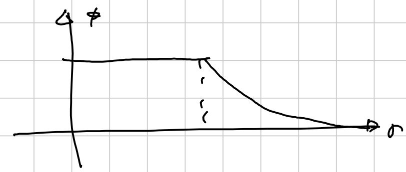


(1): $E = \begin{cases} \dfrac{q}{4\pi\varepsilon_0 r^2} & (r \geq a) \\ 0 & (0 \leq r \leq a) \end{cases}$



$\nabla \cdot E = \dfrac{\rho}{\varepsilon_0} \Rightarrow$

$\phi = -\int_{+\infty}^{r} E \cdot dr = -\left[-\dfrac{q}{4\pi\varepsilon_0 r} \right]_{\infty}^{r} = \dfrac{q}{4\pi\varepsilon_0 r}$

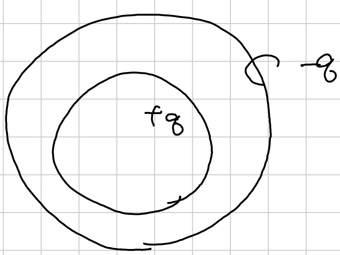


$\therefore \quad \phi = \begin{cases} \dfrac{q}{4\pi\varepsilon_0 r} & (r \geq a) \\ \dfrac{q}{4\pi\varepsilon_0 a} & (0 \leq r \leq a) \end{cases}$

(2) 静電エネルギーは

$C = \dfrac{q}{\phi} = 4\pi\varepsilon_0 a$ 故. $U = \dfrac{q^2}{2C} = \dfrac{q^2}{8\pi\varepsilon_0 a}$

(3)

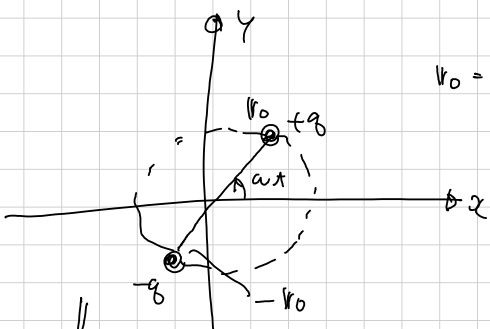


Cを求よ.

$V_{ab} = -\int_{\infty}^{a} = \phi$

$\therefore \ C = \dfrac{q}{\phi} = 4\pi\varepsilon_0 a$

[B]



$r_0 = \left(\frac{d}{2}\cos\omega t, \frac{d}{2}\sin\omega t, 0 \right)$

$\phi = \frac{1}{4\pi\varepsilon_0}\left[\frac{d}{2} \cdot \frac{(\cos\omega t) \cdot r\cos\varphi\sin\theta + \sin(\omega t) \cdot r\sin\theta\sin\varphi}{r^3} \cdot 2q \right.$

$\Downarrow$

$p = 2q r_0(t)$

?? ⇒ ただ単に計算をすれば良いのか?? θ, φのとりあつかいはどうする??

3

$(\Delta + k^2)G(\mathbf{r}) = -\delta(\mathbf{r})$ である

(1) $\tilde{G}(\mathbf{p}) = \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\mathbf{r})\, e^{-i\mathbf{p}\cdot\mathbf{r}} d\mathbf{r}$, $G(\mathbf{r}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{G}(\mathbf{p})\, e^{i\mathbf{p}\cdot\mathbf{r}} d\mathbf{r}$

$$(\Delta + k^2)\tilde{G}(\mathbf{p}) = \left(\frac{1}{2\pi}\right)^3 \iiint \left[\underbrace{\left((\Delta + k^2)G(\mathbf{r})\right)}_{-\delta(\mathbf{r})} e^{-i\mathbf{p}\cdot\mathbf{r}} + G(\mathbf{r}) \cdot (-i p + k^2)\left(e^{-i\mathbf{p}}\right) \mathbf{k} \right]$$

$$= \left(\frac{1}{2\pi}\right)^3 \cdot \left[-1 + (-p^2 + k^2) \cdot \tilde{G}(\mathbf{p}) \right] = \tilde{G}(\mathbf{p})$$

$$-1 + (-p^2 + k^2)\tilde{G} = 8\pi^3 \tilde{G}(\mathbf{p})$$

$$\tilde{G}(\mathbf{p}) = \frac{-1}{8\pi^3 + p^2 - k^2}$$

$$\therefore G(\mathbf{r}) = \iiint \frac{-1}{8\pi^3 + p^2 - k^2} e^{i\mathbf{p}\cdot\mathbf{r}} d\mathbf{p}$$

[2].

$\mathbb{P} = 2q\mathbb{r}_0 = qd(\cos\omega t, \sin\omega t, 0)$,

$$\phi(\mathbb{r}, t) = \frac{1}{4\pi\varepsilon_0}\left[\underbrace{\frac{\mathbb{P}(t-r/c)\cdot \mathbb{e}_r}{r^2}}_{\mathcal{O}\left(\frac{1}{r^2}\right)} + \frac{\dot{\mathbb{P}}(t-\frac{r}{c})\cdot \mathbb{e}_r}{cr}\right]$$

$$= \underline{\frac{1}{4\pi\varepsilon_0}\cdot\frac{\dot{\mathbb{P}}(t-r/c)\cdot\mathbb{e}_r}{cr} + \mathcal{O}\left(\frac{1}{r^2}\right)}$$

(5) $\mathbb{E} = -\nabla\phi - \frac{\partial \mathbb{A}}{\partial t}$, $\mathbb{B} = \nabla\times\mathbb{A}$

$$\nabla\phi = +\frac{1}{4\pi\varepsilon_0}\left[\cancel{\underbrace{\frac{1}{c}\left(\nabla\frac{1}{r}\right)}_{\sim\frac{1}{r^2}\text{ より}}\cdot\dot{\mathbb{P}}(t-r/c)\cdot\mathbb{e}_r} + \frac{1}{cr}\cdot\underbrace{\nabla\dot{\mathbb{P}}(t-r/c)\cdot\mathbb{e}_r}_{\Downarrow\ \nabla\dot{P}_r(t-r/c)}\right]$$

$\underset{\sim\sim\sim}{\frac{\partial\mathbb{A}}{\partial t} = \frac{\mu_0}{4\pi r}\ddot{\mathbb{P}}}$ $\boxed{r\text{は固定}}$ $= -\frac{\mu_0\omega^2}{4\pi r}\mathbb{P}$

$$\therefore \mathbb{E} = \underline{-\frac{1}{4\pi\varepsilon_0}\cdot\frac{1}{cr}\cdot\nabla\dot{\mathbb{P}}_r + \frac{\mu_0\omega^2}{4\pi r}\mathbb{P} + \mathcal{O}\left(\frac{1}{r^2}\right)}\ /\!/$$

$$\mathbb{B} = \frac{\mu_0}{4\pi}\nabla\times\left(\frac{\dot{\mathbb{P}}(t-\frac{r}{c})}{r}\right) = \underline{\frac{\mu_0}{4\pi r}\nabla\times\dot{\mathbb{P}} + \mathcal{O}\left(\frac{1}{r^2}\right)}\ /\!/$$

$$\nabla\times(fA) = \underbrace{(\nabla f)}_{\frac{\mathbb{r}}{r^3}}\times A - f(\nabla\times A)$$

$\nabla f \sim \frac{r}{r^3}$ である

[B] $S_r = |(\mathbb{E}\times\mathbb{H})_r| = |(\mathbb{E}\times\frac{\mathbb{B}}{\mu_0})_r|$

$\nabla\dot{\mathbb{P}}_r = \left(\frac{\partial}{\partial r}\dot{P}\right)\mathbb{e}_r$. $\quad \mathbb{P} = P_r\mathbb{e}_r + P_\varphi\mathbb{e}_\varphi \quad \dot{\mathbb{P}} = \dot{P}_r\mathbb{e}_r + \dot{P}_\varphi\mathbb{e}_\varphi$

$$\nabla\times\dot{\mathbb{P}} = \nabla\times(\dot{P}_r\mathbb{e}_r + \dot{P}_\varphi\mathbb{e}_\varphi) = \frac{1}{r\sin\theta}[\partial_\theta(\dot{P}_\varphi\sin\theta)]\mathbb{e}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\partial_\varphi\dot{P}_r\right]\mathbb{e}_\theta$$
$$+\frac{1}{r}[-\partial_\theta\dot{P}_r]\mathbb{e}_\varphi$$

$$(k^2+\Delta)\tilde{G} = \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Big(\overset{=-\delta(r)}{(k^2+\Delta)G(r)}\, e^{-ip\cdot r} + G(r)\,(k^2+\Delta)e^{-ip\cdot r}\Big)\,dr$$

$$= \frac{1}{(2\pi)^3}\cdot\Big(-1 + G(r)\cdot(k^2-p^2)\,e^{-ip\cdot r}\Big)dr$$

$$(k^2+\Delta)\tilde{G} = \frac{1}{(2\pi)^3}\Big(-1 + (k^2-p^2)\,\tilde{G}\Big)$$

$$(k^2+\Delta)\iiint \tilde{G}(p)\,e^{+ip\cdot r}\,dr = -\delta(r)$$

$$\iiint (k^2-p^2)\,\tilde{G}\,e^{+ip\cdot r}\,dr = -\delta(r)$$

$$(\Delta+k^2)G(r) = -\delta(r)$$

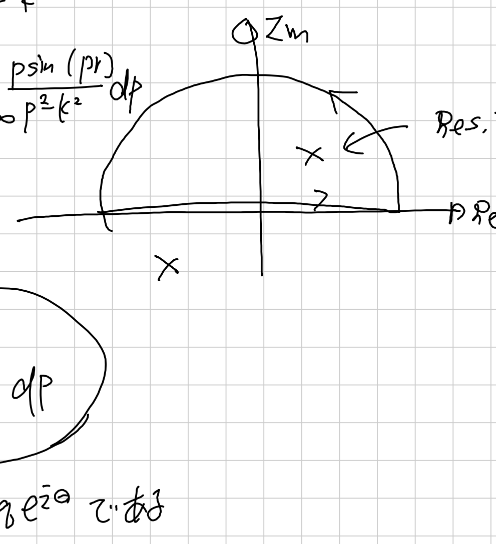
$$\to \delta(r) = \iiint \frac{1}{(2\pi)^3}\,e^{ip\cdot r}$$

$$\tilde{G}(p) = \frac{1}{(2\pi)^3} \qquad \therefore \boxed{\delta(r) = \iiint \frac{1}{(2\pi)^3}\,e^{ip\cdot r}}\ //$$

(2) $\tilde{G}(p) = \frac{1}{(2\pi)^3}\cdot\frac{1}{p^2-k^2}$

$$G(r) = \frac{1}{(2\pi)^3}\int_0^{2\pi}d\varphi\int_0^{\pi}d\theta\int_0^{\infty}dp\ p^2\sin\theta\cdot\frac{e^{-ipr\cos\theta}}{p^2-k^2}$$

$$= \frac{1}{2\pi^2 r}\int_0^{\infty}\underbrace{\frac{p\sin pr}{p^2-k^2}}_{f(p)}dp\ // = \frac{1}{4\pi^2 r}\frac{1}{2i}\int_{-\infty}^{\infty}\frac{p\sin(pr)}{p^2-k^2}dp$$



Res. をもとめたい.

$$\operatorname*{Res}_{p\to k+i\varepsilon} f(p) = \frac{1}{4\pi^2 r}\frac{1}{2i}\int_C\left[\frac{p e^{ipr}}{p^2-k^2} - \frac{p e^{-ipr}}{p^2-k^2}\right]dp$$

$$\int_C \frac{p e^{ipr}}{p^2-k^2} = \int_{-\infty}^{\infty}dp\cdot\frac{p e^{ipr}}{p^2-k^2} + \int_{C_R} \cdots\, dp$$

$\Downarrow$ $\operatorname*{Res}_{p\to k+i\varepsilon} f(p)$

$\Downarrow$ $p = q e^{i\theta}$ である

$$\left|\int_C f(p)dp\right| = \left|\int_0^{\pi}\frac{q e^{i\theta}}{q^2 e^{2i\theta}-k^2}\,d\theta\right| \le \int_0^{\pi}\frac{q e^{i\theta}}{|\ \ |}dp \le \frac{q^2}{q^2-k^2}\int e^{-pr\cos\theta} \le \frac{\pi}{ri}$$

$O(q)$ より収束する

$$C(\mathbf{r}) = \frac{1}{4\pi^2 r}\cdot\frac{1}{2i}\, 2\times 2\pi i\cdot\frac{e^{ikr}}{2} = \frac{e^{ikr}}{4\pi r} \;/\!/ \times$$

[B]

$\frac{\cos kr}{4\pi r}$ となる. //

$$\left(\Delta - \frac{1}{c}\frac{\partial^2}{\partial t^2}\right) G(\mathbf{r}, t) = -\delta(\mathbf{r})\delta(t) \leftarrow 付け, \frac{1}{2\pi}$$

(4) $G(\mathbf{r}, t)$

$$\begin{cases} \tilde{G}(\mathbf{r}, \omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\mathbf{r}, t)\, e^{i\omega t}\, dt \\ G(\mathbf{r}, \omega) = \int_{-\infty}^{\infty} \tilde{G}(\mathbf{r}, t)\, e^{-i\omega t}\, d\omega \end{cases} \quad \text{II}$$

$$\left(\Delta - \frac{1}{c}\frac{\partial^2}{\partial t^2}\right) G = -\delta(\mathbf{r})\delta(t)$$

$$\left(\Delta + \frac{\omega^2}{c^2}\right)\tilde{G} = -\frac{\delta(\mathbf{r})}{2\pi}$$

$\left(\frac{\omega}{c}\right)^2 = k^2$, $2\pi\tilde{G} \Rightarrow G$, [A] から $\boxed{\tilde{G} = \frac{\cos(kr)}{4\pi r}\cdot\frac{1}{2\pi}}$ が成立する

$$G(\mathbf{r}, t) = \frac{1}{8\pi^2 r}\int \frac{e^{ikr} + e^{-ikr}}{2}\, e^{-i\omega t}\, d\omega$$

$$= \frac{1}{16\pi^2 r}\int \left(e^{ikr} + e^{-ikr}\right) e^{-i\omega t}\, d\omega$$

$$= \frac{1}{16\pi^2 r}\int e^{-i\omega(t - r/c)} + e^{-i\omega(t + r/c)}\, d\omega$$

$$= \frac{1}{8\pi^2 r}\cdot\frac{1}{2}\left(2\pi\, \delta(t - r/c) + 2\pi\, \delta(t + r/c)\right) = \frac{1}{8\pi r}\delta(t - r/c) \;/\!/$$