[I]

[A]

(1) $\int_{-\infty}^{\infty} \exp(-ax^2+bx)\,dx = \int_{-\infty}^{\infty} \exp\left(-(a-\frac{b}{2a})^2\right)\cdot \exp\left(\frac{b^2}{4a}\right)dx = \underline{\sqrt{\frac{\pi}{a}}\exp\left(\frac{b^2}{4a}\right)}$ //

[B]

(1) $\Gamma(\frac{1}{2}) = \int_0^{\infty} e^{-t}\, t^{-1/2}\,dt = 2\int_0^{\infty} e^{-\tau^2}\,d\tau = \underline{\sqrt{\pi}}$ // $\boxed{t^{1/2}=\tau,\ \frac{1}{2}t^{-1/2}dt = d\tau}$

(2) $\Gamma(\frac{1}{2}) = \int_0^{\infty} e^{-t}\underbrace{t^{-1/2}}_{=2(t^{1/2})'}dt = 2\underbrace{\int_0^{\infty} e^{-t}\,t^{1/2}\,dt}_{\Gamma(\frac{3}{2})}$ $\therefore\ \underline{\Gamma(\frac{3}{2}) = \frac{\sqrt{\pi}}{2}}$ //

(3) $\Gamma(n) = \int_0^{\infty} \underbrace{e^{-t}}_{=(-e^{-t})'} t^{n-1}\,dt = (n-1)\int_0^{\infty} e^{-t}\,t^{n-2}\,dt = (n-1)\Gamma(n-1)$

$= (n-1)(n-2)\cdots 3\cdot 2\cdot 1\,\underbrace{\Gamma(1)}_{=1} = \underline{(n-1)!}$ //

$\Gamma(n+\frac{1}{2}) = \int_0^{\infty} \underbrace{(e^{-t})}_{=(-e^{-t})'}\, t^{n-\frac{1}{2}}\,dt = (n-\frac{1}{2})\int_0^{\infty} e^{-t}\,t^{n-\frac{3}{2}}\,dt$

$= \cdots = \underline{(n-\frac{1}{2})(n-\frac{3}{2})(n-\frac{5}{2})\cdots(\frac{5}{2})(\frac{3}{2})(\frac{1}{2})\cdot\sqrt{\pi}}$ //

(4) $\Gamma(z) = \frac{\Gamma(z+1)}{z} = \frac{\Gamma(z+2)}{z\cdot(z+1)} = \cdots = \frac{\Gamma(z+n+1)}{z(z+1)\cdots(z+n)}$

$\therefore\ \lim_{z\to -n}\cancel{(z+n)}\frac{\Gamma(z+n+1)}{z(z+1)\cdots\cancel{(z+n)}} = \frac{\Gamma(1)=1}{-n(-n+1)(-n+2)\cdots(-n+n-1)}$

$= \frac{1}{(-1)^n\cdot n(n-1)(n-2)\cdots(1)} = \underline{\frac{1}{(-1)^n\cdot n!}}$ //

[C]

変数分離で解く.

$f(r,t) = R(r)\cdot g(t)$ で分けると. $f(r,t)=0$ も解であり (この時 $v=0$), $f(r,t)\neq 0$ の時等.

$\frac{1}{g(t)}g''(t) = \frac{1}{R(r)}v^2\frac{1}{r^2}(2rR' + r^2R'') = \lambda$ (const)

$\therefore\ g(t) = Ae^{-\sqrt{\lambda}t} + Be^{\sqrt{\lambda}t}$

$R'' + \frac{2}{r}R' - \underbrace{\frac{\lambda}{v^2}}_{=\alpha}R = 0 \Rightarrow \rho = \sqrt{\alpha}\,r$ とすると. $R'' + \frac{2}{\rho}R' + R = 0$

$l=0$ の解であり, $r=0$ で発散しない $= J_0(\rho)$

$\therefore\ f(r,t) = (Ae^{-\sqrt{\lambda}t} + Be^{\sqrt{\lambda}t})J_0(\rho)$.

(6) $f(r, t=0) = \dfrac{\sin kr}{r} = (A+B)\dfrac{\sin \theta}{\theta} = \dfrac{(A+B)}{k} \cdot \dfrac{\sin(kr)}{r}$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\;\; 1 \;\; {}''$

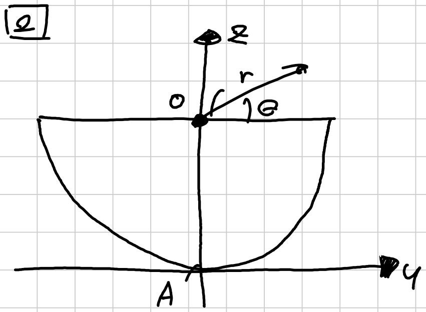
$\dfrac{\partial}{\partial t} f(r, t=0) = (A-B) = 0 \quad \therefore A = B \quad \therefore \quad A = B = \dfrac{k}{2}$

また. $k = \sqrt{\alpha} \quad \therefore \quad k^2 = -\dfrac{\lambda}{v^2} \quad \lambda = -k^2 v^2$

$\therefore \; f(r, t) = \dfrac{\cancel{k}}{2}\left(\exp(-kv\sqrt{-1}\,t) + \exp(kv\sqrt{-1}\,t)\right) \cdot \dfrac{\sin kr}{\cancel{k} r}$

$= \dfrac{\cos(kvt)\sin(kr)}{r}$ //

2



(1) $I_0 = \int r^2\, dm = \int r^2 \sigma \cdot dV = \sigma \int_0^a dr\, r^3 \int_{-\pi}^{0} \theta$

$\frac{2M}{\pi a^2} \leftarrow \sigma \cdot \pi \cdot \frac{1}{4} a^4$

$= \underline{\frac{1}{2} M a^2}$ //

(2) $\mathbf{r}_c = \dfrac{\int \mathbf{r}\, dm}{\int dm} = \dfrac{\int (r\cos\theta, r\sin\theta)\, dm}{\int dm} = \dfrac{\sigma \boxed{\int (r\cos\theta, r\sin\theta)\, r\, dr\, d\theta}}{M}$

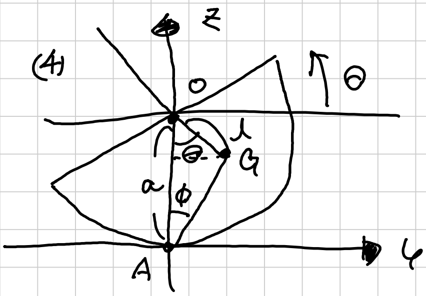
$\int (r\cos\theta, r\sin\theta)\, r\, dr\, d\theta = \frac{1}{3} a^3 \left(\int_{-\pi}^{0} \underset{(\sin\theta)'}{\cos\theta}\, d\theta,\ \int_{-\pi}^{0} \underset{(-\cos\theta)'}{\sin\theta}\, d\theta \right) = \frac{1}{3} a^3 (0, -2)$

$= \left(0, -\frac{2}{3} a^3\right)$

$\therefore\ \mathbf{r}_c = \frac{2}{\pi a^2} \cdot \left(0, -\frac{2}{3} a^3\right) = \boxed{-\frac{4a}{3\pi}}$ $\quad \therefore\ \underline{\ell = \frac{4a}{3\pi}}$ //

(3) (I)

(4)



$Mg(a - \ell\cos\theta_0) = Mg(a - \ell) + \frac{1}{2} I_O \cdot \left(-\frac{v_G}{\ell}\right)^2$ < Oをキジュとの 非慣性系

$I_O = I_G + M\ell^2.$

$I_A = I_G + M(a-\ell)^2$

$I_O = I_A - M(a-\ell)^2 + M\ell^2 = (I_A - Ma^2 + 2Ma\ell)$

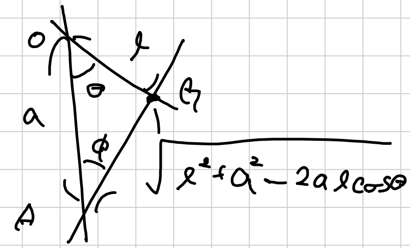
$\therefore\ Mg\ell(1 - \cos\theta_0) = \frac{1}{2}(I_A - Ma^2 + 2Ma\ell)\frac{v_G^2}{\ell^2}$

$\sqrt{\dfrac{2Mg\ell(1-\cos\theta_0)}{I_A - Ma^2 + 2Ma\ell} \cdot \ell^2} = v_G$ //

(5)

$$I_A = I_G + M(l^2 + a^2 - 2al\cos\theta)$$

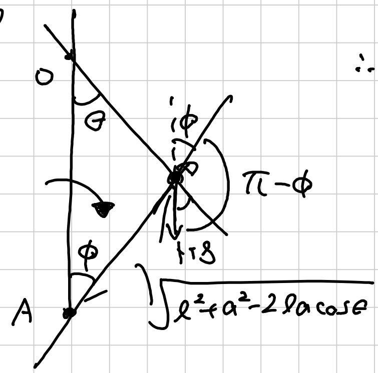
$$I_O = I_G + Ml^2$$



$$\therefore I_A = \underset{\frac{1}{2}Ma^2}{\underset{\shortparallel}{I_O}} + M(a^2 - 2al\cos\theta)$$

$$= \frac{1}{2}M(3a^2 - 4al\cos\theta)$$

(6)



$\sin(\pi - \phi) = -\sin\phi$

$$\therefore \text{Torque} = \int \mathbf{r} \times \mathbf{F} = -\sqrt{l^2+a^2-2al\cos\theta} \cdot Mg\sin\phi$$

$$\therefore I_A(\theta) \cdot \ddot{\phi} = \sim Mg\sin\theta$$

$$\simeq \sim Mg\sin\theta$$

$$\therefore \ddot{\phi} \approx \frac{-\sqrt{\quad}}{\frac{1}{2}M(3a^2 - 4al\cos\theta)} Mg\phi$$

$$\therefore \omega = \sqrt{\frac{\sqrt{\quad} Mg}{\sim}}$$

わかったら一巾でOK

$\boxed{3}$

(1) $m\ddot{x} = \underbrace{-\frac{m}{\tau}\dot{x}}_{\text{周りの電荷による抵抗}} - m\omega_0^2 x - eE$ である

$-m\omega_0^2 x$：電磁場の振動

(2) $\tau \to \infty$

∴ $m\ddot{x} = -m\omega_0^2 x - eE$

$m(-\omega^2)X(\omega) = -m\omega_0^2 X(\omega) - eE_0$

$\hookrightarrow X(\omega) = \frac{eE_0}{m(\omega^2-\omega_0^2)}$

$J = -nev = -ne\dot{x}$

$= -neX(\omega)\cdot(-i\omega)\, e^{-i\omega t}$

$= i\omega\varepsilon_0 \frac{\omega_p^2}{\omega^2-\omega_0^2} E_0\, e^{-i\omega t}$

(3) Maxwell eq より. $\nabla\times H = J + \frac{\partial D}{\partial t}$,

$D = \varepsilon E(r)$, $H = \frac{1}{\mu}B$

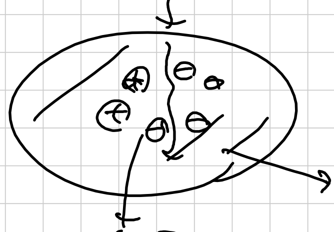
∴ $\frac{1}{\mu_0}\nabla\times B = J + \varepsilon_0\frac{\partial E}{\partial t}$

$\nabla\times B = \mu_0 J + \mu_0\varepsilon_0\frac{\partial E}{\partial t} = \mu_0\varepsilon_0\left[i\omega\frac{\omega_p^2}{\omega^2-\omega_0^2}\cancel{\varepsilon_0}E_0 e^{-i\omega t} + (-i\omega)E_0 e^{-i\omega t}\right]$

[例題1]

$E = E_0\cos(\omega t)$, $m\dot{v} = eE = eE_0\cos(\omega t)$

誘電体 → 真空中では



左辺は $\frac{1}{\mu}\nabla\times B = \varepsilon\frac{\partial D}{\partial t} = -i\omega\cdot\varepsilon E_0 e^{-i\omega t}$

∴ $\mu\varepsilon = \mu_0\varepsilon_0\left[1 - \frac{\omega_p^2}{\omega^2-\omega_0^2}\right]$

∴ $\sqrt{1-\frac{\omega_p^2}{\omega^2-\omega_0^2}}$ //

(4) $\omega < \omega_0$

nに分散公式.

午後 Ⅱ

(1) (ア) $\hat{S}^{-1}\psi(x) = \psi(x+L)$ (イ) 小回等 (ウ) ω (エ) ± 1.

$1, \omega, \omega^2$

(オ) 交換 (カ) $[S, H] = 0, [P, H] = 0$

$$\hat{P}^{-1} S \hat{P}|u\rangle = \hat{P}^{-1} S \psi(-x) = \hat{P}^{-1}\psi(-x-L) = \psi(x+L) = S^{-1}\psi(x)$$

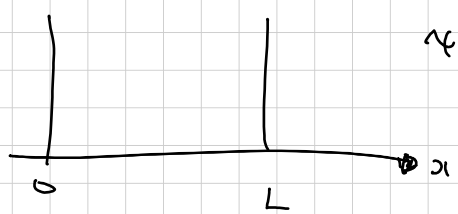
$$\therefore \hat{P}^{-1} S P = S|u\rangle = \lambda|\psi'\rangle$$

$|\psi\rangle$
u

(キ)

$S\psi = \lambda\psi$

$\frac{1}{\lambda}\psi = S^{-1}\psi$



$\psi = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi}{L}x\right)$ $(0 \le x \le L)$ で与えられる

(2)

$H\Phi_1(x) = E_1\Phi_1(x)$

$H\psi_1(x) = e\psi(x)$, とける

$H\psi_2(x) =$

$$H\psi = -\frac{\hbar^2}{2m}\cdot\left(\frac{\pi}{L}\right)^2\cdot(-1)\psi$$

$$= \frac{\hbar^2}{2m}\cdot\frac{\pi^2}{L^2}\psi(x)$$

e 〃

$x+L$でも変わらない

??

(3)

$$\phi_1 - \omega\phi_2 = \frac{1}{\sqrt{3}}\left((1-\omega)\psi_1 + (1-\omega^2)\psi_3\right),$$

⇒ 与式について、 $\psi_1 = \frac{1}{\sqrt{3}}(\phi_1 + \phi_2 + \phi_3)$

$$-i\hbar\partial_t \psi = E\psi \quad \therefore \quad \psi(t) = \exp\left(-i\frac{E}{\hbar}t\right)\psi(0) \ \text{だね}$$

$$\psi(t,x) = \frac{1}{\sqrt{3}}\left(\phi_1 e^{-iE_1 t/\hbar} + \phi_2 e^{-iE_3 t/\hbar} + \phi_3 e^{-iE_3 t/\hbar}\right)$$

$$\therefore \quad |\psi|^2 \sim \left|\phi_1 + (\phi_2+\phi_3)\underbrace{\exp\left((E_1-E_2)i\,t/\hbar\right)}_{\substack{= \\ 1 \\ \text{の時}}}\right|$$

$$-\Delta E\cdot i t/\hbar = -2\pi i \quad \therefore \quad \underline{t = \frac{2\pi\hbar}{\Delta E}}_{//}$$

(5) $t = \frac{T}{2}$ の時

$$\psi = (\underbrace{\phi_1 - \phi_2 - \phi_3}_{=}) \qquad , \qquad \phi_1+\phi_2+\phi_3 = \psi_1$$

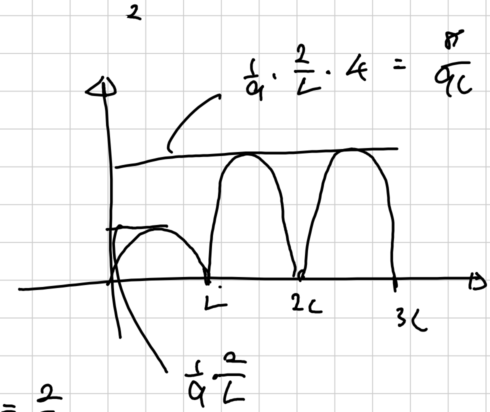
$$(1-\omega^2)\psi_2 + (1-\omega)\psi_3$$

$$-\psi_1 - \omega\psi_2 - \omega^2\psi_3$$

$$-\psi_1 + (\underbrace{1-\omega-\omega^2}_{=2})\psi_2 = (\underbrace{1-\omega^2-\omega}_{=2})\psi_3$$

$$\frac{1}{3}\left(-\psi_1 + 2\psi_2 + 2\psi_3\right)$$

ϕ の発生 $\therefore$



$$|\psi_1|^2 = \frac{2}{L}$$

2

(1) $z = \exp\left(\frac{\mu_0 B}{k_B T}\right) + \exp\left(-\frac{\mu_0 B}{k_B T}\right) = 2\cosh\left(\frac{\mu_0 B}{k_B T}\right)$

$\therefore Z = z^N = 2^N \cosh^N(\beta \mu_0 B)$

(2) $\langle E \rangle = -\frac{\partial}{\partial \beta} Z = -2^N \cdot \mu_0 B \cdot \cosh^{N-1}(\beta \mu_0 B) \cdot \sinh(\beta \mu_0 B)$

$\therefore M = -\frac{E}{B} = \underline{2^N \mu_0 \cosh^{N-1}(\beta \mu_0 B) \sinh(\beta \mu_0 B) \cdot N}$ //

(3) $M = \chi B \quad \therefore \quad \chi = \frac{M}{B} = \frac{2N}{B} \mu_0 \underbrace{\cosh^{N-1}(\beta \mu_0 B)}_{\sim 1} \cdot \underbrace{\sinh(\beta \mu_0 B)}_{\sim \beta \mu_0 B} \cdot N$

$\sim \underline{N \cdot 2^N \cdot \mu_0^2 \beta}$ //

(4) キュリーの法則

(5)

エネルギーが $\varepsilon + d\varepsilon$ の範囲にある量子状態の数が $D(\varepsilon)d\varepsilon$ で与えられるとすると

$D(\varepsilon) \sim$ $\varepsilon \sim d\varepsilon$ の間の量子状態の数

⊙ グランドカノニカル分布

$P = \frac{1}{\Xi} \exp\left[-\frac{1}{k_B T}(E - \mu N)\right]$. $\Xi = \sum_N \sum_n \exp\left[-\frac{1}{k_B T}(E_n(N) - \mu N)\right]$

⇓

grand canonical partial = 大きな正準分布.

7-2 Fermi statistics and Bose statistics

量子状態 i をしめる粒子数を n_i とおく. $\{n_i\} = (n_1, n_2, n_3, \cdots)$

$\Xi(T, \mu) = \sum_{N=0}^{\infty} \sum_{\sum n_i = N} \exp\left[-\frac{1}{k_B T} \sum_i (\varepsilon_i - \mu) n_i\right]$ である.

◎ 理想 Fermi 気体

エネルギー ε の1粒子状態を占める粒子の平均数は.

$$f(\varepsilon) = \frac{1}{\exp\left((\varepsilon-\mu)/k_BT\right)+1}$$

$\Rightarrow$ $\Omega \simeq \dfrac{V}{(2\pi\hbar)^3}$ ← 1粒子状態は [illegible]のように

◎ 有限温度のフェルミ分布

$k_BT \ll \mu$ の低温を考える

$$\boxed{\int_0^\infty D(\varepsilon) f(\varepsilon) = N}, \quad D_+(\varepsilon) = \frac{dN(\varepsilon)}{d\varepsilon}$$

← 平行な場合は

$$\#\{\tilde\varepsilon \mid \tilde\varepsilon - \mu_0 B \le \varepsilon\}$$

反平行で ⊖ される

$$= \#\{\tilde\varepsilon \mid \tilde\varepsilon \le \varepsilon + \mu_0 B\}$$

$$= D(\varepsilon + \mu_0 B)$$

$$D_-(\varepsilon) = D(\varepsilon - \mu_0 B)$$

$$\therefore\ N = \int D(\varepsilon + \mu_0 B) f(\varepsilon)\, d\varepsilon + \int D(\varepsilon - \mu_0 B) \overset{f(\varepsilon)}{\overline{}}\, d\varepsilon$$

$$M = \mu_0 \left[\int D(\varepsilon + \mu_0 B) f(\varepsilon)\, d\varepsilon - \int D(\varepsilon - \mu_0 B)\, d\varepsilon\right]$$

$(\because$ $f = \dfrac{1}{e^{(\varepsilon-\mu)/k_BT}+1} \simeq e^{(\mu-\varepsilon)/k_BT}$ ($\gg$)

$$D(\varepsilon + \mu_0 B) + D(\varepsilon - \mu_0 B) \approx 2D(\varepsilon)$$

??

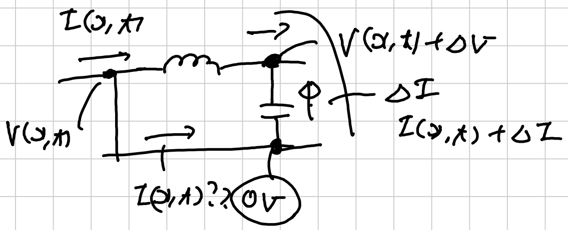
③

回路の方程式より

$$-\Delta V = L\Delta x \frac{dI}{dt} = L\Delta x\, \dot{I}(x,t)$$

$$\Delta I = \frac{dQ}{dt} = \frac{d}{dt}(C\Delta V)$$

(1)



$$\Delta V = -L\Delta x\, \dot{I}, \qquad -\Delta I = \frac{d}{dt}\left(C\Delta x\cdot\left(V(x,t)+\Delta V\right)\right)$$

$$= C\Delta x \cdot \dot{V}$$

$$\frac{\partial V}{\partial x} = -L\frac{dI}{dt}, \qquad -\frac{\partial I}{\partial x} = C\frac{\partial V}{\partial t}$$

$$\frac{\partial^2 V}{\partial x^2} = -L\frac{d^2 I}{dt\,dx} = -L\frac{d}{dt}\left(-C\frac{dV}{dt}\right) = LC\frac{d^2 V}{dt^2}$$

$$\frac{\partial^2 V}{\partial x^2} = -C\frac{\partial^2 V}{\partial t\,\partial x} = -C\frac{\partial}{\partial t}\left(-L\frac{dI}{dt}\right) = LC\frac{d^2 I}{dt^2}$$

(2) $\frac{1}{\sqrt{LC}}$ が速度 $\qquad \underline{l\cdot\sqrt{LC}}$

(3) $f(x-vt)$ であるかとわる

t

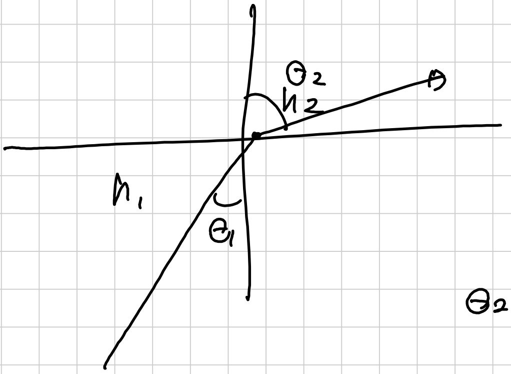
$f(-vt) = F(t')$ $\qquad \underline{V(x,t) = F\left(t - \frac{x}{v}\right)}$

$-vt' = x - vt$

$t' = -\frac{x}{v} + t$

(4) 「ドライトモニ ダンスせいごうの言葉」

[B]



(5)

$\theta_2 = \frac{\pi}{2}$, ??

(6)