(1) = $A^TA = \begin{pmatrix} a^2 & ab \\ ab & b^2 \end{pmatrix}$. $\Rightarrow$ a^2+b^2. (0の[illegible])

$$\begin{pmatrix} -b^2 , & ab \\ ab , & -a^2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0. \quad \therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ab \\ b^2 \end{pmatrix} = \underline{b\begin{pmatrix} a \\ b \end{pmatrix}}$$

(2) $A = BV$

$\Leftrightarrow (a, b) = (c, 0)\begin{pmatrix} v_1 & v_2 \\ v_3 & v_4 \end{pmatrix} = (cv_1, \ cv_2)$ $\quad a = cv_1$, $b = cv_2$.

Vが直交 $\Leftrightarrow V^TV = I$

$$\begin{pmatrix} v_1 & v_3 \\ v_2 & v_4 \end{pmatrix} \cdot \begin{pmatrix} v_1 & v_2 \\ v_3 & v_4 \end{pmatrix} = \begin{pmatrix} v_1^2 + v_3^2 , & v_1v_2 + v_3v_4 \\ v_1v_2 + v_4v_3 , & v_2^2 + v_4^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$A^TA\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}$

$$\boxed{\begin{aligned} v_1 &= \left(\frac{a}{c}\right), \ v_2 = \left(\frac{b}{c}\right) \\ v_3 &= \sqrt{1-\left(\frac{a}{c}\right)^2} \\ v_4 &= \sqrt{1-\left(\frac{b}{c}\right)^2} \end{aligned}}$$

$A^TAA^T = E$, $\quad AV^T = B$,

$A^T = V^TB^T$

$A = BV$

$A^TA = V^T\underbrace{B^TB}_{\shortparallel}V$

$\begin{pmatrix} a^2 & ab \\ ab & b^2 \end{pmatrix} = V^T\begin{pmatrix} c^2 & 0 \\ 0 & 0 \end{pmatrix}V$

$V\begin{pmatrix} a^2 & ab \\ ab & b^2 \end{pmatrix}V^T = \begin{pmatrix} c^2 & 0 \\ 0 & 0 \end{pmatrix} \rightarrow c \neq 0$ $\quad c^2 = a^2+b^2$

(3) $(c, 0)\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = (cb_1) = 1. \quad b_1 = (1/c), \ , b_2 = 0$

(4) $A\tilde{A} = 1,$ $\qquad \tilde{B} = (1/c, 0)$

$B\tilde{B} = B(VV^{-1})\tilde{B} = AV\tilde{B}$

Vでおこるく $\tilde{A}$??

$(\tilde{A}A)^T = (V\tilde{B}BV)^T$

$= V(\tilde{B}B)V = V\tilde{B}BV = \tilde{A}A$ $\quad \therefore A\tilde{A}$は対称 $\therefore A\tilde{A} = 1$

$\rightarrow \tilde{A} = V\tilde{B}$

(5)

(5) $A_n x = v$. $A_n x = \sum_i a_i x_i = v$

$$\sum_i x_i^2 = \min$$

Lagrange の未定乗数法に $F = x^T x + \lambda(A_n x - v) = \sum_{i=1}^{n} x_i^2 + \lambda\left(\sum_i a_i x_i - v\right)$

$\frac{\partial}{\partial x_i} F = 0$ (for all i),

$\left(\frac{\partial F}{\partial x} = \text{Const}\right)$

$$2x_i + \lambda(a_i) = 0$$

$$x_i = -\frac{\lambda a_i}{2}, \rightarrow x = -\frac{\lambda}{2} A^T$$

$\frac{\partial F}{\partial \lambda} = 0 \Leftrightarrow -\frac{\lambda}{2}\sum a_i^2 = v$

$$\therefore \lambda = \frac{-2v}{A^T A} \qquad \therefore x_0 = \frac{v}{A^T A}\cdot A^T$$

$$= v\left(\underbrace{\frac{1}{A^T A} A^T}_{\tilde{A}}\right)$$

(6): $v^T(M^T M)v = v'^T(MM^T)v'$

$M^T M v = \lambda v$. $MM^T(\underbrace{Mv}_{u}) = \lambda(\underbrace{Mv}_{u})$

λは一致

$a_j u_j = 0$ より

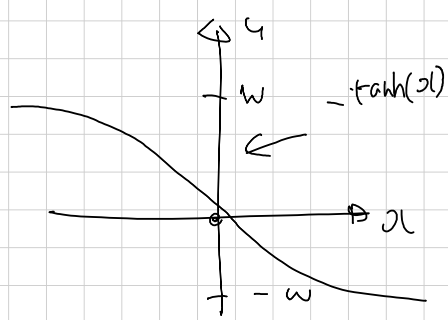
第2問

$\frac{\partial u}{\partial x} = -\frac{\partial u}{\partial t}$, $u(x,t) = f(x-t)$

(2) $u\frac{du}{dx} = \frac{d^2 u}{dx^2}$,

(i) $\frac{d}{dx}(F) = 0$. $F(u,x)$

$$\frac{d}{dx}\left(\frac{du}{dx} - \frac{1}{2}u^2\right) = 0$$



$$\frac{1}{2}u^2 - \frac{du}{dx} = C$$

先に境界条件を入れる

$x = 0$ … $-\frac{du}{dx} = C$. → 0になる. uは一定

$x = +\infty$ … $\left.\frac{1}{2}u^2 - \cancel{\frac{du}{dx}}\right|_{x\to\infty} = C$ → $C = \frac{1}{2}W^2$

$\frac{1}{2}u^2 - \frac{du}{dx} = \frac{1}{2}w^2 \longleftrightarrow \quad \frac{1}{2}(u^2-w^2) = \frac{du}{dx} \quad \Rightarrow \int \frac{2}{u^2-w^2}\,du = \int dx$

変数分離

$$\int \left(\frac{1}{u-w} - \frac{1}{u+w}\right)\frac{2}{2w}\,du = \int dx$$

$$\frac{1}{w}\{\log(w-u) - \log(u+w)\} = x$$

$(\because u-w<0 \ (\ (-w<u<w)))$

$$\log\left(\frac{w-u}{u+w}\right) = xw \qquad \frac{w-u}{u+w} = e^{xw} \quad \therefore u = -w\,\frac{e^{xw}-1}{e^{wx}+1} = -w\tanh\left(\frac{xw}{2}\right)$$

---

3. $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = D\frac{\partial^2 u}{\partial x^2}.$

(i): $\frac{\partial S}{\partial t} = \frac{1}{2}\left(\frac{\partial S}{\partial x}\right)^2 + D\frac{\partial^2 S}{\partial x^2}$

$u^* = -\frac{\partial S}{\partial x}, \Rightarrow \quad \frac{\partial u^*}{\partial t} + u\frac{\partial u^*}{\partial x} = D\frac{\partial^2 u^*}{\partial x^2}$ であるか.

$-\frac{\partial S}{\partial t\partial x} - u\frac{\partial^2 S}{\partial x^2}$

$-\frac{\partial}{\partial x}\left(\frac{1}{2}\left(\frac{\partial S}{\partial x}\right)^2 + D\frac{\partial^2 S}{\partial x^2}\right) - u\frac{\partial^2 S}{\partial x^2}$

$$\frac{\partial}{\partial t}\left(\frac{\partial S}{\partial x}\right) = \underbrace{\frac{1}{2}\cdot\frac{\partial}{\partial x}\left(-u^{*2}\right)}_{\shortparallel} + D\frac{\partial^2}{\partial x^2}\left(-u^*\right)$$

$$\frac{\partial u^*}{\partial t} = -u^*\cdot\frac{\partial u^*}{\partial x} - D\frac{\partial^2 u^*}{\partial x^2}$$

(ii): $\frac{\partial \phi}{\partial x} = D\frac{\partial^2\phi}{\partial x^2}$ になる.

$S^*(t,x) = S(\phi(x))$

$$\frac{\partial S(\phi)}{\partial t} = \frac{1}{2}\cdot\left(\frac{\partial S(\phi)}{\partial x}\right)^2 + D\left(\frac{\partial^2 S(\phi)}{\partial x^2}\right)$$

$\shortparallel \qquad \shortparallel$

$\frac{\partial S(\phi)}{\partial \phi}\cdot\frac{\partial\phi}{\partial t} \qquad \left(\frac{\partial\phi}{\partial x}\cdot\frac{\partial S}{\partial\phi}\right)$

$\cancel{D}\,\frac{\partial^2\phi}{\partial x^2}\cdot\cancel{\frac{\partial S(\phi)}{\partial\phi}}$

$= \frac{1}{2}\cdot D^{\cancel{2}}\left(\frac{\partial^2\phi}{\partial x^2}\right)^2\cdot\left(\frac{\partial S}{\partial\phi}\right)^{\cancel{2}}$

$+\cancel{D}\left(\frac{\partial^2\phi}{\partial x^2}\cdot\cancel{\frac{\partial S}{\partial\phi}} + \left(\frac{\partial\phi}{\partial x}\right)^2\cdot\left(\frac{\partial^2 S}{\partial\phi^2}\right)\right)$

$D\frac{\partial^2\phi}{\partial x^2}$

$$\frac{\partial\phi}{\partial t}\cdot\frac{\partial S(\phi)}{\partial\phi} = \frac{1}{2}\cdot\left(\frac{\partial\phi}{\partial x}\cdot\frac{\partial S}{\partial\phi}\right)^2 + D\left(\frac{\partial^2\phi}{\partial x^2}\cdot\frac{\partial S}{\partial\phi} + \left(\frac{\partial\phi}{\partial x}\right)^2\cdot\frac{\partial^2 S}{\partial\phi^2}\right)$$

$D\ \frac{\partial^2\phi}{\partial x^2}\frac{\partial S}{\partial\phi} + \left(\frac{\partial\phi}{\partial x}\right)^2\cdot\frac{\partial^2 S}{\partial\phi^2}$

$$-\frac{1}{2}\left(\frac{\partial S}{\partial \phi}\right)^2 = D\frac{\partial^2 S}{\partial \phi^2} \longrightarrow \phi\ で微分$$

$$-\left(\frac{\partial^2 S}{\partial \phi^2}\right)\cdot\left(\frac{\partial S}{\partial \phi}\right) = D\frac{\partial^3 S}{\partial \phi^3} \qquad \left(u = \frac{\partial S}{\partial \phi}\right)であると、$$

$$\underline{-\left(\frac{\partial u}{\partial \phi}\right)\cdot u = D\cdot\frac{\partial^2 u}{\partial \phi^2}} \rightarrow u\frac{\partial u}{\partial \phi} = -D\cdot\frac{\partial^2 u}{\partial \phi^2}$$

---

(1) $H = \begin{pmatrix} \langle 1|H|1\rangle & \langle 1|H|2\rangle \\ \langle 2|H|1\rangle & \langle 2|H|2\rangle \end{pmatrix} = \begin{pmatrix} E_1 & V \\ V & E_2 \end{pmatrix}$

$\langle 2|(H - E_1 I)|2\rangle = E_2 - E_2 = 0,\quad V^2$

(2) $\lambda = \frac{1}{2}\left((E_1+E_2) \pm \sqrt{(E_1-E_2)^2+4V^2}\right)$

$\langle 2|\psi_+\rangle / \langle 1|\psi_+\rangle = \frac{1}{2V}\left(E_2 - E_1 + \sqrt{(E_1-E_2)^2+4V^2}\right)$

$\langle 2|\psi_-\rangle / \langle 1|\psi_-\rangle = \frac{1}{2V}\left(E_2 - E_1 - \sqrt{(E_1-E_2)^2+4V^2}\right)$

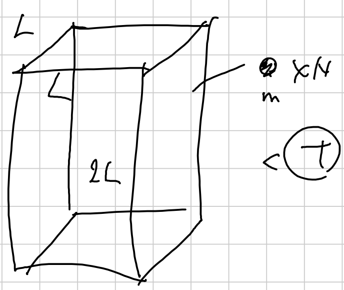
→ ミニット

(4) $E_{1-} - E_1 = \sqrt{(\ )^2+4V^2}$

$\geq 2V$

ok

$\frac{d}{dt}\tilde{C}_- = \frac{i}{2}\xi\cdot\frac{\dot{\theta}}{T}\tilde{C}_+$



I,

(i): $Z = \frac{1}{N!}\frac{1}{h^{3N}}\iint dr_1^x dr_1^y \cdots dp_1^x \cdots$

$= Z = \frac{V^{3N}}{N!}\cdot h^{3N}\cdot\left(\frac{1}{\beta}2m\pi\right)^{3/2N}$

$U = -\frac{\partial}{\partial\beta}\log Z = +\frac{V^{3N}}{N!}\cdot\frac{1}{h^{3N}}\cdot\frac{3}{2}N\frac{\partial}{\partial\beta}(\log\beta + \log\cdots)$

$= \cancel{\frac{V^{3N}}{N!}\cdot h^{3N}}\cdot\frac{3}{2}N\cdot\frac{1}{\beta} = \frac{3}{2}N\beta^{-1}$

$C = \frac{\partial U}{\partial T} = \frac{3}{2}Nk_B$

(ii) $P = -\left(\frac{\partial F}{\partial V}\right)_T \qquad F = -\frac{1}{\beta}\log Z = -\frac{1}{\beta}N\log V$

$= +\frac{1}{\beta}N\frac{1}{V}$

$$S(T_0) - S\left(\frac{T_0}{2}\right) = \int_{\frac{T_0}{2}}^{T_0} \frac{C}{T'} dT' = \frac{3}{2} N k_B \log 2$$

2.

(i) $$\Xi = \sum_{N=0}^{\infty} Z e^{\beta \mu N} = \exp\left(\frac{V}{h^3} \left(\frac{2\pi m}{\beta}\right)^{3/2} e^{\beta\mu} \right)$$

$$J = -\frac{1}{\beta} \log \Xi = -\frac{1}{\beta} \cdot \frac{1}{h^3} \left(\frac{2\pi m}{\beta}\right)^{3/2} e^{\beta\mu}$$ が成立する

$$dJ = -p\,dV - S\,dT - N\,d\mu$$

$$N = -\left(\frac{\partial J}{\partial \mu}\right)_{V,T} = \frac{V}{h^3} \left(\frac{2\pi m}{\beta}\right)^{3/2} e^{\beta\mu}$$

$$N(T_1)/N(T_2) = \left(\frac{\beta_2}{\beta_1}\right)^{3/2} e^{(\beta_1-\beta_2)\mu} = \left(\frac{T_1}{T_2}\right)^{3/2} \cdot e^{\frac{\mu}{k_B}\left(\frac{1}{T_1}-\frac{1}{T_2}\right)}$$ が成立する.

$$p = -\left(\frac{\partial J}{\partial V}\right)_{T,\mu} = \frac{1}{\beta} \cdot \frac{1}{h^3} \cdot \left(\frac{2\pi m}{\beta}\right)^{3/2} e^{\beta\mu}$$ である

$$n = \frac{N}{V} = \frac{1}{h^3} \cdot \left(\frac{2\pi m}{\beta}\right)^{3/2} e^{\beta\mu} \quad \therefore n = \beta p \Rightarrow \boxed{pV = N k_B T}$$ になっている

(i) $$E_0 = \frac{\hbar^2}{2m} \cdot \frac{(1+1+\frac{1}{4})}{L^2} = \hbar^2 \frac{9\pi^2}{8L^2} N \quad , \quad E_1 = \frac{9}{4} \cdot \frac{\pi^2\hbar^2}{2mL^2}(N-1) + 3\frac{\pi^2\hbar^2}{2mL^2}$$

$$f_B(E_0) + f_B(E_1) = N$$ である.

全エネルギー $$U = \left(E_0 f_0(E_0) + E_1 f_1(E_1) + \cdots\right) N$$

$$= E_0 N + (E_1 - E_0) f_B = E_0 N + \frac{1}{e^{\beta(E_1 - E_0)} - 1}(E_1 - E_0)$$

$$\longrightarrow E_0 N \quad (T \to 0)$$