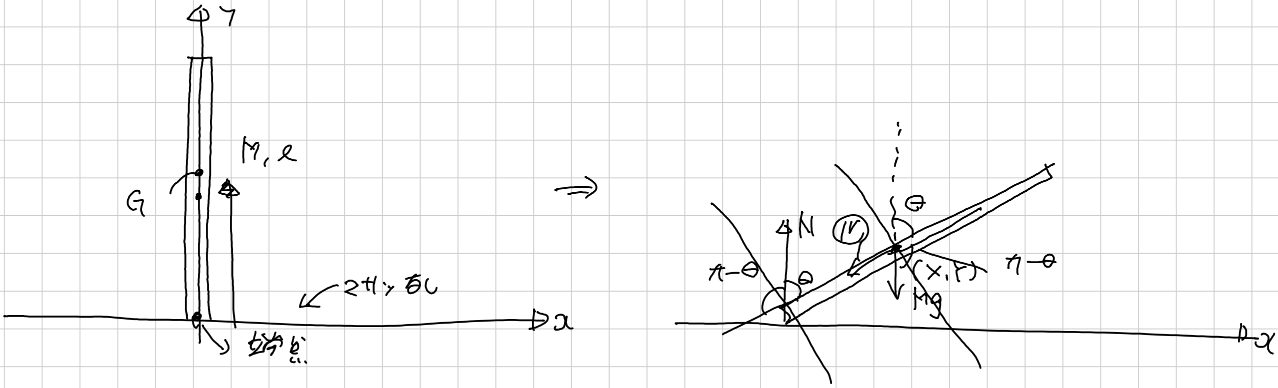


(1)

$I_O = \int_0^l \rho x^2 dx$　　$\rho = \frac{M}{l}$, $= \frac{1}{3}x^3$

$= \frac{M}{3l}\cdot l^3 = \frac{1}{3}Ml^2$

$I_G = \int_{-l/2}^{l/2} \rho x^2 dx = \frac{M}{l}\cdot\frac{1}{3}\cdot\left(\frac{l^3}{8}+\frac{l^3}{8}\right) = \frac{M}{12}l^2$　　$\left(\frac{l^3}{4}\right)$

(2)

M.E：　$M\ddot{x} = 0$.　$M\ddot{y} = -Mg + N$

回転の方程式：　$\underbrace{\sum r\times F_G}_{-\frac{l}{2}\cdot N\cdot\sin\theta} = I_G(-\ddot{\theta})$　∴　$I_G\ddot{\theta} = \frac{l}{2}N\sin\theta$ ①

(3) 剛体の回転エネルギー.

$$\begin{cases} T = \frac{1}{2}\omega^T I \omega = \frac{1}{2}(\dot{\theta})^2\cdot I_G + \frac{1}{2}M(\dot{x}^2+\dot{y}^2) \\ U = Mgy \\ T_0 = 0 \\ U_0 = Mg\cdot\frac{1}{2}l \end{cases}$$

$\ddot{x} = 0$ より $\dot{x} = 0$　($\dot{x}(0) = 0$)

∴ $\frac{1}{2}(\dot{\theta})^2 I_G + \frac{1}{2}M(\dot{x}^2+\dot{y}^2)$ $+\frac{1}{2}Mgl\cos\theta$ $= Mg\cdot\frac{1}{2}l$

(4) ①より

$\ddot{\theta} = \frac{l}{2I_G}N\sin\theta = \frac{6}{Ml}\cdot N\sin\theta$

支点の方程式より.　$+\frac{l}{2}\cdot Mg\sin\theta = I_O\cdot(+\ddot{\theta})$　∴　$\boxed{\ddot{\theta} = \frac{l}{2I_O}\cdot Mg\sin\theta}$

$\frac{l}{2I_O}Mg\cancel{\sin\theta} = \frac{l}{2I_G}N\cancel{\sin\theta}$　　$N = \frac{I_G}{I_O}\cdot Mg$

$\ddot{\theta} = \frac{3}{2Ml}\cancel{Mg\sin\theta} = \cancel{\frac{3g}{2l}\sin\theta}$

$N = \frac{Ml\ddot{\theta}}{6\sin\theta} = \frac{Ml}{6\sin\theta}\dot{\omega}$

$= \boxed{}$

→ $\ddot{\theta}$ を考えること!!

→ $\dot{y}$ は, $y = \frac{l}{2}\cos\theta$ より $\dot{y} = -\frac{l}{2}\sin\theta\cdot\omega$ である!!

(5) $N = \frac{\frac{M}{12}l^2}{\frac{1}{3}Ml^2}\cdot Mg = \frac{1}{4}Mg$

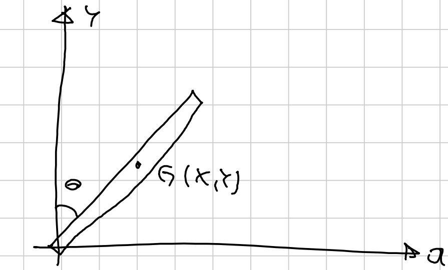
$\frac{1}{24}\omega^2\cdot Ml^2 + \frac{1}{2}Mgl\cos\theta + \frac{1}{2}M\cdot\frac{l^2}{4}\sin^2\theta\,\omega^2 = \frac{1}{2}Mgl$

$\omega^2(l + 3l\sin^2\theta) = 12g - 12g\cos\theta$

$\omega = \sqrt{\frac{12g}{l}\cdot\frac{1-\cos\theta}{1+3\sin^2\theta}}$

$\dot{\omega} = \frac{1}{2}(\quad)^{-1/2}$

[B]



(6) すべっていない場合でもすべっている場合でも同じなので、エネ保より

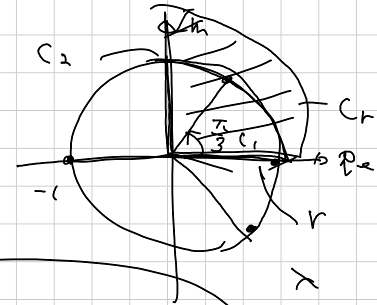
$$Mg\cdot\frac{1}{2}l = \frac{1}{2}(\underset{\omega_f}{\dot{\theta}})^2\cdot I_G + \frac{1}{2}Mv_f^2 \qquad \therefore\ \underline{\frac{1}{2}Mgl = \frac{1}{2}\omega_f^2\cdot\frac{M}{12}l^2 + \frac{1}{2}Mv_f^2}$$

$$\boxed{\frac{1}{2}l\omega_f = v_f}$$

(7) (ウ)：v_fの値は2つの場合で変わらないため

[2]

(A) $f(z) = \frac{1}{z^3+1} = \frac{1}{(z+1)(z-e^{\frac{\pi}{3}i})(z-e^{-\frac{\pi}{3}i})}$

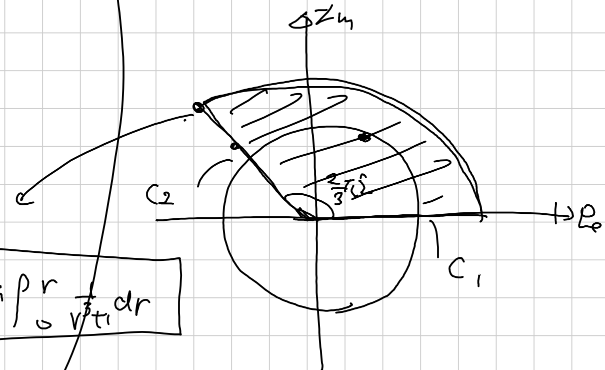


右図の領域での積分を考えて留数定理を使うことを考える

$\int_C f(z)dz = 2\pi i \, Res(z=e^{\frac{\pi}{3}i}) = 2\pi i \frac{1}{(e^{\frac{\pi}{3}i}+1)(e^{\frac{\pi}{3}i}-e^{-\frac{\pi}{3}i})}$

$\int_{C_r}|f(z)dz| \leq \int_{C_r} \frac{1}{r^3-1}dr \sim O(r^{-2}) \to 0 \quad (r\to\infty)$

$|z|^3 - 1 \leq |z^3+1| \leq |z|^3+1$



$C_2 = \int_{re^{\frac{2}{3}\pi i}}^{0} \frac{1}{z^3+1}dz = \int_r^0 \frac{1}{r^3+1}\cdot e^{i\frac{2}{3}\pi}dr$

$r\cdot e^{i\frac{2}{3}\pi} = z \ : \ e^{i\frac{2}{3}\pi}dr = dz$

$\boxed{-e^{i\frac{2}{3}\pi}\int_0^r \frac{1}{r^3+1}dr}$

$C_1 = \int_0^r \frac{1}{r^3+1}dr$

$\therefore \quad (1-e^{i\frac{2}{3}\pi})\int_0^\infty \frac{1}{x^3+1}dx =$

$e^{\frac{\pi}{3}i} = \frac{1}{2}+\frac{\sqrt{3}}{2}i$

$e^{-\frac{\pi}{3}i} = \frac{1}{2}-\frac{\sqrt{3}}{2}i$

$e^{i\frac{2}{3}\pi} = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$

$(\frac{3}{2}-\frac{\sqrt{3}}{2}i)(\quad) = \frac{2\pi i}{(\frac{3}{2}+\frac{\sqrt{3}}{2}i)(\sqrt{3}i)}$

$(\quad) = \frac{2\pi}{\underbrace{(\frac{9}{4}+\frac{3}{4})}_{3}\cdot\sqrt{3}} = \boxed{\frac{2}{9}\sqrt{3}\pi}$

[B]

$F(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-ikx}dx$

(2) $\int_{-\infty}^{\infty} F(k)\cdot(G(k))^* dk = \int_{-\infty}^{\infty} f(x)(g(x))^* dx$ を示す

$\shortparallel$

$\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\left(\int_{-\infty}^{\infty} f(x)e^{-ikx}dx\right)\left(\int_{-\infty}^{\infty} g^*(y)e^{iky}dy\right)\right]dk$

$\shortparallel$

$\infty \quad \infty$

$f(x)=1$の時

$F(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-ikx}dx$

$1 = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(x)e^{ikx}dk$

$\int_{-\infty}^{\infty} e^{-ikx}$

$$\int_{-\infty}^{\infty} f(x)\cdot\underbrace{\left(\int_{-\infty}^{\infty} G^*(k)\, e^{-ikx}\, dk\right)}_{g^*(x)} dx = \int_{-\infty}^{\infty} dk \int_{-\infty}^{\infty} dx\ G^*(k)\ f(x) e^{-ikx}\, dx$$

$$= \underline{\int_{-\infty}^{\infty} dk\ G^*(k)\cdot F(k)} \quad \text{ok}$$

(別に $\int \frac{dk}{2\pi} e^{i(y-x)k} dk = \delta(y-x)$ をつかって良い

(3)

$$F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-|x| - ikx}\, dx = \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-x-ikx}\, dx + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} e^{x-ikx}\, dx$$

$$\frac{1}{\sqrt{2\pi}} \left[-\frac{1}{1-ik}\, e^{-x-ikx} \right]_0^{\infty} = \frac{1}{\sqrt{2\pi}} \left(\frac{1}{1+ik} \right)$$

$|\ | \to 0$

$$\frac{1}{\sqrt{2\pi}} \left[\frac{1}{1-ik} e^{x-ikx} \right]_{-\infty}^{0} = \frac{1}{\sqrt{2\pi}} \left(\frac{1}{1-ik} \right)$$

$$\therefore F(k) = \frac{1}{\sqrt{2\pi}} \left(\frac{1}{1+ik} + \frac{1}{1-ik} \right) = \underline{\frac{2}{1+k^2} \cdot \frac{1}{\sqrt{2\pi}}}$$

$\frac{4}{2\pi}$

(4)

$F(k) = \frac{2}{1+k^2}\frac{1}{\sqrt{2\pi}}\ G^*(k) = \frac{2}{1+k^2}\frac{1}{\sqrt{2\pi}}$ とする.

$$\frac{2}{\pi} \int_{-\infty}^{\infty} \frac{1}{(1+k^2)^2} dk = \int_{-\infty}^{\infty} e^{-2|x|} dx = 2 \int_0^{\infty} e^{-2x} dx = 2\left[-\frac{1}{2} e^{-x} \right]_0^{\infty}$$

$= 1$

$$\therefore \int_{-\infty}^{\infty} \frac{1}{(1+k^2)^2} dk = \underline{\frac{\pi}{2}}$$

I[ ]

$$\exp\left(\frac{x}{2}\left(z - \frac{1}{z}\right) \right) = \sum_{n=-\infty}^{\infty} J_n(x) z^n \to e^{ix\sin\theta} = \sum_{n=-\infty}^{\infty} J_n(x) e^{in\theta}$$

(5) $$\frac{1}{2\pi} \int_{-\pi}^{\pi} e^{ix\sin\theta - in\theta} d\theta = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} J_n(x) \underbrace{\int_{-\pi}^{\pi} \left(e^{in'\theta - in\theta} \right)}_{} d\theta$$

$\delta(n', n)$ になれば良い

$$\int_{-\pi}^{\pi} \exp(i(n'-n)\theta)\, d\theta = \frac{1}{i(n'-n)} \left[\exp(i(n'-n)\theta) \right]_{-\pi}^{\pi} = 0 \ (\text{for } n' \neq n)$$

$$\exp(in\pi) - \exp(-in\pi)$$

$$(-1)^n - (-1)^{-n} = (-1)^n (1 - (-1)^{-2n}) = 0$$

1

$= 2\pi \ (\text{for } n' = n)$

$$\therefore \quad = \sum_{n=-\infty}^{\infty} J_{n'}(x)\, \delta(n', n) = \underline{J_n(x)} \quad \text{ok}$$

(6)

$$J_0(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{ix\sin\theta}\, d\theta$$

$$F(k) = \frac{1}{\sqrt{2\pi}}\cdot\frac{1}{2\pi}\int_{-\infty}^{\infty} dx \int_{-\pi}^{\pi} e^{ix\sin\theta}\, d\theta \cdot e^{-ikx}$$

$F(k) = \delta(k-k')$ 일때

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \delta(k-k')\cdot e^{+ikx} = \frac{1}{\sqrt{2\pi}} e^{+ik'x}$$

$$\delta(k-k') = \cancel{\frac{1}{\sqrt{2\pi}}}\ \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i(k-k')x}\, dx$$

$$F(k) = \frac{1}{(2\pi)^{3/2}}\cdot\int_{-\pi}^{\pi}\left[\boxed{\int_{-\infty}^{\infty} e^{-i(k-\sin\theta)x}\, dx}\right] d\theta$$

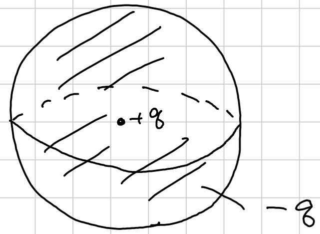
$$= \cancel{\frac{1}{2\pi}}\ \frac{1}{\sqrt{2\pi}}\cdot\int_{-\pi}^{\pi} \delta(k-\sin\theta)\, d\theta$$

$k = \sin\theta$ 일때 $\quad \theta' = \sin^{-1}(k)$

$$\begin{cases} -\pi \le \sin^{-1}(k) \le \pi \text{ 일때} & F[J_0] = \frac{1}{2\pi} \\ \text{그 외} & F[J_0] = 0 \end{cases}$$

[3]

(1)



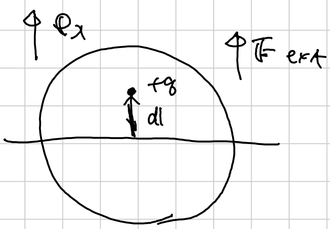
ガウスの法則を用いる. $\int E\cdot dS = \int \frac{\rho}{\varepsilon_0} dV$

$+q$に関して. $E_{+,r}\cdot 4\pi r^2 = \frac{1}{\varepsilon_0} q$ ∴ $\underline{E_{+,r} = \frac{q}{4\pi r^2 \varepsilon_0}}$ //

$-q$に関して. $\int E ds = \int \frac{\rho}{\varepsilon_0} dv = \begin{cases} \frac{1}{\varepsilon_0}\cdot\left(\frac{r}{a}\right)^3\cdot(-q) & (r\le a) \\ \frac{1}{\varepsilon_0}(-q) & (r\ge a)\end{cases}$

∴ $E\cdot 4\pi r^2 = \begin{cases} \frac{1}{\varepsilon_0}\left(\frac{r}{a}\right)^3(-q) \\ \frac{1}{\varepsilon_0}(-q)\end{cases} \longrightarrow E_{-,r} = \begin{cases} -\frac{1}{4\pi\varepsilon_0}\cdot\frac{r}{a^3}q & (r\le a) \\ -\frac{1}{4\pi\varepsilon_0}\cdot\frac{1}{r^2}q & (r\ge a)\end{cases}$ //

(2)



$+q$が動かないための条件は $F = qE_{-,in}$ 等

$F = -q\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{d}{a^3}q\, e_x$

∴ $0 = \left(-\frac{q^2}{4\pi\varepsilon_0 a^3}d + qE_{ext}\right)e_x$

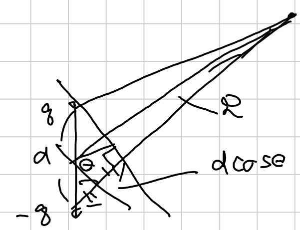
∴ $E_{ext} = \frac{qd}{4\pi\varepsilon_0 a^3}$ $(4\pi a^3)\varepsilon_0 E_{ext} = qd$

α ∴ $\underline{\alpha = 4\pi a^3}$ //

$f'(x) = n(1+ax)^{n-1}a$
$f''(x) = n(n-1)(1+ax)^{n-2}a^2$

$(1+ax)^n$
$\simeq 1 + a\cdot nx$
$+\frac{1}{2!}\cdot(n)(n-1)a^2x^2$

(3)



$-q$ ↖点電荷とみなしていい?

$\phi_P = -\int_\infty^r E(r)dr \quad -\int_\infty^r \frac{1}{4\pi\varepsilon_0}\cdot\frac{1}{r^2}(-q)dr$

$\frac{1}{4\pi\varepsilon_0}\int_\infty^r \frac{1}{r^2}dr = -\frac{1}{4\pi\varepsilon_0 r}$

$\phi_P = -\frac{q}{4\pi\varepsilon_0(R + d/2\cos\theta)} + \frac{q}{4\pi\varepsilon_0(R - d/2\cos\theta)} = \frac{q}{4\pi\varepsilon_0}\Big($

$R \gg d$ より $\longrightarrow \frac{q}{4\pi\varepsilon_0}\frac{1}{R}\cdot\left(-\left(1 - \frac{d}{2R}\cos\theta\right) + \left(1 + \frac{d}{2R}\cos\theta\right)\right)$

$= \frac{q}{4\pi\varepsilon_0}\left(-\left(R - \cancel{\frac{d}{2}\cos\theta}\right) + R\left(+\frac{d}{2}\cos\theta\right)\right) = \underline{\boxed{\frac{q}{4\pi\varepsilon_0}\frac{d}{R^2}}\cos\theta}$ //

(4) $E_P = \left(-\frac{q}{4\pi\varepsilon_0(R + d/2\cos\theta)^2} + \frac{q}{4\pi\varepsilon_0(R - \frac{d}{2}\cos\theta_1)^2}\right)e_R$

$\simeq \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{R^2}\left(-\left(1 - \frac{d}{R}\cos\theta\right) + \left(1 + \frac{d}{R}\cos\theta\right)\right)$

$p\cdot E_P = (\quad)e_R\cdot d\,e_x = (\quad)d\cos\theta_1$

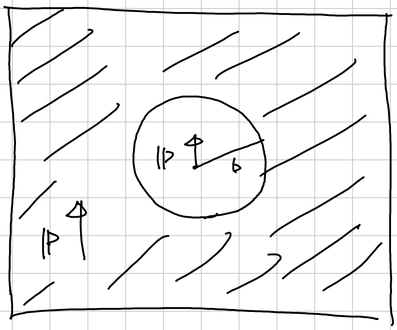
$= \frac{q}{4\pi\varepsilon_0 R^2}\cdot\frac{2d^2}{R}\cos^2\theta_1 = \cancel{0 \; 近似?}$

$= \frac{q}{4\pi\varepsilon_0 R^2}\cdot\frac{2d}{R}\cos\theta_1$

$\cos\theta_1 = 0$??

[B]

(5)



?? (6) ??

② → 家にかえって教科書を読む.

⇒ (4) $E_p = -\nabla_{R,\theta}\phi_p$ である

$$-\left(e_R\partial_R + \frac{1}{R}e_\theta\partial_\theta\right)\frac{qd}{4\pi\epsilon_0}\cdot\frac{1}{R^2}\cos\theta = +e_R\left(\frac{2qd}{4\pi\epsilon_0}\frac{1}{R^3}\cos\theta\right)$$

$$+\frac{1}{R}e_\theta\cdot\frac{qd}{4\pi\epsilon_0}\cdot\frac{1}{R^2}\sin\theta$$

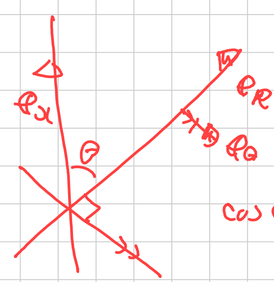
$$=\frac{qd}{4\pi\epsilon_0}\cdot\frac{1}{R^3}\left(2\cos\theta\, e_R + \sin\theta\, e_\theta\right)$$

$e_R\cdot e_x = \cos\theta \qquad e_\theta\cdot e_x = -\sin\theta$

$E_p\cdot e_x = \sim(2\cos^2\theta - \sin^2\theta) = 0$ なので

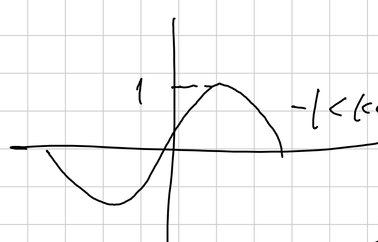
$2\cos^2\theta - (1-\cos^2\theta) = 0$

$\cos\theta = \pm\frac{1}{\sqrt{3}}$



$\cos(\theta + 90^\circ) = -\sin$

$$\frac{1}{\sqrt{2\pi}}\int_{-\pi}^{\pi} d\theta\ \delta(\sin\theta - k)$$



$-1 < k < 1$

$$\hat{j}_k(k) = \frac{1}{\sqrt{2\pi}}\int_{-\pi}^{\pi} d\theta\ \frac{1}{\sqrt{1-k^2}}\left(\delta(\theta-\theta_1) + \delta(\theta-(\pi-\theta_1))\right)$$

$$\cos\theta\Big|_{\sin\theta = k} = \sqrt{1-k^2}$$

IV

(1) $I = \int r^2 dm$ $\therefore I_0 = \frac{Ml^2}{3}$, $I_G = \frac{Ml^2}{12}$

$\to I_0 = I_G + M(\frac{l}{2})^2$: 平行軸の定理

(2) $M\ddot{x} = 0$. $M\ddot{y} = Mg - N$

$\to$ $-\frac{l}{2}\sin\theta N = (-\ddot{\theta}) \Rightarrow \underline{\frac{l}{2}\sin\theta N = \ddot{\theta}}$ //

(3)

$\therefore \frac{Mg}{2}l = \frac{M}{2}(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}I_G\dot{\theta}^2 + \frac{Mgl\cos\theta}{2}$ //

(4)

$\int \mathbb{D} \cdot dS = Q_{true}$.

$\varepsilon_0 \int \mathbb{E} \cdot dS = Q_{true} - \int \mathbb{P} \cdot dS$. $\int \rho_{por} \, dS \to Q_{por}$

$\boxed{\rho_{por} = -\nabla \cdot \mathbb{P} = 0\ ??}$

(6)

$\mathbb{P} = n\mathbb{p}$