H30 (午後)

II

(1) $\sigma_j\sigma_k = (m+(\sigma_j-m))\cdot(m+(\sigma_k-m)) = m^2 + m(\sigma_j-m) + m(\sigma_k-m) + 0$

$= -m^2 + m(\sigma_j+\sigma_k)$

$H = -J\underbrace{\sum_{\langle j,k\rangle}(-m^2)}_{4\cdot\frac{N}{2}\cdot(-m^2)} + \sum_j(-H+2m)\sigma_j = 4\cdot\frac{N}{2}Jm^2 - \sum(+4m^J+H)\sigma_j$

$= 2NJm^2 - \sum(+4m^J+H)\sigma_j$

(2)

$Z_{MF} = \mathrm{tr}\exp(-\beta H_{MF}) = \prod_{\sigma_j}\exp(-\beta 2NJm^2 + \beta(-4m+H)\sigma_j)$

$= \exp(-\beta 2NJm^2)\prod_j\sum_\sigma \exp(+4m^J+H)\sigma_j) = (\ )\cdot\sum_j(\exp(-4m+H) + \exp(-\textcircled{})))$

$2\cosh(\frac{-4m+H}{k_BT})$

$= \exp(-\beta 2NJm^2)[2\cosh(+4m^J+H)]^N$ k_BT

$F_{MF} = -k_BT\log Z_{MF} = -$

"

(3) $\langle H_{MF}\rangle = 2NJm^2 - (-4m+H)Nm$

$\langle E\rangle = \sum_n E_n\cdot P_n = \frac{1}{Z}\sum_n E_n e^{-E_n/k_BT} = \frac{1}{Z}\sum_n E_n e^{-\beta E_n} = -\frac{1}{Z}\frac{\partial}{\partial\beta}\sum_n e^{-\beta E_n}$ ($=Z$)

$= -\frac{1}{Z}\left(\frac{\partial}{\partial\beta}Z\right)$ である

$\cosh\ \frac{e^x+e^{-x}}{2}$

$\sinh\ \frac{e^x-e^{-x}}{2}$

$\therefore -\frac{1}{Z}\cdot\frac{\partial}{\partial\beta}Z = \exp(\ )\cdot(-\frac{1}{Z}\cdot\frac{\partial}{\partial\beta}[2\cosh(\beta(-4m+N))]^N]$

$= -\frac{1}{Z}\exp(\ )\cdot N\cdot\left(2\cosh(\beta(-4m+N))^{N-1}\cdot 2\sinh(\beta(-4m+N))\cdot(-4m+N)\right)$

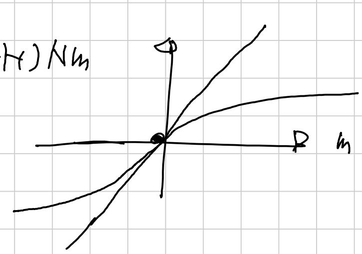
$= -\frac{2N[2\cosh[\beta(-4m+N)]]^{N-1}\cdot\sinh[\beta(-4m+H)]\cdot(-4m+H)}{[2\cosh[\beta(-4m+H)]]^N}$

$= 2NJm^2 - (-4m+H)Nm$ $\qquad +4m^2N$

(4) $-N\cdot\tanh(\beta(-4m+H))\cdot(-4m+H) = 2NJm^2 - (-4m+H)Nm$

$-N\tanh(-4m\beta)(-4m) = 2Nm^2(J+2)$

$+N\tanh(-4m\beta) = +\frac{1}{2}Nm(J+2)$



III ぶつりきそ

(1) $H_{MF} = 2NJm^2 - \Sigma(4mJ - H)\sigma_j$

(2) $Z_{MF} = \mathrm{Tr}\exp(-\beta H_{MF}) = \prod_j \sum_{\sigma_j} \exp(-\beta 2NJm^2 + \beta(4mJ - H)\sigma_j)$

$$= \underline{\exp(-2\beta NJm^2)\,[\,2\cosh\beta(4mJ - H)\,]^N}$$

$$F_{MF} = -\frac{1}{\beta}\log Z = \underline{2NJm^2 - \frac{N}{\beta}\log(2\cosh(\beta(4mJ - H)))}$$

①

(3) $\langle \Sigma_j \sigma_j \rangle = Nm \quad \therefore \quad \langle \sigma_j \rangle = m$

$$\langle \sigma_j \rangle = \frac{\exp(\beta(4mJ - H)) - \exp(-\beta(4mJ - H))}{\exp(\beta(4mJ - H)) + \exp(-\beta(4mJ - H))}$$

$$= \underline{\tanh\left(\frac{4mJ - H}{k_BT}\right) = m}$$

(4) $\tanh\left(\underset{x}{\underset{\shortparallel}{\frac{4mJ}{k_BT}}}\right) = m \qquad \tanh(x) = \frac{k_BT}{4J}x$

$\frac{k_BT}{4J}x$

$\frac{k_BT}{4J} < 1$ の時

$$\therefore\ \underline{T_c = \frac{4J}{k_B}}$$

(5) $\overset{1}{x} - \frac{1}{3}x^{\cancel{3}\,2} = \frac{k_BT}{4J}x = \frac{T}{T_c}\cancel{x}$

$$\left(1 - \frac{T}{T_c}\right) = \frac{1}{3}x^2 \quad \therefore \quad x = 3\left(\frac{T_c - T}{T_c}\right)^{1/2} \quad \therefore\ \underline{\beta = 1/2}$$

(6) $\chi = \frac{\partial m}{\partial H} =$

$$\left(\frac{T_c}{T}m - \frac{H}{k_BT}\right) = m \quad \therefore \quad \left(\frac{T_c + T}{T}\right)$$

$$m\left(\frac{T_c + T}{T}\right) = \frac{H}{k_BT} \quad \therefore \quad m = \frac{T}{T - T_c}\cdot\frac{H}{k_BT} \quad \therefore \quad \underline{\chi = (T - T_c)^{-1}}$$

$\boxed{2}$

○ おなじみ 中心力ポテンシャルの復習

2粒子の Schrödinger eqは

$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2m_1}\Delta_1 - \frac{\hbar^2}{2m_2}\Delta_2 + V(\mathbf{r}_1-\mathbf{r}_2)\right]\psi(\mathbf{r}_1, \mathbf{r}_2, t) \longrightarrow \text{1体問題にしたい}$$

$\downarrow$

正準量子化を利用する

$$L = \frac{m_1}{2}\dot{\mathbf{r}}_1^2 + \frac{m_2}{2}\dot{\mathbf{r}}_2^2 - V(\mathbf{r}_1-\mathbf{r}_2), \rightarrow \mathbf{P}_1 = \frac{\partial L}{\partial \dot{\mathbf{r}}_1} = m_1\dot{\mathbf{r}}_1, \quad \mathbf{P}_2 = m_2\dot{\mathbf{r}}_2$$

$\left(\mathbf{R} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1+m_2}, \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2\right)$ である

$$\dot{\mathbf{R}} = \frac{m_1\dot{\mathbf{r}}_1 + m_2\dot{\mathbf{r}}_2}{m_1+m_2}, \quad \dot{\mathbf{r}} = \dot{\mathbf{r}}_1 - \dot{\mathbf{r}}_2 \quad \therefore \begin{bmatrix}\dot{\mathbf{R}}\\ \dot{\mathbf{r}}\end{bmatrix} = \begin{bmatrix}\frac{m_1}{m_1+m_2} & \frac{m_2}{m_1+m_2}\\ 1 & -1\end{bmatrix}\begin{bmatrix}\dot{\mathbf{r}}_1\\ \dot{\mathbf{r}}_2\end{bmatrix}$$

$$\begin{bmatrix}\dot{\mathbf{r}}_1\\ \dot{\mathbf{r}}_2\end{bmatrix} = +1\cdot\begin{bmatrix}+1 & +\frac{m_2}{m_1+m_2}\\ +1 & -\frac{m_1}{m_1+m_2}\end{bmatrix}\begin{bmatrix}\dot{\mathbf{R}}\\ \dot{\mathbf{r}}\end{bmatrix}$$

$$\dot{\mathbf{r}}_1 = \dot{\mathbf{R}} + \frac{m_2}{m_1+m_2}\dot{\mathbf{r}}, \quad \dot{\mathbf{r}}_1^2 = \dot{\mathbf{R}}^2 + \frac{m_2^2}{(m_1+m_2)^2}\dot{\mathbf{r}}^2, \quad \dot{\mathbf{r}}_2 = \dot{\mathbf{R}}^2 + \frac{m_1^2}{(m_1+m_2)^2}\dot{\mathbf{r}}^2$$

$$\therefore \frac{m_1}{2}\dot{\mathbf{r}}_1^2 + \frac{m_2}{2}\dot{\mathbf{r}}_2^2 = \frac{1}{2}(m_1+m_2)\dot{\mathbf{R}}^2 +$$

$$\frac{m_1m_2^2 + m_2m_1^2}{m_1^2+m_2^2+2m_1m_2} = m_1m_2\frac{1}{(m_1+m_2)}$$

$$\therefore L = \frac{M}{2}\dot{\mathbf{R}}^2 + \frac{\mu}{2}\dot{\mathbf{r}}^2 - V(\mathbf{r}) \qquad \mu = \frac{m_1m_2}{m_1+m_2}$$

$\mathbf{P} = M\dot{\mathbf{R}}, \quad \mathbf{p} = \mu\dot{\mathbf{r}}$

$\downarrow$

$$\mathbb{H} = \frac{\mathbf{P}^2}{2M} + \frac{\mathbf{p}^2}{2\mu} + V(\mathbf{r}) \longrightarrow \underline{H = \left[-\frac{\hbar^2}{2M}\Delta_R - \frac{\hbar^2}{2\mu}\Delta_\mu + V(\mathbf{r})\right]}$$

$\swarrow$ Sのみすことで.

$\rightarrow H = \left[-\frac{\hbar^2}{2\mu}\Delta_{\mathbf{r}} + V(\mathbf{r})\right]$ にすればいい

$$\hat{L}_x = \hat{y}\hat{p}_z - \hat{z}\hat{p}_y \quad \longleftarrow \quad \hat{\mathbf{L}} = \hat{\mathbf{x}}\times\hat{\mathbf{p}} = \begin{pmatrix}\hat{x}\\ \hat{y}\\ \hat{z}\end{pmatrix}\times\begin{pmatrix}\hat{p}_x\\ \hat{p}_y\\ \hat{p}_z\end{pmatrix}$$

※ $\hat{L}_x, \hat{L}_y, \hat{L}_z$ の同時固有値は存在しない.

計算すると. $\mathbf{L}^2 = \mathbf{r}^2\cdot\mathbf{p}^2 - \mathbf{r}(\mathbf{r}\cdot\mathbf{p})\cdot\mathbf{p} + 2i\hbar(\mathbf{r}\cdot\mathbf{p})$

$\downarrow$

計算していくと. $\underline{\mathbf{p}^2 = -\frac{\hbar^2}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{\mathbf{L}^2}{r^2}}$

$$\therefore \underline{H = \left[-\frac{\hbar^2}{2\mu}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{\mathbf{L}^2}{2\mu r^2} + V(\mathbf{r})\right]}$$

$$\mathbb{L} = \mathbb{r} \times \frac{\hbar}{i}\nabla = \frac{\hbar}{i}\, r\, e^{(r)} \times [\, e^{(r)} \wedge \,] = \frac{\hbar}{i}\left[e^{(\varphi)} \cdot \frac{\partial}{\partial\theta} - e^{(\theta)} \cdot \frac{1}{\sin\theta}\frac{\partial}{\partial\varphi} \right]$$ となる

ゆえに $L_z = \frac{\hbar}{i}\frac{\partial}{\partial\varphi}$ である

$$\mathbb{L}^2 = -\hbar^2\left[\frac{1}{\sin\theta} \cdot \frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta} \cdot \frac{\partial^2}{\partial\varphi^2} \right] \longrightarrow -\frac{\hbar^2}{2\mu r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\phi\right) + \left(\frac{L^2}{2\mu r^2} + V(r)\right)\phi = E\phi$$

(18.40)