第2問

1.

T 熱浴と接触している場合

$$Z = \frac{1}{h^{3N} \cdot N!} \int \cdots \int dp_{1x} dp_{2x} dp_{3x} \cdots dp_{Nz} \cdot (dq_1 q_y q_z) \cdots_{N \text{個}} \exp\left(-\frac{\beta}{2m}(p_{1x}^2 + p_{1y}^2 + p_{1z}^2)\right) \cdot \exp\left(-\frac{\beta}{2m}(p_{2x}^2 + p_{2y}^2 + p_{3z}^2)\right) \cdots$$

$$\boxed{\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}}$$

$$Z = \frac{1}{h^{3N} \cdot N!} \left(\sqrt{\frac{2m}{\beta} \cdot \pi}\right)^{3N} \cdot V^N$$

2. $F = -\frac{1}{\beta} \log Z$

$$F = -\frac{1}{\beta} \cdot \left(N \log V - (N \log N - N) + \cdots \right)$$

$$-\frac{1}{\beta} N \left(\log\left(\frac{V}{N}\right)\right) + \cdots$$

1: $1 + \omega + \omega^2 + \ldots \omega^{d-1} = \frac{1(1-\omega^d)}{1-\omega} = 0$

$Z|a\rangle = \omega^a |a\rangle$

2: $ZX|d\rangle = Z|d+1\rangle = \omega^{d+1}|d+1\rangle$

$XZ|d\rangle = X\omega^d|d\rangle = \omega^d|d+1\rangle \Rightarrow$ $c = \omega$

3: 同時対角化可能 $\Leftrightarrow$ $U^{(n,m)} \cdot U^{(n',m')} = U^{(n',m')} \cdot U^{(n,m)}$

4:

$X Z^m \cdot X^{n'} \cdot Z^{m'} |d\rangle = X^{n'} Z^{m'} \cdot X Z^m$ となるような m', n'

$\omega^{d \cdot m} |d+1\rangle$

$X Z^m \cdot X^{n'} \cdot \omega^{d \cdot m'} |d\rangle$

$X Z^m \cdot \omega^{d \cdot m'} |d+n'\rangle$

$\omega^{d+m} \cdot \omega^{m'(d+1)} |d+1\rangle$

$\omega^{d \cdot m'} \cdot X \cdot \omega^{(d+n')m} |d+n'\rangle$

$$\boxed{\omega^{d \cdot m' + (d+n')m} |d+n'+1\rangle}$$

$\therefore$ $d \cdot m' + (d+n')m = dm + m'(d+1)$

$d(\cancel{m'} + m - 1 - \cancel{m'}) = -n'm + m + n'$

$n'm = m' \pmod d$

$\Rightarrow$ $Z \begin{bmatrix} 1 & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \, \\ 1 \end{bmatrix}$

$X|a\rangle = |a+1 (\mathrm{mod}\ d)\rangle$

$\vdots$

$X^{2}|a\rangle = |a+\overset{x}{\overset{\shortparallel}{2}} (\mathrm{mod}\ d)\rangle$

$a = a + x\ (\mathrm{mod}\ d)) \rightarrow \underline{x = d}$ //

$\underline{\omega^{\cdot}\ -m\cdot}$ //

$U^{(n',m')}$. $n', m' = d$ の時. E然、$|0\rangle, |1\rangle$ は固有

$X Z^m |a\rangle = X\, \omega^m |a\rangle = \omega^m |a+1\rangle$

$c(XZ)ZZ\cdots Z = Z(XZ)Z\cdots Z$

c^{2}.

$X Z^m = C^m Z^m X$

$$Z^m \sum_{i=0}^{d} C_i |i\rangle = \sum_{i}^{d} C_i \omega^i |i\rangle$$

$$X \qquad = \sum_{i=0}^{d} C_i \omega^2 |i+1\rangle$$

$$C_{i+1}\omega^{i+1} = C_i \omega^i$$

$$C_{i+1}\cdot \omega = C_i \qquad \therefore \frac{C_{i+1}}{C_i} = \omega^{-1}$$

$$C_i = \omega^{-i}$$

$$\therefore \boxed{\sum_{i=0}^{d} \omega^{-i}|i\rangle}$$ //

相関系. 平成26年度. 総合科目.

物理 その1

I

(1) $\hat{a}+\hat{a}^\dagger = \sqrt{\frac{2m\omega}{\hbar}}\hat{q}$, $\hat{a}-\hat{a}^\dagger = \underset{\overset{\shortparallel}{i}}{-\frac{1}{i}}\sqrt{\frac{2}{m\hbar\omega}}\hat{p}$

$$\boxed{\hat{a} = \frac{1}{2}\left(\sqrt{\frac{2m\omega}{\hbar}}\hat{q} + i\sqrt{\frac{2}{m\hbar\omega}}\hat{p}\right),\quad \hat{a}^\dagger = \frac{1}{2}\left(\sqrt{\frac{2m\omega}{\hbar}}\hat{q} - i\sqrt{\frac{2}{m\hbar\omega}}\hat{p}\right)}$$

また. $[\hat{q},\hat{p}]=i\hbar$ であるため. $[\hat{a},\hat{a}^\dagger]=$

$$i\hbar = [\hat{q},\hat{p}] = -i\cdot\frac{1}{2}\hbar\cdot[(\hat{a}+\hat{a}^\dagger),(\hat{a}-\hat{a}^\dagger)] = -\frac{1}{2}i\hbar\left([\hat{a},-\hat{a}^\dagger] + \underbrace{[\hat{a}^\dagger,\hat{a}]}_{-[\hat{a},\hat{a}^\dagger]}\right)$$

$$= i\hbar\underline{[\hat{a},\hat{a}^\dagger]}_{1} \quad //$$

$aa^\dagger - a^\dagger a = 1$

(2) $[\hat{n},\hat{a}] = [\hat{a}^\dagger\hat{a},\hat{a}] = \hat{a}^\dagger\hat{a}\hat{a} - \hat{a}\hat{a}^\dagger\hat{a} = \underline{-\hat{a}}\ //$

$\hat{a}\hat{a}^\dagger - 1$

$[\hat{n},\hat{a}^\dagger] = \hat{a}^\dagger\underbrace{\hat{a}\hat{a}^\dagger}_{1+a^\dagger a} - \hat{a}^\dagger\hat{a}^\dagger\hat{a} = \underline{+\hat{a}^\dagger}\ //$

(3) $\hat{n}|n\rangle = n|n\rangle$ はみとめる. $(\underbrace{\hat{n}\hat{a} - \hat{a}\hat{n}})|n\rangle = -\hat{a}|n\rangle$

$\boxed{\text{◎：調和振動子く習}}$

$\hat{n}(\hat{a}|n\rangle) - \hat{a}\cdot n|n\rangle = -\hat{a}|n\rangle$

$\hat{n}(\hat{a}|n\rangle) = \hat{a}(n-1)|n\rangle = (n-1)(\hat{a}|n\rangle)$

$\boxed{\therefore \hat{a}|n\rangle = |n-1\rangle}$

$(\hat{n}\hat{a}^\dagger - \hat{a}^\dagger\hat{n})|n\rangle = +\hat{a}^\dagger|n\rangle$

$\hat{n}(\hat{a}^\dagger|n\rangle) = (n+1)(\hat{a}^\dagger|n\rangle)$

$\therefore \boxed{\hat{a}^\dagger|n\rangle = |n+1\rangle}\ //$

$\hat{a}|n\rangle = a_n|n-1\rangle$ である

$\hat{a}^\dagger|n\rangle = a^\dagger_{n+1}|n+1\rangle$ である

$\hat{n} = \quad a_n$

(4) $\langle q\rangle = \langle 0|\hat{q}|0\rangle = \langle 0|\left(\sqrt{\frac{\hbar}{2m\omega}}(\hat{a}+\hat{a}^\dagger)|0\rangle = 0\right.$

$\langle p\rangle = 0$

(5) $\hat{q}^2 = \frac{\hbar}{2m\omega}\cdot(\cancel{\hat{a}\hat{a}} + \cancel{\hat{a}^\dagger\hat{a}^\dagger} + \hat{a}\hat{a}^\dagger + \hat{a}^\dagger\cancel{\hat{a}})$, $\hat{p}^2 = -\frac{m\hbar\omega}{2}(\cancel{\hat{a}\hat{a}} + \cancel{\hat{a}^\dagger\hat{a}^\dagger} - \hat{a}\hat{a}^\dagger - \cancel{\hat{a}^\dagger\hat{a}})$

$\hat{a}\hat{a}^\dagger|0\rangle = \hat{a}\cdot|1\rangle = |0\rangle$,

~~$\hat{a}^\dagger\hat{a}|0\rangle =$~~

$\delta q = \sqrt{\frac{\hbar}{2m\omega}}\ //$

$\delta p = \sqrt{\frac{m\hbar\omega}{2}}\ //$

II