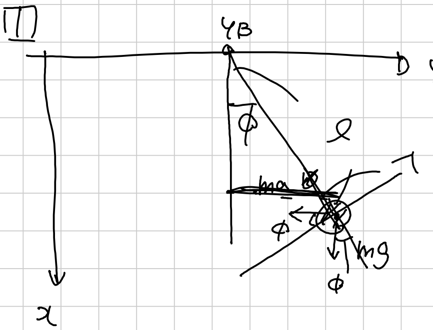
[I]



∴ (aが大きすぎると、φが微小にならないかも)

(1) $m\ell\ddot{\phi} = -ma\cos\phi - mg\sin\phi$ ∴ $\phi = -\frac{a}{g} + \frac{a}{g}\cos\left(\sqrt{\frac{g}{\ell}}t\right)$

(2)

$\mathbf{r} = (\ell\cos\phi,\ \ell\sin\phi + \varphi_B)$ ∴ $\dot{\mathbf{r}} = (-\ell\sin\phi\cdot\dot{\phi},\ \ell\cos\phi\cdot\dot{\phi} + \dot{\varphi}_B)$

∴ $|\dot{\mathbf{r}}|^2 = \ell^2\dot{\phi}^2 + 2\dot{\varphi}_B\ell\cos\phi\dot{\phi} + \dot{\varphi}_B^2$

∴ $L = \frac{1}{2}m|\dot{\mathbf{r}}|^2 + mg\ell\cos\phi.$ $\boxed{V = -mg\ell\cos\phi}$

$= \frac{1}{2}m(\ell^2\dot{\phi}^2 + 2\dot{\varphi}_B\ell\cos\phi\dot{\phi} + \dot{\varphi}_B^2) + mg\ell\cos\phi$

∴ $\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\phi}}\right) = \frac{\partial L}{\partial\phi}$ ∴ $\underline{m\ell^2\ddot{\phi} + m\ddot{\varphi}_B\ell\cos\phi = -mg\ell\sin\phi}$ //

(3) 計算すると、 $m\ell^2\ddot{\phi} - m\omega_0^2\varphi_0\cos(\omega_0 t)\ell\underbrace{\cos\phi}_{1} + mg\ell\underbrace{\sin\phi}_{\phi} = 0$

$\boxed{= m\ell^2\ddot{\phi} - m\omega_0^2\varphi_0\cos(\omega_0 t)\ell + mg\overset{\ell}{\phi} = 0}$

$\phi = \Phi + A\cos(\omega_0 t)$

$m\ell^2\ddot{\phi} - \overset{m\ell^2}{A}\omega_0^2\cos(\omega_0 t) - m\omega_0^2\varphi_0\cos(\omega_0 t)\ell$

$+ mg\phi + mgA\cos\cdot\ell$

$-A\omega_0^2 - m\omega_0^2\varphi_0\ell + mgA = 0$

$\underset{\neq 0}{A}(mg - \underset{m\ell^2}{\omega_0^2}) = m\omega_0^2\varphi_0\ell$ ∴ $A = \frac{m\omega_0^2\varphi_0\ell}{mg\ell - m\ell^2\omega_0^2}$

(でないといけない)

$A = \left(\frac{\omega_0^2\varphi_0}{g - \ell\omega_0^2}\right)$

DE,

∴ $m\ell^2\ddot{\Phi} + mg\overset{\ell}{\Phi} = 0$ ∴ $\Phi = -A\cos\left(\sqrt{\frac{g}{\ell}}t\right)$

∴ $\phi = \frac{\omega_0^2\varphi_0}{g - \ell\omega_0^2}\cdot\left(-\cos\left(\sqrt{\frac{g}{\ell}}t\right) + \cos(\omega_0 t)\right)$

(4)

$\phi = A\cos(\omega_0 t) + B$ とすると. $mg\ell B - mg y_0 = 0 \therefore B = \frac{y_0}{\ell}$

$\therefore A = -\frac{y_0}{\ell} \quad \therefore \underline{\phi = \frac{y_0}{\ell}(1-\cos(\omega_0 t))}$ //

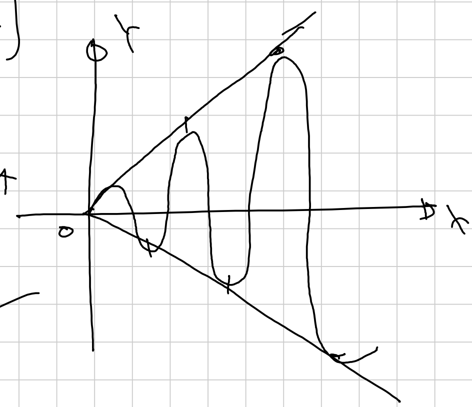
$\therefore \ell\phi = y_0(1-\cos(\omega_0 t))$

$\phi(t) = \frac{\omega_0^2}{\omega_0^2-\omega^2}\cdot\frac{g}{\ell}(\cos(\omega t) - \cos(\omega_0 t))$,$\left(\omega = \sqrt{\frac{g}{\ell}}\right)$

$|\omega_0 - \omega| \ll 1$ の極限.

$\lim_{\omega_0\to\omega}\phi(t) = \lim_{\omega\to\omega_0}\frac{\omega_0^2}{(\omega_0-\omega)(\omega_0+\omega)}\cdot\frac{y_0}{\ell}\cdot\frac{\cos(\omega t)-\cos(\omega_0 t)}{\omega t - \omega_0 t}(\omega-\omega_0)t$

$= -\frac{\omega_0 t}{2}\frac{y_0}{\ell}\cdot(-\sin\omega_0 t) = \frac{y_0}{2\ell}\cdot\omega_0 t\sin\omega_0 t$



「振幅が大きい」

[2]
[A]

(1) $a \leq r \leq b$ の時. Gauss' theorem より. $\int E\cdot dS = \int\frac{\rho}{\varepsilon}dV \therefore E\times 2\pi r\times\ell = \frac{1}{\varepsilon}\cdot\lambda\ell$

$\therefore E = \frac{\lambda}{2\pi\varepsilon r}$

$\therefore \mathbb{E} = \begin{cases} 0 & (r\leq a) \\ \frac{\lambda}{2\pi\varepsilon r}e_r & (a\leq r\leq b) \\ 0 & (b\leq r)\end{cases}$

(2) $V = -\int_b^a \mathbb{E}\cdot d\mathbb{r} = -\int_b^a E dr = \underline{\frac{\lambda}{2\pi\varepsilon}\log\frac{b}{a}}$ //

$\lambda\cdot\ell = C\times V \therefore \underline{\frac{C}{\ell} = \frac{2\pi\varepsilon}{\log b/a}}$ //

[B]

(3) $\nabla\times\mathbb{B} = \mu_0 \mathbb{j} + \frac{1}{c^2}\cancel{\frac{\partial\mathbb{E}}{\partial t}}^{\,0}$ より $\int_V \nabla\times\mathbb{B}\,dV = \int_V \mu_0\mathbb{j}\,dV \therefore \oint_{\partial S}\mathbb{B}\cdot d\mathbb{r} = \begin{cases} 0 & r\leq a \\ \mu_0 I & a\leq r\leq b \\ 0 & b\leq r\end{cases}$

$B\cdot 2\pi r$

$\therefore \mathbb{B} = \begin{cases} 0 & r\leq a \\ \frac{\mu_0 I}{2\pi r} & a\leq r\leq b \\ 0 & b\leq r\end{cases}$ e_θ に回転する向き 磁束を求める

$\Phi_m = -\int_b^a \mathbb{B}\cdot d\mathbb{r}$

(4) $\int dV \cdot \frac{1}{2\mu} B^2 = \int r dr dz d\theta \cdot \frac{1}{\cancel{2\mu}} \cdot \frac{\mu^2 I^2}{4\pi^2 r^2} = \frac{I^2}{8\pi^2}\mu \int_a^b \frac{1}{r} dr dz d\theta$

$= \frac{I^2}{8\pi^2}\mu \log\left(\frac{b}{a}\right) \cdot 2\pi \cdot \left(\int dz\right)$

$\therefore \underline{\frac{I^2}{4\pi}\mu \log\left(\frac{b}{a}\right)}$ //

(5) $E = \frac{1}{2} L I^2 \quad \therefore \quad L = \frac{2}{\cancel{I^2}} \times \frac{\cancel{I^2}}{4\pi}\mu \log\left(\frac{b}{a}\right) = \underline{\frac{\mu}{2\pi} \log\left(\frac{b}{a}\right)}$ //

[C]

(6) (ア) : $\mathrm{rot}\, \mathbb{E} = -\frac{\partial \mathbb{B}}{\partial t}$, (イ) $\nabla \times \mathbb{B} = \varepsilon\mu \cdot \frac{\partial^2 \mathbb{E}}{\partial t^2}$

(ウ) : $\frac{1}{\varepsilon_0\mu_0}\left(\frac{\partial^2}{\partial t^2} E_x t \cdots\right)$ $\quad$ エ $= \underline{\frac{1}{\varepsilon\mu}}$ //

$= \frac{d^2}{dt^2}(\quad)$

(7)

$$S \frac{1}{\mu}\mathbb{E} \times \mathbb{B} = \begin{cases} 0 & r \leq a \\ - & \\ 0 & b \leq a \end{cases}$$ //

$V \cdot \frac{1}{\log\frac{b}{a}} = \frac{\lambda}{2\pi\varepsilon}$

(オ) $V \times \frac{1}{\log\frac{b}{a}} \cdot \frac{1}{r} \times \frac{\mu I}{2\pi r} = \boxed{\frac{\cancel{\mu} I V}{2\pi r^2 \cdot \log\left(\frac{b}{a}\right)}} \cdot \mathbf{e}_z$

(カ) $\int r dr \cancel{dz} d\theta |S| = \frac{IV}{2\pi \log\left(\frac{b}{a}\right)} \times \log\left(\frac{b}{a}\right) \times 2\pi = \underline{IV}$ //

3

[A]

(1) $\det A = -1\cdot 1 + 1\cdot(-1) = \underline{\underline{-2}}$ （余因子展開）

余因子行列より

(2) $A^{-1} = -\frac{1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 1 & -1 & -1 \\ -1 & -1 & 1 \end{bmatrix} = \underline{\underline{\frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix}}}$ → 対称になっている

(3)

$$\begin{vmatrix} x-1 & 0 & -1 \\ 0 & x-1 & -1 \\ -1 & -1 & x \end{vmatrix} = (x-1)\cdot(x(x-1)-1) + (-1)\cdot(x-1) = 0$$

$$(x-1)(\underbrace{x^2-x-1-1}_{\substack{x^2-x-2 \\ (x-2)(x+1)}}) = 0$$

$\therefore x = \pm 1, 2$

$x=1$の時　$\underline{\underline{\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}}}$

$x=-1$の時

$-2x - z = 0$　　$z = -2x$

$-2y - z = 0$

$-x - y - z = 0$　　$y = -x + 2x = x$

$\therefore \underline{\underline{\frac{1}{\sqrt{6}}\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}}}$

$x=2$の時

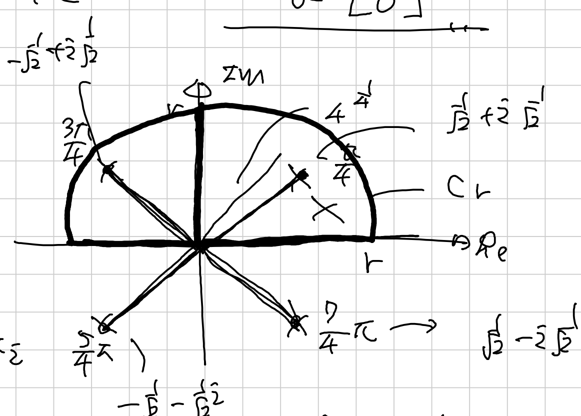
$x - z = 0$　　$x = z$

$y - z = 0$　　$y = z$

$-x - y + 2z = 0$

$\underline{\underline{\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}}}$

[B] $\displaystyle\int_C \frac{e^{iz}}{z^4+4}dz$ の線積分を考える.



$z^4 = -4 = 4e^{i n\pi}$ (n=奇)

$\therefore z = 4^{\frac{1}{4}}\cdot e^{\frac{\pi}{4}i + \frac{2n\pi}{4}i}$　$\frac{5}{4}\pi$)

$\therefore \displaystyle\int_{-r}^{r} f(z) + \underbrace{\int_{C_r} f(z)\,dz}_{②}$　　$\therefore$ ②: $\displaystyle\int_0^{\pi} \frac{e^{ir\cos\theta - r\sin\theta}}{r^4e^{4i\theta}+4}\cdot i r e^{i\theta}d\theta$

$$|②| \leq \frac{e^{-r\sin\theta}}{r^4-4} r\,d\theta \leq \frac{\pi}{r}\cdot\frac{r}{r^4-4} \sim O\left(\frac{1}{r^4}\right) \to 0$$

$\because$ ジョルダンの不等式.

$$\therefore \int_C f(z)\,dz = 2\pi i\cdot\frac{1}{4^{\frac{3}{4}}(e^{\frac{\pi}{4}i}-e^{\frac{3}{4}\pi i})(e^{\frac{\pi}{4}i}-e^{\frac{5}{4}\pi i})(e^{\frac{\pi}{4}i}-e^{\frac{7}{4}\pi i})}$$

$$+2\pi i\cdot\frac{1}{4^{\frac{3}{4}}(e^{\frac{3}{4}\pi i}-e^{\frac{\pi}{4}i})(e^{\frac{3}{4}\pi i}-e^{\frac{5}{4}\pi i})(e^{\frac{3}{4}\pi i}-e^{\frac{7}{4}\pi i})}$$

$$\int_C f(z)\,dz = 2\pi i \cdot \frac{1}{4^{\frac{3}{4}}\left(e^{\frac{\pi}{4}i}-e^{\frac{3}{4}\pi i}\right)\left(e^{\frac{\pi}{4}i}-e^{\frac{5}{4}\pi i}\right)\left(e^{\frac{\pi}{4}i}-e^{\frac{7}{4}\pi i}\right)}$$

$$+2\pi i \cdot \frac{1}{4^{\frac{3}{4}}\left(e^{\frac{3}{4}\pi i}-e^{\frac{\pi}{4}i}\right)\left(e^{\frac{3}{4}\pi i}-e^{\frac{5}{4}\pi i}\right)\left(e^{\frac{3}{4}\pi i}-e^{\frac{7}{4}\pi i}\right)}$$

$$= \frac{1}{4^{\frac{3}{4}}} 2\pi i \left(\frac{e^{i(\sqrt{2}e^{\frac{\pi}{4}i})} = e^{i(1+i)}}{\frac{2}{\sqrt{2}}\cdot\left(\frac{2}{\sqrt{2}}+\frac{2}{\sqrt{2}}i\right)\left(\frac{2i}{\sqrt{2}}\right)} + \frac{e^{i(\sqrt{2}e^{\frac{3}{4}\pi i})} = e^{i(-1+i)}}{\left(-\frac{2}{\sqrt{2}}\right)\left(\frac{2i}{\sqrt{2}}\right)\cdot\left(-\frac{2}{\sqrt{2}}+\frac{2}{\sqrt{2}}i\right)} \right)$$

$$= \frac{1}{4^{\frac{3}{4}}}\cdot 2\pi i\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}i}\cdot\left(\frac{1}{\sqrt{2}+\sqrt{2}i}-\frac{1}{-\sqrt{2}+\sqrt{2}i}\right) = 4^{-\frac{3}{4}}\cdot\cancel{2}\pi\cancel{i}\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\cancel{\sqrt{2}i}}\cdot\cancel{\frac{\sqrt{2}}{2}}$$

$$\frac{1}{\sqrt{2}+\sqrt{2}i}+\frac{1}{\sqrt{2}-\sqrt{2}i} \qquad \boxed{= 4^{-\frac{3}{4}}\cdot\pi\cdot\frac{1}{\sqrt{2}}}$$

$$\frac{1}{\sqrt{2}+\sqrt{2}i}+\frac{1}{-\sqrt{2}+\sqrt{2}i} = \frac{2\sqrt{2}}{2+2} = \frac{\sqrt{2}}{2} \qquad \frac{\pi}{\sqrt{2}^3}\cdot\frac{1}{\sqrt{2}} = \frac{\pi}{2\sqrt{2}}$$

$$\frac{2\sqrt{2}i}{-2-2} = \left(-\frac{\sqrt{2}}{2}i\right)$$

[C]

$$x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)$$

$$\frac{\pi}{2\sqrt{2}}\cdot e^{-1}\left[\left(\cos 1 + i\sin 1\right)\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\left(\cos 1 - i\sin 1\right)\right]$$

$$a_n = \frac{1}{\ell}\int_{-\ell}^{\ell} x^2\cos\frac{n\pi x}{\ell}\,dx$$

$$\rightarrow \frac{\pi}{2\cancel{\sqrt{2}}}\frac{1}{e}\left[\frac{\sqrt{2}}{2}\cos 1 + \frac{\sqrt{2}}{2}\sin 1\right]$$

$$a_n + i b_n = \frac{1}{\ell}\int_{-\ell}^{\ell} x^2 \exp\left(i\frac{n\pi}{\ell}x\right)dx$$

$$= \frac{\pi}{4}\cdot\frac{1}{e}\left[\cos 1 + \sin 1\right]$$

3

$$\frac{\ell}{n\pi}\exp(\;)' \qquad \frac{\ell}{n\pi}\exp(\;)''$$

$$= \frac{1}{\ell}\left[x^2\exp\left(i\frac{n\pi}{\ell}x\right)\cdot\frac{\ell}{n\pi}\right]_{-\ell}^{\ell} - \frac{2}{\ell}\int_{-\ell}^{\ell} x\,\frac{\ell}{n\pi}\exp\left(i\frac{n\pi}{\ell}x\right)dx$$

$$= \frac{1}{\ell}\ell^2\cdot\frac{\ell}{n\pi}\cdot 2i\sin\left(\frac{n\pi}{\cancel{\ell}}\cancel{\ell}\right) = 0$$

$$= -\frac{2}{n\pi}\int_{-\ell}^{\ell} x\exp\left(i\frac{n\pi}{\ell}x\right)dx$$

$$= -\frac{2}{n\pi}\left[\frac{\ell}{n\pi}\cdot x\exp\left(i\frac{n\pi}{\ell}x\right)\right]_{-\ell}^{\ell} + \frac{2}{n\pi}\int_{-\ell}^{\ell}\frac{\ell}{n\pi}\cdot\exp(\;)\,dx$$

$$= -\frac{2}{n\pi}\cdot\frac{\ell}{n\pi}\cdot\ell\left[2\cos(n\pi)\right] + \frac{2}{n\pi}\cdot\left[\exp\left(i\frac{n\pi}{\ell}x\right)\right]_{-\ell}^{\ell}$$

$$= -\frac{2\ell^2}{n^2\pi^2}\cdot 2\cos(n\pi) + \cancel{\frac{2}{n\pi}\cdot 2i\cdot\underbrace{\sin(n\pi)}_{0}}$$

$$\boxed{a_0 = \frac{1}{\ell}\cdot\frac{2}{3}\ell^3 = \frac{2}{3}\ell^2 = \frac{2}{3}\pi^2}$$

$$\therefore \boxed{a_n = -\frac{2\ell^2}{n^2\pi^2}\cdot 2\cos(n\pi) \rightarrow -\frac{4}{n^2}\cos(n\pi)}$$

$$\boxed{b_n = 0}$$

$$\cancel{4}\;(-1)^{n-1}$$