カンニングシート

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極座標

3 次元

$$\nabla = \boldsymbol{e}_r \frac{\partial}{\partial r} + \boldsymbol{e}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + \boldsymbol{e}_\phi \frac{1}{r \sin\theta} \frac{\partial}{\partial \phi} \tag{1}$$

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2}{\partial \phi^2} \tag{2}$$

ヤコビアンは $r^2 \sin\theta$

2 次元

$$\nabla = \boldsymbol{e}_r \frac{\partial}{\partial r} + \boldsymbol{e}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} \tag{3}$$

$$\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta^2} \tag{4}$$

ヤコビアンは r

円柱座標

$$\nabla = \boldsymbol{e}_r \frac{\partial}{\partial r} + \boldsymbol{e}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + \boldsymbol{e}_z \frac{\partial}{\partial z} \tag{5}$$

$$\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \tag{6}$$

ヤコビアンは r

デルタ関数

$$\delta(ax) = \frac{1}{|a|} \delta(x) \tag{7}$$

$$\delta(f(x)) = \sum_i \frac{1}{|f(a_i)|} \delta(x - a_i) \tag{8}$$

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} dk \tag{9}$$

$$\Delta \left(\frac{1}{r} \right) = -4\pi\delta(r) \tag{10}$$

$$\nabla \left(\frac{1}{r} \right) = -\frac{\boldsymbol{e}_r}{r^2} = -\frac{\boldsymbol{r}}{r^3}, \quad \therefore \nabla \cdot \left(\frac{\boldsymbol{r}}{r^3} \right) = 4\pi\delta(r) \tag{11}$$