解答

91-12 = 79

[1] $|\psi(t)\rangle = \exp\left(\frac{1}{i\hbar}\hat{H}(t-t_0)\right)|\psi_0\rangle$

[2] エルミートより $\hat{H}^\dagger = \hat{H}$ ∴ $\hat{U}^\dagger\hat{U} = \exp\left(-\frac{1}{i\hbar}\hat{H}^\dagger(t-t_0)\right)\cdot\exp\left(\frac{1}{i\hbar}\hat{H}(t-t_0)\right)$

$= \exp(0) = \mathbb{I}$ $\quad \boxed{\hat{H}^\dagger - \hat{H} = 0}$

[3] $\langle\psi(t)|\psi(t)\rangle = \langle\psi_0|\hat{U}^\dagger\hat{U}|\psi_0\rangle = \langle\psi_0|\psi_0\rangle = \mathrm{Const}$ なので示せた

[4] $\hat{U}(t) = \exp\left(\frac{1}{i\hbar}a\hat{\sigma}_z t\right) = \exp\left(\frac{t}{i\hbar}a\cdot\hat{\sigma}_z\right) = \exp\left(-i\frac{t}{\hbar}a\cdot\hat{\sigma}_z\right)$

$\exp(iA) = \mathbb{I} + (iA) + \frac{1}{2!}(-A^2) + \frac{1}{3!}(-A^3 i) + \frac{1}{4!}(A^4) + \cdots$

$\left(\exp(x) = 1 + \frac{1}{1!}\cdot x + \frac{1}{2!}x^2 + \cdots\right)$

$\mathbb{I} - \frac{1}{2!}A^2 + \frac{1}{4!}A^4 + \cdots$
$= \cos A$

$i\left(A - \frac{1}{3!}A^3 + \cdots\right) = i\sin A$

∴ $\exp(iA) = \cos A + i\sin A$ である

$\exp(ia\sigma_z) = \mathbb{I} + (ia\sigma_z) + \frac{1}{2!}(-a^2\mathbb{I}) + \frac{1}{3!}(-a^3 i\sigma_z) + \cdots$

$= \mathbb{I}\left(\underbrace{1 - \frac{1}{2!}a^2 + \frac{1}{4!}a^4 + \cdots}_{=\cos(a)}\right)$

$+ i\sin(a)\sigma_z \quad = \boxed{\cos(a)\sigma_z + i\sin(a)\sigma_z}$

∴ $\exp\left(i\left(-\frac{t}{\hbar}a\right)\sigma_z\right) = \cos\left(-\frac{t}{\hbar}a\right)\sigma_z + i\sin\left(-\frac{t}{\hbar}a\right)\sigma_z$

$= \cos\left(\frac{t}{\hbar}a\right)\sigma_z - i\sin\left(\frac{t}{\hbar}a\right)\sigma_z$

$= \begin{bmatrix}\cos\left(\frac{t}{\hbar}a\right) - i\sin\left(\frac{t}{\hbar}a\right) & 0 \\ 0 & \cos\left(\frac{t}{\hbar}a\right) + i\sin\left(\frac{t}{\hbar}a\right)\end{bmatrix}$

$e^{-i\frac{t}{\hbar}a} \qquad e^{i\frac{t}{\hbar}a}$

$\begin{bmatrix} e^{-i\frac{t}{\hbar}a} & 0 \\ 0 & e^{i\frac{t}{\hbar}a}\end{bmatrix} = e^{-i\frac{t}{\hbar}a}\begin{bmatrix}1 & 0 \\ 0 & e^{2i\frac{t}{\hbar}a}\end{bmatrix}$

$2\frac{t}{\hbar}a = \phi$

[5] $\hat{U}_\phi = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{bmatrix} = \exp(-i\frac{t}{\hbar}H)$ から. log をとる.

$= e^{\frac{i\phi}{2}} \begin{bmatrix} e^{-i\frac{\phi}{2}} & 0 \\ 0 & e^{i\frac{\phi}{2}} \end{bmatrix} = \exp(\frac{i\phi}{2}) \cdot \exp(-i\frac{\phi}{2})\sigma_z$ $\quad \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} =$

[7] $\hat{U}(t) = \exp(-i\frac{t}{\hbar}\hat{H}) = \sigma_H$ の形

$\{\sigma_i, \sigma_j\} = 0$

$+ \sigma_i\sigma_j + \sigma_j\sigma_i$

$\exp(ia(\sigma_i + \sigma_j)) = I + (ia(\sigma_i+\sigma_j)) + \frac{1}{2!}(-a^2 \cdot (1+1)) + \frac{1}{3!}(-a^2 i(\sigma_i + \sigma_j)) + \cdots$

$\exp(ia\sigma_i) = I + (ia\sigma_i) + \frac{1}{2!}(-a^2 I) + \frac{1}{3!}(-a^3 i \sigma_i) + \cdots$

$= I(\underbrace{1 - \frac{1}{2!}a^2 + \frac{1}{4!}a^4 + \cdots}_{\cos(a)})$

$+ i\sin(a)\sigma_i \quad = \boxed{\cos(a)\sigma_I + i\sin(a)\sigma_i}$

$(\sigma_i + \sigma_j)^2 = 1 + 1 + \underbrace{\sigma_i\sigma_j + \sigma_j\sigma_i}_{\{\sigma_i, \sigma_j\} = 0} = 2I$

これが2倍にならないのかも..?? まいいか...

理想 $\boxed{\cos(a)(\sigma_I + \sigma_I) + i\sin(a)(\sigma_i + \sigma_j)}$

$\exp(i(a\sigma_i + b\sigma_j)) = I + (i(a\sigma_i + b\sigma_j)) + \frac{1}{2!} \cdot (-1 \cdot (a^2 + b^2)) + \cdots$

$\sigma_I \cdot (a\sigma_i + b\sigma_j)$

$= -I + (\cos(a) + \cos(b))\sigma_I + i\sin$

→ けいさんめんどう...

[8]

$\sigma_H^2 = \sigma_I$ なので、同様にして、

$U(\tau) = \exp\left(-i\frac{H\tau}{\hbar}\right) = \sigma_H$ の時

$H = a\sigma_z$

$\underbrace{\cos\left(-\frac{a\tau}{\hbar}\right)}_{0}\sigma_I + i\sin\left(-\frac{a\tau}{\hbar}\right)\sigma_z$

$\therefore -\frac{a\tau}{\hbar} = \frac{\pi}{2}$ $\therefore$ $\boxed{a\tau = -\frac{\pi}{2}\hbar}$

$\boxed{a = -\frac{\pi\hbar}{2\tau}}$

$\boxed{\therefore H = -\frac{\pi\hbar}{2\tau}\sigma_H}$ //

[9]

$\hat{U}_{C-\phi} = \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&e^{i\phi} \end{bmatrix}$ $\hat{U}_{C-\phi}^2 = \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&e^{2i\phi} \end{bmatrix}$

[5]. $\hat{U}_\phi = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{bmatrix}$ $\hat{U}_\phi^2 = \begin{bmatrix} 1 & 0 \\ 0 & e^{2i\phi} \end{bmatrix}$ である。

$\hat{U}_\phi = e^{+\frac{i\phi}{2}} \underbrace{\begin{bmatrix} e^{\frac{i\phi}{2}} & 0 \\ 0 & e^{-i\frac{\phi}{2}} \end{bmatrix}}_{A}$ $A^2 = I$ であるため $\left(\begin{bmatrix} \exp(\frac{i\phi}{2}) & 0 \\ 0 & \exp(-i\frac{\phi}{2}) \end{bmatrix} \right] \left[\exp \right.$

$e^{\frac{i\phi}{2}} \cdot \left(\cos\left(\frac{-a\tau}{\hbar}\right)\sigma_I + i\sin\left(\frac{-a\tau}{\hbar}\right)\sigma_z \right) = \exp\left(-i\frac{a\tau}{\hbar}\sigma_\Sigma\right) = \exp\left(-i\frac{H}{\hbar}\tau\right)$

$e^{\frac{i\phi}{2}} \cdot$

$\therefore \frac{H}{\hbar}\tau = \frac{a\tau}{\hbar}\sigma_A + \left(-\frac{\phi}{2}\right)$

[5]. $\boxed{H = a\sigma_A - \frac{\phi}{2}\cdot\frac{\hbar}{\tau}}$ //

[9] $= e^{\frac{i\phi}{2}} \begin{bmatrix} e^{-\frac{i\phi}{2}} & 0 & 0 & 0 \\ & e^{-\frac{i\phi}{2}} & & \\ & & \ddots & \end{bmatrix}$

$$u = u^T \exp(i(k^T x - \omega^T t)) + u^P e^{i(k^P x - \omega^P t)}$$

Tranverse perallel

$$u^T = (0, u_2^T, u_3^T). \qquad u^P = (u^P, 0, 0)$$