数学 カンニングシート

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極座標

3 次元

$$\nabla = \boldsymbol{e}_r \frac{\partial}{\partial r} + \boldsymbol{e}_\theta \frac{1}{r}\frac{\partial}{\partial \theta} + \boldsymbol{e}_\phi \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi} \tag{1}$$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2} \tag{2}$$

ヤコビアンは $r^2\sin\theta$

2 次元

$$\nabla = \boldsymbol{e}_r \frac{\partial}{\partial r} + \boldsymbol{e}_\theta \frac{1}{r}\frac{\partial}{\partial \theta} \tag{3}$$

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta^2} \tag{4}$$

ヤコビアンは r

円柱座標

$$\nabla = \boldsymbol{e}_r \frac{\partial}{\partial r} + \boldsymbol{e}_\theta \frac{1}{r}\frac{\partial}{\partial \theta} + \boldsymbol{e}_z \frac{\partial}{\partial z} \tag{5}$$

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \tag{6}$$

ヤコビアンは r

デルタ関数

$$\delta(ax) = \frac{1}{|a|}\delta(x) \tag{7}$$

$$\delta(f(x)) = \sum_i \frac{1}{|f(a_i)|}\delta(x - a_i) \tag{8}$$

$$\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx} dk \tag{9}$$

$$\Delta\left(\frac{1}{r}\right) = -4\pi\delta(r) \tag{10}$$

$$\nabla\left(\frac{1}{r}\right) = -\frac{\boldsymbol{e}_r}{r^2} = -\frac{\boldsymbol{r}}{r^3}, \quad \therefore \nabla\cdot\left(\frac{\boldsymbol{r}}{r^3}\right) = 4\pi\delta(r) \tag{11}$$

また、クロネッカーのデルタに関しては次が有名である：

$$\delta_{n,m} = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{i(n-m)\theta} d\theta \tag{12}$$

三角関数・双曲線関数

展開

$$\sin(x) \sim x - \frac{x^3}{3!} + \cdots \tag{13}$$
$$\cos(x) \sim 1 - \frac{x^2}{2!} + \cdots \tag{14}$$
$$\tan(x) \sim 1 + \frac{x^3}{3!} + \cdots \tag{15}$$

$$\sinh(x) \sim x + \frac{x^3}{3!} + \cdots \tag{16}$$
$$\cosh(x) \sim 1 + \frac{x^2}{2!} + \cdots \tag{17}$$
$$\tanh(x) \sim 1 - \frac{x^3}{3!} + \cdots \tag{18}$$

微分とかの性質

$$\cosh^2(x) - \sinh^2(x) = 1 \tag{19}$$
$$1 - \tanh^2(x) = \frac{1}{\cosh^2(x)} \tag{20}$$

微分は

$$(\cosh(x))' = \sinh(x) \tag{21}$$
$$(\sinh(x))' = \cosh(x) \tag{22}$$
$$(\tanh(x))' = \frac{1}{\cosh^2(x)} \tag{23}$$