

数学 カンニングシート

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極座標

3次元

$$\nabla = \mathbf{e}_r \frac{\partial}{\partial r} + \mathbf{e}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + \mathbf{e}_\phi \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \quad (1)$$

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \quad (2)$$

ヤコビアンは $r^2 \sin \theta$

2次元

$$\nabla = \mathbf{e}_r \frac{\partial}{\partial r} + \mathbf{e}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} \quad (3)$$

$$\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \quad (4)$$

ヤコビアンは r

円柱座標

$$\nabla = \mathbf{e}_r \frac{\partial}{\partial r} + \mathbf{e}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + \mathbf{e}_z \frac{\partial}{\partial z} \quad (5)$$

$$\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \quad (6)$$

ヤコビアンは r

デルタ関数

$$\delta(ax) = \frac{1}{|a|} \delta(x) \quad (7)$$

$$\delta(f(x)) = \sum_i \frac{1}{|f'(a_i)|} \delta(x - a_i) \quad (8)$$

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} dk \quad (9)$$

$$\Delta \left(\frac{1}{r} \right) = -4\pi \delta(r) \quad (10)$$

$$\nabla \left(\frac{1}{r} \right) = -\frac{\mathbf{e}_r}{r^2} = -\frac{\mathbf{r}}{r^3}, \quad \therefore \nabla \cdot \left(\frac{\mathbf{r}}{r^3} \right) = 4\pi \delta(r) \quad (11)$$