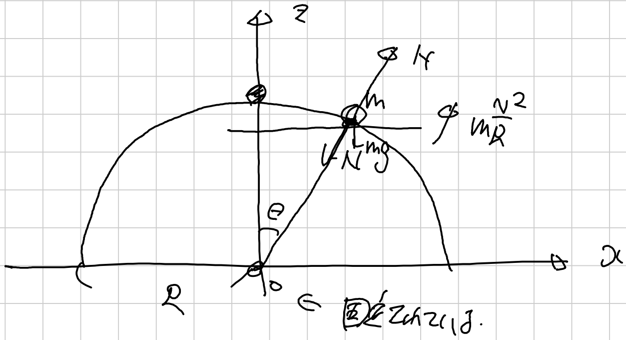


[1].

[1.1]. エネルギー保存則より. $mgR = mgR\cos\theta + \frac{1}{2}mv^2$ $\therefore v = \sqrt{2gR(1-\cos\theta)}$

[1.2]. 回転系でみると.

$$m\frac{v^2}{R} + N = mg\cos\theta \qquad \therefore N = mg\cos\theta - m\frac{v^2}{R}$$

$$= mg\cos\theta - m\frac{1}{R}(2gR(1-\cos\theta))$$

$$= 3mg\cos\theta - 2mg = 0 \text{ の時}$$

$$\boxed{\cos\theta = \frac{2}{3}}$$

$\therefore \cos\theta_{c1} = \frac{2}{3}$

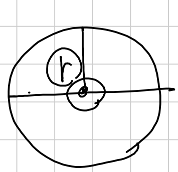
$v_{c1} = \sqrt{2gR(1-\frac{2}{3})} = \sqrt{\frac{2}{3}gR}$

$\int_0^{\pi} \sin\theta \, d\theta = [-\cos\theta]_0^{\pi} = 1-(-1) = 2$

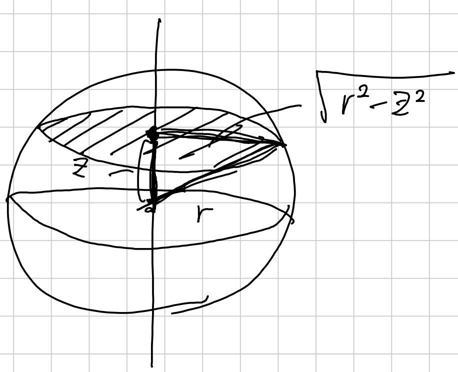
[2]

[2.1]

$$I = \int dm\, a^2 = \int_{-r}^{r} \boxed{\frac{3m}{4\pi r^3}\int (r^2 - z^2)\, dV}$$



$$\left(\frac{m}{\frac{4}{3}\pi r^3} \times dV\right)$$



$$\frac{3m}{4\pi r^3} \times 2\pi \times 2 \times \frac{1}{4}\int_0^r (r^4 - 2r^2z^2 + z^4)\, dr$$

$$r^5 - \frac{2}{3}r^5 + \frac{1}{5}r^5$$

$$= \frac{15 - 10 + 3}{15}$$

$$= \frac{8}{15}$$

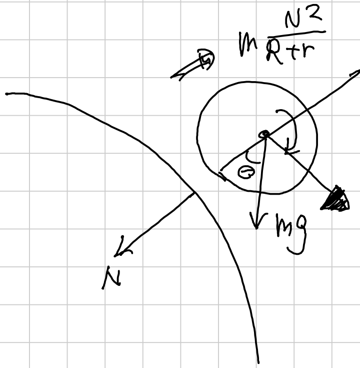
$$\frac{3m}{r^3} \times \cancel{\frac{1}{4}} \times \frac{\cancel{8}^{2}}{15}r^5 =$$

[2.2]

→ エネルギー保存則

$$\boxed{mg(R+r) = mg(R+r)\cos\theta + \underbrace{\frac{1}{2}m\omega^2}_{\text{重心の回転}} + \underbrace{\frac{1}{2}mv^2}_{\text{重心の運動エネルギー}}}$$

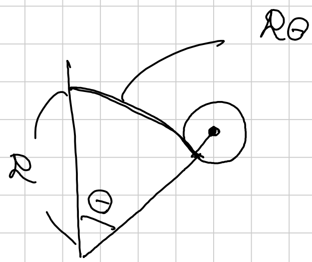
[2.3]



この時, 回転に効いてくる力は何なのか??

この運動を解いてた つまり 摩擦がある

mg と N は 回転に効いてこないと与えられない



s. s. 3500

L

(1.0074)

拘束条件の復習

N個の質点の系がp個の拘束条件: $f^\mu(r_1, r_2, \cdots, r_N, t) = 0$ の時

($\mu = 1, 2, \cdots p$)

↓

拘束力: $F^\mu = \lambda_\mu \nabla f^\mu(x, t)$ で表わされる

↓

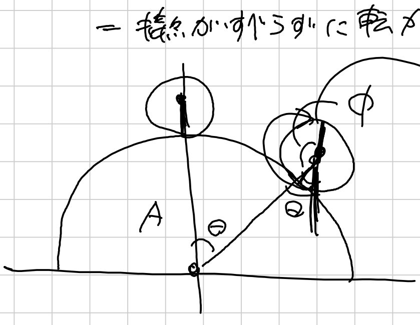
Lagrange eq: $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{r}}\right) + \frac{\partial U}{\partial r_\alpha} - \lambda_\mu \frac{\partial f^\mu}{\partial r_\alpha} = 0$ ($\alpha = 1, 2, \cdots N$)

↓より, 拡大した Lagrangian

$$\bar{L} = L' - U' = L^0(x, \dot{x}) + \lambda_\mu f^\mu(x, t)$$

→ 球状円筒の中心が $r = A + a$ の運動 → $f(r) = r - (A + a) = 0$

— 接点がすべらずに転がる条件:



基準がおかしくないのか?

$A\theta = a(\phi - \theta)$ ∴ $(A+a)\dot{\theta} = a\dot{\phi}$

$L^{\circ} = \underbrace{\frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2)}_{\text{重心の運動 eq}} + \underbrace{\frac{1}{2}\cdot I\omega^2}_{\text{回転の運動 eq}} \Rightarrow \boxed{\frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{1}{4}ma^2\dot{\phi}^2}$

$\frac{1}{2}\cdot\frac{1}{2}ma^2\cdot\dot{\phi}^2$

$-mgr\cos\theta$

$\varepsilon_r[L^{\circ}] = \lambda_1\frac{\partial f}{\partial r}$

$\varepsilon_\theta[L^{\circ}] = \lambda_2\frac{\partial f}{\partial \theta}$

$= \lambda_2(A+a)$

$\therefore \frac{d}{dt}(mr^2\dot{\theta}) - mgr\sin\theta = (A+a)\lambda_2$

$\varepsilon_\phi[L^{\circ}] = \lambda_2(-a) \quad \therefore \frac{m}{2}a^2\ddot{\phi} = -a\lambda_2$

$f(r) = r-(A+a) = 0$

$df_1 = dr = 0, \quad df_2 = (A+a)d\theta - a\,d\phi = 0$

$\boxed{\text{しい数で書く}}$

$\dot{r}=0, \ddot{r}=0, \quad a\ddot{\phi} = (a+A)\ddot{\theta}$

$2m\cancel{r\dot{r}\dot{\theta}} + mr^2\ddot{\theta} - mgr\sin\theta = (A+a)\lambda_2$

$\frac{m}{2}a^2\cdot\left(\frac{a+A}{a}\right)\ddot{\theta} = -a\lambda_2$

$\lambda_2 = -\frac{m}{2}\cdot(a+A)\ddot{\theta}$

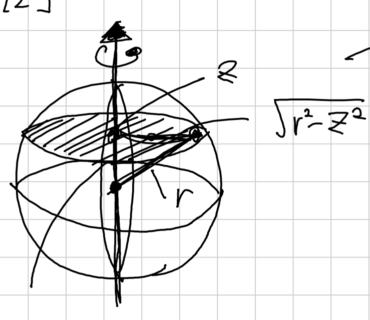
$\ddot{\theta}mr^2 + (A+a)\cdot\frac{m}{2}(a+A)\ddot{\theta} - mgr\sin\theta = 0$

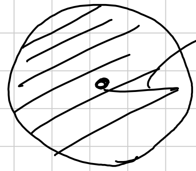
$\boxed{\frac{3}{2}m(A+a)\ddot{\theta} - mg\sin\theta = 0}$

$\boxed{\Rightarrow T_r = \lambda_1\frac{df}{dr} \text{ で求まる;}}$

---

[2]





$\sqrt{r^2-z^2} = R$

$0 \leq r' \leq \sqrt{r^2-z^2}$

$\left(\int dm\,R^2\right)$

$dm = \frac{m}{\frac{4}{3}\pi r^2}dV \Rightarrow I = \left(\int r'^2\,dV\right)\cdot\frac{3m}{4\pi r^2}$

$r'dr'd\theta dz$

$= \left(\int_0^{\sqrt{r^2-z^2}} r'^3\,dr'\right)\cdot 2\pi\cdot\frac{3m}{4\pi r^2}$

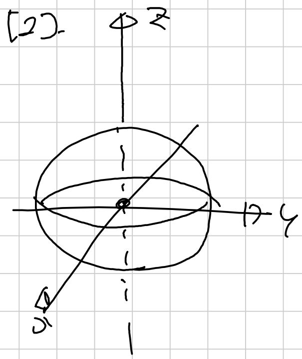
$= 2\int_0^r \frac{1}{4}\cdot(r^2-z^2)^2dz\cdot\frac{3m}{2r^2}$

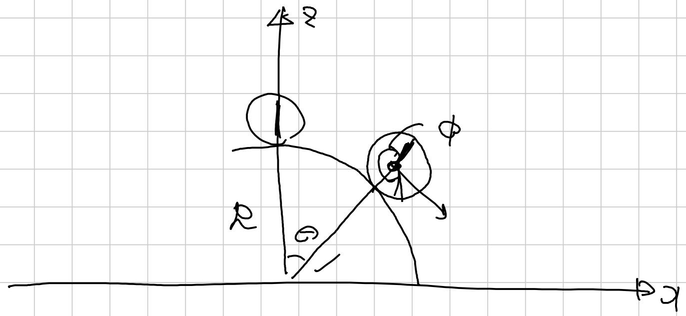
$\frac{1}{2}\cdot\frac{3m}{2r^2}\int_0^r(r^4 - 2r^2z^2 + z^4)dz$

$\left(r^4 - \frac{2}{3}r^4 + \frac{1}{5}r^4\right)$

$\frac{15-2+1}{15} = \frac{14}{15}r^4 \times \frac{3m}{4r^2}$

[2].





$\therefore\ R\theta = r(\phi - \theta)$

$R\dot{\theta} = r(\dot{\phi} - \dot{\theta})$ $\qquad \omega = \dot{\phi}\ ,\ (R+r)v = \dot{\theta}$

$(R+r)\dot{\theta} = r\omega \qquad \therefore \boxed{(R+r)^2 v = r\omega}$

[2.3]. 拘束をまとめる

$f_r(r') = r' - (r+R) = 0, \qquad f_{\theta,\phi}(\theta,\phi) = (R+r)\theta - r\phi = 0$

$L_0 = \frac{1}{2} m \Big(\frac{d}{dt}(r'\sin\theta), \frac{d}{dt}(r'\cos\theta) \Big)$

$\left| \left(\dot{r}'\sin\theta + r'\cos\theta\cdot\dot{\theta},\ \dot{r}'\cos\theta - r'\sin\theta\cdot\dot{\theta} \right) \right|^2$

$$\boxed{\frac{1}{2} m(\dot{r}'^2 + r'^2\dot{\theta}^2) + \frac{1}{2}\left(\frac{2}{5} m r^2\right)\cdot\dot{\phi}^2 - mg\,r'\cos\theta}$$

$\mathcal{E}[\mathcal{L}_0]_r = \underbrace{m\ddot{r}'}_{=0} - (m r'\dot{\theta}^2 - mg\cos\theta) = \lambda_1$

$\mathcal{E}[\mathcal{L}_0]_\theta = m r'^2\cdot\ddot{\theta} - mg r'\sin\theta = \lambda_2\cdot(R+r)$

$\mathcal{E}[\mathcal{L}_0]_\phi = \frac{1}{5} m r^2\ddot{\phi} = \lambda_2(-r) \qquad \therefore\ \lambda_2 = -\frac{mr}{5}\ddot{\phi} = -\frac{m}{5}\cdot(R+r)\ddot{\theta}$

$(R+r)\ddot{\theta} = r\ddot{\phi}$

$m r'^2\ddot{\theta} - mg r'\sin\theta = -\frac{m}{5}(R+r)^2\ddot{\theta}$

$\Downarrow$ 拘束より $R+r = r'$

$\frac{6}{5} m r'\ddot{\theta} - mg\sin\theta = 0$

$\Downarrow$

$\frac{6}{5} m r'\dot{\theta}\ddot{\theta} - \dot{\theta} mg\sin\theta = 0 \Rightarrow \frac{3}{5} m r'\dot{\theta}^2 + mg\cos\theta = 0$

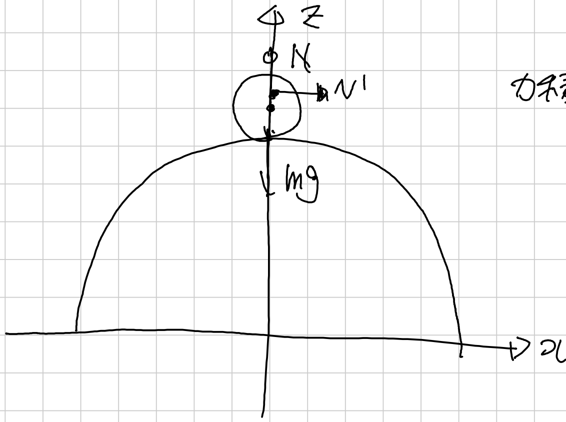
$-mg$

$\therefore\ \dot{\theta}^2 = \frac{5}{3r'} g(1-\cos\theta)$

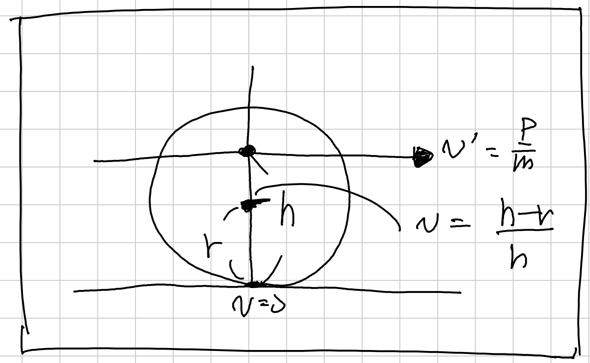
$\therefore\ -m r'\cdot\frac{5}{3r'} g(1-\cos\theta) + mg\cos\theta = \lambda_1$

$= \frac{5}{3} mg(\cos\theta) - \frac{2}{3} mg \qquad \therefore\ \cos\theta = \frac{3}{5}\cdot\frac{2}{3} = \frac{2}{5}$

ここに代入して

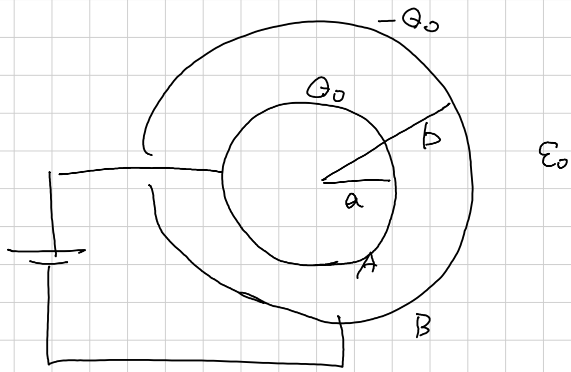


力積： $0+P=mv'$



[3]??

[第2問]

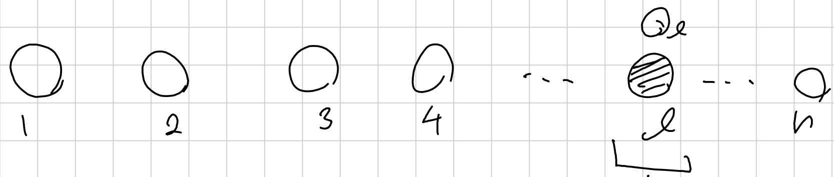


[1.1]

問題：Greenの相反定理

<復習>

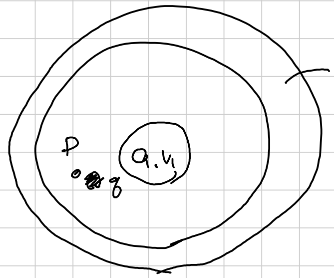
n個の導体があるけど、



$p_{kl}Q_l$

各導体の電位は
$$
\left.\begin{aligned}
V_1 &= p_{11}Q_1 + p_{12}Q_2 + \cdots + p_{1n}Q_n\\
V_2 &= p_{21}Q_1 + p_{22}Q_2 + \cdots + p_{2n}Q_n\\
&\ \vdots\\
V_n &= p_{n1}Q_1 + p_{n2}Q_2 + \cdots + p_{nn}Q_n
\end{aligned}\right\} \rightarrow
\begin{matrix} Q_1 \\ Q_1 \\ \vdots \end{matrix} = \underbrace{\begin{bmatrix} \quad \end{bmatrix}}_{\text{容量係数}} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}
$$

[4]



両導体の電位を V_1, V_2 にした時の電荷を Q_1, Q_2、
求める電荷を q_1, q_2 とする

$$V_1 \cdot q_1 + V_2 \cdot q_2 + V' \cdot q = 0 \cdot Q_1 + 0 \cdot Q_2 + V'' \cdot Q = 0$$

相反定理より。 $\sum V_k^{(1)} Q_k^{(2)} = \sum V_k^{(2)} Q_k^{(1)}$

点Pに電荷qをおいた時. $q_1 + q_2 + q = 0$

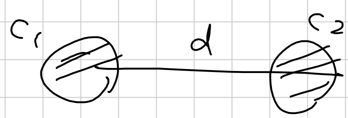
$$\left(V_1 = p_{11} Q_1 + p_{12} Q_2, \quad V_2 = p_{21} Q_1 + Q_{22} Q_2 \quad , Q_1 + Q_2 = Q \right)$$

孤立した時の静電容量が C_1, C_2 である2つの導体. $q_{11} = C_1$, $q_{22} = C_2$

[8.1]

$$Q_1 = C_1 V_1 + \beta \underset{V}{V_2}, \quad Q_2 = \beta V_1 + C_2 \underset{V.}{V_2} \qquad Q_1 = (C_1 + \beta) V, \quad Q_2 = (\beta + C_2) V$$

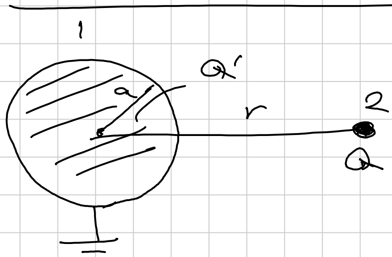
$$Q = Q_1 + Q_2 \quad \therefore \quad C = \frac{Q}{V} = C_1 + C_2 + 2\beta$$



$q_{11} = C_1$ $q_{22} = C_2$ の時 $C_1 \to Q$

$$V_2 = \frac{Q}{4\pi\varepsilon_0 d}$$

$$V_1 = \frac{Q_1}{C_1}, \quad V_2 = \frac{Q}{4\pi\varepsilon_0 d} \quad \therefore \quad Q_2 = q_{21} V_1 + q_{22} V_2$$

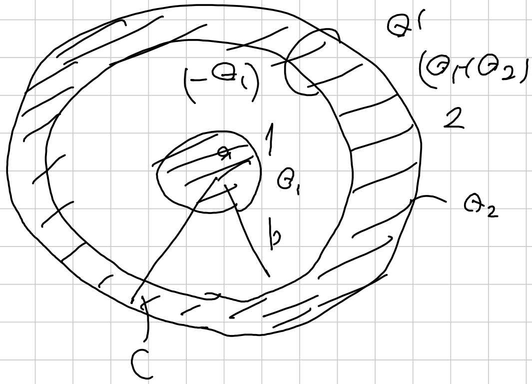


→ 相反定理をつかってとく.

$$\left(1 に 1,\ Q に 0 の時 \quad V_1 = \frac{1}{4\pi\varepsilon_0 a}, \ V_2 = \frac{1}{4\pi\varepsilon_0 r} \right) \rightarrow 接地 \quad V_1 = 0, \ V_2 = \frac{Q}{4\pi\varepsilon_0 r}$$

1にQ'. 2にQ

$$\frac{Q'}{4\pi\varepsilon_0 a} + \frac{Q}{4\pi\varepsilon_0 r} = 0$$

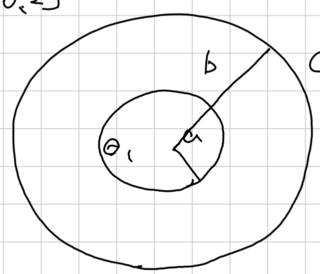


$Q = Q_1 + Q'$ ← Gauss

(B.1)

$$\therefore\ V_1 = \frac{Q_1}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b} + \frac{1}{c}\right) + \frac{Q_2}{4\pi\varepsilon_0}\frac{1}{c},\quad V_2 = \frac{Q_1}{4\pi\varepsilon_0 c} + \frac{Q_2}{4\pi\varepsilon_0 c}$$

[16.2]



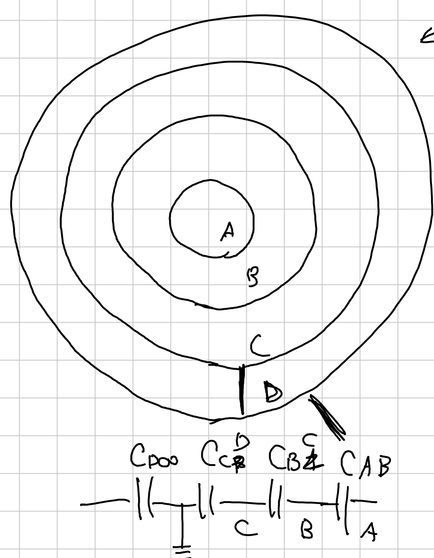
「方3の問題 電荷の45どうかあり」

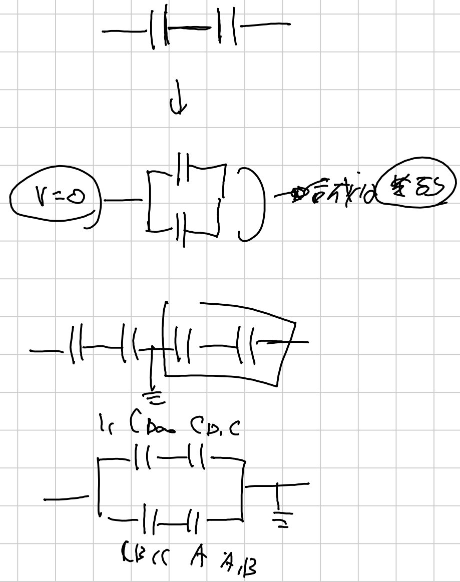
$$V_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{a} + \frac{Q_2}{b}\right)\quad V_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{b} + \frac{Q_2}{b}\right)$$

「静電容量之3 直己列」

[33]

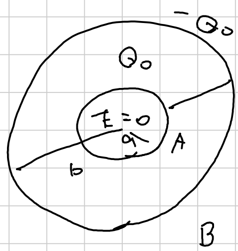
← 静電容量.





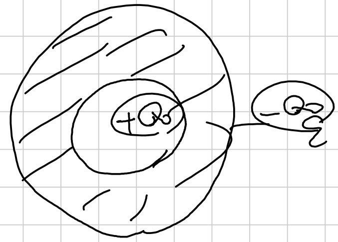
第2問

[1].

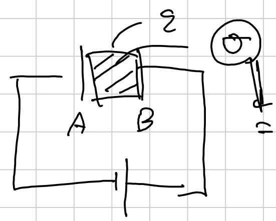


の時 $V = \frac{Q_0}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$ ∴ $Q = 4\pi\varepsilon_0 V\cdot\frac{ab}{b-a}$

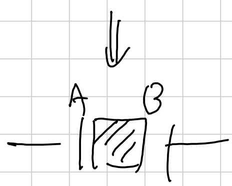
??というとこと?

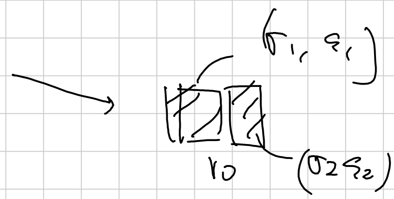


→ ε_0 を ε_1 にかえる

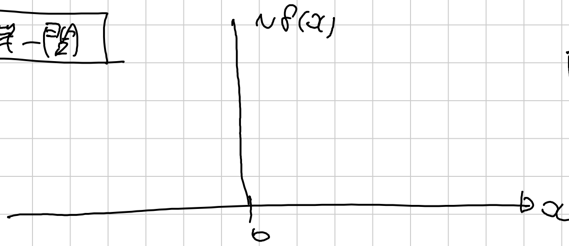


ここの抵抗を求めれば良い。





第一問



[1]. $-\frac{\hbar^2}{2m}\phi'' = E\phi(x)$

$$\phi'' = -\frac{2mE}{\hbar^2}\phi(x)$$

$$\phi(x) = A\exp\left(\underbrace{\sqrt{-\frac{2mE}{\hbar^2}}}_{\chi}x\right) + B\exp\left(-\sqrt{\frac{-2mE}{\hbar^2}}x\right)$$

$x>0$ の時

$$\phi(x) = B\exp\left(-\sqrt{\frac{-2mE}{\hbar^2}}x\right)$$

∴ $\boxed{\xi = \sqrt{\frac{-2mE}{\hbar^2}}}$

$$\int\phi^*\phi = 2|B|^2\int_0^\infty \exp\left(-2\sqrt{\frac{-2mE}{\hbar^2}}x\right)dx$$

$$= 2|B|^2\cdot\left(+\frac{1}{2}\sqrt{\frac{\hbar^2}{-2mE}}\right) = 1$$

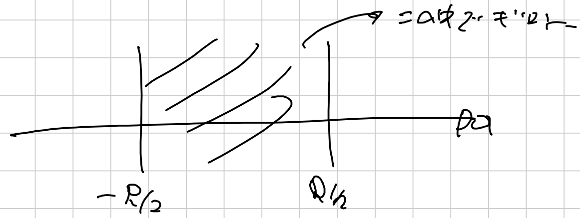
∴ $\boxed{|B|^2 = \sqrt{\frac{-2mE}{\hbar^2}}}$

それぞれの境界条件より

$-\frac{\hbar^2}{2m}\left(\phi'(\xi>0) - \phi'(\xi<0)\right) = V\phi(0)$, $\boxed{A=B}$

→ 連立で解ける

→ 2体の1次元 Shrödinger



$$(a+b)\hat{H}(a'+b') \to \underbrace{a\hat{H}a'}_{(A)} + a\hat{H}b' + \underbrace{b\hat{H}a'}_{=(B)} + \underbrace{b\hat{H}b'}_{=}$$

$|c_1|^2$ $\quad c_1^* c_2 \quad$??