Ugeopgave 3 - Chillaxgruppen

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1 Part 1

We have been asked to determine which of $n + log_2 n$, $n^2 + 2^n$, $n^2 + nlog_1 0n$, $n^2(3 + \sqrt{n})$, $(n + \sqrt{n})^2$ are of some order of magnitude as n^2.

- $n + log_2 n$ is not the same order of magnitude as n^2, because due to rule S2, $log_2 n$ is a smaller order of magnitude than n, which by S8 means $n + log_2 n$ has the same order of magnitude as n, and by S3 n^1 has lower order of magnitude than n^2.

- $n^2 + 2^n$ has larger order of magnitude than n^2, because by rule S5, n^2 has a smaller order of magnitude than 2^n, which by S8 implies that $n^2 + 2^n$ has the same order of magnitude as 2^n, which we've just established is bigger than n^2.

- $n^2 + n\ log_{10} n$ is the same order of magnitude as n^2, because by rule S2 $log_{10} n$ is of smaller order of magnitude than n, and by rule S7 we can multiply this by n, resulting in the conclusion that $n\ log_{10} n$ is of smaller order of magnitude than n^2; this lets us use rule S8 to conclude that $n^2 + n\ log_{10} n$ has same order of magnitude as n^2.

- $n^2(3 + \sqrt{n})$ can be reduced to $3n^2 + n^{2.5}$. We can apply rule S6 to eliminate the constant 3 and rule S3 to conclude that n^2 has lower order of magnitude than $n^{2.5}$. This lets us apply S8 to conclude that $3n^2 + n^{2.5}$ has the same order of magnitude as $n^{2.5}$, which we've just concluded has higher order of magnitude than n^2.

- $(n + \sqrt{n})^2$ can be restated as $n^2 + 2n^{1.5} + n$. Consider the fragment $2n^{1.5} + n$. This has order of magnitude $n^{1.5}$, as we can apply rule S6 and S3 to conclude that n has smaller order of magnitude than $n^{1.5}$, and S8 and S6 to conclude that $2n^{1.5} + n$ is therefore of magnitude $n^{1.5}$. Now we can apply S3 and S8 to conclude that n^2 is of larger magnitude than $n^{1.5}$ and that therefore $n^2 + O(n^{1.5})$ has order of magnitude n^2.

To conclude, $n^2 + n\ log_{10} n$ and $(n + \sqrt{n})^2$ have the same order of magnitude as n^2.

2 Part 2

We have been asked to consider the sequences:

- $a_1 = 10$, $a_n = a_{n-1}$
- $b_n = \sum\limits_{k=1}^{n} k^2$
- $c_n = \frac{n^2}{10}$
- $d_n = (\frac{3}{2})^n$

2.1 (a)

First, we have been asked to compute the first three numbers in each sequence.

n	1	2	3
a_n	10	10	10
b_n	1	5	14
c_n	0.1	0.4	0.9
d_n	$\frac{3}{2}$	$2 + \frac{1}{4}$	$3 + \frac{3}{8}$

2.2 (b)

Sequence a is the smallest sequence, as it is constant. By rule S5, d is of greater magnitude than both b and c. $b_n = \sum\limits_{k=1}^{n} k^2 = \frac{2n^3+3n^2+n}{6}$, which means it is of order of magnitude n^3, making it greater than c by rule S3.

This means that the ordering is a, c, b and d, in increasing order of magnitude.

3 Part 3

We have been asked to find a closed form for $\sum\limits_{k=0}^{n} (2k+1)$.

$$\begin{aligned}
\sum_{k=0}^{n} (2k+1) &= \sum_{k=0}^{n} 1 + \sum_{k=0}^{n} 2k \\
&= 1 + n + \sum_{k=0}^{n} 2k \\
&= 1 + n + 2\sum_{k=0}^{n} k \\
&= 1 + n + 2\frac{n^2+n}{2} \\
&= 1 + n + n^2 + n \\
&= n^2 + 2n + 1
\end{aligned}$$