Lista de Exercícios 3a

1. Considere a função resposta: $E(Y) = 25 + 3X_1 + 4X_2 + 1{,}5X_1X_2$

a) Faça o gráfico de $E(Y) \times X_1$ quando $X_2 = 3$ e $X_2 = 6$.

b) Os efeitos de X_1 e X_2 são aditivos? Como você identificou isto no gráfico obtido no item ***a***.

2. Estabeleça a matriz $\mathbf{X}$ e os vetores $\mathbf{Y}$ e $\boldsymbol{\beta}$ para os seguintes modelos (assuma que $i = 1,2,3,4$).

a) $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i1} X_{i2} + \varepsilon_i$

b) $\sqrt{Y_i} = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \varepsilon_i$

3. Por que não é siginificativo atribuir um sínal ao coeficiente de correlação múltipla, embora façamos isso para o coeficiente de correlação linear simples?

4. Exercícios 6.5 a 6.8 do livro-texto.

6.5. **Brand preference.** In a small-scale experimental study of the relation between degree of brand liking (Y) and moisture content (X_1) and sweetness (X_2) of the product, the following results were obtained from the experiment based on a completely randomized design (data are coded):

i:	1	2	3	...	14	15	16
X_{i1}:	4	4	4	...	10	10	10
X_{i2}:	2	4	2	..	4	2	4
Y_i:	64	73	61	...	95	94	100

a. Obtain the scatter plot matrix and the correlation matrix. What information do these diagnostic aids provide here?

b. Fit regression model (6.1) to the data. State the estimated regression function. How is b_1 interpreted here?

c. Obtain the residuals and prepare a box plot of the residuals. What information does this plot provide?

d. Plot the residuals against $\hat{Y}$, X_1, X_2, and X_1X_2 on separate graphs. Also prepare a normal probability plot. Interpret the plots and summarize your findings.

e. Conduct the Breusch-Pagan test for constancy of the error variance, assuming $\log \sigma_i^2 = \gamma_0 + \gamma_1 X_{i1} + \gamma_2 X_{i2}$; use $\alpha = .01$. State the alternatives, decision rule, and conclusion.

f. Conduct a formal test for lack of fit of the first-order regression function; use $\alpha = .01$. State the alternatives, decision rule, and conclusion.

6.6. Refer to **Brand preference** Problem 6.5. Assume that regression model (6.1) with independent normal error terms is appropriate.

a. Test whether there is a regression relation, using $\alpha = .01$. State the alternatives, decision rule, and conclusion. What does your test imply about β_1 and β_2?

b. What is the P-value of the test in part (a)?

c. Estimate β_1 and β_2 jointly by the Bonferroni procedure, using a 99 percent family confidence coefficient. Interpret your results.

6.7. Refer to **Brand preference** Problem 6.5.

a. Calculate the coefficient of multiple determination R^2. How is it interpreted here?

b. Calculate the coefficient of simple determination R^2 between Y_i and $\hat{Y}_i$. Does it equal the coefficient of multiple determination in part (a)?

6.8. Refer to **Brand preference** Problem 6.5. Assume that regression model (6.1) with independent normal error terms is appropriate.

a. Obtain an interval estimate of $E\{Y_h\}$ when $X_{h1} = 5$ and $X_{h2} = 4$. Use a 99 percent confidence coefficient. Interpret your interval estimate.

b. Obtain a prediction interval for a new observation $Y_{h(\text{new})}$ when $X_{h1} = 5$ and $X_{h2} = 4$. Use a 99 percent confidence coefficient.

5. Exercícios 7.3, 7.12, 7.16, 7.24 e 7.30 do livro-texto.

7.3. Refer to **Brand preference** Problem 6.5.

a. Obtain the analysis of variance table that decomposes the regression sum of squares into extra sums of squares associated with X_1 and with X_2, given X_1.

b. Test whether X_2 can be dropped from the regression model given that X_1 is retained. Use the F^* test statistic and level of significance .01. State the alternatives, decision rule, and conclusion. What is the P-value of the test?

7.12. Refer to **Brand preference** Problem 6.5. Calculate R^2_{Y1}, R^2_{Y2}, R^2_{12}, $R^2_{Y1|2}$, $R^2_{Y2|1}$, and R^2. Explain what each coefficient measures and interpret your results.

7.16. Refer to **Brand preference** Problem 6.5.

a. Transform the variables by means of the correlation transformation (7.44) and fit the standardized regression model (7.45).

b. Interpret the standardized regression coefficient b_1^*.

c. Transform the estimated standardized regression coefficients by means of (7.53) back to the ones for the fitted regression model in the original variables. Verify that they are the same as the ones obtained in Problem 6.5b.

7.24. Refer to **Brand preference** Problem 6.5.

a. Fit first-order simple linear regression model (2.1) for relating brand liking (Y) to moisture content (X_1). State the fitted regression function.

b. Compare the estimated regression coefficient for moisture content obtained in part (a) with the corresponding coefficient obtained in Problem 6.5b. What do you find?

c. Does $SSR(X_1)$ equal $SSR(X_1|X_2)$ here? If not, is the difference substantial?

d. Refer to the correlation matrix obtained in Problem 6.5a. What bearing does this have on your findings in parts (b) and (c)?

7.30. Refer to **Brand preference** Problem 6.5.

a. Regress Y on X_2 using simple linear regression model (2.1) and obtain the residuals.

b. Regress X_1 on X_2 using simple linear regression model (2.1) and obtain the residuals.

c. Calculate the coefficient of simple correlation between the two sets of residuals and show that it equals $r_{Y1|2}$.