LISTA 2

ANÁLISE DE SÉRIES TEMPORAIS

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Descrição da atividade

- Exercícios 2.1, 2.2, 2.4, 2.5 e 2.6 do Cap.2 (pag.20) de Cryer & Chan (2008)

Exercício 2.1

Suppose $E(X) = 2$, $Var(X) = 9$, $E(Y) = 0$, $Var(Y) = 4$, and $Corr(X, Y) = 0.25$. Find:

a. $Var(X + Y)$
b. $Cov(X, X + Y)$
c. $Corr(X + Y, X - Y)$

- Respostas:

Item A.

(I) $Var(X + Y) = Var(X) + Var(Y) + Cov(X + Y)$

Para encontrar $Cov(X + Y)$, tem-se:

$$Cov(X + Y) = Corr(X, Y)\sqrt{Var(X)Var(Y)}$$

$$Cov(X + Y) = 0.25 \cdot \sqrt{9 \cdot 4}$$

$$Cov(X + Y) = 1.5$$

Substituindo em (I), tem-se:

$$Var(X + Y) = 9 + 4 + 1.5$$

$Var(X+Y) = 14.5$

Item B.

$Cov(X, X+Y) = Cov(X, X) + Cov(X, Y)$

$Cov(X, X+Y) = Var(X) + Cov(X, Y)$

$Cov(X, X+Y) = 9 + 1.5$

$Cov(X, X+Y) = 10.5$

Item C.

Tem-se que:

(I) $Corr(X+Y, X-Y) = \frac{Cov(X+Y, X-Y)}{\sqrt{Var(X+Y) \cdot Var(X-Y)}}$

Assim:

i. Pelo *Item A*,

$Var(X+Y) = 14.5$

ii. Calcula-se a variância de $X-Y$

$Var(X-Y) = Var(x) + Var(Y) - Cov(X, Y)$

$Var(X-Y) = 9 + 4 - 1.5$

$Var(X-Y) = 11.5$

iii. Calcula-se a covariância de $X+Y$ e $X-Y$:

$Cov(X+Y, X-Y) = Cov(X, X) - Cov(X, Y) + Cov(X, Y) - Cov(Y, Y)$

$Cov(X+Y, X-Y) = Cov(X, X) - Cov(Y, Y)$

$Cov(X+Y, X-Y) = Var(X) - Var(Y)$

$Cov(X+Y, X-Y) = 9 - 4$

$Cov(X+Y, X-Y) = 5$

Por fim, é possível substituir em (I) de tal forma que:

$Corr(X+Y, X-Y) = \frac{Cov(X+Y, X-Y)}{\sqrt{Var(X+Y) \cdot Var(X-Y)}}$

$Corr(X+Y, X-Y) = \frac{5}{14.5 \cdot 11.5}$

$Corr(X+Y, X-Y) \approx 0.3872$

Exercício 2.2

If X and Y are dependent but $Var(X) = Var(Y)$, find $Cov(X + Y, X - Y)$.

- Respostas:

Em passos similares na resolução do *Item C, (iii)*, tem-se que:

$$Cov(X + Y, X - Y) = Cov(X, X) - Cov(X, Y) + Cov(X, Y) - Cov(Y, Y)$$

$$Cov(X + Y, X - Y) = Cov(X, X) - Cov(Y, Y)$$

$$Cov(X + Y, X - Y) = Var(X) - Var(Y)$$

Como agora $Var(X) = Var(Y)$, então:

$$Cov(X + Y, X - Y) = 0$$

Exercício 2.4

Let e_t be a zero mean white noise process. Suppose that the observed process is $Y_t = e_t + \theta e_{t-1}$, where θ is either 3 or 1/3.

a. Find the autocorrelation function for Y_t both when $\theta = 3$ and when $\theta = 1/3$.

b. You should have discovered that the time series is stationary regardless of the value of θ and that the autocorrelation functions are the same for $\theta = 3$ and $\theta = 1/3$. For simplicity, suppose that the process mean is known to be zero and the variance of Y_t is known to be 1. You observe the series Y_t for $t = 1, 2, ..., n$ and suppose that you can produce good estimates of the autocorrelations ρ_k. Do you think that you could determine which value of θ is correct (3 or 1/3) based on the estimate of ρ_k? Why or why not?

Exercício 2.5

Suppose $Y_t = 5 + 2t + X_t$, where X_t is a zero-mean stationary series with autoco- variance function γ_k.

a. Find the mean function for Y_t.

b. Find the autocovariance function for Y_t.

c. Is Y_t stationary? Why or why not?

Exercício 2.6

Let X_t be a stationary time series, and define

$$Y_t = \begin{cases} X_t & \text{for } t \text{ odd} \\ X_t + 3 & \text{for } t \text{ even} \end{cases}$$

a. Show that $Cov(Y_t, Y_t{-}k)$ is free of t for all lags k.

b. Is Y_t stationary?