

LISTA 2

ANÁLISE DE SÉRIES TEMPORAIS

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Descrição da atividade

- Exercícios 2.1, 2.2, 2.4, 2.5 e 2.6 do Cap.2 (pag.20) de Cryer & Chan (2008)

Antes de iniciar os exercícios propostos, carrega-se os pacotes necessários:

```
if (!require(pacman)) install.packages("pacman")
pacman::p_load(tidyverse)
```

Exercício 2.1

Suppose $E(X) = 2$, $Var(X) = 9$, $E(Y) = 0$, $Var(Y) = 4$, and $Corr(X, Y) = 0.25$. Find:

- $Var(X + Y)$
- $Cov(X, X + Y)$
- $Corr(X + Y, X - Y)$

Exercício 2.2

If X and Y are dependent but $Var(X) = Var(Y)$, find $Cov(X + Y, X - Y)$.

Exercício 2.4

Let e_t be a zero mean white noise process. Suppose that the observed process is $Y_t = e_t + \theta e_{t-1}$, where θ is either 3 or $1/3$.

- Find the autocorrelation function for Y_t both when $\theta = 3$ and when $\theta = 1/3$.
- You should have discovered that the time series is stationary regardless of the value of θ and that the autocorrelation functions are the same for $\theta = 3$ and $\theta = 1/3$. For simplicity, suppose that the process mean is known to be zero and the variance of Y_t is known to be 1. You observe the series Y_t for $t = 1, 2, \dots, n$ and suppose that you can produce good estimates of the autocorrelations ρ_k . Do you think that you could determine which value of θ is correct (3 or $1/3$) based on the estimate of ρ_k ? Why or why not?

Exercício 2.5

Suppose $Y_t = 5 + 2t + X_t$, where X_t is a zero-mean stationary series with autocovariance function γ_k .

- Find the mean function for Y_t .
- Find the autocovariance function for Y_t .
- Is Y_t stationary? Why or why not?

Exercício 2.6

Let X_t be a stationary time series, and define

$$Y_t = \begin{cases} X_t & \text{for } t \text{ odd} \\ X_t + 3 & \text{for } t \text{ even} \end{cases}$$

- Show that $Cov(Y_t, Y_{t-k})$ is free of t for all lags k .
- Is Y_t stationary?