CS225 Homework Keisuke Yasuda

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1 a CID = 1375 b CID 732

$=$

2 a $\prod_{k=2}^{n}\left(1-\frac{1}{k^2}\right) = \prod_{k=2}^{n}\left(\frac{k^2-1}{k^2}\right)$

$= \prod_{k=2}^{n}\left(\frac{(k+1)(k-1)}{k^2}\right)$

$= \frac{\cancel{3}\times 1}{2\times\cancel{2}} \cdot \frac{\cancel{4}\times\cancel{2}}{\cancel{3}\times 3} \cdot \frac{\cancel{5}\times\cancel{3}}{\cancel{4}\times\cancel{4}} \cdot \frac{6\times 4}{\cancel{5}\times 5} \cdots \frac{(n-1+1)(n-1-1)}{(n-1)^2} \cdot \frac{(n+1)(n-1)}{n^2}$

$= \frac{1}{2}\left(\frac{n+1}{n}\right)$

b $3^{1000} \bmod 7$

$3 \equiv 3 \bmod 7$

$3^2 = 9 \equiv 2 \bmod 7$

$3^3 = 27 \equiv 6 \bmod 7$

$3^4 = 81 \equiv 4 \bmod 7$

$3^5 = 243 \equiv 5 \bmod 7$

$3^6 = 729 \equiv 1 \bmod 7$

$3^7 = 2187 \equiv 3 \bmod 7$

$3^8 = 6561 \equiv 2 \bmod 7$

cycle per 6 steps

$1000 = 6(166) + 4$

$3^{1000} \bmod 7 \equiv 3^{6(166)+4} \bmod 7$

$\equiv (3^6)^{166} \times 3^4 \bmod 7$

$\equiv 1 \bmod 7 \times 4 \bmod 7$

$\equiv 4 \bmod 7$

$\equiv 4$

c $\sum_{n=1}^{n}\left(\frac{1}{2}\right)^{k} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$

d $\frac{\log_7 81}{\log_7 9} = \log_9 81 = 2$

e $\log_2 4^{2n} = \log_2 (2^2)^{2n} = \log_2 2^{4n} = 4n \log_2 2$

$= 4n$

f $\log_{17} 221 - \log_{17} 13 = \log_{17}\left(\frac{221}{13}\right)$

$= \log_{17} 17$

$= 1$

3 $1 + \sum_{j=1}^{n} j!j = 1 + 1 + 2\times 2 + 2\times 3\times 3 + 2\times 3\times 4\times 4 \cdots n!n$

$= 2\left(\frac{1}{2} + \frac{1}{2} + 1\times 2 + 1\times 3\times 3 + 1\times 3\times 4\times 4 \cdots \frac{n!n}{2}\right)$

$= 2\cdot 3\left(1 + 3 + 4\times 4 \cdots \frac{n!n}{2\cdot 3}\right)$

$= n!\left(1 + \frac{n!n}{n!}\right)$

$= n! + n!n$

$= (n+1)!$

4 a $f(n) = 4^{\log_4 n} = n$ $\quad$ $f(n)$ is $O(g(n))$

$g(n) = 2n+1$ $\quad$ $g(n)$ is $O(f(n))$

$\therefore f(n) \Theta g(n)$

b $f(n) = n^2$ $\quad$ $g(n) = (\sqrt{2})^{\log_2 n}$ $\quad$ $0 \leq g(n) \leq (f(n)$

$= 2^{\frac{1}{2}\log_2 n}$

$= n^{\frac{1}{2}}$ $\quad$ $f(n) \Omega g(n)$

c $f(n) O g(n)$

d $f(n) O g(n)$

5 a $T(n) = T\left(\frac{n}{2}\right) + 5 = T\left(\frac{n}{2^2}\right) + 5 + 5$ $\quad$ $2^k = n$

$= T\left(\frac{n}{2^3}\right) + 5 + 5 + 5$ $\quad$ $k = \log_2 n$

$= T\left(\frac{n}{2^k}\right) + 5\times k$

$T(n) = T\left(\frac{n}{n}\right) + 5\log_2 k$

$= 1 + 5\log_2 h$

b $T(n) = T(n-1) + \frac{1}{n}$

$= T(n-2) + \frac{1}{n} + \frac{1}{n-1}$ $\qquad n-k=0$

$= T(n-3) + \frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2}$ $\qquad k=n$

$= T(n-k) + \sum_{j=1}^{k} \frac{1}{j}$

$T(n) = T(0) + \sum_{j=1}^{n} \frac{1}{j}$

$= \sum_{j=1}^{n} \frac{1}{j}$

c If that use 2 for fomala

$T(\frac{n}{2}) + 5 = 1 + 5\log_2 n$

$T(1) + 5 = 1 + 5\log_2 2$

$1+5 = 1+5$

$6 = 6$

6

7a $x = 2 \quad n = 12$

$x = 4 \quad n = 6$

$x = 16 \quad n = 3$ $\qquad 16 \times 256 = 4096$

$x = 256 \quad n = 1$

$x = 65536 \quad n = 0$

b This program calculate x^n and returns value

c

d