

Instructions

Use any programming language that can do the task. A code needs to be commented and results must be nicely presented. The outcome of your solutions must be in .pdf file.

- Problem 1 (50pts)

1. Implement the Explicit Finite Difference (EFD) method to price both European Call and Put options. Follow Chapter 3 in Clewlow and Strickland.
2. Implement the Implicit Finite Difference (IFD) method to price European Call and Put options.
3. Implement the Crank-Nicolson Finite Difference (CNDF) method to price both European Call and Put options.
4. Consider $S_0 = 100$; $K = 100$, $T = 1$ year, $\sigma = 25\%$; $r = 6\%$; $\delta = 0 : 0.03$. Calculate and report the price for European Call and Put using all the FD methods. Put the results of the 3 methods (EFD, IFD, CNDF) side by side in a table and write your observations.
5. Using the parameters from the previous part, plot on the same graph the implicit finite difference updating coefficients A ; B ; C as a function of σ , with $\sigma \in \{0.05, 0.1, 0.15, \dots, 0.6\}$. Interpret your observations with comments.
6. Calculate the hedge sensitivities for the European call option using the explicit finite difference method. You need to calculate Delta, Gamma, Theta, and Vega.

- Problem 2 (30pts)

1. Download Option prices (you can use the Bloomberg Terminal, Yahoo! Finance, etc.) for AMZN, for 3 different maturities (1 month, 2 months, and 3 months) and 10 strike prices. Use the same method from Homework 1 to calculate the implied volatility. Set the current short-term interest rate equal to the one from the day you downloaded the data.
2. Use the Explicit, Implicit, and Crank-Nicolson Finite Difference schemes implemented in Problem 1 to price European Call and Put options. Use the calculated implied volatility to obtain a space parameter Δx that insures stability and convergence with an error magnitude of no greater than 0.001.
3. For the European style options above (puts and calls) calculate the corresponding Delta, Gamma, Theta, and Vega using the Explicit finite difference method.

- Problem 3 (20pts+10pts bonus)

Implement an explicit finite difference scheme for the Heston model following Chapter 10 in F. D. Rouah, The Heston Model and its Extensions in Matlab and C (2013). Use this scheme to price a put option with the information: $S_0 = 100$, $K = 100$, $T = 1$ year, $v_0 = 0.0625$, $r = 6\%$. The Heston model

$$\begin{aligned}dS_t &= rS_t dt + \sqrt{v_t}S_t dW_t^1 \\ dv_t &= \kappa(\theta - v_t)dt + \sigma\sqrt{v_t}dW_t^2\end{aligned}$$

with parameters: $\kappa = 2$, $\theta = 0.09$, $\sigma = 0.5$, $\rho = 0.6$